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ICTP 40th Anniversary

SMR.1572 - 10

Workshop on
Novel States and Phase Transitions in Highly Correlated Matter

12 - 23 July 2004

Quantum criticality: wide open field

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Quantum Criticality: Wide open field

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Trieste, July 12th 2004.

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Rutgers Center for Materials Theory

N. Andrei, Rutgers CMT

C. Pépin, CEA, Saclay
I. Paul, CEA, Saclay.

J. Hopkinson, Sherbrooke

R. Ramazashvili, Argonne

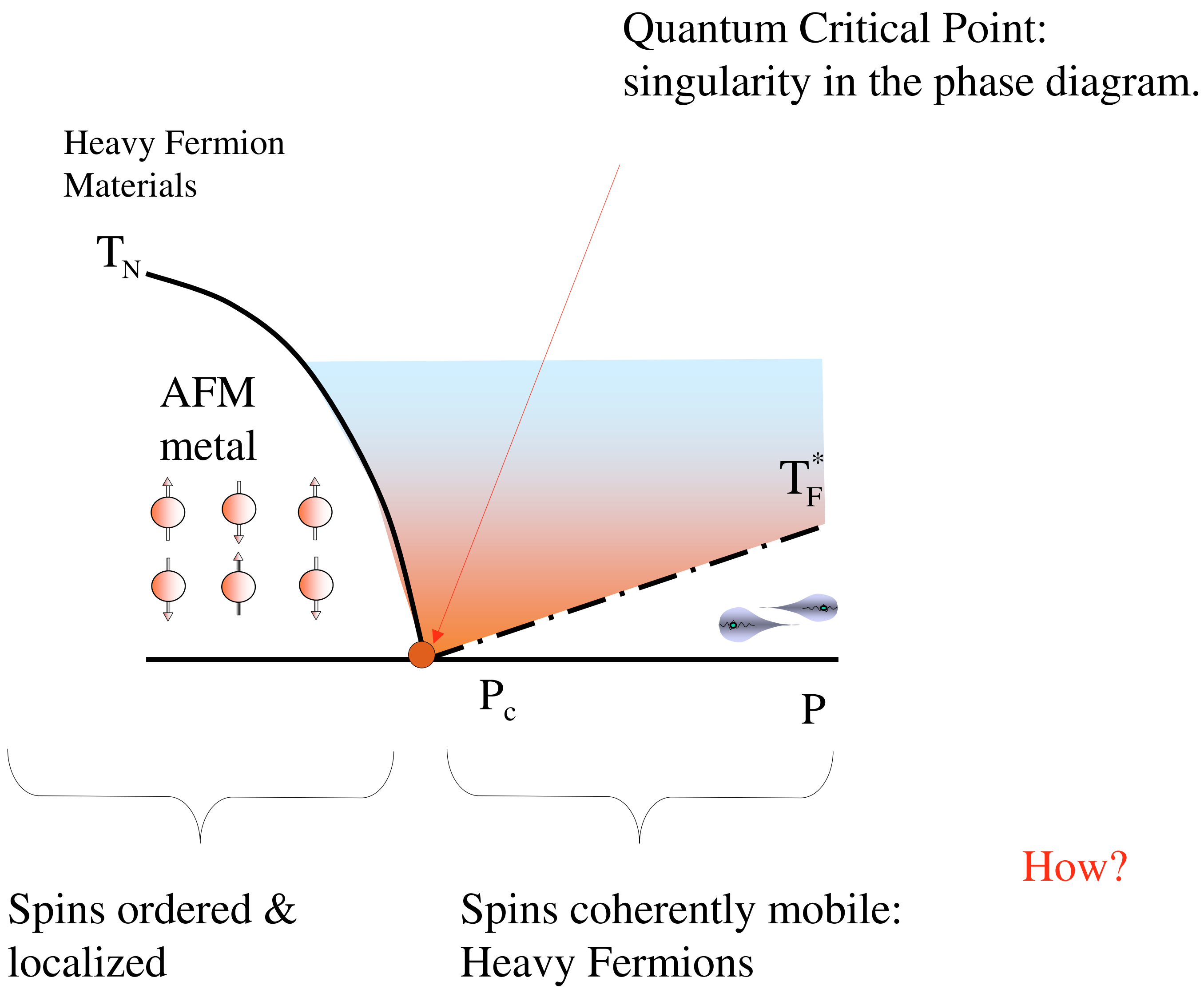
Q. Si, Rice

G. Zarand, Budapest

S. Paschen, J. Custers, P. Gegenwart, C. Geibel, H. Wilhelm and F. Steglich, MPICPS, Dresden

G. Aeppli (UCL), A. Schroeder (Kent State)

Quantum Criticality



Questions for discussion

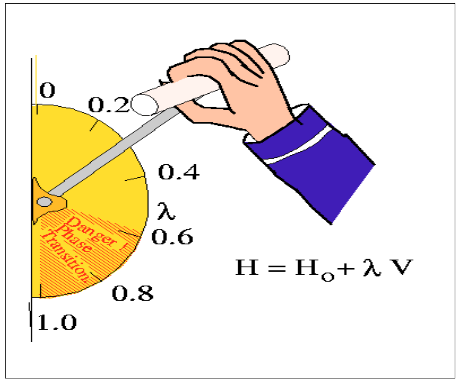
- What Landau Theory describes a heavy electron quantum critical point – and is this a good starting point?
- Why does the Fermi Temperature appear to go to zero when Neel order develops?
- Why is there scaling at heavy electron QCP?
- What happens to the Fermi surface at a QCP?
- What is the relationship of superconductivity to a QCP?

- Quantum Criticality: singularity in the phase diagram.

- Key Properties

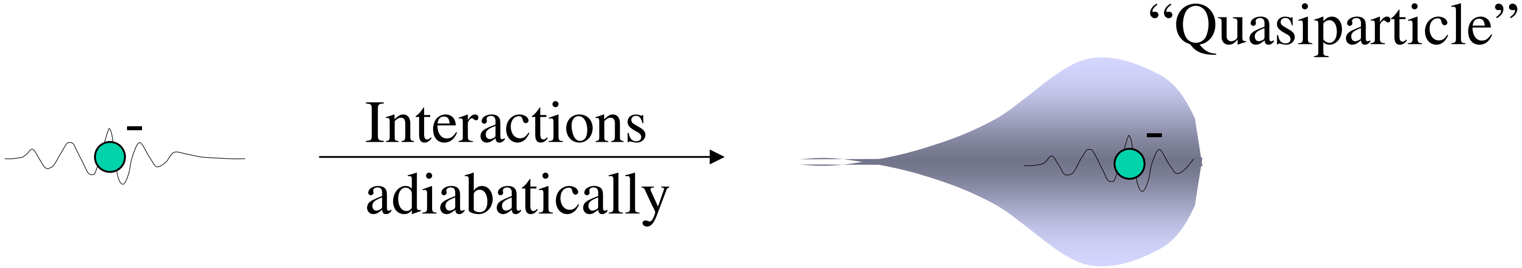
- Open questions.

Singularity in the phase diagram



Landau: interactions can be turned on adiabatically, preserving the excitation spectrum.

Landau, JETP 3, 920 (1957)

