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Workshop on
Novel States and Phase Transitions in Highly Correlated Matter

12 - 23 July 2004

New quantum phase transitions in a noisy mesoscopic world

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“Trieste 2004: Novel phase transitions in highly correlated matter”

Fonds de recherche sur la nature et les technologies Québec

NSERC CRSNG

CIAR The Canadian Institute for Advanced Research

In Brief

Target: Large quantum dot (single electron box)

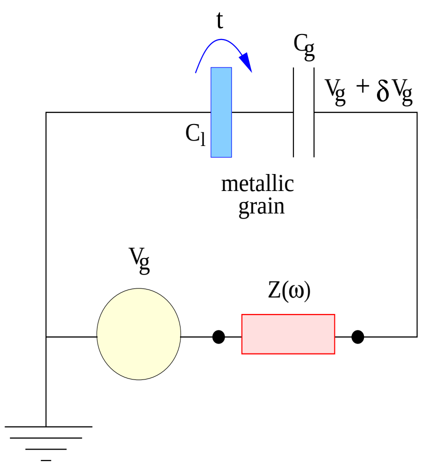
Charge fluctuations on a large dot coupled to a 2DEG: Mapping on Fermi-Kondo models

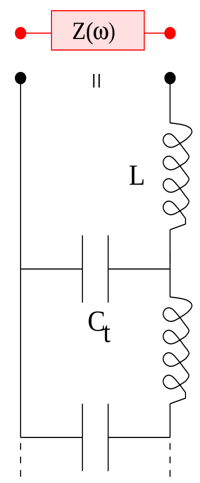
Dissipative environment: Bosonic bath?

Zero-point fluctuations?

Bose-Fermi Kondo models?

Quantum critical points?



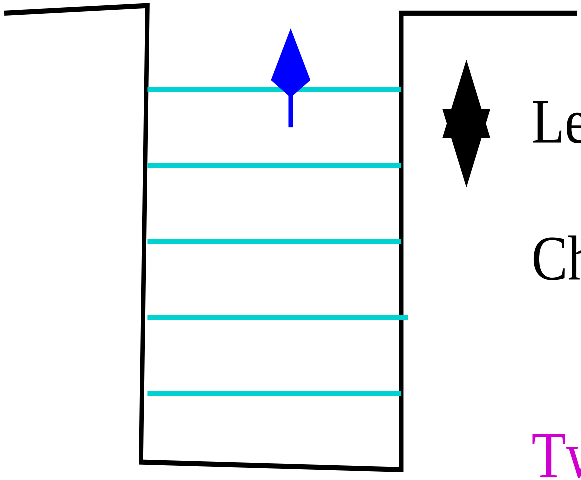


Josephson junction array

K. Le Hur, PRL 92, 196804, 2004

small/large dot?

Energy spectrum in dot

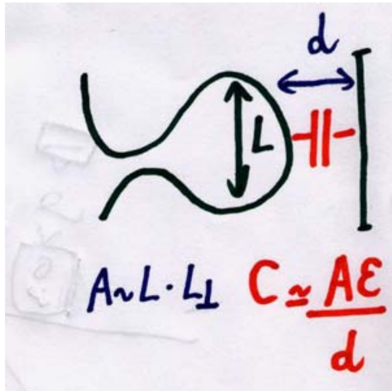


Level spacing $\Delta \sim L^{-d}$

Charging energy $E_c \sim L^{-1}$

$$\frac{\Delta}{E_c} \sim \left(\frac{\lambda_F}{L}\right)^{d-1}$$

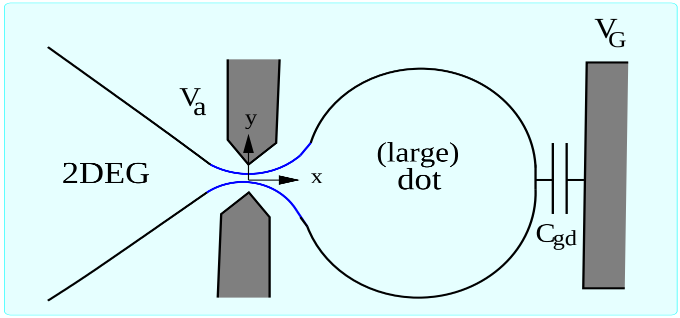
Two−dimensional dot: $\Delta << E_c$

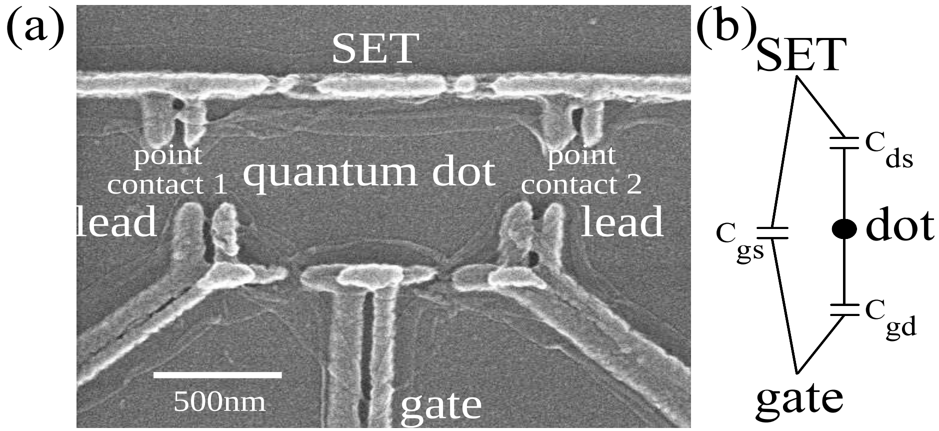


Small dot (Nano): Spin-1/2
Large dot (Micron): “Many-body”

Coulomb Blockade

Glazman and Matveev, JETP 71, 1031 (1990)



