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Workshop on
Novel States and Phase Transitions in Highly Correlated Matter

12 - 23 July 2004

Anisotropic properties of nodal superconductors in a magnetic field

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Anisotropic properties of nodal superconductors in a magnetic field

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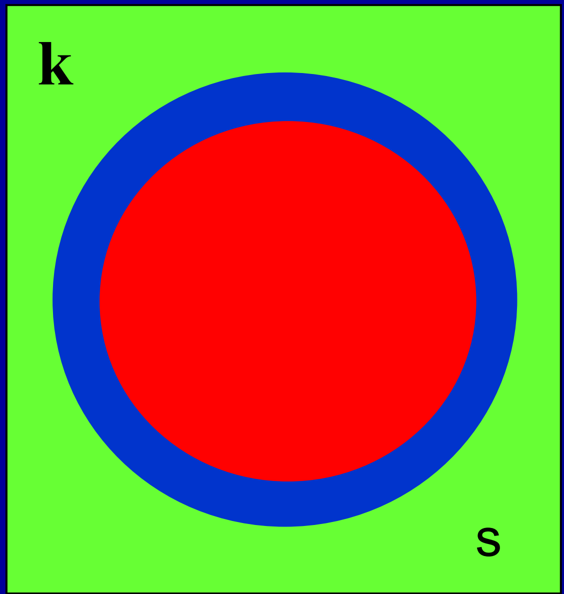
Jules Carbotte (McMaster)

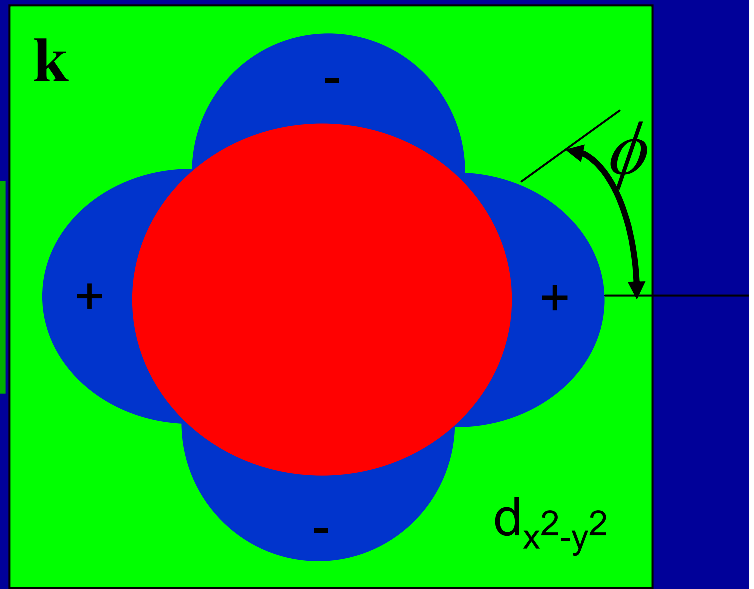
Many thanks to :

Yuji Matsuda (U. Tokyo)

Anisotropic Superconductors



$$E(\mathrm{k}) = \sqrt{\xi_{\mathrm{k}}^2 + |\Delta(\hat{\mathrm{k}})|^2}$$
$$\xi_{\mathrm{k}} = \mathrm{v}_F \cdot (\mathrm{k} - \mathrm{k}_F)$$



isotropic gap Δ

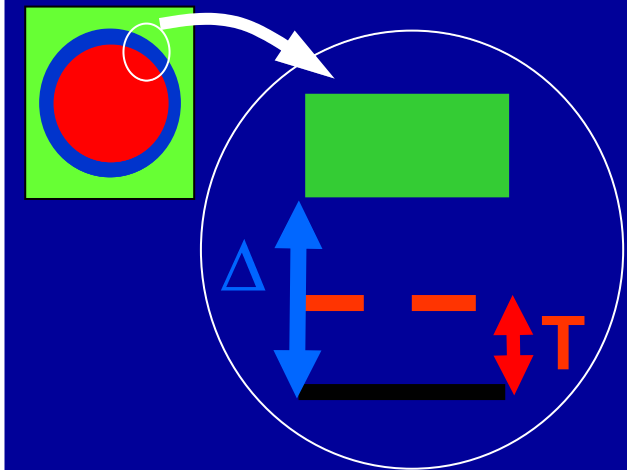
anisotropic $\Delta = \Delta_0 \cos 2\phi$

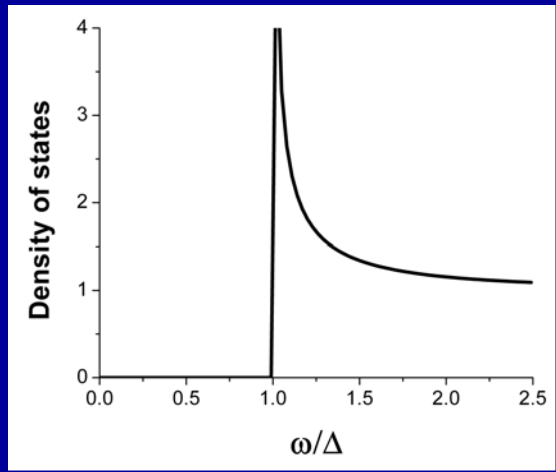
Anisotropic: heavy fermions, organics, ruthenates, high-T_c …

Basic question: how to determine the symmetry of the superconducting gap in the bulk?

Low energy excitations

s



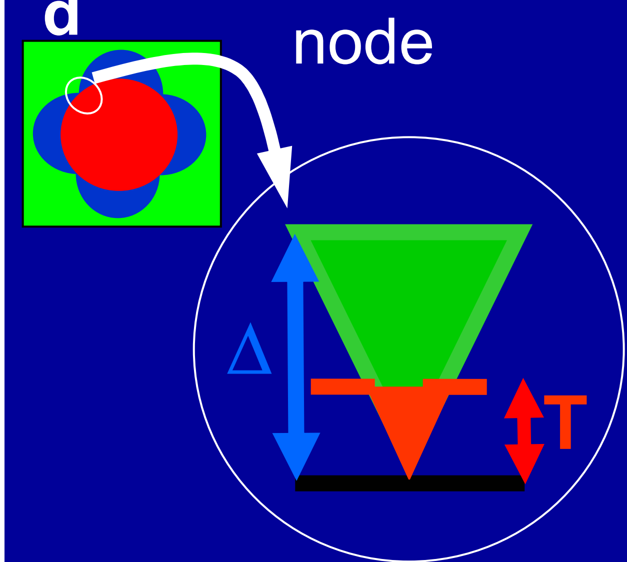


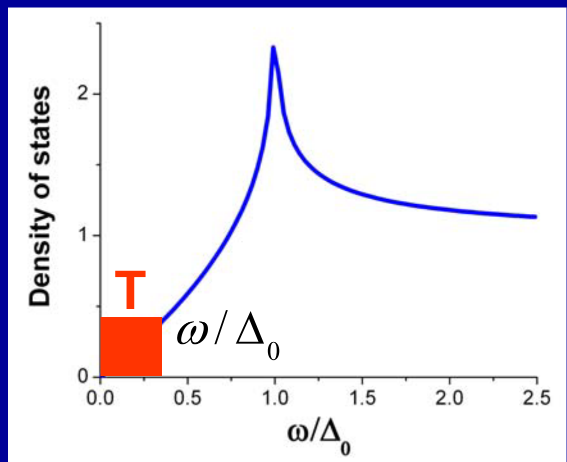
No excitations at low T

Activated behavior of thermal properties

$\exp(-\Delta/T)$

d





Density of qp $\propto T$

Power laws:

Specific heat $C(T) \propto T^2$

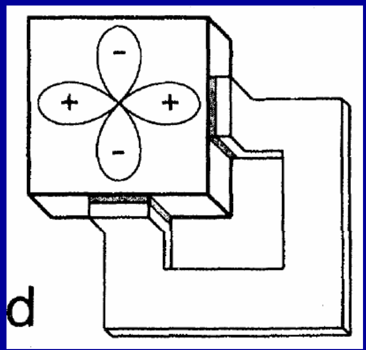
NMR $T_1^{-1} \propto T^3$

Existence but not position of nodes

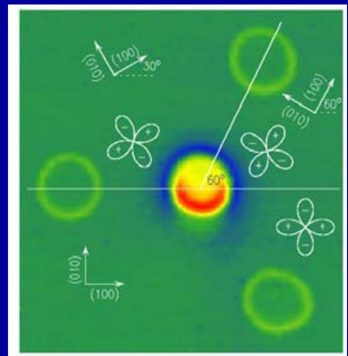
Sign of Δ(k): surface probes

Cuprates: change in the phase of the gap

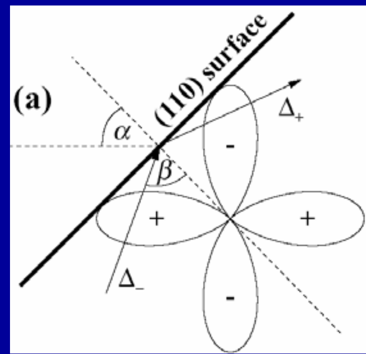
corner junctions, tricrystal, Andreev bound states



Van Harlingen et al.



