Workshop on Novel States and Phase Transitions in Highly Correlated Matter

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Towards a theory of the non-Fermi liquid phase in MnSi

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These are preliminary lecture notes, intended only for distribution to participants
Towards a theory of the non-Fermi liquid phase in MnSi

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- A non-Fermi liquid **phase** in MnSi?
  Review of experiments

- Topological defects?

- Anomalous overdamped (pseudo-) Goldstone modes?

- Band structure in a spiral

Disclaimer:
Work in progress: many questions – few answers
Towards "novel phases" in metals

from Fermi liquids to non-Fermi liquid behavior:

- instability between two phases by fine-tuning
- new stable phases
- known
- new fixpoint
- known
- new line

critical fluctuations, power-laws, scaling,...
dozens of systems, e.g. CeCu$_{6-x}$Au$_x$, YbRh$_2$Si$_2$, CePd$_2$Si$_2$, NiS$_{2-x}$Se$_x$,...
diverging Grüneisen parameter

new quasiparticles, quantum number fractionalization
Luttinger liquids, fractional QHE, nematic metal, Griffiths singularities...

low dimensions!
quantum-critical \( \rho \sim T, \ c/T \sim \ln[1/T] \)

CeCu\(_{6-x}\)Au\(_x\) metal antiferromagnet

Fermi liquid

Löhneysen et al. (96)

HTc cuprates:

“non-Fermi liquid” regime

pseudogap regime

Fermi liquid?

hidden quantum critical point?

hidden new phase?

spectroscopy of Luttinger liquids by parallel tunneling

Auslaender, Yacoby et al.
MnSi – a standard itinerant magnet

- Text-book band-magnet below 30K
  (see Landau-Lifschitz, Vol.8, 3rd edition)

- Ginzburg-Landau theory for helical spin-density wave:
  Bak, Jensen (1980), Nakanishi et al. (1980)

- Extremely clean (mean free path 3000-10000 Å)

- Cubic but no inversion symmetry (P2_13)

- Standard example for spin-fluctuation theory (Lonzarich, Moriya)

New physics under pressure!

in MnSi for wide pressure range, \( p > p_c \)

\[ \rho(T) - \rho_0 \sim T^{3/2} \]

almost 3 decades in \( T \), very clean system

A genuine non-Fermi liquid phase?
resistivity $\rho(T) \sim T^{3/2}$ for almost 3 decades (10mK to 5K):

$\rho(T) \sim T^{3/2}$ for $p > p_c$ up to 28 kbar (Lonzarich)
in MnSi for wide pressure range, $p > p_c$

\[ \rho(T) - \rho_0 \sim T^{3/2} \]
almost 3 decades in $T$, very clean system

A genuine non-Fermi liquid phase?

Alternative: Quantum critical behavior?

contra: NFL observed even for $p \gg p_c$ and low $T$

first order transition close to $p_c$

jump of $\chi$ and of local moment in NMR, $\mu$SR (Thessieu et al. 98)
Susceptibility, helix formation and first order transition

Pfleiderer et al.

pronounced 1st order transition

maximum: helix formation?
MnSi

in MnSi for wide pressure range, $p > p_c$

\[
\rho(T) - \rho_0 \sim T^{3/2}
\]

almost 3 decades in $T$, very clean system

A genuine non-Fermi liquid phase?

Alternative: **Quantum critical** behavior?

**contra:** NFL observed even for $p \gg p_c$ and low $T$

**first order** transition close to $p_c$

jump of $\chi$ and of local moment in NMR, $\mu$SR (Thessieu et al. 98)

**pro:** maybe 2nd order endpoint at $p = p_c$?

$A$-coefficient, $\Delta \rho(T) \approx AT^2$, diverges for $p \rightarrow p_c$ in ord. phase

Neutrons: $T = 0$ ordered moment vanishes continuously for $p \rightarrow p_c$; \Rightarrow chimera of first and second order? (cf. $d = \infty$ Mott)
Quantum critical theory

Joerg Schmalian and Misha Turlakov (03):

NEXT TALK
Three distinct scales in MnSi:

- **dominant:** itinerant ferromagnet with large ordered moment \((0.4\mu_B)\)  
  \[\Rightarrow\] fixes amplitude of local magnetization

- **but:** instable to formation of chiral helix  
  due to weak **spin-orbit** coupling in  
  non-centrosymmetric crystal:  
  \[ q_0 \int \vec{S} \cdot (\vec{\nabla} \times \vec{S}) \]  
  **linear** in \(\vec{k}\)  
  \[\Rightarrow\] fixes pitch \(1/q_0 \approx 150\text{Å}\) of helix

