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WORKSHOP ON

QUANTUM SYSTEMS OUT OF EQUILIBRIUM

(14 – 25 June 2004)

" Electron injection in a nanotube with leads:
finite frequency noise-correlations and anomalous charges"

presented by:

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Electron injection in a nanotube with leads: finite frequency noise correlations and anomalous charges

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with

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PRB 205408 (2003), cond-mat/0405325

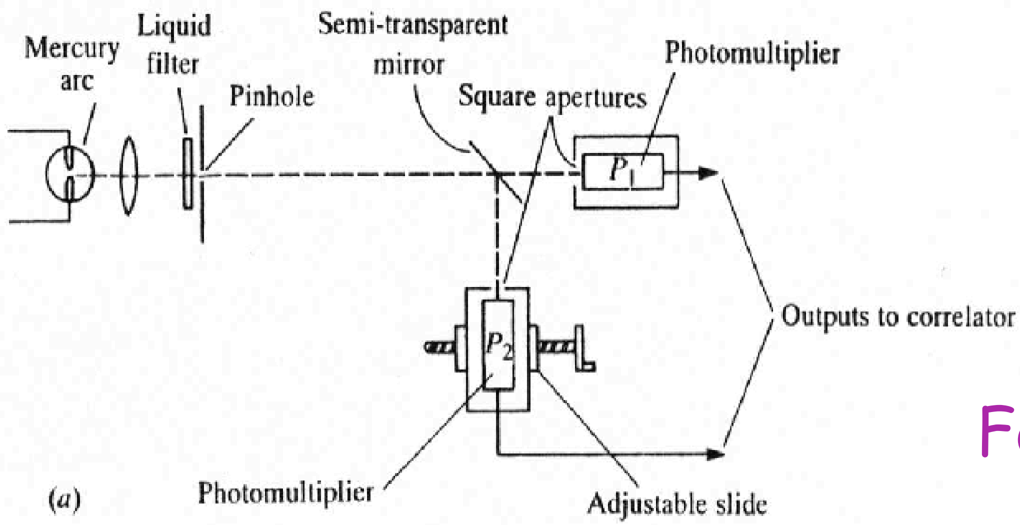
Outline:

- Positive correlations in fermionic systems.

- Injection of e- in a nanotube
  - tunneling current and tunneling noise
  - current and noise along the nanotube
  - noise correlations
  - entanglement

- Nanotube with leads: finite frequency noise correlations

Hanbury Brown and Twiss experiment

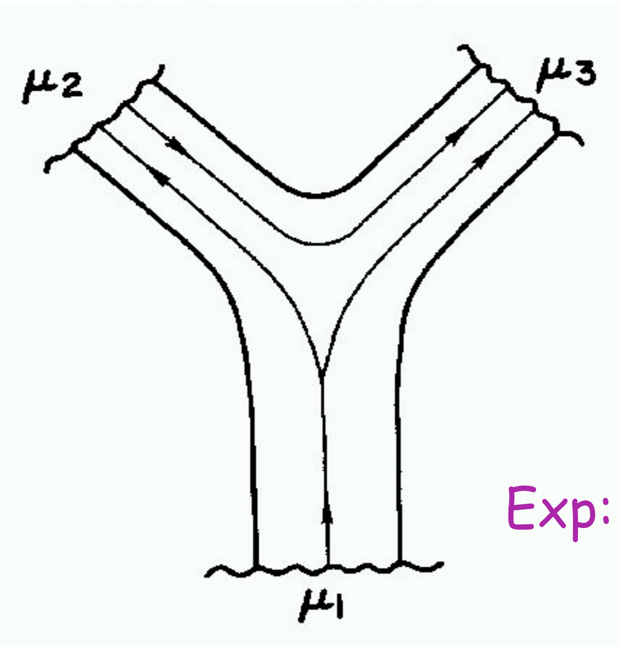


Bunching effect:
positive correlations

For fermions:

Negative correlations

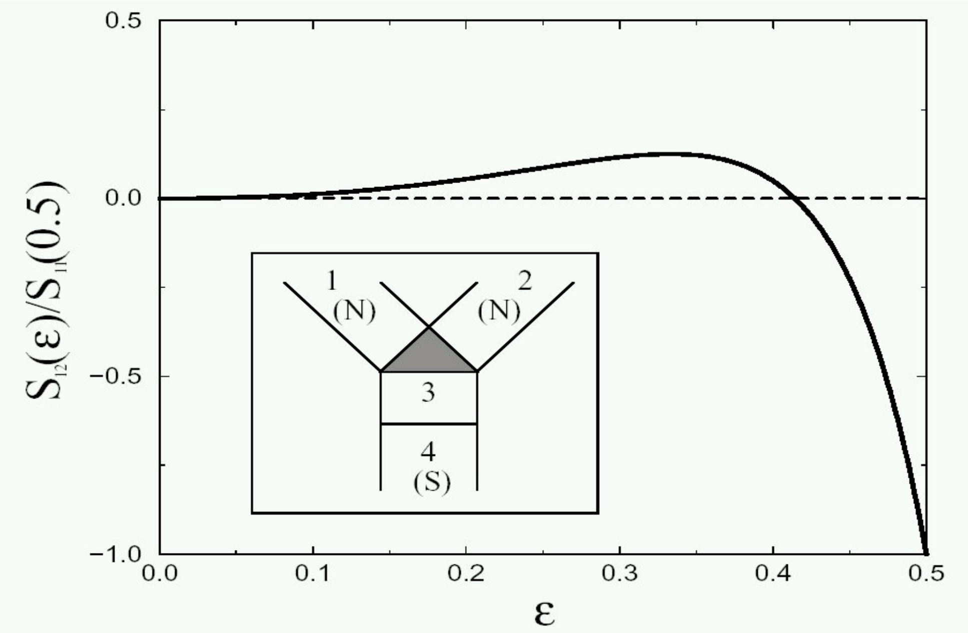
regardless of scattering properties



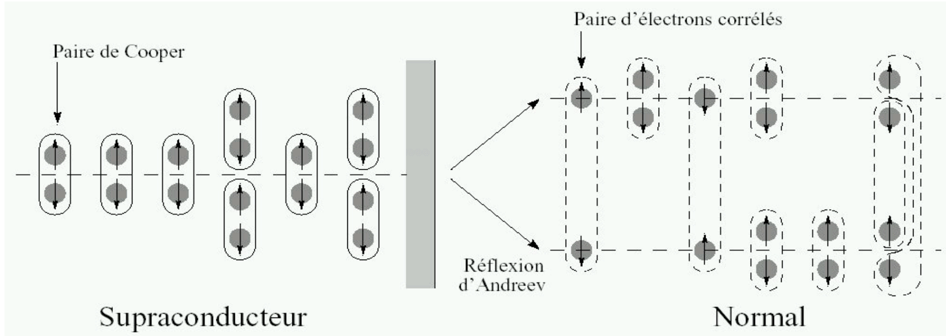
$$S_{23} = -(4e^2/h)\mathrm{Tr}[s_{21}s_{21}^{*}s_{31}s_{31}^{*}]$$

Exp: Schonenberger 99, Yamamoto 99 (Science)

Positive noise correlation in an « NS fork »



J.Torrès+TM, EPJB (99)

