



the  
**abdus salam**  
international centre for theoretical physics

*ICTP 40th Anniversary*

SMR.1572 - 11

**Workshop on  
Novel States and Phase Transitions in Highly Correlated Matter**

**12 - 23 July 2004**

---

**Towards a theory of the non-Fermi liquid phase in MnSi**

**Achim ROSCH  
University of Cologne  
Institute of Theoretical Physics  
Zulpicher Strasse 77  
D-50937 Cologne  
GERMANY**

---

These are preliminary lecture notes, intended only for distribution to participants

# Towards a theory of the non-Fermi liquid phase in MnSi

Achim Rosch, Markus Garst, Inga Fischer

Institut für Theoretische Physik, University of Cologne

Experiments: Christian Pfleiderer, University of Karlsruhe

- A non-Fermi liquid **phase** in MnSi?  
Review of experiments
- Topological defects?
- Anomalous overdamped (pseudo-) Goldstone modes?
- Band structure in a spiral

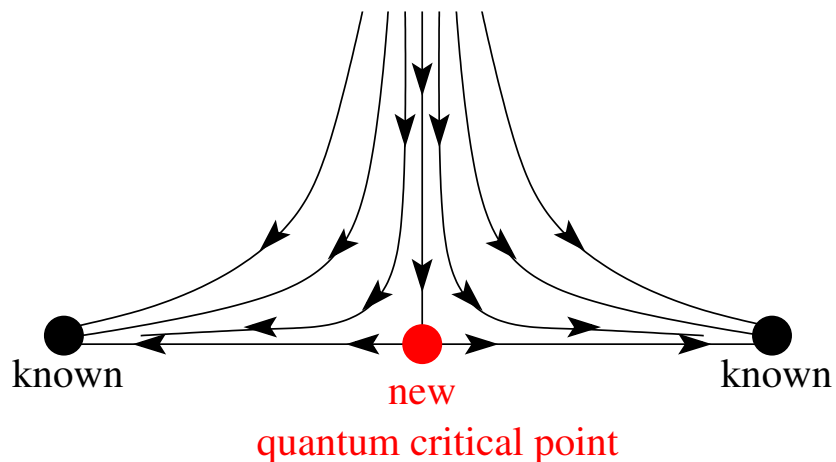
Disclaimer:

Work in progress: many questions – few answers

# Towards "novel phases" in metals

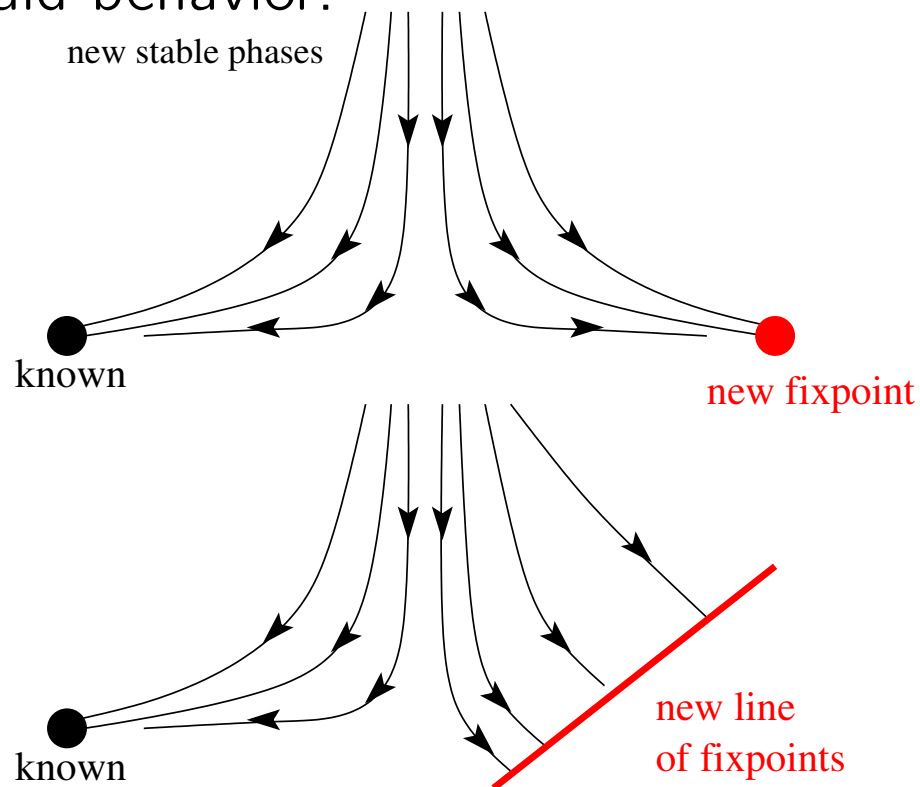
from Fermi liquids to non-Fermi liquid behavior:

instability between two phases by fine-tuning

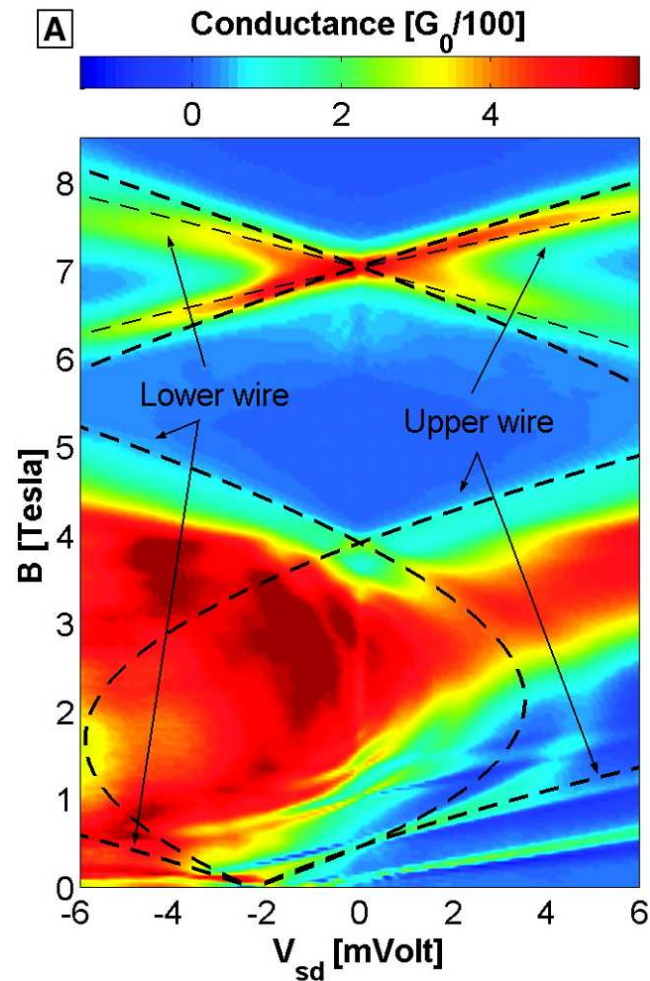
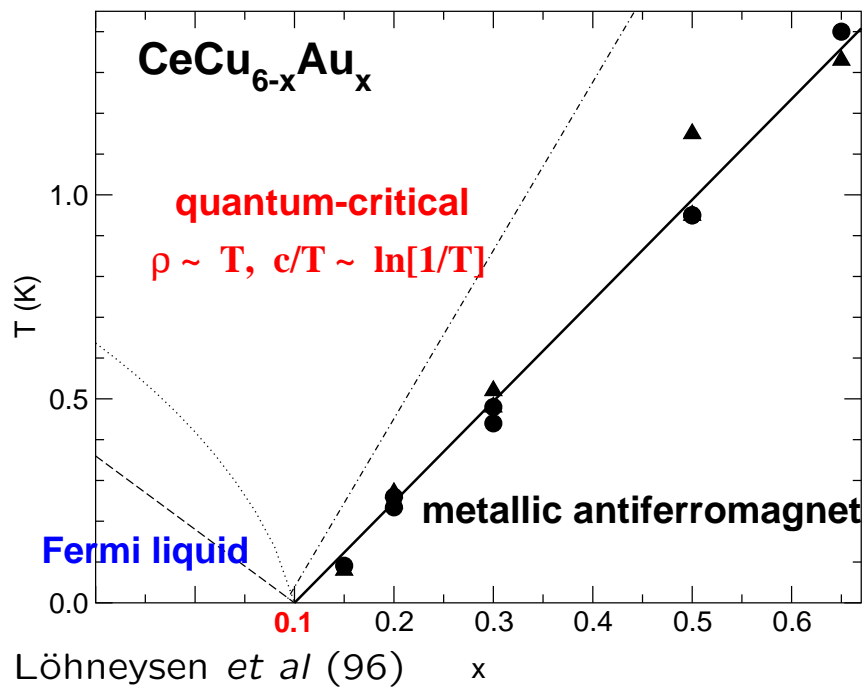


critical fluctuations, power-laws, scaling, ...  
dozens of systems, e.g.  $\text{CeCu}_{6-x}\text{Au}_x$ ,  $\text{YbRh}_2\text{Si}_2$ ,  $\text{CePd}_2\text{Si}_2$ ,  $\text{NiS}_{2-x}\text{Se}_x$  ...  
diverging Grüneisen parameter