Kirtley et al.



C.R. Hu, L. Greene, L. Alff

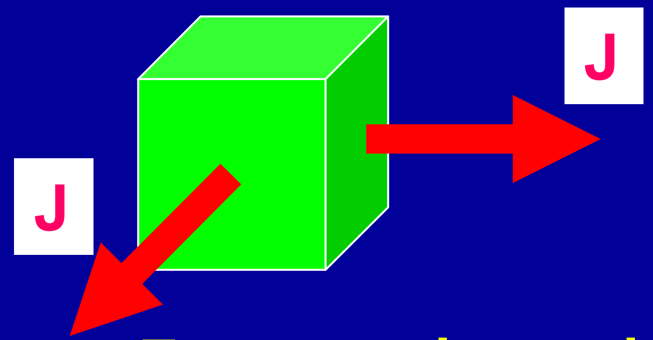
Not always easy / possible

Bulk measurements bargain: shape of the gap in the bulk,

Bulk: thermodynamics and transport
(specific heat, NMR, thermal conductivity) → $|\Delta(\hat{k})|$

Why magnetic field?

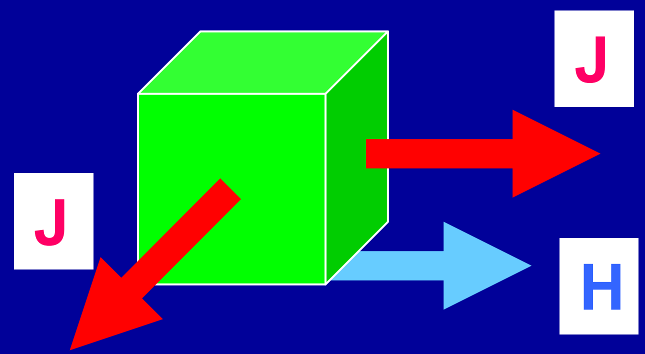
- Need **directional probe** that provides anisotropy of transport/thermodynamics.



Transport isotropic

- **Magnetic field?**

- But also need a directional probe that **couples to nodal quasiparticles**

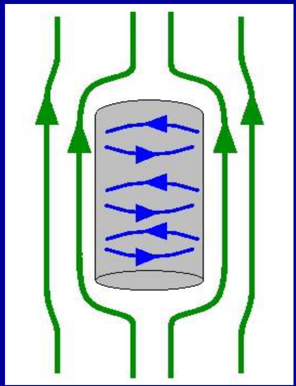


Transport anisotropic

- **How does H do that?**

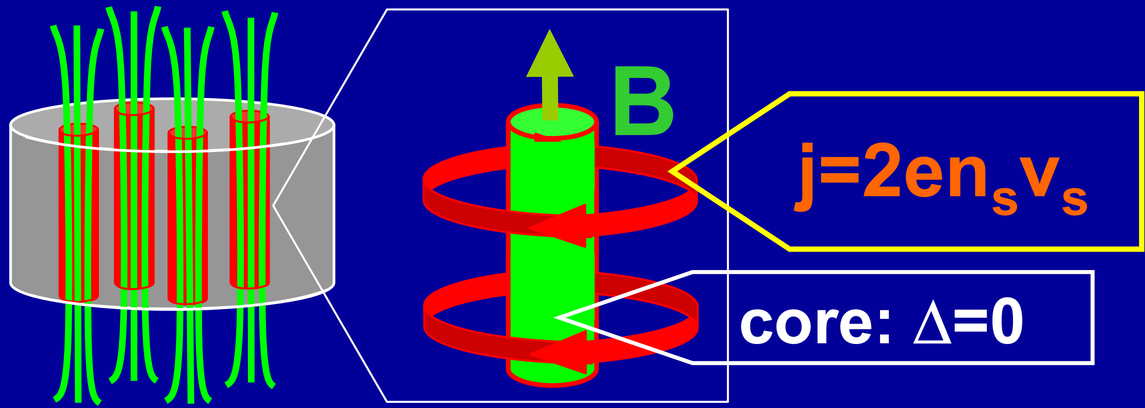
Magnetic field

- Magnetic field induces vortices (type-II superconductors)

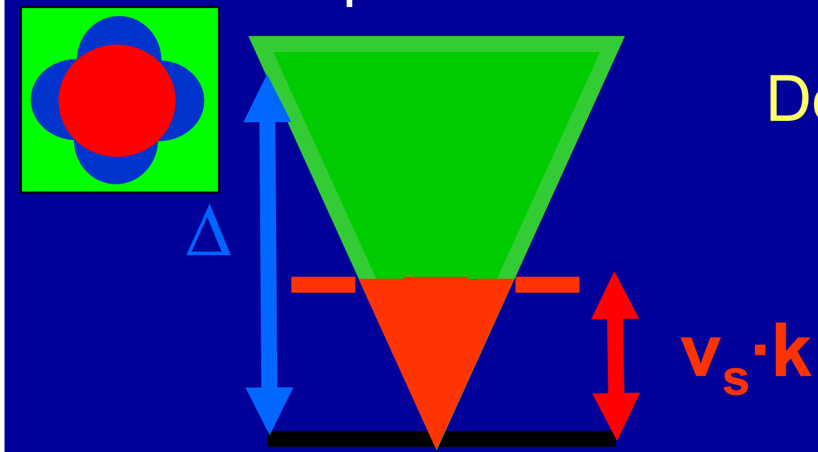


Energy cost $H^2/8\pi$.

When $H>H_{c1}$



- In a superconductor with nodes field excites quasiparticles



Doppler shift $E'(\mathrm{k},\mathrm{r}) = E(\mathrm{k}) + \mathrm{v}_s(\mathrm{r}) \cdot \mathrm{k}$

Contribution of Doppler shifted quasiparticles outside of vortex cores exceeds that of the bound states in the cores

G. Volovik, 1993

"Volovik" Effect

Magnetic field probes mostly nodal quasiparticles.

Caveat 1. True for low fields $\mathbf{H \ll H_{c2}}$

Caveat 2. True for **line** and **quadratic point** nodes.

For linear point nodes cores and nodes contribute almost equally.

Caveat 3. Semiclassical approximation: $\mathbf{v}_s$ nearly uniform

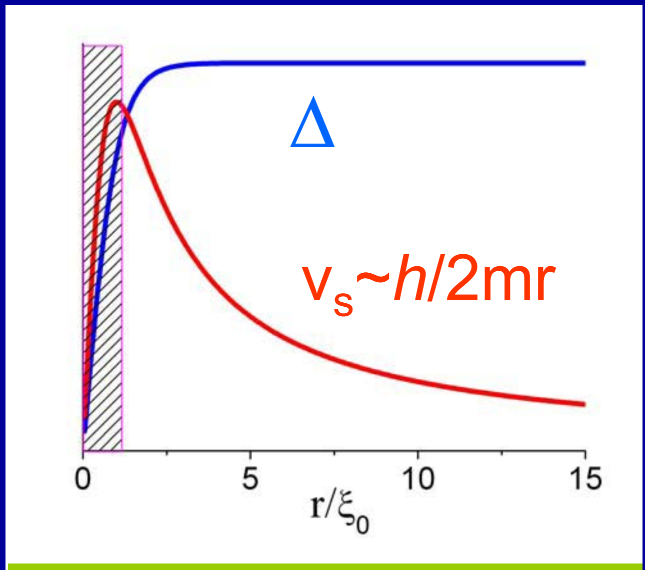
no Aharonov-Bohm phases *(cf Franz and Tesanovic)*

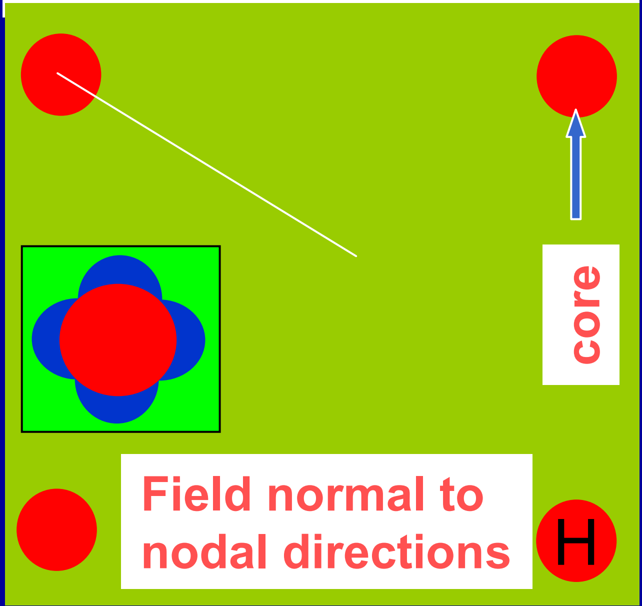
Caveat 4. Infinite lifetime: no scattering on vortices.