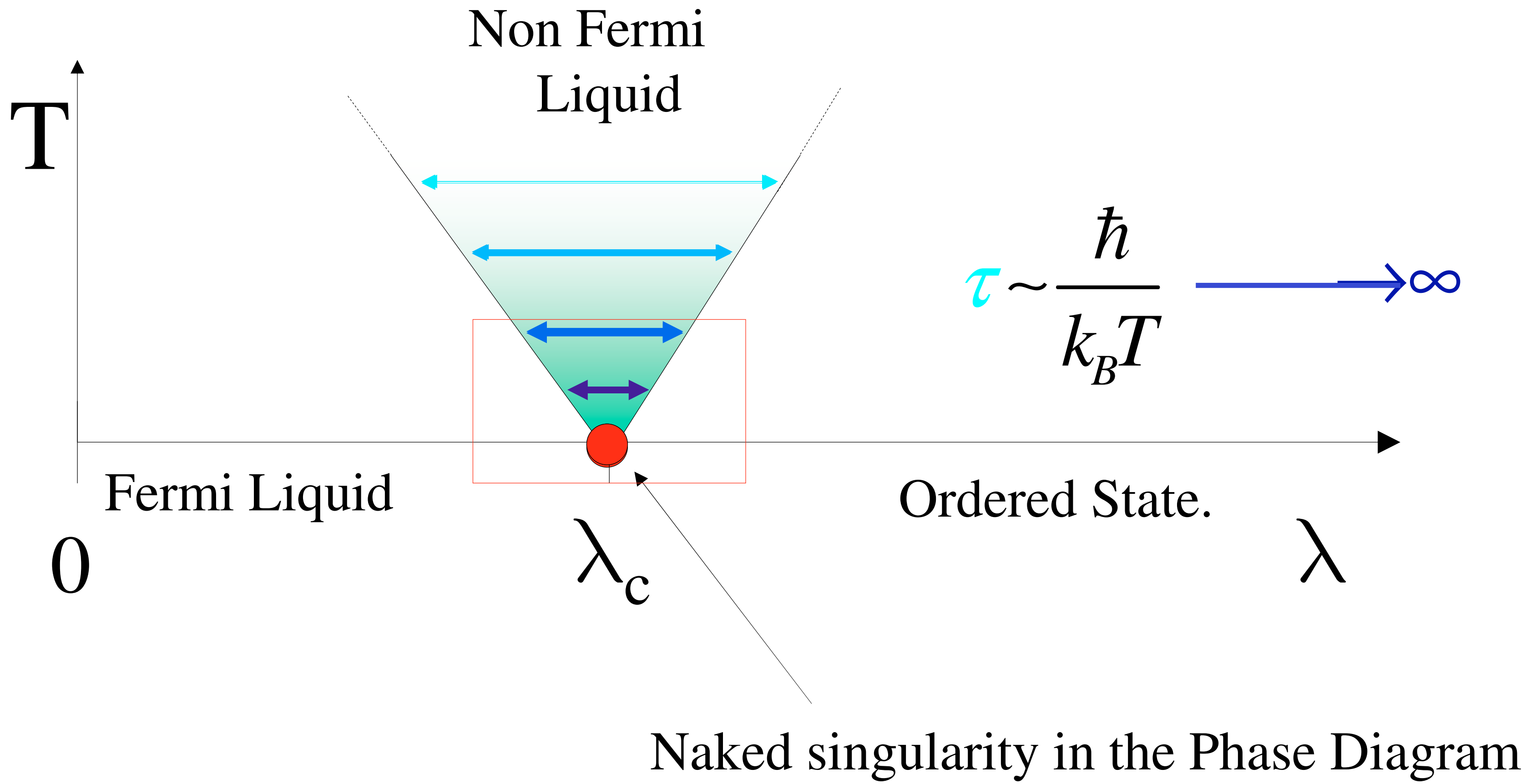


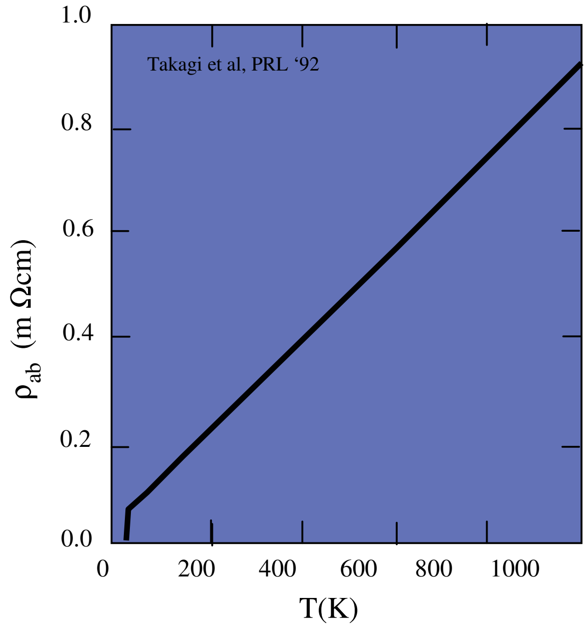
$$|e^-\rangle \qquad |qp^-\rangle$$

$$\frac{m^*}{m} = \frac{N(0)^*}{N(0)} = 1 + \frac{F_1^s}{3}$$

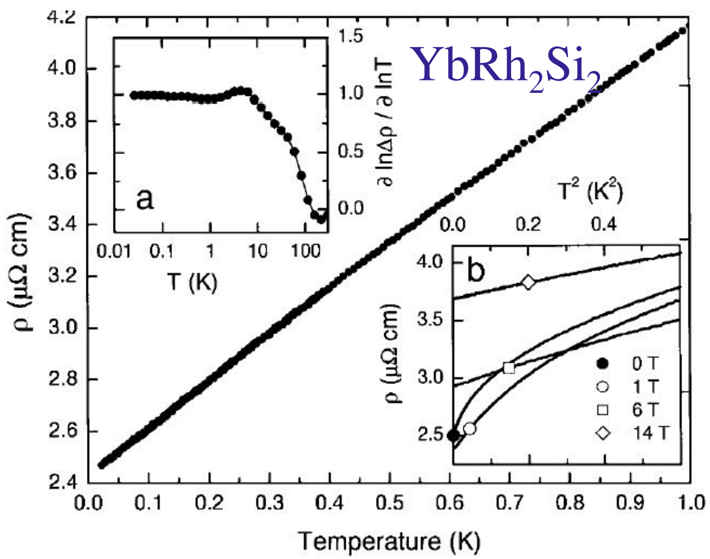
When the interaction becomes too large….

…what happens?



