united nations educational, scientific and cultural organization
international atomic energy agency
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ICTP 40th Anniversary

SMR.1573 - 6

SUMMER SCHOOL AND CONFERENCE ON DYNAMICAL SYSTEMS

Evolutionary Dynamics

(Lecture 3)

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Asymmetric Games

Asymmetric Games

- Owner vs. Intruder
- Female vs. Male
- Young vs. Old
- Buyer vs. Seller
- Host vs. Parasite

Male vs. Female: Battle of the Sexes

- Males invest little
- Females invest much
- Females must be choosy
- long engagement periods?
- Females: coy or fast
- Males: faithful or philandering

Battle of the Sexes

- Females coy
- Males faithful
- Females fast
- Males philanderers
- Females coy
- ...

Battle of the Sexes

		females	
		coy	fast
males	phil	$(0,0)$	$(G, G-C)$
	faith	$(G-\frac{C}{2}-E, G-\frac{C}{2}-E)$	$(G-\frac{C}{2}, G-\frac{C}{2})$

G Gain from raising offspring

$-C$ Parental investment

$-E$ Cost of engagement

with $0 < E < G < C < 2(G-E)$

Battle of the Sexes

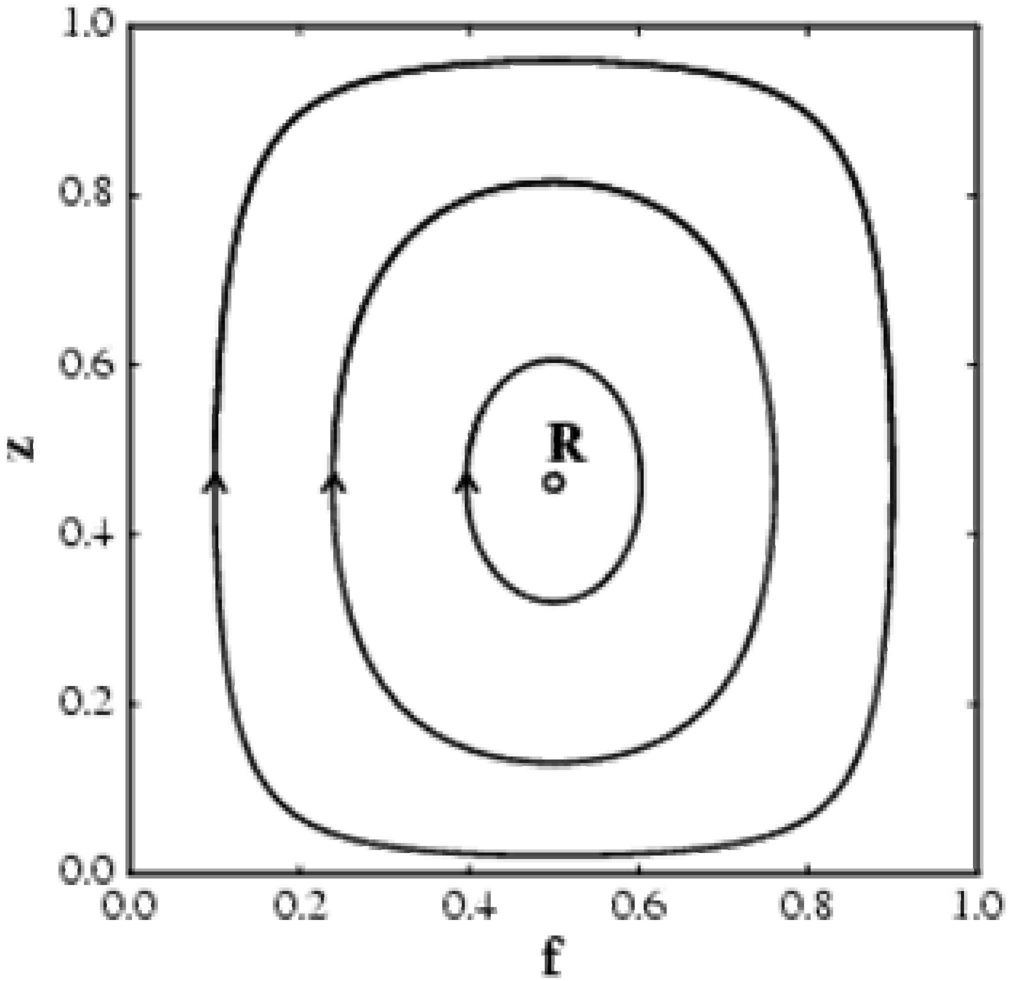
x frequ. of philanderers (males)