Good when the mean free path $l \leq 2R$

Intervortex distance $R \approx \sqrt{\Phi_0 / H} \approx 450\,\text{Å} / \sqrt{H, \text{Tesla}}$

Density of states





Characteristic supervelocity

$$v_s \approx \hbar / 2mR$$

Characteristic Doppler energy

$$E_H = v_s k_F \approx v_F / R \propto \sqrt{H}$$

Competes with **T.**

Density of states

$$N(\omega, H) / N_0 \approx \omega / \Delta_0$$ **E_H<< ω**

$$N(\omega, H) / N_0 \approx E_H / \Delta_0$$ **E_H>> ω**

Specific heat for **E_H>> T**

$$C(T, H) \propto T\sqrt{H}$$

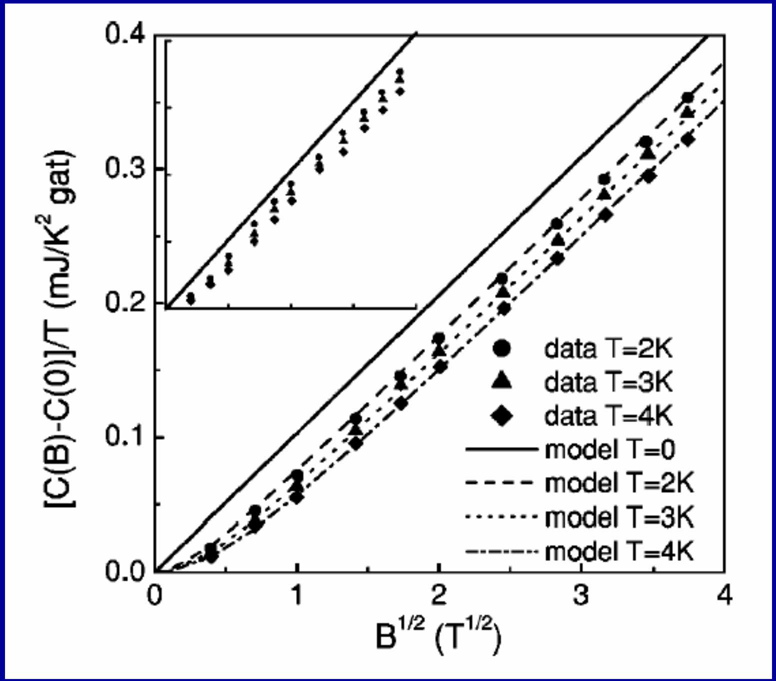
G.Volovik 1993

More formally $$G^{-1}(k,\omega_n) = i\omega_n - \xi_k \tau_3 - \Delta(k)(i\tau_2)$$

In field $$G(k, i\omega_n; r) = G(k, i\omega_n + \mathrm{v}_s(r)\cdot k)$$

Take a realistic **$v_s(r)$,** compute **local** ***N(r)*,** average

$$N(\omega, H) = A^{-1}\int d^2 r\, N(\omega, r)$$



Quantitative agreement in YBCO

Expt.:

K. A. Moler et al 1994, B. Revaz et al. 1998

D. Wright et al. 1999, Y. Wang et al. 2001

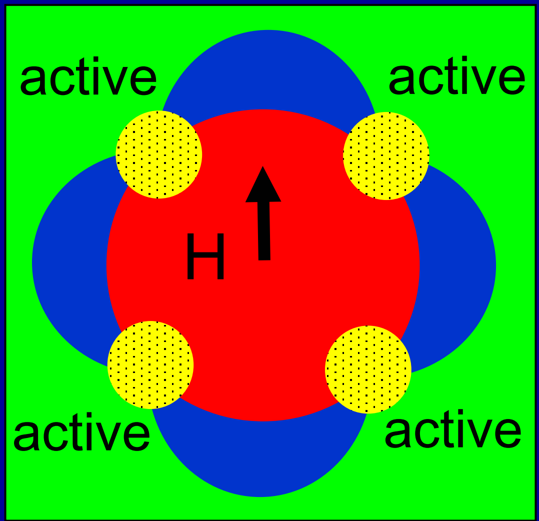
Theory:

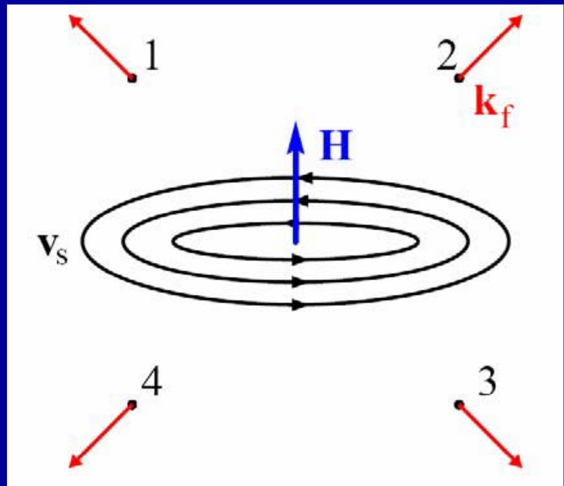
Kübert, Hirschfeld 1998,

Vekhter et al. 1998-2001

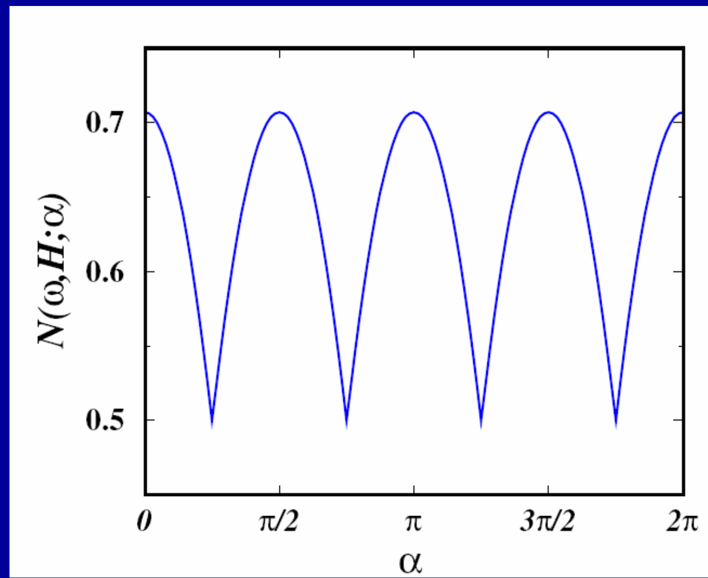
Anisotropic specific heat

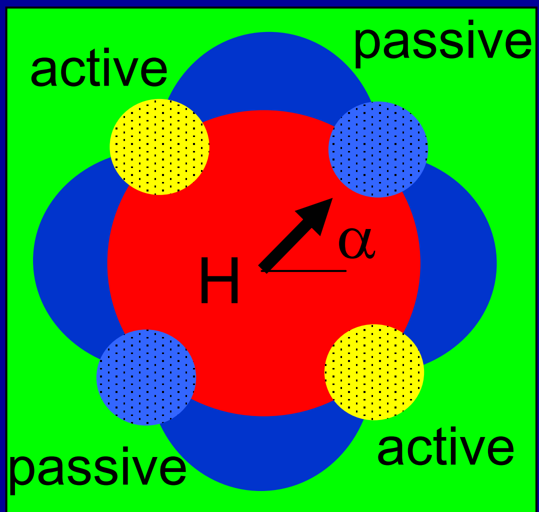
Supervelocity $\mathbf{v}_s(r) \perp \mathbf{H}$ and Doppler shift is $\mathbf{v}_s(r)\cdot\mathbf{k}_{nodal}$

