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Pseudogap in strongly coupled superconductors

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These are preliminary lecture notes, intended only for distribution to participants

Pseudogap in strongly coupled superconductors.

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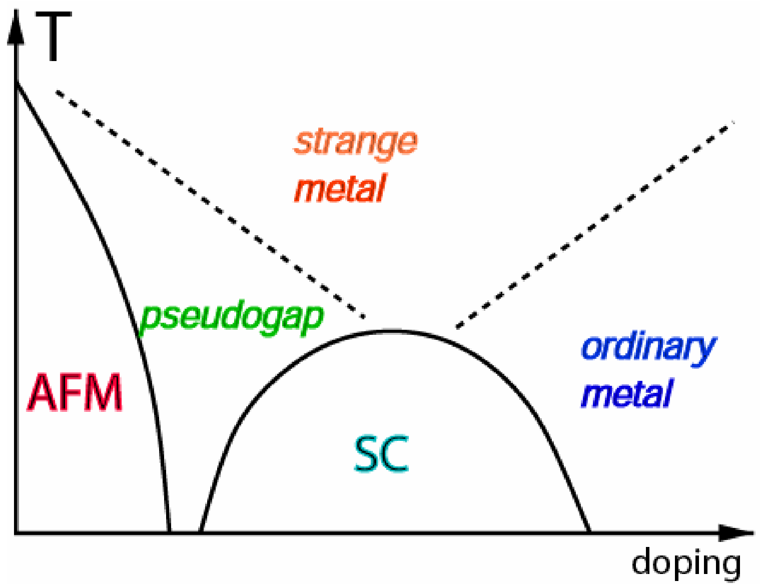
Princeton

Inspired by Rolan Combescot

ENS, Paris

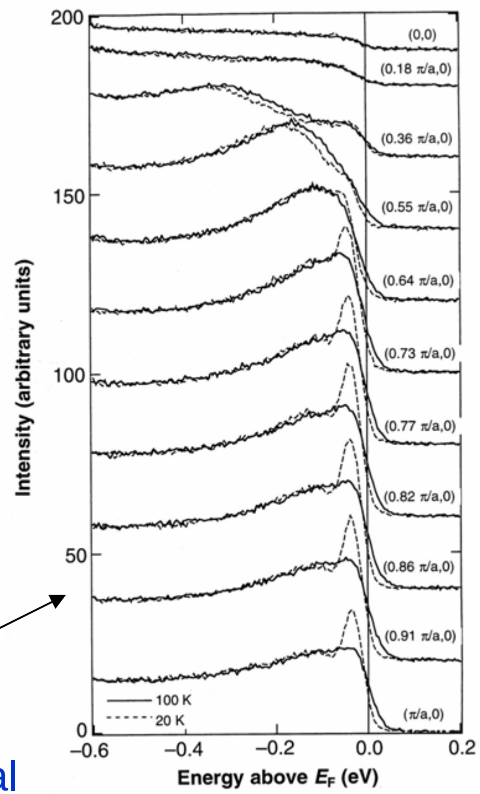
Why there is still an interest in high temperature superconductivity?

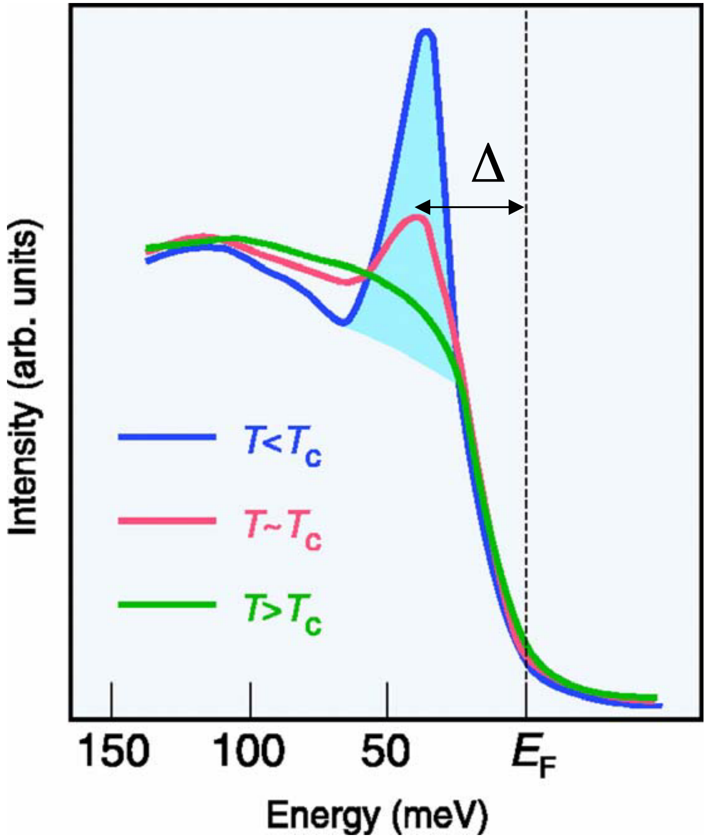
- Non-Fermi liquid behavior in the normal state
- Pseudogap