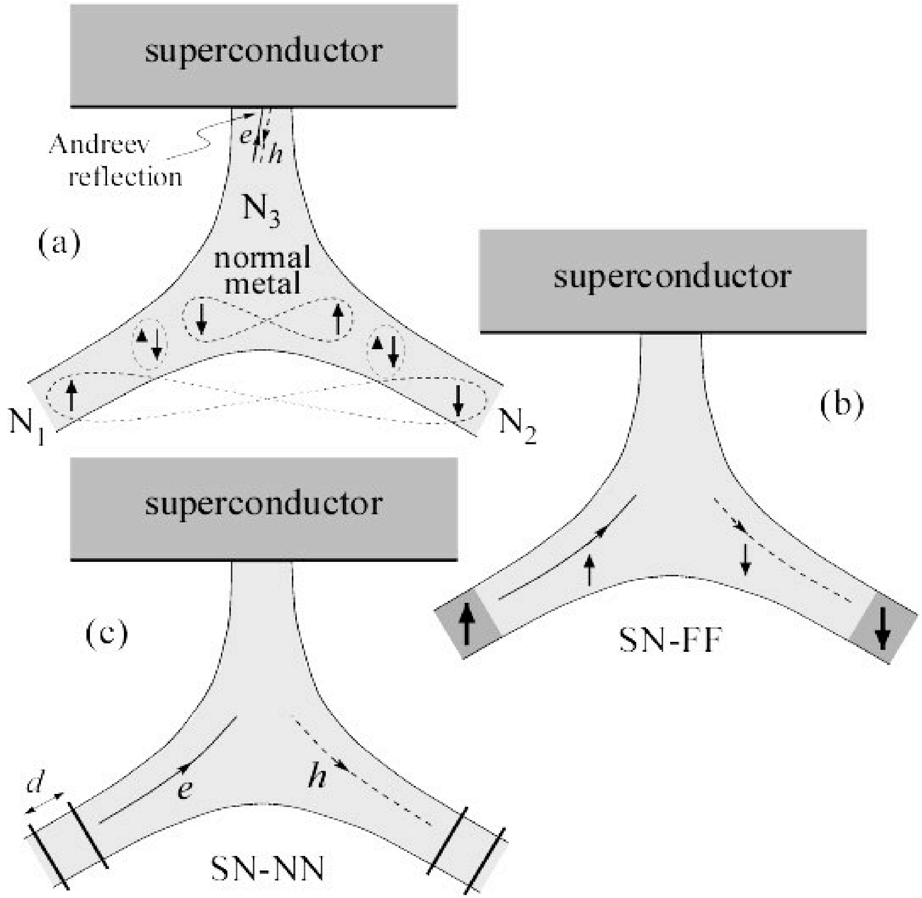


$$S_{12}(0) = \frac{2e^2}{h}\int_0^{eV} \mathrm{d}E \sum_{i=1,2}$$
$$\times \Big[\sum_{j=1,2} \left(s^*_{1iee}s_{1jeh} - s^*_{1ihe}s_{1jhh}\right)\left(s^*_{2jeh}s_{2iee} - s^*_{2jhh}s_{2ihe}\right)$$
$$+ \sum_{\alpha=e,h} \left(s^*_{1iee}s_{14e\alpha} - s^*_{1ihe}s_{14h\alpha}\right)\left(s^*_{24e\alpha}s_{2iee} - s^*_{24h\alpha}s_{2ihe}\right)\Big],$$

Andreev

quasiparticles

Entanglement: Lesovik, Martin, Blatter, EPJB(2001)



energy filters
(spin entanglement)

or

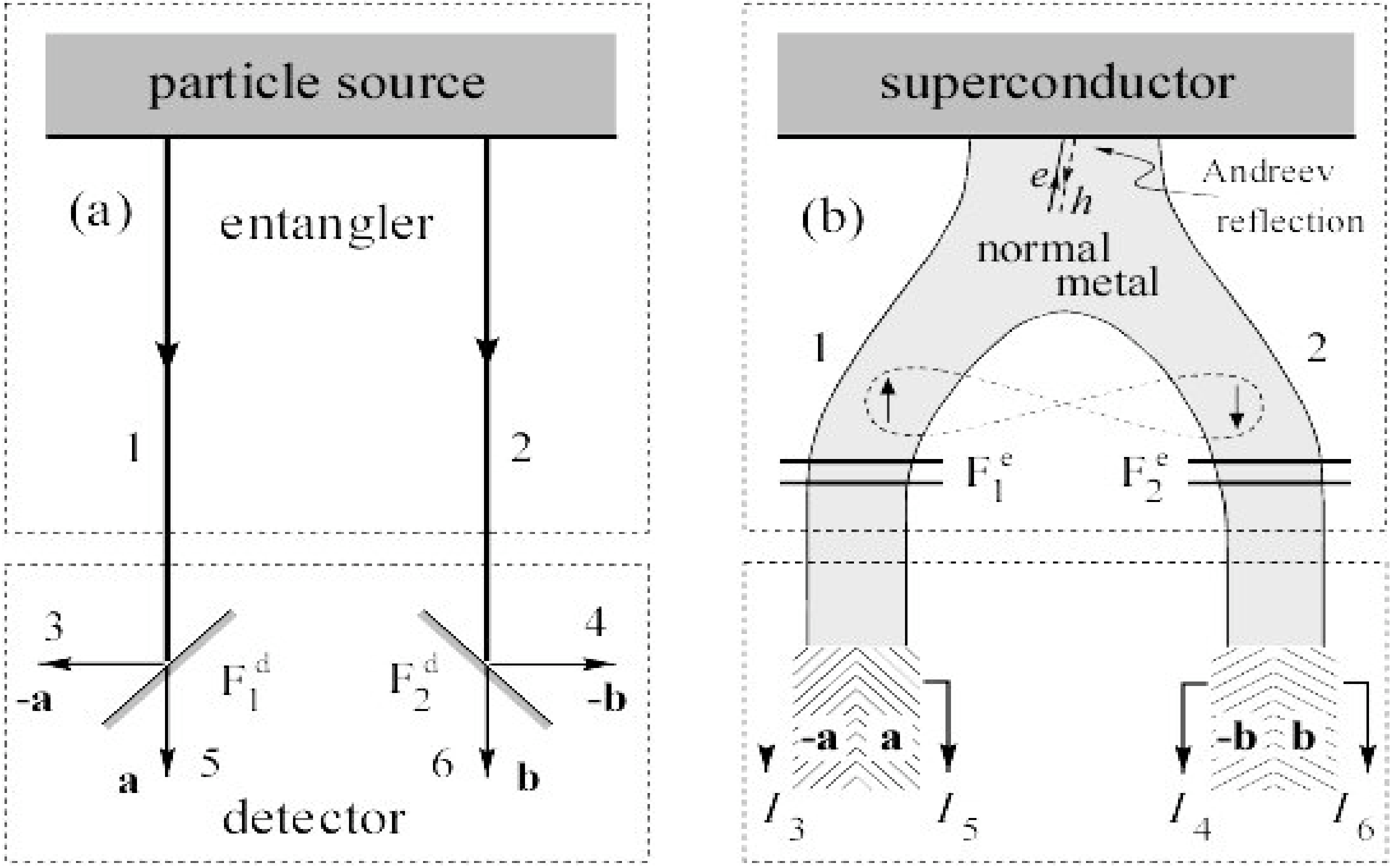
spin filters
(energy entanglement)

POSITIVE CORRELATIONS ONLY!
also:
Texier Buttiker PRB
(no superconductor)

ALSO:
Edge states (e-/h+) in the quantum Hall effect (Beenakker et al. PRL2003), orbital entanglement NS (Samuelson et al…PRL2003)…

Test entanglement: Bell (Clauser-Horne) inequalities in NS

Chtchelkatchev, Blatter, Lesovik, Martin PRB2002



Energy filters +E -E on each arm

Only split Cooper pairs in the two arms

2 spin filters with opposite directions on each arm

$$\rho = \int d\lambda f(\lambda)\rho_\alpha(\lambda)\otimes\rho_\beta(\lambda),$$

$$\langle \delta N_\alpha(\tau)\delta N_\beta(\tau)\rangle = \int_{-\infty}^{\infty}\frac{d\omega}{2\pi}S_{\alpha\beta}(\omega)\frac{4\sin^2(\omega\tau/2)}{\omega^2}.$$

$$|F(\mathbf{a},\mathbf{b})-F(\mathbf{a},\mathbf{b}')+F(\mathbf{a}',\mathbf{b})+F(\mathbf{a}',\mathbf{b}')|\leqslant 2,$$

$$F(\mathbf{a},\mathbf{b}) = \frac{S_{56}-S_{54}-S_{36}+S_{34}+\Lambda_-}{S_{56}+S_{54}+S_{36}+S_{34}+\Lambda_+},$$

$$S_{\alpha\beta} = S_{\alpha\beta}^{(a)}\sin^2\left(\frac{\theta_{\alpha\beta}}{2}\right)$$

Angle between spin filters.

Specific angles

Time window for single Cooper pair emission

- Assume (local) density matrix

- Convert particle number into noise correlators

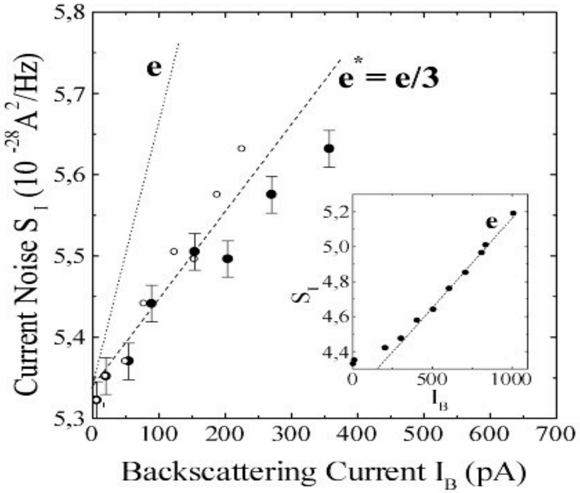
- Derive corresponding inequality for number correlators.