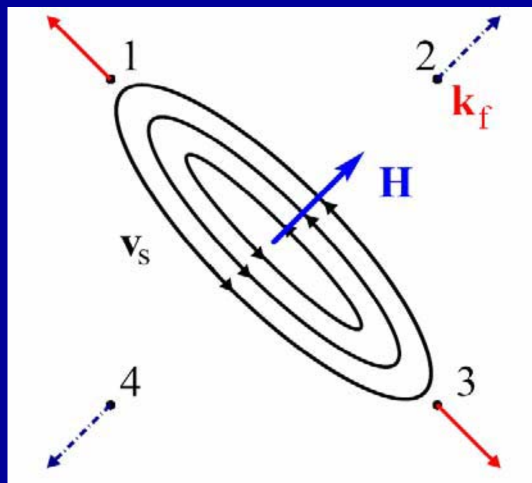




Minima in DOS for H || nodes



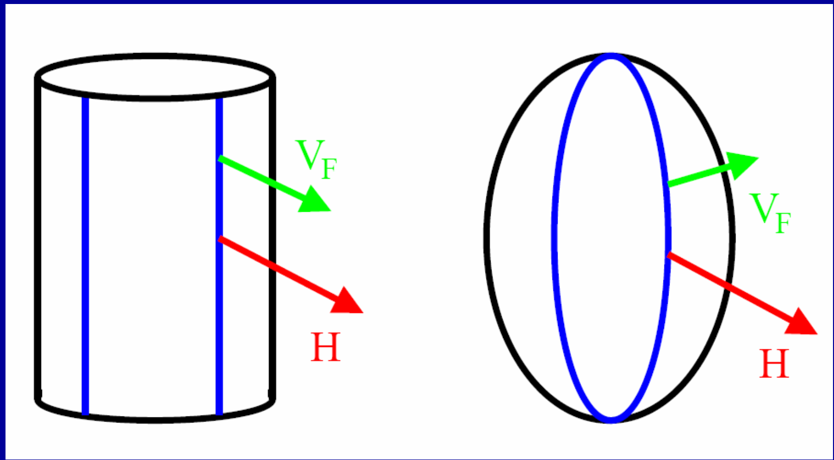




$$E_H >> T$$

I. Vekhter et al. 1999-2002,
see also T. Nakai et al. 2004

3D vs 2D



Anisotropy:

Amplitude is smaller in 3D than in **2D**: nodal lines are only partially inactive

Anisotropy amplitude depends on the shape of the Fermi surface: (not the salient features)

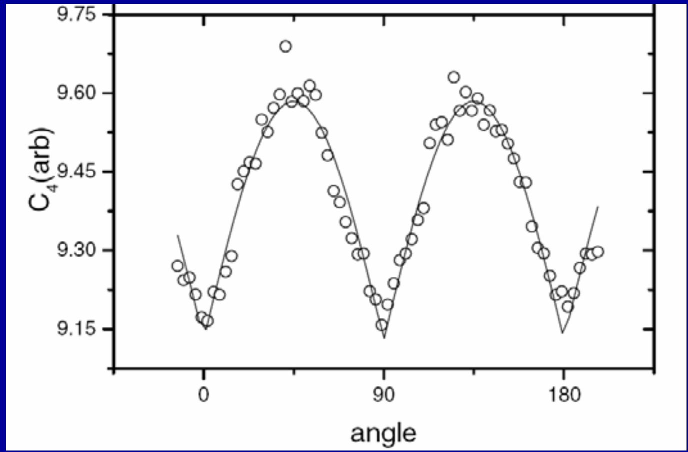
a) what areas have $\mathbf{v_F}$ parallel to **H**;

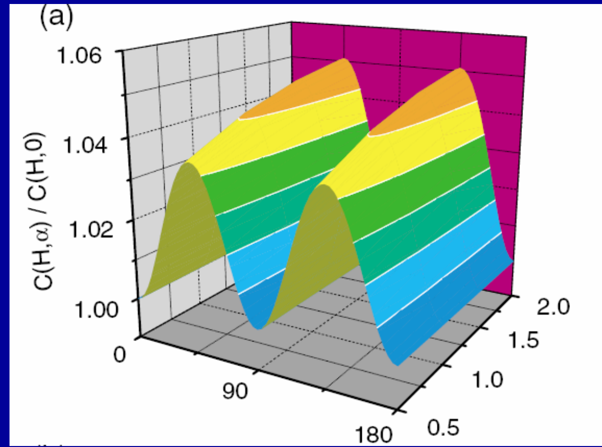
b) how close these areas are to the nodes.

I. Vekhter et al `99, K. Maki and H. Won, 2001; K. Maki and P. Thalmeier 2003

S. Graser, T. Dahm, and N. Schopohl, 2003, ….

Experiment: borocarbides





YNi_2B_2C and $LuNi_2B_2C$: T.Park et al, PRL 2003, 2004

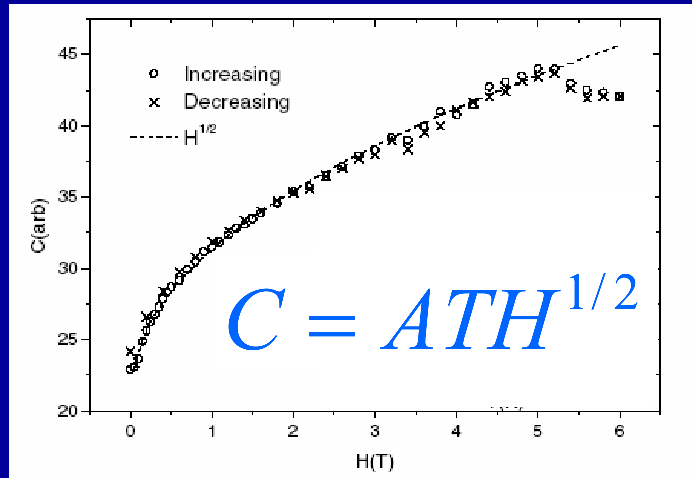
Nodes or deep minima?

$$E_H \approx \Delta\sqrt{H/H_{c2}}$$ **suggests** $$\Delta_{min} \le 0.1\Delta_{max}$$

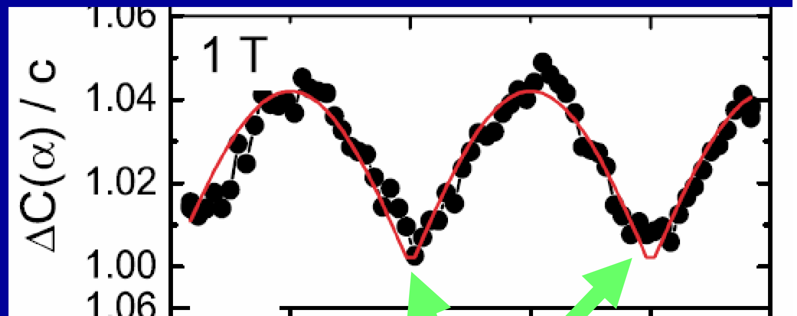
- Not a phase sensitive experiment: only anisotropy of $|\Delta(\hat{k})|$
- Upper limit: E_H is a moderately high energy scale
- Combine with other measurements (low-T NMR, penetration depth, etc.) to improve and decide on true nodes.

Specific heat: summary

- Anisotropic superconductor with a known Fermi surface in a field far below $\mathbf{H_{c2}}$.



- **Measure:**
  - Field dependence of **C/T**;
  - Dependence on the angle



T.Park et al. 04

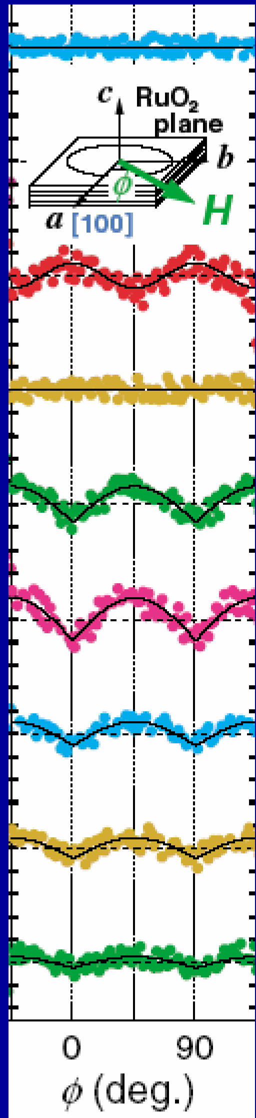
When C/T is at a minimum: H || v_F at 'nodes'.

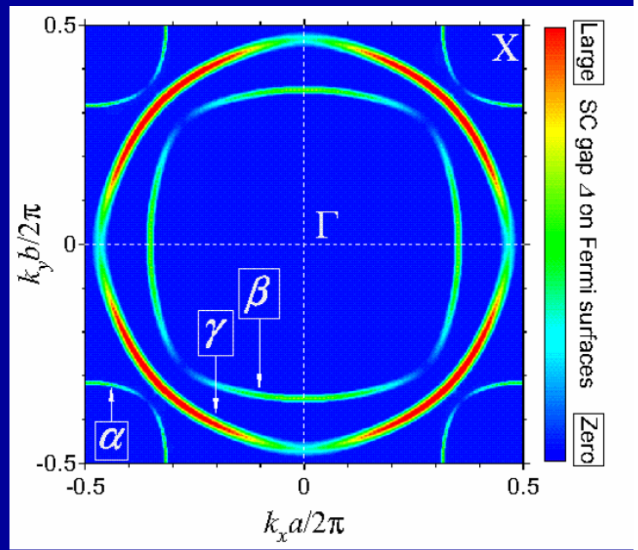
Dirty details

H_{c2}

1. **Multiband SC:** interpretation is more difficult.

2. **Don't go too close to H_{c2}:** it may be anisotropic.