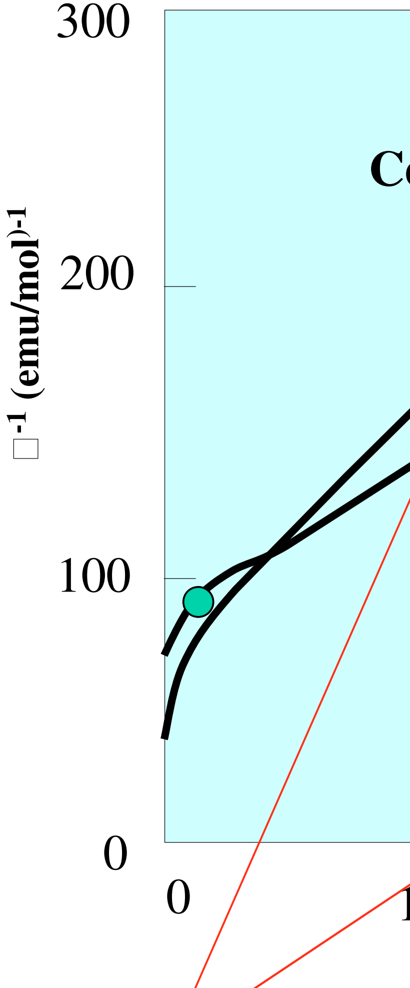


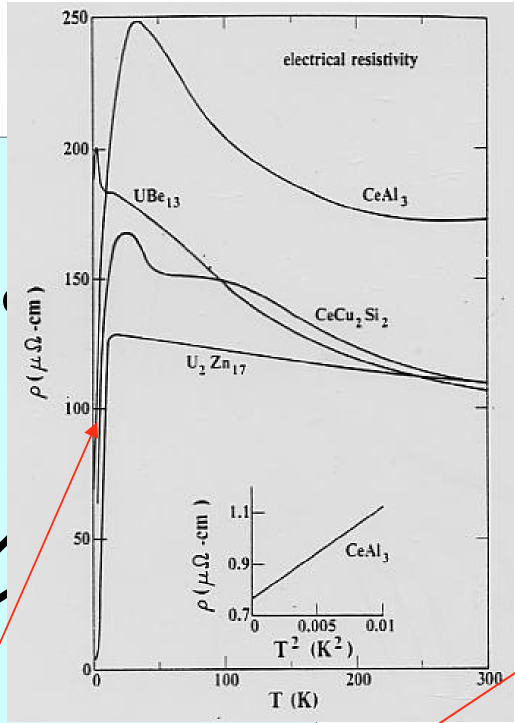
Anomalous normal state of cuprates

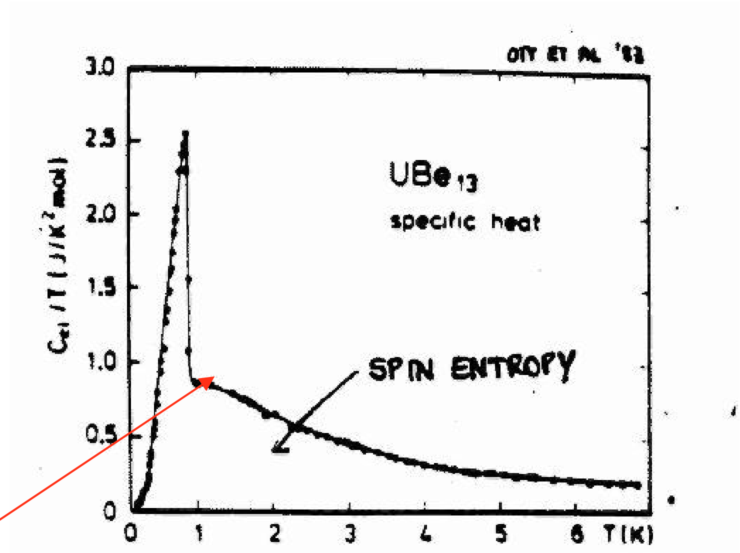


Key Observations

Heavy Fermions

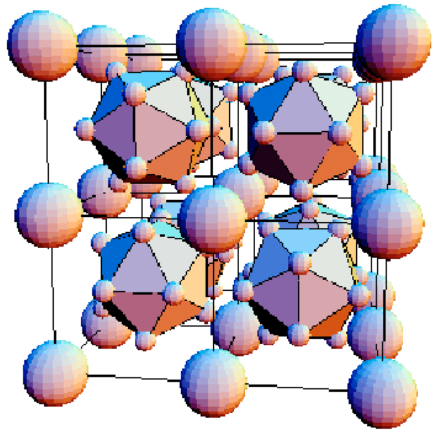






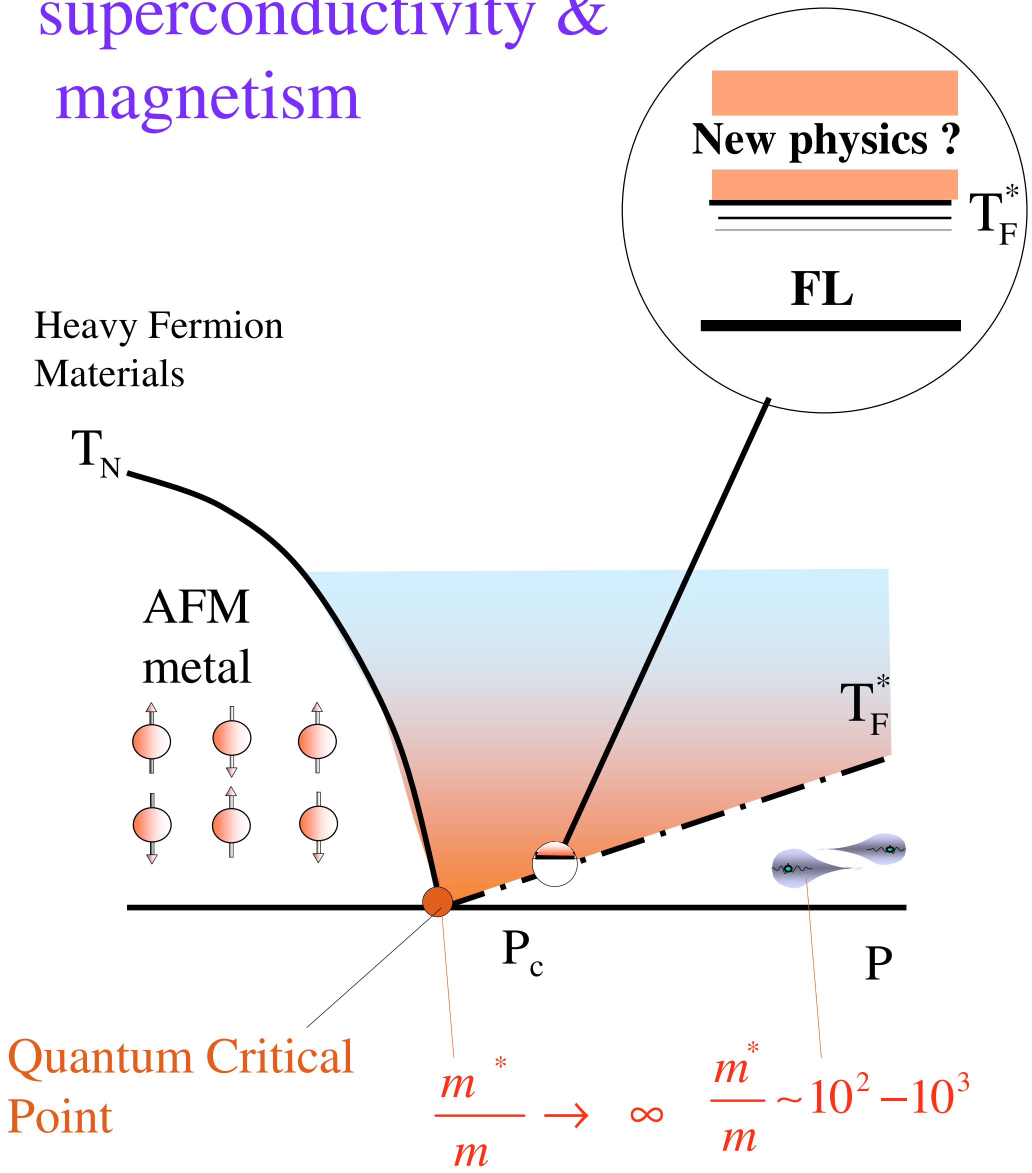
Heavy Electrons
m*/m ~ 100-1000

Extreme limit of
Fermi liquid behavior



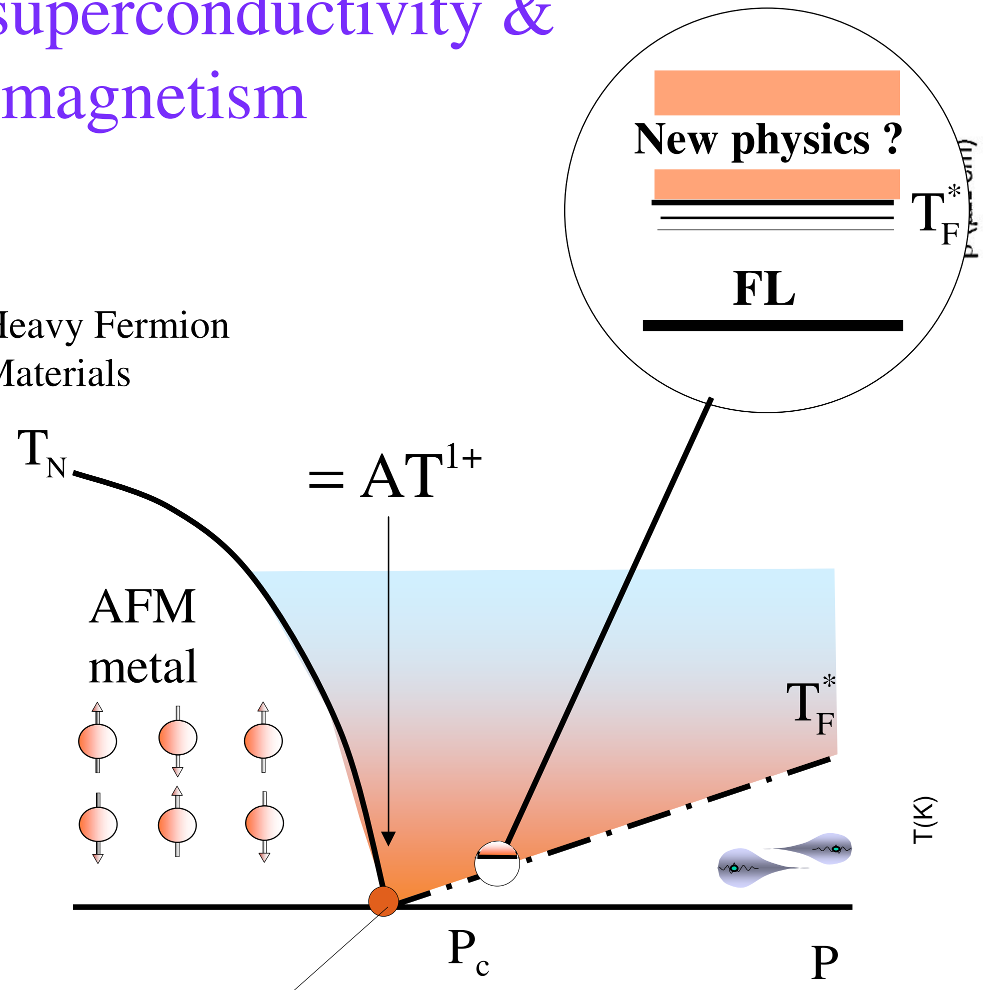
UBe_{13}

Quantum Criticality: superconductivity & magnetism

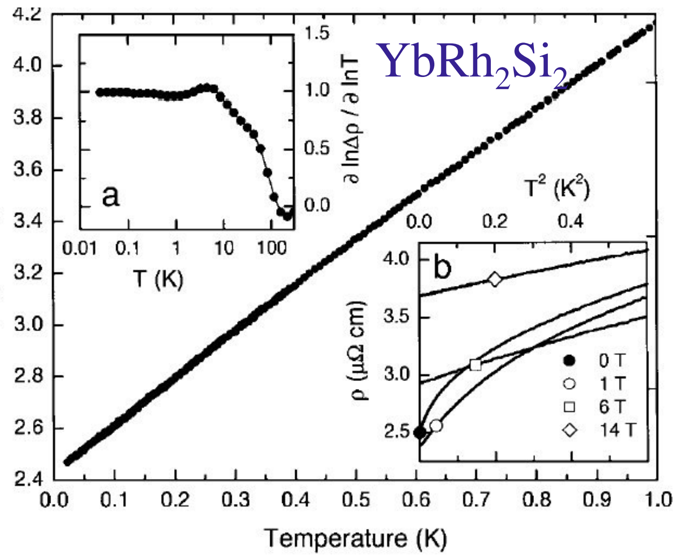


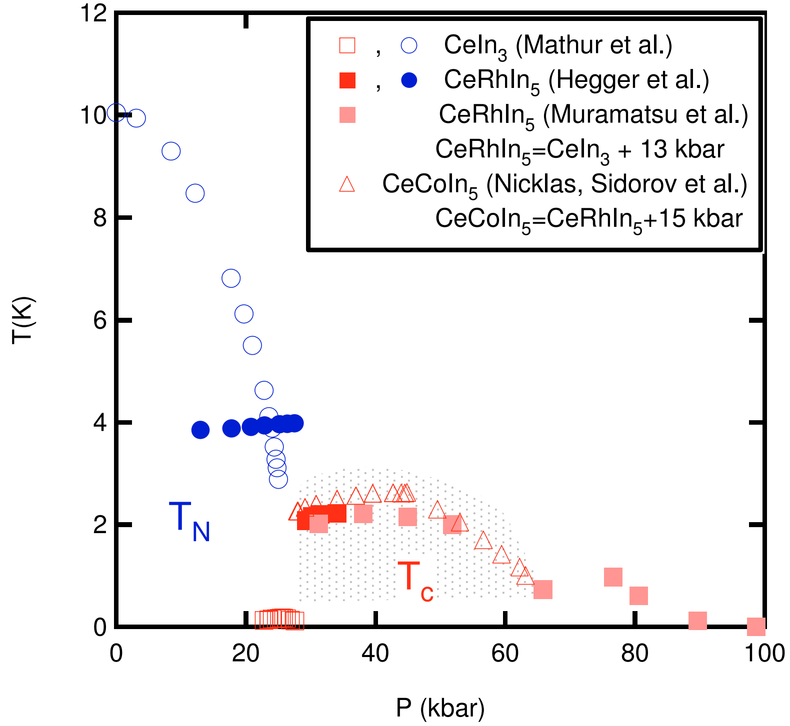
Quantum Criticality: superconductivity & magnetism

Heavy Fermion Materials

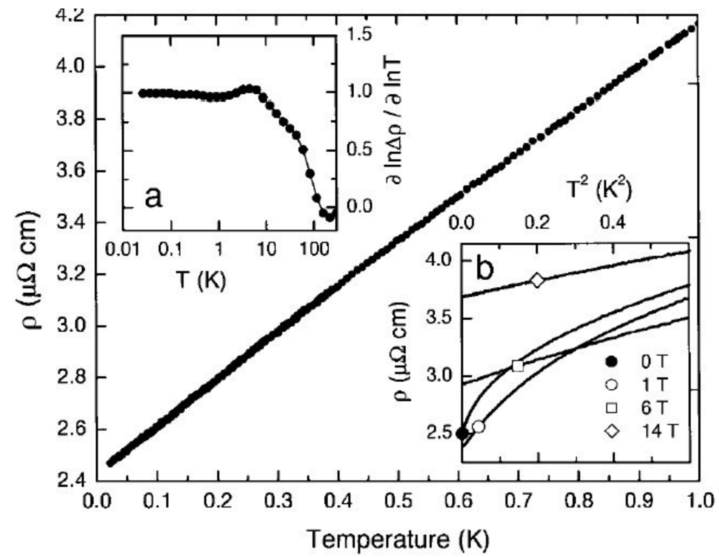


Quantum Critical Point





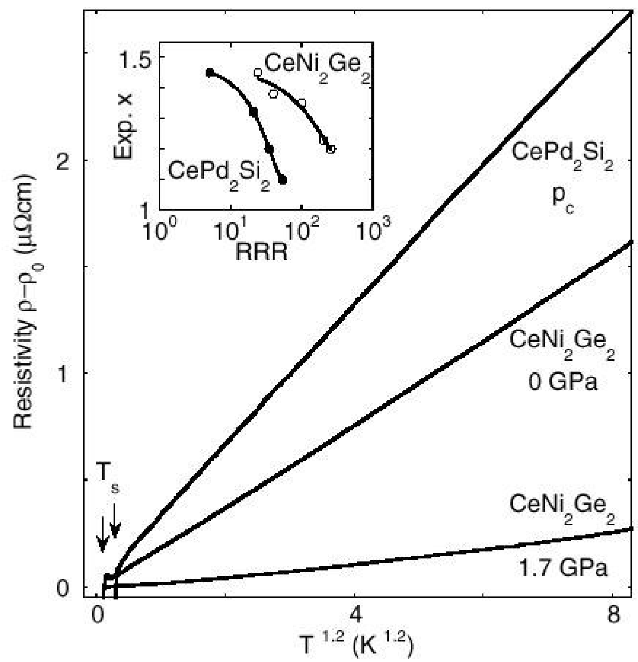
Quasi-linear
Resistivity.



Trovarelli et al (2000).

YbRh$_2$Si$_2$

$$\rho \propto T^{1+\varepsilon}$$



Grosche et al, (2000).

CeNi$_2$Ge$_2$

CePd$_2$Si$_2$

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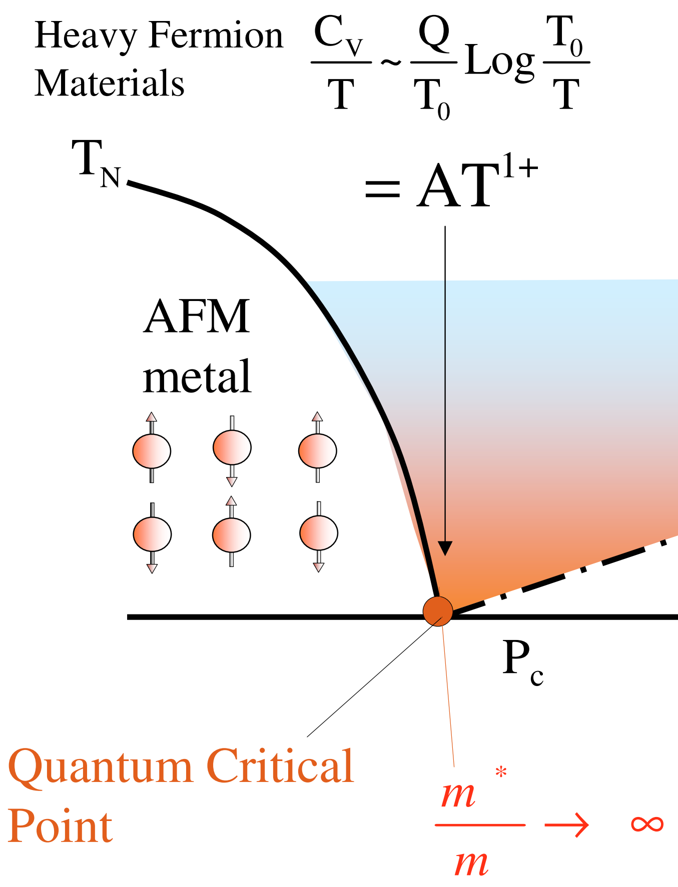
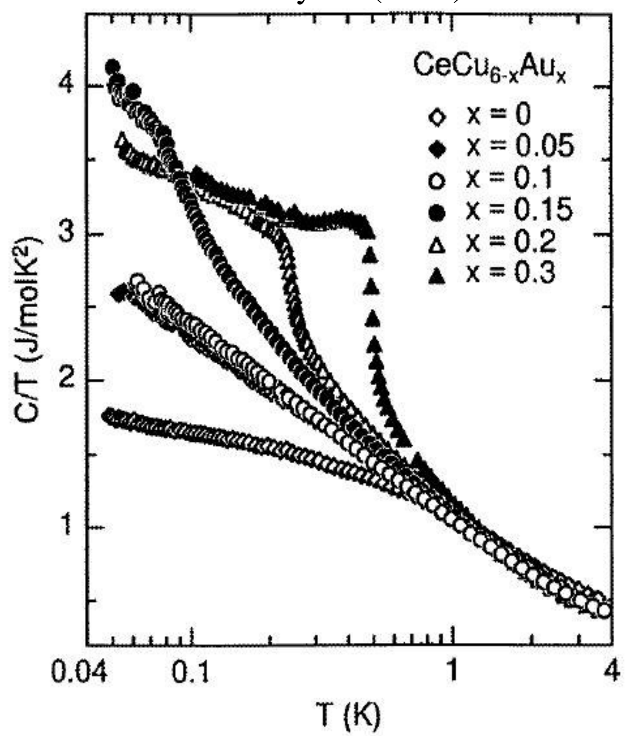
Quantum Criticality: divergent specific heat capacity



