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Workshop on
Novel States and Phase Transitions in Highly Correlated Matter

12 - 23 July 2004

Interaction Effects in High-Mobility Si MOSFETs

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These are preliminary lecture notes, intended only for distribution to participants

Interaction Effects

in High-Mobility Si MOSFETs

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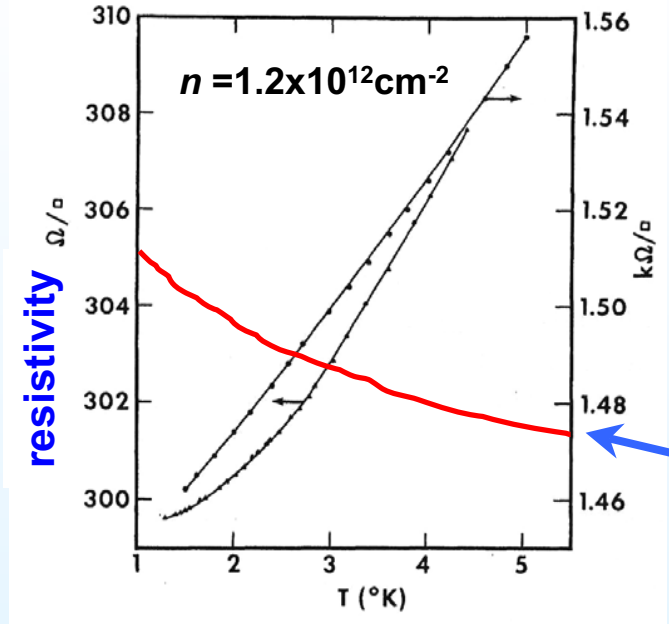
Collaborators: **N. Klimov and H. Kojima (*Rutgers*)**

V. Pudalov (*Lebedev Inst., Moscow*)

ICTP, July 2004

The State University of New Jersey
Rutgers
Department of Physics and Astronomy



Cham and Wheeler, *PRL* **44**, 1472 (1980)

The scaling theory of localization for non-interacting electrons (Abrahams *at al*., '79):

all 2D electron states are localized!

prediction of the WL theory

σ (T) and σ ($B_{||}$) of the most ubiquitous 2D system – electrons in Si MOSFETs – defied predictions of the WL+Int. theory

Introduction: a 20+ years old puzzle

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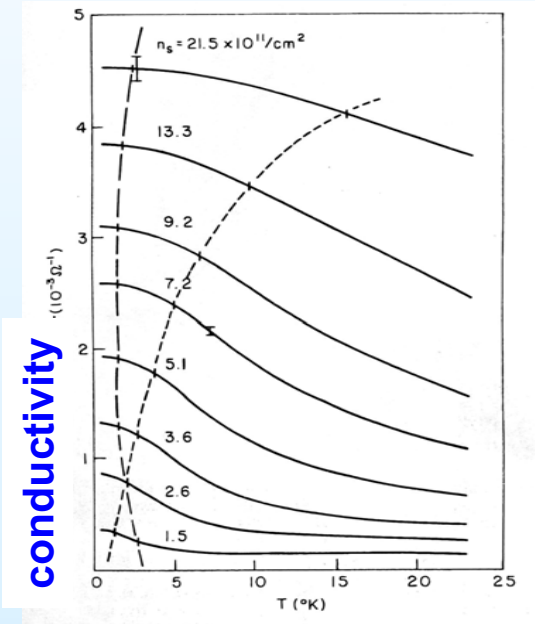
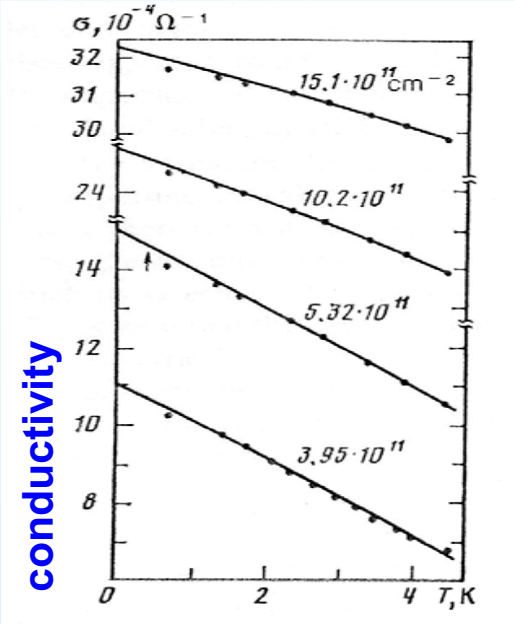
Reluctance of two-dimensional systems

A. I. Larkin
L. D. Landau Institute of Theoretical Physics, USSR Academy of Sciences

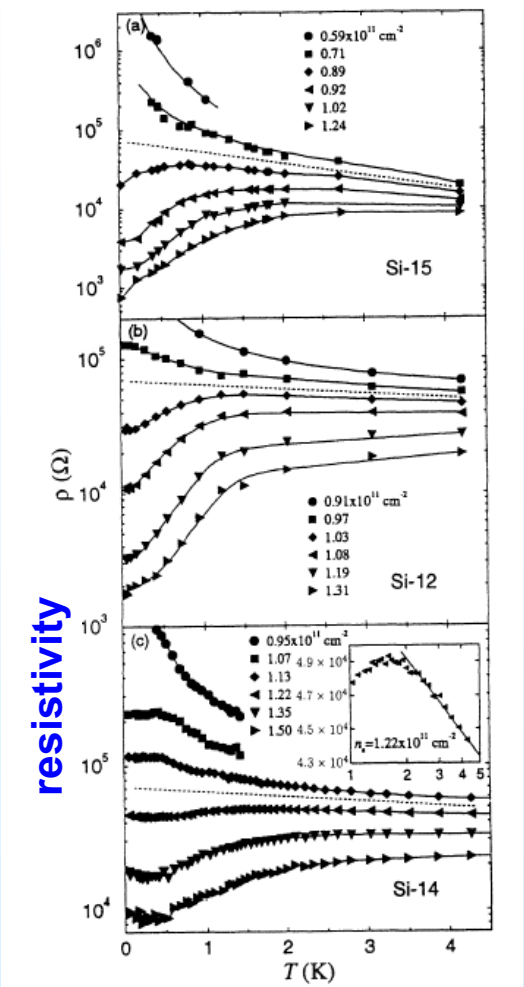
(Submitted 17 January 1980)
Pis'ma Zh. Eksp. Teor. Fiz. **31**, No. 4, 239–243 (20 February 1980)



Smith and Stiles,
SSC **58**, 511 ('86)
μ = 2.5x10^4 cm^2/Vs @ 0.2K



Vyrodov *et al.*,
JETP **67**, 998 ('88)
μ = 1.7x10^4 cm^2/Vs



Kravchenko *et al.*,
PRB **50**, 8039 ('94)
μ = (1.9-7)x10^4 cm^2/Vs

"Metallic" behavior has been observed in all **high-mobility** systems in the **low-density** regime: *p*- and *n*-type GaAs/GaAlAs, *n*-type AlAs, *p*-type Si/SiGe, etc.

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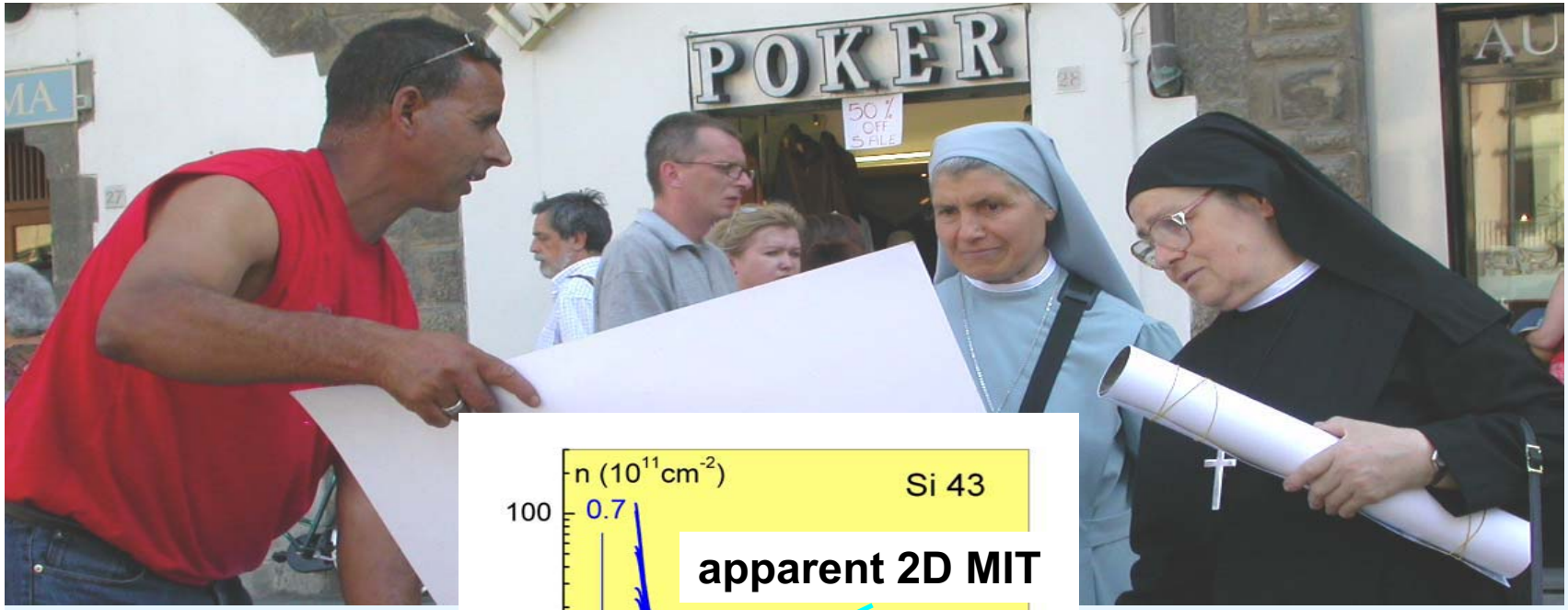
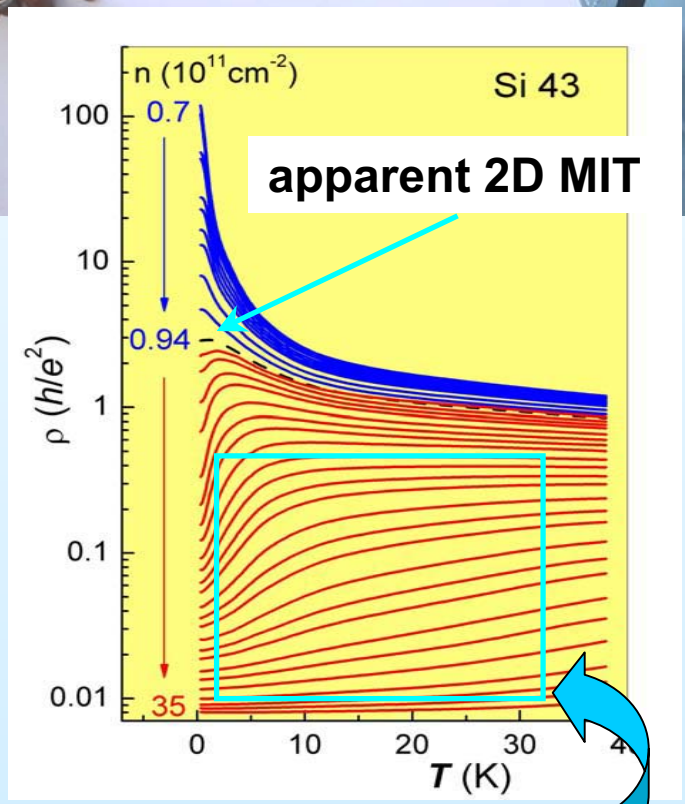


Pudalov *et al.*, circa 2000



"metallic" behavior

Puzzling – the "metallicity" rather than "2D MIT"

A great deal of interest –

the ground state of 2D systems is at stake!

The "metallic" behavior ≡ $d\rho/dT > 0$

The "insulating" behavior ≡ $d\rho/dT < 0$

The apparent 2D MIT ≡ the crossover from "metallic" to "insulating" behavior

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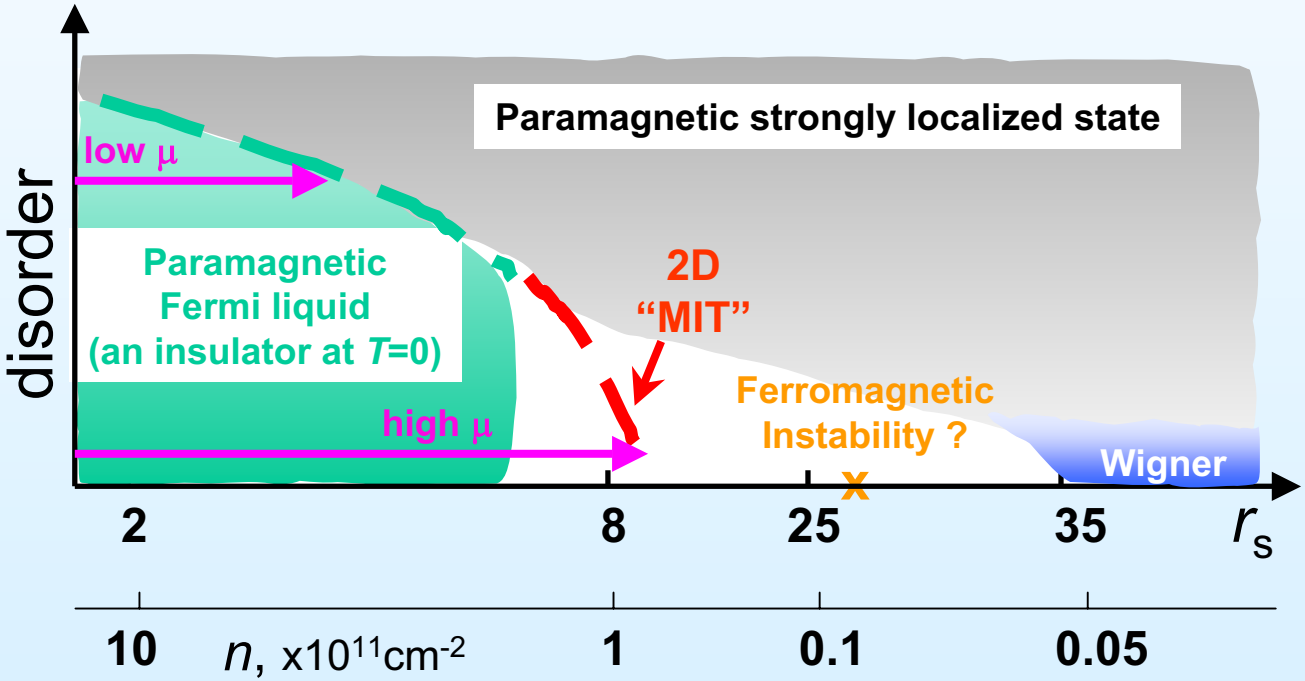
$$r_s = \frac{E_C}{E_F} \left(\frac{\propto \sqrt{n}}{\propto n} \right) = \frac{a_B}{\sqrt{\pi n}}$$

The lower *n*, the stronger the effective interactions



Primary suspect - electron-electron interactions

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renormalization of el. parameters

“metallic” conductivity

crossover “metallic” - ”insulating” conductivity

Three inter- related (?) phenomena: what is the role of interactions?