y frequ. of coy females

$$\dot{x} = x(1-x)\left(\frac{C}{2} - (G-E)y\right)$$

$$\dot{y} = y(1-y)(-E + (C+E-G)x)$$

Battle of the Sexes



Example: Owner-Intruder

Chicken Game, escalate or not

G gain, - C cost of injury, C > G

	escalate	yield
escalate	$(\frac{G-C}{2}, \frac{G-C}{2})$	$(G,0)$
yield	$(0,G)$	$(\frac{G}{2}, \frac{G}{2})$

Example: Chicken

x prob that player I escalates

y prob that player II escalates

(1,0) and (0,1) are Nash equilibria

but who is I and who is II?

symmetric Nash equilibrium (x, x)

with $x = \frac{G}{C}$

Example: Owner-Intruder

x prob that player I escalates

y prob that player II escalates

(1,0) and (0,1) are Nash equilibria

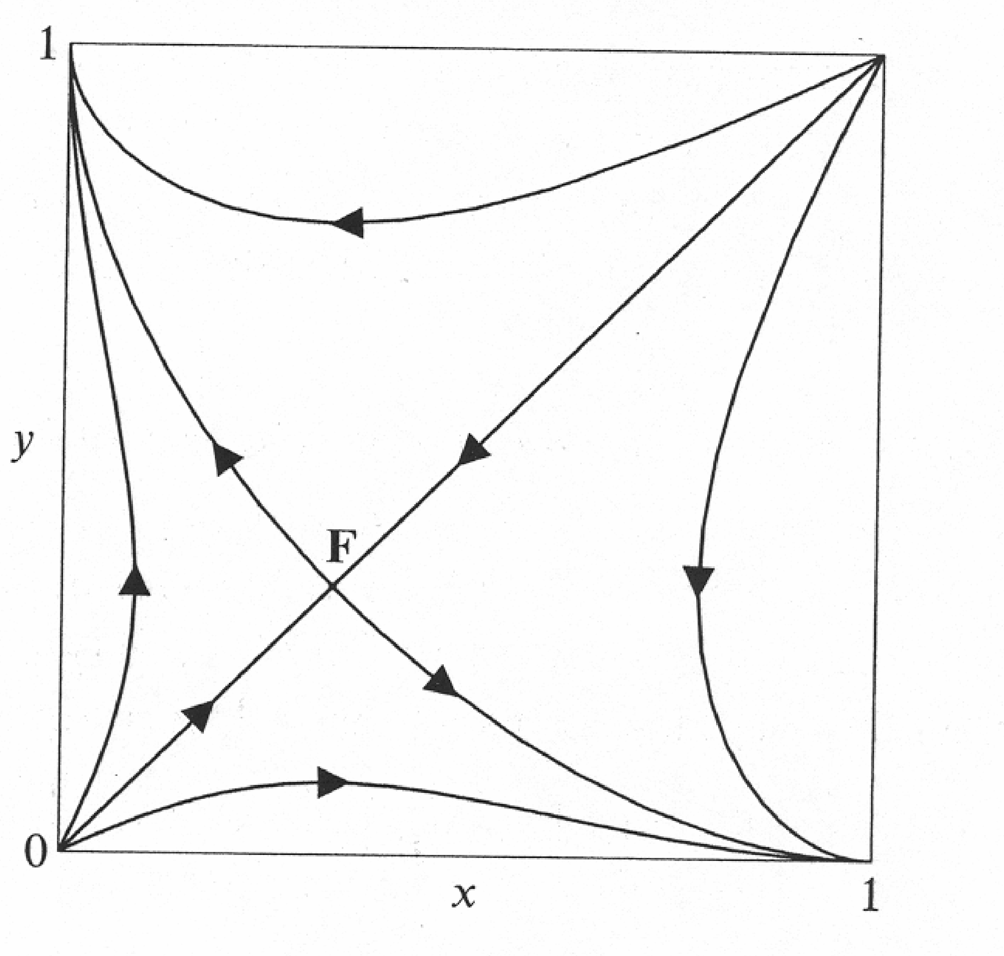
but who is I and who is II?

