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ICTP 40th Anniversary

SMR.1572 - 21

Workshop on Novel States and Phase Transitions in Highly Correlated Matter

12 - 23 July 2004

Quantum phase transitions of magnetic rotons (Part I)
Hierarchy of energy scales (Part II)

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Quantum phase transitions of magnetic rotons

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Acknowledgements:
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G. Lonzarich
Ch. Pfleiderer

Review of possible 1-order magnetic transitions

! quantum phase transitions are frequently weakly 1-order

Examples: MnSi, UGe2,...

- Band structure effects (minimum in the electron density of states)

G.Lonzarich, H.Yamada,...

$$F(M) = \frac{a}{2} M^2 - \frac{b}{4} M^4 + \frac{d}{6} M^6$$

Why *weakly 1*-order?

- Additional degrees of freedom or fields (gauge fields, coupling to strain, ...)

Halperin, Lubensky, Ma (1974)
Larkin, Pikin (1969)

- Non-analytic Ginsburg-Landau theory (integrating electrons out)

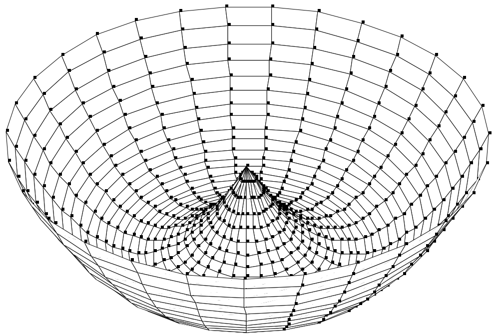
S. Misawa (1988,1993)
D.Belitz,T.R.Kirkpatrick, T.Vojta (1999)

- **Large phase volume of soft modes** *S. Brazovskii (1974)*

Fluctuational 1-order *classical* phase transition

S. Brazovskii (1974)
[crystallization of a liquid]

- Isotropic roton excitations with minima at finite momenta



Local fluctuations diverge if roton gap $\Delta \Rightarrow 0$

$$\left\langle \phi^2 \right\rangle_{r=0} = \int d^3 q \frac{T}{\Delta + (|q| - q_0)^2} = 4\pi q_0^2 \int d|q| \frac{T}{\Delta + (|q| - q_0)^2} \approx \frac{T}{\sqrt{\Delta}}$$

Finite contribution to free energy difference

finite phase volume of fluct. modes $\propto q_0^2$ → 1-order

small vanishing volume of fluct. modes (FM and AFM) $\propto q^2$ → 2-order

Physical systems:

Crystallisation of He-3 and He-4 — *Brazovskii(1974), Dyugaev (1976)*

Rayleigh-Benard convection — *Hohenberg, Swift (1995)*

Motivation to study *quantum phase transitions* of Brazovskii type

MnSi

helical itinerant ferromagnet

Two main experimental puzzles appear to be connected

- *Weak quantum* 1-order ferromagnetic transition
- Non-Fermi liquid paramagnetic phase above critical pressure

Magnetic rotons were predicted
and observed directly by C.Pfleiderer et al, Nature (2004)

Outline

- Review of possible reasons for 1-order transitions
- Classical fluctuational 1-order transition *(S.Brazovskii)*
- Motivation (itinerant helical ferromagnet -MnSi)

