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SMR.1573 - 7

SUMMER SCHOOL AND CONFERENCE
ON DYNAMICAL SYSTEMS

Evolutionary Dynamics

(Lecture 4)

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The evolution of cooperation

Altruism and the selfish gene

- Altruism: benefit b to recipient at cost -c to the donor
- `get the altruism out of altruism...´ (Trivers)

- kin selection
- Hamilton´s rule: c< b.r
- in humans co-operation among non-relatives

Darwin:

- The small strength and speed of man, his want of natural weapons, etc., are more than counterbalanced ... by his social qualities, which led him to give and receive aid from his fellow men.

Cooperation among non-relatives

- Direct reciprocation and the Prisoner´s Dilemma
- Indirect reciprocation
- Public Goods

Reciprocity

- Reciprocal altruism...the trading of altruistic acts in which benefit is larger than cost, so that over a period of time both parties enjoy a net gain.

  (Trivers)

The Prisoner´s Dilemma game

- To cooperate or to defect
- (example: cooperate means to give a gift of value b at cost -c)

Prisoner´s Dilemma

		Oskar cooperates	Oskar defects
Johnny	cooperates	10	-5
Johnny	defects	15	0

Prisoner´s Dilemma

Strategies C and D (cooperate and defect)

Payoff matrix

$$\begin{bmatrix} R & S \\ T & P \end{bmatrix}$$

R Reward, T Temptation, P Punishment, S Sucker

$R > P$ and $T > R$

if also $P > S$ Prisoner's Dilemma $T > R > P > S$

if $S > P$ Snowdrift Game (like Chicken)

The iterated Prisoner´s Dilemma

- probability w for another round
- (for instance, toss a coin, stop game if 6)
- shadow of the future (Axelrod)

The iterated Prisoner´s Dilemma

	Tit For Tat	always defect
Tit For Tat	60	-5
always defect	15	0

The iterated Prisoner´s Dilemma

	Tit For Tat	AllD
Tit For Tat	$\frac{R}{1-w}$	$S + w\frac{P}{1-w}$
AllD	$T + w\frac{P}{1-w}$	$\frac{P}{1-w}$

Bistable if $w > \frac{T-R}{T-P}$ $\left(= \frac{c}{b}\right)$

The iterated Prisoner´s Dilemma

memory one strategies

(p_R, p_S, p_T, p_P)