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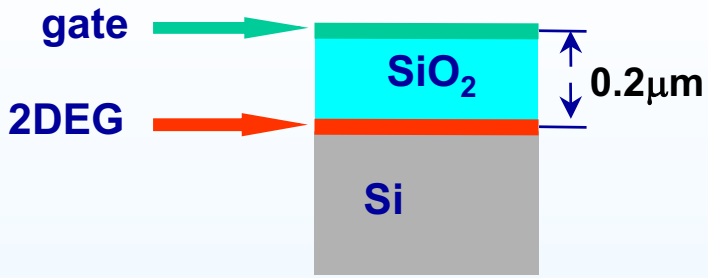
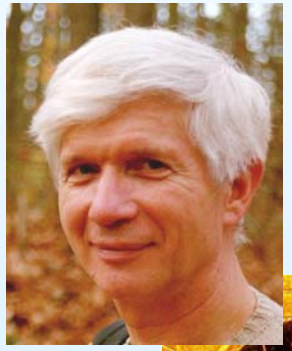
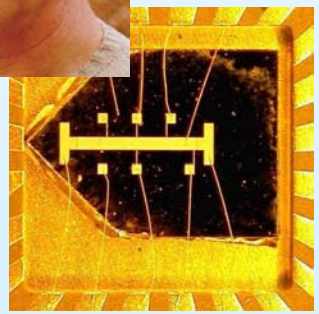
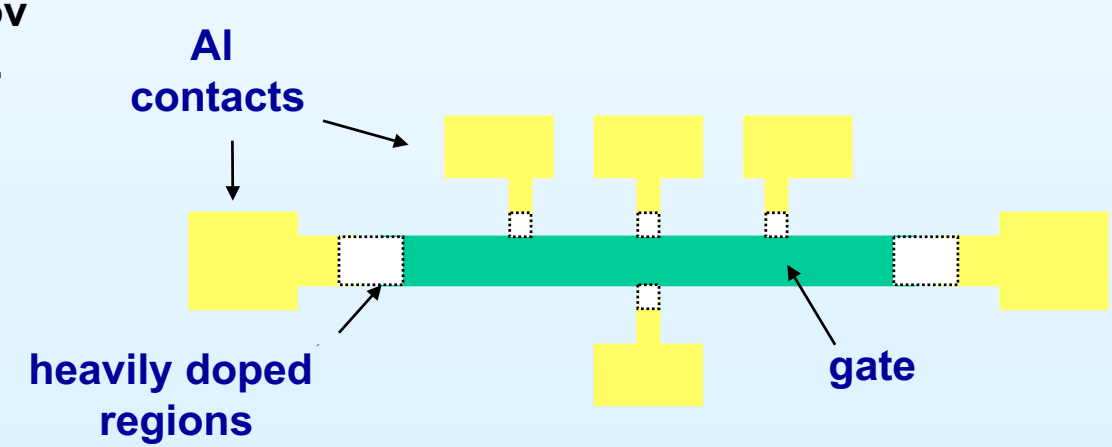


- $\mu \sim (2\text{-}4)\cdot 10^4\ cm^2/Vs$ @ $2\cdot 10^{11}\ cm^{-2}$
- inter-electron distance < distance to the gate (0.2 μm)



Vladimir Pudalov (Lebedev Inst. Moscow)





Part I: **"Quantification" of interactions: measurements of the interaction-driven renormalization of the electron parameters**

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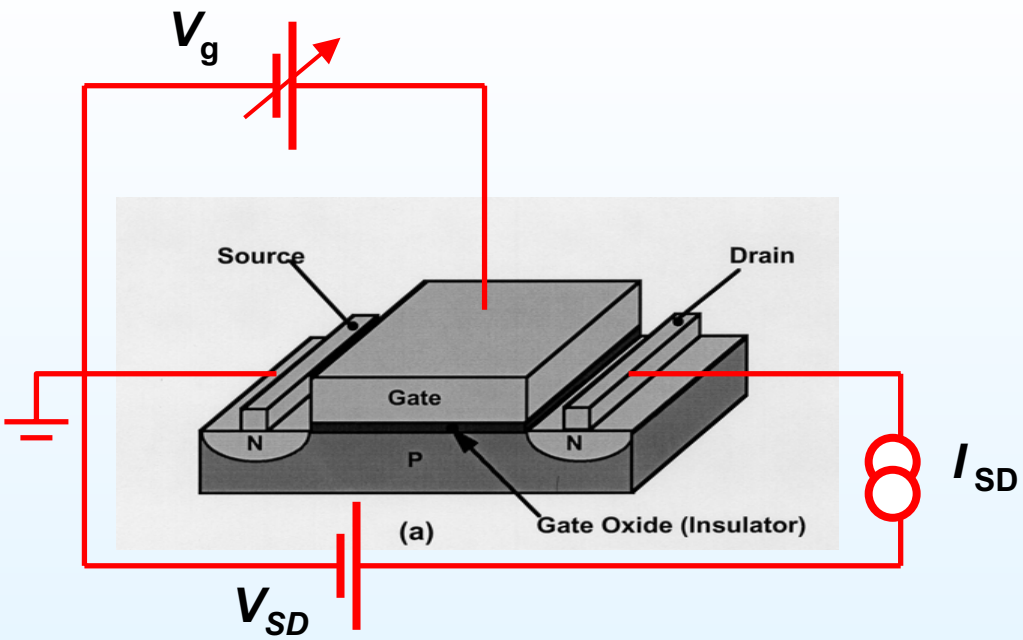

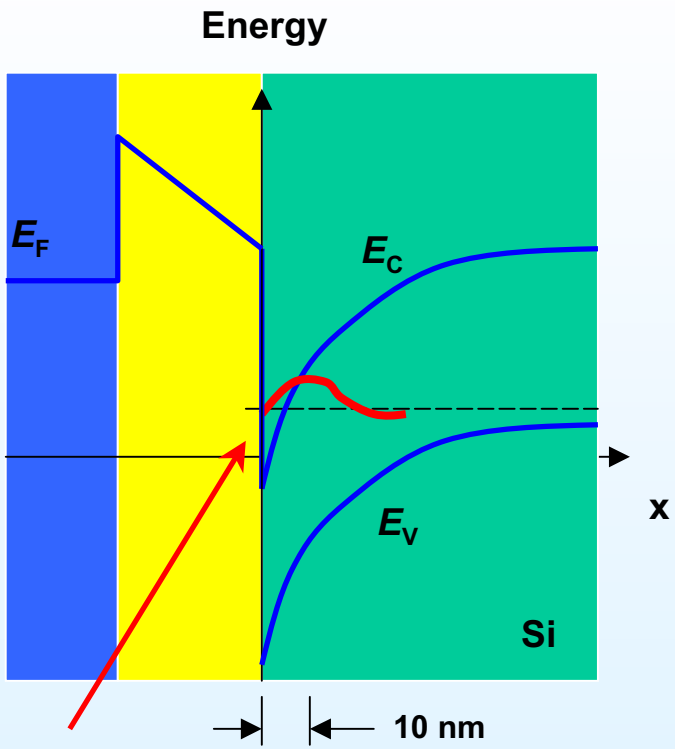

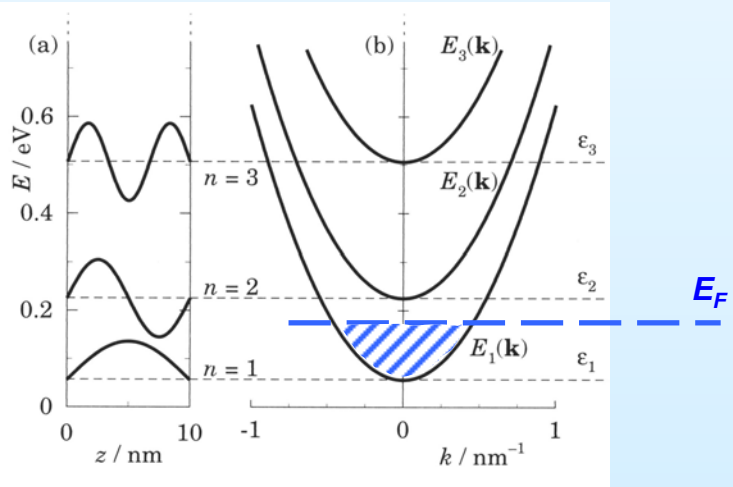

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2E_F m^*}} \sim 10nm$$

$$E_F \approx 7 \times \left(\frac{n}{1 \times 10^{11} cm^{-2}} \right) \quad [K]$$

Si MOSFET operation

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The interaction-driven renormalization of *m, *g**, etc.**

is controlled by the Fermi-liquid parameters F_0^a and F_1^s:

$$g^* = \frac{g_b}{1+F_0^a}$$

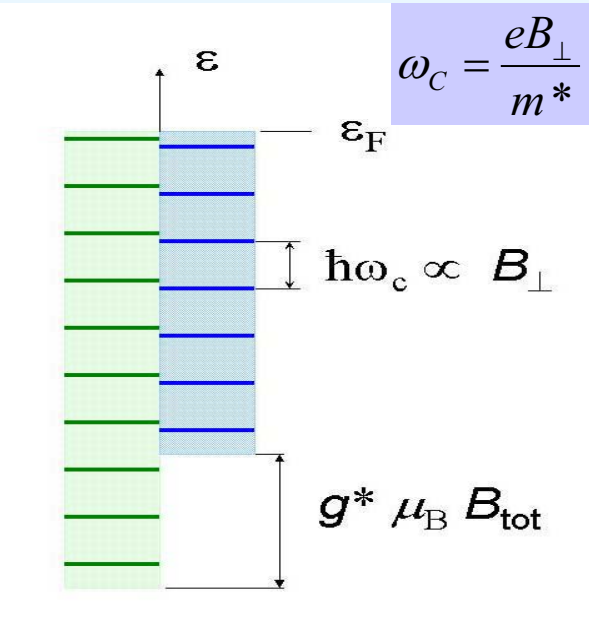
the g-factor

$$m^* = m_b(1+F_1^s/2)$$

the mass

$$\chi^* = g^* m^* \frac{g_b \mu_B^2}{\pi \hbar^2}$$

the spin susceptibility



The experimental tool:

<u>Shubnikov - de Haas effect</u>

The interaction parameters

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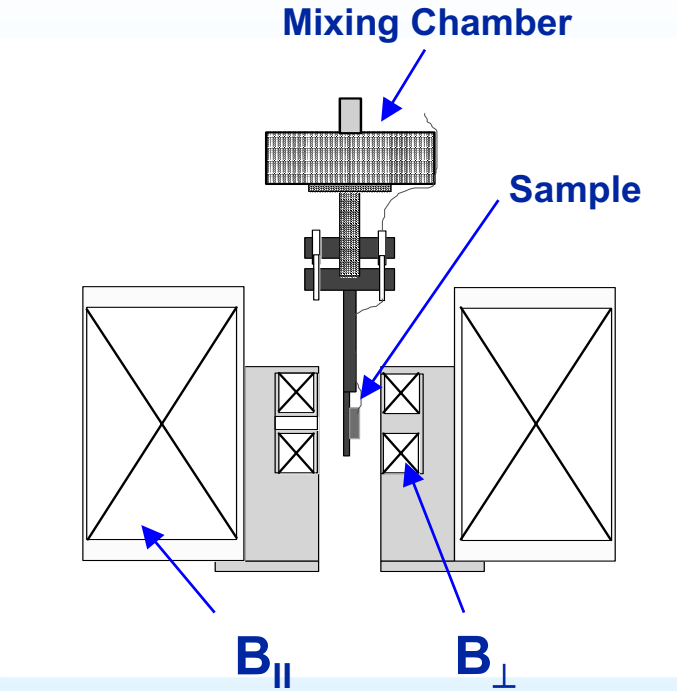
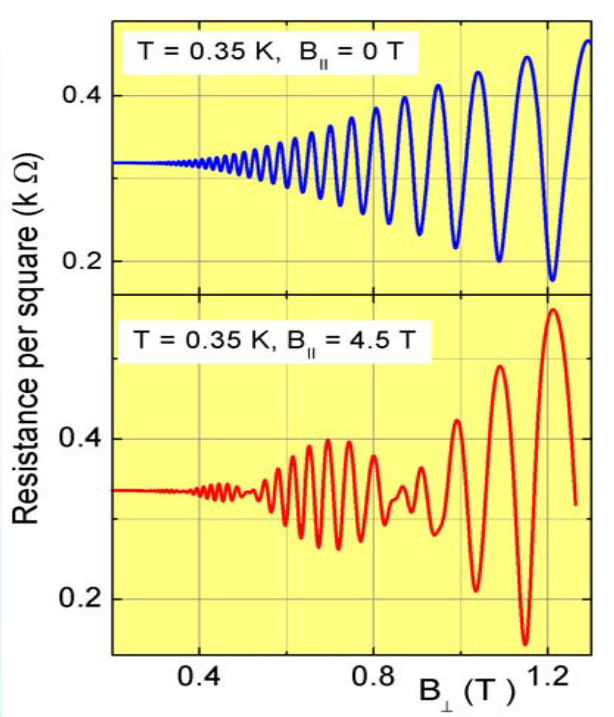


Gershenson *et al., Physica* E **12**, 585 (2002)



Pudalov *et al., PRL* **88**, 196404 ('02)

χ^* - from the position of nodes
m* **- from the *T*-dependence of SdH amplitude**
g* **- from the ratio χ^* / *m*****

$$P \equiv \frac{n_\uparrow - n_\downarrow}{n} = \frac{\chi^* B_{tot}}{g_b \mu_B n} = g^* m^* \frac{eB_{tot}}{n\hbar}$$

Crossed magnetic fields – the tool for probing orbital and spin properties of 2D electrons

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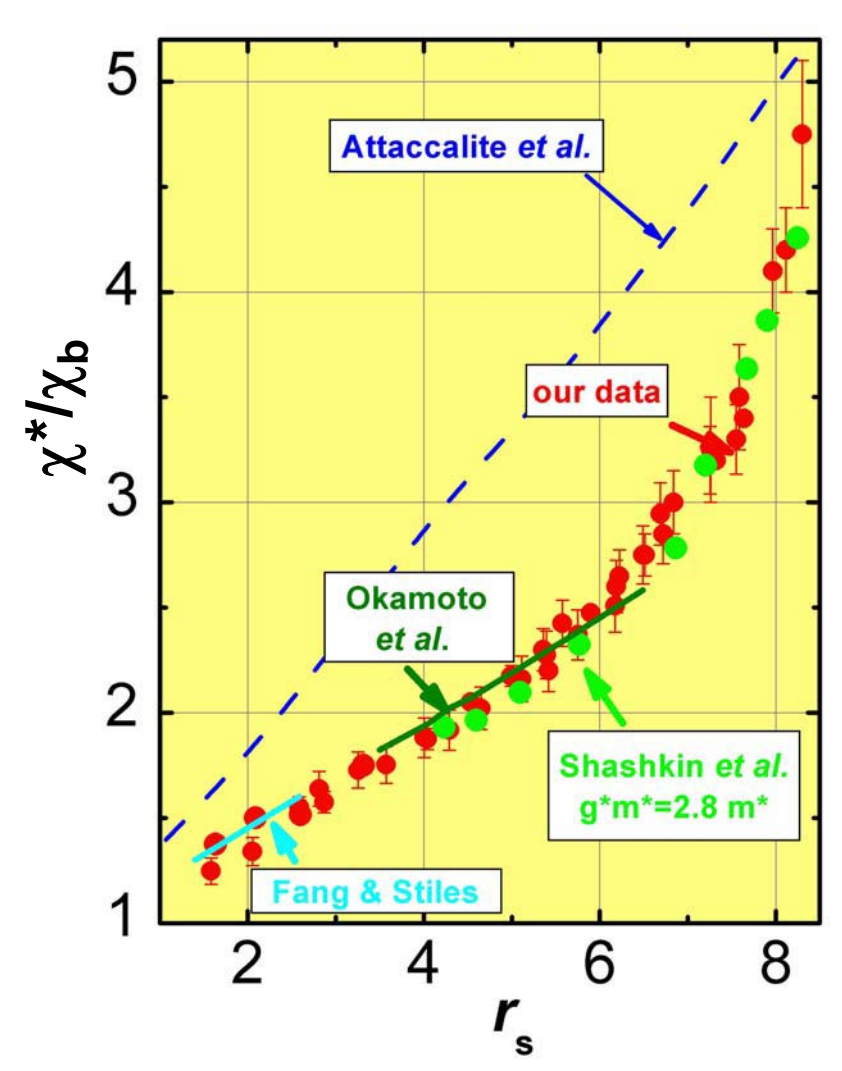
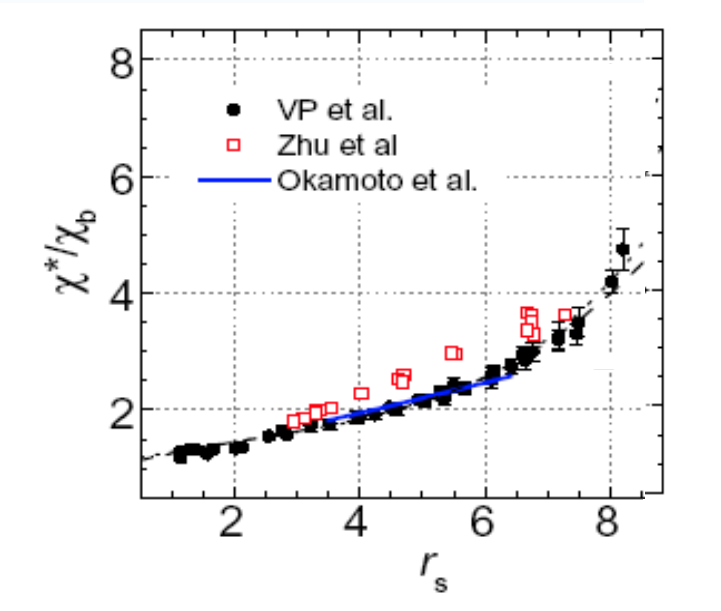