Quantum fluctuational 1-order transitions *(J. Schmalian, M.T.)*

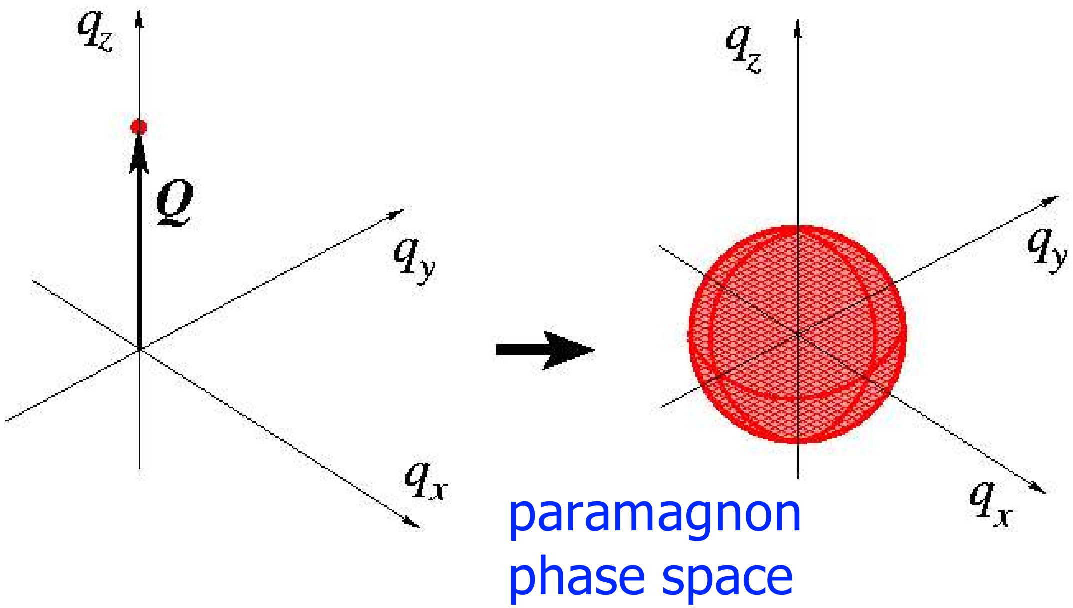
- Derivation of magnetic rotons due to spin-orbit coupling
- Self-consistent Hartree theory
- Roton-roton interactions
- RG analysis and tricritical points
- Classification of these type of transitions
- Disordered and metastbale states

Fermion-roton scattering and coupling

- Electron self-energy
- Resistivity

Magnetic rotons: (J.Schmalian, M.T.)

dramatic increase in the phase space of magnetic excitations



magnetic rotons:
degenerate low energy
excitations
with finite momentum

specific application: quantum phase transition in MnSi

Dynamics of rotons

$$\chi(q,\omega) = \frac{1}{E_+(q) - i\Gamma\omega}$$

$$\omega \rightarrow \omega^{2/z}$$

with z arbitrary

Z=2 diffusive due to particle-hole excitations

$$E_+(q = q_0) = \Delta$$

$\Delta \rightarrow 0$ at the critical point

challenges

roton-roton interaction ➡ nature of the phase transition

roton-electron interaction ➡ non-Fermi liquid beh.