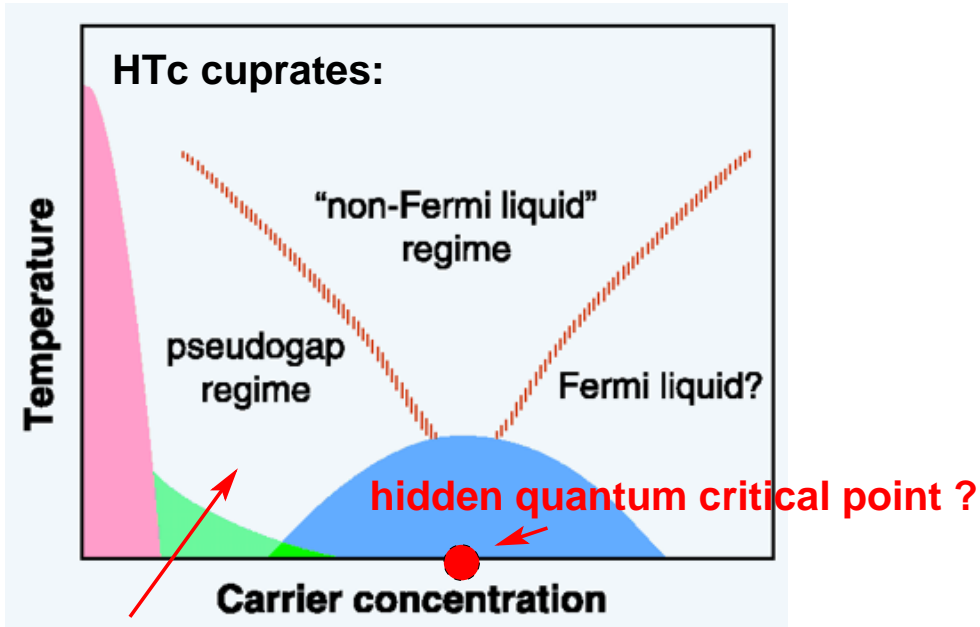
new stable phases



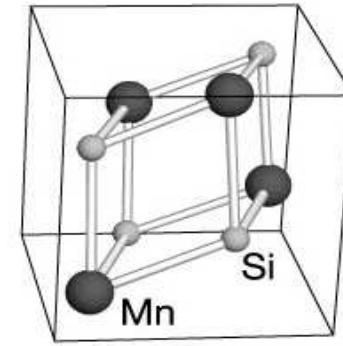
new quasiparticles, quantum number fractionalization  
Luttinger liquids, fractional QHE, nematic metal, Griffiths singularities...  
**low dimensions!**



spectroscopy of Luttinger liquids  
 by parallel tunneling  
 Auslaender, Yacoby *et al.*

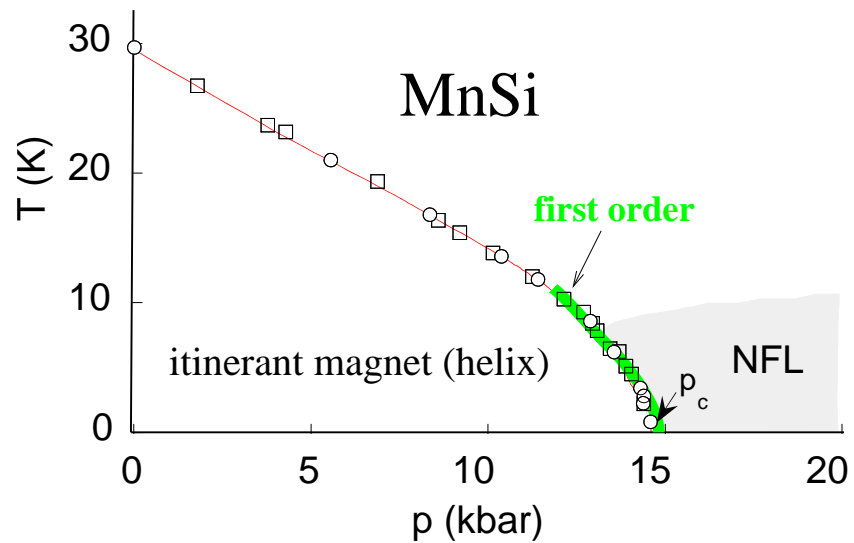


## MnSi – a standard itinerant magnet



- text-book band-magnet below 30K  
(see Landau-Lifschitz, Vol.8, 3<sup>rd</sup> edition)
- Ginzburg-Landau theory for helical spin-density wave:  
Bak, Jensen (1980), Nakanishi *et al.* (1980)
- extremely clean (mean free path 3000-10000 Å)
- cubic but no inversion symmetry ( $P2_13$ )
- standard example for spin-fluctuation theory (Lonzarich, Moriya)

**New physics under pressure!**



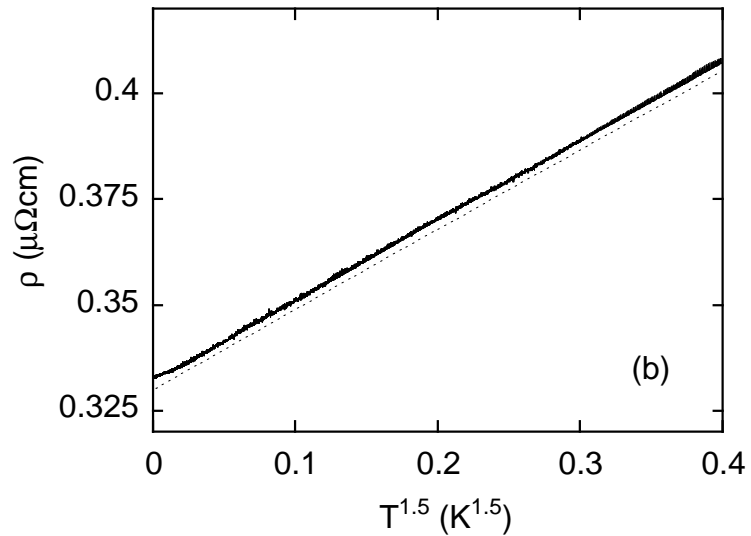
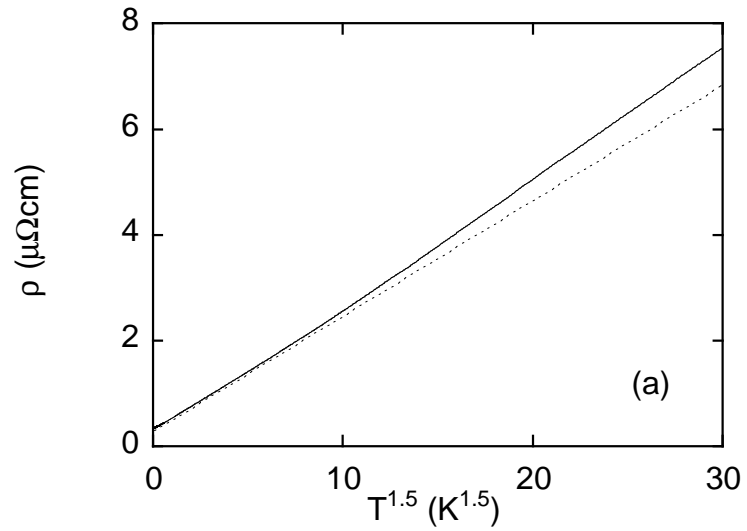
C. Pfleiderer, Julian, Lonzarich, Nature (2001):  
 in MnSi for wide pressure range,  $p > p_c$

$$\rho(T) - \rho_0 \sim T^{3/2}$$

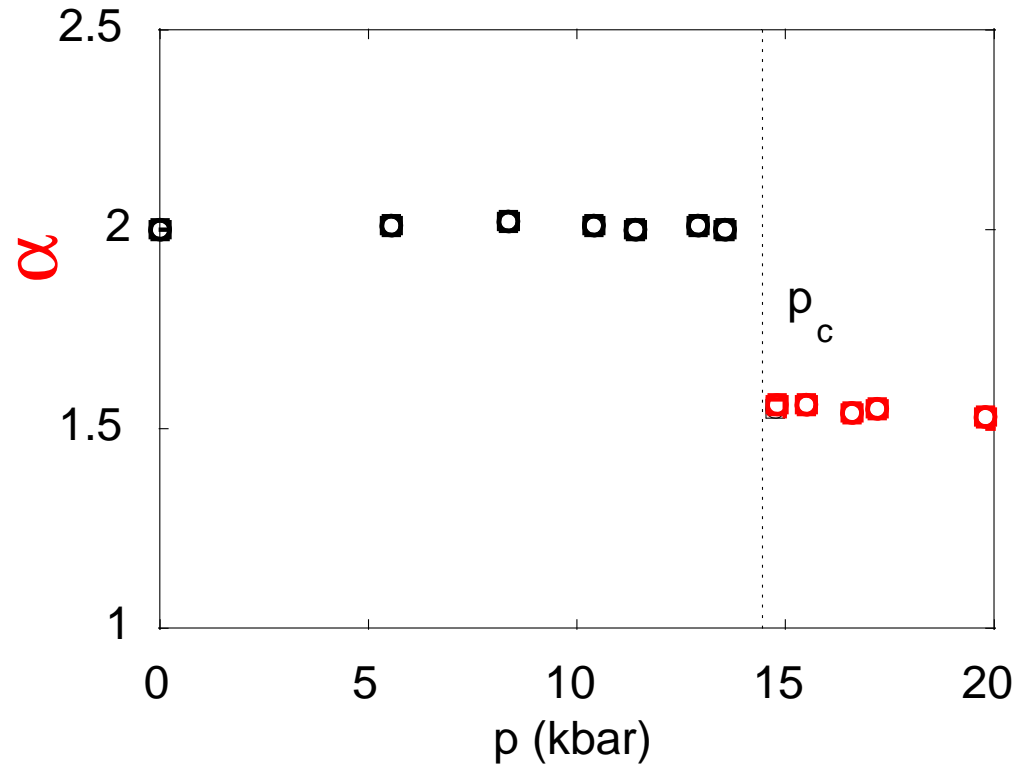
almost **3 decades** in  $T$ , very clean system

A genuine non-Fermi liquid phase?

resistivity  $\rho(T) \sim T^{3/2}$  for almost 3 decades (10mK to 5K):