H. Von Lohneyson (1996)



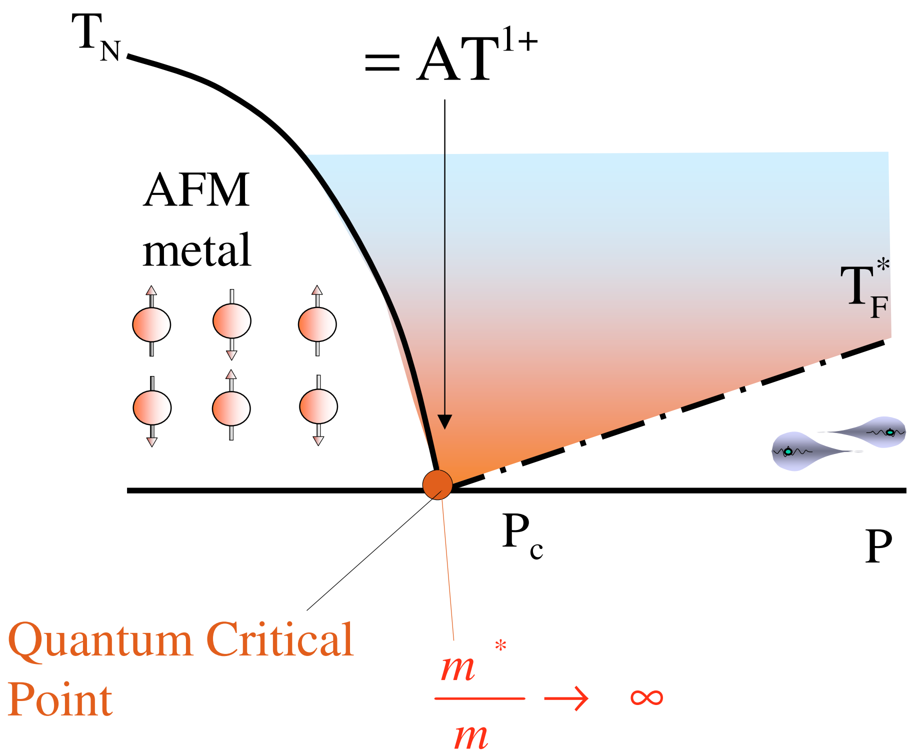
Quantum Criticality:
E/T scaling

Temperature the only scale.

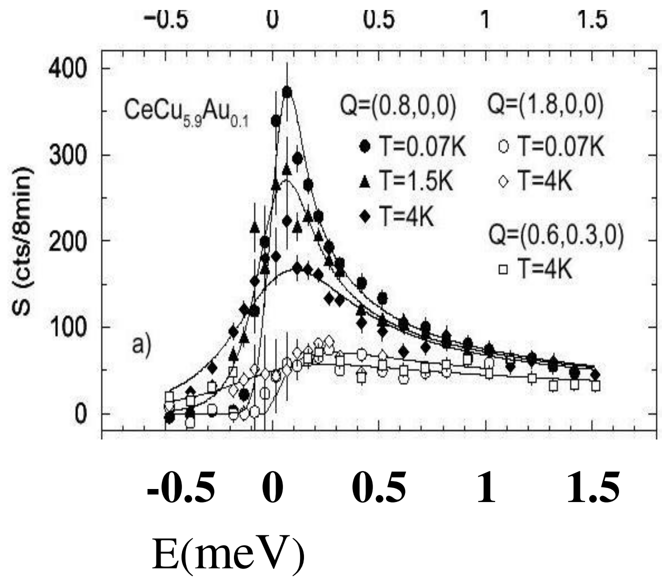
E/T Scaling

Heavy Fermion Materials $\frac{C_V}{T} \sim \frac{Q}{T_0} \mathrm{Log} \frac{T_0}{T}$

$= AT^{1+}$

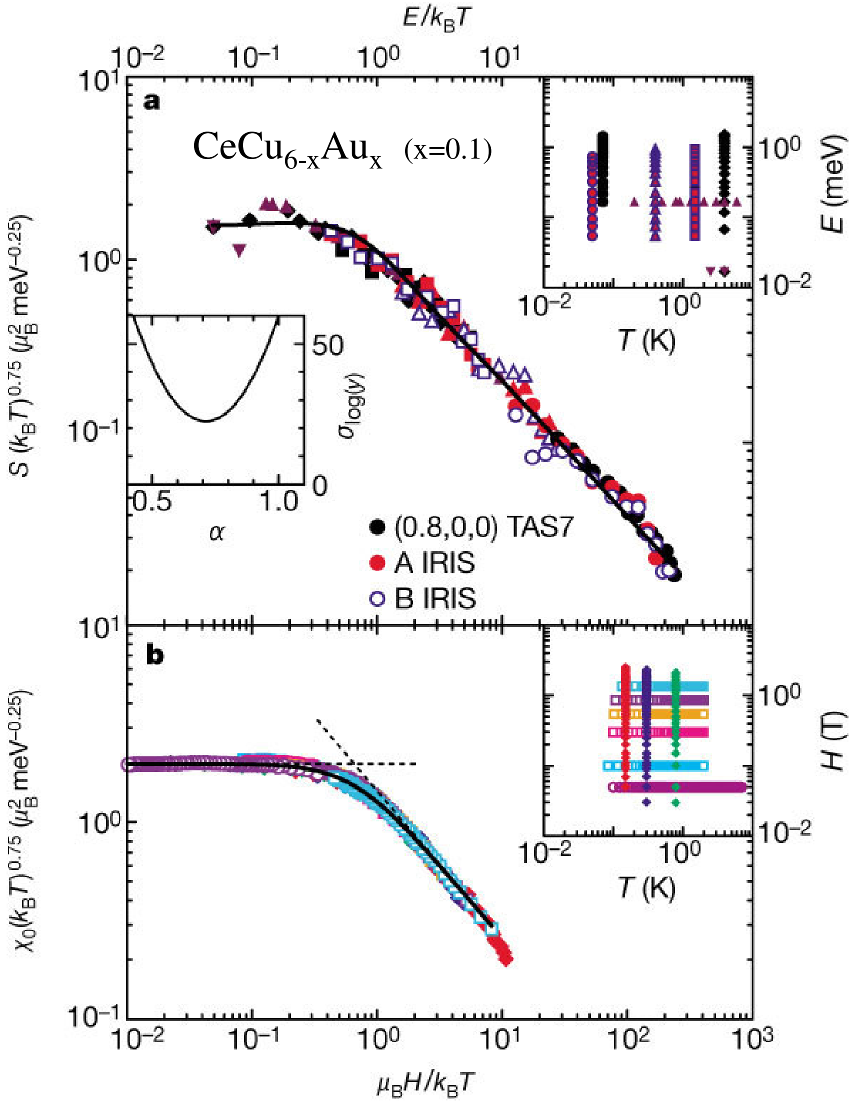


Schroeder et al , PRL (1999), see also
Aronson et al, PRL (1996).



E/T Scaling:

Schroeder et al, Nature 407,351 (2000).



$$\chi''(E) = \frac{1}{E^{1-\alpha}} G(\frac{E}{T})$$

Physics Below the upper Critical Dimension.

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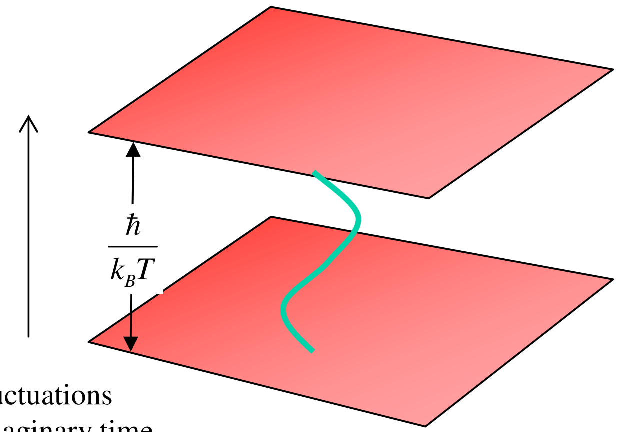
Temperature : finite size in imaginary time

(Hertz, 76)



[t] =[L]^z

$$d_{eff} = d + z$$

Quantum fluctuations
evolve in imaginary time

d+z < d_u

$$\tau^{-1} \propto \frac{k_B T}{\hbar}$$

$$\chi''(E) = \frac{1}{E^{1-\alpha}} G(\frac{E}{T})$$

α>0, E/T Scaling.
One energy scale- the temperature.

Sachdev and Ye, PRL 69, 2411 (92),
Sachdev, QPT, pp234 (Cambridge, 99)

d +z > d_u

$$\tau^{-1} \propto T^{d/2} U, \quad (z = 2)$$

$$\alpha = 0$$

“Gaussian fixed point” α =0.

T is not the only energy scale.

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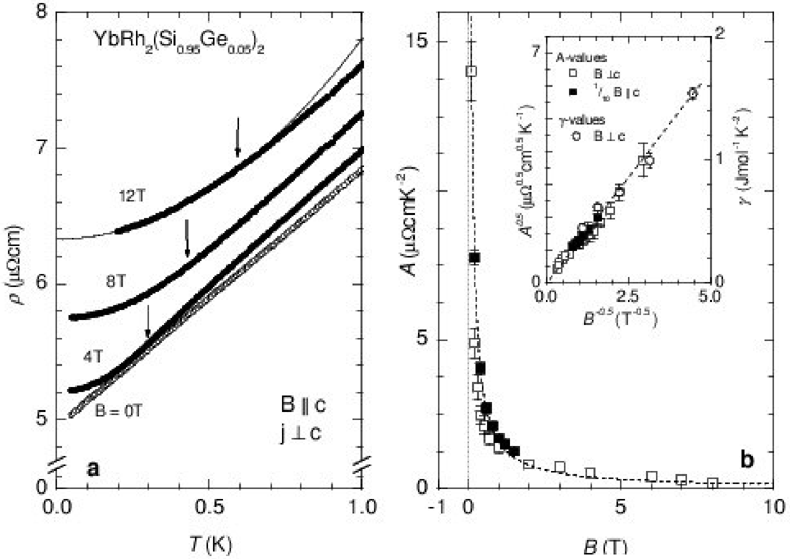
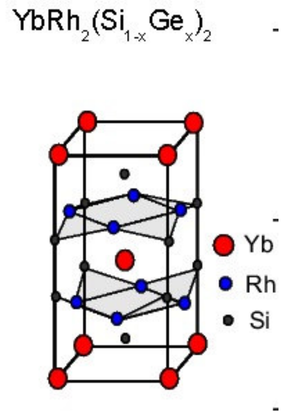
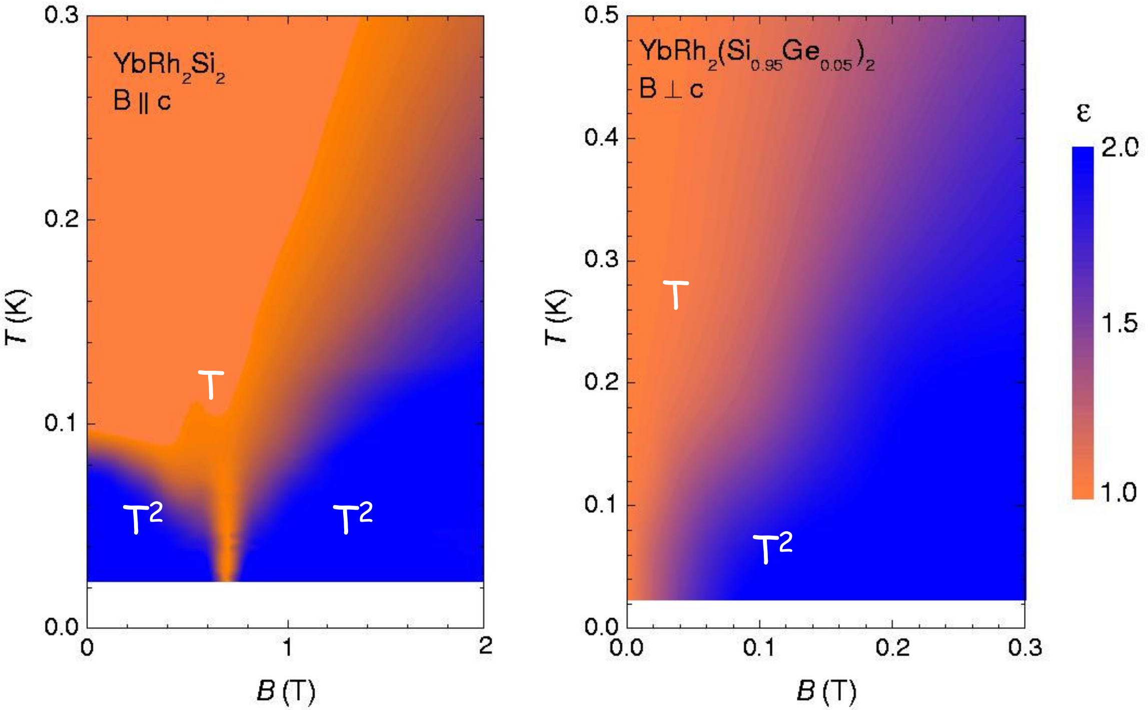