where p_o prob to play C after outcome o

(1,1,1,1) is AllC

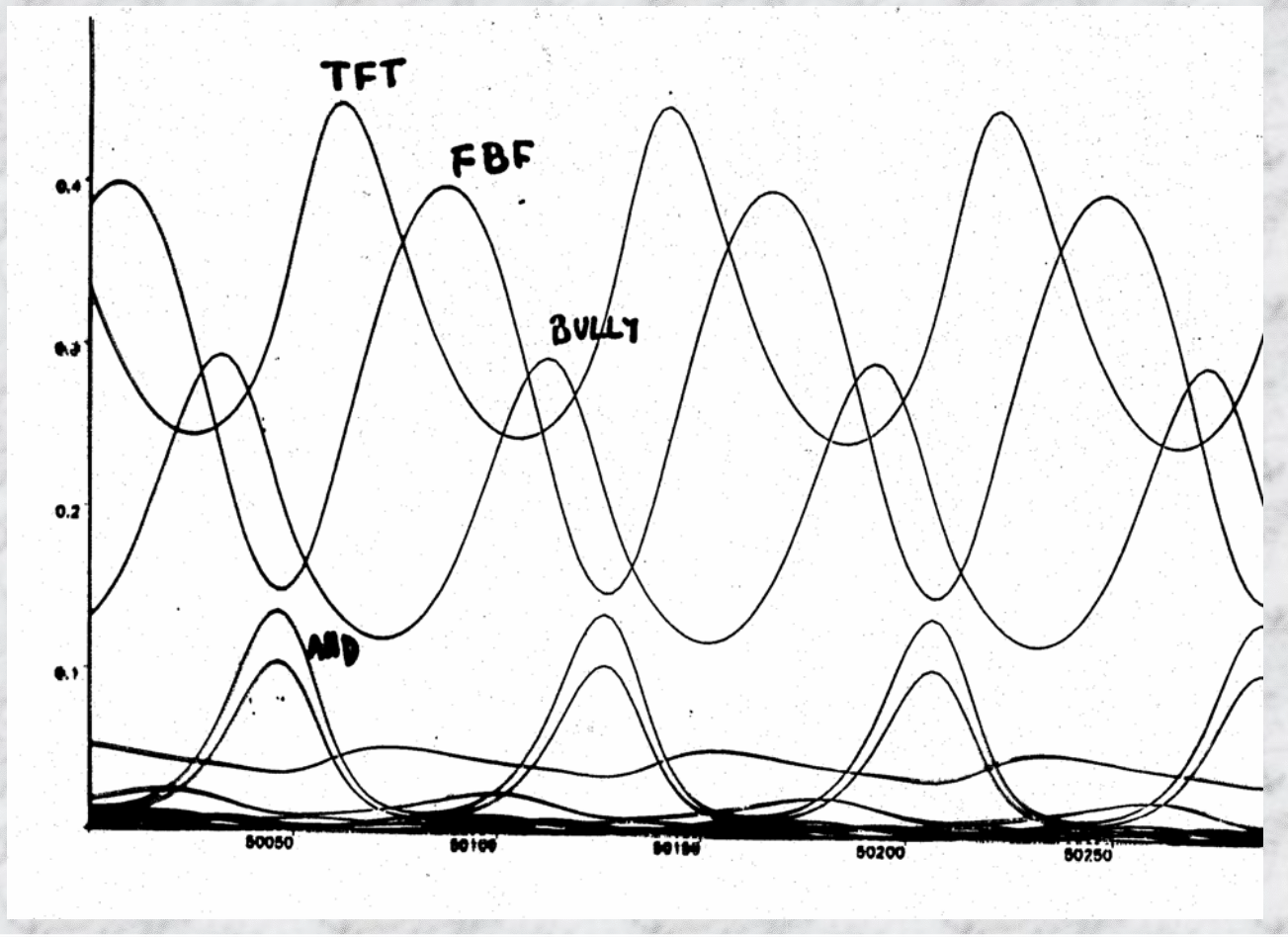
(0,0,0,0) is AllD

(1,0,1,0) is TFT

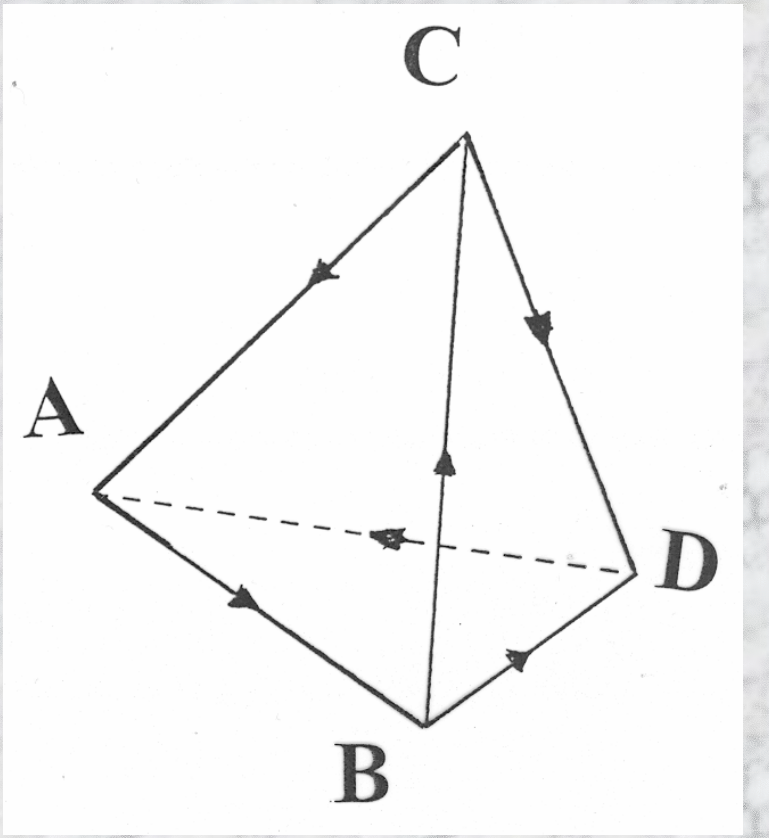
(1,0,1,1) is Firm but Fair

