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Electron self-energy near a quantum critical point

Conflicting stories of the divergent effective electron mass

With applications to the metal-insulator transition in a two-dimensional electron gas (MIT in 2DEG)

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1. Experimental overview of the metal-insulator transition and divergence of the effective mass in 2DEG

2. Divergence of the effective mass near a density-wave instability within the Landau theory of Fermi liquids
   - Shaginyan, *JETP Lett.* **77**, 99 (2003)
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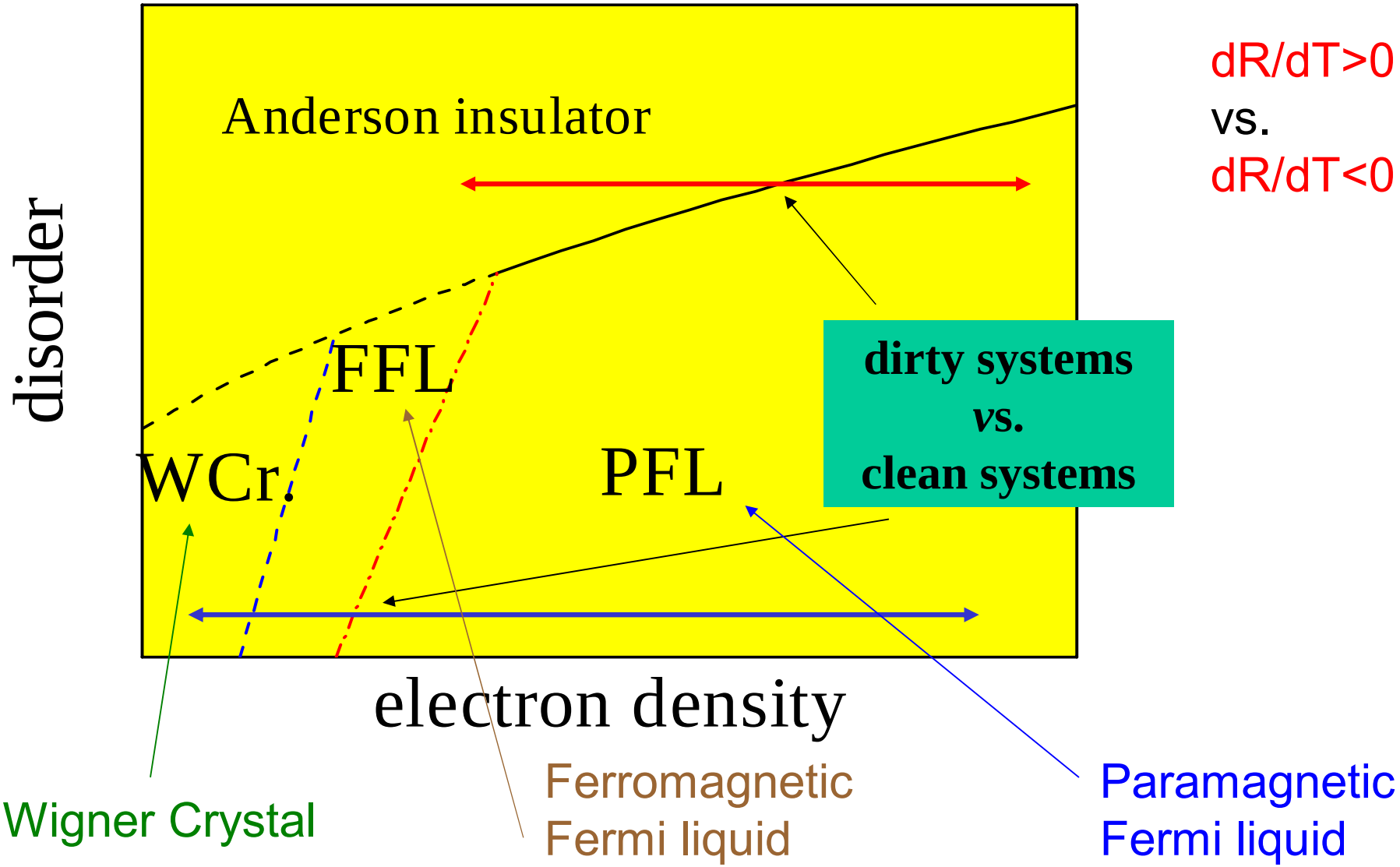
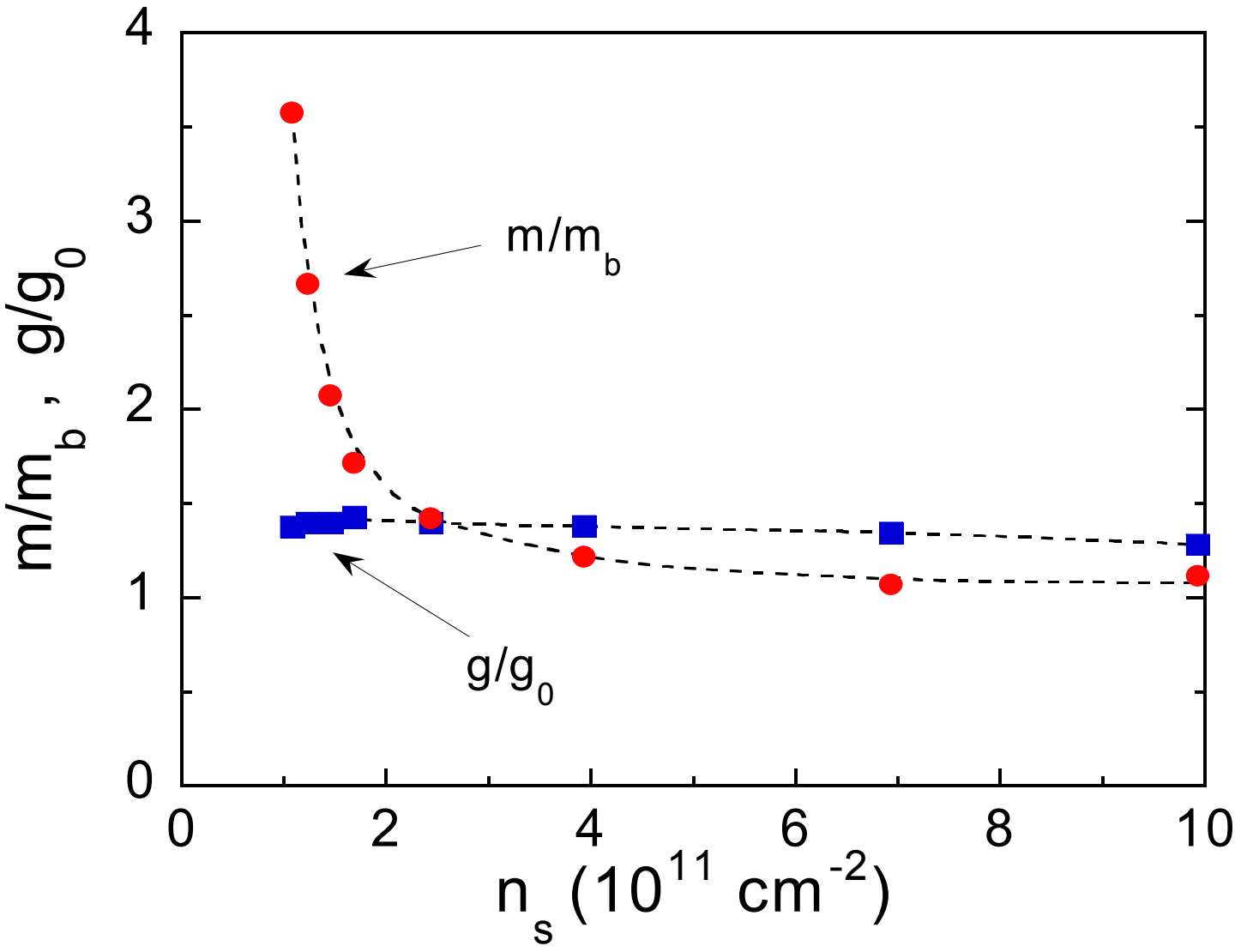
   - T. M. Rice, *Annals of Physics* **31**, 100 (1965)