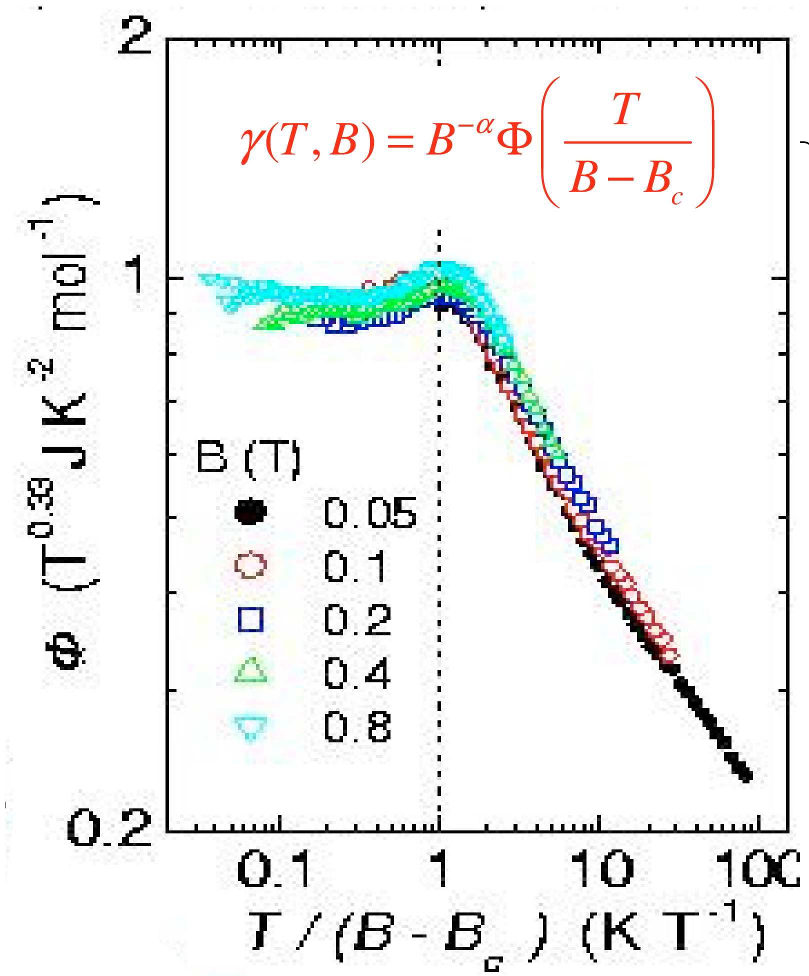
Gegenwart et al (2002)

$$\rho = AT^2 + \rho_0$$

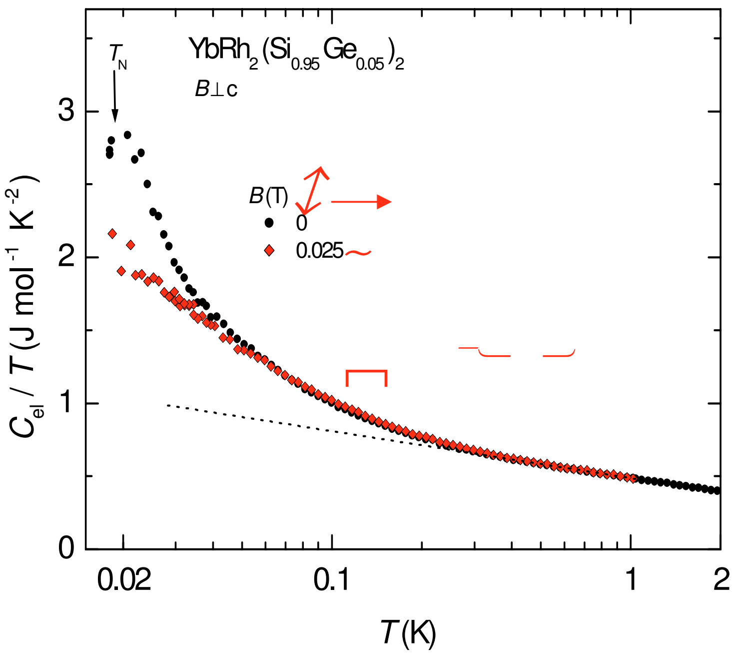
$$A \propto \frac{1}{T_F^{\,2}} \propto \frac{1}{B - B_c}$$



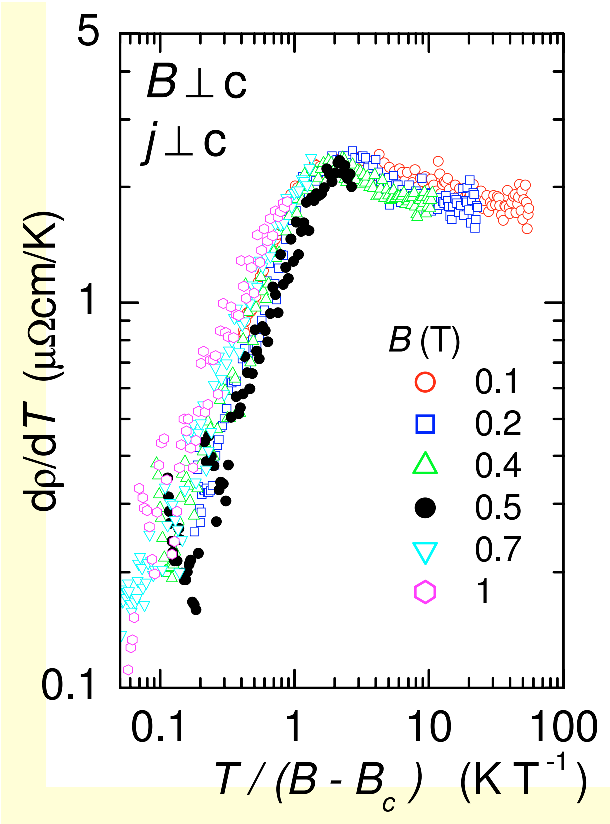


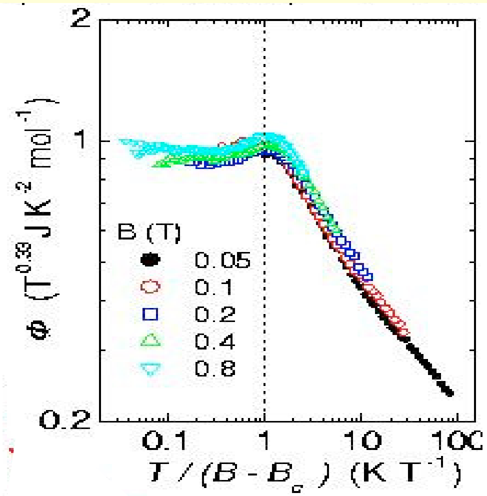


$\gamma(T,B) = B^{-\alpha}\Phi\left(\frac{T}{B-B_c}\right)$
Φ (T^0.33 J K^-2 mol^-1)
B (T)
0.05
0.1
0.2
0.4
0.8
2
1
0.2
0.1
1
10
100
T / (B - B_c) (K T^-1)



T_N
YbRh_2(Si_0.95Ge_0.05)_2
B⊥c
B(T)
0
0.025
C_el / T (J mol^-1 K^-2)
3
2
1
0
0.02
0.1
1
2
T (K)



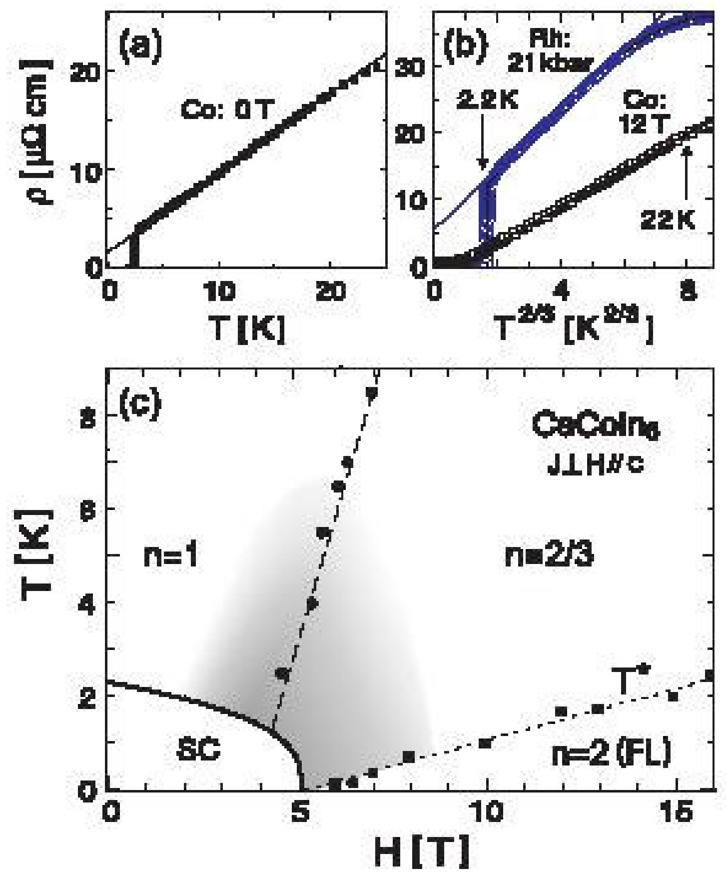


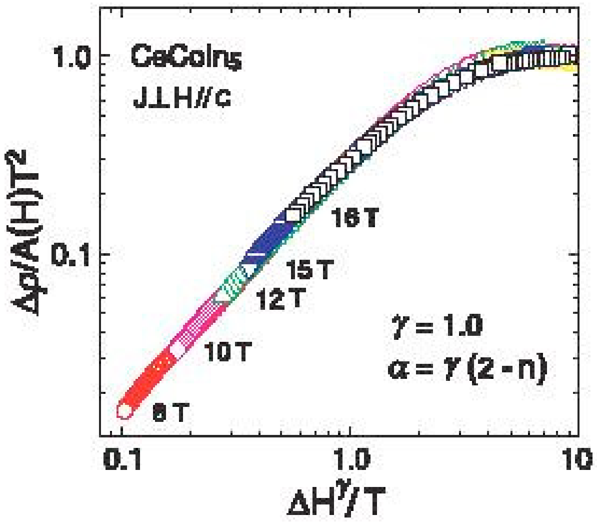
One scale, in transport and the thermodynamics…. going to zero at the QCP

$$\gamma(T,B) = B^{-\alpha}\Phi\left(\frac{T}{T_0(B)}\right)$$

$$\frac{d\rho}{dT} = \Theta\left(\frac{T}{T_0(B)}\right)$$

$$\alpha \approx 0.33, \quad T_0(B) \sim B - B_c.$$





$$\rho = T^n \Lambda\left(\frac{h^\gamma}{T}\right)$$

Jp Paglione et al, '04

Open questions

Questions for discussion

- What Landau Theory describes a heavy electron quantum critical point – and is this a good starting point?
- Why does the Fermi Temperature appear to go to zero when Neel order develops?
- Why is there scaling at heavy electron QCP?
- What happens to the Fermi surface at a QCP?
- What is the relationship of superconductivity to a QCP?