(0,0,0,1) is Bully

The iterated Prisoner´s Dilemma



The iterated Prisoner´s Dilemma



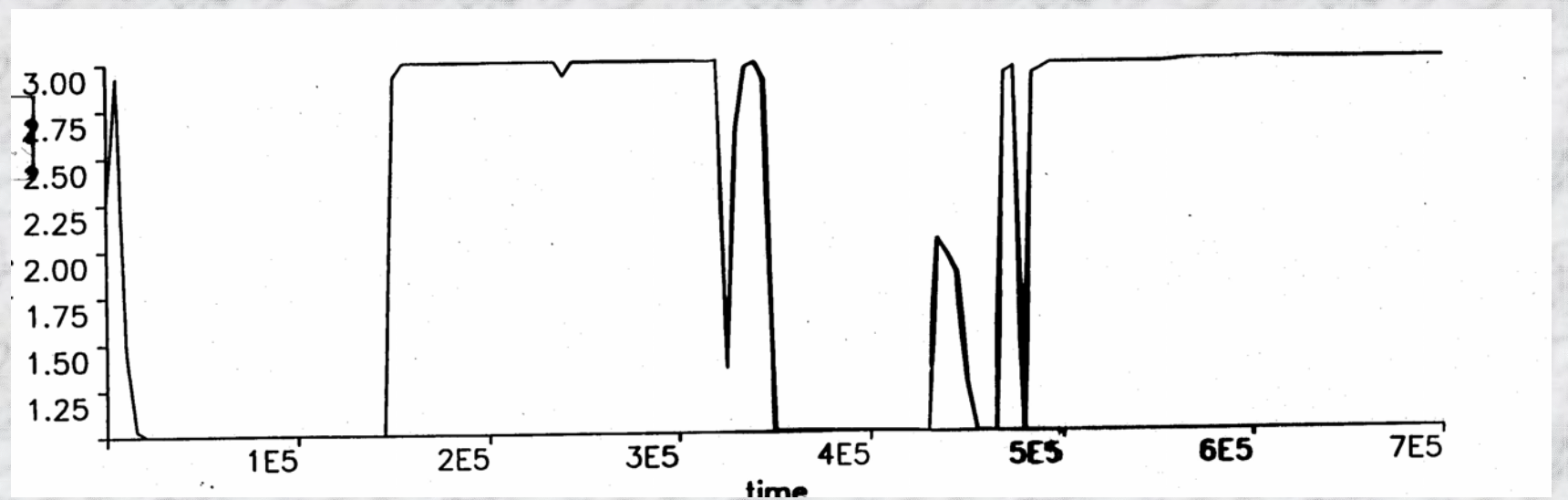
- Heteroclinic network
- A = Tit or Tat
- B = Firm But Fair
- C = Bully
- D = AllD