Pudalov *et al., PRL* **88**, 196404 ('02)



n-type GaAs/AlGaAs

Zhu *et al., PRL* **90**, 056805 ('03)

Renormalization of χ^*

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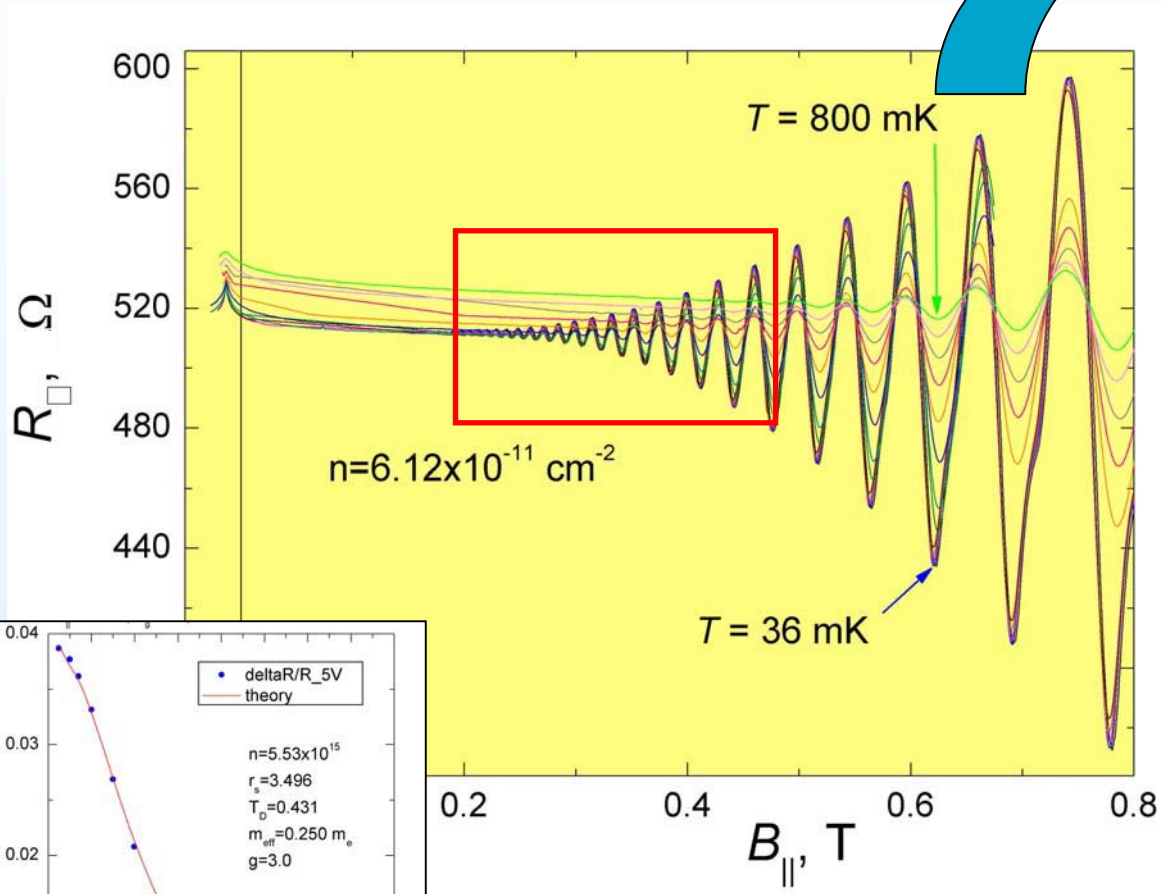
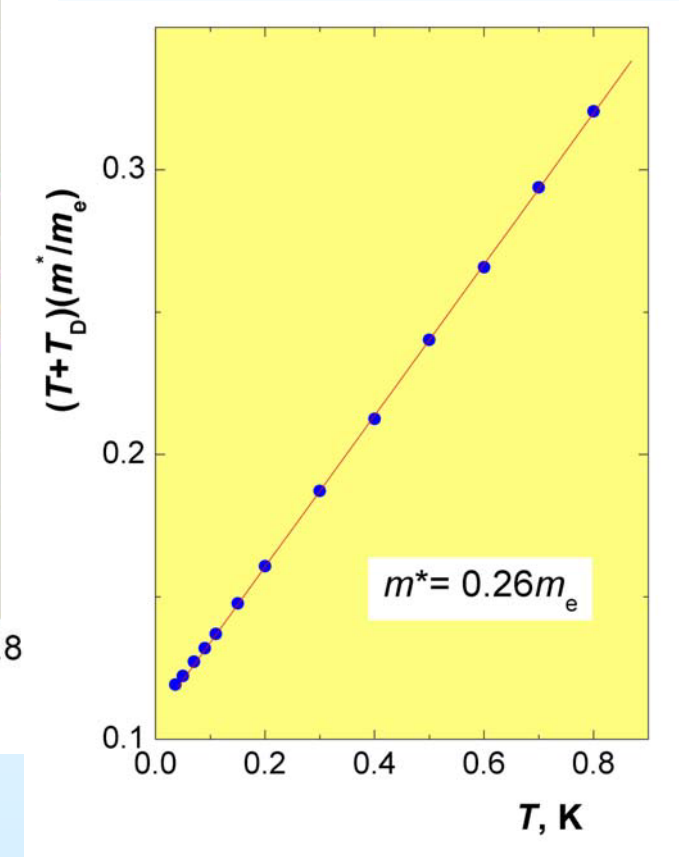
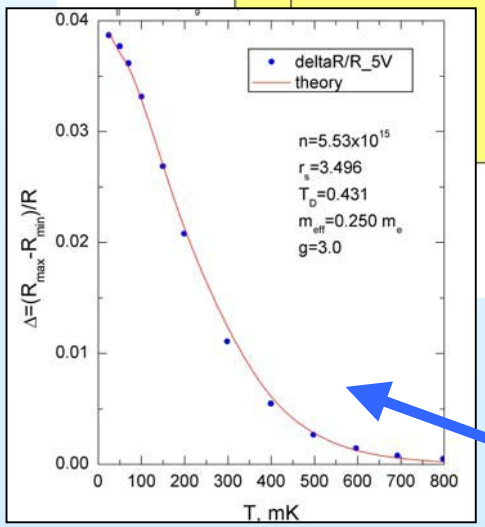
Amplitude

$\propto (T + T_D)m^*$,

$T_D = \hbar/k_B\tau$







Klimov *et al.*, unpublished

Electron temperature – down to 30 mK.

*m** from SdH amplitude

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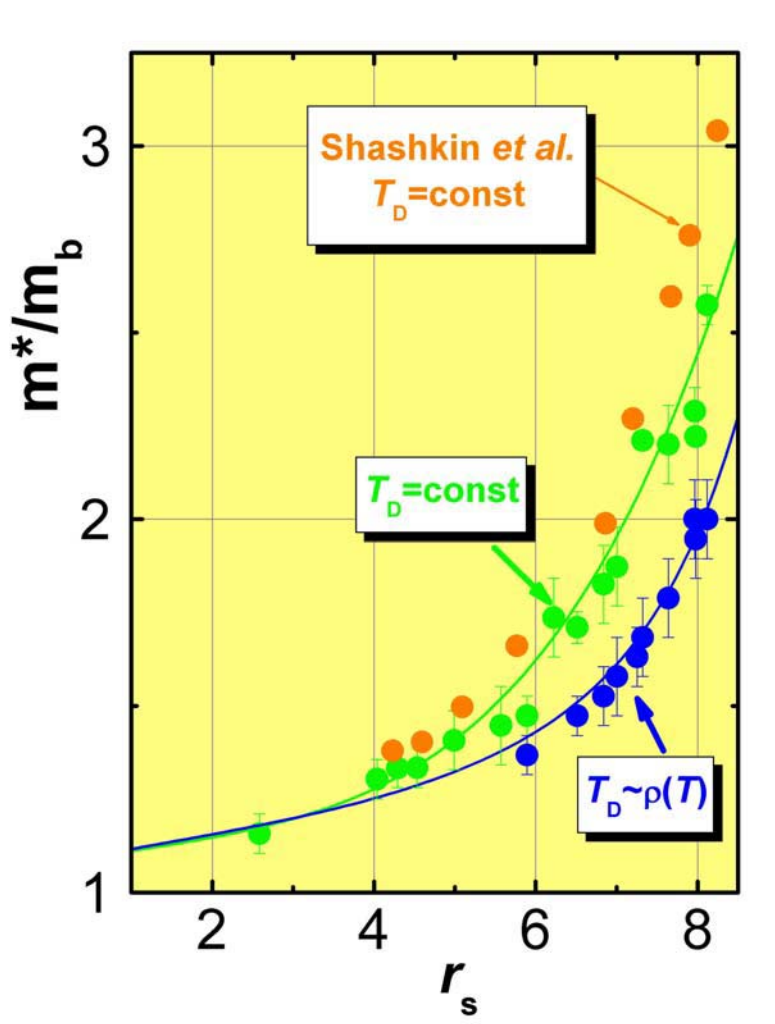

At low *n*, where $\Delta\rho(T)/\rho$ is not small, two approaches:

- $T_D \neq f(T)$
- $T_D \propto \rho(T)$

The difference between them becomes significant at $r_s > 6$.

The second approach seems to make more sense:

One of the empirical procedures used in Ref. 6 to account for the observed deviations from the LK formula was to assume that the effective Dingle temperature has the same T dependence as the zero-field resistivity. Comparing Eq. (10) with the result for the interference correction to the resistivity,[15] we see that, although this recipe is not precise, it has some theoretical justification: the general structure of the $T_D(T)$ and $\rho(T)$ depedencies is similar in the ballistic regime, except for a factor of $\ln(\varepsilon_F/T)$ present in $T_D(T)$ but not in $\rho(T)$.

Martin *et al.*, *PRB* **68**, 241309 ('03)

Also, the "$T_D \propto R(T)$" data, being used as the input parameters for the interaction theory, describe $\sigma(T)$ much better.

Renormalization of *m** and *g**

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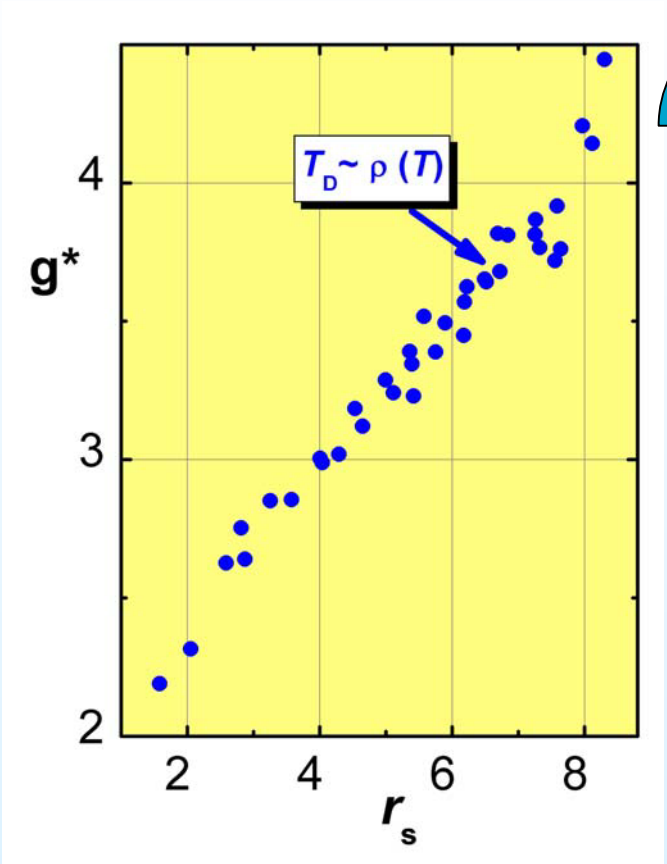
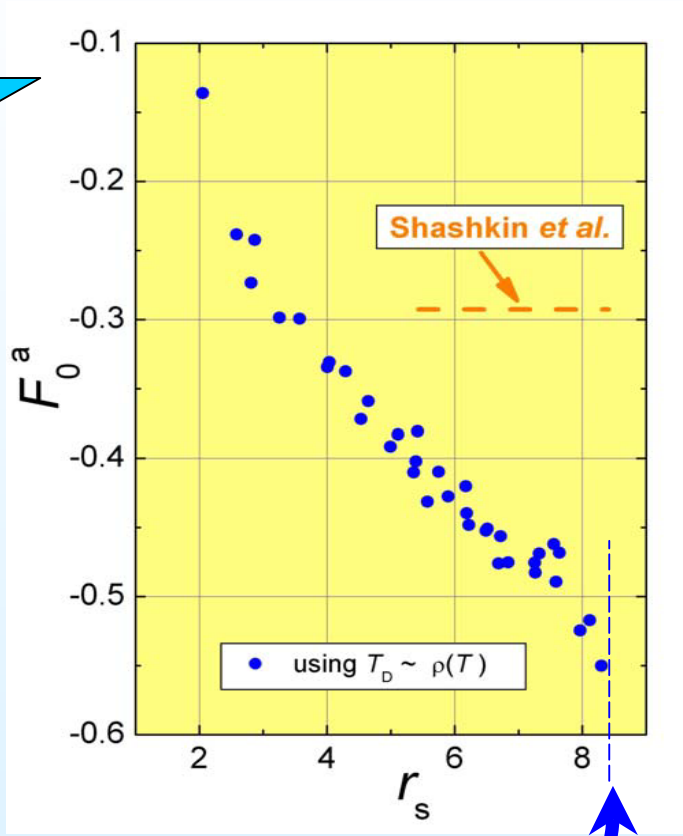


$$g^* = \frac{g_b}{1 + F_0^a}$$



"2D MIT"

(Stoner instability $\Rightarrow F_0^a = -1$)

The g^* renormalization is the link to the $\sigma(T)$ analysis – the interaction corrections are expressed in terms of the Fermi-liquid constant F_o^a.