- What Landau Theory describes a heavy electron quantum critical point – and is this a good starting point?

Various points of view:

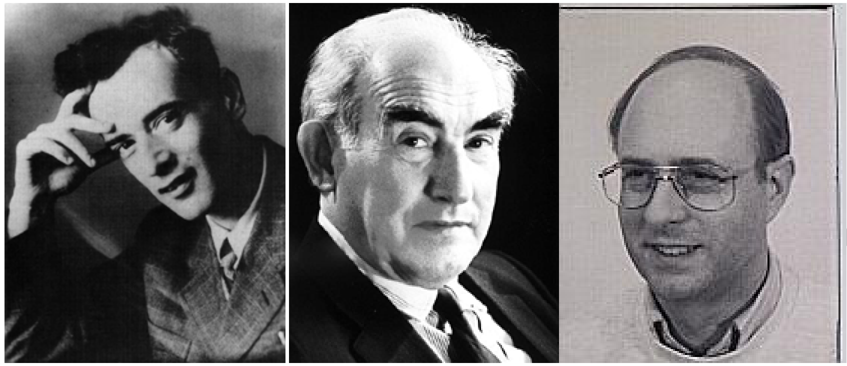
- Landau is good, but the devil is in the fermions.

- Dimensionality of the spin fluid is not three, but two or two/zero (Rosch et al, Ingersent, Si, Rabello)

- We need a new Landau Theory.

- Landau is irrelevant (Senthil et al.)

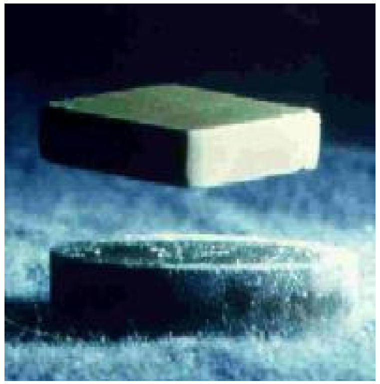
Why bring in Landau?

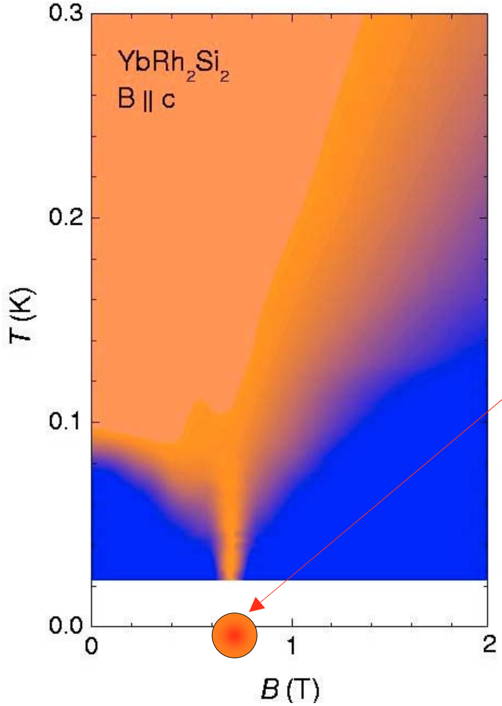


Landau & Ginzburg '51 Hertz '76

$$F = \int d\tau d^3x \left[(x_c - x)|\psi|^2 + b|\psi|^4 + |\nabla \psi|^2 + disspn \right]$$

Quantum Criticality



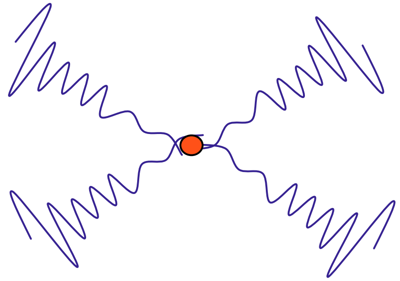


Quantum Critical Point

Standard Model

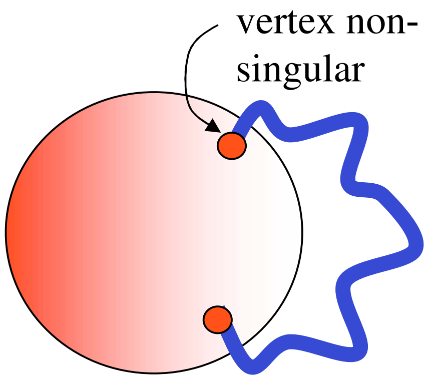
Standard Model: QSDW

- Moriya, Doniach,Schrieffer (60s)
- Hertz (76)
- Millis (93)



$$d_{eff} = d + z$$

If $d_{eff} > 4$, $\square^4$ terms "irrelevent"
Critical modes are <u>Gaussian</u>.



$$\chi_0^{-1} = -J(Q)$$

F.S. instability

$$\tau^{-1} \propto \xi^{-2} \Rightarrow z = 2$$

<u>NO E/T SCALING , NO MASS DIVERGENCE IN 3D</u>

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$$V_{eff}(\vec{q},\omega) = g^2 \frac{\chi_o}{(\vec{q}-\vec{Q})^2 - i\omega/\Gamma_Q}$$

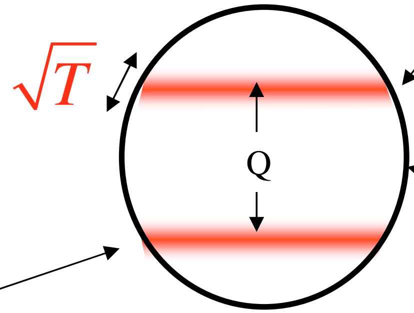
$$V_{eff}(\vec{r},\omega=0) \propto \frac{1}{r} e^{i\vec{Q}.\vec{r}}$$

Singular potential is rapidly modulated: only affects electrons along hot-lines.

$$\varepsilon_{k_F} = \varepsilon_{k_F+Q}$$

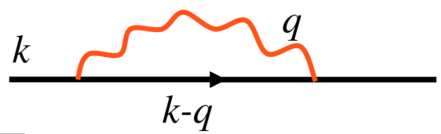
$$\frac{m^*}{m} \sim 1 - \frac{\partial\Sigma}{\partial\omega} \sim \ln(\Gamma/T)$$

divergent



$$\frac{m^*}{m}$$ finite

$$\Sigma(k,\omega) = -T\sum_{q,\nu} g^2 \chi_o(q,\nu) G(k-q,\omega-\nu)$$



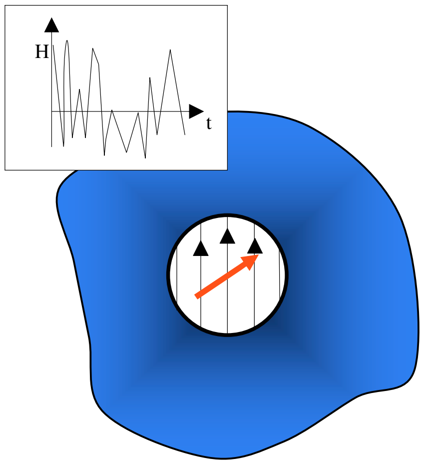
$$F_{Singular} \sim T\sum\int d^3q \log[\chi^{-1}(q,\omega)]$$
$$\sim T(T^{3/z}) \sim T^{5/2}$$

$$\frac{C_V}{T} \sim \gamma_o - A\sqrt{T}$$

What makes the mass diverge?

• **Frustrated spin layers.** (Rosch et al, 1998):
Decoupled layers of spin fluid in a 3D charge fluid.

• **Local quantum criticality** (Si et al, Nature 2001):
Spin is the critical mode.



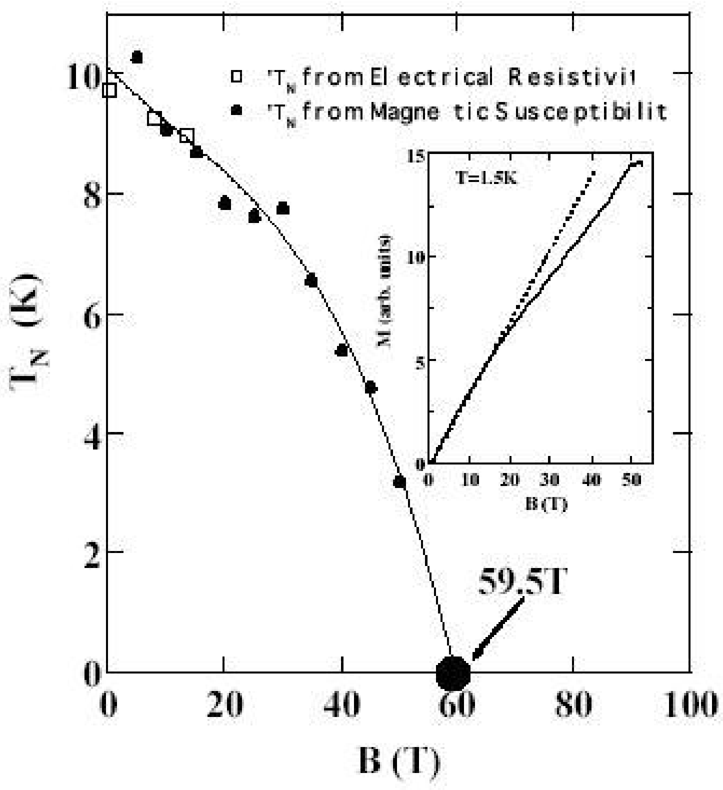
Requires a 2D spin fluid for a divergent local spin susceptibility

Qualitative departure from Wilson Kadanoff approach to criticality.

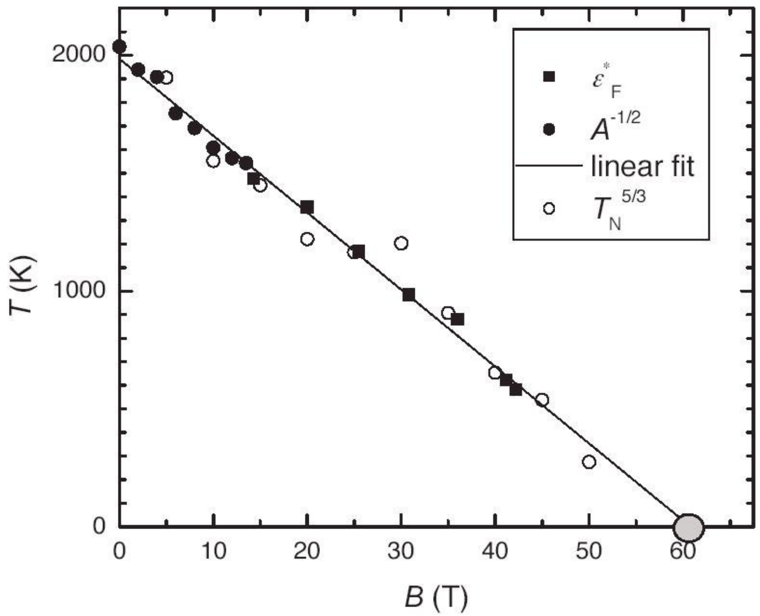
• **Spin Charge separation at QCP.** (PC, Pepin, 2001)

CeIn3: cubic, 3D QCP.