The iterated Prisoner´s Dilemma

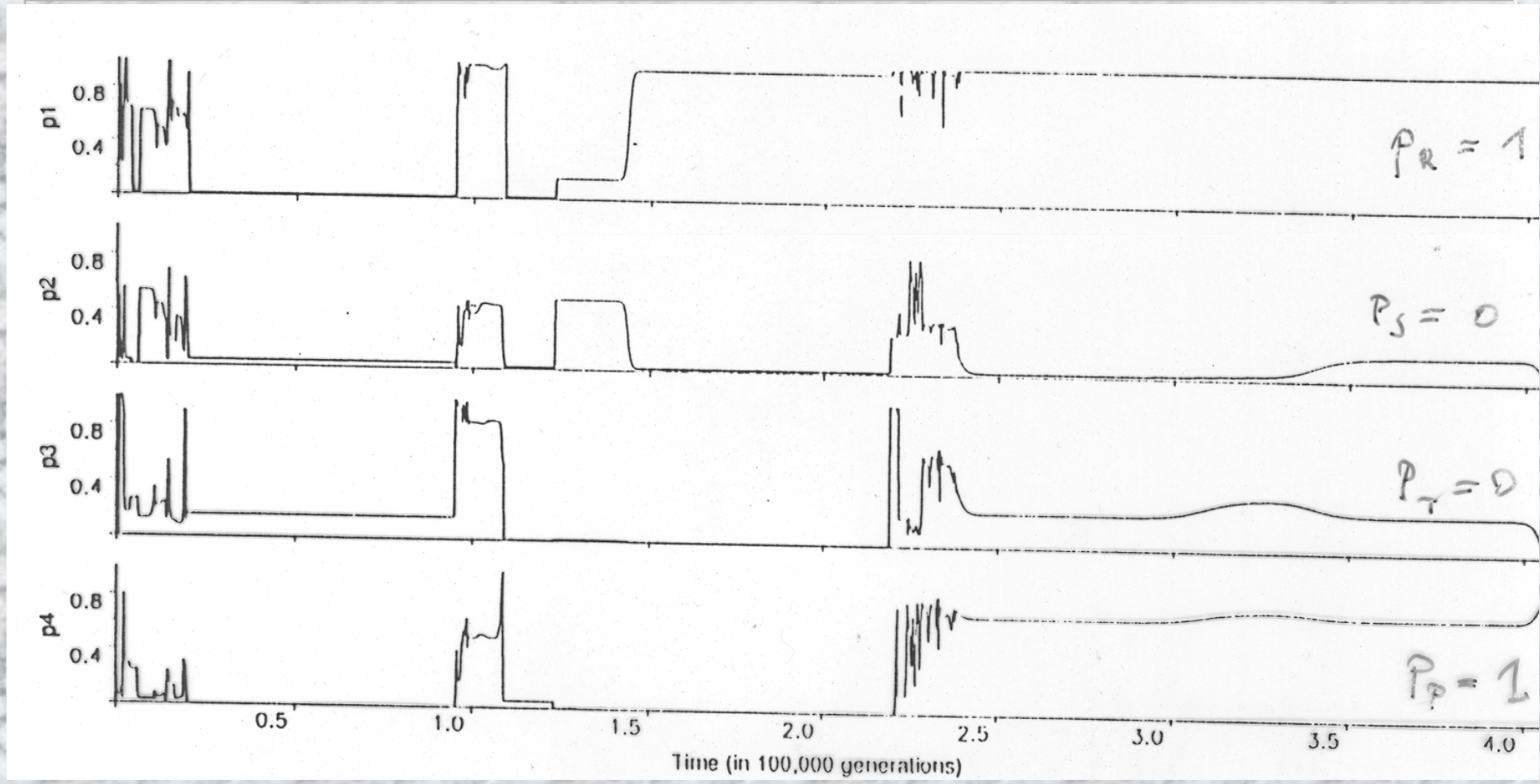
- Start with randomly chosen strategies
- Run replicator equation (selection)
- Occasionally add minority of new strategy (mutant)

Individual-based modelling

The iterated Prisoner´s Dilemma



The iterated Prisoner´s Dilemma



Pavlov

$(1,0,0,1) = (p_R, p_S, p_T, p_P)$ Pavlov

cooperates iff co-player used same move

win-stay, lose-shift :

$\binom{C}{C}$	$\binom{C}{D}$	$\binom{D}{C}$	$\binom{D}{D}$
R	S	T	P
C	D	D	C

Pavlov

Pavlov is error - correcting

If Pavlov against Pavlov

C C C...C D D C C

C C C...C C D C C

If TFT against TFT

C C C ... C D C D C D ...

C C C ... C C D C D C ...

Pavlov

Pavlov is a 'simpleton'

against AllD

C D C D C D C ...