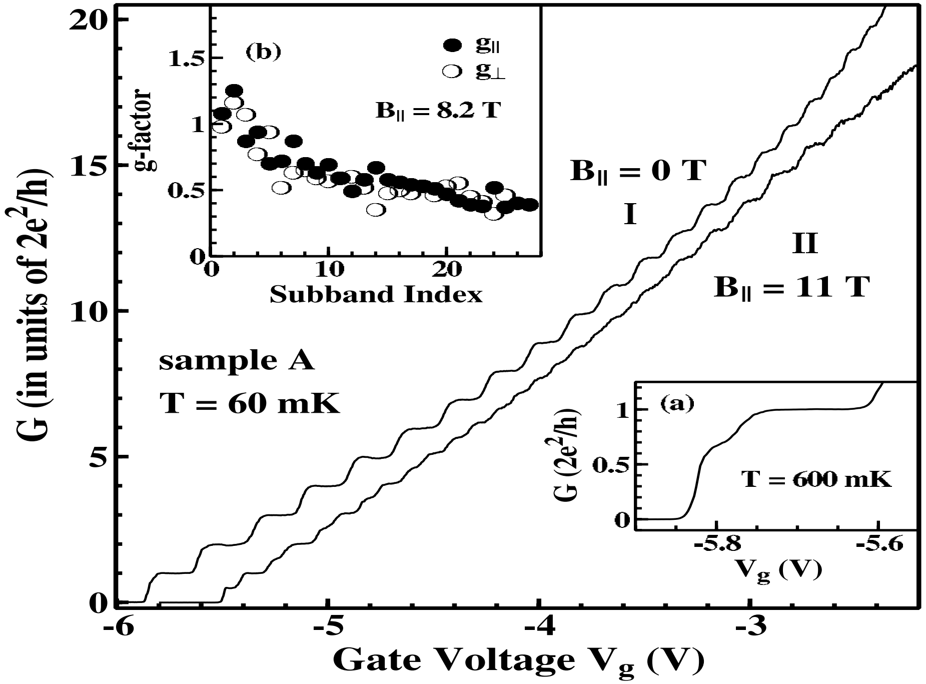


Ashoori sample in GaAs

Single-mode QPC with 2 spin channels

In-plane magnetic field: Zeeman $t_{\uparrow} > t_{\downarrow}$

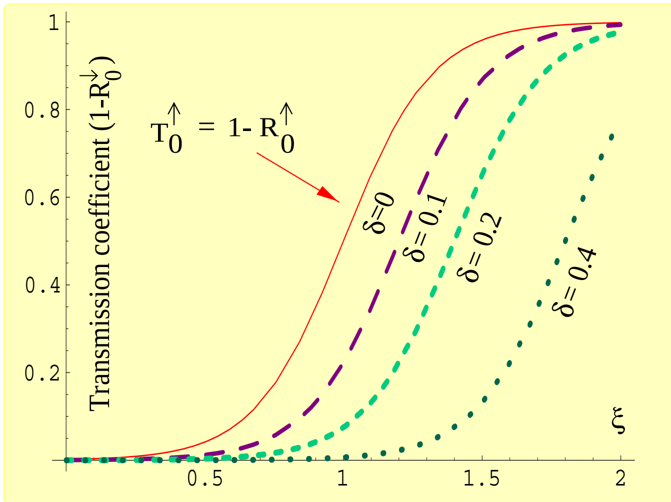
Strong-field: Spin filter!



Pepper's group

Zero field: extra plateau 0.7?

Strong field: plateau $0.5(2e^2/h)$



QPC; harmonic potential
ξ linear in V_g

K. Le Hur, PRB 161302R (2001)

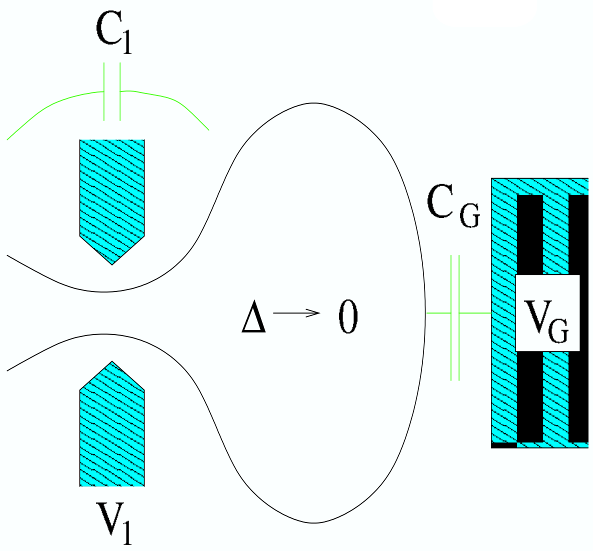
Connor (1968)

Glazman, Lesovik, Khmel'nitskii, Shekter (1988)

Cyclotron effects: Fertig and Halperin (1987), Büttiker (1990)

COULOMB BLOCKADE

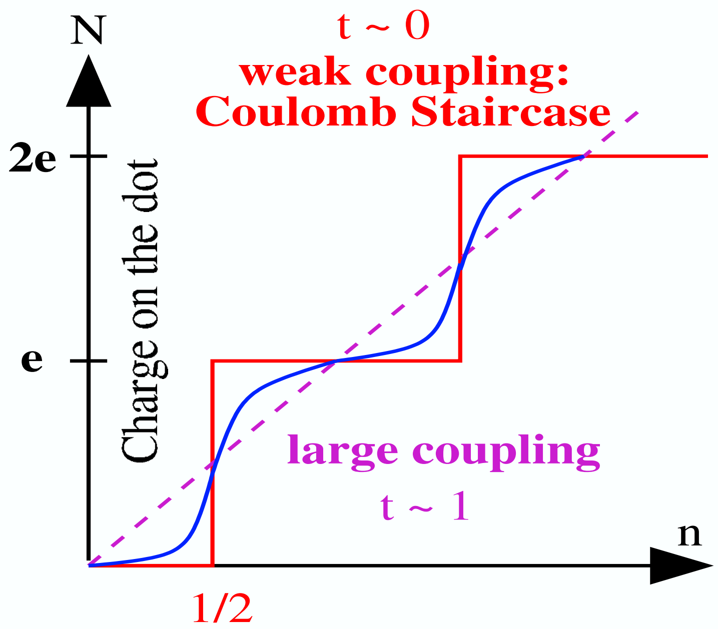
Large dot, low T: $\Delta \to 0, \quad T << E_C$



one-terminal geometry:
study equilibrium properties

$$\langle Q \rangle, \quad C = \partial \langle Q \rangle / \partial V_G$$

two limits: $t \to 0, \quad r \to 0$



Hamiltonian:

$$H = H_{2DEG} + H_{QD} + H_T + H_C$$

map on Kondo!

MAPPING ON KONDO

mapping of the tunneling Hamiltonian:

$$\psi^{+}_{\sigma D}\psi_{\sigma L}P_0 \Rightarrow \psi^{+}_{\sigma D}\psi_{\sigma L}S^{+} \Rightarrow s^{-}_{\sigma}S^{+}$$

$$\psi^{+}_{\sigma L}\psi_{\sigma D}P_1 \Rightarrow \psi^{+}_{\sigma L}\psi_{\sigma D}S^{-} \Rightarrow s^{+}_{\sigma}S^{-}$$

$$|Q=0\rangle = |\downarrow\rangle$$

$$|Q=1\rangle = |\uparrow\rangle$$

the Coulomb Hamiltonian becomes

$$H_C \approx 0\cdot\hat{P}_0 - eU\hat{P}_1 = -\frac{eU}{2}(\hat{P}_1 - \hat{P}_0) - \frac{eU}{2}(\hat{P}_0 + \hat{P}_1) = \frac{eU}{2}(2S_z - 1)$$

charge on the dot:

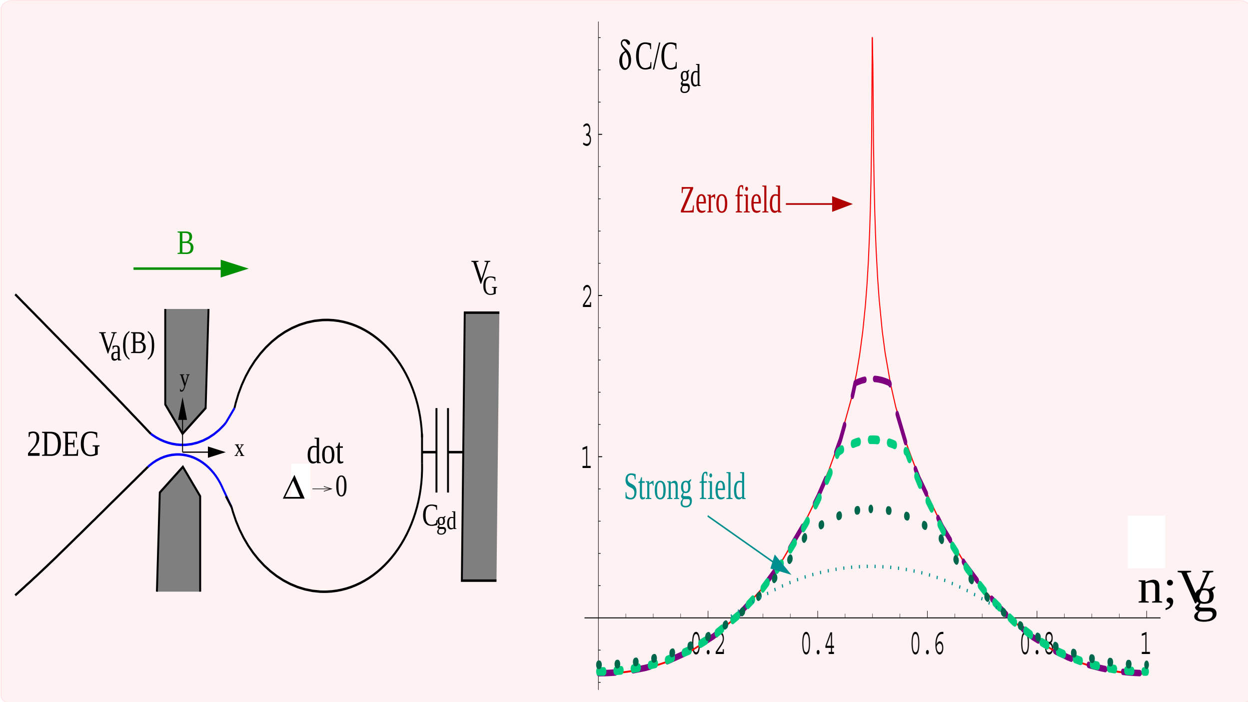
$$\langle Q\rangle = e\langle\hat{P}_1\rangle = e\left(\frac{1}{2} - \langle S_z\rangle\right)$$