RG analysis

two stable fixed points

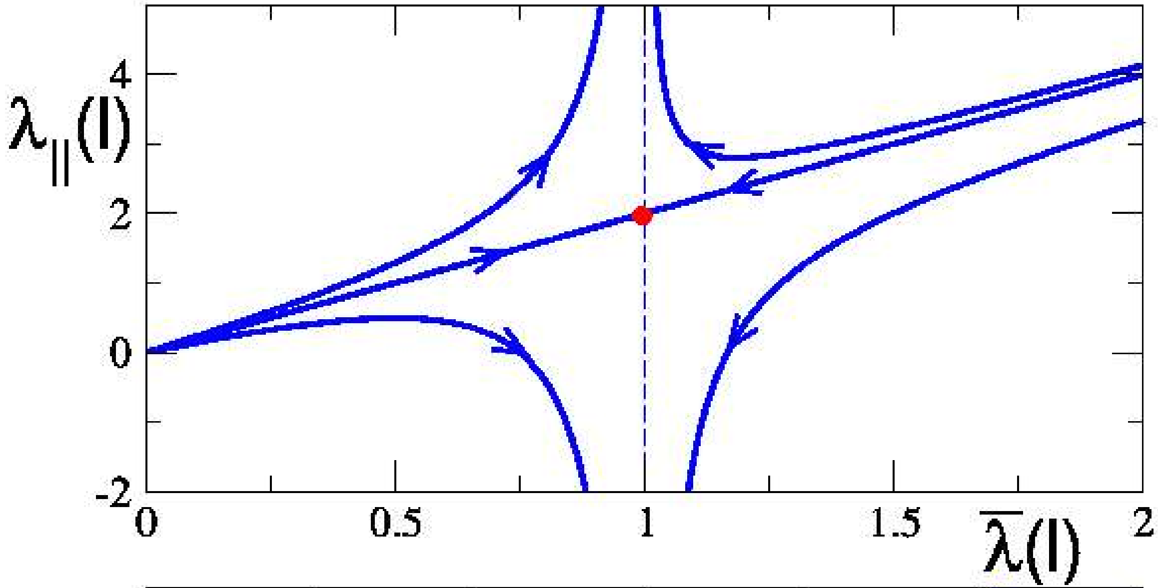
$\lambda_{\|} \to -\infty$ 1-order

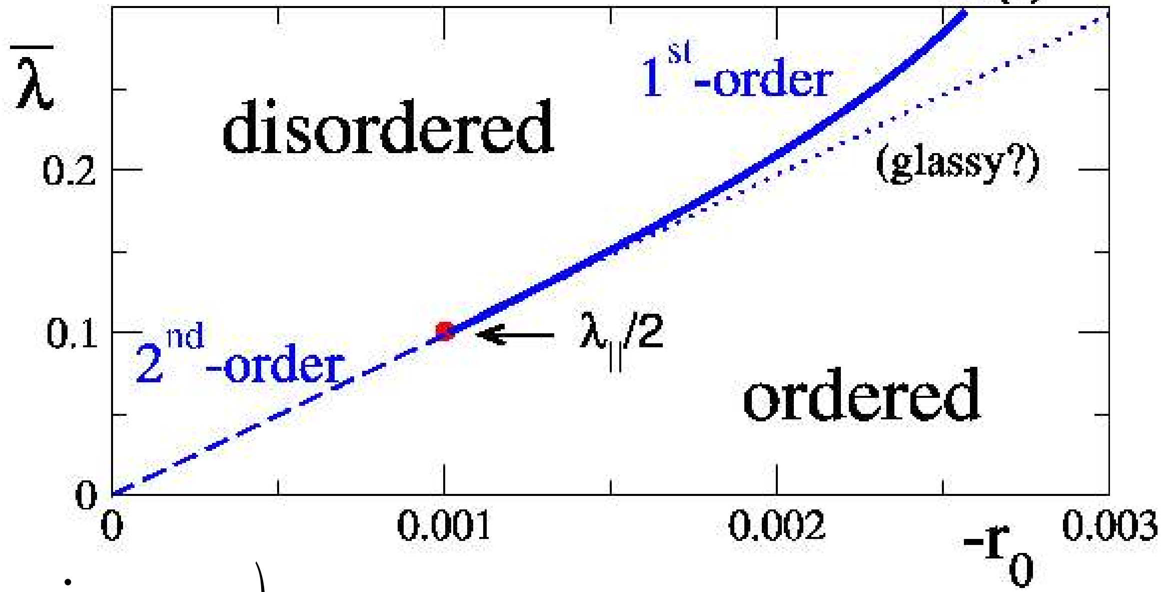
$\lambda_{\|} \to +\infty$ 2-order

Line of unstable tricritical fixed points at

$$\epsilon = 3 - z \to 0$$

$$\epsilon \propto \lambda_{\|} = 2\bar{\lambda}$$





order parameter

$$\mathbf{M}(\mathbf{x}) = M_0(\mathbf{m}_1 \cos \mathbf{q}_0 \cdot \mathbf{x} + \mathbf{m}_2 \sin \mathbf{q}_0 \cdot \mathbf{x})$$

Open problems

- Disorder and fluctuational 1-order transitions
- Finite temperatures (crossover from 1^{st} to 2-order)?
- Metastable states at finite temperatures
- Resistivity of glassy state with spin disorder

$$\rho \sim T^{3/2} ?$$

Several hand-written slides are ommitted

- Hartree solution
- short review of experiments (see C. fleiderer's talk)