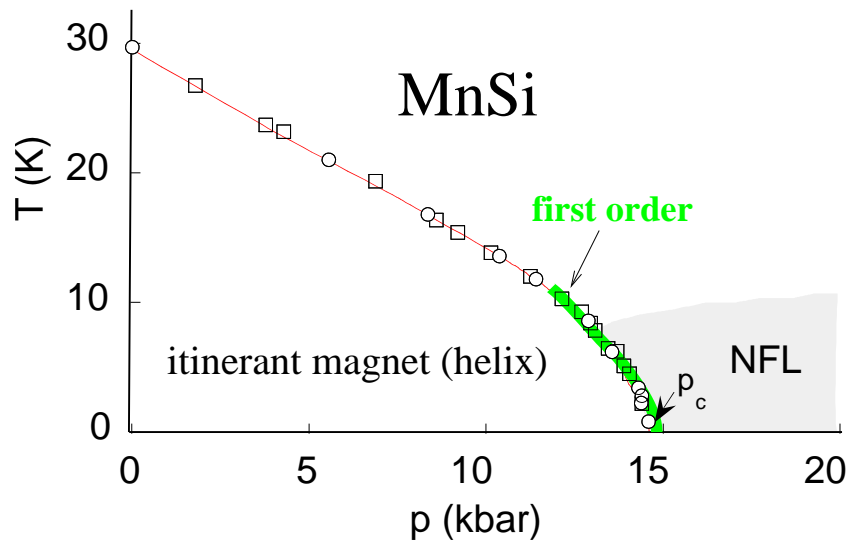


exponent  $\alpha$  for  $\Delta\rho(T) \sim T^\alpha$  for  $T \rightarrow 0$



$\rho(T) \sim T^{3/2}$  for  $p > p_c$  up to 28 kbar

(Lonzarich)



C. Pfleiderer, Julian, Lonzarich, Nature (2001):  
 in MnSi for wide pressure range,  $p > p_c$

$$\rho(T) - \rho_0 \sim T^{3/2}$$

almost **3 decades** in  $T$ , very clean system

A genuine non-Fermi liquid phase?

Alternative: **Quantum critical** behavior?

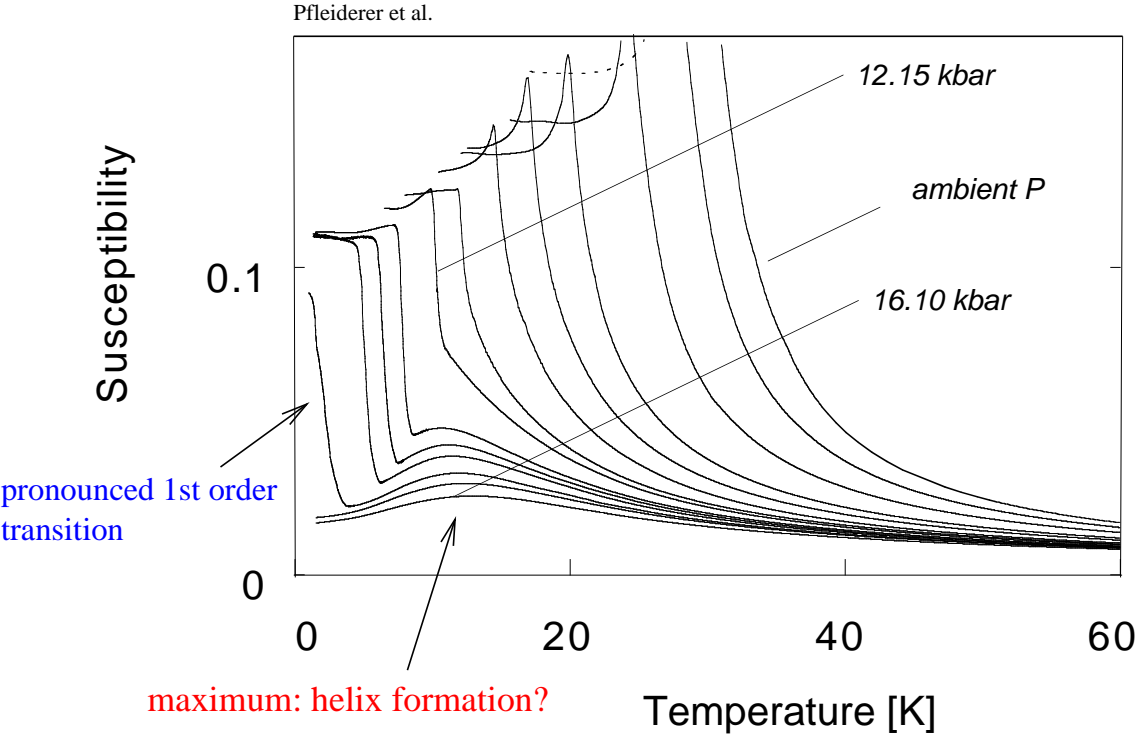
**contra**: NFL observed even for  $p \gg p_c$  and low  $T$

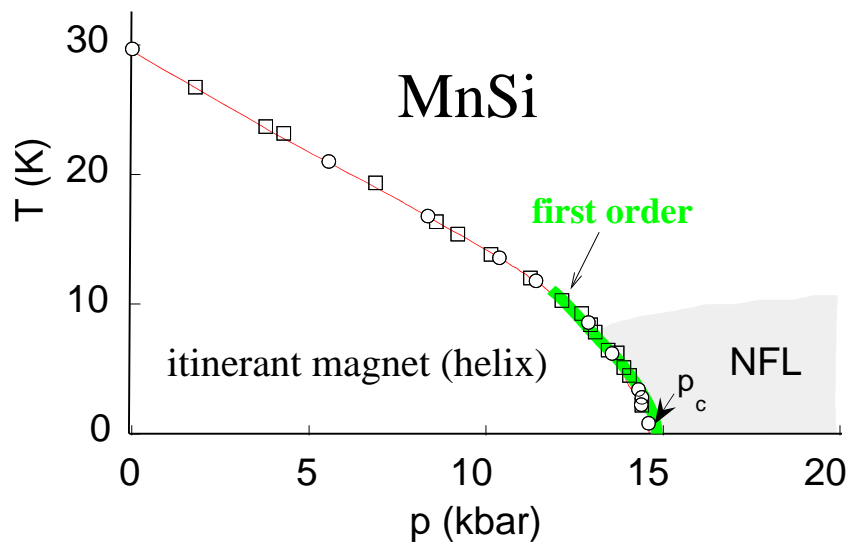
**first order** transition close to  $p_c$

jump of  $\chi$  and of local moment in NMR,  $\mu$ SR (Thessieu *et al.* 98)



# Susceptibility, helix formation and first order transition





C. Pfleiderer, Julian, Lonzarich, Nature (2001):  
 in MnSi for wide pressure range,  $p > p_c$

$$\rho(T) - \rho_0 \sim T^{3/2}$$

almost **3 decades** in  $T$ , very clean system

A genuine non-Fermi liquid phase?

Alternative: **Quantum critical** behavior?

**contra:** NFL observed even for  $p \gg p_c$  and low  $T$

**first order** transition close to  $p_c$

jump of  $\chi$  and of local moment in NMR,  $\mu$ SR (Thessieu *et al.* 98)

**pro:** maybe 2nd order endpoint at  $p = p_c$ ?

$A$ -coefficient,  $\Delta\rho(T) \approx AT^2$ , diverges for  $p \rightarrow p_c$  in ord. phase

Neutrons:  $T = 0$  ordered moment vanishes continuously for  $p \rightarrow p_c$ ;

$\Rightarrow$  chimera of first and second order? (cf.  $d = \infty$  Mott)

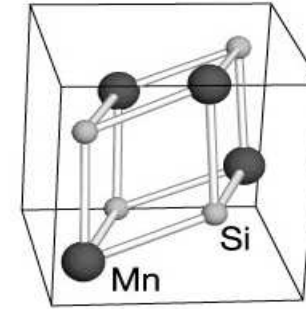
## Quantum critical theory

Joerg Schmalian and Misha Turlakov (03):

**NEXT TALK**

## Three distinct scales in MnSi:

- dominant: itinerant ferromagnet with large ordered moment ( $0.4\mu_B$ )  
 $\Rightarrow$  fixes amplitude of local magnetization



- but: instable to formation of chiral helix due to weak **spin-orbit** coupling in non-centrosymmetric crystal:

Dzyaloshinskii-Moriya interaction

$$q_0 \int \vec{S} \cdot (\vec{\nabla} \times \vec{S}) \quad \text{linear in } \vec{k}$$

$\Rightarrow$  fixes pitch  $1/q_0 \approx 150\text{\AA}$  of helix

- small correction: **spin-orbit** coupling breaks rotational symmetry in cubic crystal  
 $\Rightarrow$  fixes direction of helix

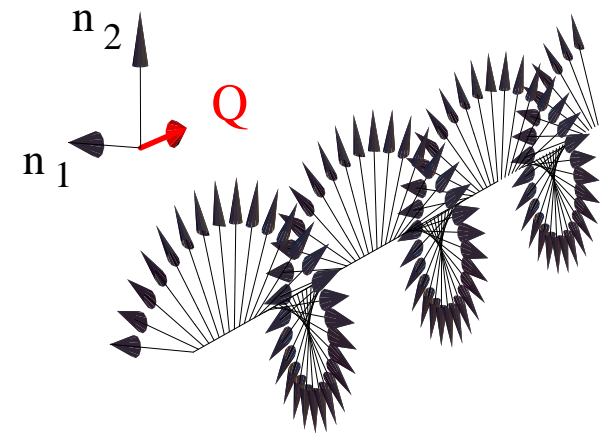
$$F = F(\vec{\Phi}^2) + \vec{k}^2 |\vec{\Phi}_{\vec{k}}|^2 + q_0 \vec{k} \cdot (\vec{\Phi}_{\vec{k}} \times \vec{\Phi}_{\vec{k}}^*) + k_x^4 |\vec{\Phi}_{\vec{k}}|^2 + k_x^2 \Phi_y^2 + \Phi_x^4 + \text{cycl.} + \dots$$

$$F = F(\vec{\Phi}^2) + \vec{k}^2 |\vec{\Phi}_{\vec{k}}|^2 + q_0 \vec{k} \cdot (\vec{\Phi}_{\vec{k}} \times \vec{\Phi}_{\vec{k}}^*) + k_x^4 |\vec{\Phi}_{\vec{k}}|^2 + k_x^2 \Phi_y^2 + \Phi_x^4 + \text{cycl.} + \dots$$