Metal-insulator transition in 2DEG



Effective mass divergence in 2DEG



Shashkin, Kravchenko, Dolgopolov, Klapwijk, *PRB* **66**, 073303 (2002)

$$\frac{1}{m} = \frac{v_F}{p_F}$$

Effective mass divergence in 2D He3

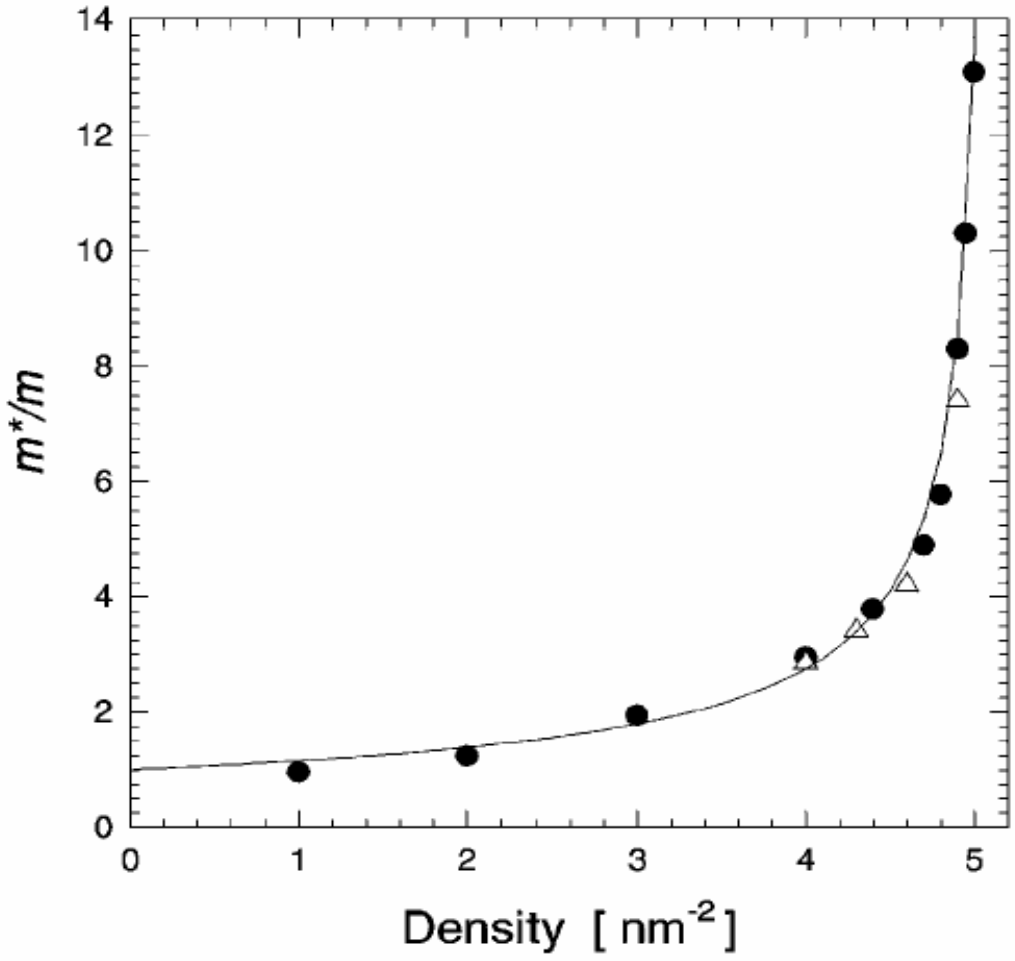


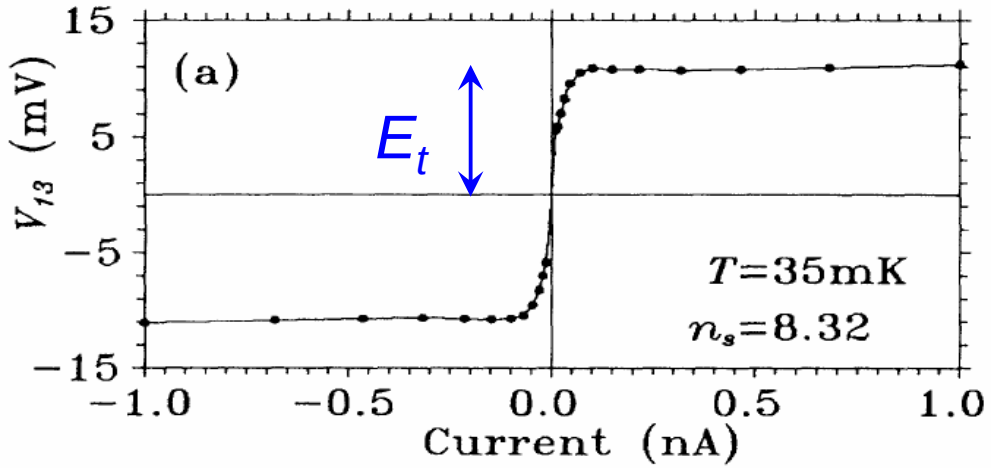
FIG. 2. Effective mass ratio as a function of ^{3}He fluid density inferred from heat capacity (●), magnetization (△), showing apparent divergence. Solid line is fit to data (see text).