Pseudogap

Photoemission intensity in high Tc





Z-X Shen et al
Campuzano et al
Johnson et al
Kordyuk et al

The gap does not vanish at Tc.

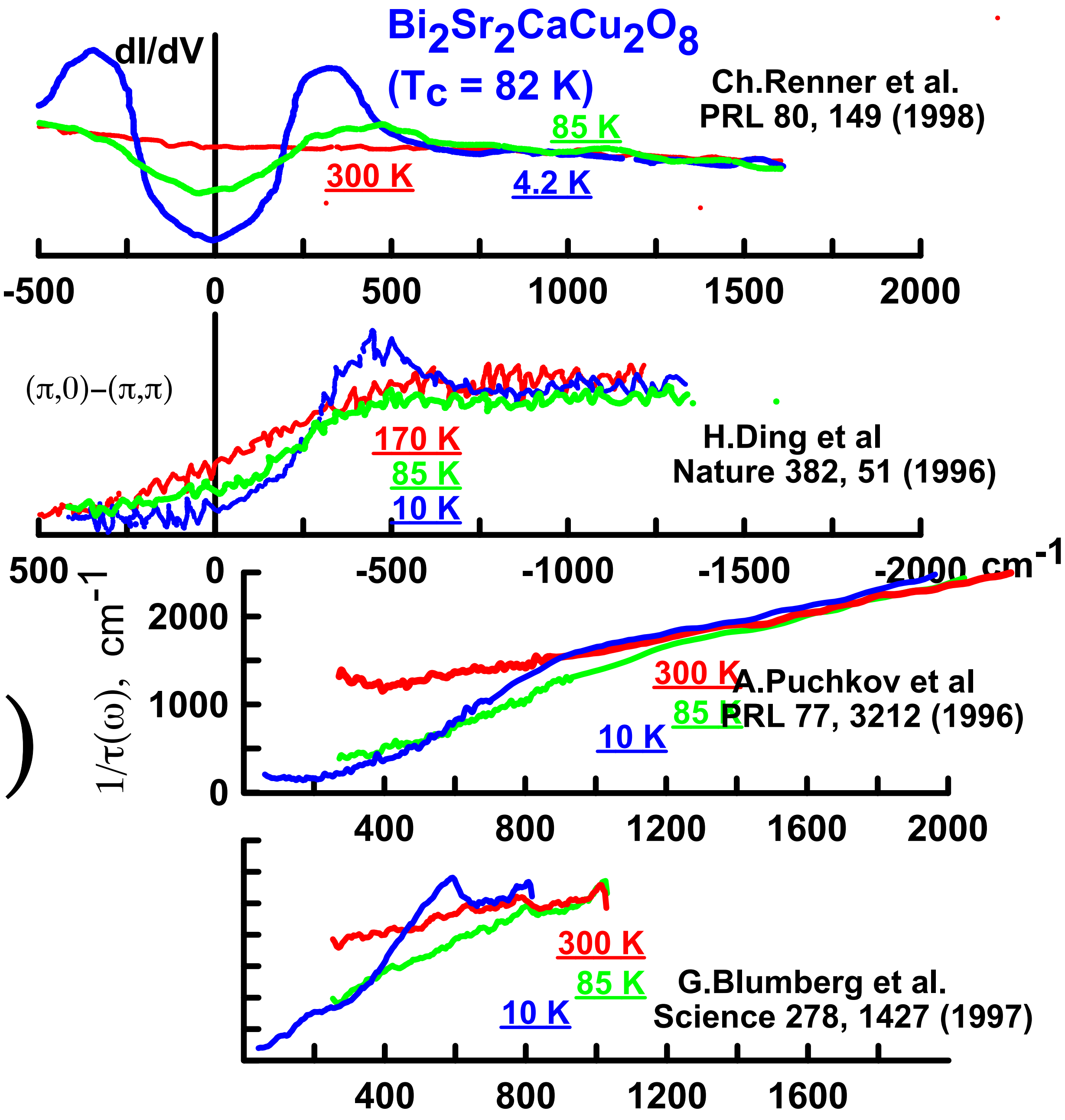
Pseudogap

STM

ARPES

IR:$1/\tau(\omega)$

Raman



What is pseudogap?

a novel order in the particle hole channel,

or

an instability in the particle-particle channel
(a precursor to superconductivity)?

FACTS:

- pseudogap transforms into a superconducting gap below Tc
- pseudogap has d-wave symmetry
- there is only one peak in the density of states below Tc

↓

Pseudogap is a precursor to superconductivity

Conventional reasoning

Superfluid stiffness is small

$$\rho_s\,(T=0) << \Delta$$

Phase fluctuations: $\rho_s\,(T) = \rho_s\,(T=0) - T$

$$T_c \sim \rho_s\,(T=0) << T_c \sim \Delta$$

Can small stiffness be an intrinsic property of a strongly coupled superconductor?

- Eliashberg theory $\lambda > 1$

Stiffness

$\rho_s \sim E_F$ weak coupling

$\rho_s \sim E_F/\sqrt{\lambda}$ strong coupling

Looks, the stiffness can be made as small as possible.

Migdal condition prevents this

Phonon superconductors

Three parameters: interaction, Debye frequency, and Fermi energy

$$\lambda = \left(\frac{g}{\omega_D}\right)^2$$