K.A. Matveev

JETP 72, 892 (1991)

PRB 51, 1734 (1995)

Capacitance versus field



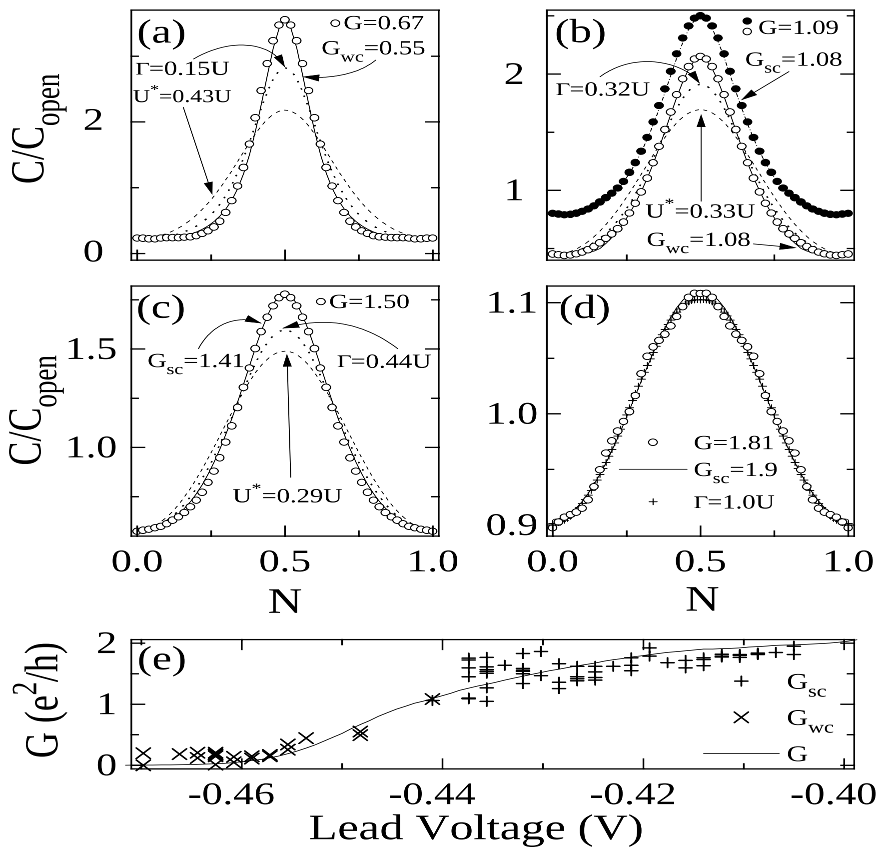
Magnetic field = anisotropic two-channel Kondo model!

Strong magnetic field = one-channel Kondo model (Fermi liquid)

K. Le Hur and G. Seelig, PRB 65, 165338 (2002)

Coulomb peaks subsist until $t_\uparrow \to 1$

Observat^o of 2-channel Kondo realm?



PRL 82, 161 1999
MIT group - Ashoori

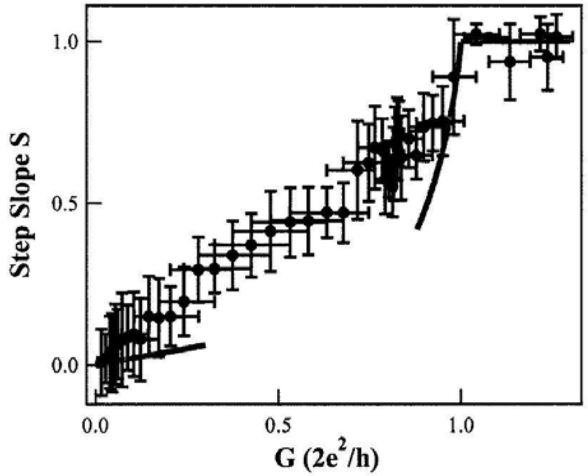
Datas from Berman et al.: No Magnetic field!
Problem: To find a reasonable T_K!

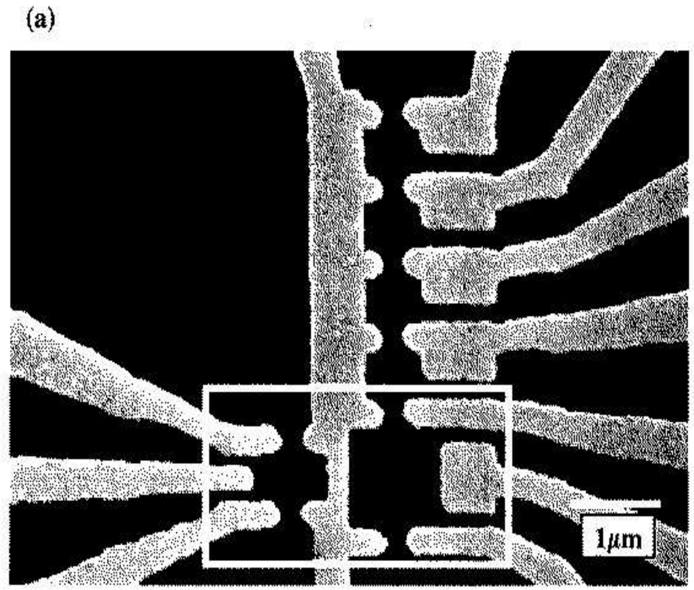
dots: perturbative results of Kostya Matveev and Hermann Grabert

More experiments

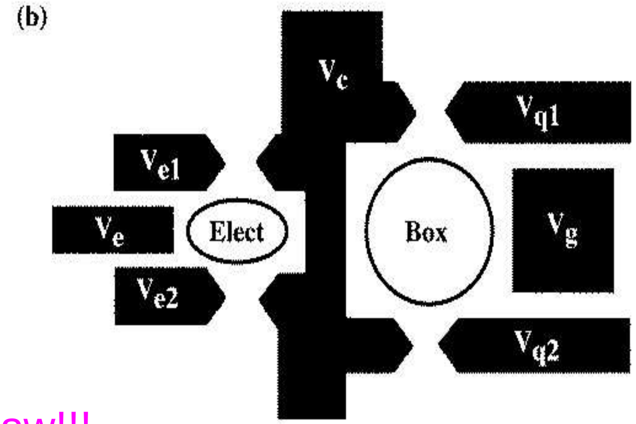
Westervelt et al. (Harvard)

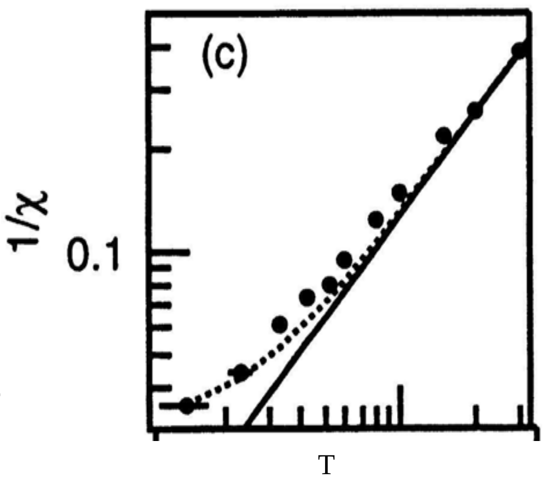
staircase still close to perfect transmission





Lehnert-Schoelkopf (SC qubit in field)
(many modes)





Log-deviations from Curie law!!!