Casey, Patel, Nyéki, Cowan, & Saunders, *PRL* **90**, 115301 (2003)

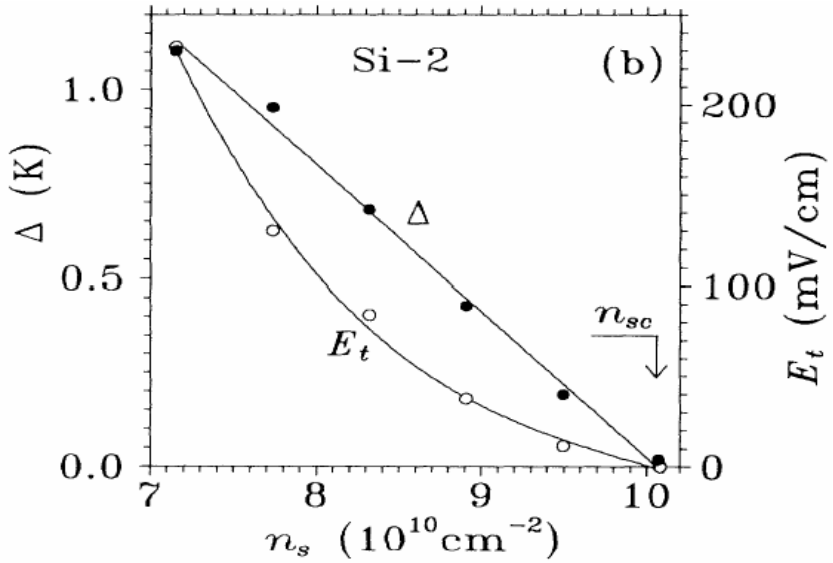
Effective mass divergence is a precursor of the crystallization transition from liquid to solid He3.

Highly nonlinear *I-V* in the insulating phase

Pudalov, D'Iorio, Kravchenko, & Campbell, *PRL* **70**, 1866 (1993)



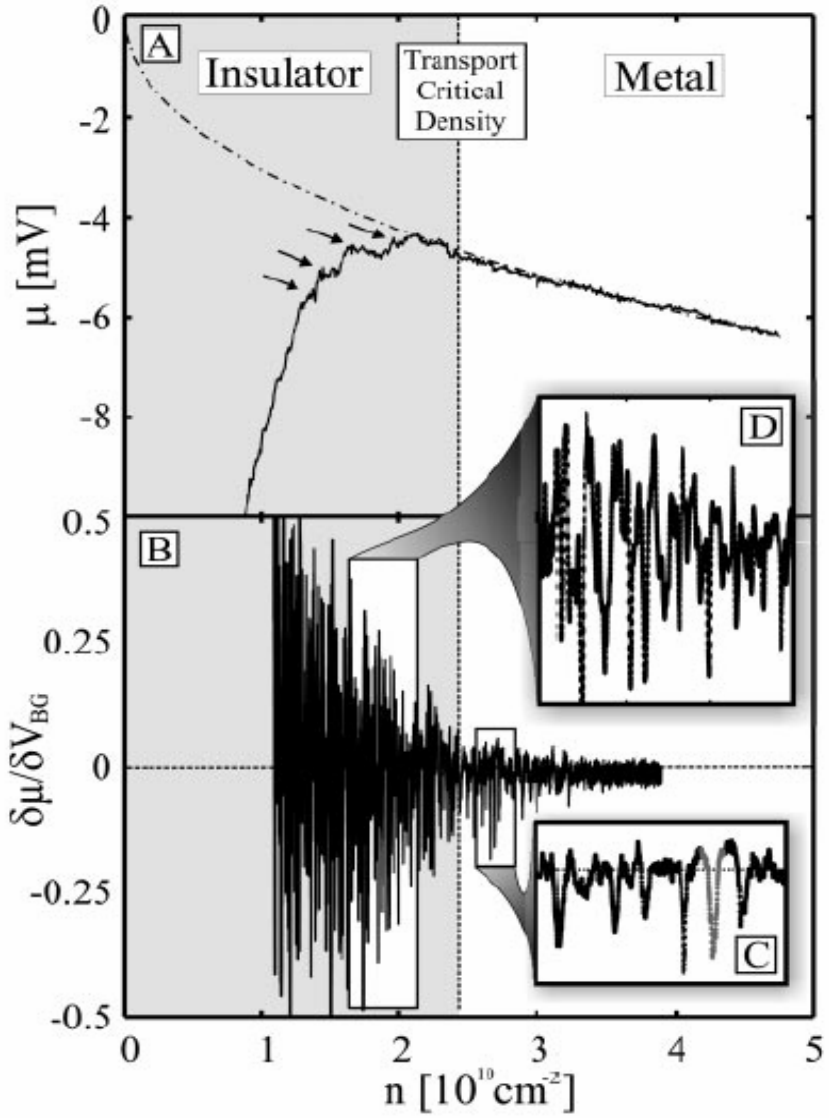
Differential conductance increases by the factor of 10^6 when $E>E_t$