THEN

- Compute noise correlations for an NS fork

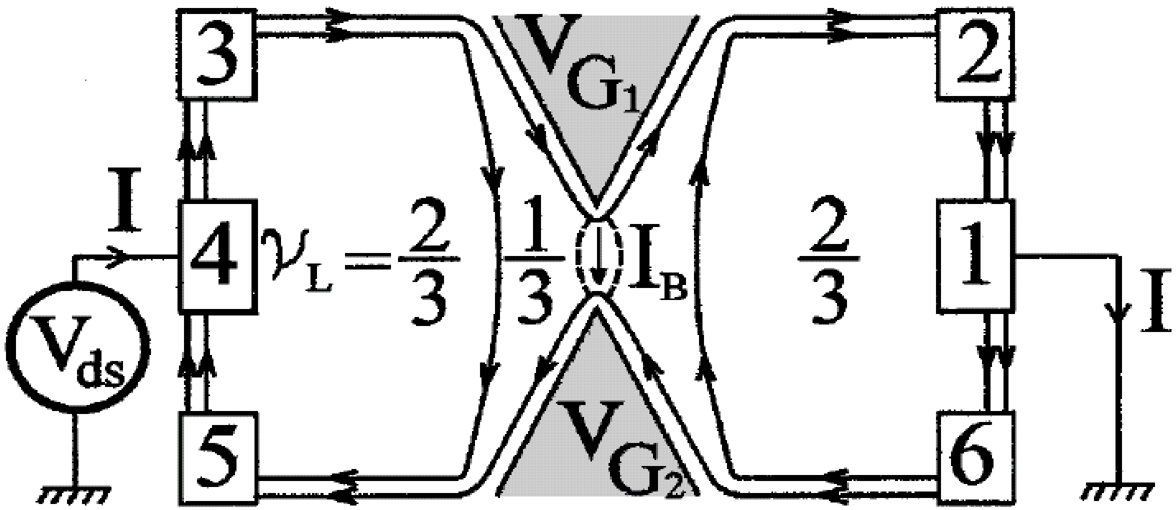
RESULT: **maximal violation of Bell inequality**.

CHIRAL Luttinger Liquids:
Fractionnal Quantum Hall effect



$$S = e^* \langle I \rangle = \frac{e}{3} \langle I \rangle$$

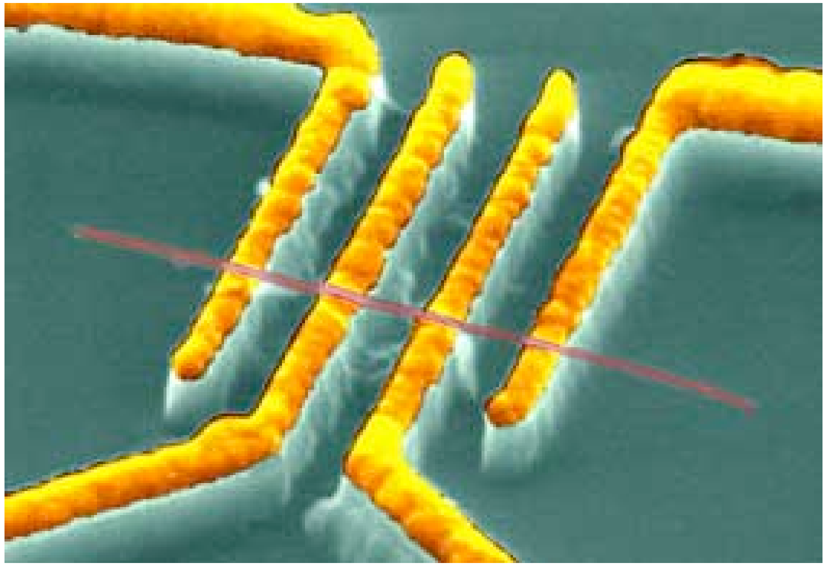
Kane PRL 1994, Chamon PRB1995
Reznnikov Nature 1997,
Saminadayar PRL 1997



BUT
no proposal for detecting charges in NON-CHIRAL Luttinger liquids

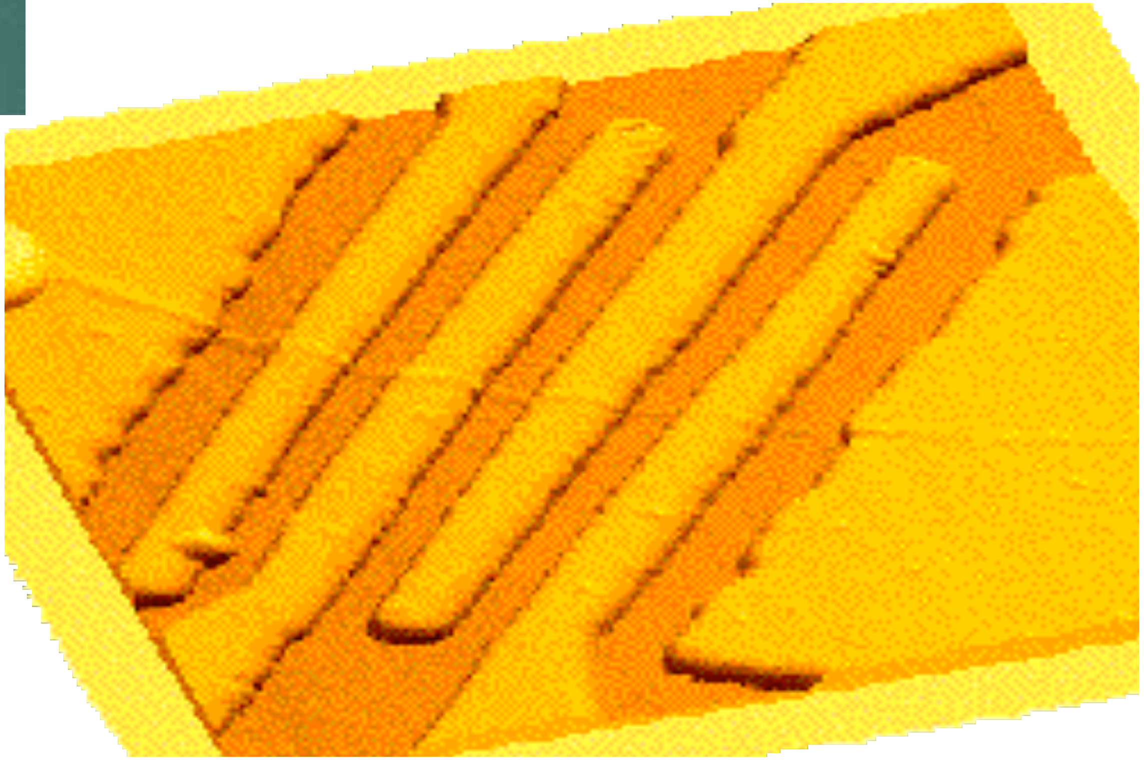
(no separation between chiral excitations unlike FQHE)

Experiments on carbon nanotubes



Tunnel contacts

(Schonenberger…)



…contacts over an extended distance

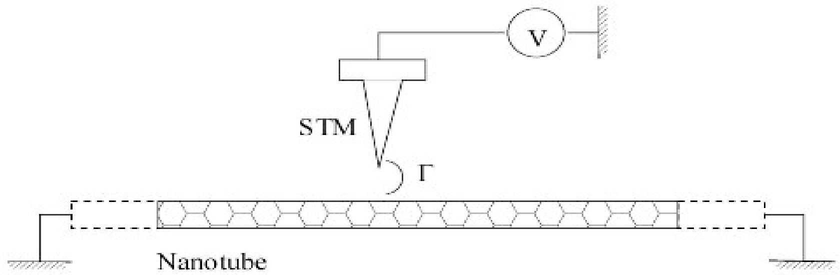
(McEuen…)

+

embedded contacts

Injection of an e- in the bulk of a nanotube

Tunnel Hamiltonian:



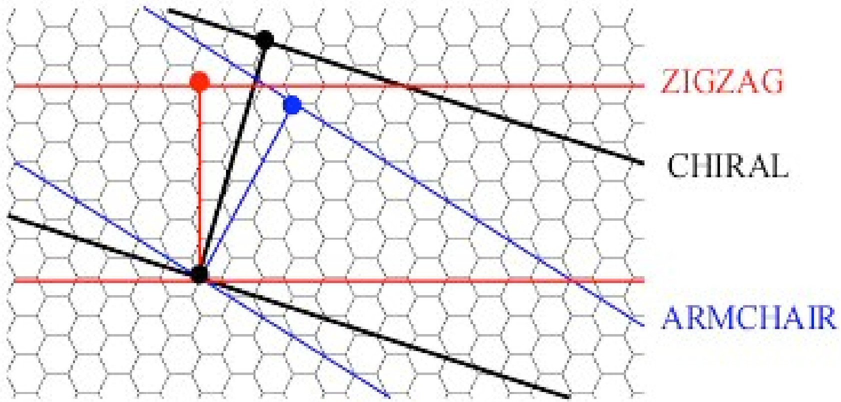
$$H_T(t) = \Gamma(t) \sum_{r\alpha\sigma} \Psi^{+}_{r\alpha\sigma}(0,t) c_\sigma(t) + h.c.$$

$$\Gamma(t) = \Gamma \exp\left(i\frac{eV}{\hbar}t\right) = \Gamma \exp\left(i\omega_0 t\right)$$

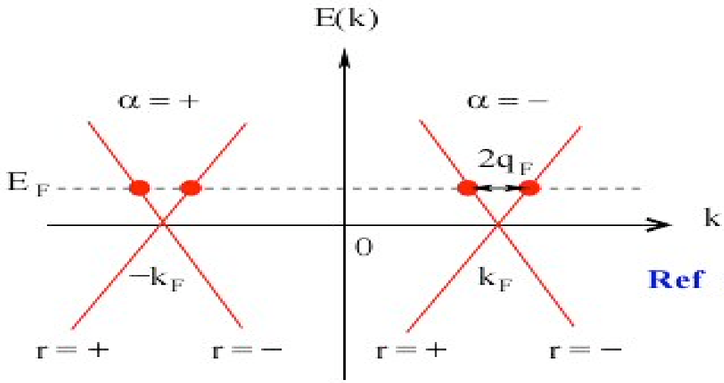
HERE:

a) electron transfer from STM tip to bulk of nanotube

b) metallic nanotube,described by bosonization.



Justification: linear dispersion at the Fermi level



$$\begin{cases} r = \pm \ \text{(branche)} \\ \alpha = \pm \ \text{(mode)} \\ \sigma = \pm \ \text{(spin)} \end{cases} \qquad \begin{cases} j = \text{c (charge) ou s (spin)} \\ \delta = \pm \ \text{(flavor)} \end{cases}$$

Ref : Egger et Gogolin
Eur. Phys. J. B 3, 281 (1998)

Luttinger liquid Hamiltonian :

$$H_N(t) = \sum_{j\delta} \frac{v_{j\delta}}{2} \int dx \left(K_{j\delta}^N \left(\partial_x \phi_{j\delta}(x,t)\right)^2 + \frac{1}{K_{j\delta}^N} \left(\partial_x \theta_{j\delta}(x,t)\right)^2 \right)$$

4 possibilities for $j\delta$:

- $j\delta =$ c+ with $K_{c+}^N = 1/\sqrt{1 + 4V_0(\mathbf{k}=0)/\pi v_F} < 1$
- $j\delta =$ c- with $K_{c-}^N = 1$
- $j\delta =$ s+ with $K_{s+}^N = 1$
- $j\delta =$ s- with $K_{s-}^N = 1$

Bosonized fermion operator :

$$\Psi_{r\alpha\sigma}(x,t) = \frac{M_{r\alpha\sigma}}{\sqrt{2\pi a}} e^{irq_F x + i\alpha k_F x + i\varphi_{r\alpha\sigma}(x,t)}$$

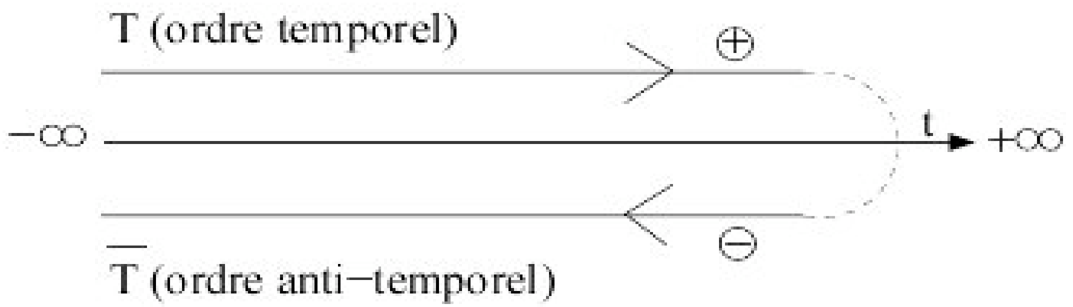
Bosonic field operator :

$$\varphi_{r\alpha\sigma} = \sqrt{\frac{\pi}{2}} \sum_{j\delta} (\phi_{c+} + r\theta_{c+} + \alpha\phi_{c-} + r\alpha\theta_{c-} + \sigma\phi_{s+} + r\sigma\theta_{s+} + \sigma\alpha\phi_{s-} + r\sigma\alpha\theta_{s-})$$

Average tunnel current :

$$\langle I_T(t)\rangle = \frac{1}{2}\sum_\eta \langle T_C\{I_T(t^\eta)e^{-i\int_C dt_1 H_T(t_1)}\}\rangle$$

where C is the Keldysh contour



Green's function:

$$G^{\theta\theta}_{j\delta}(x,t;x',t') = \langle \theta_{j\delta}(x,t)\theta_{j\delta}(x',t')\rangle$$

Differential equation:

$$\left(\frac{\omega^2}{v^N_{j\delta}K^N_{j\delta}} - \partial_x \frac{v^N_{j\delta}}{K^N_{j\delta}}\partial_x\right) G^{\theta\theta}_{j\delta}(x,x';\omega) = 4\pi\delta(x-x')$$

Ref. : Maslov and Stone, PRB 52, 5539 (1995)

Solution :

$$G^{\theta\theta}_{j\delta}(x,t;x',t') = -\frac{K^N_{j\delta}}{8\pi}\sum_r \ln\left(1 + i\frac{v_F(t-t')}{a} + ir\frac{K^N_{j\delta}(x-x')}{a}\right)$$

$\longrightarrow$ same for $G^{\phi\phi}_{j\delta}$; $G^{\phi\theta}_{j\delta}$; $G^{\theta\phi}_{j\delta}$

Tip: trivial chiral Luttinger liquid

Fermionic operator :

$$c_\sigma(t) = \frac{N_\sigma}{\sqrt{2\pi a}}\, e^{i\tilde{\varphi}_\sigma(t)}$$

Chiral Green's function :

$$g_\sigma(t;t') = \langle \tilde{\varphi}_\sigma(t)\tilde{\varphi}_\sigma(t')\rangle = -\frac{1}{2\pi}\ln\left(1 + i\frac{u_F^\sigma(t-t')}{a}\right)$$

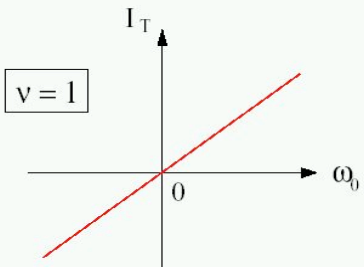
$O(\Gamma^2)$ tunneling current :

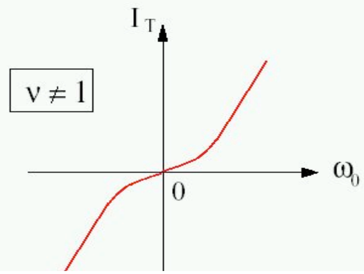
$$\langle I_T\rangle = \frac{2e\Gamma^2}{\pi a}\left(\sum_\sigma \frac{1}{u_F^\sigma}\right)\left(\frac{a}{v_F}\right)^\nu \frac{\mathrm{sgn}(\omega_0)|\omega_0|^\nu}{\boldsymbol{\Gamma}(\nu+1)}$$

with :

$$\nu = \frac{1}{8}\sum_{j\delta}\left(\frac{1}{K_{j\delta}^N} + K_{j\delta}^N\right) \approx 1,2$$

because $K_{c+}^N \approx 0,28$ et $K_{c-}^N = K_{s+}^N = K_{s-}^N = 1$





Tunneling noise:

$$S_T(t,t') = \frac{1}{2}\sum_{\eta} \langle T_K\{I_T(t^{\eta})I_T(t'^{-\eta})e^{-i\int_K dt_1 H_T(t_1)}\}\rangle,$$

No surprise: Schottky 's formula

$$S_T(\omega=0) = e|\langle I_T\rangle|$$

Charge current and noise along the nanotube:

$$\begin{aligned} I_\rho(x,t) &= ev_F \sum_{r\alpha\sigma} r\Psi^{\dagger}_{r\alpha\sigma}(x,t)\Psi_{r\alpha\sigma}(x,t) \\ &= 2ev_F\sqrt{\frac{2}{\pi}}\partial_x\phi_{c+}(x,t). \end{aligned}$$