- **small correction:** **spin-orbit** coupling breaks rotational symmetry  
  in cubic crystal  
  \[\Rightarrow\] fixes direction of helix

\[
F = F(\vec{\Phi}^2) + k^2|\vec{\Phi}_k|^2 + q_0 k \cdot (\vec{\Phi}_{k} \times \vec{\Phi}^*_{k}) + k_x^4|\vec{\Phi}_k|^2 + k_x^2 \Phi_y^2 + \Phi_x^4 + \text{cycl.} + \ldots
\]
\[ F = F(\Phi^2) + k^2|\Phi_k|^2 + q_0 \vec{k} \cdot (\Phi_k \times \Phi_k^*) + k_x^4|\Phi_k|^2 + k_x^2\Phi^2 + \Phi^4 + \text{cycl.} + ... \]

- first 3 terms minimized by chiral helix:
  \[ \vec{\Phi}(\vec{x}) = \Phi_0 \left[ \hat{n}_1 \cos(\vec{Q} \vec{x}) + \hat{n}_2 \sin(\vec{Q} \vec{x}) \right] \]

- \( \hat{n}_1 \perp \hat{n}_2 \perp \vec{Q} \) form chiral “Dreibein”
  \( \hat{n}_1(\hat{n}_2 \times \vec{Q}) = \pm 1 \) dep. on sign of \( q_0 \)

- pitch \( 1/|\vec{Q}| = 1/q_0 \) large (150Å) as spin-orbit coupling weak

- direction of \( \vec{Q} \) determined by cubic terms;
  \( \Rightarrow \vec{Q} \parallel (1,0,0) \) or \( \vec{Q} \parallel (1,1,1) \)
  \( (\vec{Q} \parallel (1,1,0) \) only possible if \( \Phi_6, k_6 \) important)
What is origin of NFL behavior?

neutron scattering in disordered phase:

- spiral survives into disordered phase with **full** moment
- signal on **surface** of tiny sphere in reciprocal space
- **static** on neutron timescale close to \( p_c \), (but note: **static** signal vanishes deep in NFL phase, fluctuations faster?)
- pitch \( 1/|\vec{Q}| \) unchanged (resolution limited) \( \Rightarrow \) spiral intact
- direction \( \vec{Q}/|\vec{Q}| \) fluctuating [predominantly in (1,1,0) directions] no signal left in (1,1,1) direction

Momentum dependence:

- resolution limited in radial direction
- broad in tangential direction, width increases towards lower T
Temperature dependence:

crossover or phase transition? QCP?
Magnetic order and transport

Major mystery:
No signature in $\rho$ at onset of **partial** order despite the large moment involved!

- in contrast: huge and sharp drop in $\rho$ at onset of **long-range** order for $p < p_c$, main scattering mechanism frozen out
- Does fluctuating partial order exist in full NFL phase? maximum in $\chi$ at 10K: helix formation?
NMR ($^{29}$Si)


powdered samples ⇒ precise comparison to neutrons difficult
suggests partial order static on NMR scales?
Origin of NFL phase? Partial order

- Order parameter survives on intermediate (> 2000Å) length and time (neutron ω-resolution) scales
  exp. confirmed close to \( p_c \) ⇒ our assumption: valid also for \( p \gg p_c \)

- NFL behavior seems to occur only when spiral is formed
  (indications: behavior in large magn. fields, maximum in \( \chi(T) \))

Two scenarios:

1. scattering from soup of fluctuating topological defects

2. scattering from anomalous (pseudo-) Goldstone modes in “almost” ordered state
First scenario: scattering from topological defects

topological structure similar to cholesteric liquid crystals (replace director by vector)
(review on topological defects: Mermin RMP 1979; blue phases: Wright, Mermin RMP 1989)

- order parameter exists locally but not globally
  ⇒ finite density of topological defects
- domain walls, line defects, point defects?
Blue phases: networks of topological defects

cholesteric liquid crystals: complex phase diagram
blue phase III: no long-range order
Classification of topological defects: homotopy groups

- neglect pinning of $\vec{Q}$ to cubic lattice $(1/|\vec{Q}| \gg a)$
- order parameter: 3 orthogonal vectors
  $\Phi(\vec{x}) = \hat{n}_1 \cos[\vec{Q} \vec{x}] + \hat{n}_2 \sin[\vec{Q} \vec{x}]$
- invariant under rotation by $\pi$ around $\hat{n}_1$: $\hat{n}_2 \rightarrow -\hat{n}_2$, $\vec{Q} \rightarrow -\vec{Q}$
- groundstate manifold: $SO(3)/Z_2$
  $$\Pi_1(SO(3)/Z_2) = \Pi_1(SU(2)/Z_4) = \mathbb{Z}_4$$
- 3 types of line defects
  (in $SU(2)$ pathes from 1 to $i\sigma_x$, to $-i\sigma_x$, to $-1$, i.e. rotations by $\pi$, $-\pi$ and $2\pi$)
- domainwalls, no point defects
- warning: top. classification here not reliable
  small change of $\vec{Q}$ may lead to large change of $\Phi(\vec{x})$
  $\Rightarrow$ large energy cost ?? $\Rightarrow$ some defects not realized, novel defects?
Line defects:
shown: directions of magnetization (black arrows) and $\vec{Q}$ vector (red)