$\lambda > 1$ - interaction is larger than Debye frequency

Eliashberg theory (no vertex corrections) is valid when $\lambda \frac{\omega_D}{E_F} << 1$

$$\omega_D << g << \left(E_F\, \omega_D\right)^{1/2} << E_F$$

interaction is much smaller than E_F

How this affects the stiffness?

$$\rho_s \sim \frac{E_F}{\sqrt{\lambda}} \sim g \left(\frac{\sqrt{E_F\, \omega_D}}{g} \right)^2 >> g$$

Pairing gap $\Delta \leq g \;\; \Rightarrow \;\; \rho_s >> \Delta$

phase fluctuations
are irrelevant

Eliashberg theory is inconsistent with phase fluctuation scenario

[phase fluctuations are small in Eliashberg theory]

Downward renormalization of ρ_s is not enough.

A similarity with an s-wave superconductor with impurities

- γ normal state damping
- Δ superconducting gap

clean limit $\gamma << \Delta \quad (\Delta\tau >> 1)$ $\rho_s \sim E_F$

dirty limit $\gamma >> \Delta \quad (\Delta\tau << 1)$ $\rho_s \sim E_F \dfrac{\Delta}{\gamma} << E_F$

However, even in dirty limit, $\rho_s \sim \Delta \dfrac{E_F}{\gamma} >> \Delta$

(Tc is not affected by impurieies)

Two ways out (cuprates)

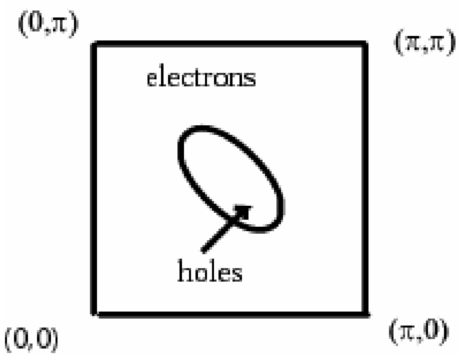
1. break $g << E_F$ (make interaction larger than the bandwidth)