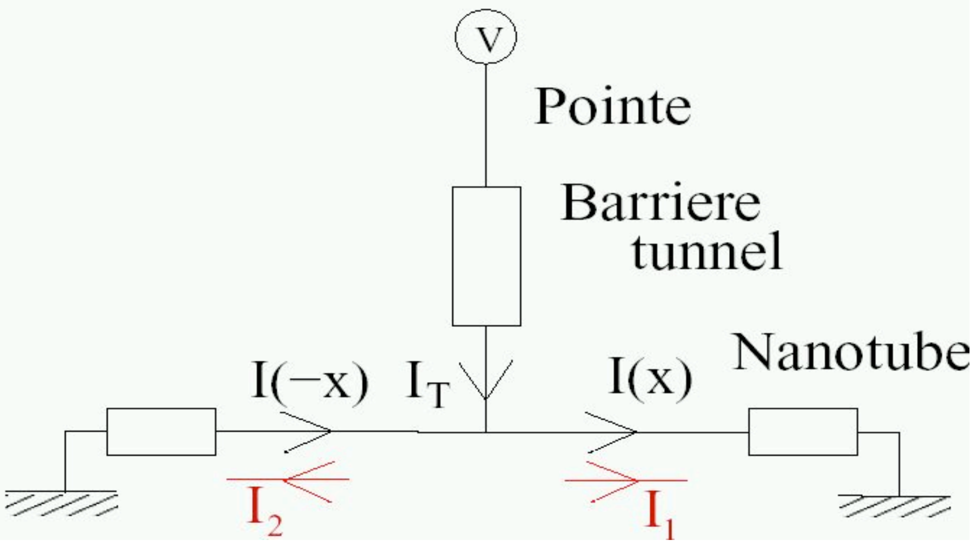
$$\begin{aligned} S_\rho(x,t;x',t') = -\frac{1}{4}\sum_{\eta\eta_1\eta_2} \eta_1\eta_2 \Big\langle T_K\Big\{ I_\rho(x,t^{\eta})I_\rho(x',t'^{-\eta}) \\ \times \int\int dt_1 dt_2 H_T(0,t_1^{\eta_1})H_T(0,t_2^{\eta_2})\Big\}\Big\rangle, \end{aligned}$$

Second order charge current

$$\langle I(x)\rangle = \frac{e\Gamma^2}{\pi a}\left(\sum_\sigma \frac{1}{u_F^\sigma}\right)\left(\frac{a}{v_F}\right)^\nu \frac{\mathrm{sgn}(\omega_0)|\omega_0|^\nu}{\mathbf{\Gamma}(\nu+1)}\mathrm{sgn}(x)$$

Current conservation:

$$|\langle I(x)\rangle| = \frac{\langle I_T\rangle}{2}$$



Usual convention:
Current in each
direction
along the nanotube
SHOULD be measured
AWAY from the
injection
location (lower arrows).

Noise + noise cross-correlations:

Auto-correlation x' = x :

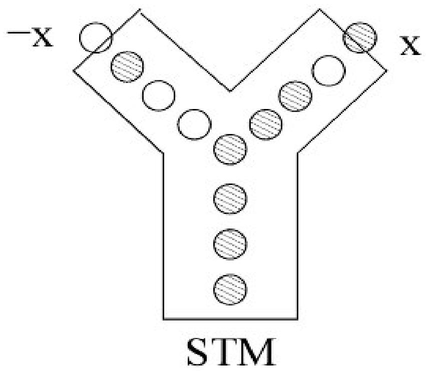
$$S(x,x,\omega=0) = \frac{1+(K_{c+}^N)^2}{2} e|\langle I(x)\rangle| \qquad (\Gamma^2)$$

Cross-correlations x' = - x :

$$S(x,-x,\omega=0) = -\frac{1-(K_{c+}^N)^2}{2} e|\langle I(x)\rangle| \qquad (\ \Gamma^2)$$

(Opposite sign with HBT convention)

2nd order in the tunneling amplitude
Usual situation for non-interacting Fermions: 4th order.



Buttiker, (IQHE, PRL 90), (PRB 92).
Martin Landauer (PRB 92)

HERE, POSITIVE CORRELATIONS FOR AN INTERACTING FERMIONIC SYSTEM !!!

Interpretation: anomalous charges:

Injection of a charge e in a Luttinger liquid:

$$Q_1 = \frac{1 - K_{c+}^N}{2} e \quad \text{in one direction}$$

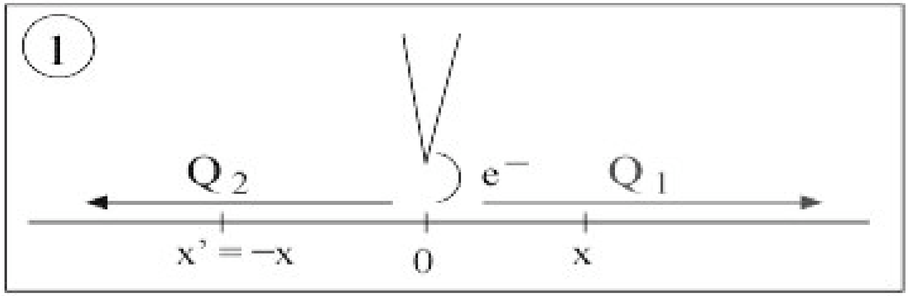
$$Q_2 = \frac{1 + K_{c+}^N}{2} e \quad \text{other direction}$$

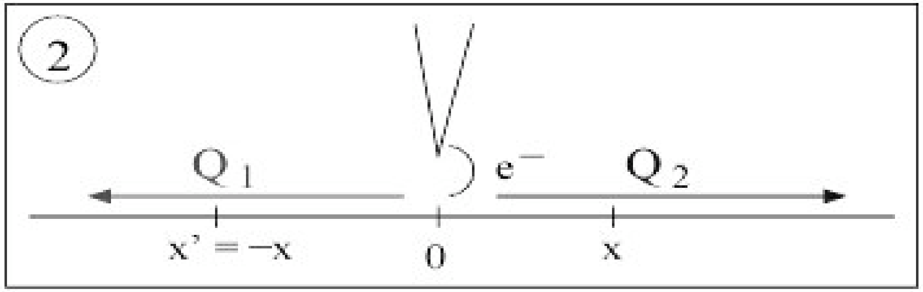
Ref. : Safi, Ann. Phys. Fr. 22, 463 (1997)

Ref. : Imura et al., PRB 66, 035313 (2002)

What does it imply for the noise ?

$$S(x, x') \propto Q(x)Q(x') \ ?$$





$$S(x, x) \propto \frac{1}{2}\left(Q_1^2 + Q_2^2\right) = \frac{1 + (K_{c+}^N)^2}{4} e^2$$

$$S(x, -x) \propto \frac{1}{2}\left(Q_1 Q_2 + Q_2 Q_1\right) = \frac{1 - (K_{c+}^N)^2}{4} e^2$$

Entanglement ?

Symptom: positive cross correlations, as in NS forks

e- injected at x=0

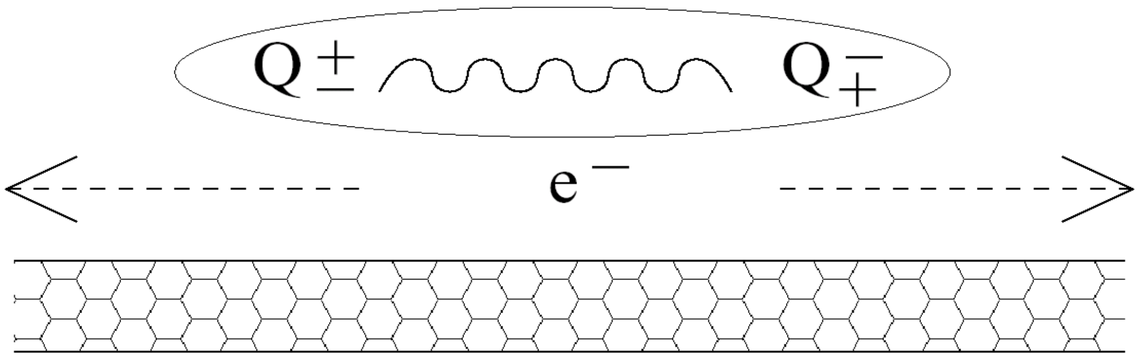
$$\sum_{r\alpha}\Psi^{\dagger}_{r\alpha\sigma}|O_{LL}\rangle = \frac{M_{r\alpha\sigma}}{2\pi a}\sum_{r\alpha} e^{-i\sum_{j\delta}\sqrt{\frac{\pi}{2K^N_{j\delta}}}h_{\alpha\sigma j\delta}\left(\frac{1+rK^N_{j\delta}}{2}\tilde{\varphi}^{+}_{j\delta}+\frac{1-rK^N_{j\delta}}{2}\tilde{\varphi}^{-}_{j\delta}\right)}|O_{LL}\rangle$$

with : $h_{\alpha\sigma c+}=1,\ h_{\alpha\sigma c-}=\alpha,\ h_{\alpha\sigma s+}=\sigma,\ h_{\alpha\sigma s-}=\sigma\alpha,$

use the underlying chiral bosonic fields :

$$\tilde{\varphi}^{r}_{j\delta}(x) = \frac{1}{4\sqrt{K^N_{j\delta}}}\sum_{r'\alpha\sigma} h_{\alpha\sigma j\delta}(r + r'K^N_{j\delta}$$

$$\times \sum_{(r'k)>0}\sqrt{\frac{1}{|k|L}}\left(d^{\dagger}_{\alpha\sigma}(k)e^{-ik}\right.$$