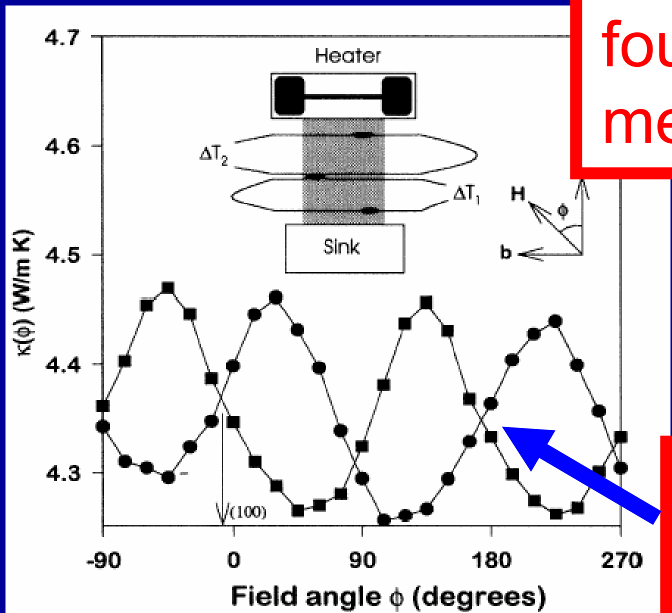
Sr_2RuO_4

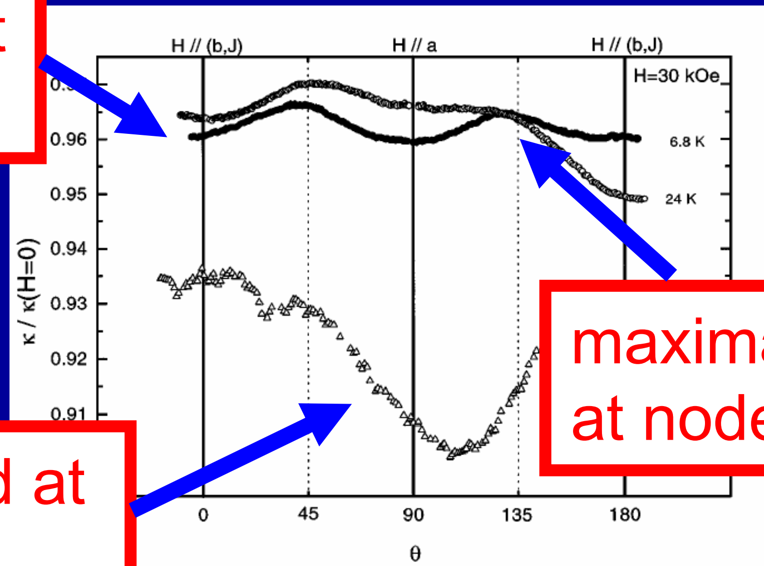
K. Deguchi et al. 2004

Thermal conductivity

Entropy transport: **only unpaired qps contribute**

Cuprates: experiment predates theory





fourfold at med. T

twofold at low T

maxima at nodes

F. Yu, M. Salamon et al. 1995;

H. Aubin, K. Behnia et al. 1997

Not at all what is expected from the density of states

Transport: a challenge

- Depends on density of states and lifetime

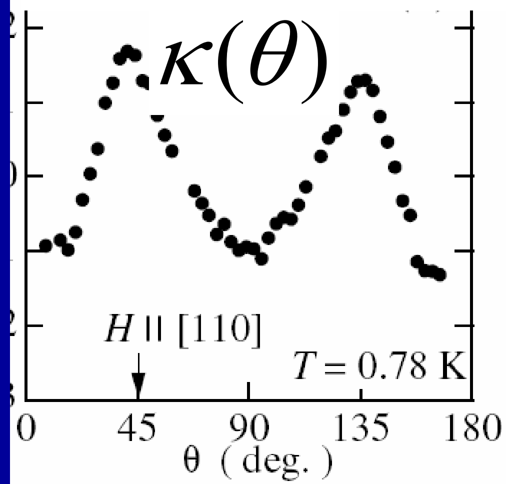
$$\kappa \propto T N(0)\, v_F^2\, \tau$$

- Applied magnetic field
  - enhances the local density of states;
  - modifies scattering;

    Kübert and Hirschfeld, Vekhter and Hirschfeld
  - **introduces vortex scattering**

    Yu et al., Aubin et al.

Yu. Matsuda et al.



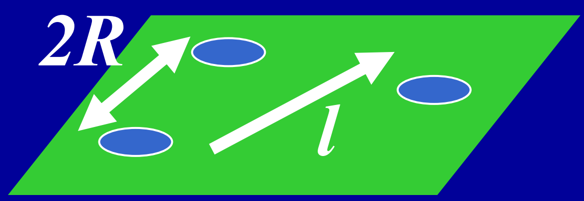
Low DOS → poor transport for H || nodes

Reduced scattering → good transport for H II nodes

Minima or maxima correspond to nodes?

Semiclassical analysis: questions

1. Is there a well-defined **local thermal conductivity** $\kappa(r)$?

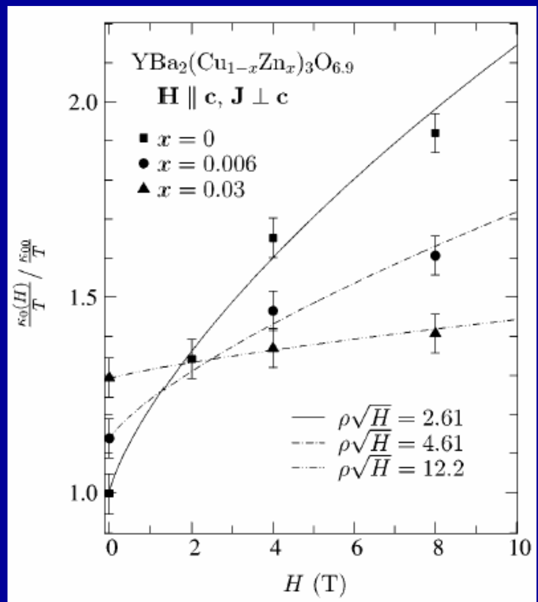


- yes, if $l \leq 2R$ P. Hirschfeld, P. Hirschfeld and I. Vekhter
- possibly otherwise if one takes $\tau^{-1} = \int d^2 r\, \tau^{-1}(r)$ K. Maki et al.

2. No vortex scattering in the model

Fit to data with field normal to the 2D planes in YBCO at low T and H.

May well descibe situation when vortex scattering is unimportant



M. Chiao et al.

What is measured?

$\kappa(r)$:

$$\frac{\kappa(T,r)/T}{\kappa_n/T_c} = \frac{3}{2\pi^2}\int_0^\infty \frac{d\omega}{T}\left(\frac{\omega}{T}\right)^2 \operatorname{sech}^2\frac{\omega}{2T} K(\omega)$$

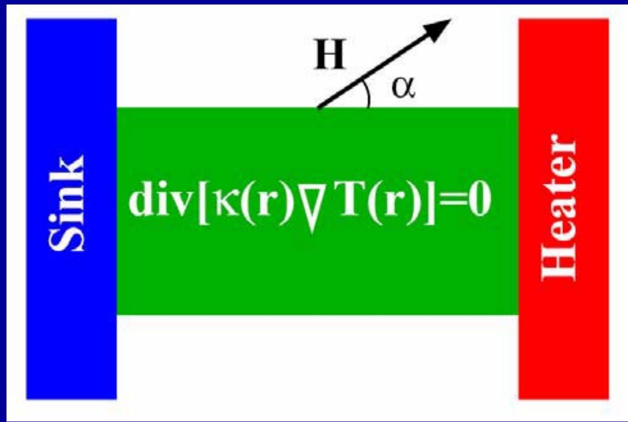
$$K(\omega) = \frac{\Gamma}{\tilde{\omega}'\tilde{\omega}''}\operatorname{Re}\left\langle \frac{(\tilde{\omega}^2 + |\tilde{\omega}|^2 - 2|\Delta_k|^2)k_x^2}{\sqrt{\tilde{\omega}^2 - \Delta_k^2}} \right\rangle_{FS}$$

Input: form of the gap,
impurity scattering,
Doppler shift.

↓

Mimics gap symmetry:
4-fold for d-wave etc.

κ:



Input: local conductivity
direction of net current.