“Quantum noise”: A brief detour

Identifications:

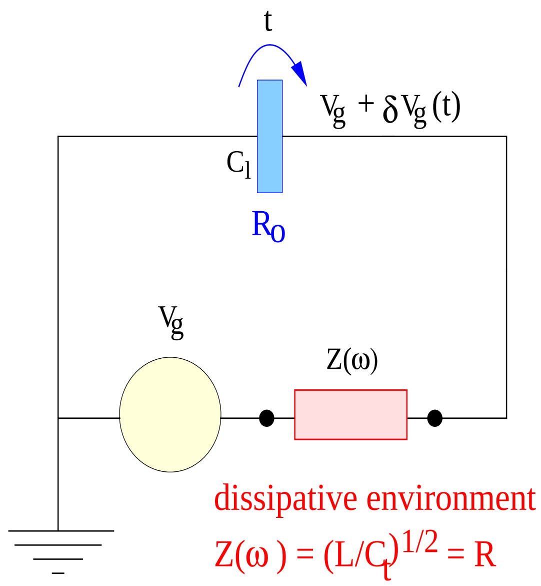
After diagonalization,

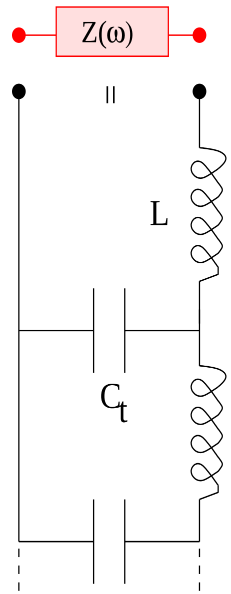
$\delta V_g(t) = \gamma \sum_j \lambda_j X_j(t)$

$\varphi(t) = (e/\hbar) \int_{-\infty}^{t} \delta V_g(t') dt'$

sum of the momenta P_j

Important: δV_g & φ conjugate!





e.g. Josephson junction array

Quantum Hamiltonian for the transmission line:

$$H_B = \sum_{j=1}^{+\infty} \left(\frac{{P_j}^2}{2M} + \frac{M{\omega_j}^2}{2} {X_j}^2 \right)$$

Ohmic bath: $J(\omega) = \gamma^2 \sum_j {\lambda_j}^2 \pi \delta(\omega - \omega_j)/(2M\omega_j) = \hbar R \omega$

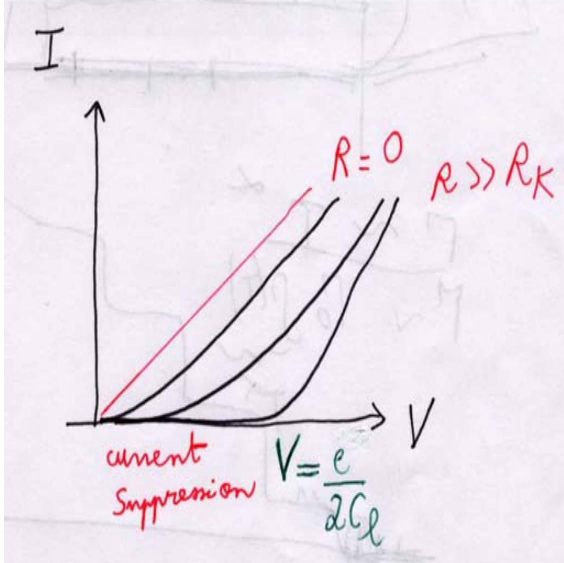
Charge accumulation at the junction

$\hat{H}_t = t\sum_{k,q} c_k^\dagger c_q e^{i\varphi} + h.c.$

Important:

$P(E) = \frac{1}{2\pi\hbar}\int dt e^{i\frac{E}{\hbar}t} < e^{i\varphi(t)} e^{-i\varphi(0)} >$

probability that an inelastic tunnel event occurs or "photon" of energy E is emitted to the bath



$$P(E) \approx \frac{\exp -(\gamma/\alpha)}{\Gamma(\alpha)} \frac{1}{E} \left|\frac{2\pi}{\alpha} E\right|^{1/\alpha}$$

$I(V \to 0) \sim |V|^{1+1/\alpha}$ **and** $\alpha = R_K/(2R)$

$R_K = h/e^2 = 26k\Omega$ quantum of resistance

Clear observation in Cleland et al. PRL (1990) using thin film resistors
Devoret, Esteve, Grabert et al. (1990,1998); Nazarov and Ingold (1992)

VOLUME 64, NUMBER 13 PHYSICAL REVIEW LETTERS 26 MARCH 1990

Charge Fluctuations in Small-Capacitance Junctions

A. N. Cleland, J. M. Schmidt, and John Clarke

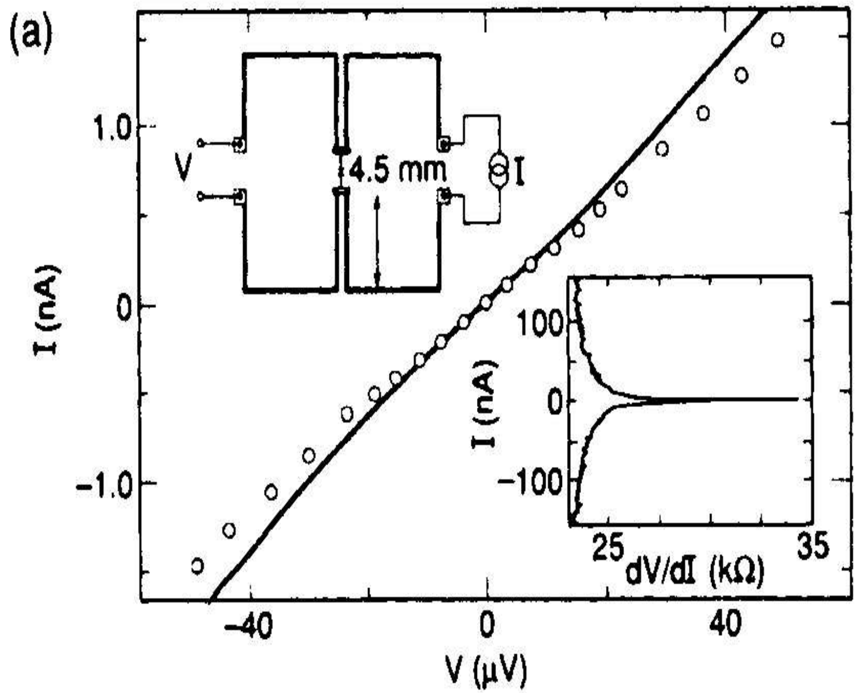
Department of Physics, University of California, Berkeley, California 94720
and Materials and Chemical Sciences Division, Lawrence Berkeley Laboratories, Berkeley, California 94720

(Received 18 December 1989)

The current-voltage characteristics of submicron normal-metal tunnel junctions at millikelvin temperatures are observed to exhibit a sharp Coulomb blockade with high-resistance thin-film leads, but to be heavily smeared for low-resistance leads. As the temperature is lowered, the zero-bias differential resistance tends asymptotically to a limit that is greater for junctions with high-resistance leads. Both observations are explained in terms of a model in which quantum fluctuations in the external circuit enhance the low-temperature tunneling rate. The predictions are in reasonable agreement with the data.