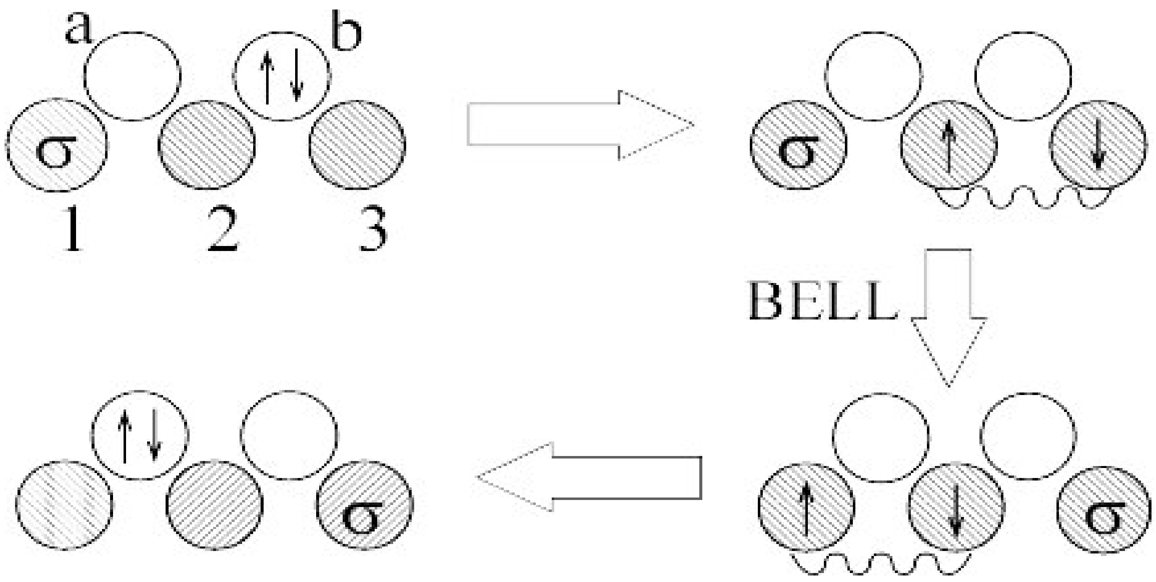
« Triplet » wavefunction!

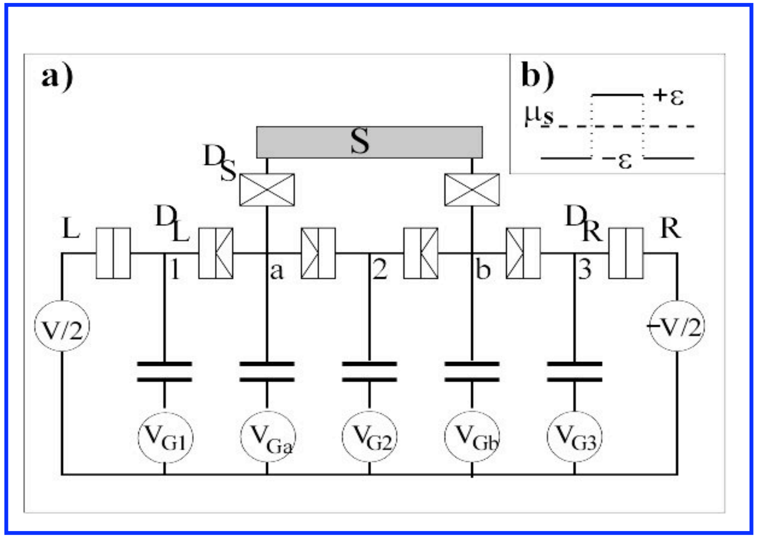
$$\sum_{r\alpha}\Psi^{\dagger}_{r\alpha\sigma}|O_{LL}\rangle = \frac{1}{\sqrt{2\pi a}}\sum_{\alpha}\prod_{j\delta}\Big[(\tilde{\psi}^{\dagger}_{j\delta+})_{\sigma j\delta+}(\tilde{\psi}^{\dagger}_{j\delta-})_{\sigma j\delta-}$$
$$+(\tilde{\psi}^{\dagger}_{j\delta+})_{\sigma j\delta-}(\tilde{\psi}^{\dagger}_{j\delta-})_{\sigma j\delta+}\Big]|O_{LL}\rangle$$

$|Q_+Q_-> + |Q_-Q_+>$

Chiral fields $\tilde{\psi}^{\dagger}_{j\delta\pm}(x) = \exp\left[-i\sqrt{\frac{\pi}{2K^N_{j\delta}}}h_{\alpha\sigma j\delta}\tilde{\varphi}^{\pm}_{j\delta}(x)\right]$

When you have entanglement…why not **teleportation** ?



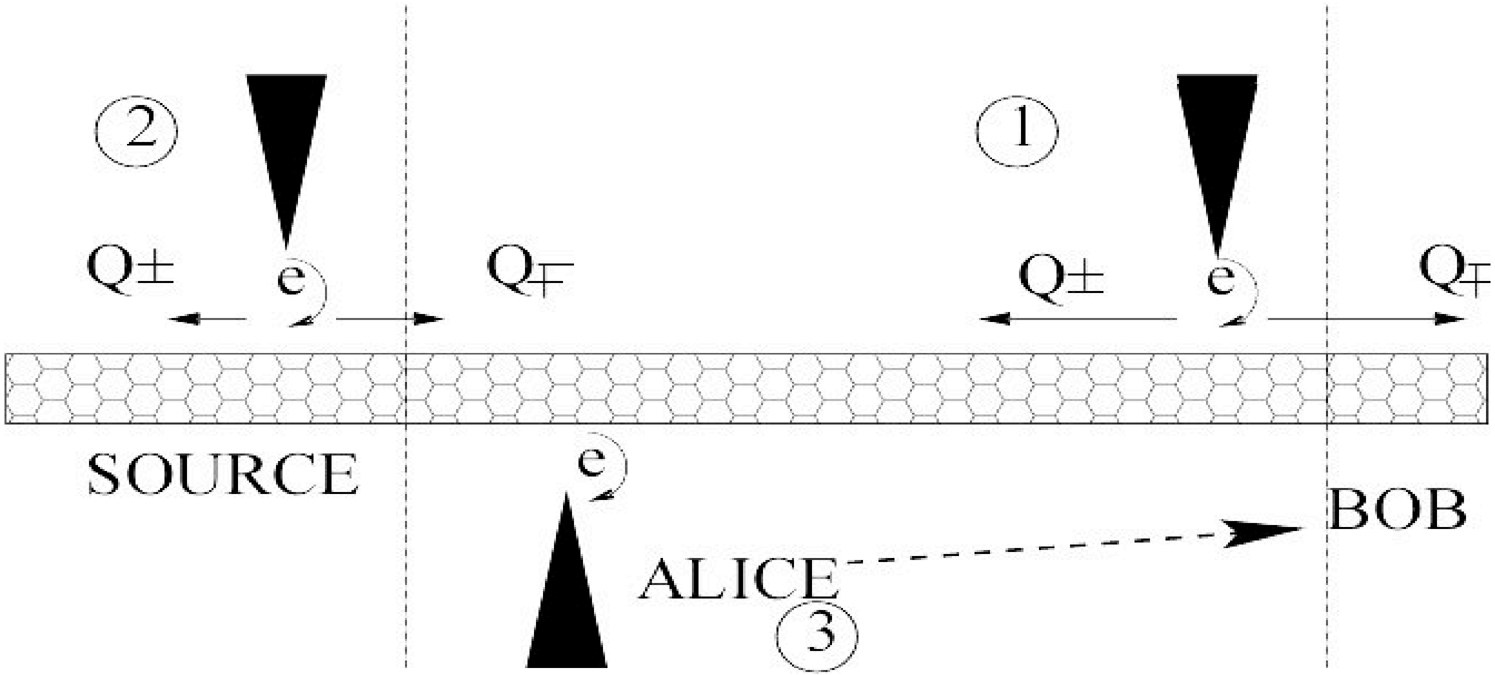


Teleportation with superconductors and quantum dots:

- dots for filtering e- one by one
- superconductor as source and detectors of singlet e- pairs

Sauret, Feinberg, Martin, EPJB 2003 + PRB 2004
Beenakker et al., e-h+ pairs in IQHE, PRL 2003

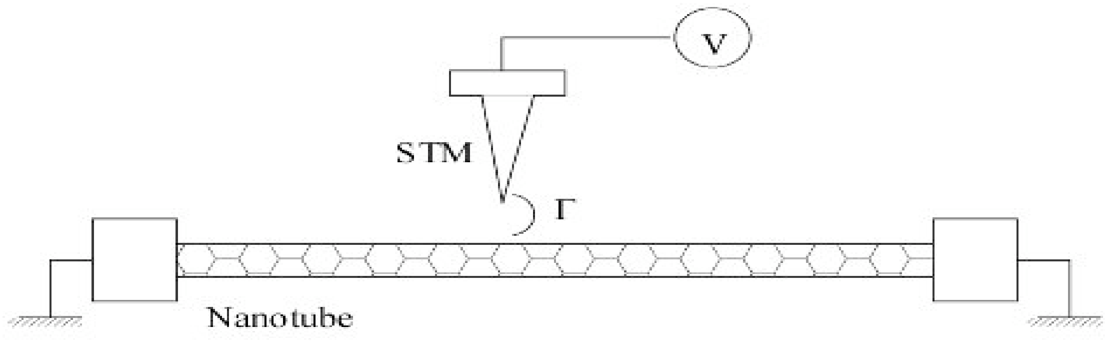
Teleportation of anomalous charge states in nanotube ?
This requires 3 contacts.



Non-local transfer of anomalous charge states:
1 inject Q+ Q- pair shared by Alice and Bob.
2 inject pair with anomalous charge to be teleported.
3 anihilate the latter: Alice detects an e-, and tells Bob.
Coincidences in injected/detected tunneling currents.

What about contacts?

Do they spoil everything?



Almost!

Conductance: Safi-Schulz, Maslov Stone, Ponomarenko

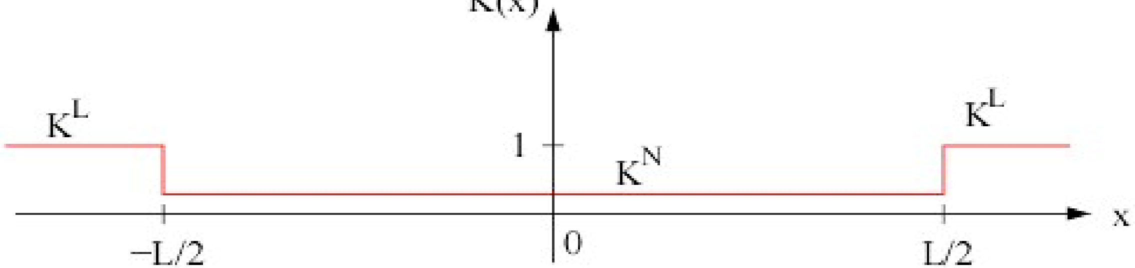
Noise: Ponomarenko... PRB, Trauzettel Egger Grabert PRL

No renormalization due to interactions !

Inhomogeneous Luttinger liquids:

1D Fermi liquid leads

(Safi-Schulz, Maslov, Ponomarenko)



$$-v_{j\delta}(x)K_{j\delta}(x)\partial_x G^{\phi\phi}_{j\delta}(x,x',\bar{\omega})\Big|_{x=x'-\epsilon}^{x=x'+\epsilon} = 1$$

Multiple Andreev-like reflection at the boundaries

Finite frequency noise

Unsymmetrized correlator

$$S_{xx'}(\omega) = -\frac{e^2 v_F^2 \Gamma^2}{2(\pi a)^2} \sum_{\eta_1 \eta_2 r \sigma} \eta_1 \eta_2$$

$$\times \Big(\tilde{A}^{r\sigma}_{\eta_1\eta_2}(0) \tilde{B}^{r\sigma}_{-,\eta_1}(x,0,\omega) \tilde{B}^{r\sigma}_{+,\eta_1}(x',0,-\omega)$$

$$- \tilde{A}^{r\sigma}_{\eta_1\eta_2}(-\omega) \tilde{B}^{r\sigma}_{-,\eta_2}(x,0,\omega) \tilde{B}^{r\sigma}_{+,\eta_1}(x',0,-\omega)$$

$$- \tilde{A}^{r\sigma}_{\eta_1\eta_2}(\omega) \tilde{B}^{r\sigma}_{-,\eta_1}(x,0,\omega) \tilde{B}^{r\sigma}_{+,\eta_2}(x',0,-\omega)$$

$$+ \tilde{A}^{r\sigma}_{\eta_1\eta_2}(0) \tilde{B}^{r\sigma}_{-,\eta_2}(x,0,\omega) \tilde{B}^{r\sigma}_{+,\eta_2}(x',0,-\omega) \Big),$$

Tunneling density of states

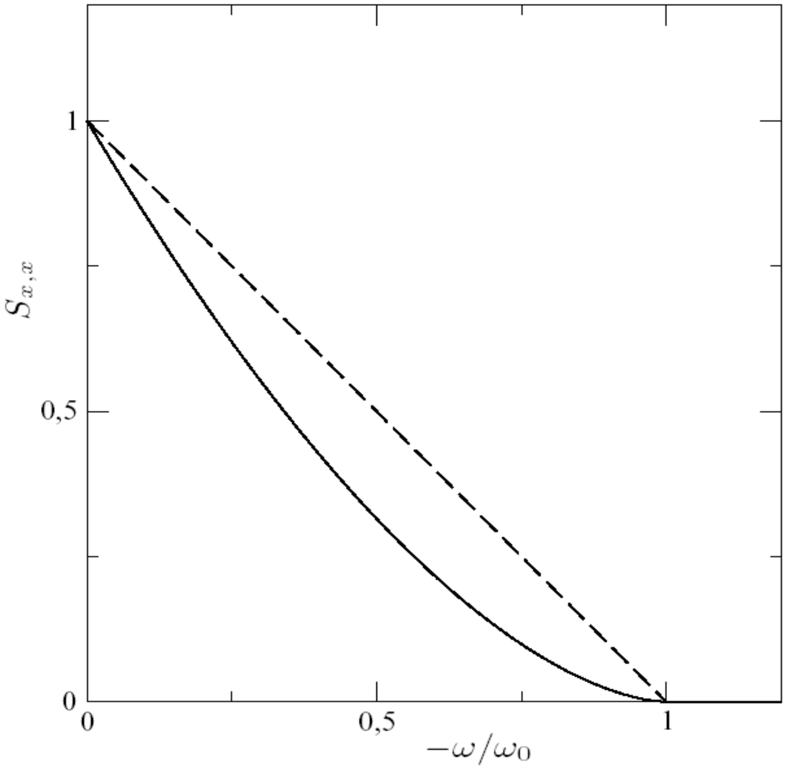
$$A^{r\sigma}_{\eta\mu}(t) = \cos\omega_0 t \, e^{g^{\sigma}_{\eta\mu}(t)} \exp\Big[\frac{\pi}{4} \sum_{j\delta} \tilde{G}^{\phi\phi}_{j\delta,\eta\mu}(0,0,t)$$

$$+ r\tilde{G}^{\phi\theta}_{j\delta,\eta\mu}(0,0,t) + r\tilde{G}^{\theta\phi}_{j\delta,\eta\mu}(0,0,t) + \tilde{G}^{\theta\theta}_{j\delta,\eta\mu}(0,0,t) \Big]$$

Propagation along the nanotube

$$B^{r\sigma}_{\eta\mu}(x,0,t) = \partial_x \Big[\tilde{G}^{\phi\phi}_{c+,\eta\mu}(x,0,t) + r\tilde{G}^{\phi\theta}_{c+,\eta\mu}(x,0,t) \Big]$$

Autocorrelation noise in an infinite nanotube



$$S_{xx'}(\omega) = \frac{e^2\Gamma^2}{(\pi a)^2} \frac{K_{c+}^2 + sgn(x)sgn(x')}{2}$$
$$\times\Theta(-\omega)\, e^{i\omega\tau_-}\left(\tilde{A}_{+-}(|\omega|) + \tilde{A}_{-+}(-|\omega|)\right)$$