↓

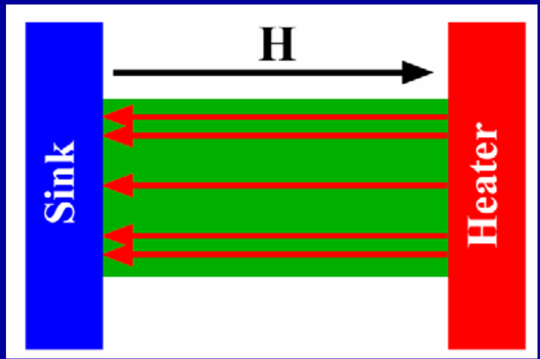
Does not mimic gap symmetry

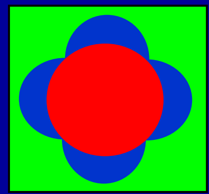
More complicated dependence

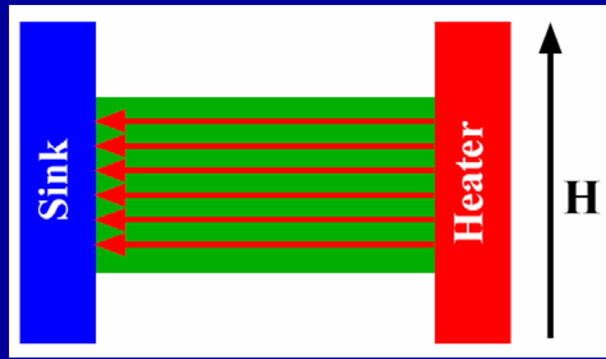
$$\kappa \neq \int d^2r\, \kappa(r)$$

Twofold angle-dependence

Quasi-2D system: analytic solution possible when





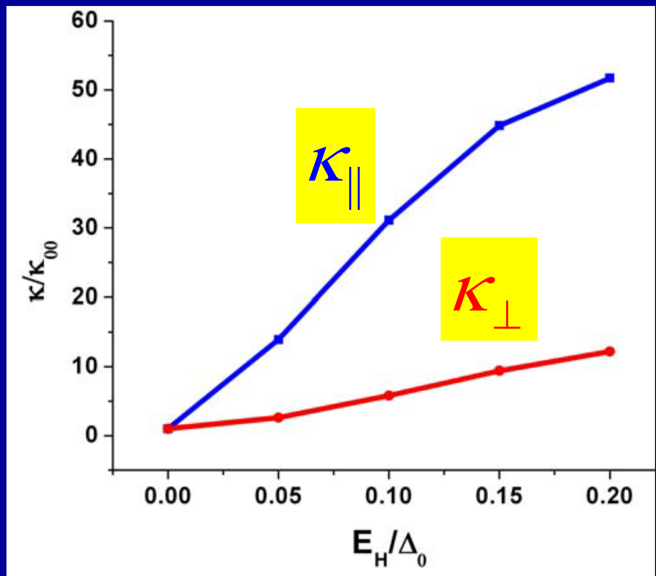


$$\kappa_{\parallel} = \int dy\,\kappa(y)$$

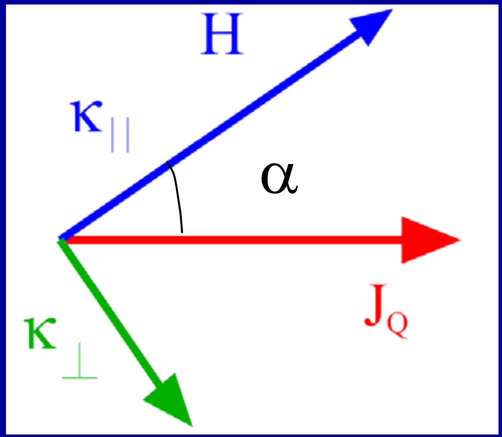
series

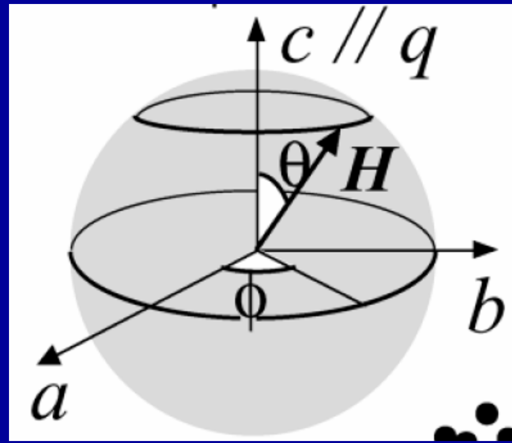
$$\kappa_{\perp}^{-1} = \int dx\,\kappa^{-1}(x)$$

parallel



3D vs 2D





2D:

Need to rotate **H** wrt **J**,

Measured κ is some convolution of

2-fold (vortex scattering) and

nodal patterns (4-fold)

3D:

conical rotation

Directly nodal patterns

For all angles ϕ convoluted in the same way with vortex scattering.

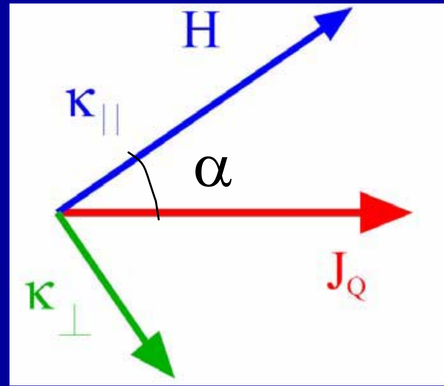
Yu. Matsuda et al. 01-04

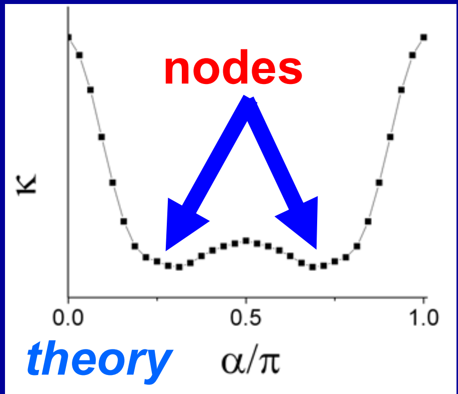
Effective medium approach

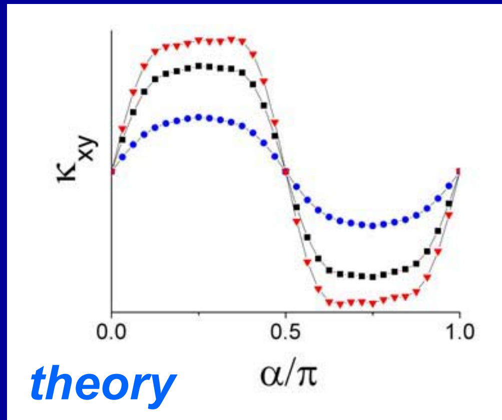
- Treat $\kappa_{||}$ and $\kappa_{\perp}$ as **principal axes**
- **Steady state technique: fixed J_Q**

$$\kappa_{xx} = \frac{\kappa_{||}\kappa_{\perp}}{\kappa_{\perp}\cos^2\alpha + \kappa_{||}\sin^2\alpha}$$

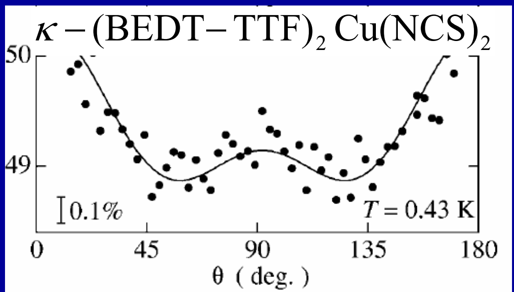
$$\kappa_{xy} = \frac{\kappa_{||}\kappa_{\perp}(\kappa_{||} - \kappa_{\perp})\sin 2\alpha}{(\kappa_{\perp}\cos^2\alpha + \kappa_{||}\sin^2\alpha)^2}$$

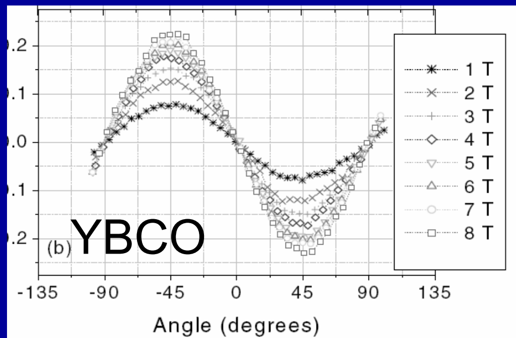






Minima almost always remain at the nodes (tentative).





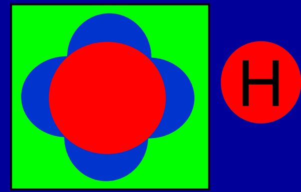
Thermal Hall of the same order as 4-fold part of κ_{xy}

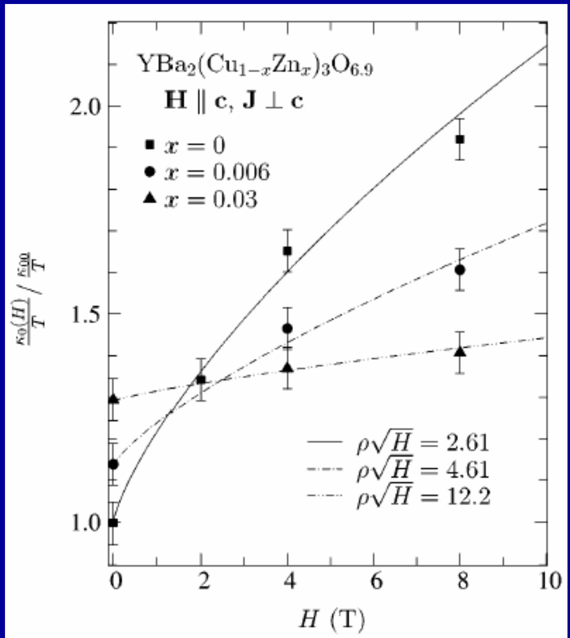
K.Izawa et al.