It is predicted theoretically[1-4] and well established experimentally[5-14] that submicron tunneling junctions at low temperatures T can, under appropriate conditions, exhibit charging effects due to the discreteness of the electronic charge. In particular, when the charging energy $E_C = e^2/2C$ associated with the tunneling of a single electron across a capacitance C becomes large compared with $k_B T$, one may observe suppression of the tunnel current I at voltages $V < e/2C$. As a result, the I-V characteristic at higher voltages is offset by the Coulomb gap, $e/2C$. However, observation of these effects depends strongly on the nature of the environment coupled to the junction. For example, Delsing *et al.*[12] varied the environment by studying two kinds of circuits. In the first, metallic leads were coupled directly to the junction while in the second, linear arrays of submicron junctions were placed in each lead to the junction under study.

Zero-bias anomaly!



Noise in the single-electron box...

What's happening when combining "noise-induced" (resistor) and double-junction Coulomb effects? *amplification of Coulomb blockade?*

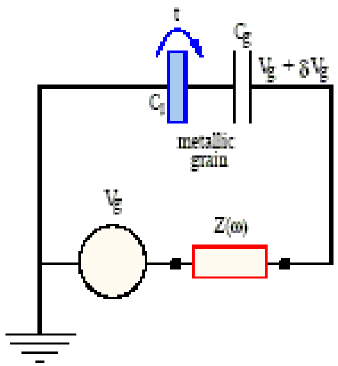
Interesting points

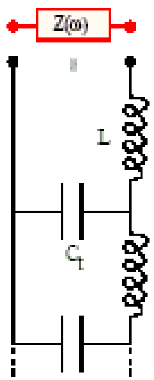
How to include noise effect in the Matveev scheme?

Close to perfect transmission?

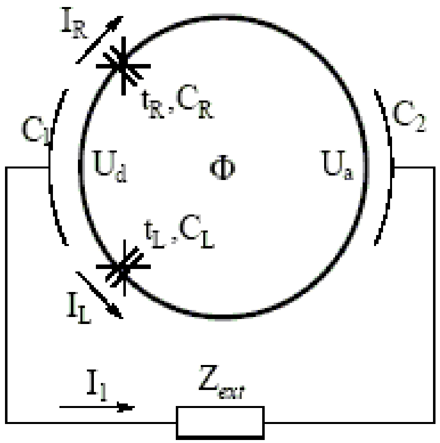
P. Cedraschi and M. Büttiker

Annals of Physics **289**, *1-23 (2001)*

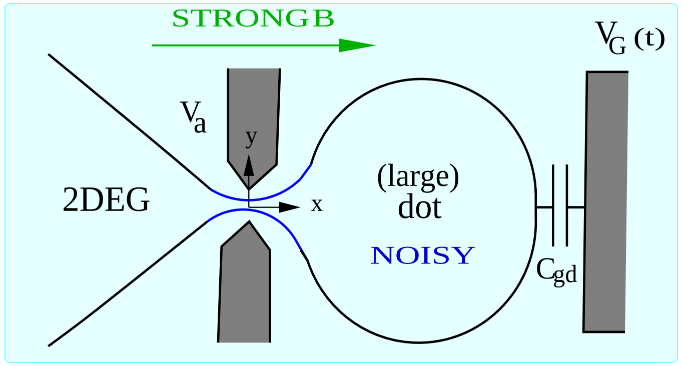




very dense spectrum
and semi-∞ lead



Here is a useful mapping



Ohmic dissipative bath:

- $\gamma^2 = R/R_K$ and $R_K = 26K\Omega$
- $\delta V_g(t) = \gamma \sum_j \lambda_j X_j(t) = \gamma\Phi(t)/e$
- $J(\omega) = \gamma^2 \sum_j {\lambda_j}^2 \pi\delta(\omega - \omega_j)/(2M\omega_j) = \hbar R\omega$

$$\mathbf{H} = \left(\sum_{\alpha=L,D} H^{\alpha}_{Kin} - hS^z\right) + H_B + \frac{J_\perp}{2}\left(s^+S^- + h.c.\right) - \gamma\Phi S^z$$

Map on anisotropic Bose-Fermi Kondo model

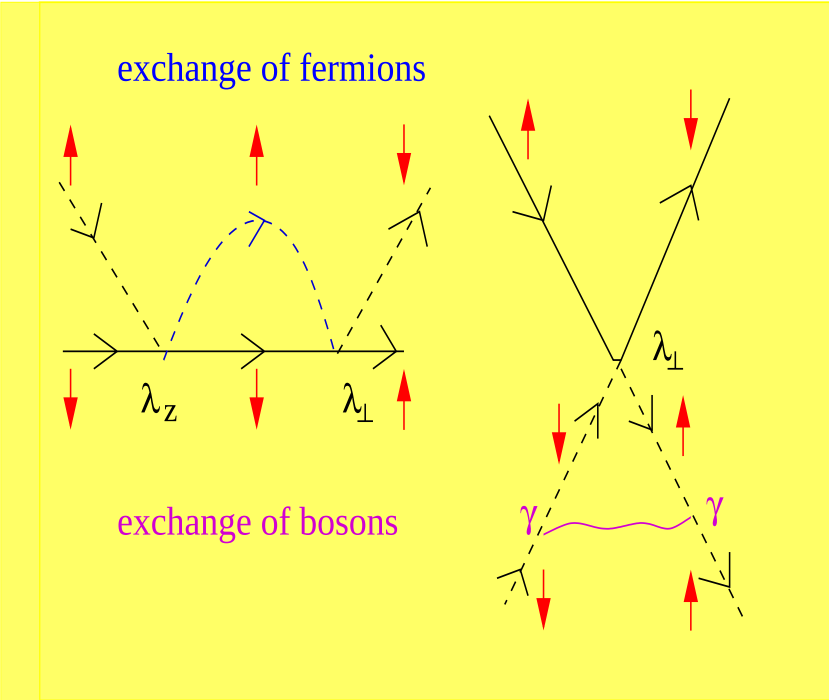
Zarand & Demler, PRB 66, 024427 (2002)
Zhu & Si, PRB 66, 024426 (2002)
Kircan & M. Vojta, cond-mat/0312150

K. Le Hur, cond-mat/0312292 and PRL 2004

RG analysis

Pedestrian RG equations:

$$\frac{d\lambda_\perp}{dl} = \lambda_\perp \lambda_z - \frac{\nu}{2}\lambda_\perp g_z$$
$$\frac{d\lambda_z}{dl} = \lambda_\perp{}^2$$
$$\frac{dg_z}{dl} = -g_z \lambda_\perp{}^2$$