g^* and F_o^a

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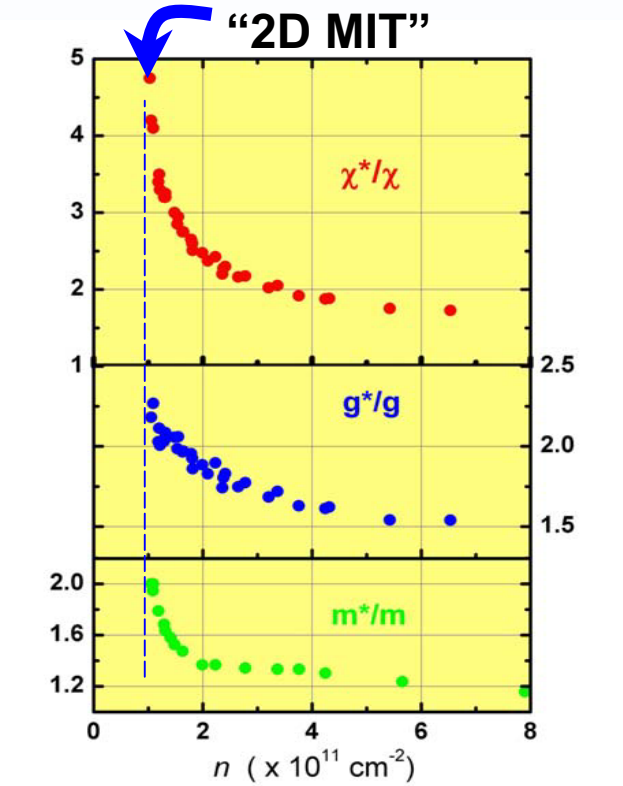
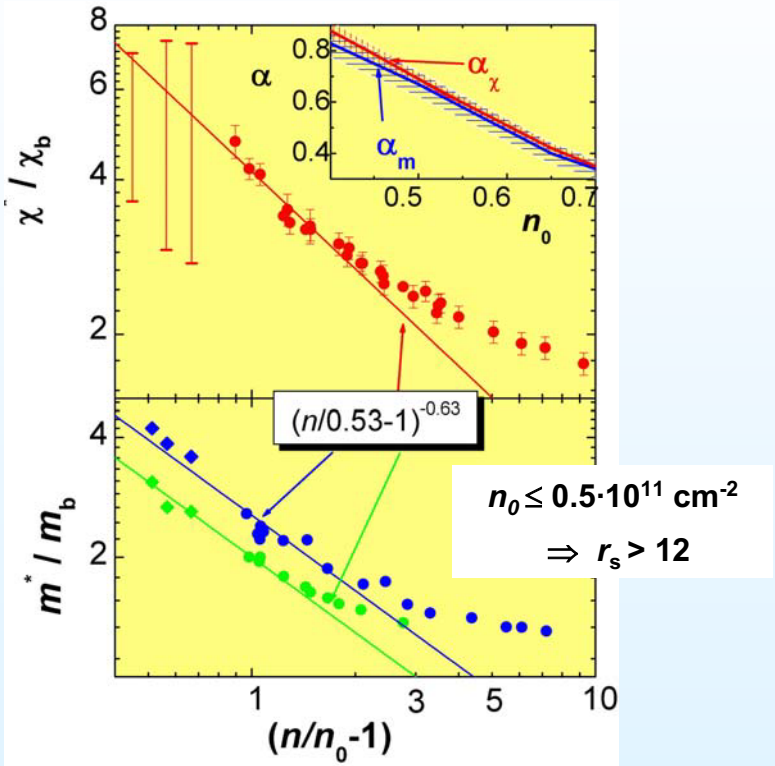


Pudalov et al., PRL **88**, 196404 ('02)



Pudalov et al., cond-mat/0110160

- **The electron parameters χ^*, m^*, and g^* are strongly renormalized at low n; this renormalization does not depend on the proximity to the apparent 2D MIT.**
- **The critical behavior of χ^*, if exists, is buried deeply in the "insulating" regime.**

Enhancement, not divergency

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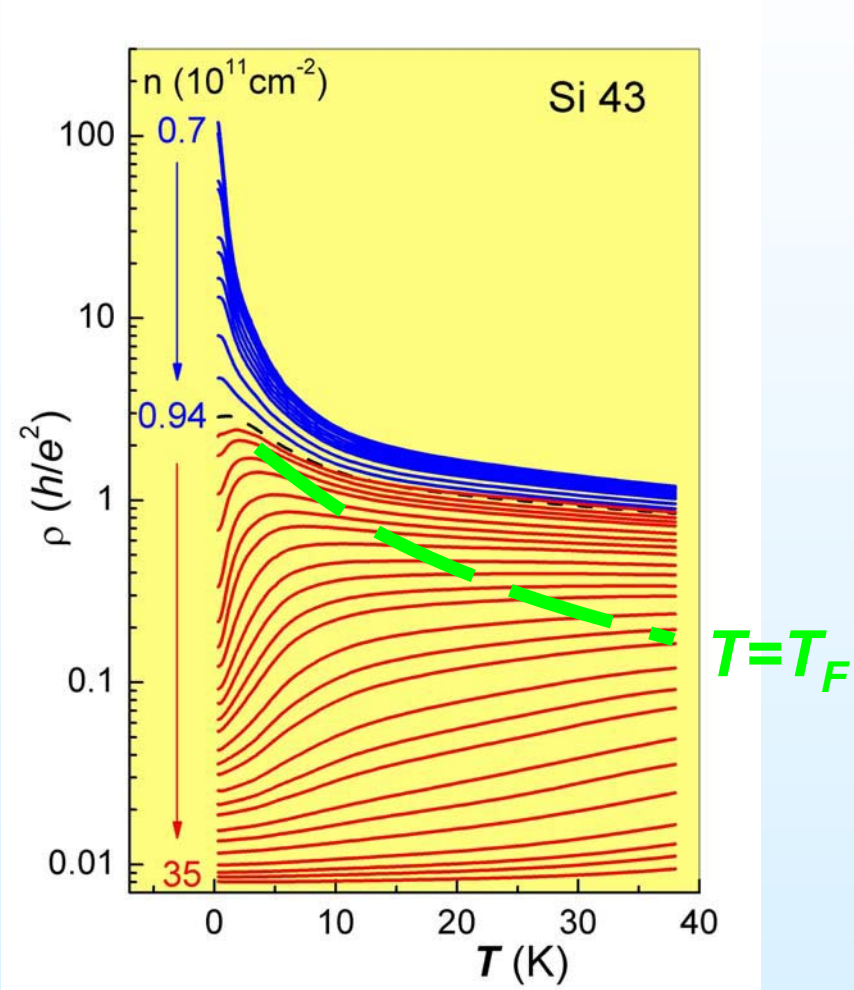
For 2D electron system, strong renormalization of χ^*, m^*, and g^* is observed in the regime of low n.

The experimental methods are inadequate to address the issue of a possible critical behavior/new ground states, which are "buried" deeply in the insulating regime.

An open issue: to understand better how the system specifics (valley degeneracy, different type of carriers, etc,) affect the renormalization of these quantities, and to describe quantitatively the renormalization (it is still a challenge even at not-too-large r_s).

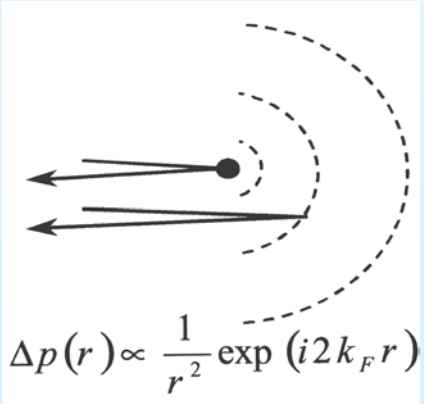
Part I: Conclusion



Stern ('80), Gold & Dolgopolov ('86), and Das Sarma & Hwang ('99):

$\Delta\rho(T) \propto T$ is due to the T-dependent screening of disorder.

Zala, Narozhny, & Aleiner (2001): interaction corrections to the conductivity - the model of interference between the waves scattered from the short-range potential and Friedel oscillations.



Part II: Anomalous "metallic" behavior of $R(T)$

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Ballistic regime:

***T*τ >> 1, the scattering acts can be treated independently**

Valley degeneracy
[Punnoose and Finkelstein, 2002]

$$\partial\sigma_{\text{int}} = -\frac{e^2}{\pi\hbar}\left\{1 + \mathbf{15}\,\frac{F_0^{\,a}}{1+F_0^{\,a}}\right\}\frac{T\tau}{\hbar}$$

Diffusive regime: Altshuler, Aronov, & Lee ('80); Finkelstein ('83)

***T*τ << 1, the scattering acts interfere with each other**

$$\partial\sigma_{\text{int}} = -\frac{e^2}{2\pi^2\hbar}\left\{1 + \mathbf{15}\left[1 - \frac{\ln\left(1+F_0^{\,a}\right)}{F_0^{\,a}}\right]\right\}\ln\left(\frac{\hbar}{k_B T\tau}\right)$$

2 spins ⊗ 2 valleys ⇒ **16 (1 singlet +15 triplets)**

Advantages of the ZNA theory:

- it provides unified description of both **ballistic** and **diffusive** regimes
- it allows expressing Δ ρ (*T*) in terms of F_0^a that can be found independently

Theoretical ingredients

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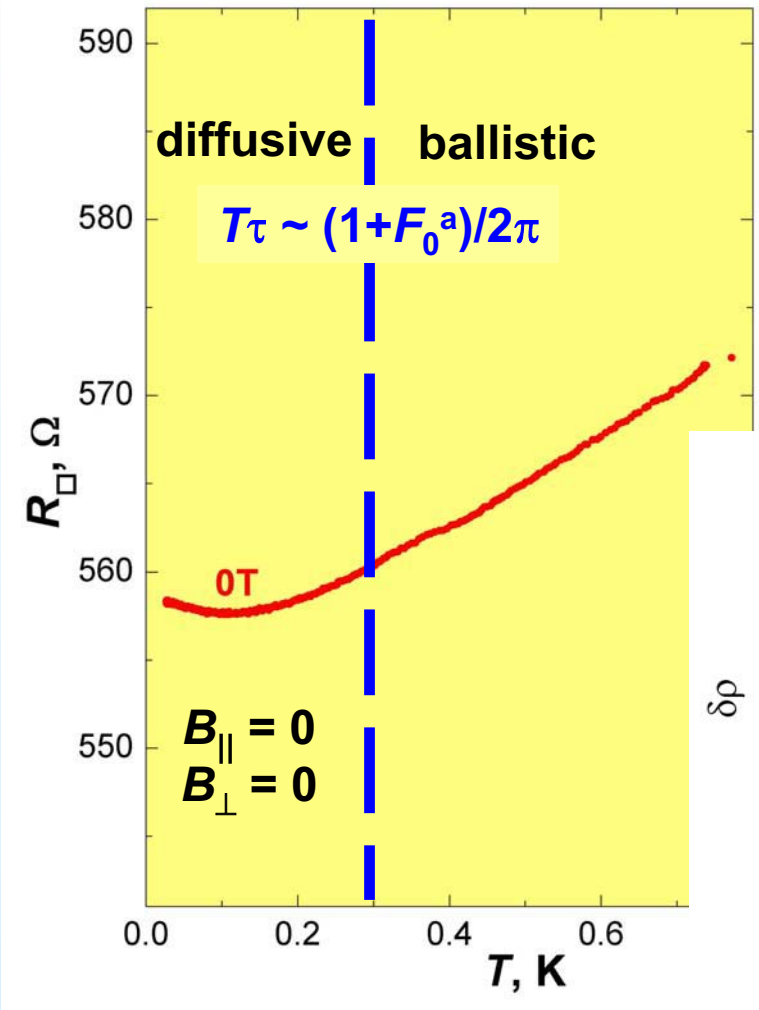
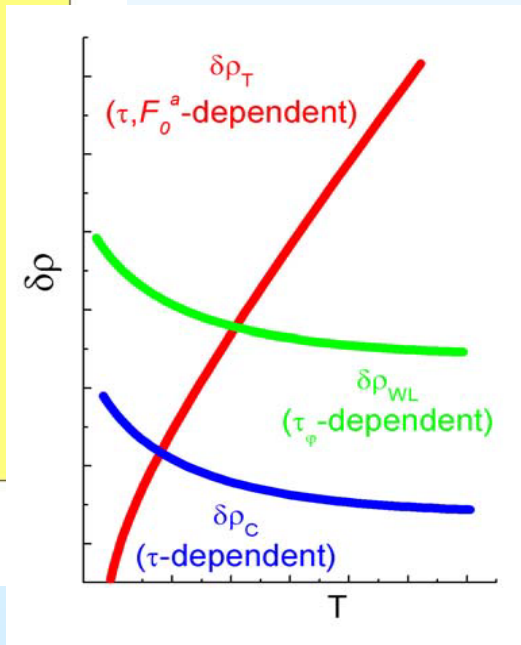
Klimov *et al.*, unpublished





$n = 6\cdot10^{11}\,\mathrm{cm}^{-2}$ (far from the "2D MIT")

$\Rightarrow$ the quantum corrections are small, though the "metallicity" is clearly seen.

- $B_\perp$ separates the WL and interaction corrections
- $B_\parallel$ elucidates the structure of interaction corrections

Evolution of $R(T)$ with B

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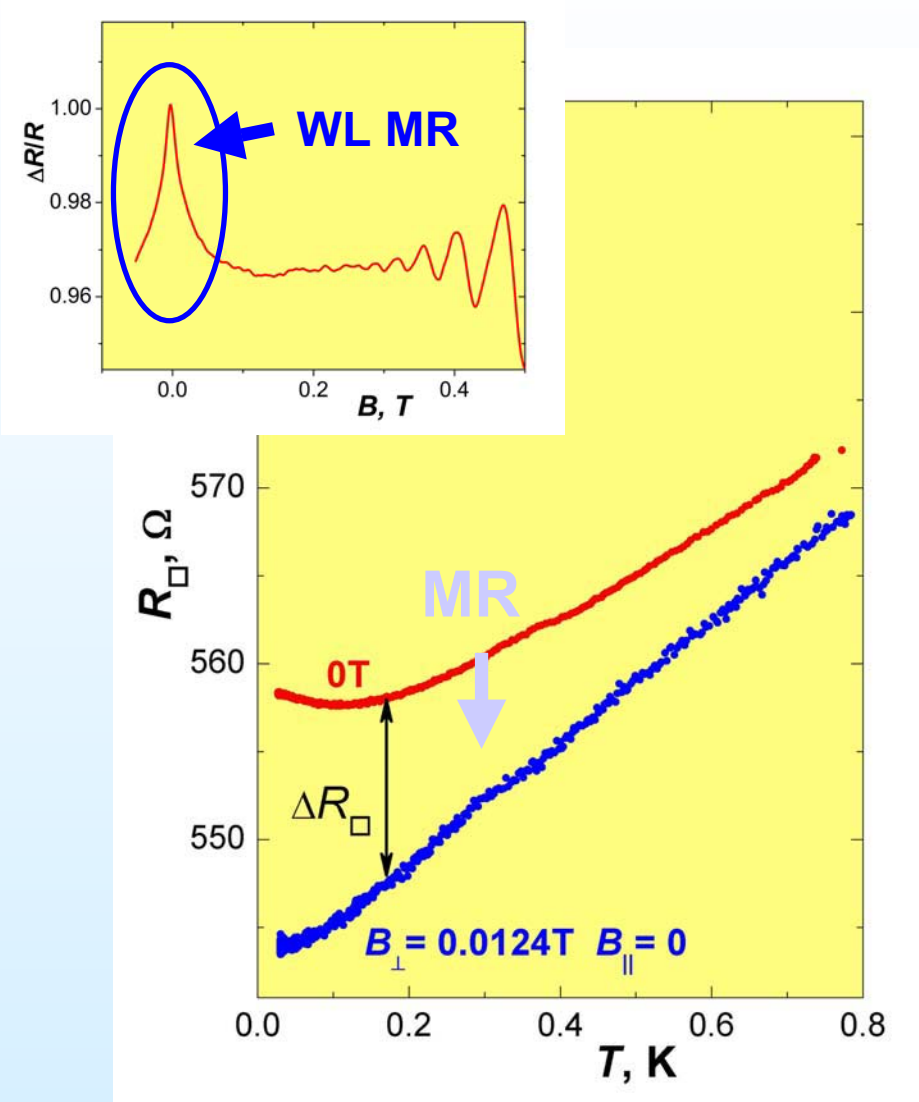