Plus:

no double occupancy constraint enhances phase fluctuations

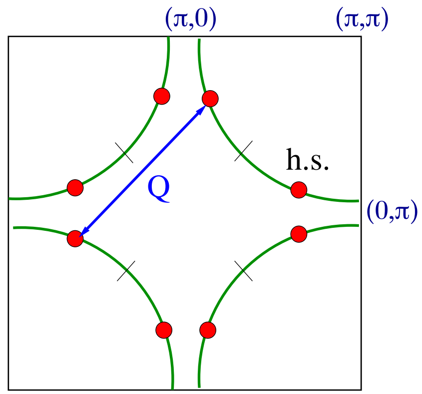
Minus:

Luttinger theorem is violated in the normal state above T_{ins}



physics of x

ARPES above T_{ins}



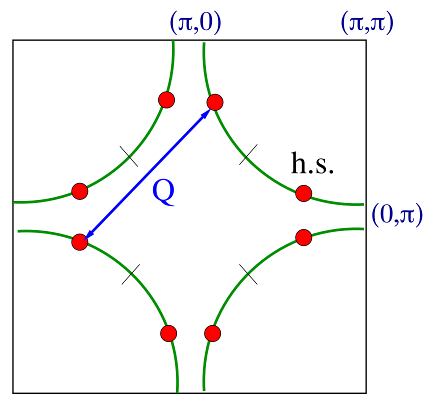
physics of 1-x

Two ways out (cuprates)

2. keep $g << E_F$ (i.e., keep interaction smaller than the bandwidth),

and more carefully analyze Eliashberg theory

Plus: large, Luttinger type Fermi surface in the normal state



If the pseudogap exists in the Eliashberg theory,
then the crossover from the physics of x to the physics of 1-x
should happen inside the pseudogap phase

Eliashberg theory (again). Phonons (again) as an example.

Previous works:

Allen & Dynes, Carbotte & Marsiglio, Combescot, Maksimov et al

gap equation

$$\Delta_{\omega_m} = \pi T g^2 \sum_n \frac{\Delta_{\omega_n} - \Delta_{\omega_m} \frac{\omega_n}{\omega_m}}{\sqrt{\omega_n^2 + \Delta_{\omega_n}^2}} \frac{1}{(\omega_n - \omega_m)^2 + g^2/\lambda}$$

onset of pairing

$$T_c \sim \omega_D e^{-\frac{1}{\lambda}}$$

weak coupling

$$T_c \sim \omega_D \sqrt{\lambda} \sim g$$

strong coupling

gap $$\Delta(\omega = 0) \sim T_c$$

condensation energy $$E_c \sim -N_0 \Delta^2 < 0$$

Strong coupling

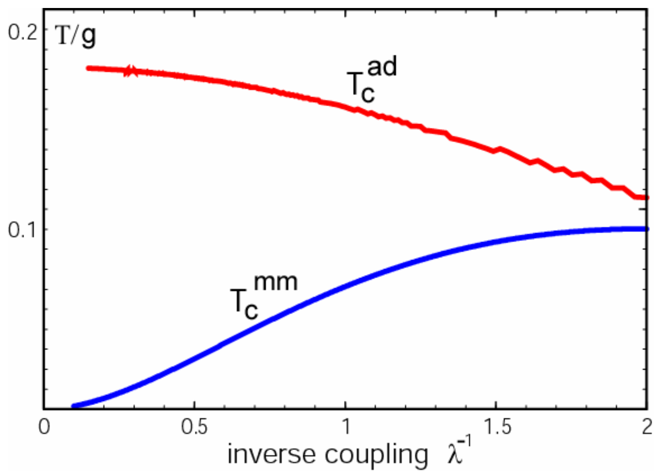
$$\lambda = \left(\frac{g}{\omega_D}\right)^2$$

$$T_c^{ad} \sim \omega_D \sqrt{\lambda} \sim g$$

(Allen-Dynes formula, 1975)

$$T_c^{mm} \sim \omega_D\, e^{-\frac{\lambda}{1+\lambda}} \sim \frac{g}{\lambda^2} << g$$

McMillan formula, 60th



We argue:

T_c^{mm} is the true superconducting Tc

T_c^{ad} is the onset of the pseudogap

Where do these two temperatures come from?