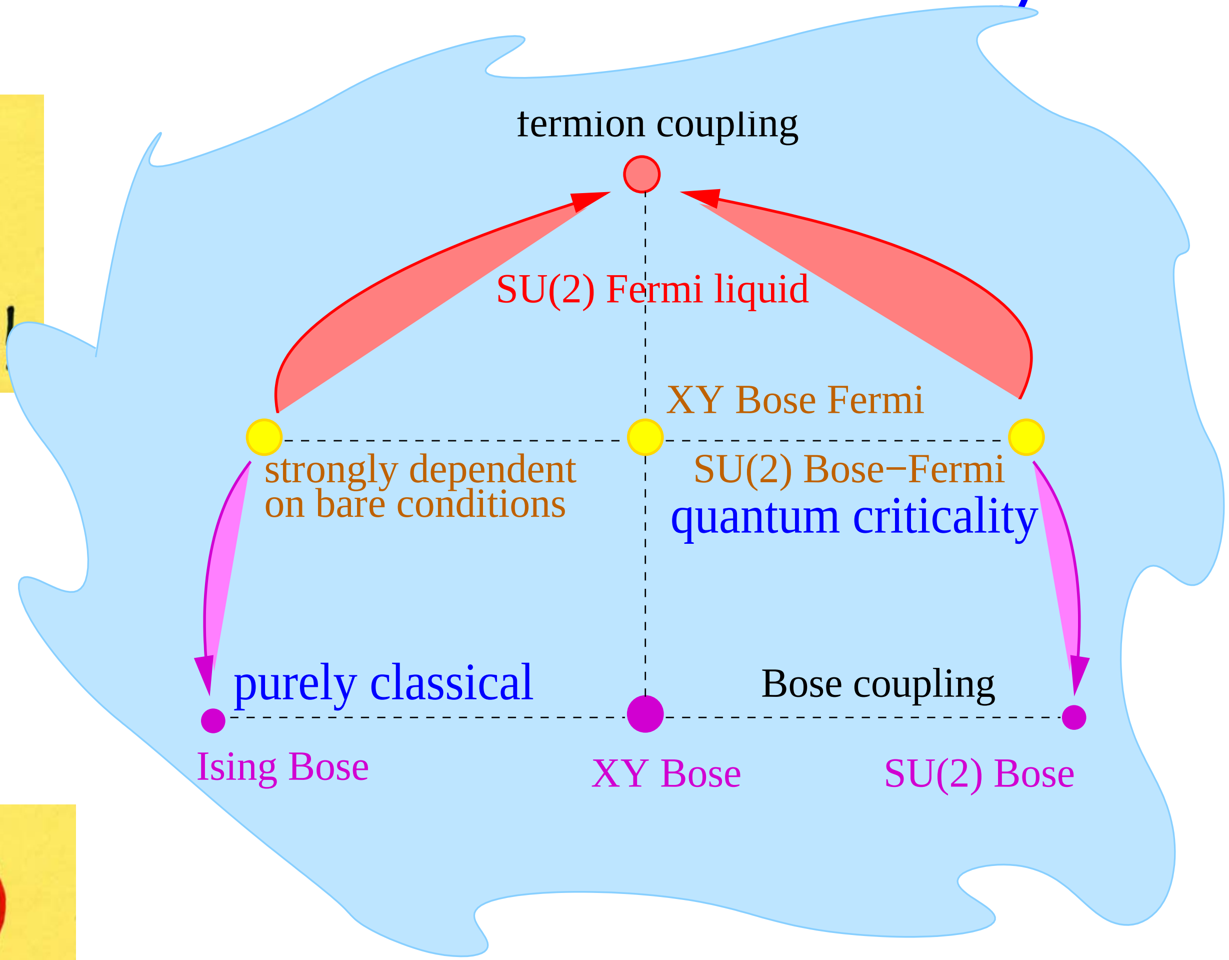


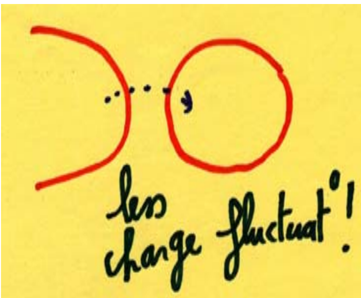
$l = \ln(D_o/T)$, $\mathbf{g_z} = \gamma^2 = \frac{\mathbf{R}}{\mathbf{R_K}}$, and $\lambda_\perp = 2|t|N_{dot}$

No transverse coupling between orbital spin and bosons

Kosterlitz-Thouless flow in terms of $\lambda_\perp$ and $\hat{\lambda}_z = \lambda_z - (\nu/2)\gamma^2$

Sketch of the story

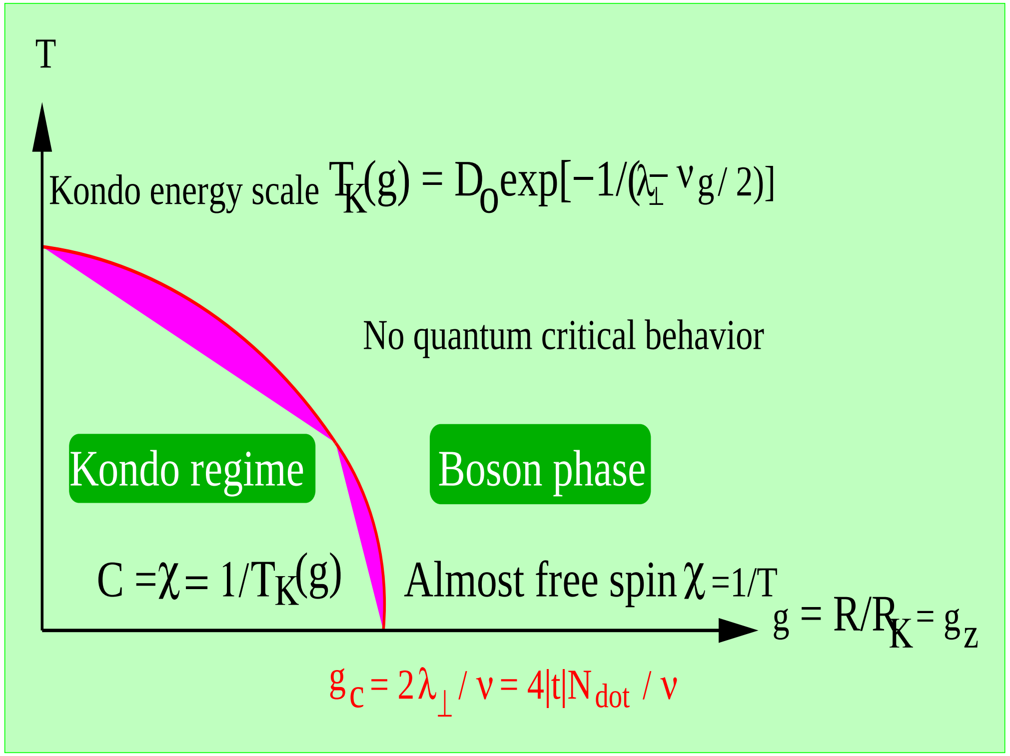




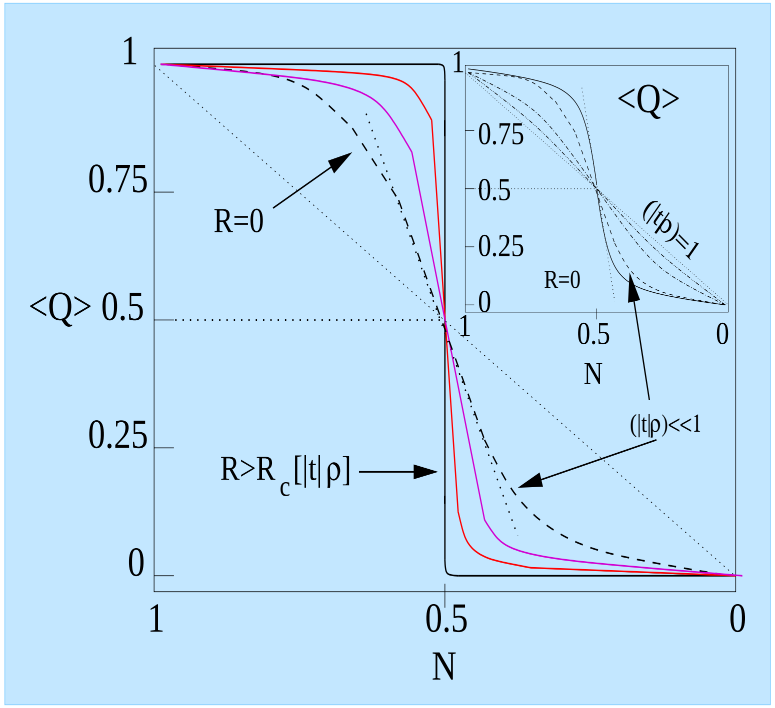
$g < g_c = 2\lambda_\perp/\nu$: SU(2) Fermi liquid $\lambda_\perp(l) = \lambda_z(l)$ large and $g_z(l) = 0$