energy/length:
$\ln L$

experiments: evidence for rotation of $\vec{Q}$? GL: $\vec{Q} \perp (1,1,1)$:
closer look: probably not
Effective theory in presence of large local order parameter $\hat{\Phi}(\vec{x}, t)$:

- electrons follow adiabatically large OP $\hat{\Phi}(\vec{x}, t)$
  natural “quasi-particle”: spin quantization axis $\parallel$ to $\hat{\Phi}(\vec{x}, t)$

- new “holons”:
  $$\tilde{c}_\sigma(\vec{x}, t) = U(\vec{x}, t)c_\sigma(\vec{x}, t)U^\dagger(\vec{x}, t)$$
  with $U(\vec{x}, t) \left( \tilde{S}(\vec{x}, t)\hat{\Phi}(\vec{x}, t) \right) U^\dagger(\vec{x}, t) \equiv S_z(\vec{x})$

- by construction $[\tilde{S}, \tilde{c}_\sigma] = 0 \Rightarrow$ holons do not transform under global spin-rotation $\Rightarrow$ spin-charge separation (spin eaten up by OP)

- eff. field theory: gauge theory of topological defects
  interacting with $\tilde{c}$ (not worked out, possibly $U(1)$??)
  Physics: “holons” acquire Berry phases when encircling OP textures

- deconfining phase of this gauge theory: non Fermi liquid
  similar to other gauge theories, $Z_2$ or $U(1)$
  (Balents, Nayak, Senthil, Fisher, Sachdev, Muramatsu, Zaanen, Franz, Tešanović,...)
Is scattering from topological defects relevant?

Problem: distance of defects > 2000Å
but: inelastic mean free path can be smaller in $T^{3/2}$ regime

What are electrons scattering from?

**Scenario 2:** Scatter from small fluctuations of OP on lengthscales smaller than distance of topological defects

(pseudo-) Goldstone modes?

- in “disordered” phase, chiral spirals not pinned effectively by cubic lattice, stronger local fluctuations possible?

- calculate Goldstone modes in ordered phase neglecting (for a beginning) pinning to cubic lattice (pitch 150Å ≫ lattice spacing)
Anomalous Goldstone modes in a metallic chiral helix: $\vec{k}$ dependence

$F = F(\Phi^2) + \vec{k}^2|\Phi_\vec{k}|^2 + q_0 \vec{k} \cdot (\Phi_\vec{k} \times \Phi_\vec{k}^*) + k_x^4|\Phi_\vec{k}|^2 + k_x^2\Phi_y^2 + \Phi_x^4 + \text{cycl.} + \ldots$

- energy of Goldstone mode: $k_{||}^2 + k_{\perp}^4 / q_0^2$ (like in smectics)

- correction from pinning to cubic background (cubic terms): $q_0^2k_{\perp}^2$ relevant only for $k_{\perp} \ll q_0^2$
Anomalous Goldstone modes: damping

- “phonon-like” Goldstone mode: inconsistent with $\rho \sim T^{3/2}$

- overdamped Goldstone modes?

smectic-A liquid crystal

$$\omega \sim q|\sin 2\phi| - i\eta q^2$$

$^3$He-A

$$\omega \sim iq^3 \frac{\ln T}{T^2}$$  (Wölfle 75)

nematic Fermi liquid

$$G^{-1} = q^2 - \omega^2 - i\omega \sin^2 2\phi$$  

Physics? $^3$He-A: point node moves

nematic metal: gapless Fermi surface moves

$\Rightarrow$ many p-h pairs

Fermi surface of nematic metal:
Anomalous Goldstone modes in a metallic chiral helix: damping

Electrons in chiral helix:

- in antiferromagn. metal: multiple gaps open (translational invariance broken)
- chiral helix: translation+simultaneous rotation unbroken $U(1)$ symmetry $T_zU_z$ remains, $U_z = e^{izQs_z}$
- simple description in comoving frame distortion of FM Fermi surface
- similar to nematic metal?
- **warning**: spin-orbit in band-structure neglected (see later)
Anomalous Goldstone modes in a metallic chiral helix: damping

within RPA: lowest eigenvalue of matrix of susceptibilities in ordered phase
exact cancelations due to Goldstone theorem

\[ G^{-1} \approx q_{||}^2 + q_{\perp}^4/q_0^2 - \omega^2/q_0^2 - i\omega|q|q_0^2 \]

corresponding resistivity (including vertex corrections ect.):

\[ \Delta \rho \sim T^{2.5} \]

incompatible with experimental \( \rho \sim T^{1.5} \)

open questions: more realistic band structure
contributions from massive modes (Vekhter, Chubukov 04)
finite size of domains?
Electrons, helical order and spin-orbit coupling

up to now: only DM-interaction, other spin-orbit effects neglected

dominant contribution for $P2_1 3 \ (T^4)$:

$$H_{SOC} = \delta \sum_{i=1}^{3} k_i \sigma_{\alpha\beta}^i c_{k\alpha}^\dagger c_{k\beta}$$

(with $\delta \sim \vec{Q}$.)

in comoving coordinate system for $\vec{Q} \parallel z$:

$$
\begin{pmatrix}
\vdots \\
d_{+,k}^\dagger \\
\vdots \\
d_{+,k+\vec{Q}}^\dagger \\
\vdots \\
d_{-,k}^\dagger \\
\vdots \\
d_{-,k+\vec{Q}}^\dagger \\
\vdots \\
\end{pmatrix}
\begin{pmatrix}
E_1(k) & -k_z\delta & -k_x\delta & -k_x\delta \\
-k_z\delta & E_2(k) & k_x\delta & k_x\delta \\
-k_x\delta & k_x\delta & E_1(k+\vec{Q}) & -(k_z+\vec{Q})\delta \\
-k_x\delta & k_x\delta & -(k_z+\vec{Q})\delta & E_2(k+\vec{Q}) \\
\vdots & & & \\
\end{pmatrix}
\begin{pmatrix}
\vdots \\
d_{+,k} \\
\vdots \\
d_{+,k+\vec{Q}} \\
\vdots \\
d_{-,k} \\
\vdots \\
d_{-,k+\vec{Q}} \\
\vdots \\
\end{pmatrix}

$$

where $E_{1,2}(k) = \frac{|k|^2}{2m} + \frac{k_F^2}{2m} \pm \sqrt{\left(\frac{k_z\vec{Q}}{2m}\right)^2 + |\Phi|^2 - \delta k_z}$

$H_{SOC}$ induces mini-bands (breaks residual U(1) symmetry)
Electrons, helical order and spin-orbit coupling

bandstructure non-perturbative in small SO-interaction map to tight-binding model in band-index space:

\[ H_{TB} = \sum_n -\frac{1}{2m} \left(n\vec{Q} + k_z - m\delta \right)^2 d_n^\dagger d_n + k_x\delta d_n^\dagger d_{n-1} + k_x\delta d_n^\dagger d_n. \]

\[ \text{• for } \vec{k}_F \perp \vec{Q} \text{ superflat mini-bands} \]

\[
\text{bandwidth } \propto e^{-c\sqrt{\delta \langle Q \rangle}} \sim e^{-\frac{c'}{\sqrt{\delta}}} \]

\[ \text{bandgaps } \sim Q\sqrt{\delta} \]

\[ \text{• electron motion } \parallel \vec{Q} \text{ stopped for large fraction of Fermi-surface: } \frac{k_z}{k_F} \lesssim \sqrt{\delta} \]
Electrons, helical order and spin-orbit coupling

experimental consequences:

- de-Haas van-Alphen: unrealistically clean samples required
- resistivity: $\frac{\rho_{\parallel} - \rho_{\perp}}{\rho} \sim \delta^{3/2}$ small effect
- huge change in anomalous skin effect determined by electrons moving parallel to surface with $v_{\perp}/v_{\parallel} < \Delta/l$,

rotate spirals by $B$ field $\parallel / \perp$ to surface

compare skin depth $\Delta_{\parallel}, \Delta_{\perp}$
large for $k_Fl_0 \gtrsim \frac{(\lambda k_F/\alpha)^{1/3}}{\sqrt{\delta}}$

- Hall effect?
- damping of Goldstone modes?
- inelastic scattering?
Conclusions? – No conclusions yet

- genuine NFL phase in MnSi?
  unique: clear exp. evidence for NFL phase AND hint towards origin

- scenario 1: local order remains, spin-charge separation, scattering from topological defects, Gauge theory

- scenario 2: scattering from (pseudo-) Goldstone modes
  anomalous $k_\perp$ dependence, *overdamped*
  but: wrong power-law for $\rho(T)$

- other options: scattering from domain-walls, texture-glass, ...

- large effects of spin-orbit coupling