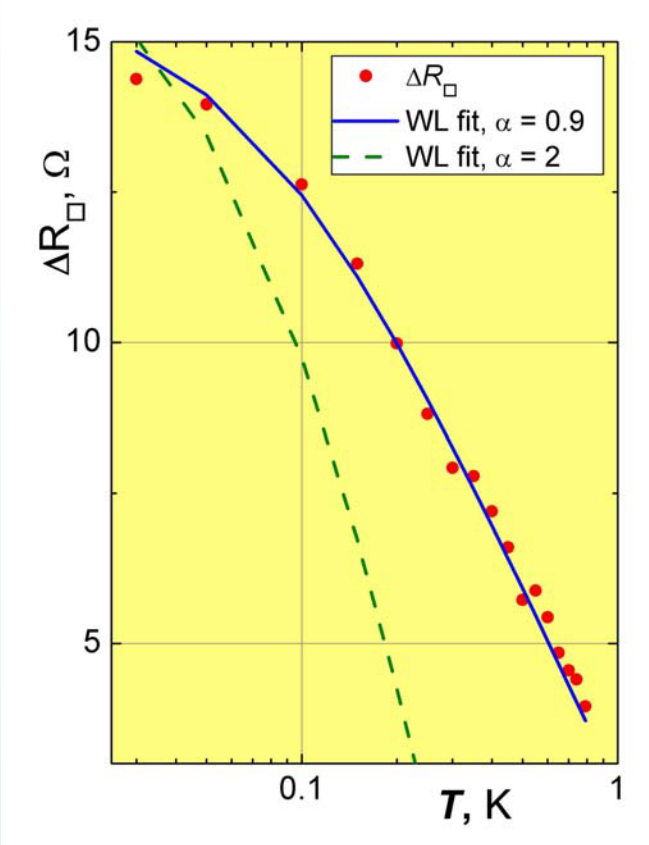

$$\Delta\sigma_{WL} = \alpha \frac{e^2}{2\pi^2\hbar} \ln\left[\tau_\varphi(T)\right]$$

Suppression of the WL correction by $B_\perp$

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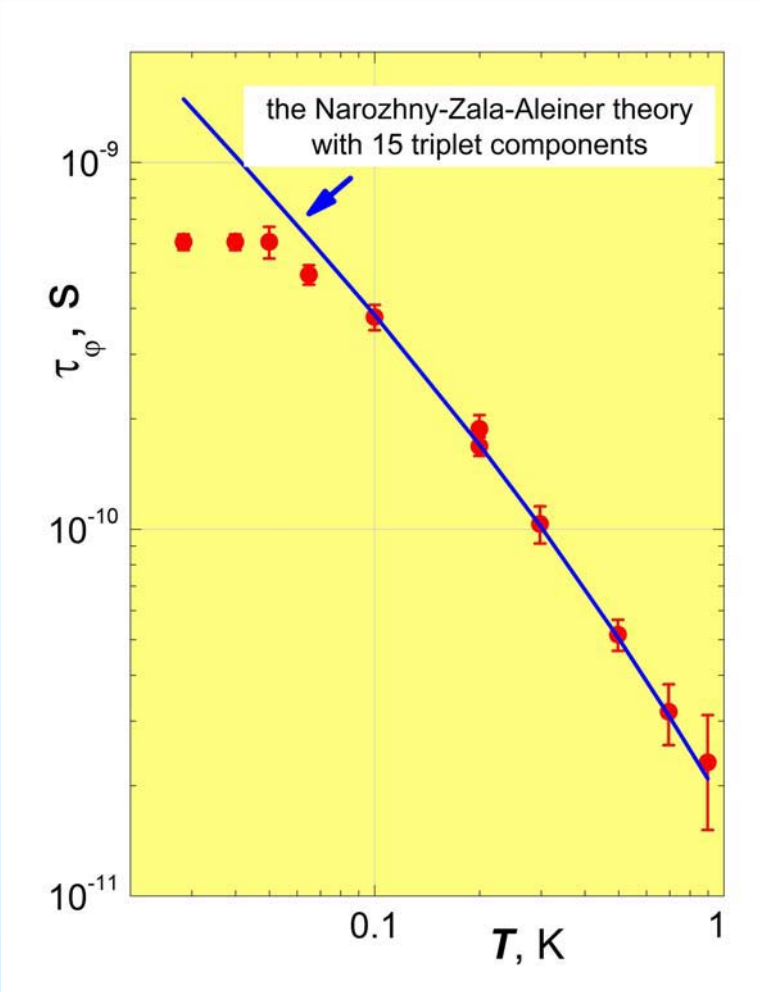
The dephasing rate due to both singlet and triplet channels (Narozhny, Zala, Aleiner, '02)

$$\frac{1}{\tau_\varphi} = (\ldots)\frac{T}{g}\ln g + (\ldots)\frac{\pi}{4}\frac{T^2}{E_F}\ln(E_F\tau)$$

$$g = \frac{2\pi\hbar\sigma}{e^2}$$

a strongly interacting system – is still a Fermi-liquid system



The dephasing time in the dilute regime

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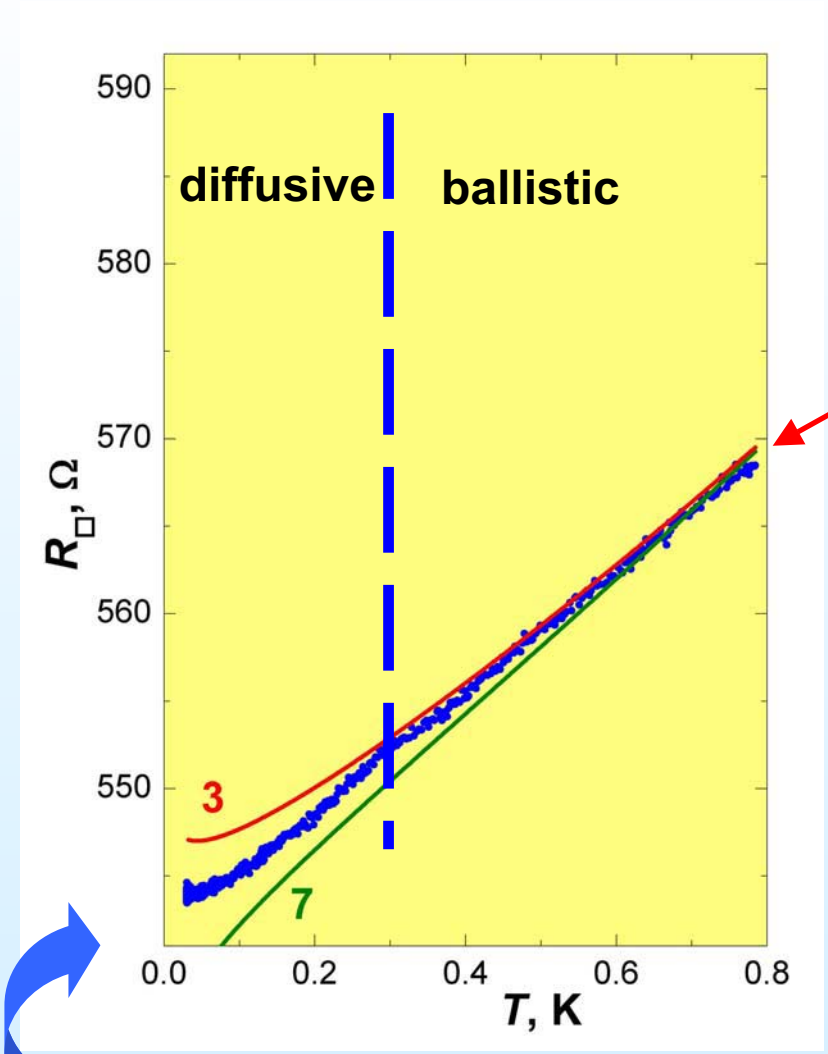

# of triplet components in the diff. term

$$\Delta\sigma_{\text{int}}^{ZNA} = f(F_0^{\,a})\frac{T\tau}{\hbar} + g(F_0^{\,a})\ln\left(\frac{\hbar}{T\tau}\right)$$

ZNA '01
no fitting parameters in the ballistic regime!

Good agreement in the ballistic regime (higher T).

More complicated situation in the diffusive regime (lower T).

Comparison with the ZNA and DSH theories

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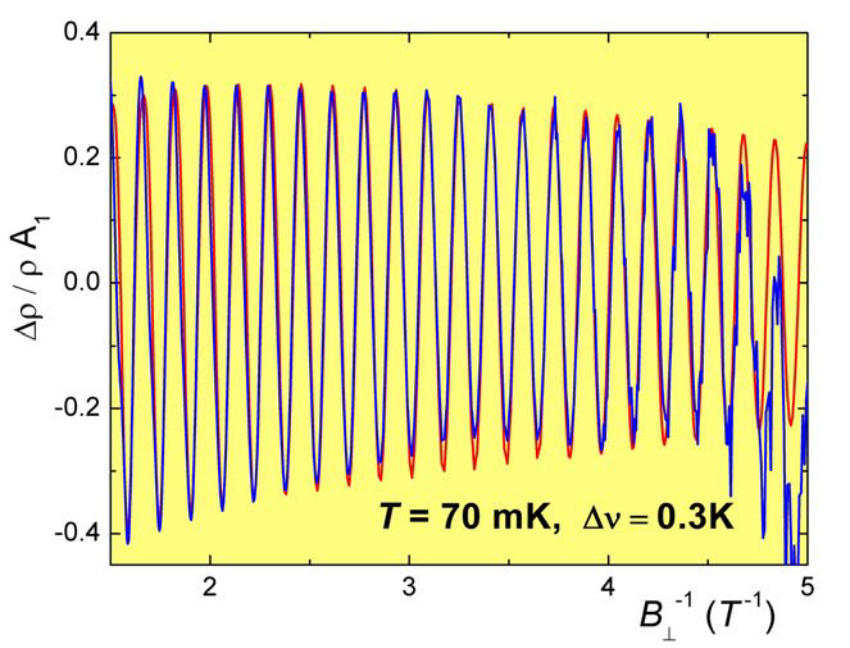
Mechanisms of the "prefactor 15" reduction:

- **valley splitting** $\Delta_V > T$ **(15 ⇒ 7)**
- **inter-valley scattering** $\hbar/\tau_V > T$ **(15 ⇒ 3)**



$\Delta_V \approx 0.3$K fits the data

better than $\Delta_V = 0$

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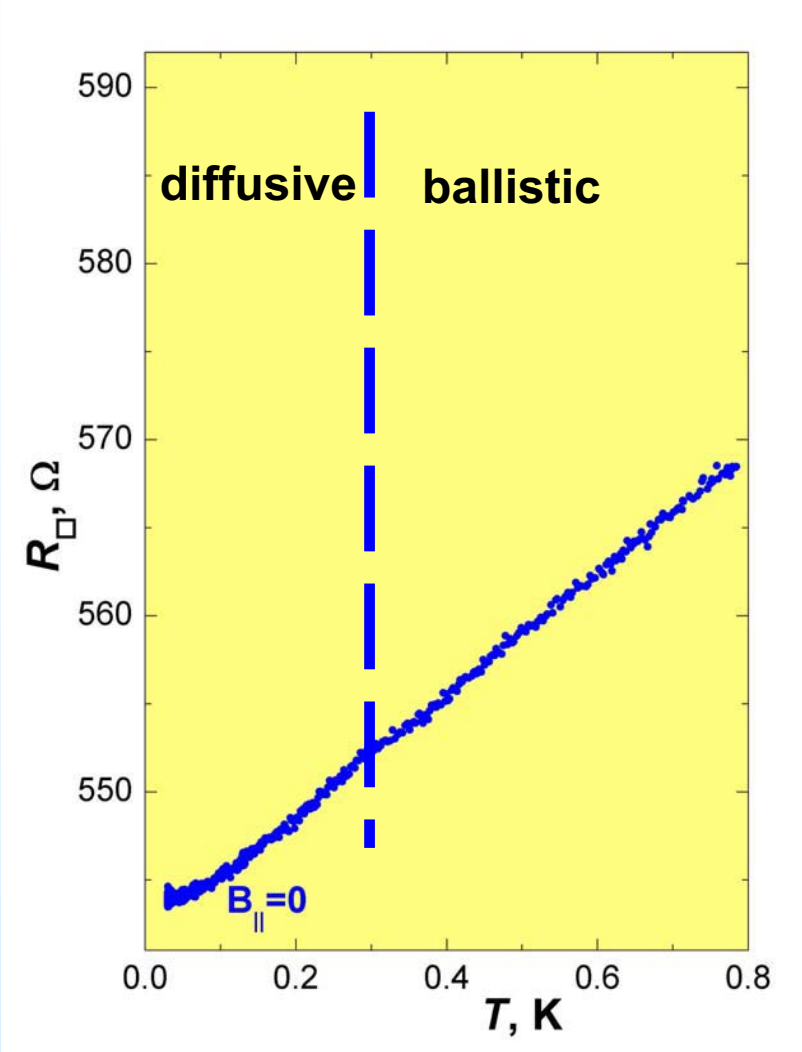


- If the Zeeman energy

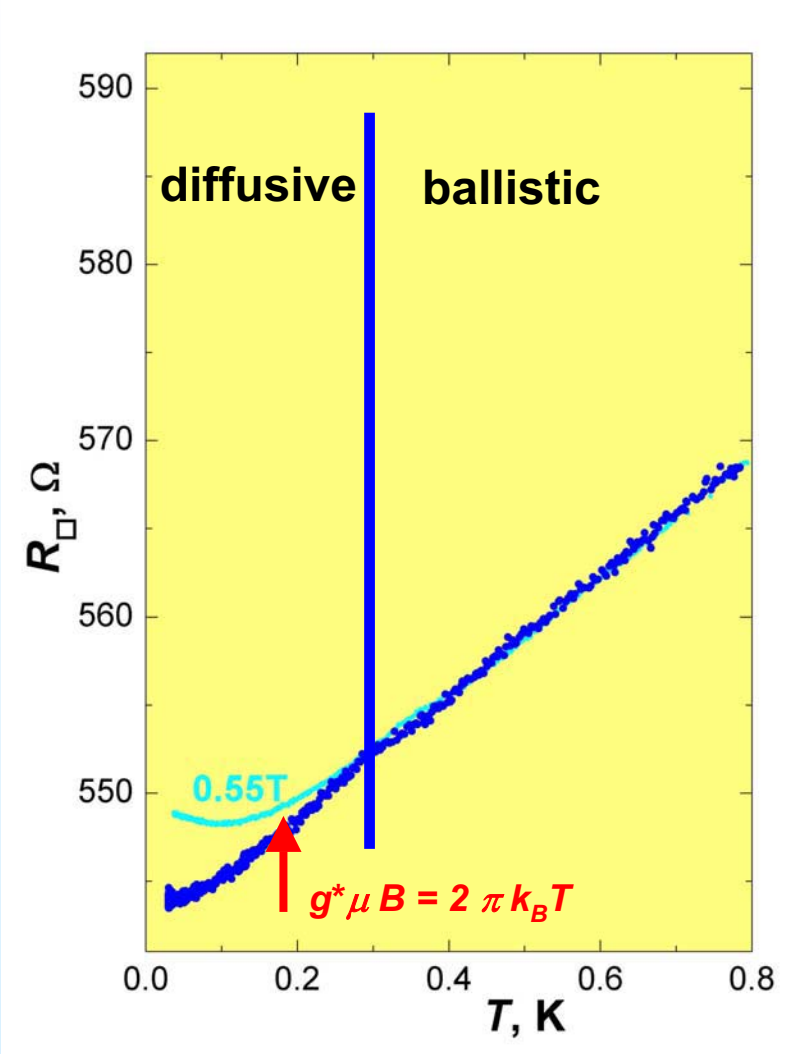
$$g^*\mu B_{||} > 2\pi k_B T \quad - \quad \mathbf{15 \Rightarrow 7}$$