Classification of quantum fluctuational 1-order transitions

$$\langle M^2 \rangle \propto \Sigma_H \propto \Delta^{\frac{z-1}{2}}$$

$$\Pi \propto \Delta^{\frac{z-3}{2}}$$

1) Local magnetization diverges $z < 1$

(similar to classical Brazovskii transition)

--> strong coupling to disorder

depends only on dynamical critical exponent z

2) Local magnetization does not diverge
Polarization operator diverges $1 < z < 3$

$$\omega \sim (\delta |q|)^z$$

3) ordinary 2nd-order transition $z > 3$

Compare with a condition for non-mean field critical QPT for small (vanishing) phase volume of critical soft modes

$D + z < 4$

Differences between roton-field theory and ordinary ϕ^4-theory

two-loop approximation

$$\Lambda_e \sim \lambda^3 (\Pi(\Delta))^2$$

$$\Lambda_f \sim \frac{\lambda^3}{\Delta} \Delta^{(D-1)/2}, \; \Lambda_e \gg \Lambda_f$$

small factor $\Delta^{(D-1)/2}$

1-order transition occurs at

$$1 \simeq \lambda_{\|} \Pi(\Delta)$$

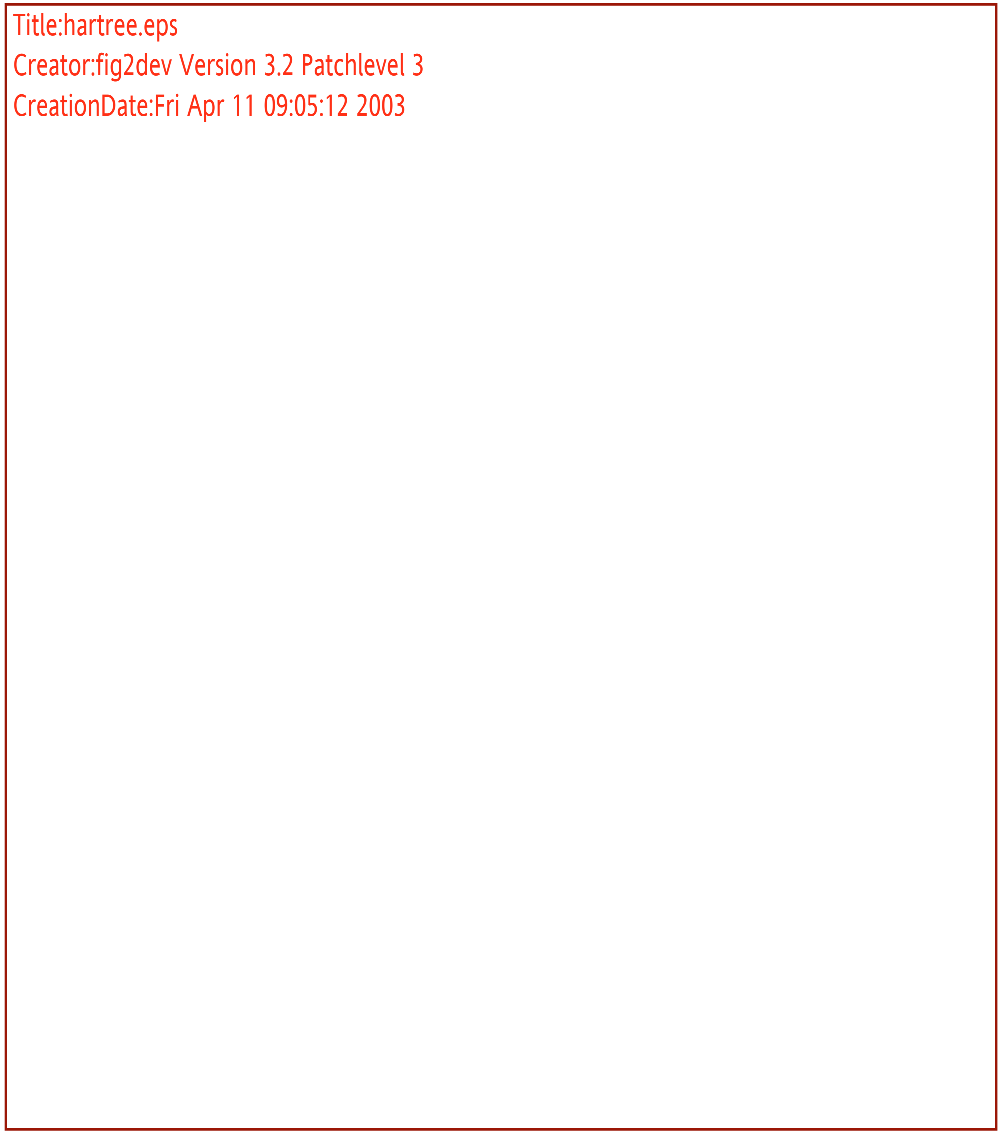
$$\Delta^{(3-z)/2} \simeq \lambda_{\|}$$