Non-interacting case, dashed: singularity (Yang, Lesovik)

Interacting case, full

Cross correlation in a nanotube with leads: two time scales

1) Injection (voltage) time $\tau_{\mathrm{V}} = \hbar/eV$

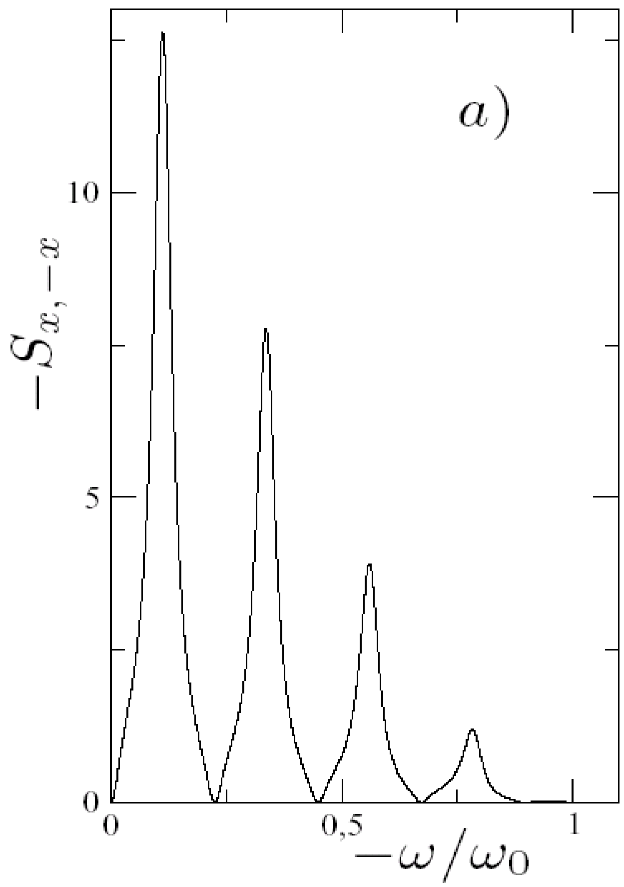
time spread of elctron wave packet

(Lesovik Levitov)

2) Time of flight $\tau_{\mathrm{L}} = L/2v_{c+}$

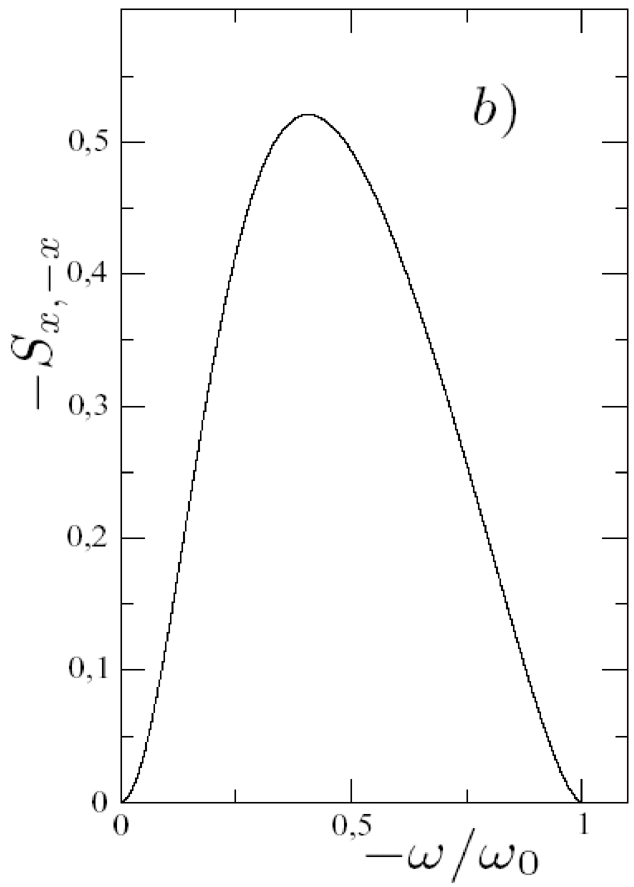
$$S_{xx'}(\omega) = \frac{e^2\Gamma^2}{\pi v_{\mathrm{F}}^2}\left(\frac{a}{v_{\mathrm{F}}}\right)^{\nu-1} e^{i\omega\tau_-}\Theta(|\omega_0|-|\omega|)\frac{(|\omega_0|-|\omega|)^\nu}{\Gamma(\nu+1)} \times\left(\frac{1}{1-(1-K_{c+}^{-2})\sin^2\omega\tau_L}+\frac{sgn(x)sgn(x')}{1-(1-K_{c+}^{2})\sin^2\omega\tau_L}\right),$$

Cross correlations in a nanotube with leads



$\tau_L = 14\tau_V$

Several round trips



$\tau_L = \tau_V$

No round trips

Detection of anomalous charges ?

$$Q_{\pm} = (1 \pm K_{c+})/2$$

Specify frequency associated with peaks

$$\omega \tau_L = (2p + 1)\pi/2 \ (p \text{ integer})$$

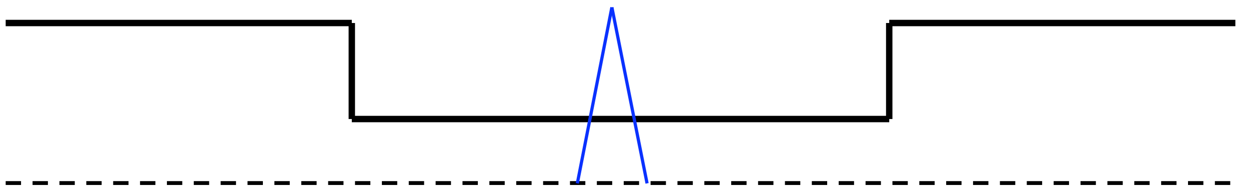
Ratio of cross correlations to autocorrelation noise

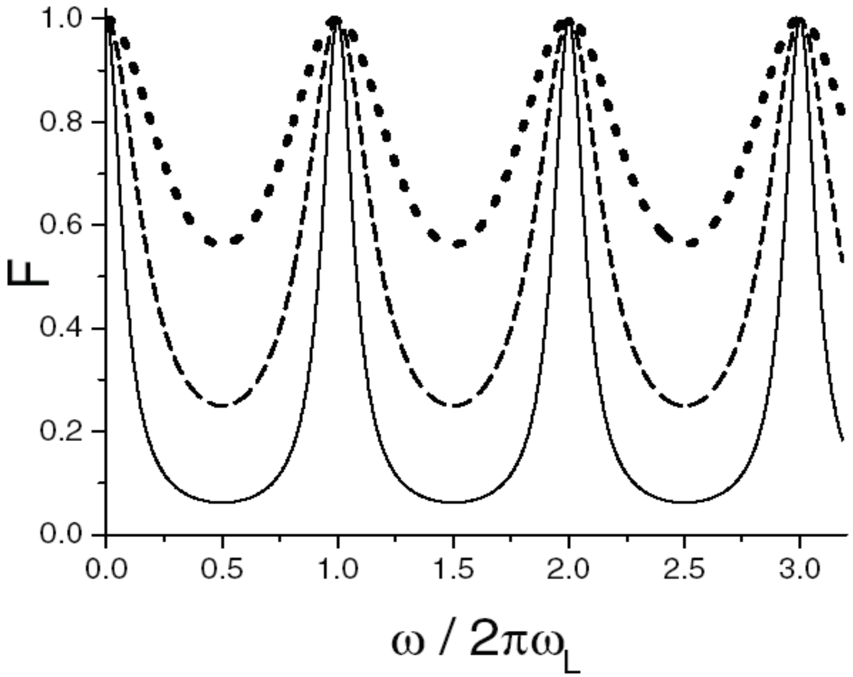
$$|S_{x,-x}/S_{x,x}| = (1 - K_{c+}^4)/(1 + K_{c+}^4)$$

Impurities ? Secondary peaks in the noise

Alternative: LL with leads with an impurity in the middle

(Trauzettel et al. PRL04)





$$S(x, x; \omega) \simeq 2eF(\omega)I_{\mathrm{BS}}$$

Requires an averaging

of the noise over τ_L^{-1}

CONCLUSION:

- Noise auto AND cross correlations allow to detect anomalous charges.

- Some entanglement scenarios in condensed matter systems based on an interaction (superconductor, dots).

- Nanotube: electrons are « not welcome ».
  Separation into right and left moving charges which are entangled.

- Leads: need to probe charges at finite frequency.

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