The threshold electric field E_t vanishes at $n \to n_c$

Formation of a density wave by the second order phase transition at $n=n_c$ and its depinning by electric field $E>E_t$?

Jumps in the local $\mu(n)$ dependence



Ilani, Yacoby, Mahalu, Shtrikman, *Science* **292**, 1354 (2001)

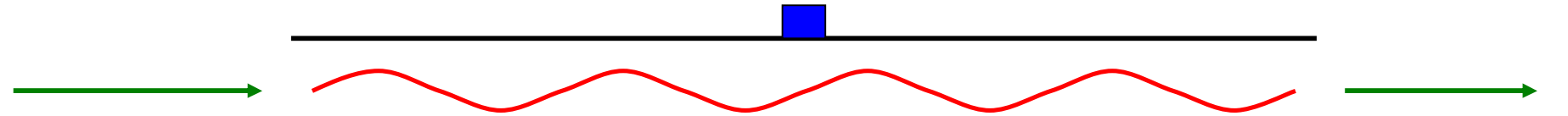
The $\mu(n)$ dependence was measured by a local SET (single-electron transistor), where
μ is the local chemical potential,
n is the electron concentration.

Jumps in $\mu(n)$ – rearrangments of a pinned density wave upon changes in n?

Is the insulating phase of the MIT really a charge-density wave?

Proposed experiment: modification of the local SET measurements by Yacoby *et al.*, *Science* (2001)

Radio-frequency single-electron transistor (RF-SET)



Velocity v | Charge-density wave | Pulling electric field

Applying a pulling electric field forces CDW to slide with velocity v.

The sliding CDW should generate an ac signal on the RF-SET with the frequency $\nu \sim j/e\sqrt{n}$ (the narrow-band noise).

For typical values j = 1 nA/mm, $n = 10^{10}$ cm^{-2}, and $1/\sqrt{n}$ = 100 nm, we find $\nu \sim 600$ kHz and $v \sim 6$ cm/sec.

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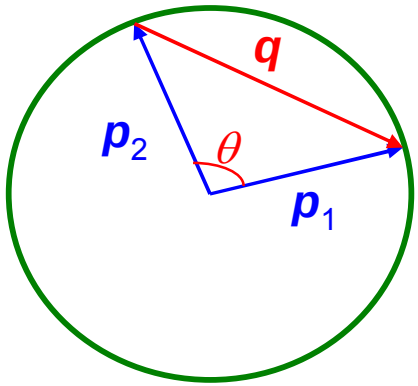
   - T. M. Rice, *Annals of Physics* **31**, 100 (1965)

Landau theory of Fermi liquids

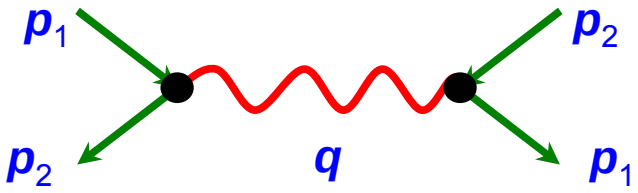
$$\frac{1}{m^*} = \frac{1}{m} - C\int \cos\theta\, f(\theta)\, d\theta$$



Fermi surface

$f(\theta)$ is the Landau interaction function

$f(\boldsymbol{p}_1,\boldsymbol{p}_2) = f(\boldsymbol{q}), \quad \boldsymbol{q} = \boldsymbol{p}_1 - \boldsymbol{p}_2$



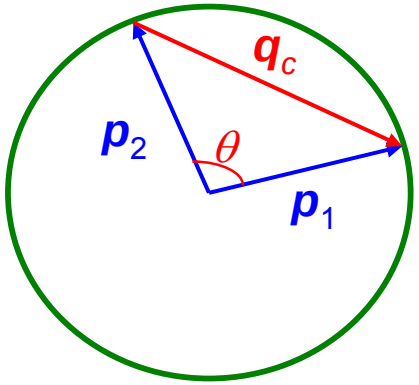
For the screened Coulomb interaction: $f(q) = -\dfrac{2\pi e^2}{q + \kappa}$.

$f<0$, and $|f|$ is maximal at $q=0$, i.e. at $\theta=0$, where $\cos\theta>0$. So, m^* decreases: $m^*<m$, as observed for small $r_s \propto a/a_B$.