- first 3 terms minimized by **chiral** helix:

$$\vec{\Phi}(\vec{x}) = \Phi_0 [\hat{n}_1 \cos(\vec{Q}\vec{x}) + \hat{n}_2 \sin(\vec{Q}\vec{x})]$$

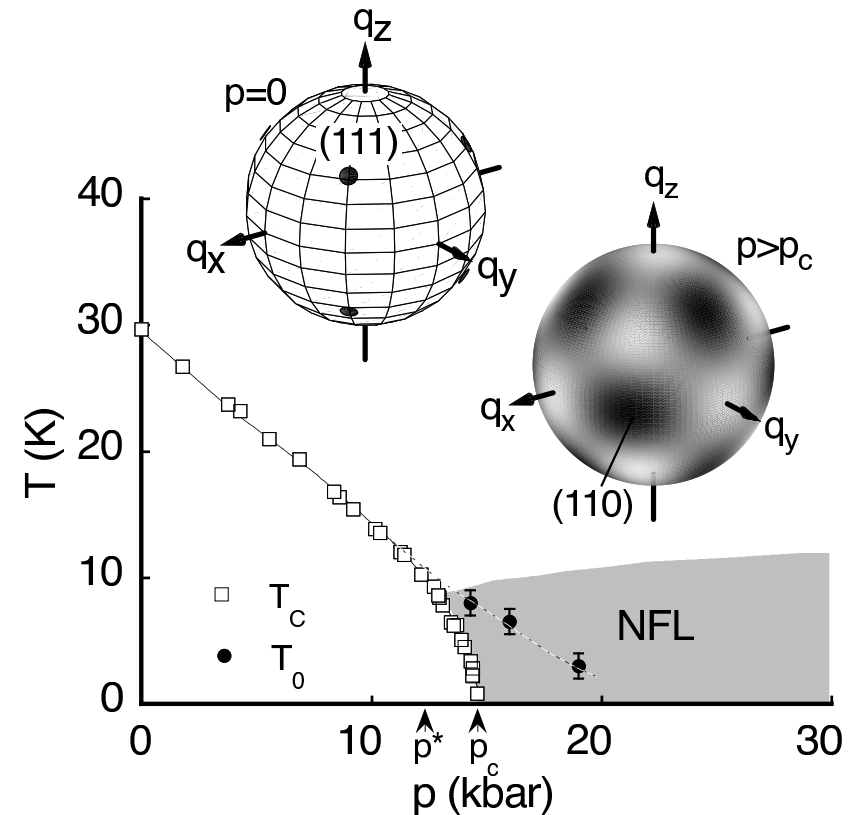
- $\hat{n}_1 \perp \hat{n}_2 \perp \vec{Q}$  form chiral “Dreibein”  
 $\hat{n}_1(\hat{n}_2 \times \vec{Q}) = \pm 1$  dep. on sign of  $q_0$
- pitch  $1/|\vec{Q}| = 1/q_0$  large (150Å) as spin-orbit coupling weak
- direction of  $\vec{Q}$  determined by **cubic terms**;  
 $\Rightarrow \vec{Q} \parallel (1, 0, 0)$  or  $\vec{Q} \parallel (1, 1, 1)$   
 $(\vec{Q} \parallel (1, 1, 0))$  only possible if  $\Phi^6, k^6$  important)



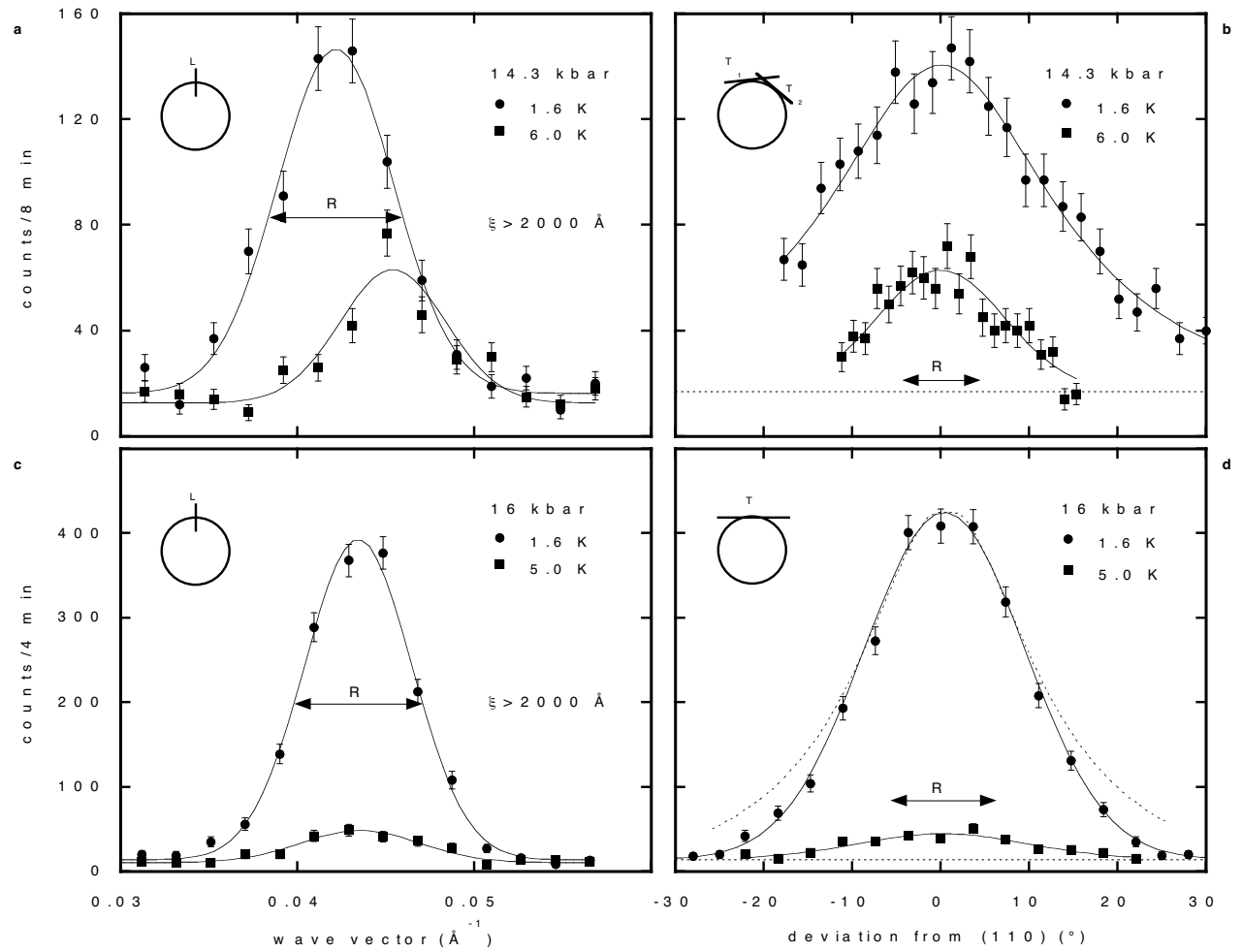
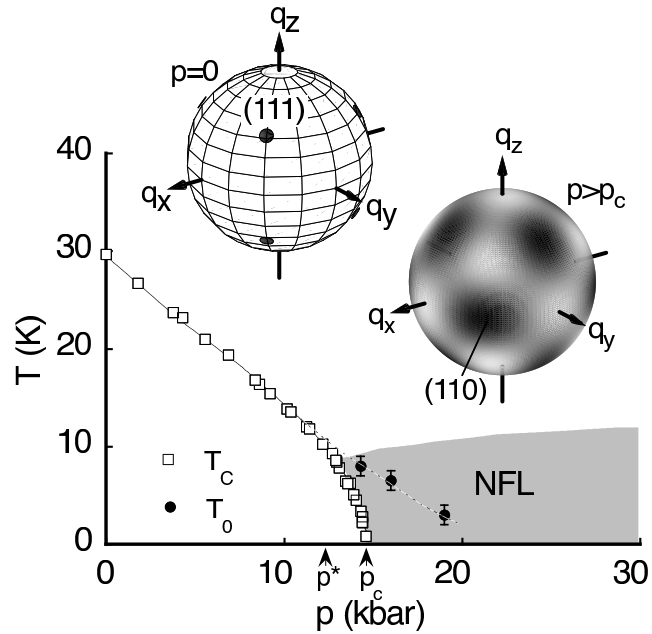
## What is origin of NFL behavior ?

neutron scattering in disordered phase:

- spiral survives into disordered phase with **full** moment
- signal on **surface** of tiny sphere in reciprocal space
- **static** on neutron timescale close to  $p_c$ ,  
(but note: **static** signal vanishes deep in NFL phase, fluctuations faster?)
- pitch  $1/|\vec{Q}|$  unchanged (resolution limited)  
 $\Rightarrow$  spiral intact
- direction  $\vec{Q}/|\vec{Q}|$  fluctuating [predominantly in (1,1,0) directions]  
no signal left in (1,1,1) direction