<u>*condition of validity*</u>

$$\lambda_{\|}^{\frac{D-1}{3-z}} \ll 1$$

classical case

$T > 0\,K$ $\qquad \lambda_{\|}^{1/16} \ll 1$



metastable states and disorder

• First solutions appear with N-large number of spirals with order parameter

$$M_0^2 = \frac{\Delta}{(2\bar{\lambda} - \lambda_{\|})} \frac{1}{(4N-1)}$$

• Large-N solutions are metastable, because a single-spiral state has the lowest energy, but

1) disorder can lock (stabilize) metastable states with N-spirals
2) large-N spiral state is stable to thermal fluctuations (unlike 1D and 2D structures)

R.Peierls (1934), L.D.Landau (1937)
also Mermin-Wagner theorem

Magnetic droplets-defects close to the ferromagnetic transition

Larkin, Melnikov (1972)
Millis, Morr, Schmalian (2001)
Loh, Tripathi, Turlakov (2004)

important differences between Ising and XY (and Heisenberg) symmetry

"giant moment"
multi-channel Kondo effect

$$g = \frac{(JSn(\epsilon_F))^2}{1+F_a} (coupling\ constant)$$

---> contribution of magnetic droplets-impurities to the magnetic susceptibility and resistivity

Fermion-roton scattering

large phase volume of soft modes (rotons) → → **strong damping of electrons**

Electron self-energy

$$Im\Sigma(\Omega)=\int_0^{\Omega} d\omega \int dq\, q^{D-2}\, Im\chi(q,\omega)[1+n_B(\omega)]$$

$$Im\Sigma(\Omega)\propto\lambda^2\alpha q_0^2\frac{\acute{m}}{m}\sqrt{\frac{\Delta+\Omega}{\alpha q_0^2}}$$

$$Im\Sigma(\Omega)\propto\Omega^{\frac{z-1}{z}}\ \textit{for arbitrary } z$$

Resistivity

$$\rho\sim\frac{1}{\tau_{tr}}\sim\sqrt{T}\ \textit{for}\ kT\gg\Delta$$

High-temperature result

Conclusions

observed in MnSi !

- magnetic rotons form a new universality class of quantum phase transitions (fluctuation induced 1st order trans., tricritical points, scaling behavior ...)

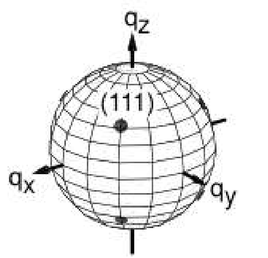
- pressure induced state in MnSi is a fluctuation induced amorphous state with helix defects causing unconventional transport