D D D D DD D...

payoff per round $\frac{P+S}{2}$

AllD gets $\frac{T+P}{2}$

Pavlov stable if $T+P<2R$ (i.e. $c<\frac{b}{2}$)

Pavlov cannot invade AllD

Pavlov

- Simple learning rule
- stable, error-correcting
- but needs retaliator to prepare the ground

Indirect reciprocity

- few rounds, as donor or receiver
- never with the same partner twice

Indirect reciprocity

- R. Alexander:

  Indirect reciprocity .. involves reputation and status, and results in everyone in the group continually being assessed and reassessed.

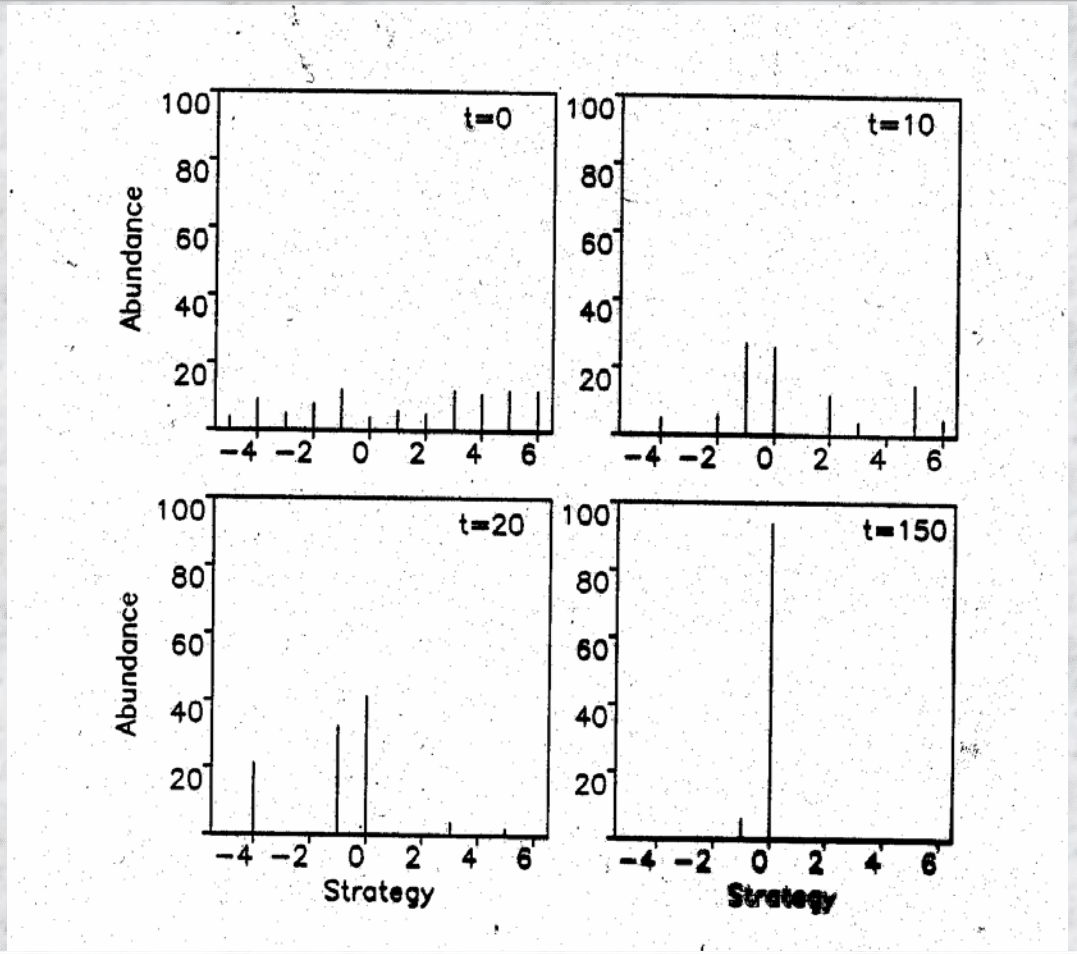
- Cooperation channelled towards cooperative members

Indirect reciprocity

- score: increases by 1 if help is given, decreases by 1 if help is withheld
- score 0 at birth

- strategy k: give only if recipient has score at least k
- k>5: always defect
- k<-5: always help