(the spin polarization is small in a weak $B_{||}$, E_F = 45K)

(a) Δ_V, $\hbar/\tau_V < k_B T \Rightarrow$ the # of triplet components is expected to be reduced from 15 to 7 (this case corresponds to the ballistic regime in our samples)

(b) Δ_V and $\hbar/\tau_V \sim k_B T \Rightarrow$ the situation is more complicated (this case corresponds to the diffusive regime in our samples)

Evolution of $R(T)$ with $B_{||}$



Evolution of $R(T)$ with $B_{||}$

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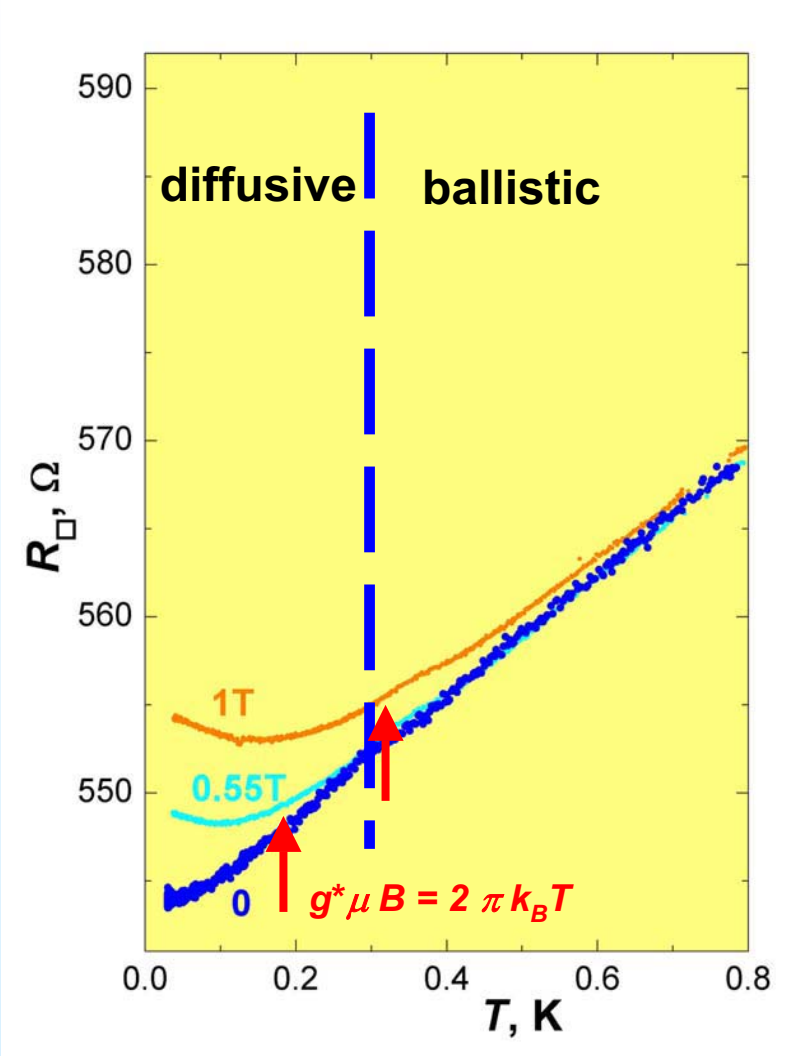

Evolution of $R(T)$ with $B_{||}$

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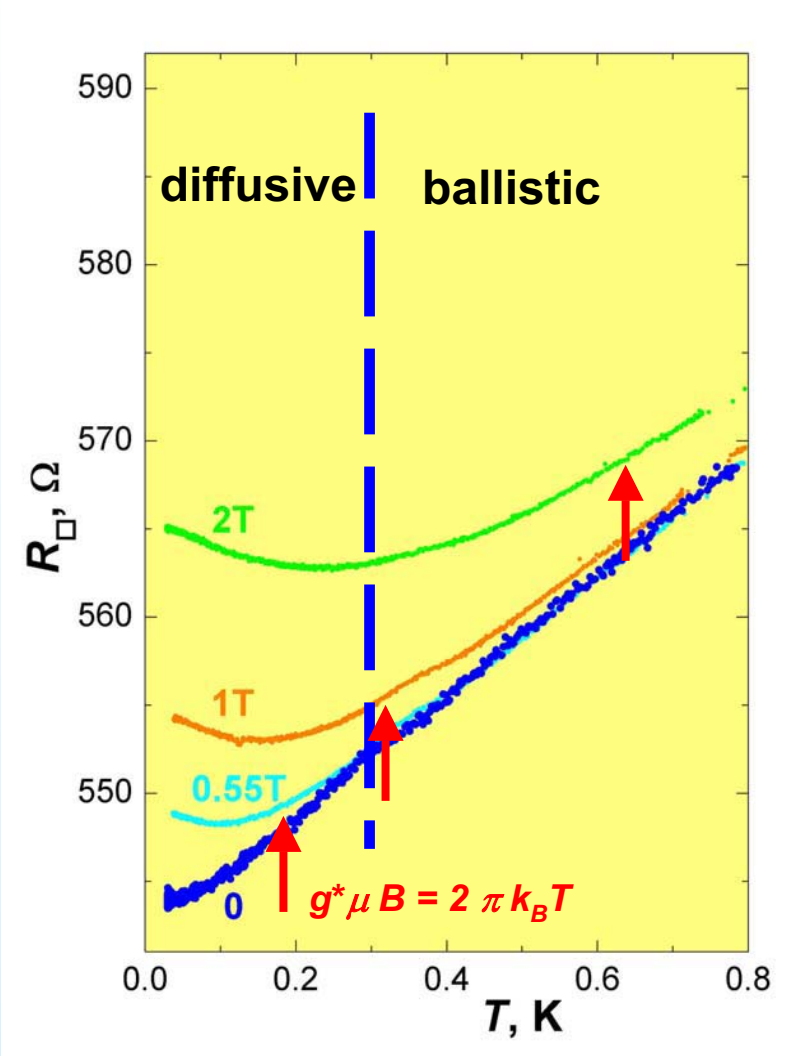

Evolution of $R(T)$ with $B_{||}$

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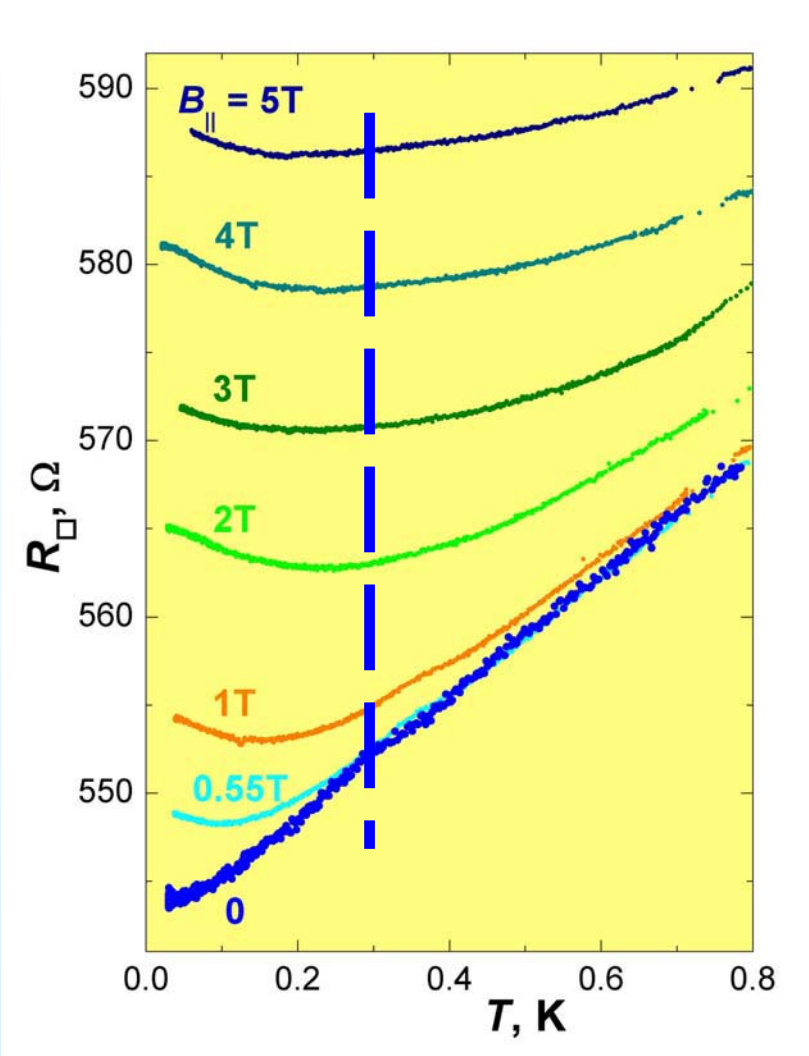

Evolution of $R(T)$ with $B_{||}$

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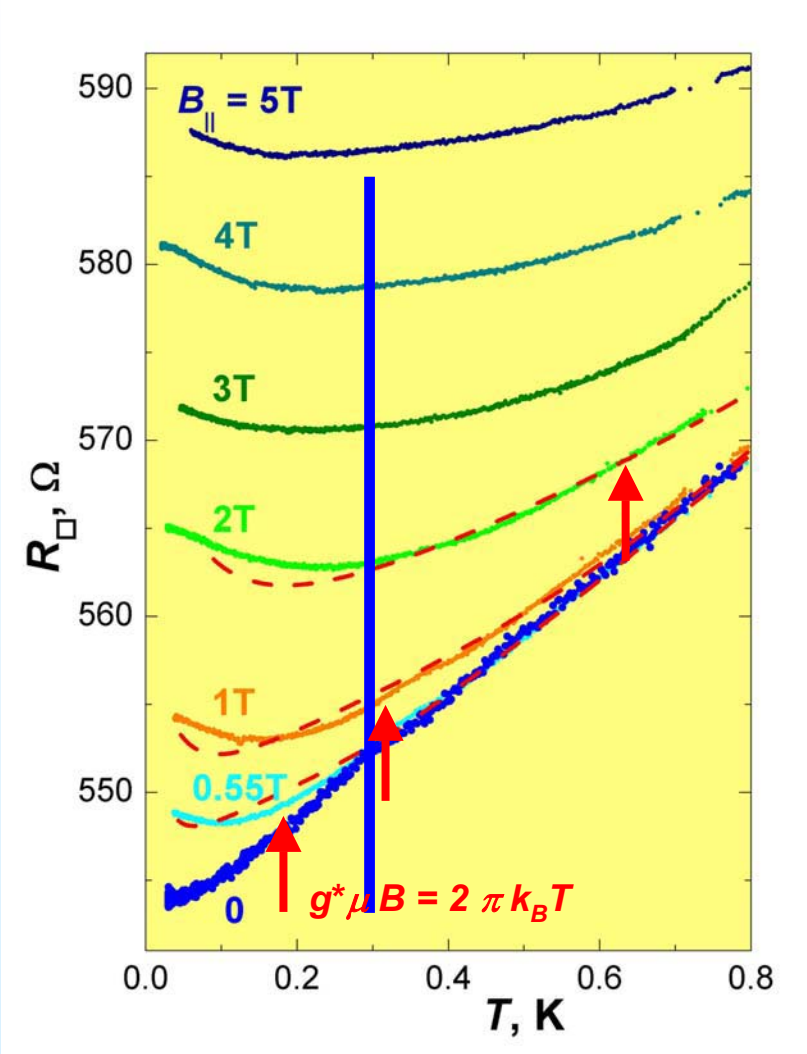

Dashed lines – the ZNA theory.

The valley degeneracy is preserved at high T (ballistic regime) - the slope in the ballistic regime at $g^*\mu B_{||} > 2\pi k_B T$ is consistent with 7 triplet components.

At lower T, the valley degeneracy is removed (diffusive regime) – 3-5 triplet components in the diffusive term.

Evolution of $R(T)$ with $B_{||}$

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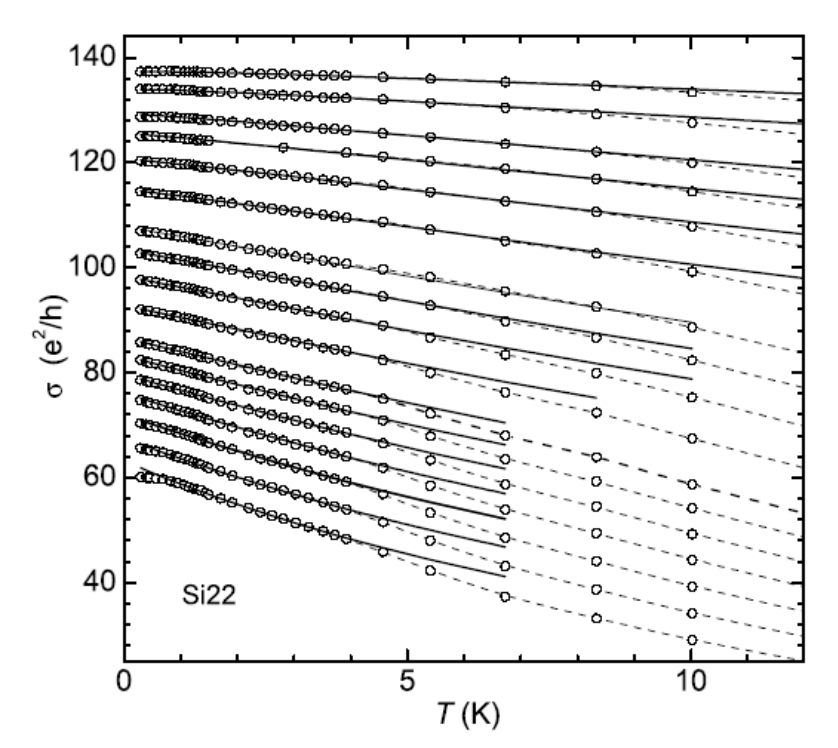

Kojima *et al.*, *J. Phys. Soc. Jpn.* **72**, 57 (2003)

Solid lines:

the ZNA theory with Δ_V=0 and F_0^a from SdH measurements

Si 22 (μ=3.3 m^2/Vs)

The electron densities:

from 35.7x10^{11} cm^{-2} (top)
to 5.7x 10^{11} cm^{-2} (bottom)

With decreasing *n*, the *T* interval of a good agreement shrinks – the ZNA theory is applicable at $T \ll (1+F_0^a)E_F$

Comparison with the ZNA theory

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The "metallic" behavior of many 2d systems in the dilute regime is consistent with predictions of the ZNA theory:

***n*-type Si:** Shashkin etal., *PRB* **66**, 73303 ('02); Kvon *et al*., *PRB* **65**, 161304 ('02); Vitkalov *et al*., *PRB* **67**, 113310 ('03)

***p*-type GaAs:** Proskuryakov *et al*., *PRL* **89**, 076406 ('02); Noh *et al*., *PRB* **68**, 165308 ('03); Yasin *et al*, cond-mat/0403411

***n*-type GaAs**: Li *et al*., *PRL* **90**, 076802 ('03); Yasin *et al*, cond-mat/0403411

***p*-type SiGe:** Coleridge *et al*., *PRB* **65**, 125328 ('02)

Other 2D dilute systems...