hierarchy of energy scales

- exchange energy of the ferromagnetism $T_c \approx 30K$

- Dzyaloshinsky-Moriya interaction $E_{inh} \approx 4K$

- crystal field potential pins $\mathbf{q_0}$ ~<111>

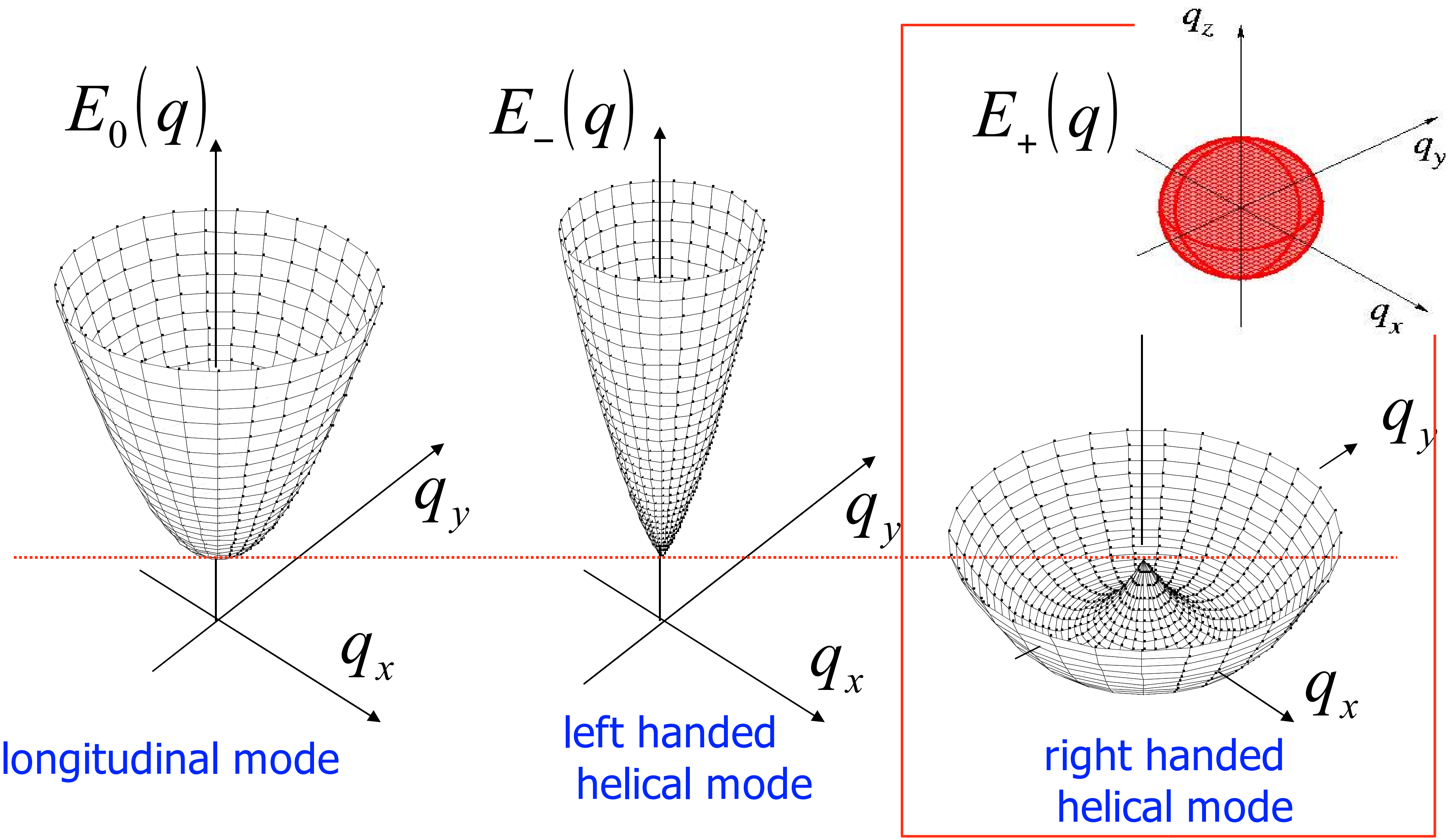


$$E_{anis.} / E_{inh.} \approx 0.05$$

key assumption:
anisotropy terms can be neglected above T_c

the paramagnons of the theory

$$L = L_{dyn.} + \gamma\, \mathbf{M}\cdot\left(\nabla\times\mathbf{M}\right) + \tfrac{d}{2}\left(\nabla\mathbf{M}\right)^2 + \tfrac{r}{2}\mathbf{M}^2 + \tfrac{\lambda}{4}\mathbf{M}^4$$