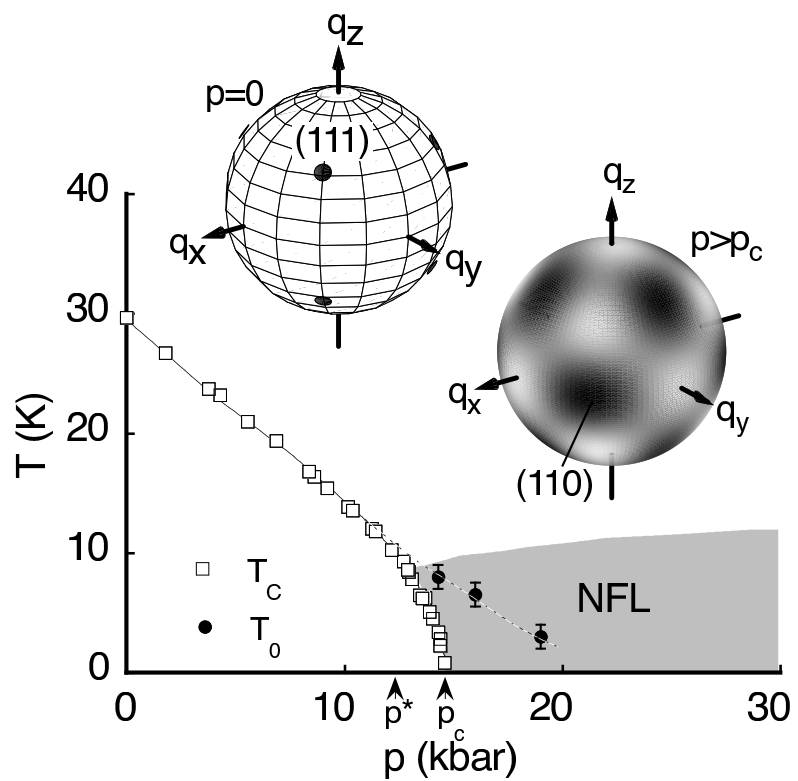


# Momentum dependence:

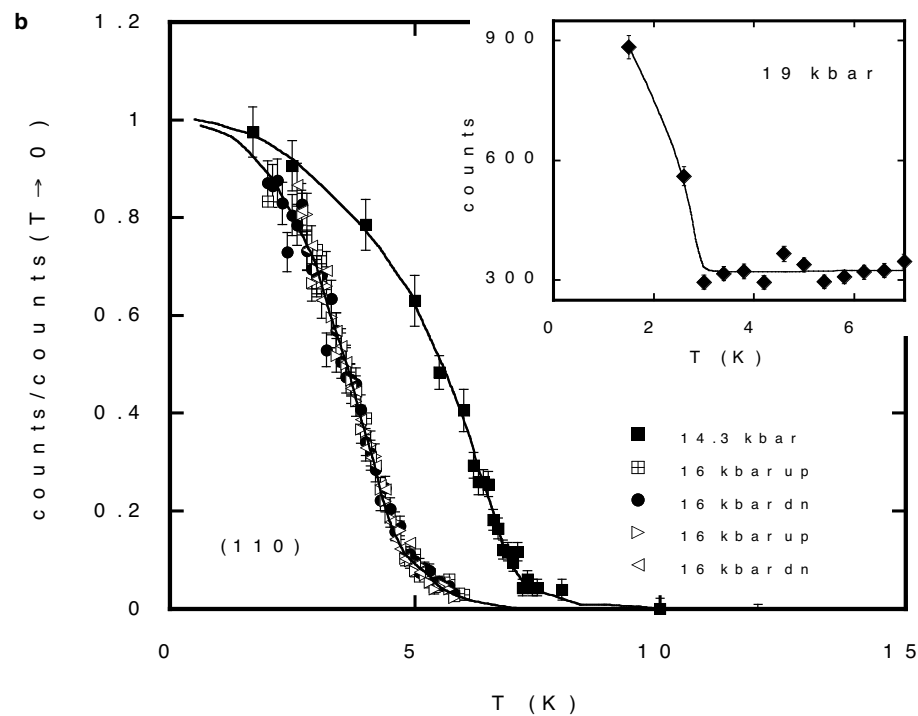
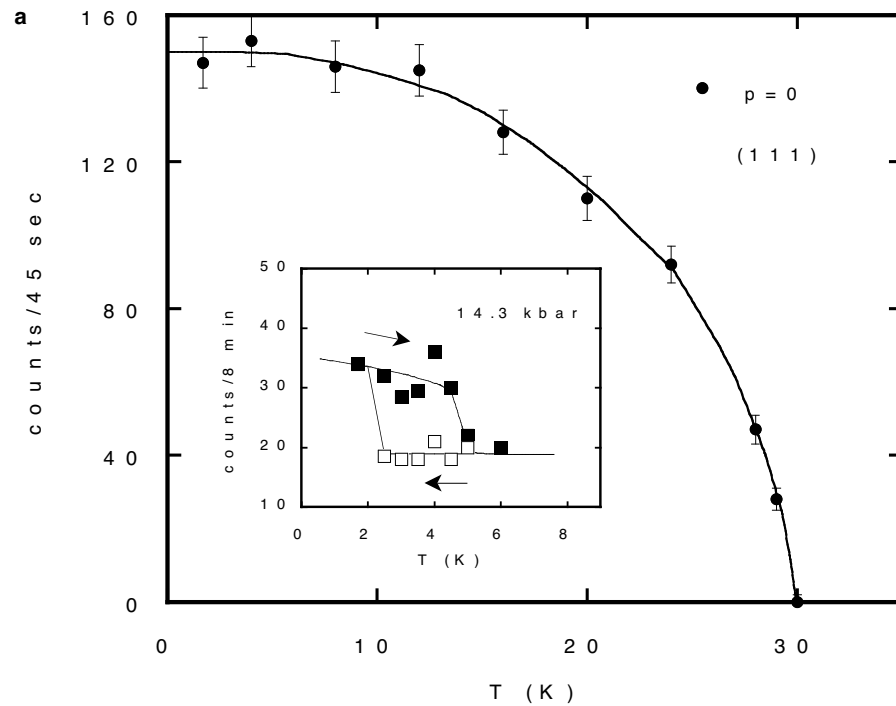


- resolution limited in radial direction
- broad in tangential direction, width increases towards lower  $T$

Temperature dependence:



crossover or phase transition?  
QCP?



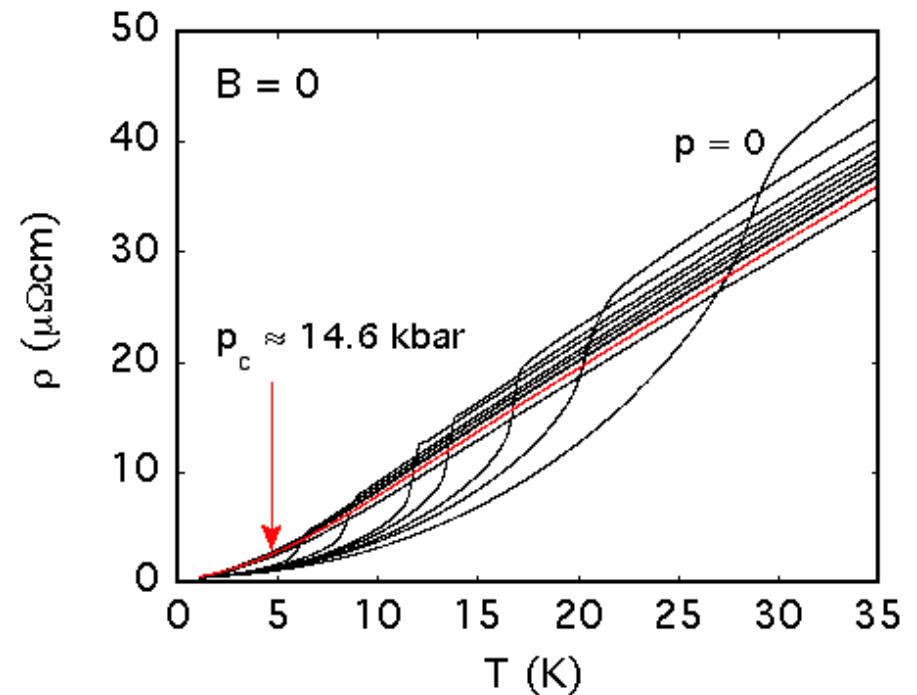


# Magnetic order and transport

## Major mystery:

No signature in  $\rho$  at onset of **partial** order despite the large moment involved!

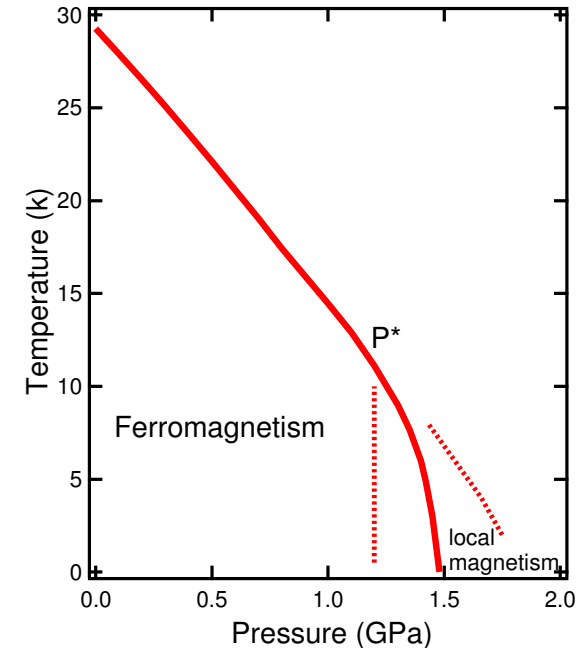
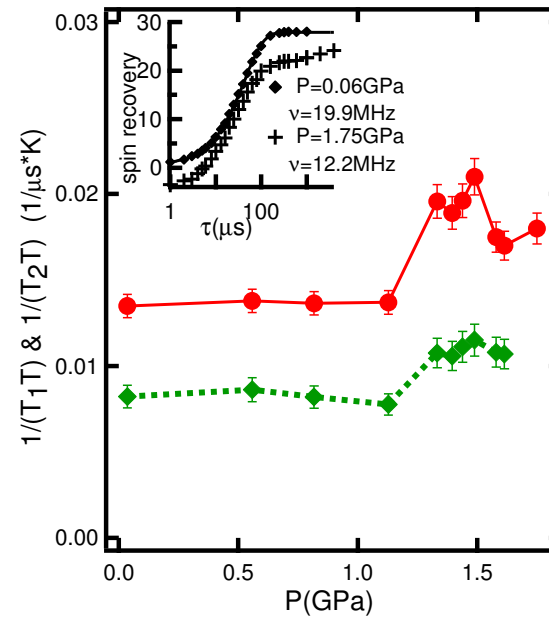
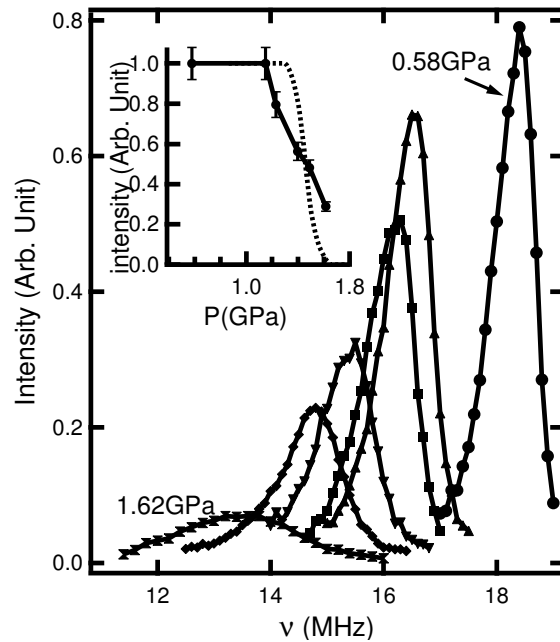
- in contrast: huge and sharp drop in  $\rho$  at onset of **long-range** order for  $p < p_c$ , main scattering mechanism frozen out
- Does fluctuating partial order exist in full NFL phase?  
maximum in  $\chi$  at 10K: helix formation?