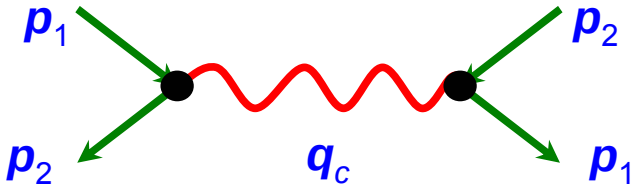
Suppose the MIT is a transition from a metal to a density wave

In this case: $f(q) = -\dfrac{C}{n - n_c + (q - q_c)^2}$, where $q_c \sim 2k_F$,

and n_c is the critical concentration.



Interaction via exchange of CDW fluctuations:



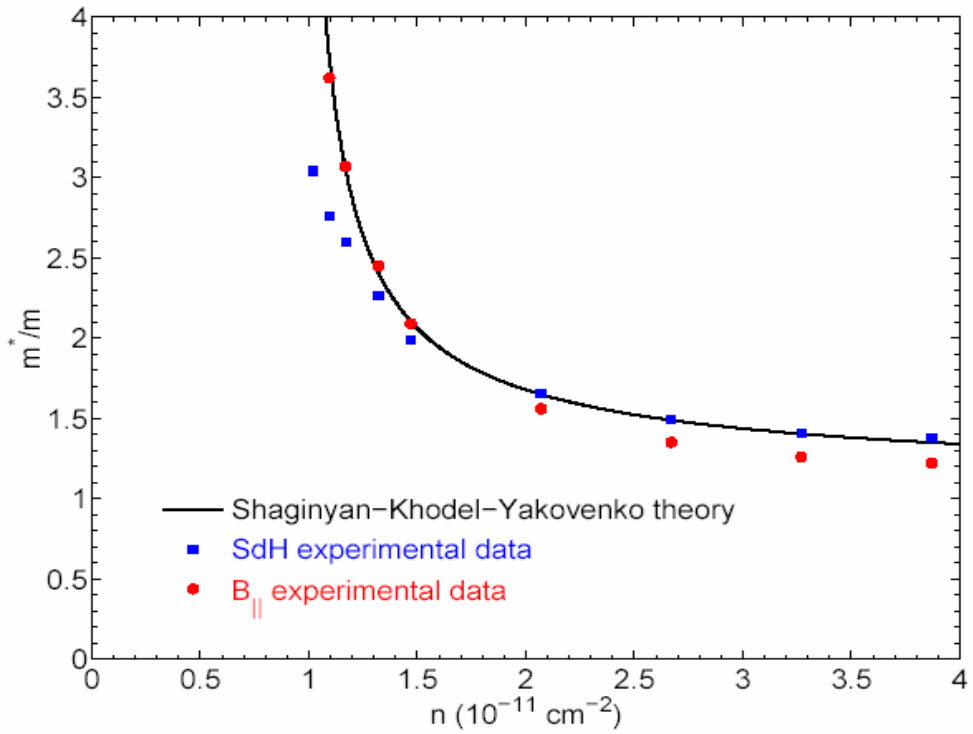
Now $|f|$ is maximal at $q_c \sim 2k_F$, where $\cos\theta < 0$, so m^* increases and diverges:

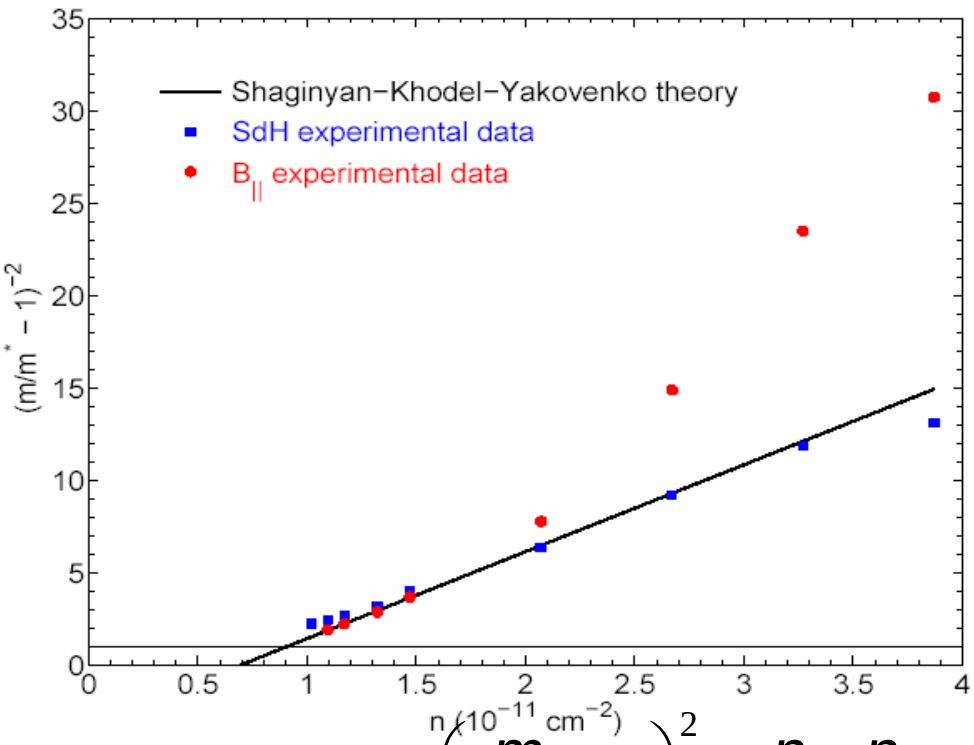
$$\frac{1}{m^*} = \frac{1}{m} - \frac{C}{\sqrt{n - n_c}}$$

Also obtained by Shaginyan, *JETP Lett.* **77**, 99 (2003)

Comparison with experiment

Data from Kravchenko & Sarachik, *Rep. Prog. Phys.* **67**, 1 (2004)





m/m* vs. carrier concentration n

$$\left(\frac{m}{m^*}-1\right)^2 = \frac{n-n_c}{C}$$

Fitting parameters: $n_c = 0.7\times10^{-11}$ cm^{-2}, C = 0.46.
m^* diverges at $n_\infty = 0.9\times10^{-11}$ cm^{-2}.