- Matsubara technique (OK for the calculations of T_{ins})

Pairing interaction: $$\chi(\omega) = -\frac{g^2}{\omega_m^2 + \omega_D^2}$$

$\omega < \omega_D$, Fermi liquid

$$\chi(\omega) \approx -\frac{g^2}{\omega_D^2} = -\lambda$$

$$\Sigma(\omega_m) = -i\int_0^{\omega_m} \chi(\Omega)\, d\Omega = i\lambda\omega_m, \quad i\omega_m + \Sigma(\omega_m) = i\omega_m(1+\lambda)$$

interaction

mass renormalization

BCS theory, pairing kernel is $$\frac{1}{\omega}\left[\frac{\lambda}{1+\lambda}\right]$$

If we stop here, we obtain $$T_c^{mm} \sim \omega_D\, e^{-\frac{\lambda}{1+\lambda}} \sim \frac{g}{\lambda^2} << g$$

only FL regime contributes

Where do these two temperatures come from?

- Matsubara technique (OK for the calculations of T_{ins})

Pairing interaction: $\chi(\omega) = -\frac{g^2}{\omega_m^2 + \omega_D^2}$

$\omega > \omega_D$, Non-Fermi liquid

$$\chi(\omega) = -\frac{g^2}{\omega_m^2} = -\lambda \frac{\omega_D^2}{\omega_m^2}$$

$$\Sigma(\omega_m) = -i \int_{\omega_D}^{\omega_m} \chi(\Omega)\, d\Omega = i\frac{g^2}{\omega_D} - i\frac{g^2}{\omega_m}$$

Pairing kernel is $\frac{1}{\omega_m^2}\omega_m = \frac{1}{\omega_m}$ up to $\omega \sim g$

numerical computation: $T_c^{ad} = 0.1827\, g$

non FL regime contributes

Same story for the spin-fermion model: two scales

$\overline{\omega}$ a residual coupling between electrons and collective spin modes

$$\omega_{sf} = \frac{\overline{\omega}}{4\lambda^2} << \overline{\omega}$$

a spin relaxation frequency

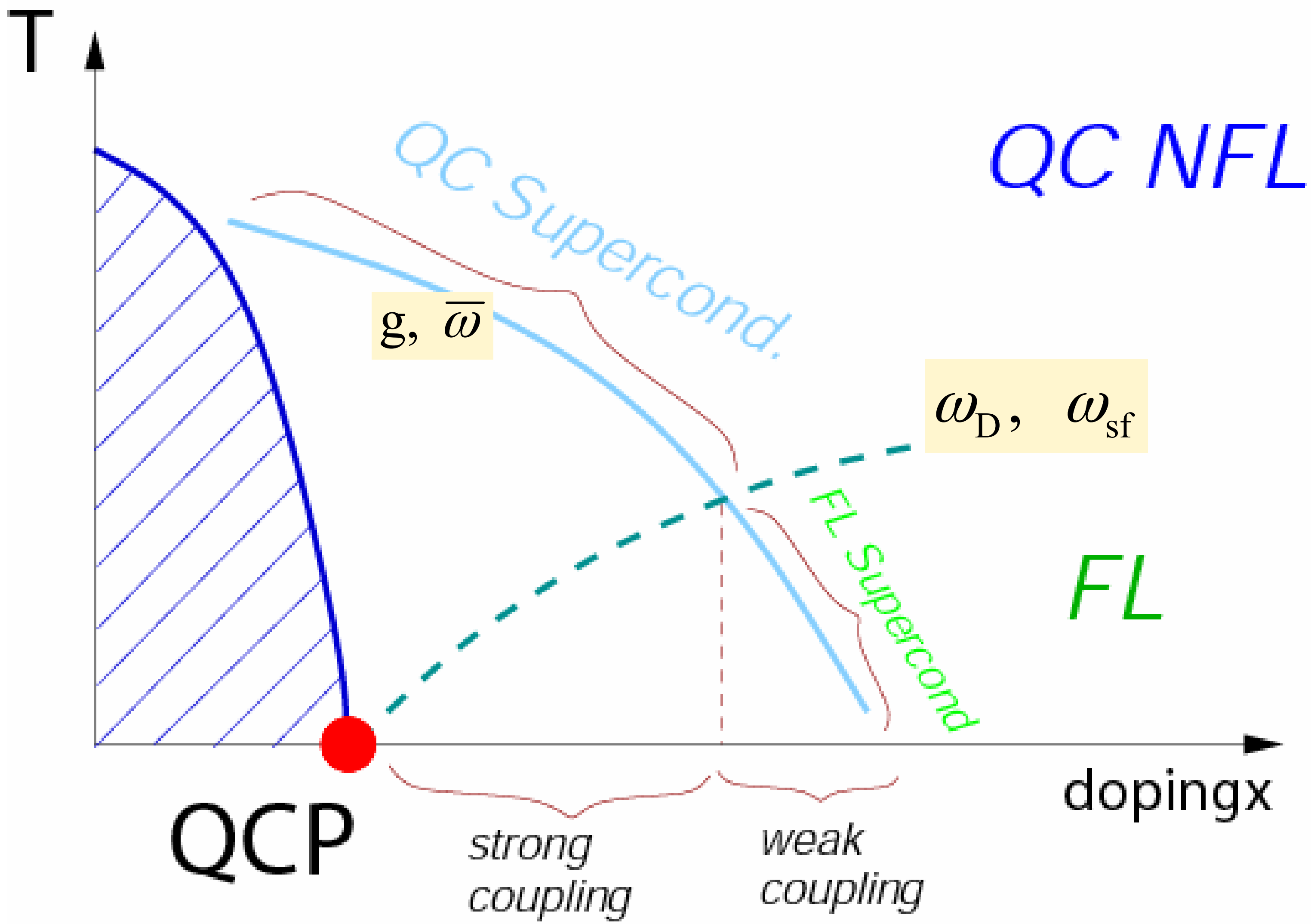
$\omega < \omega_{sf}$ $\Sigma(\omega) \approx \lambda\,\omega$ Fermi liquid