# NMR ( $^{29}\text{Si}$ )

Thessieu, Kamishima, Goto, Lapertot (1998)

Yu, Zamborszky, Thompson, Sarrao, Torelli, Fisk, Brown (2003)



powdered samples  $\Rightarrow$  precise comparison to neutrons difficult  
suggests partial order static on NMR scales?

## Origin of NFL phase? Partial order

- Order parameter survives on intermediate ( $> 2000\text{\AA}$ ) length and time (neutron  $\omega$ -resolution) scales  
exp. confirmed close to  $p_c \Rightarrow$  our assumption: valid also for  $p \gg p_c$
- NFL behavior seems to occur only when spiral is formed  
(indications: behavior in large magn. fields, maximum in  $\chi(T)$ )

Two scenarios:

1. scattering from soup of fluctuating topological defects

2. scattering from anomalous (pseudo-) Goldstone modes in “almost” ordered state

## First scenario: scattering from topological defects

topological structure similar to cholesteric liquid crystals (replace director by vector)

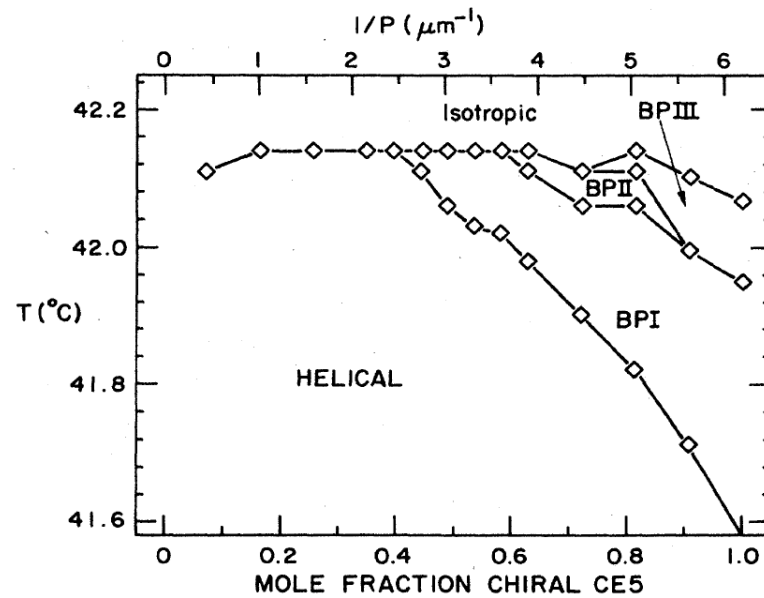
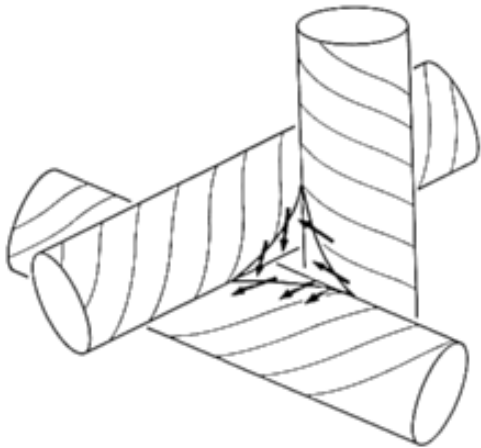
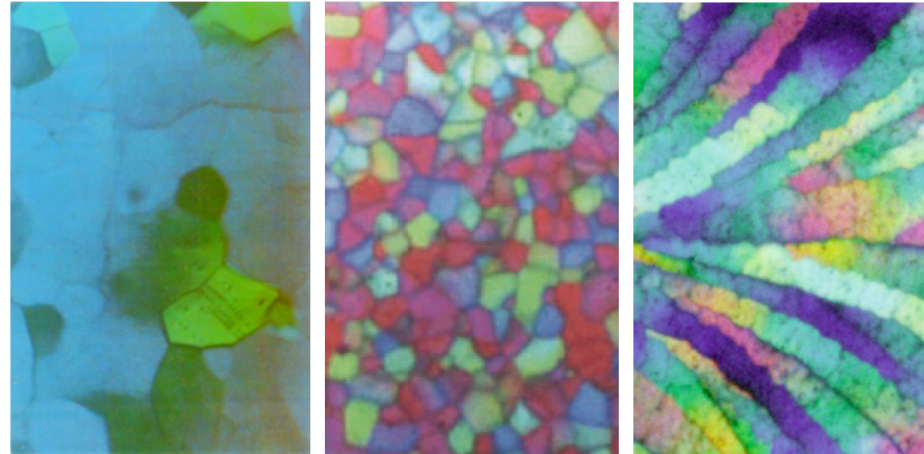
(review on topological defects: Mermin RMP 1979; blue phases: Wright, Mermin RMP 1989)

- order parameter exists locally but not globally  
⇒ finite density of topological defects
- domain walls, line defects, point defects?

# Blue phases: networks of topological defects

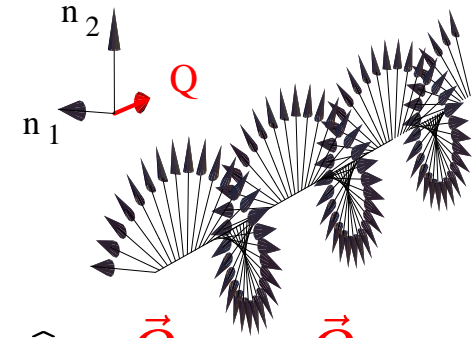
cholesteric liquid crystals: complex phase diagram

blue phase III: no long-range order



# Classification of topological defects: homotopy groups

- neglect pinning of  $\vec{Q}$  to cubic lattice ( $1/|\vec{Q}| \gg a$ )
- order parameter: 3 orthogonal vectors  
 $\hat{\Phi}(\vec{x}) = \hat{n}_1 \cos[\vec{Q}\vec{x}] + \hat{n}_2 \sin[\vec{Q}\vec{x}]$



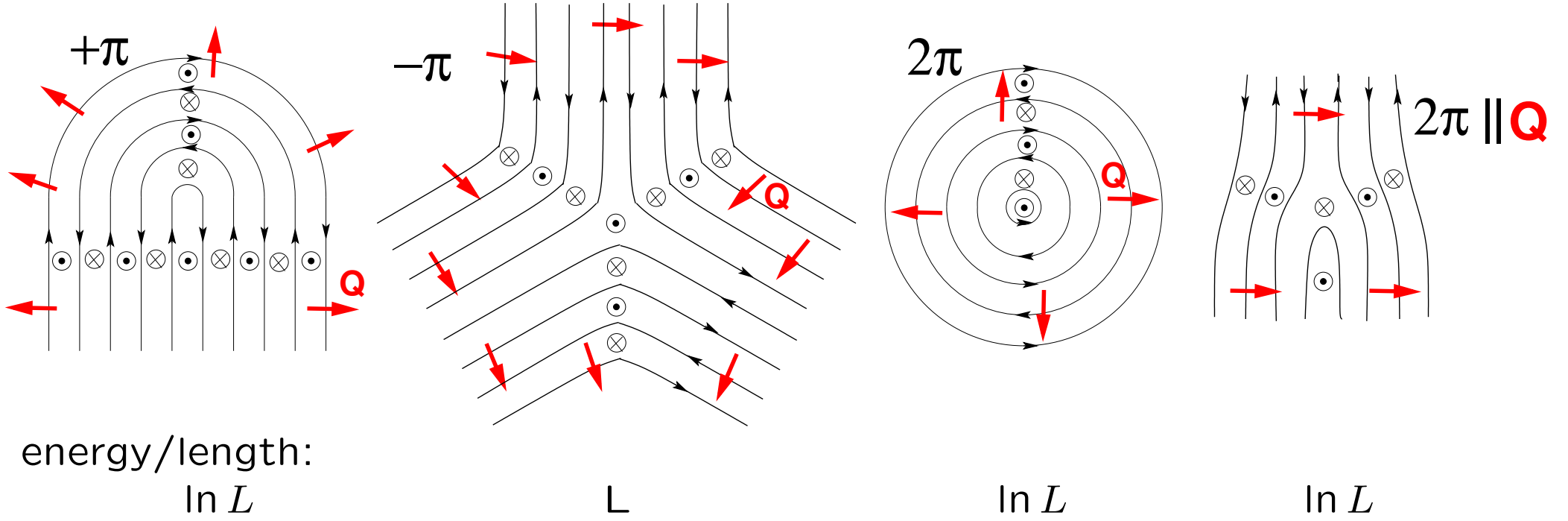
- **invariant** under rotation by  $\pi$  around  $\hat{n}_1$ :  $\hat{n}_2 \rightarrow -\hat{n}_2$ ,  $\vec{Q} \rightarrow -\vec{Q}$
- groundstate manifold: **SO(3)/Z<sub>2</sub>**

$$\Pi_1(\mathbf{SO}(3)/\mathbf{Z}_2) = \Pi_1(\mathbf{SU}(2)/\mathbf{Z}_4) = \mathbf{Z}_4$$

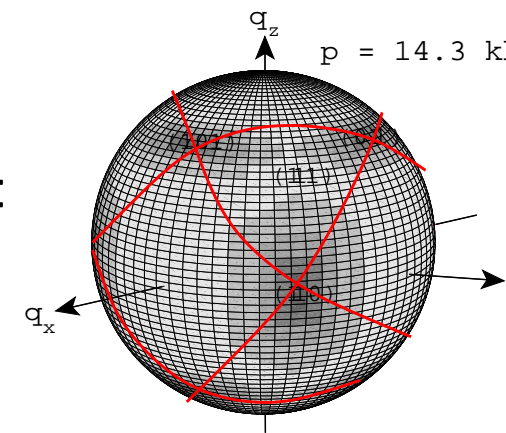
- 3 types of line defects  
 (in SU(2) paths from 1 to  $i\sigma_x$ , to  $-i\sigma_x$ , to  $-1$ , i.e. rotations by  $\pi$ ,  $-\pi$  and  $2\pi$ )
- domainwalls, **no** point defects
- **warning**: top. classification here not reliable  
**small** change of  $\vec{Q}$  may lead to **large** change of  $\vec{\Phi}(\vec{x})$   
 $\Rightarrow$  **large** energy cost ??  $\Rightarrow$  some defects not realized, novel defects?