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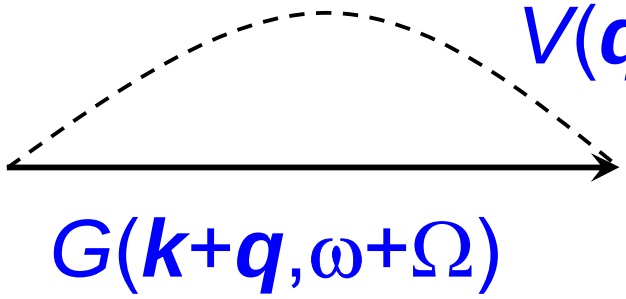
Electron self-energy $\Sigma(\boldsymbol{k},\omega)$

$\Sigma(\boldsymbol{k},\omega) =$



If the effective interaction $V(\boldsymbol{q},\Omega)\to V(\boldsymbol{q})$ does not depend on frequency Ω, then

$$\Sigma(\boldsymbol{k},\omega)\to\Sigma(\boldsymbol{k})=\int\frac{d^2q}{(2\pi)^2}\,n(\boldsymbol{k}+\boldsymbol{q})\,V(\boldsymbol{q})$$

$$\varepsilon(\boldsymbol{k})=\varepsilon_0(\boldsymbol{k})+\Sigma(\boldsymbol{k})$$

In this case, we recover the Landau theory of Fermi liquids with $f(\boldsymbol{q}) = V(\boldsymbol{q})$ and $\Sigma(\boldsymbol{k},\omega) = \Sigma(\boldsymbol{k})$.

Crossover from $\Sigma(k)$ to $\Sigma(\omega)$

Let us take into account the frequency dependence of $V(\boldsymbol{q},\Omega)$ due to the Landau damping:

$$V(\boldsymbol{q},\Omega) = \frac{g}{\xi^{-2} + (q - q_c)^2 + i\gamma\Omega}, \quad \gamma \sim \frac{g}{v_F^2}$$

We find that

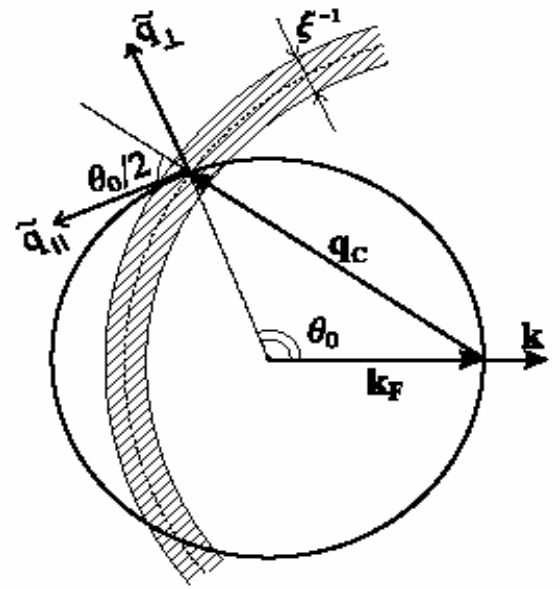
$\Sigma(\boldsymbol{k},\omega) \to \Sigma(k) = \lambda\, \varepsilon_0(k) \cos\theta_0$ when $\eta << 1$,

$\Sigma(\boldsymbol{k},\omega) \to \Sigma(\omega) = \lambda\, \omega$ when $\eta >> 1$,

where $\varepsilon_0(k) = v_F(k - k_F)$,

$\lambda \sim \int dq_{||}\, V(\boldsymbol{q},0) \sim g\, \xi / v_F$,

$\eta \sim \gamma\, E_F\, \xi^2 \sim \lambda^2\, E_F / g$



The crossover from $\Sigma(k)$ to $\Sigma(\omega)$ happens when $\xi \to \infty$ at $\eta \sim 1$, where still $\lambda << 1$ if $g << E_F$.

Divergence of the effective mass *m**

In the vicinity of the quantum critical point, where $\Sigma(k,\omega) \to \Sigma(\omega)$,

$$\frac{m^*}{m} \approx \frac{1}{Z} \approx 1 + \lambda$$

i.e. the effective mass m^* diverges together with vanishing quasiparticle residue Z at the critical point $\lambda \to \infty$, but not before.

This is in contrast to the case $\Sigma(k,\omega) \to \Sigma(k)$, where m^* would diverge before the critical point is reached, at $\lambda \cos\theta_0 = 1$:

$$\frac{m^*}{m} = \frac{1}{1 - \lambda \cos\theta_0}$$

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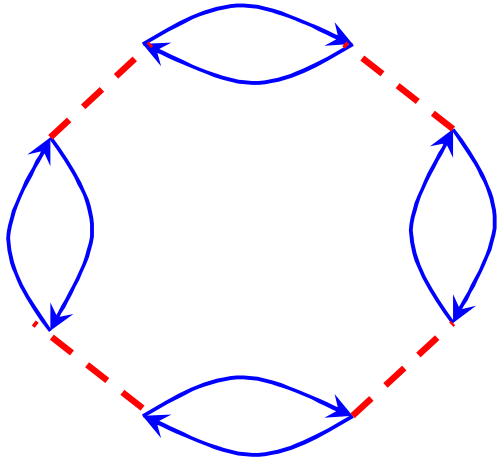
An explicit diagrammatic calculation of the quasiparticle spectrum $\varepsilon(k)$