Ebihara, Harrison and Uji: (2004)



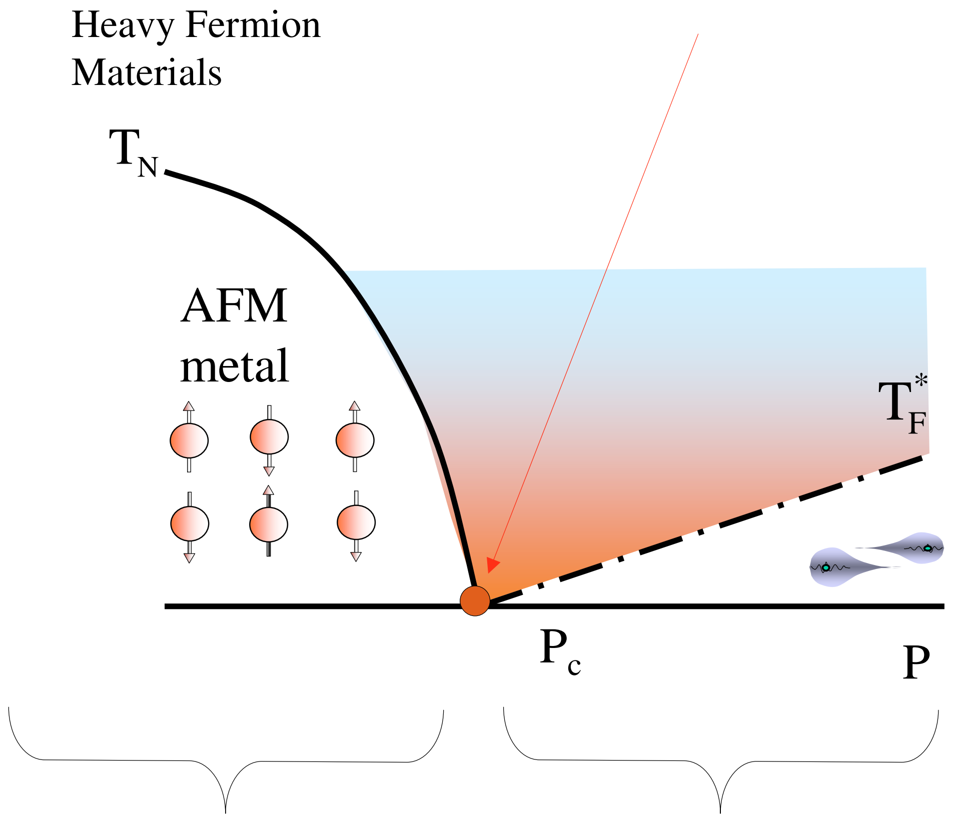
The mass divergence can not be associated with a 2D spin fluid!



$$H = \sum \varepsilon_k c_{k\sigma}^{\dagger} c_{k\sigma} + J \sum_j (\psi^{\dagger}{}_j \vec{\sigma}\, \psi_j) \bullet \vec{S}_j$$

Kondo Lattice Model
(Kasuya, 1951)

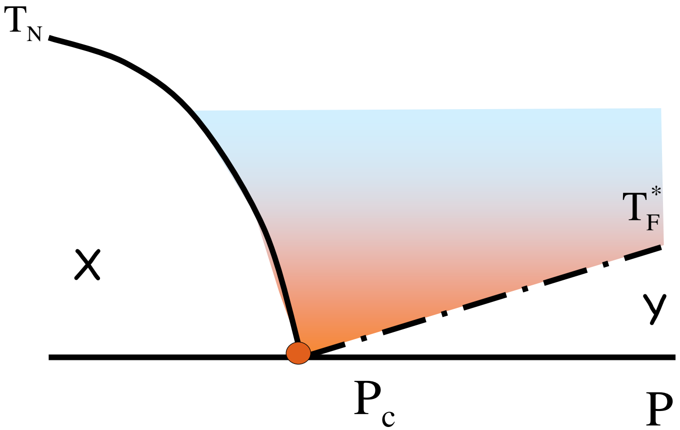
Deconfinement of spin



Spins behave as
localized moments

Spins bound to charge
as composite fermions

Search for a new Landau Theory.



$$x = (P - P_c)$$

$$F = x(X^2 - Y^2) + b(X^2 - Y^2)^2$$

Lets assume that it might be possible to produce a mean-field theory of the entire mean-field phase diagram.

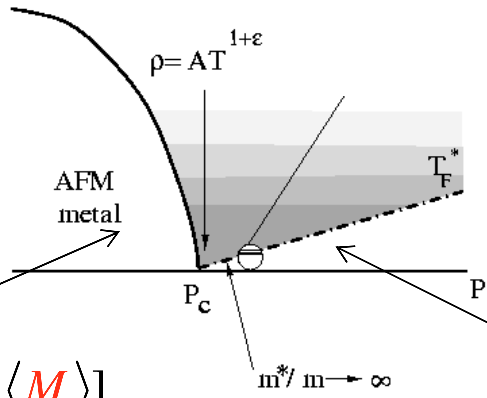
X- magnetic order
Y – Kondo order – c.f. the slave boson.

Mass of X becomes positive exactly at point where mass of Y becomes negative.

F is invariant under a non-compact group that preserves X^2-Y^2. This kind of Landau Theory appears in supersymmetric QED, where it is called a Fayet Illipolis action. In supersymmetric QED, it emerges because X and Y are the superpartners of two chiral fermions. Does this suggest a whole new class of Landau Ginzburg theories with underlying non-compact symmetries- the Stat Mech analog of the break-down of the Coleman-Mandula Theorem?

General considerations:

$$L = L_F[\psi] + L_{F-M}[\psi, M] + L_M[M]$$



$$L \sim L_F[\psi] + L_{F-M}[\psi, \langle M \rangle]$$

$$L \sim L_F[\psi]$$

S.D.W. Scenario
vector magnetism

$$L_{F-M} = g \sum_q \sigma_{-q} \cdot M_q$$

$$e^- \rightleftharpoons e^- + spin \quad fluctuation$$

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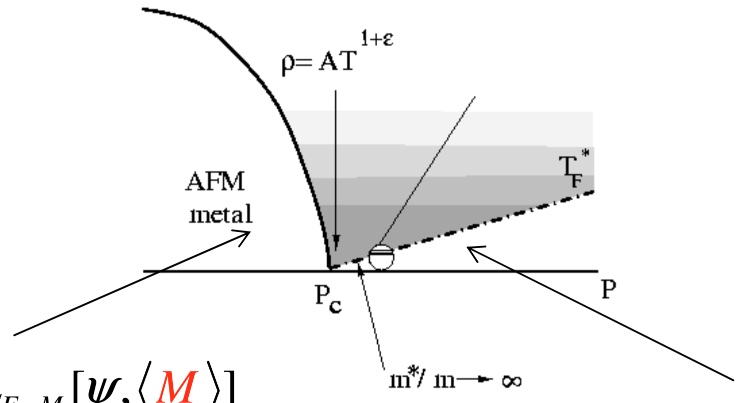
General considerations:

$$L = L_F[\psi] + L_{F-M}[\psi, M] + L_M[M]$$



$$L \sim L_F[\psi] + L_{F-M}[\psi, \langle M \rangle]$$

$$L \sim L_F[\psi]$$

S.D.W. Scenario
vector magnetism

$$L_{F-M} = g \sum_q \sigma_{-q} \bullet M_q$$

$$e^- \rightleftharpoons e^- + spin \quad fluctuation$$

or?

Composite fermions disintegrate:
Spinorial magnetism?

$$\vec{M} = \langle s_\alpha{}^\dagger \rangle \vec{\sigma}_{\alpha\beta} \langle s_\beta \rangle$$

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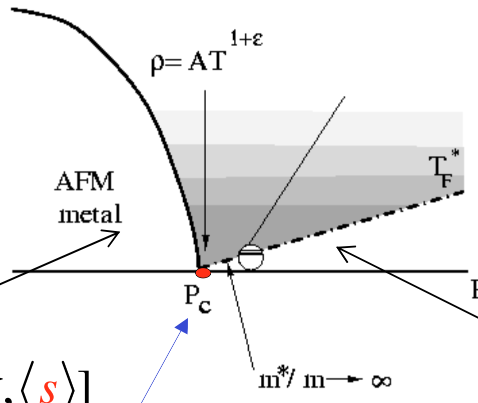
General considerations:

$$L = L_F[\psi] + L_{F-M}[\psi, s] + L_M[s]$$



$$L \sim L_F[\psi] + L_{F-M}[\psi, \langle s \rangle]$$

$$L \sim L_F[\psi]$$

Spin charge decoupling at QCP ?

C. Pepin & PC 2002

Composite fermions disintegrate:

Spinorial magnetism?

$$\vec{M} = \langle s_\alpha^\dagger \rangle \vec{\sigma}_{\alpha\beta} \langle s_\beta \rangle$$

$$e^- \rightleftharpoons s_\sigma + \chi^-$$

$$L_{F-M} = g \sum_{k,q} [s^\dagger_{k-q\sigma} \chi_q^\dagger \psi_{k\sigma} + H.c]$$

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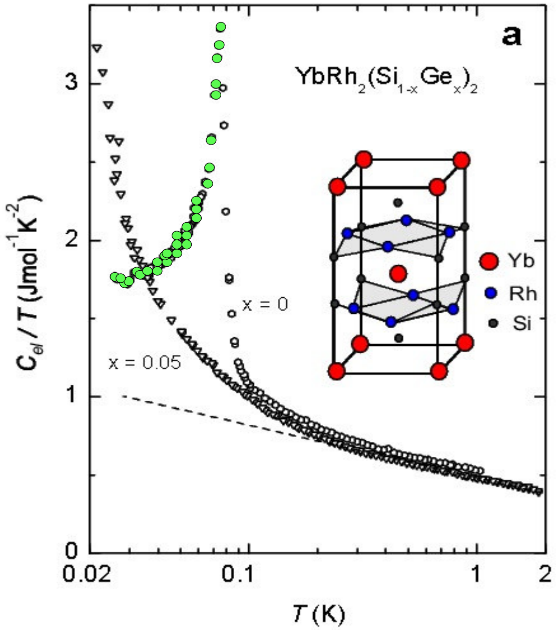
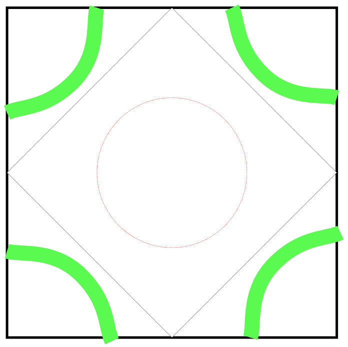
$$e^- \rightleftarrows s_\sigma + \chi^-$$

$$L_{F-M} = g\sum_{k,q}[s^{\dagger}_{k-q\sigma}\chi_q^{\dagger}\psi_{k\sigma} + H.c]$$

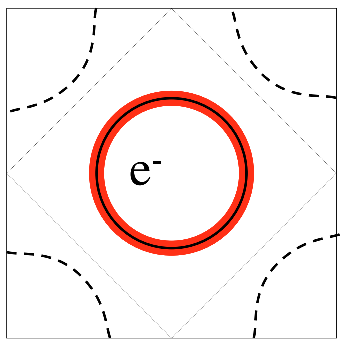
$$\xrightarrow{\langle s_\sigma\rangle \propto \sqrt{2M_Q}} \quad g\sqrt{M_Q}\sum_{k,q}[\chi^{\dagger}_{(k-\sigma Q/2)}\psi_{k\sigma} + H.c]$$