# Line defects:

shown: directions of magnetization (black arrows) and  $\vec{Q}$  vector (red)



experiments: evidence for rotation of  $\vec{Q}$ ? GL:  $\vec{Q} \perp$  to  $(1,1,1)$ :  
 closer look: probably not



## Effective theory in presence of large local order parameter $\hat{\Phi}(\vec{x}, t)$ :

- electrons follow adiabatically large OP  $\hat{\Phi}(\vec{x}, t)$   
natural “quasi-particle”: spin quantization axis  $\parallel$  to  $\hat{\Phi}(\vec{x}, t)$

- new “holons”: 
$$\tilde{c}_\sigma(\vec{x}, t) = U(\vec{x}, t)c_\sigma(\vec{x}, t)U^\dagger(\vec{x}, t)$$

with  $U(\vec{x}, t) \left( \vec{S}(\vec{x}, t) \hat{\Phi}(\vec{x}, t) \right) U^\dagger(\vec{x}, t) \equiv S_z(\vec{x})$

- by construction  $[\vec{S}, \tilde{c}_\sigma] = 0 \Rightarrow$  holons do **not** transform under global spin-rotation  $\Rightarrow$  spin-charge separation (spin eaten up by OP)
- eff. field theory: gauge theory of topological defects  
interacting with  $\tilde{c}$  (not worked out, possibly  $U(1)$ ??)  
Physics: “holons” acquire Berry phases when encircling OP textures
- deconfining phase of this gauge theory: non Fermi liquid  
similar to other gauge theories,  $Z_2$  or  $U(1)$   
(Balents, Nayak, Senthil, Fisher, Sachdev, Muramatsu, Zaanen, Franz, Tešanović, ...)



Is scattering from topological defects relevant?

Problem: distance of defects  $> 2000\text{\AA}$

but: inelastic mean free path can be smaller in  $T^{3/2}$  regime

What are electrons scattering from?

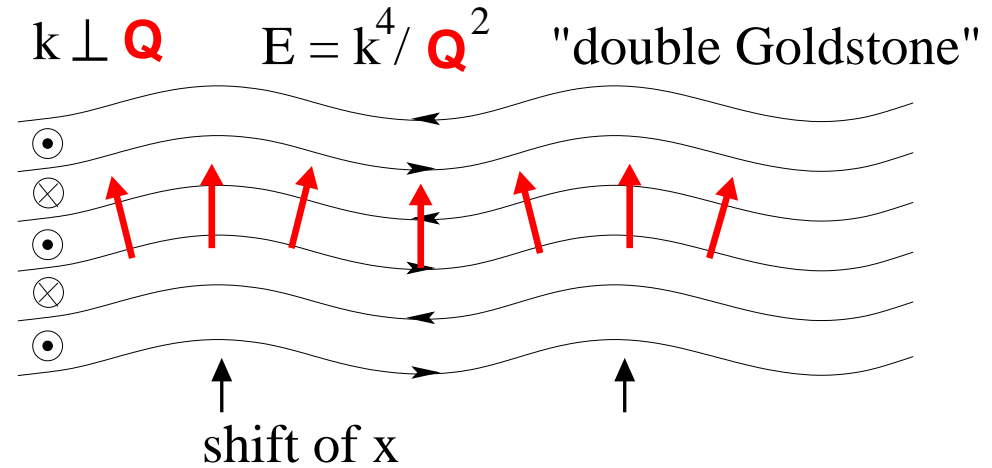
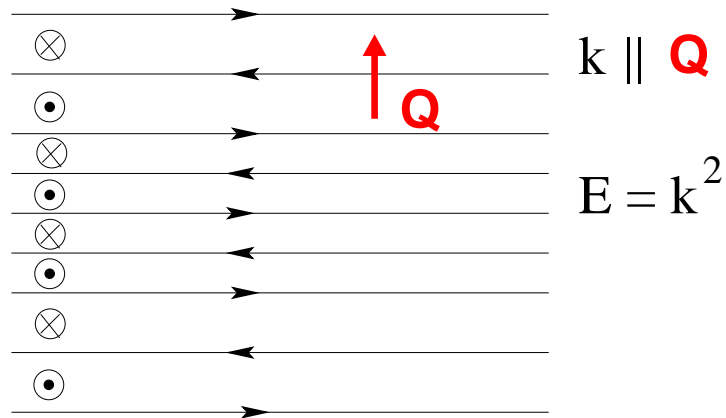
**Scenario 2:** Scatter from small fluctuations of OP on lengthscales **smaller** than distance of topological defects

(pseudo-) Goldstone modes?

- in “disordered” phase, chiral spirals not pinned effectively by cubic lattice, stronger local fluctuations possible?
- calculate Goldstone modes in ordered phase neglecting (for a beginning) pinning to cubic lattice (pitch  $150\text{\AA} \gg$  lattice spacing)

# Anomalous Goldstone modes in a metallic chiral helix:

$\vec{k}$  dependence



$$F = F(\vec{\Phi}^2) + \vec{k}^2 |\vec{\Phi}_{\vec{k}}|^2 + q_0 \vec{k} \cdot (\vec{\Phi}_{\vec{k}} \times \vec{\Phi}_{\vec{k}}^*) + k_x^4 |\vec{\Phi}_{\vec{k}}|^2 + k_x^2 \Phi_y^2 + \Phi_x^4 + \text{cycl.} + \dots$$

- energy of Goldstone mode:

$$k_{\parallel}^2 + k_{\perp}^4 / q_0^2 \quad (\text{like in smectics})$$

- correction from pinning to cubic background (cubic terms):  $q_0^2 k_{\perp}^2$   
 relevant only for  $k_{\perp} \ll q_0^2$

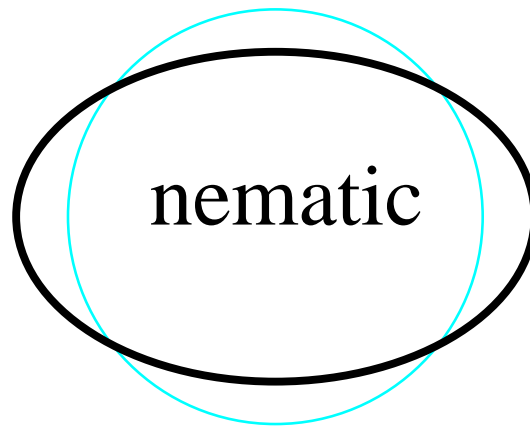
# Anomalous Goldstone modes: damping

- “phonon-like” Goldstone mode: inconsistent with  $\rho \sim T^{3/2}$
- overdamped Goldstone modes?

smectic-A liquid crystal	$\omega \sim q  \sin 2\phi  - i\eta q^2$ $\omega \sim iq^3 \frac{\ln T}{T^2}$ $G^{-1} = q^2 - \omega^2 - i\omega \sin^2 2\phi$	
$^3\text{He-A}$		(Wölfle 75)
nematic Fermi liquid		<b>talk by Hae-Young Kee</b> (Oganesyan, Kivelson, Fradkin 01)

physics?  $^3\text{He-A}$ : point node moves ⇒ many p-h pairs  
 nematic metal: gapless Fermi surface moves

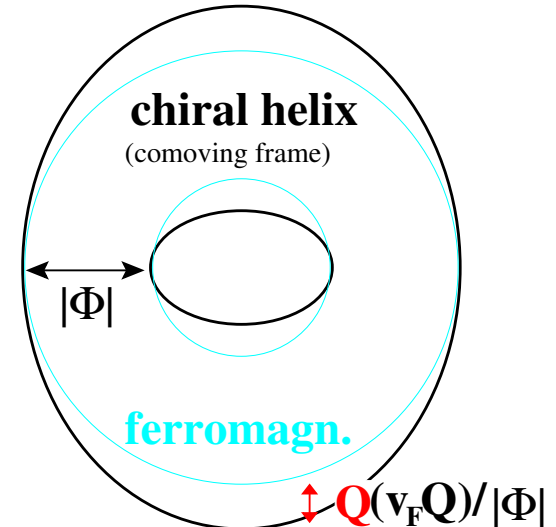
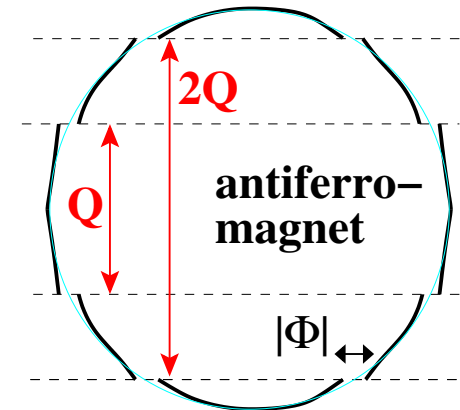
Fermi surface of nematic metal:



# Anomalous Goldstone modes in a metallic chiral helix: damping

Electrons in chiral helix:

- in antiferromagn. metal: multiple gaps open (translational invariance broken)
- chiral helix: translation + simultaneous rotation unbroken  
U(1) symmetry  $T_z U_z$  remains,  $U_z = e^{izQ} S_z$
- simple description in comoving frame  
distortion of FM Fermi surface
- similar to nematic metal?
- **warning**: spin-orbit in band-structure neglected (see later)



# Anomalous Goldstone modes in a metallic chiral helix: damping

within RPA: lowest eigenvalue of matrix of susceptibilities  
in ordered phase  
exact cancelations due to Goldstone theorem

$$G^{-1} \approx q_{\parallel}^2 + q_{\perp}^4 / q_0^2 - \omega^2 / q_0^2 - i\omega |q| q_0^2$$

corresponding resistivity (including vertex corrections ect.):

$$\Delta\rho \sim T^{2.5}$$

incompatible with experimental  $\rho \sim T^{1.5}$

open questions: more realistic band structure

contributions from massive modes (Vekhter, Chubukov 04)

finite size of domains?