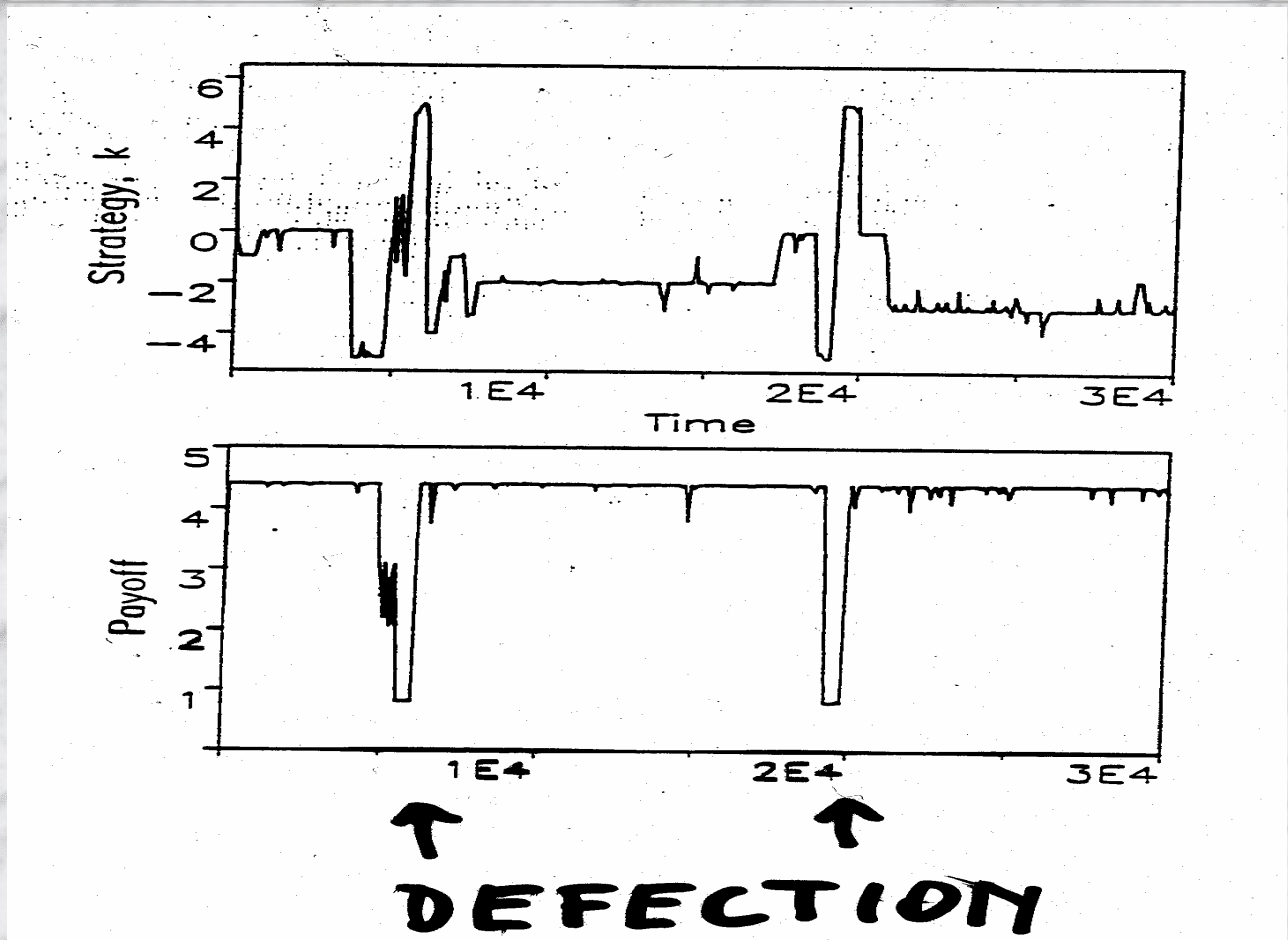
Indirect reciprocity



Indirect reciprocity

- cooperation based on discrimination
- but not stable (can occasionally break down)

Indirect reciprocity



Indirect Reciprocity