$g > g_c = 2\lambda_\perp/\nu$: Ising Bose liquid $\lambda_\perp(l) = \lambda_z(l) = 0$ and $g_z(l)$ bare

Phase diagram / Results



$$H = -hS^z - \gamma < \Phi > S^z$$



Already visible at accessible T:
Indeed suppression of the log corrections in $\chi = C$!

Another approach for mesoscopists

Absorb noise in the tunneling (Kondo) term

$$\hat{H}_t = J_\perp e^{i\varphi} s^+ S^- + h.c.$$

J_z will be renormalized by $J_\perp^2 \int d\tau \; sgn(\tau) \; G(\tau) \; e^{K(\tau)}$

$G(\tau) = 1/\tau$ *fermionic imaginary time propagator*

$K(\tau) = \langle \varphi(\tau)\varphi(0) - \varphi(0)^2 \rangle = -2(R/R_K) \ln (\omega_c|\tau|)$

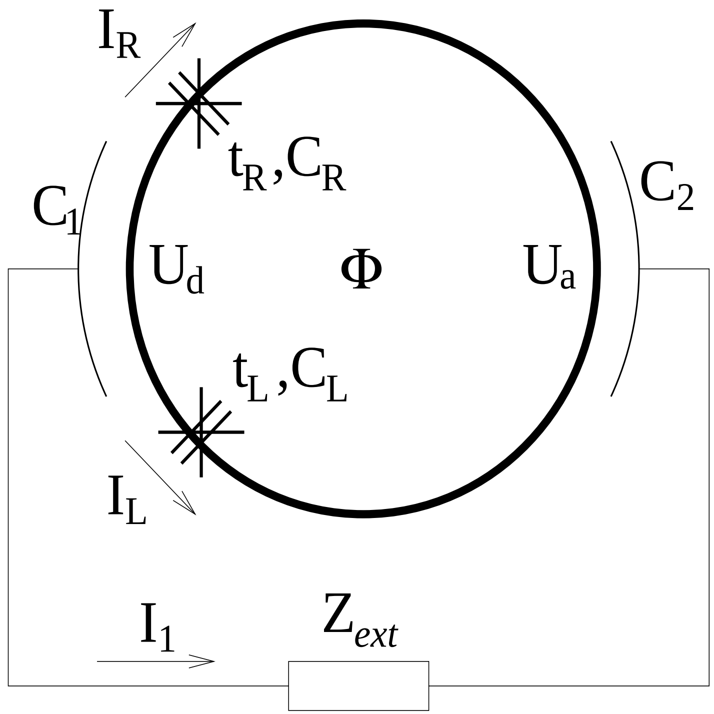
Scaling dimension $\delta = 1 + 2R/R_K$

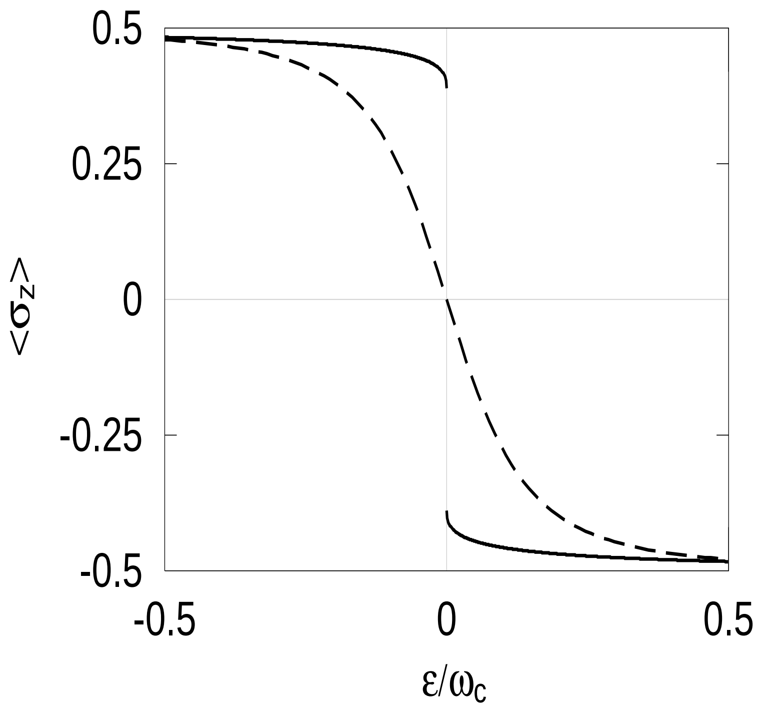
Irrelevant for sure when R/R_K is $\mathcal{O}(1)$

Restoration of a nice staircase by increasing R

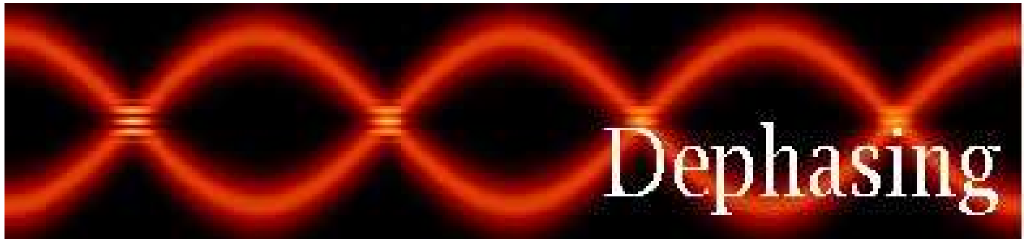
Universality in the results

Grain with dense spectrum ↔ Small dot with 1 level
Semi-∞ lead ↔ mesoscopic ring with 1 level