$\overline{\omega} > \omega > \omega_{sf}$ $\Sigma(\omega) \sim (\omega\,\overline{\omega})^{1/2} > \omega\,,\quad \chi(\omega) \propto 1/\sqrt{\omega}$ Quantum-critical, Non-Fermi liquid behavior

Pairing kernel: $\frac{1}{\sqrt{\omega}}\,\frac{1}{\sqrt{\omega}} = \frac{1}{\omega}$

$\omega > \overline{\omega}$ $\Sigma(\omega) \sim (\omega\,\overline{\omega})^{1/2} < \omega$ Fermi gas behavior

Dome of a pairing instability above QCP



Where do these two temperatures come from? (cont'd)

- Real frequencies

Pairing interaction: $\chi(\omega) = \dfrac{g^2}{\omega^2 - \omega_D^2}$

$\omega < \omega_D$, Fermi liquid

$T_c^{mm} \sim \omega_D\, e^{-\frac{\lambda}{1+\lambda}} \sim \dfrac{g}{\lambda^2} << g$

$\chi(\omega) \approx -\dfrac{g^2}{\omega_D^2} = -\lambda$ attraction

$\omega > \omega_D$, Non-Fermi liquid

$T_c^{ad} = 0.1827\, g$

$\chi(\omega) \approx +\dfrac{g^2}{\omega^2}$ repulsion

Extends to all frequencies when Debye frequency vanishes

What is the origin of T_c^{ad} ?

$$\chi(\omega) = \chi'(\omega) + i\,\chi''(\omega)$$

$$P\left(\frac{1}{\omega^2 - \omega_D^2}\right)$$

$$-i\,\frac{\pi}{2\,\omega_D}\left[\delta(\omega - \omega_D) - \delta(\omega + \omega_D)\right] \Rightarrow \pi\,\delta'(\omega)$$

- at weak coupling, $\omega > \omega_D$ are irrelevant, pairing is due to the exchange of virtual bosons

- at infinite coupling, the pairing is due to the exchange of real, on-shell bosons

Gap equation in real frequencies for vanishing Debye frequency

$$D(\omega) = \frac{\Delta(\omega)}{\omega}$$

$$D(\omega)\,(\omega + B(\omega)) = A(\omega) - i\,\frac{\pi}{2}\,\frac{d\,D(\omega)/d\omega}{\sqrt{1 - D^2(\omega)}}$$

(set g=1)

$$B(\omega) = \frac{\omega}{|\omega|}\int_0^\infty \frac{dx}{(|\omega| + x)^2}\,\mathrm{Re}\left[\frac{1}{\sqrt{1 - D^2(x)}}\right]$$

$$A(\omega) = \int_0^\infty \frac{dx}{(|\omega| + x)^2}\,\mathrm{Re}\left[\frac{D(x)}{\sqrt{1 - D^2(x)}}\right]$$

comes from the imaginary part of the interaction

come from the real (repulsive) part of the interaction

gap equation becomes differential!