I is owner, II is intruder

$$\dot{x} = x(1-x)(G-Cy)$$

$$\dot{y} = x(1-y)(G-Cx)$$

Example: Owner-Intruder



Example: Owner-Intruder

- (1,0) Bourgeois strategy
- (0,1) paradox strategy (Prudhon)
- asymmetric Nash equilibria

Conditional Strategies

- If owner, escalate; if intruder, display
- If male, philanderer; if female, coy

Conditional Strategies

- player I, strategies e_1, e_2
  player II, strategies f_1, f_2

	f_1	f_2
e_1	(A, a)	(B, b)
e_2	(C, c)	(D, d)

Conditional Strategies

- in role I, strategies e_1, e_2

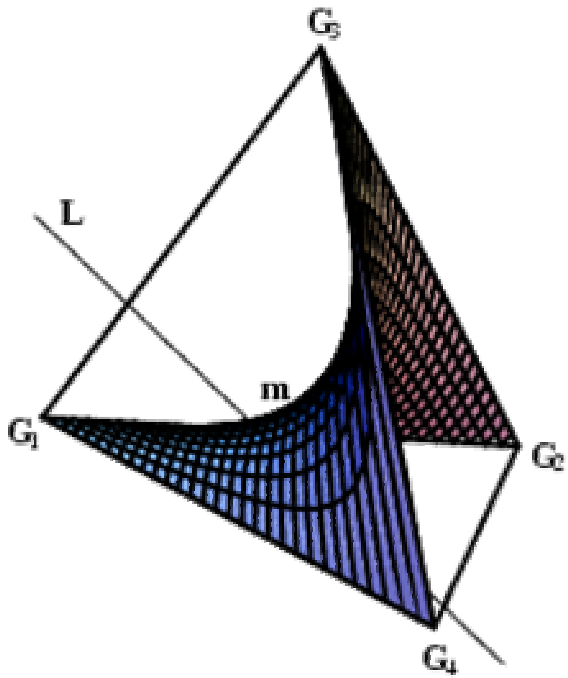
  in role II, strategies f_1, f_2

  conditional strategies

  $G_1 = e_1 f_1, G_2 = e_2 f_1, G_3 = e_2 f_2, G_4 = e_1 f_2$

$$\frac{1}{2}\begin{pmatrix} A+a & A+c & B+c & B+a \\ C+a & C+c & D+c & D+a \\ C+b & C+d & D+d & D+b \\ A+b & A+d & B+d & B+b \end{pmatrix} = M$$