At not-too-high resistances ($\rho < 0.1\ h/e^2$), the anomalous "metallic" conductivity in Si MOSFETs (as well as in the other high-mobility and low-density systems) is consistent with the theory of interaction corrections. The interaction corrections to the conductivity of Si MOSFETs are enhanced by the interaction-driven renormalization of Fermi-liquid parameters and by the valley degeneracy.

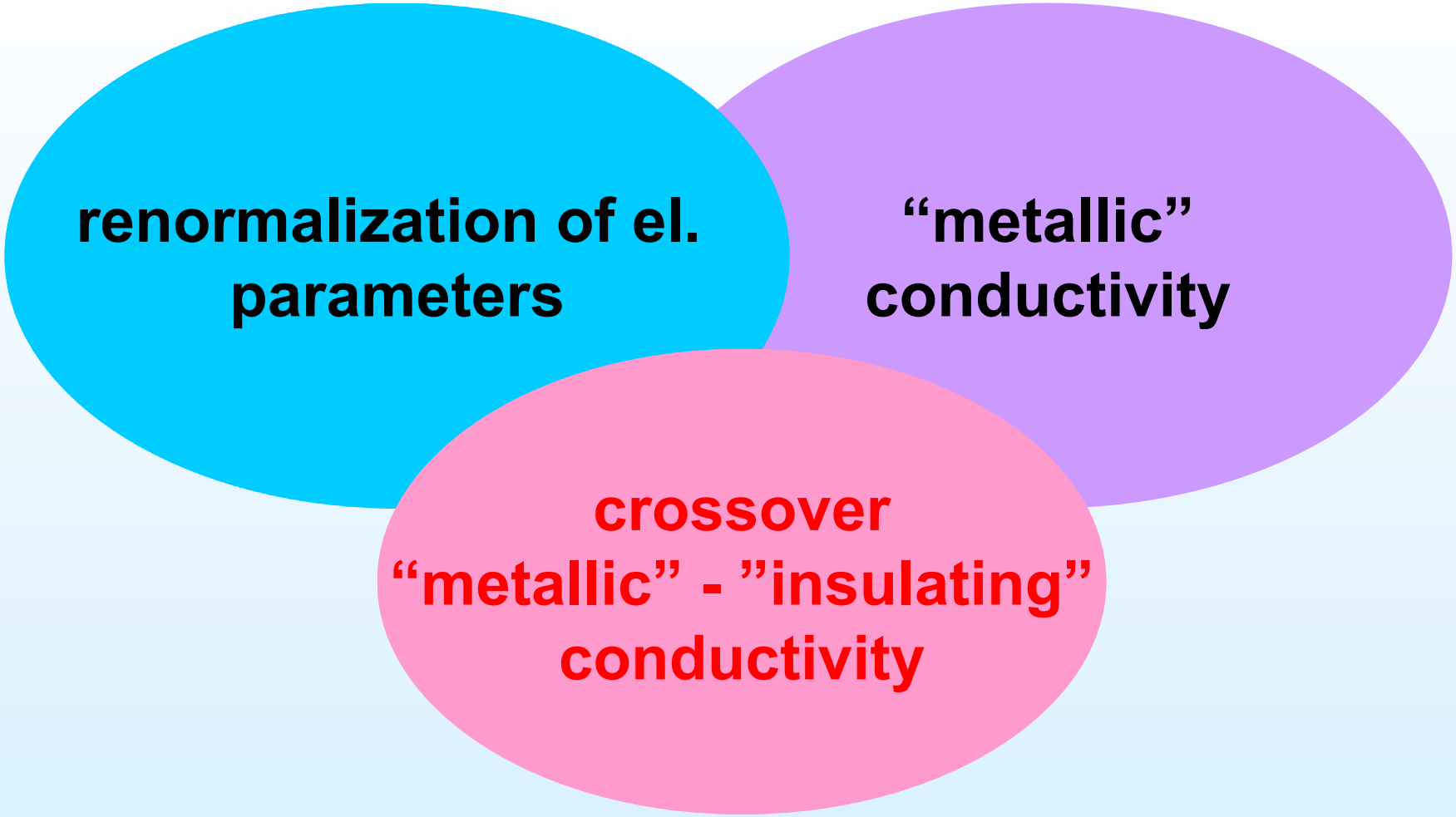
$\Downarrow$

The "metallicity" of the high-mobility and low-density 2D systems is not a signature of a novel ground state – rather, it is a finite-temperature effect, which can be understood on the basis of the Fermi-liquid theory.

Part II: Conclusion

"Entities are not to be multiplied beyond necessity"

William of Ockham (1285-1349)



<u>Part III</u>: The crossover from “metallic” to “insulating” conductivity

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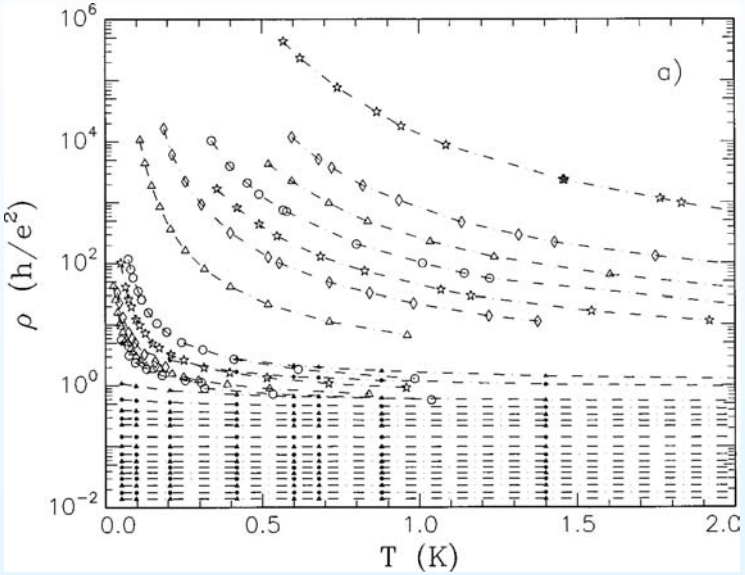

The WL-SL crossover has been observed in many quasi-1D and 2D systems (this does not mean, however, that we know how to describe $R(T)$ at the crossover quantitatively…).

Gated GaAs/AlGaAs structures,
$n = (0.65\text{-}6)\times10^{15}$ m^{-2}
Van Keuls *et al.*, *Phys. Rev.*B **56**, 13263 (1997)

Unusual features of the crossover in high-mobility Si MOSFETs:

⇒ the ρ (T) dependence changes "sign" across the "transition"

⇒ the WL and Int. corrections seem to be suppressed near the crossover, at large $\boldsymbol{R}_{\square}$, rather than enhanced...

What is so special about this crossover ?

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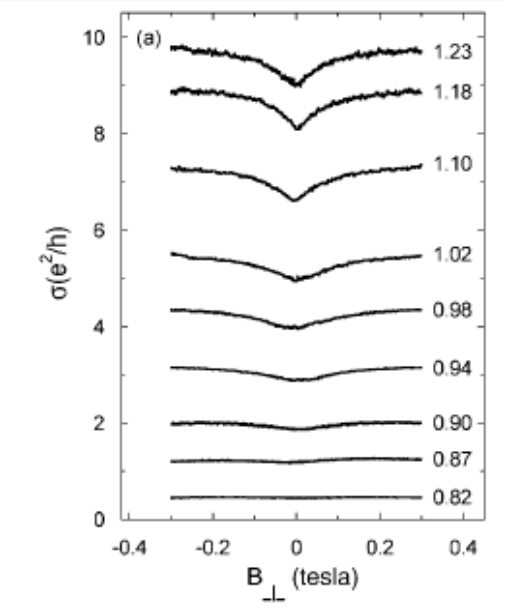
Up to ~ 5 kΩ/□, the WL corrections can be well described by the conventional theory. However, at larger $R_{\square}$, the magnitude of the WL corrections is *smaller* than the predictions for a *spatially uniform* system.



Rahimi *et al.* ('03)

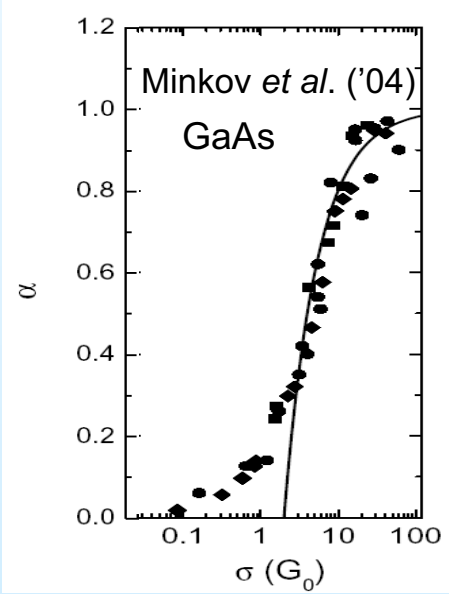
"Vanishing" WL MR at σ ⇒ e²/h is expected for both homogeneous and macroscopically inhomogeneous systems:

- **homogeneous – owing to the second-loop corrections**

$$\alpha = 1 - \frac{2G_0}{\sigma}$$
$$G_0 = e^2/(2\pi^2\hbar)$$

Gornyi ('04)

- **macro inhomogeneous – owing to the percolation effects** Aronov *et al.* ('86)



The WL corrections near the apparent 2D MIT

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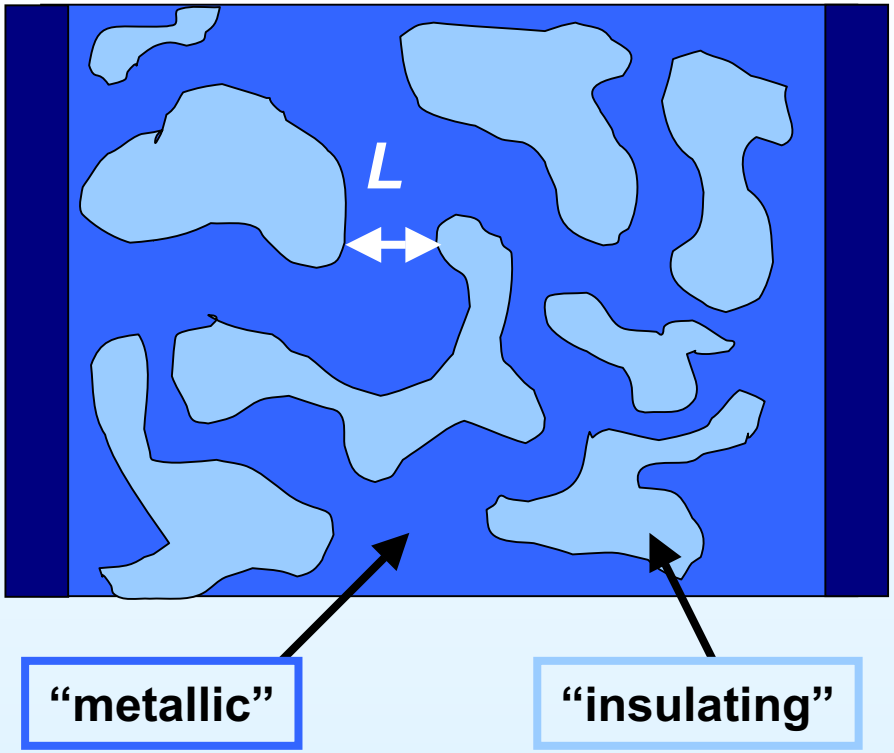

$L \gg L_\varphi$

A possible reason: a system becomes ***inhomogeneous*** at a macroscopic scale $L > L_\varphi$

Entertaining the non-homogeneous scenario…

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- The amplitude of WL and INT corrections is suppressed: $\Delta\rho_{WL,INT} / \rho \propto R_{\square\ metallic}$, but $R_{\square\ metallic} << R_{\square\ total}$ ✓

- in the conductors with percolation, the quantum corrections are smaller than one might expect for a homogeneous system with the same total resistance.

Aronov, MG, and Zhuravlev, *JETP* **60**, 554 (1984)
MG *et al., Phys.Rev.Lett.* **74**, 446 (1995).

- At the same time, τ_φ would not depend on $R_{\square\ macro}$ ✓

- The same conclusion would be applicable to the INT corrections – this percolation "reduction" of the quantum corrections might explain why the slope dR/dT in the vicinity of the apparent MIT becomes smaller instead of getting bigger (F_0^a keeps growing with decreasing n). ✓

What would be the consequences ?

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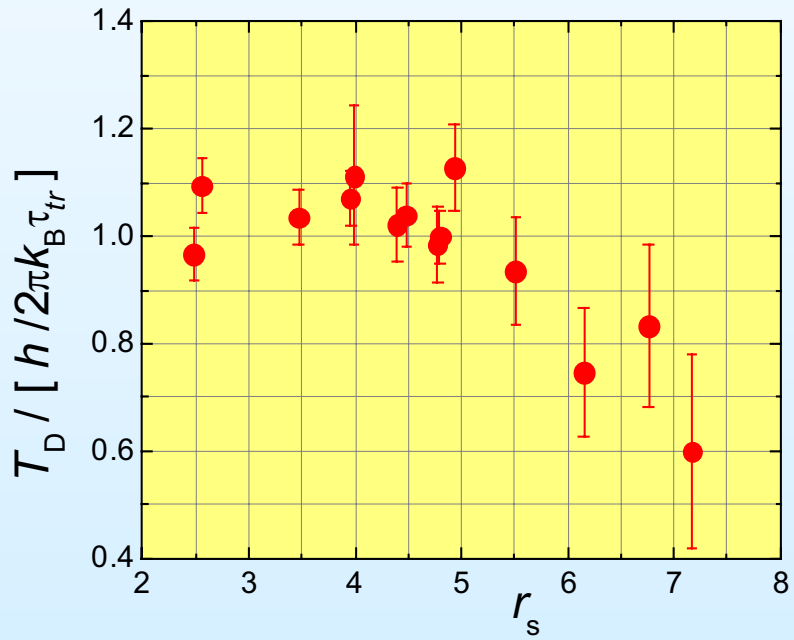
- Enhancement of the 1/f noise near the perc. threshold ✓

  D. Popovic et al., R. Leturcq et al. PRL 90, 076402 (2003)

- The amplitude of SdH oscillations becomes anomalously large: τ_q becomes greater than τ_{tr} ✓

$$\tau_q = \frac{\hbar}{2\pi\, k_B T_D}$$

$$\tau_{tr} = \frac{\sigma_{macro} m^*}{e^2 n}$$



(The WL corrections would be reduced by a large factor τ_q / τ_{tr}).