Approximate solution:

neglect contributions from the repulsive part of $\chi(\omega)$

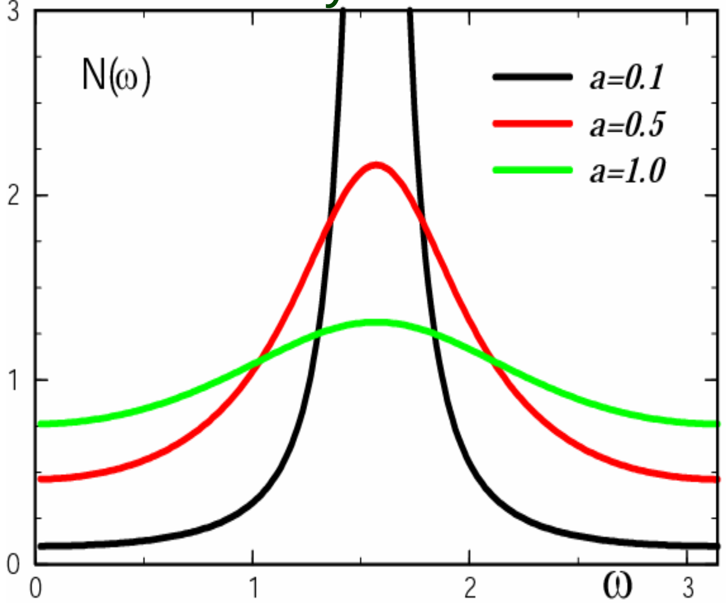
$$\Delta(\omega) = \frac{\omega}{\sin\left(\frac{2}{\pi}\omega + i\,a\right)}$$

a is a free parameter (zero mode)

a=0, $\Delta(0) \neq 0$ (Combescot, 95)

a >0, $\Delta(\omega << 1) \propto i\,\omega$ gapless sc

Density of states



- infinite degeneracy of the ground state
- DOS is finite down to zero frequency
- position of the maximum in DOS is the same for all solutions
- a linearized gap equation has a solution

at T=0 $a \Rightarrow \infty,\ \Delta(\omega) \sim e^{-a}\ \left(\omega\, e^{2i\,\omega/\pi}\right)$

Exact solution, $a \Rightarrow \infty$

$$-i\frac{\pi}{2}\frac{d\,D(\omega)}{d\,\omega} = D(\omega)\left(\omega + \frac{1}{\omega}\right) + \int_0^{\infty}\frac{d\,\omega'}{(|\omega| + \omega')^2}\,\mathrm{Re}\left[D(\omega')\right]$$

- Solution:

$$D(\omega) = 2i\,e^{-a}\,\sin\left[\beta\left(\log[-(\omega+i0)^2]+\omega^2\right)\right], \qquad \pi\beta\tanh\left[\pi\beta\right] = 1$$

(KK is satisfied)

A linearized gap equation has a solution at T=0

- Expansion in e^{-2a} -- we found the solution to order e^{-3a}

What this infinite set of solutions physically means?

Order parameter in a superconductor:

$$i F_{\alpha\beta}(t_1 - t_2) = < N+2 | T \psi_\alpha^+ \psi_\beta^+ | N >$$

Time dependence

$$F \propto \exp\left[-i\, t \left(E_{mf}(N+2) - E_{m_i}(N) - 2\mu\right)\right]$$

final initial

If m_f and m_i is the same state, $\frac{\partial E}{\partial N} = 2\mu,$ and F (0) is finite

Order parameter in a superconfuctor is F(0) (equal time correlator)

$$F(0) = |F| \exp\left[i\,\theta\right]$$

What this infinite set of solutions physically means?