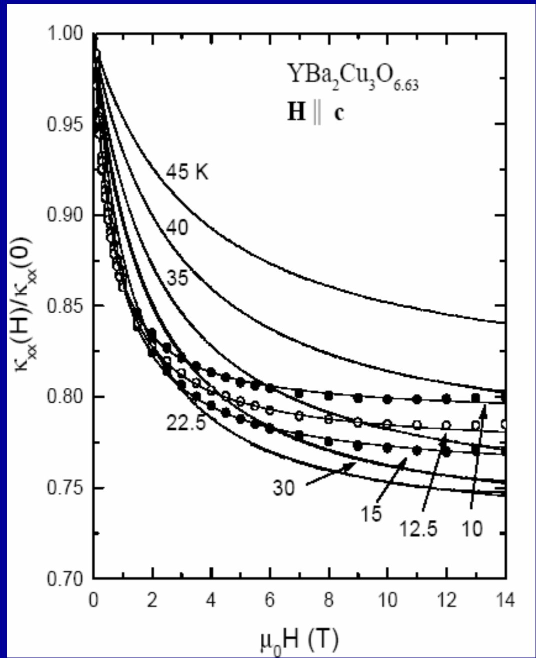
R. Ocaña and P. Esquinazi

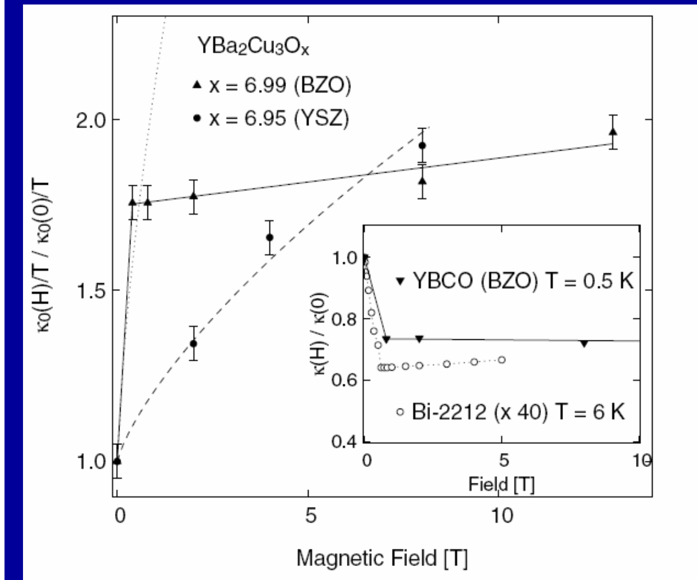
I. Vekhter, P.HIrschfeld, unpublished

Vortex scattering:
lessons from cuprates









Low T,H: density of states effect dominant. Semiclassical theory.

High T: DOS from **T**, **H** - vortex scattering

Ultrapure sample. Low T is also dominated by vortex scattering

M. Chiao et al. 1998

K. Krishana et al..1997

R. Hill et al..2004

Theory for H||c

Input: vortex **lattice**,
account for supervelocity **to all orders**,
average Green's function over vortex unit cell

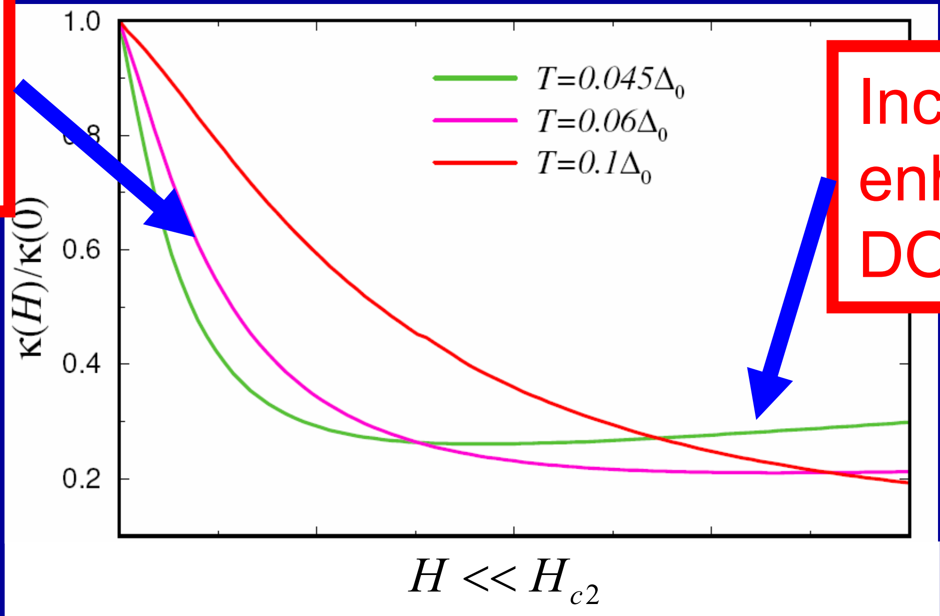
I. Vekhter and A. Houghton'99 based on U. Brandt, W. Pesch, L. Tewordt '68,

$$\tau^{-1}(\omega,\hat{k}) = \tau_{imp}^{-1}(\omega) + \Delta^2(\hat{k})F(\omega/\sqrt{H})$$

Away from node: more scattering

Nodal qp feel impurities

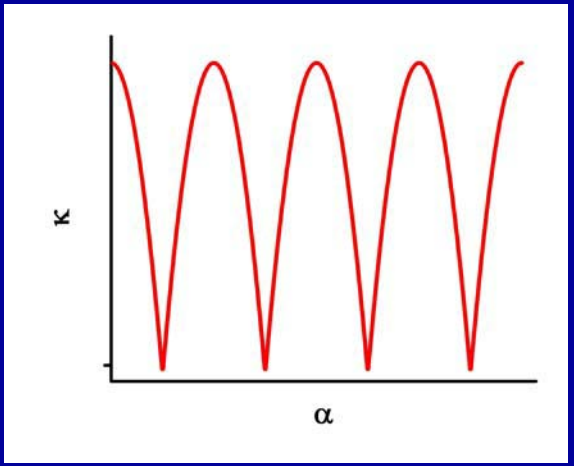
Decrease due to scattering on vortices



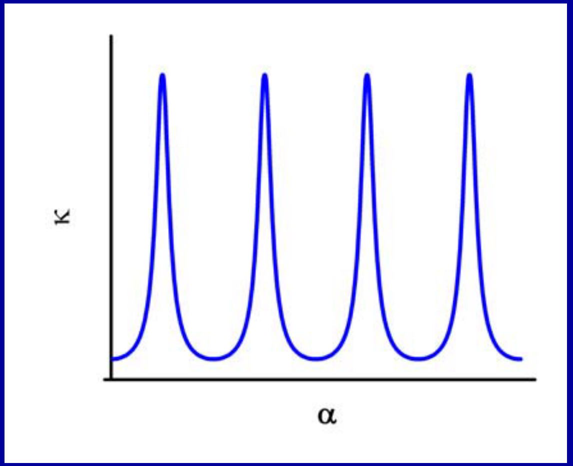
Increase due to enhancement of DOS

Minima vs. maxima: a conjecture

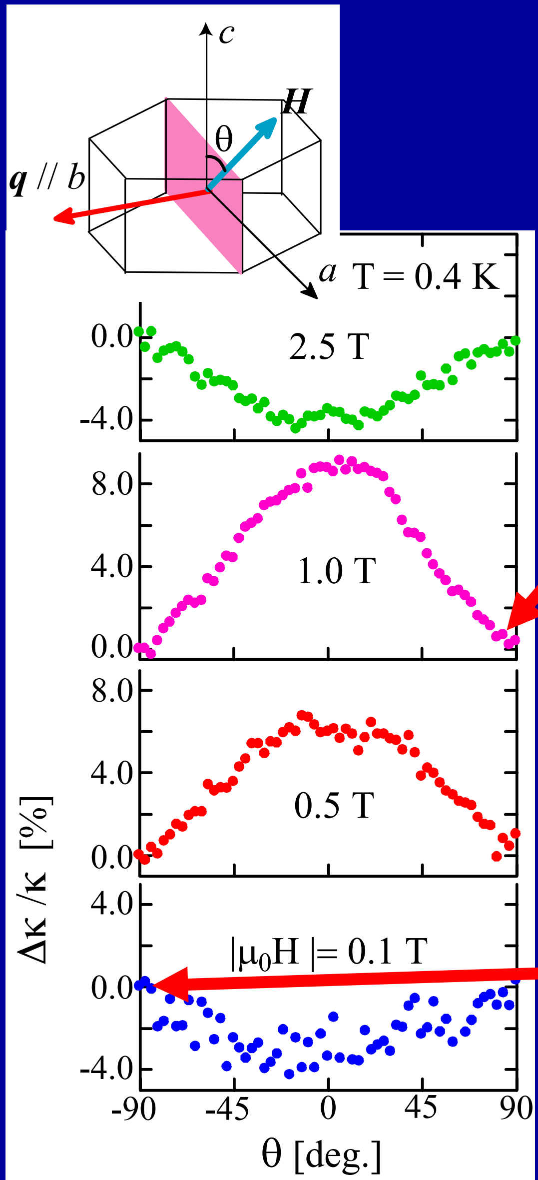
- Electronic κ **increases** with H
  - **Density of states** dominates
  - **Minima when H || nodes**