RPA – ring diagrams
(Gell-Mann & Brueckner, 1957)

E_{corr} =



The bare electron Green's function:

$$G_0(\boldsymbol{k},\omega) = \frac{n(\boldsymbol{k})}{\omega - \varepsilon_0(\boldsymbol{k}) - i\delta} + \frac{1 - n(\boldsymbol{k})}{\omega - \varepsilon_0(\boldsymbol{k}) + i\delta}$$

Variation over electron occupation numbers $n(\boldsymbol{p})$:

$$\frac{\delta G_0(\boldsymbol{k},\omega)}{\delta n(\boldsymbol{p})} = 2\pi i\, \delta(\boldsymbol{k} - \boldsymbol{p})\, \delta\left[\omega - \varepsilon_0(\boldsymbol{k})\right]$$

Landau's definition:

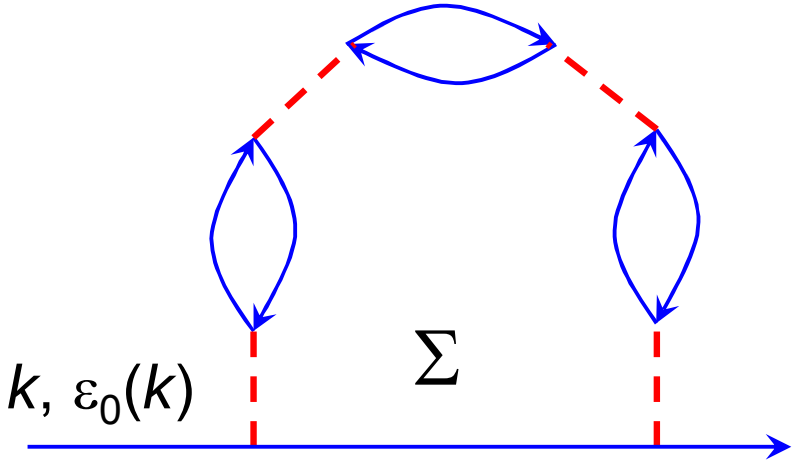
$$\varepsilon_{\text{corr}}(\boldsymbol{k}) = \frac{\delta E_{\text{corr}}}{\delta n(\boldsymbol{k})} = \frac{\delta E_{\text{corr}}}{\delta G}\frac{\delta G}{\delta n(\boldsymbol{k})} = \Sigma\left[\boldsymbol{k}, \varepsilon_0(\boldsymbol{k})\right]$$

Cut one electron line & put on shell

Quasiparticle spectrum

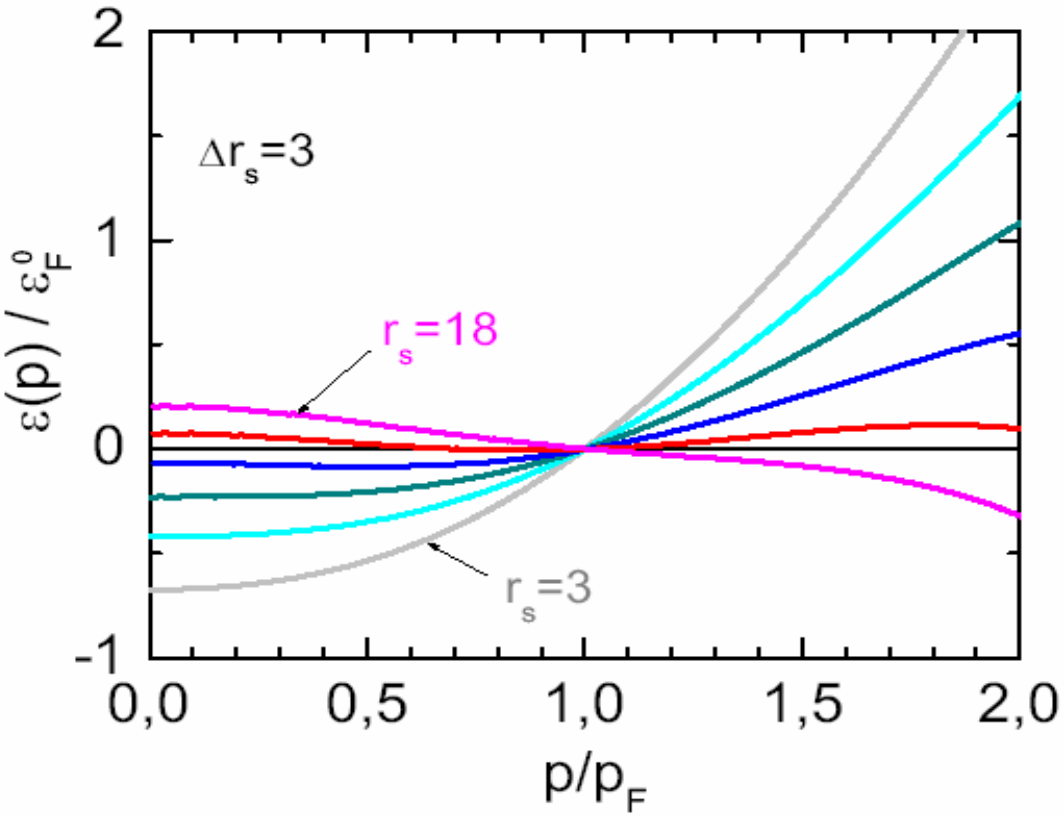
$$\varepsilon(\boldsymbol{k}) = \varepsilon_0(\boldsymbol{k}) + \Sigma\left[\boldsymbol{k}, \varepsilon_0(\boldsymbol{k})\right]$$

on-shell (T. M. Rice, 1965)



Numerical calculations done by M.Zverev and independently by Ying Zhang

We observe flattening of the electron dispersion around r_s=15. Electrons lose kinetic energy and become localized?