- conventional (equal time) interaction is repulsive (and irrelevant)

$$\Sigma(\omega) = \frac{i\pi}{2\,\omega_D} e^{-i\,\omega_D\, t} \approx i\,(\mathrm{const}) + \frac{\pi}{2}\, t$$

from on-shell bosons

interaction increases with t

- at t=0, our interaction vanishes (pure retardation)

For all our solutions, superconducting order parameter vanishes

Physics (cont'd)

A conventional sc

a single ground state
with O(N) particles

Our case, ${}_{\mathrm{D}} = 0$

a=3 a=2 a=1 a=0

An infinite set of degenerate states
with O(1) particles per state

Our case, $\omega_{\mathrm{D}} \neq 0$

a =3 a=2 a=1 a=0

now contains O(N) particles

soft longitudinal fluctuations

Stiffness at T=0

- When longutudinal gap fluctuations are not soft

$$\rho_s \sim E_F\,\omega_D/g \;>>\; T_c^{ad}$$

$$<\theta^2> \sim \frac{T}{T_c^{ad}}\left(\frac{g^2}{E_F\,\omega_D}\right) << \frac{T}{T_c^{ad}}$$

phase Migdal parameter

- When longutudinal gap fluctuations are soft

$$\rho_s \sim E_F\, e^{-2a},\;\; F = \sum_q q^2\,\theta_q^2\, e^{-2a} + \frac{g^2}{E_F}\,\frac{\omega_D^2}{\omega_D^2 + e^{-2a}}$$

soft longitudinal fluctuations

$$<\theta^2> \sim \frac{T}{T_c^{ad}}\left(\frac{g^2}{E_F\,\omega_D}\right) * \underline{\left(\frac{T\,E_F}{g\,\omega_D}\right)} \sim \left(\frac{T}{\omega_D}\right)^2$$

$<\theta^2> \sim 1$

at $T \sim \omega_D \sim T_c^{mm}$

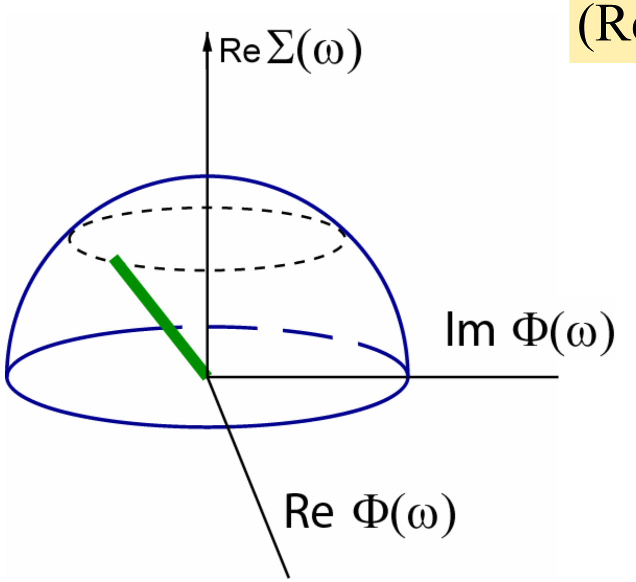
Another way to look at zero mode

order parameter symmetry is enhanced at QCP

(a balance between normal and anomalous self-energies)

$$(\mathrm{Re}\,\Sigma)^2\,(\omega) + (\mathrm{Re}\,\Phi\,(\omega))^2 + (\mathrm{Im}\,\Phi\,(\omega))^2 = \mathrm{const}$$

$$\Phi\,(\omega) = \Delta\,(\omega)\,Z\,(\omega)$$



P2 instead of O(2)

soft longitudinal gap fluctuations at QCP

Conclusions

- Two energy scales in fermionic systems near QCP

  upper edge of Fermi liquid behavior ω_D

  upper edge of quantum-critical behavior g

- Two characteristic temperatures

  $T_c^{mm} \sim \omega_D \, e^{-\frac{\lambda}{1+\lambda}} \sim \frac{g}{\lambda^2} << g$ $\quad$ $T_c^{ad} \sim \omega_D \sqrt{\lambda} \sim g$

- Gap appears at $T \sim T_c^{ad}$

- Superconductivity in a Fermi liquid regime

- Soft longitudinal fluctuations $<\theta^2> \sim 1 \;\text{ at }\; T \sim T_c^{mm}$