P. Cedraschi and M. Büttiker, Annals of Physics **289**, *1-23 (2001)*
P. Cedraschi, V. Ponomarenko, and M. Büttiker, PRL **84**, *346 (2000)*



$$H=\frac{\hbar\epsilon}{2}S^z-\hbar\Delta_o 2S^x+eS^z\delta V_g(t)+H_B$$

Analogy with a superconducting qubit

Time-evolution of the off-diagonal element

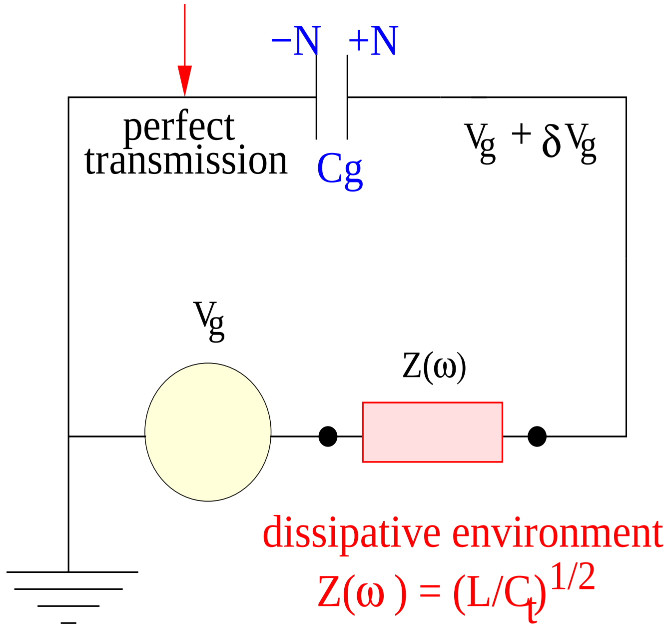
$< S^+(t) > \; = e^{i\frac{\epsilon}{2}t}G(t) < S^+(0) >$ where

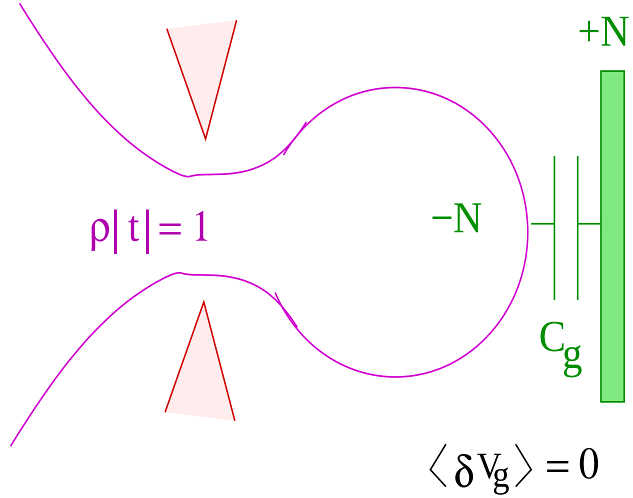
$$G(t) \; = < \exp\left(-\frac{i}{\hbar}\int^t \delta V_g(t')dt'\right) > \; \approx < \exp i\varphi(t) >$$

decays like a power-law in time at zero temperature
at finite T, dephasing rate proportional to T

Makhlin, G. Schön, and Shnirman, cond-mat/0309049
Schoelkopf, Clerk, Girvin, Lehnert, and Devoret, cond-mat/0210247

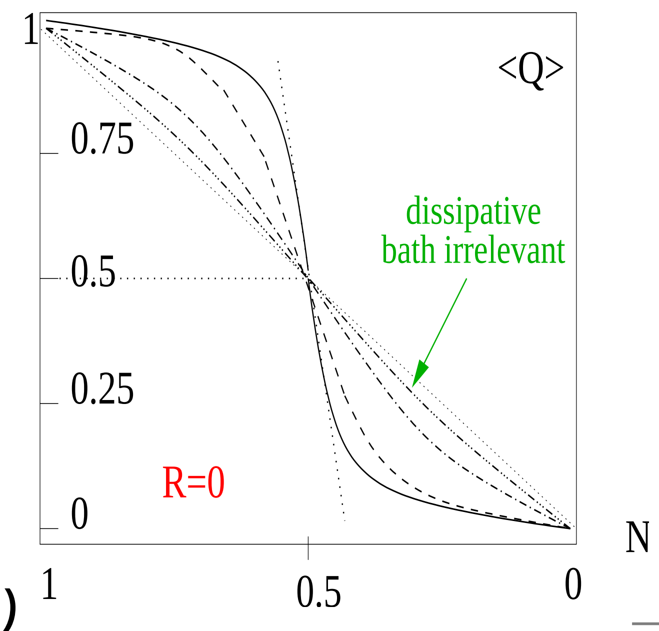
Close to perfect transmission





Friedel sum rule $\delta = \pi \langle Q \rangle / e = \pi N$

$\rho(x) \approx E_c / (\hbar v_F) \cos(2k_F|x| - 2\delta)$



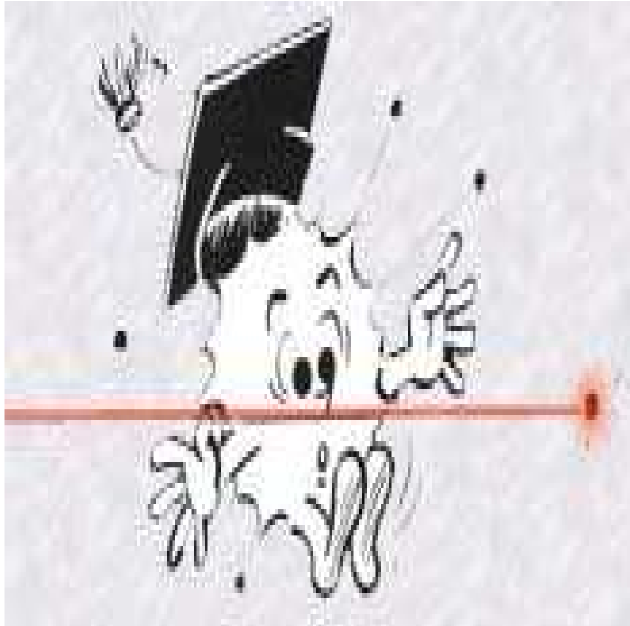
Shift in energy

$-\int dx\, \rho(x) V(x) \delta(x) \approx -|r| E_c \cos(2\pi N)$

with $|r|^2 = 1 - (\rho|t|)^2$

I. Aleiner and L. Glazman, PRB 57 9608 (1998)

In closing

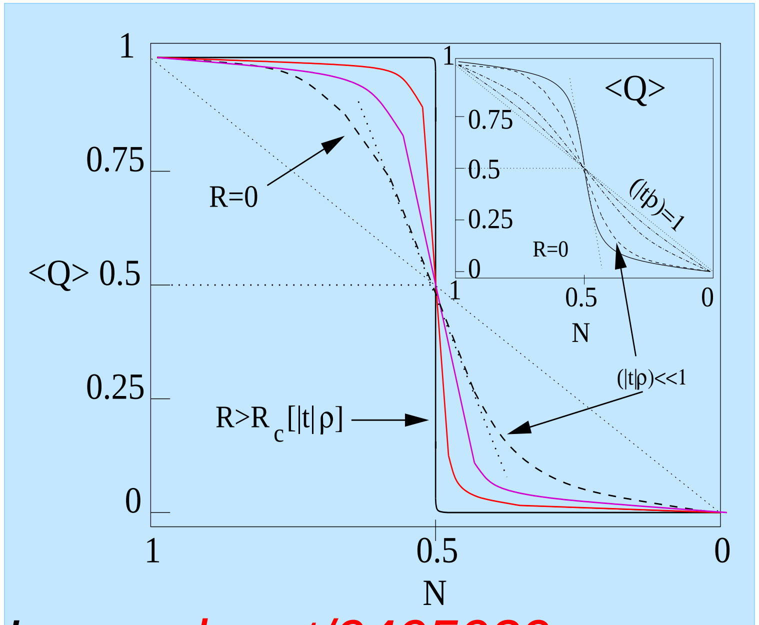


Atomic system (dot) coupled to 2 baths:
Photons (Bosons) and Matter (Electrons)

Zero-point fluctuations of "photons" affect the ground state of the dot

New effects in small tunneling regime!

Effect visible already at finite T
No log-corrections for capacitance
(free spin at zero temperature)



Sub-Ohmic bath? *Q. Si et al.*
Two-bath problem? *Meirong Li and K. Le Hur* *cond-mat/0405039*