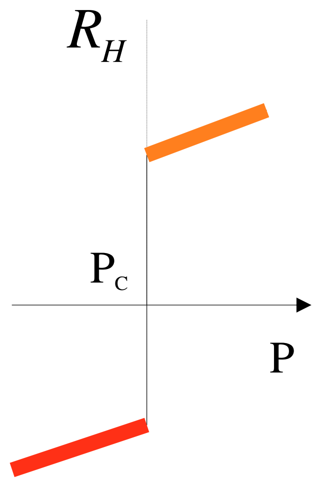
B.)



$$\frac{2v_F}{(2\pi)^d} = n_e + 1$$



$$\frac{2v_F}{(2\pi)^d} = n_e$$



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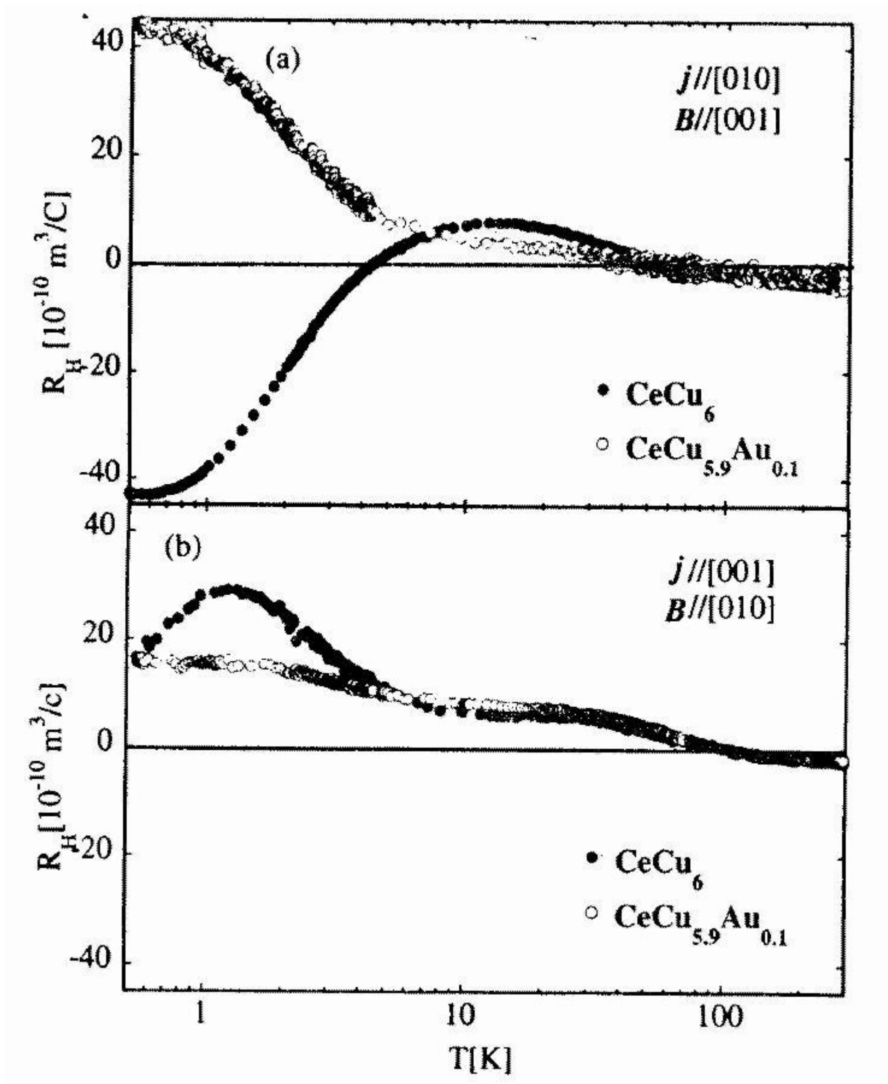
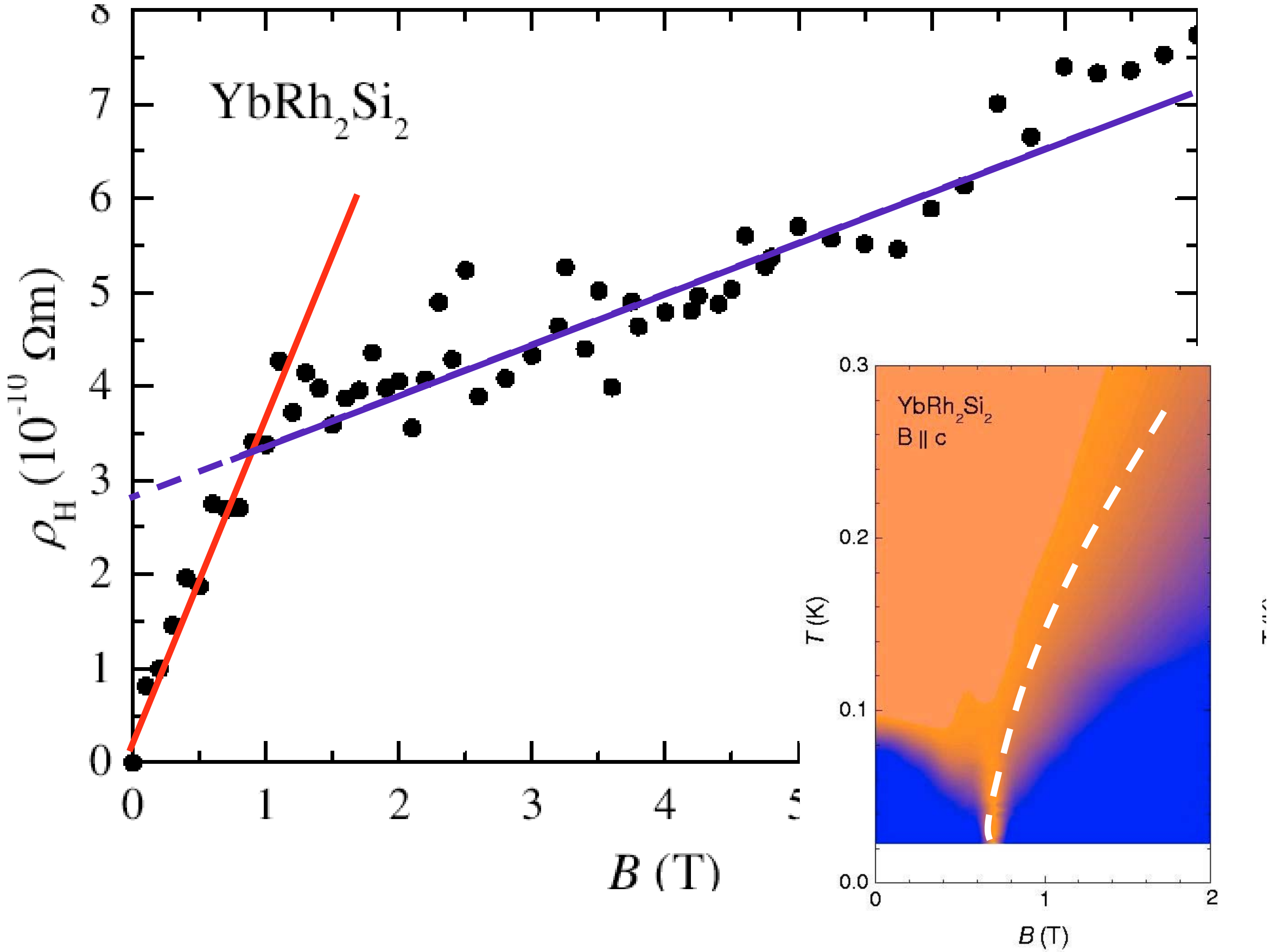
T. Namiki, H. Sato, J. Urakawa, H. Sugawara, Y. Aoki, R. Settai and Y. Onuki, Physica B 281&282 359-360 (2000).

Pfleiderer et al - this conference

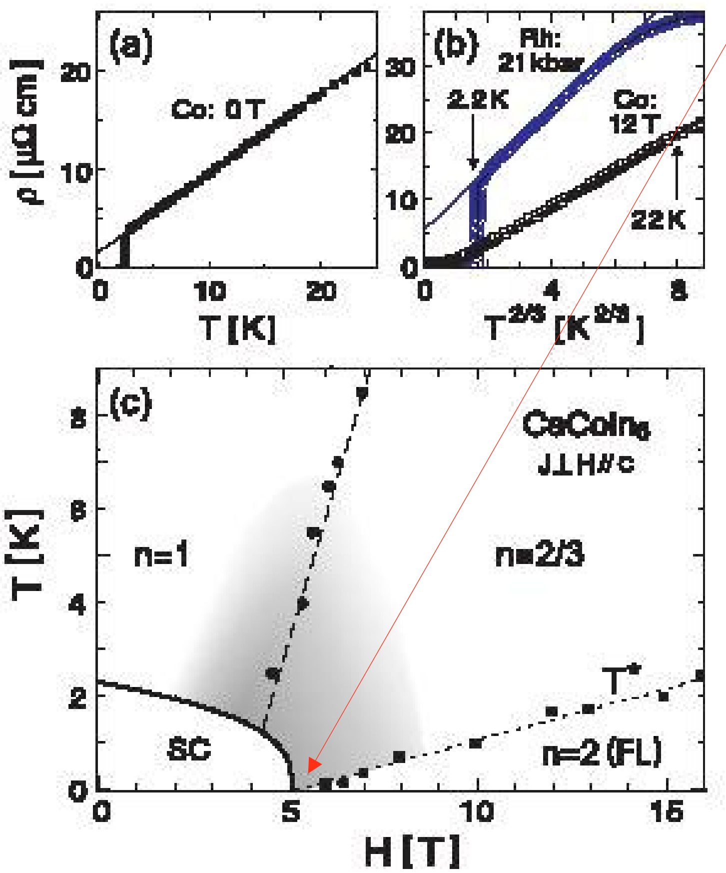


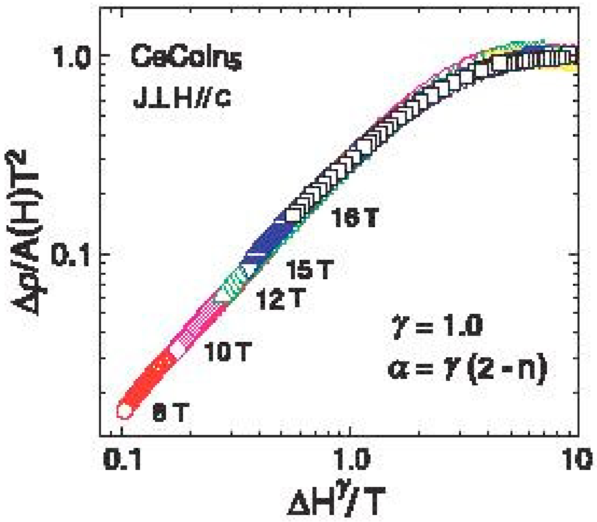
S. Paschen et al (2003).



Link with superconductivity?

Superconducting QCP?
(Los Alamos – QCP is apparently
pinned to HC2.





$$\rho = T^n \Lambda\left(\frac{h^\gamma}{T}\right)$$

Jp Paglione et al, '04

Concluding Points

• E/T scaling, anomalous dimensions, divergence of m* over whole Fermi surface and discontinuity in Hall response at heavy electron QCP suggest a fundamentally new class of phase transition and critical behavior.

• Standard model – Hertz – Moriya is the direct descendent of Landau/Weiss mean field theory. Its failure suggests we mustseek a new class of Landau Theory.

• Simultaneous collapse of Neel temperature and coherence temperature at a single point is mysterious: it can not be accounted for by appealing to reduced dimensinality, suggests a hidden symmetry at the QCP.

• One possibility – that the effective action possess a non-compact symmetry, $F(x,y)=F(x^2-y^2)$, with a fascinating resemblence to the Fayet-Illiopolis action of supersymmetric quantum electrodynamics. A coincidence ?