dynamics of rotons

For T>T$_c$

$$\chi(q,\omega) = \frac{1}{E_+(q) - i\Gamma\omega}$$

$\omega \rightarrow \omega^{2/z}$

with z arbitrary

diffusive due to particle hole excitations

$$E_+(q = q_0) = \Delta$$

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challenges

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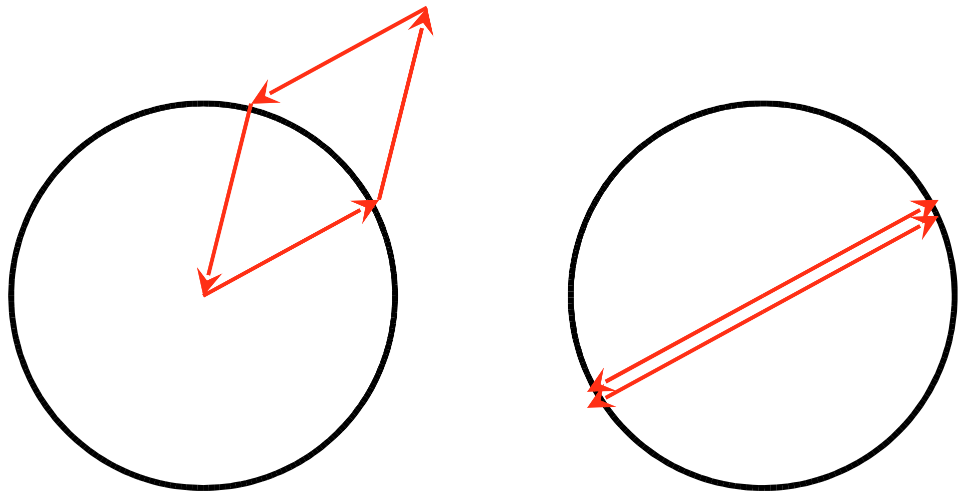
roton-roton interaction

small dimensionless coupling constant

$$\lambda \propto \left(q_0 a\right)^{D+z-4} < \; 1 \text{ if } D+z>4$$

angular dependence of the coupling constant



similar to fermions:
Shankar (1994),
Hohenberg+Swift (1995)

special case

$$\bar{\lambda} = \lambda(\mathbf{q}_0, \mathbf{q}_0', -\mathbf{q}_0, -\mathbf{q}_0')$$

$$\lambda_{\parallel} = \lambda(\mathbf{q}_0, \mathbf{q}_0, -\mathbf{q}_0, -\mathbf{q}_0)$$

roton-roton interaction- perturbation theory

leading terms of the perturbation theory

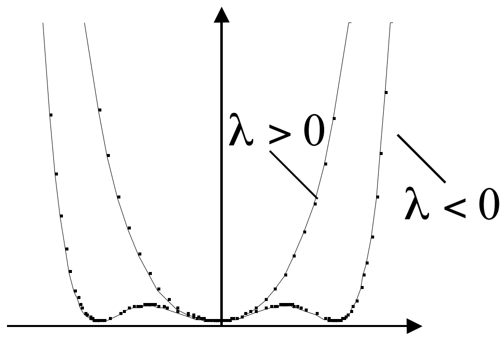
$$\bar{\lambda}^{\text{renorm}} = \bar{\lambda} - \bar{\lambda}^2 \Pi + \bar{\lambda}^3 \Pi^2 \ldots = \frac{\bar{\lambda}}{1+\bar{\lambda}\Pi}$$

polarization diagram

$$\Pi = \int d^D q \, d\omega \left(\frac{1}{|\omega|^{2/z} + \Delta + (|\vec{q}| - q_0)^2} \right)^2 \propto \Delta^{\frac{z-3}{2}} \to \infty$$

$$\lambda_{||}^{\text{renorm}} = \lambda_{||} \frac{1-(2\bar{\lambda} - \lambda_{||})\Pi}{1+\bar{\lambda}\Pi}$$

$\lambda_{||} < 2\bar{\lambda}$ changes sign!
→ 1st order transition



$\lambda_{||} > 2\bar{\lambda}$ → 2nd order transition