On shell

$$\varepsilon = \varepsilon_0(\boldsymbol{k}) + \Sigma\left[\boldsymbol{k}, \varepsilon_0(\boldsymbol{k})\right]$$

$$\frac{v_F^*}{v_F} = 1 + \frac{1}{v_F}\frac{\partial\Sigma}{\partial k} + \frac{\partial\Sigma}{\partial\omega}$$

$v_F^* \to 0$ when $\partial\Sigma/\partial\omega<0$ overcomes $\partial\Sigma/\partial k>0$.

Off-shell is essential for obtaining mass divergence.

Off shell – Dyson eqn

$$\varepsilon = \varepsilon_0(\boldsymbol{k}) + \Sigma(\boldsymbol{k}, \varepsilon)$$

$$\frac{v_F^*}{v_F} = \frac{1 + \frac{1}{v_F}\frac{\partial\Sigma}{\partial k}}{1 - \frac{\partial\Sigma}{\partial\omega}}$$

Both numerator and denominator grow, but they approximately cancel, so $v_F^* \approx v_F$.

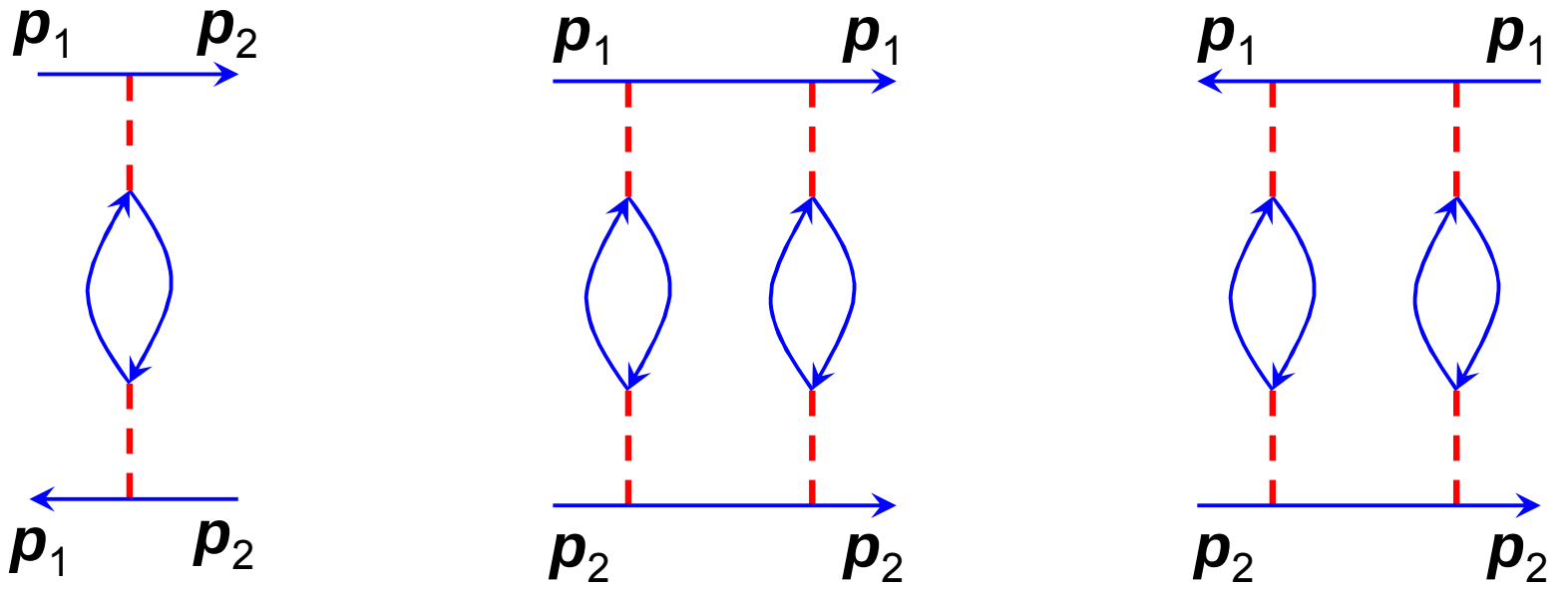
Ying Zhang & Sankar Das Sarma, cond-mat/0312565

Landau interaction function $f(\boldsymbol{p}_1,\boldsymbol{p}_2)$

Second variation over electron occupation numbers $n(\boldsymbol{p})$:

$$f(\boldsymbol{p}_1,\boldsymbol{p}_2)=\frac{\delta E_{\mathrm{corr}}}{\delta n(\boldsymbol{p}_1)\,\delta n(\boldsymbol{p}_2)}$$

Cut two electron lines & put them on shell



f < 0 and |f(θ)| has maxima at θ = 0 and θ = π. The latter results from the Cooper correlations at $\boldsymbol{p}_1 = -\boldsymbol{p}_2$. The maximum at θ = π is responsible for mass divergence.

Conclusions

- The metal-insulator transition in a clean 2DEG may be a second-order transition to a charge-density-wave state, precursor to the Wigner crystal.

- It may be possible to detect the narrow-band noise from the sliding CDW using a radio-frequency single-electron transistor as a local probe.

- Interaction between electrons via exchange of the charge-density-wave fluctuations results in divergence of the effective mass m^* at MIT.

- Divergence of the effective mass m^* at MIT can be also obtained from the ring diagrams (RPA) in the on-shell approximation, without assuming a density-wave formation.