Two-Roles Games



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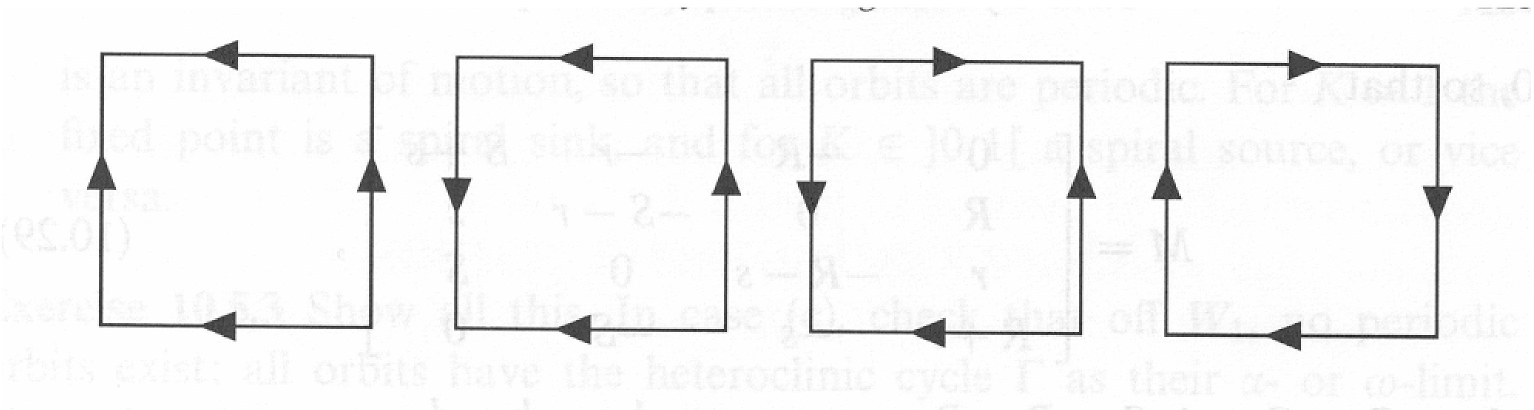
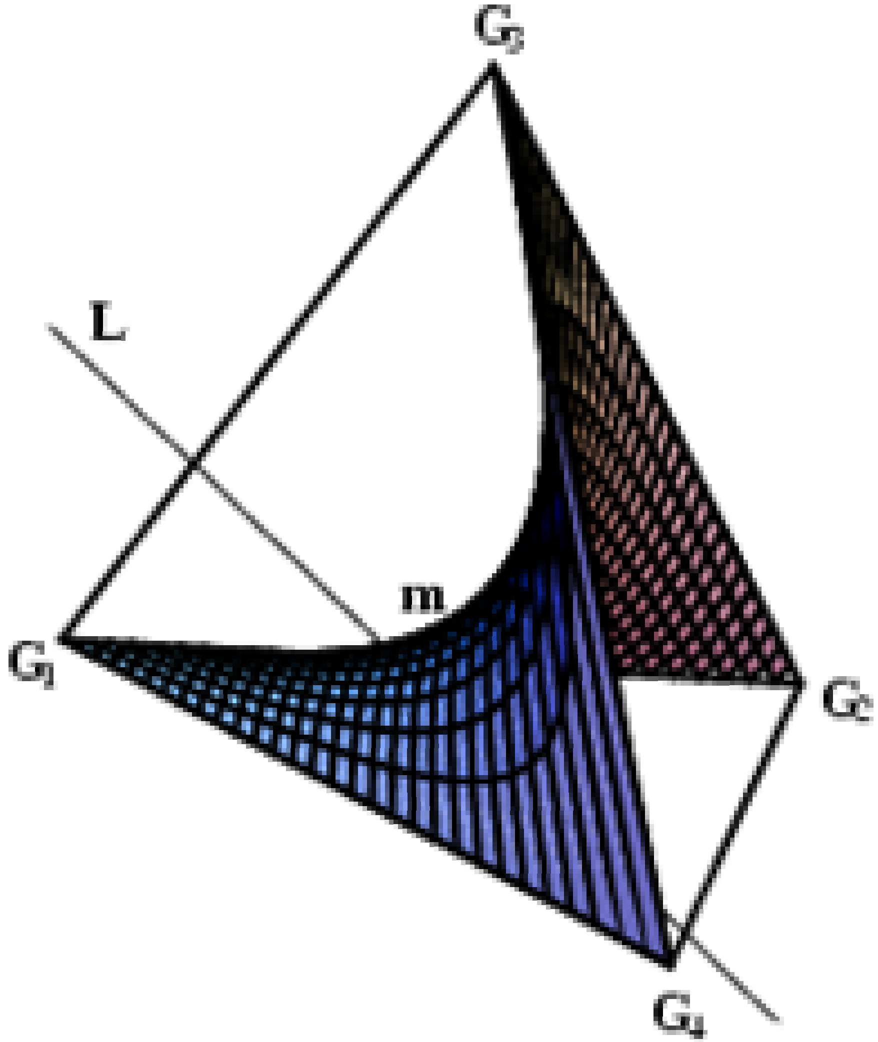
$$(Mx)_1 + (Mx)_3 = (Mx)_1 + (Mx)_3$$

$$\frac{x_1 x_3}{x_2 x_4} = const$$

$x_1 x_3 = K x_2 x_4$ invariant manifolds

Two-Roles Games




G3
L
m
G1
G2
G4