# Electrons, helical order and spin-orbit coupling

up to now: only DM-interaction, other spin-orbit effects neglected

dominant contribution for  $P2_13$  ( $T^4$ ):

$$H_{\text{SOC}} = \delta \sum_{i=1}^3 k_i \sigma_{\alpha\beta}^i c_{\vec{k}\alpha}^\dagger c_{\vec{k}\beta}$$

(with  $\delta \sim \vec{Q}$ .)

in comoving coordinate system for  $\vec{Q} \parallel z$ :

$$\begin{pmatrix} \vdots \\ d_{+,k}^\dagger \\ d_{-,k}^\dagger \\ d_{+,k+\vec{Q}}^\dagger \\ d_{-,k+\vec{Q}}^\dagger \\ \vdots \end{pmatrix} \begin{pmatrix} \dots & & & & \\ E_1(\mathbf{k}) & -k_z \delta & -k_x \delta & -k_x \delta & \\ -k_z \delta & E_2(\mathbf{k}) & k_x \delta & k_x \delta & \\ -k_x \delta & k_x \delta & E_1(\mathbf{k}+\vec{Q}) & -(k_z + \vec{Q}) \delta & \\ -k_x \delta & k_x \delta & -(k_z + \vec{Q}) \delta & E_2(\mathbf{k}+\vec{Q}) & \\ \dots & & & & \end{pmatrix} \begin{pmatrix} \vdots \\ d_{+,k} \\ d_{-,k} \\ d_{+,k+\vec{Q}} \\ d_{-,k+\vec{Q}} \\ \vdots \end{pmatrix}$$

where  $E_{1,2}(\mathbf{k}) = \frac{|\mathbf{k}|^2}{2m} + \frac{k_F^2}{2m} \pm \sqrt{\left(\frac{k_z \vec{Q}}{2m}\right)^2 + |\Phi|^2} - \delta k_z$

$H_{\text{SOC}}$  induces mini-bands (breaks residual U(1) symmetry)

# Electrons, helical order and spin-orbit coupling

bandstructure non-perturbative in small SO-interaction  
map to tight-binding model in band-index space:

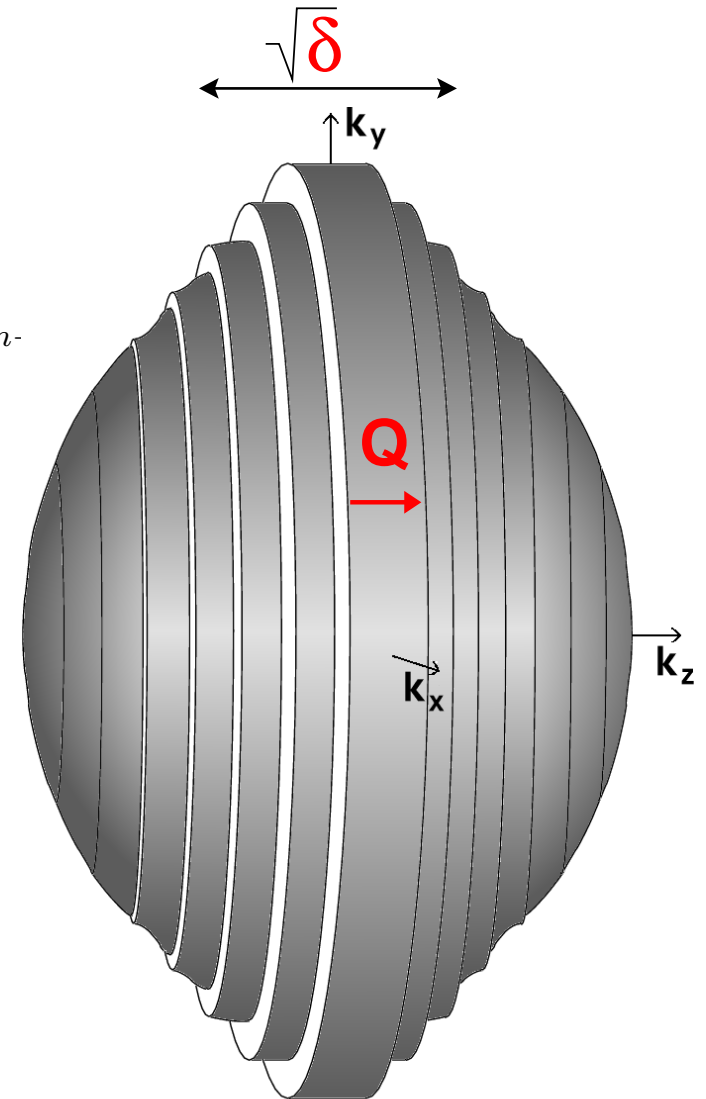
$$H_{TB} = \sum_n -\frac{1}{2m} \left( n\vec{Q} + k_z - m\delta \right)^2 d_n^\dagger d_n + k_x \delta d_{n-1}^\dagger d_n + k_x \delta d_n^\dagger d_{n+1}$$

- for  $\vec{k}_F \perp \vec{Q}$  superflat mini-bands

$$\text{bandwidth} \propto e^{-c \frac{\sqrt{\delta}}{Q}} \sim e^{-\frac{c'}{\sqrt{\delta}}}$$

bandgaps  $\sim Q\sqrt{\delta}$

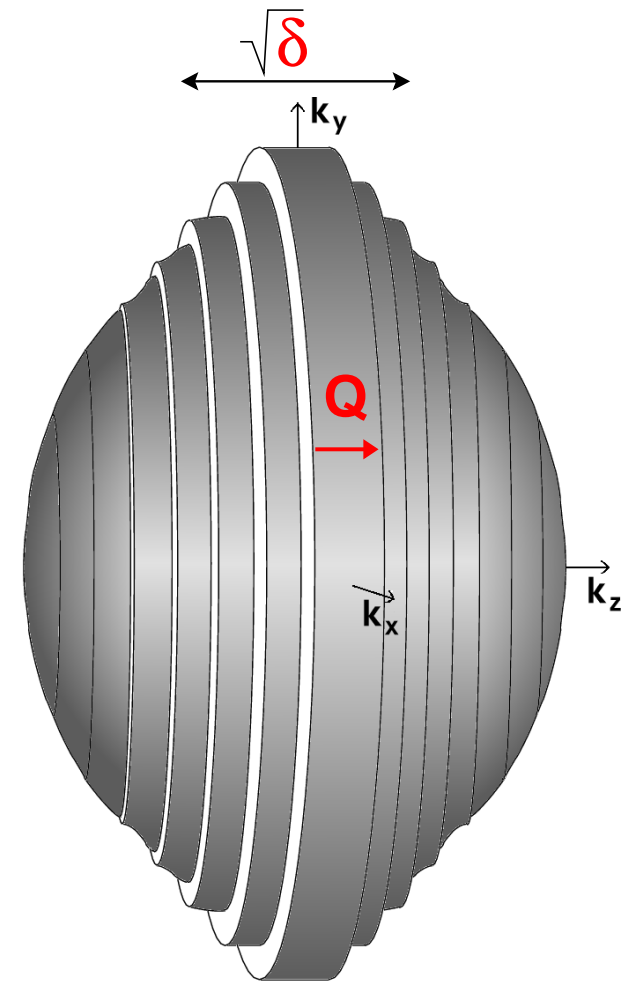
- electron motion  $\parallel \vec{Q}$  stopped for large fraction of Fermi-surface:  $\frac{k_z}{k_F} \lesssim \sqrt{\delta}$



# Electrons, helical order and spin-orbit coupling

experimental consequences:

- de-Haas van-Alphen: unrealistically clean samples required
- resistivity:  $\frac{\rho_{\parallel} - \rho_{\perp}}{\rho} \sim \delta^{3/2}$  small effect
- **huge** change in anomalous skin effect determined by electrons moving parallel to surface with  $v_{\perp}/v_{\parallel} < \Delta/l$ , rotate spirals by  $B$  field  $\parallel / \perp$  to surface compare skin depth  $\Delta_{\parallel}, \Delta_{\perp}$  large for  $k_F l_0 \gtrsim \frac{(\lambda k_F / \alpha)^{1/3}}{\sqrt{\delta}}$
- Hall effect?
- damping of Goldstone modes?
- inelastic scattering?





## Conclusions? – No conclusions yet

- genuine NFL **phase** in MnSi?  
unique: clear exp. evidence for NFL phase AND hint towards origin
- scenario 1: local order remains, spin-charge separation,  
scattering from topological defects, Gauge theory
- scenario 2: scattering from (pseudo-) Goldstone modes  
anomalous  $k_{\perp}$  dependence, **overdamped**  
but: wrong power-law for  $\rho(T)$
- other options: scattering from domain-walls, texture-glass, ...
- large effects of spin-orbit coupling