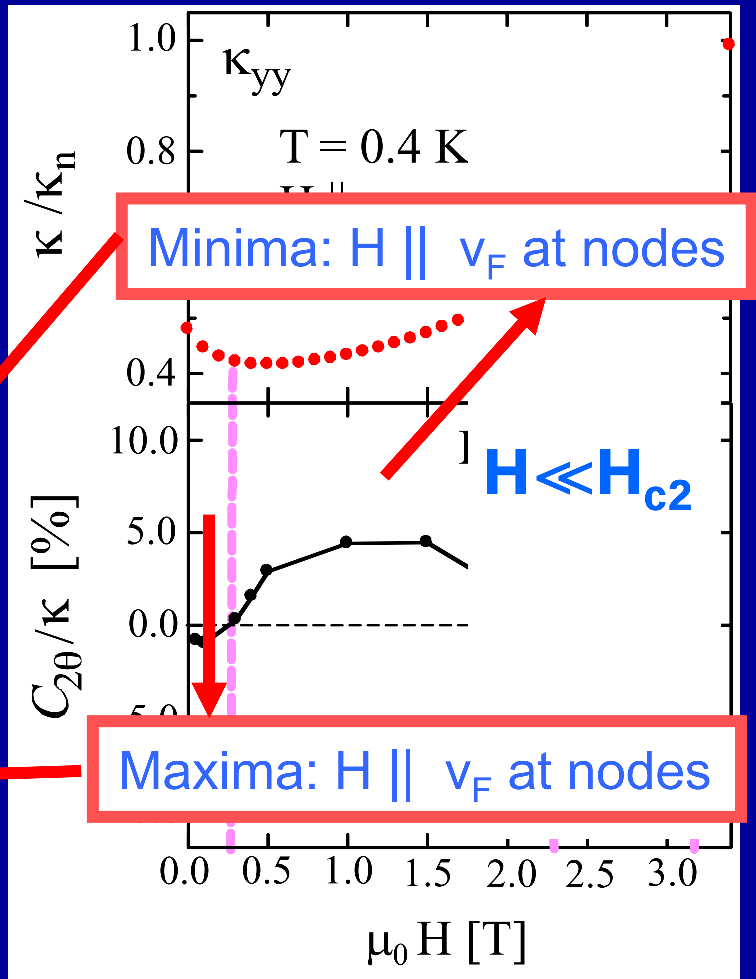
- Electronic κ **decreases** with H
  - **Scattering** dominates
  - **Maxima when H || the nodes**



Example: UPd_2Al_3

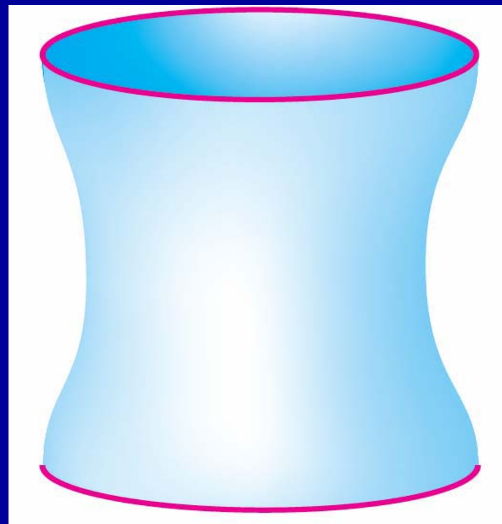


$\kappa_{yy}(\theta) = \kappa_0 + C_{2\theta} \cos 2\theta$



H≪Hc2

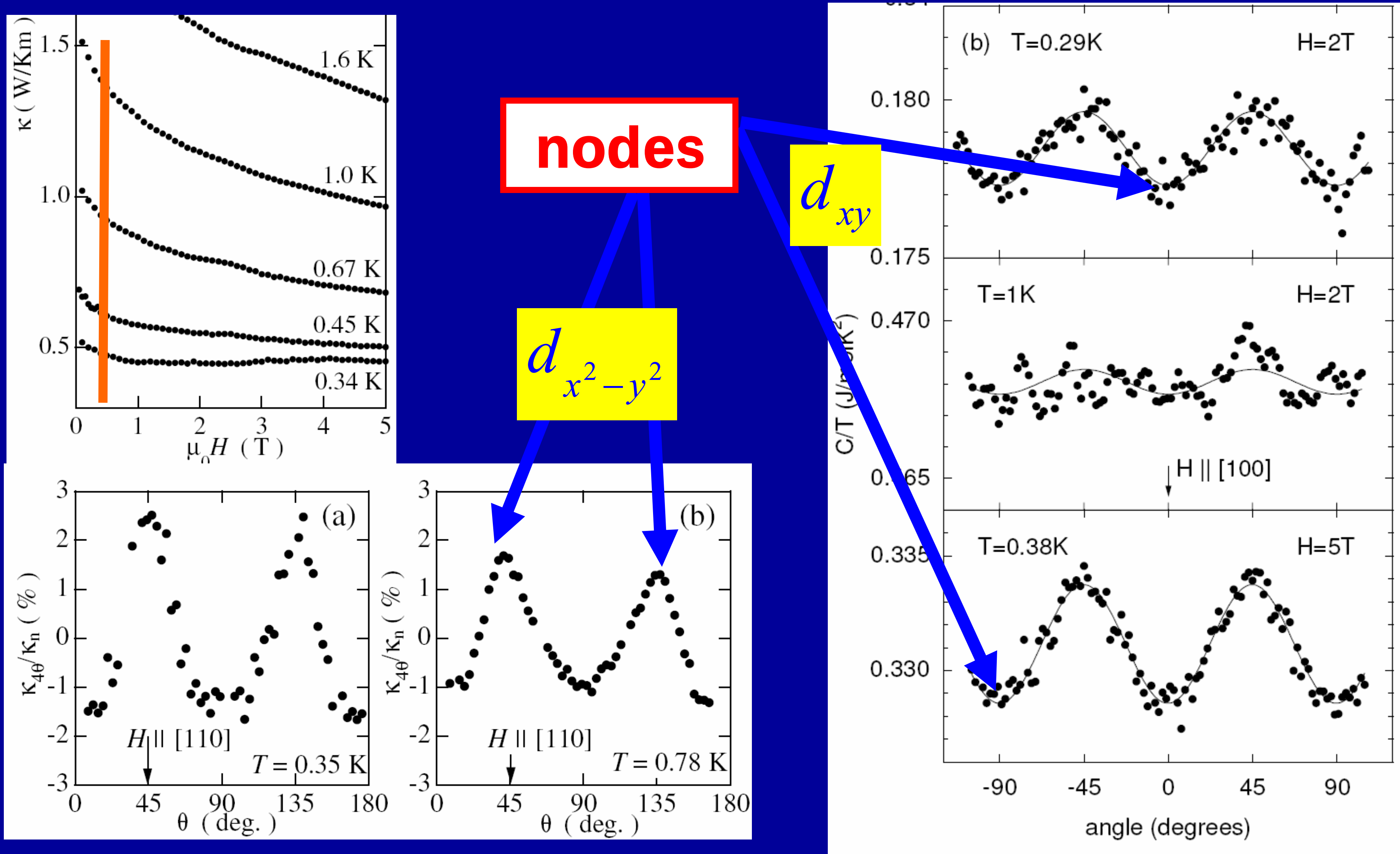
nodes



Yu.Matsuda et al. '04

Also for YNiBC

$CeCoIn_5$: a puzzle



K. Izawa et al. '01

H. Aoki et al. '03

Summary

- **System: nodal superconductors**
- **Foundation:**
  "Volovik Effect": magnetic field probes near-nodal quasiparticles.
- **Rotation** of magnetic field with respect to nodes:
- **Provides:** map of the amplitude of the gap.
- **Specific heat:** direct probe
- **Thermal transport**: vortex scattering?