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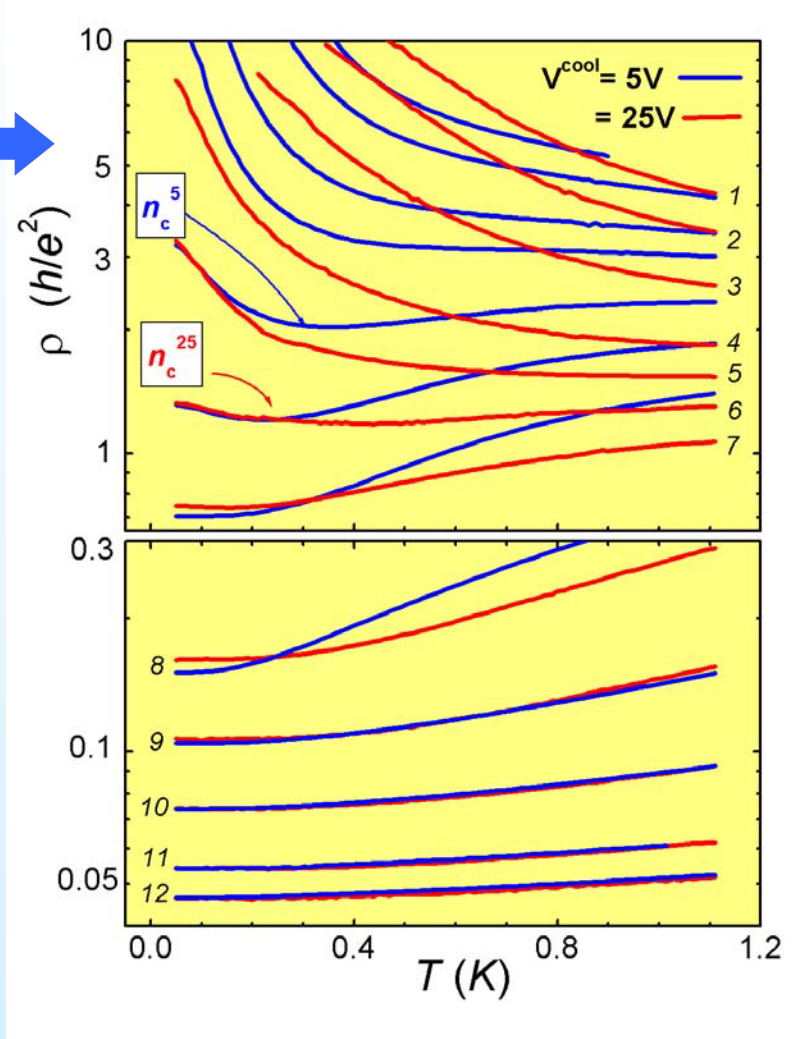
The same device, cooled down to 4K at different fixed values of V_g.

Important: different V_g^{cool} did not affect significantly the parameters which control the interaction effects (τ and F_0^a)

Universal behavior $\Rightarrow$ sufficiently far from the apparent 2D MIT ($\rho \leq 0.1 h/e^2$, or $n > 1.3 \times 10^{11}$ cm^{-2}).

VP, MG, and HK, cond-mat/0201001



Universal vs sample-specific behavior

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The crossover from "metallic" to "insulating" conductivity in high-mobility Si MOSFETs is consistent with a classical percolation scenario.

Close to the crossover, the dilute 2D systems may become. If this is the case, the temperature and magnetic-field dependences of σ are still due to the quantum corrections, but the magnitude of these corrections is diminished by development of the macroscopic non-homogeneity at low electron densities.

In the percolation scenario, the "apparent 2D MIT" has nothing to do with interactions at all.

Probing the macroscopic homogeneity of dilute 2D systems – the key issue for better understanding of the apparent 2D MIT.

At preset, it is unclear how the large-scale fluctuations can emerge from the short-range scattering in Si MOSFETs

Part III: Conclusion

Observation	Plausible explanation	Relevance to strong interactions
Renormalization of electron parameters	Electron-electron interactions (not all the details are clear)	Yes
"Metallic" behavior of conductivity	Interference corrections due to interactions, not a signature of a new ground state	Only via interaction-driven renormalization of F_0^a
Crossover "metal" – "insulator" in 2D	Development of macroscopic non-homogeneity?	No?

The main message:

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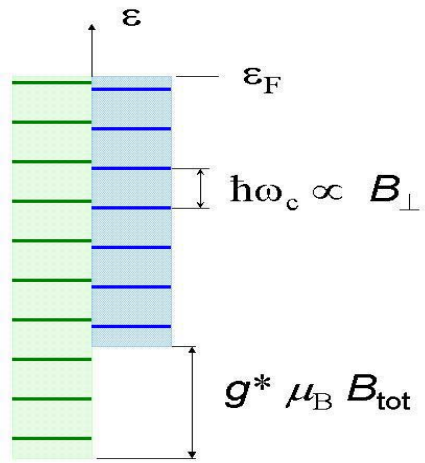
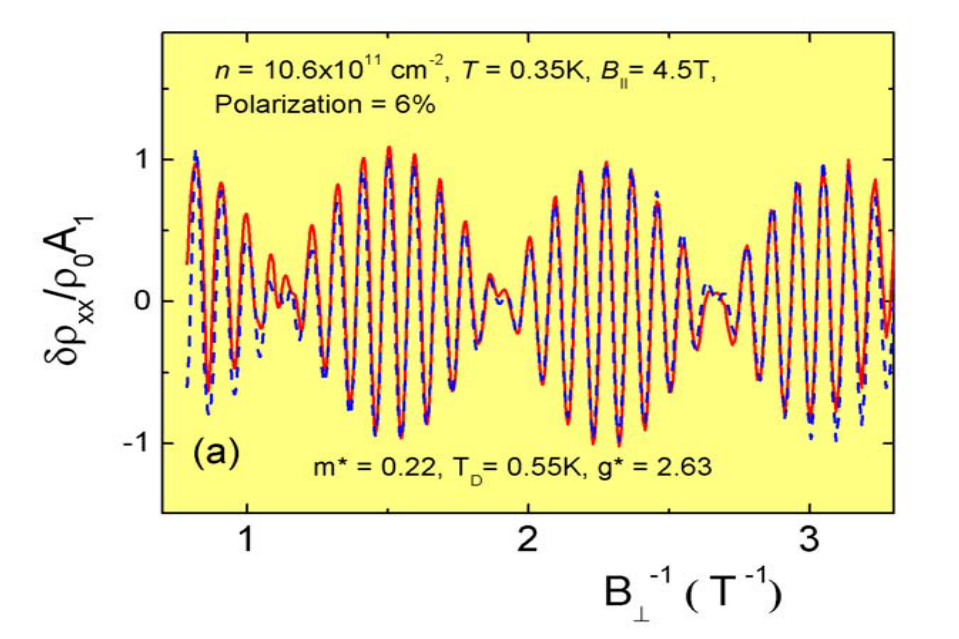
$$\frac{\partial \rho_{xx}}{\rho_0} = \sum_s A_s \cos\left[\pi s\left(\frac{\hbar \pi n}{eB_\perp} - 1\right)\right] \cos\left[\pi^2 s \hbar \frac{(n_\uparrow - n_\downarrow)}{eB_\perp}\right]$$

$$A_s = 4 \exp\left(-2\pi^2 s \frac{k_B T_D}{\hbar \omega_c}\right) \frac{2\pi^2 s \frac{k_B T}{\hbar \omega_c}}{\sinh\left(2\pi^2 s \frac{k_B T}{\hbar \omega_c}\right)}$$





$$P \equiv \frac{n_\uparrow - n_\downarrow}{n} = \frac{\chi^* B_{tot}}{g_b \mu_B n} = g^* m^* \frac{eB_{tot}}{n\hbar}$$

Spin Susceptibility

$$\chi^* = g^* m^* \frac{g_b \mu_B^2}{\pi \hbar^2}$$

Analysis of SdH oscillations

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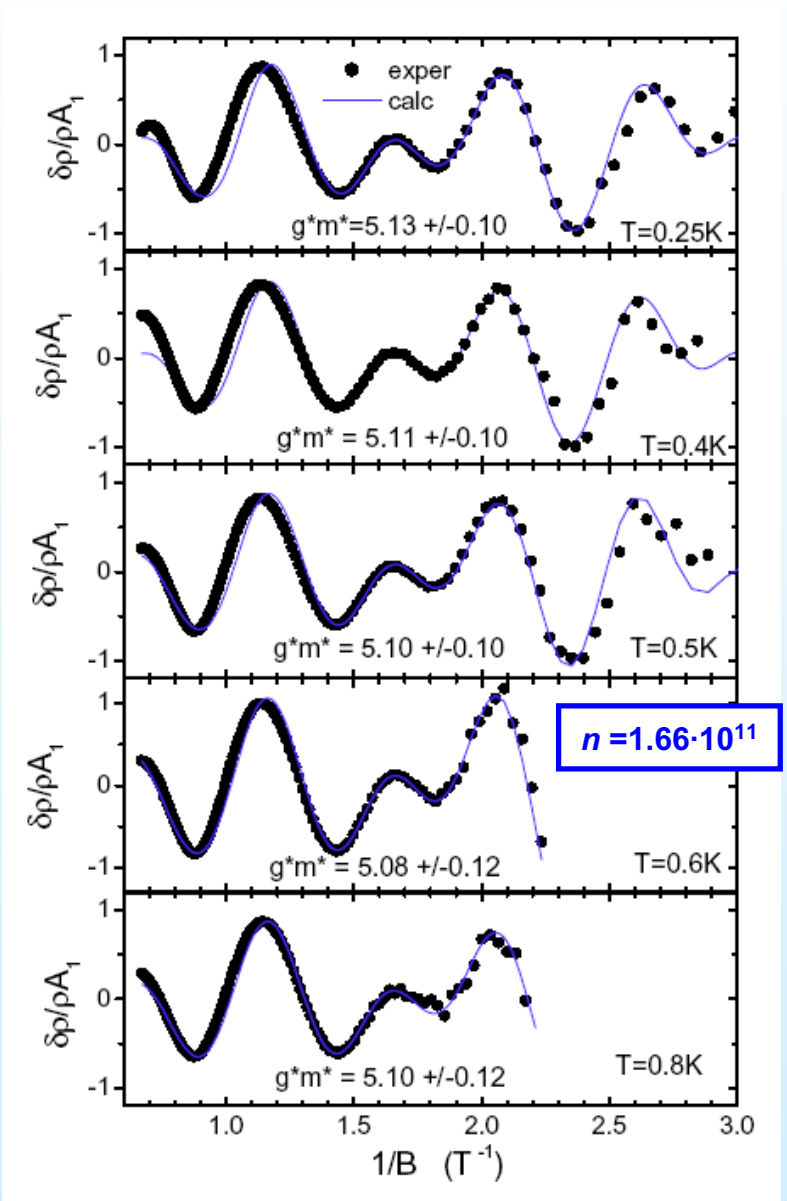
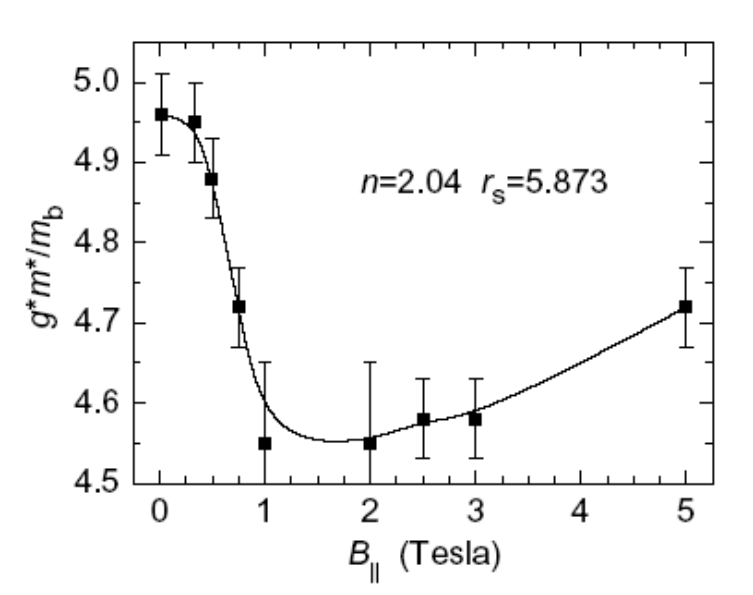


Pudalov, MG, Kojima, cond-mat/0401396

$\chi^*(T, B_{||})$



χ^* is T - independent (within 2%), though it does depend on $B_{||}$

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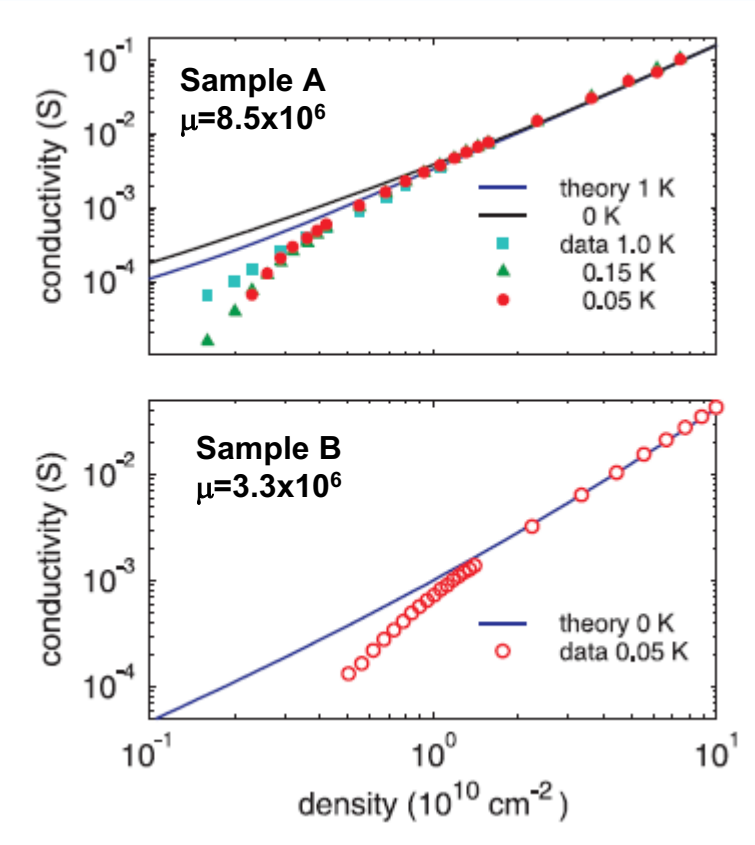

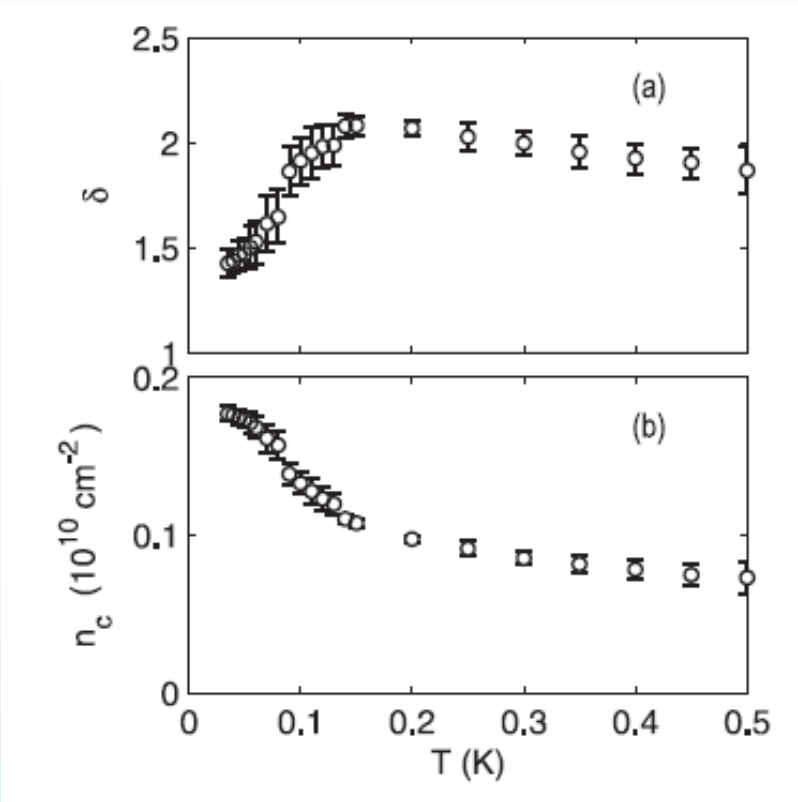

$$\sigma(n) = A(n - n_c)^{\delta}$$

Das Sarma *et al*. cond-mat/0406655

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