



the  
**abdus salam**  
international centre for theoretical physics

40<sup>th</sup> anniversary  
1964  
2004

SMR/1578 - 2

**SCHOOL AND CONFERENCE  
ON  
FUNDAMENTAL ASPECTS OF COMPLEXITY**

(6 - 10 September 2004)

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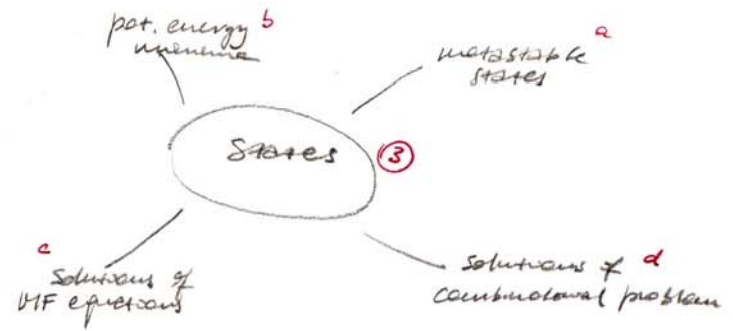
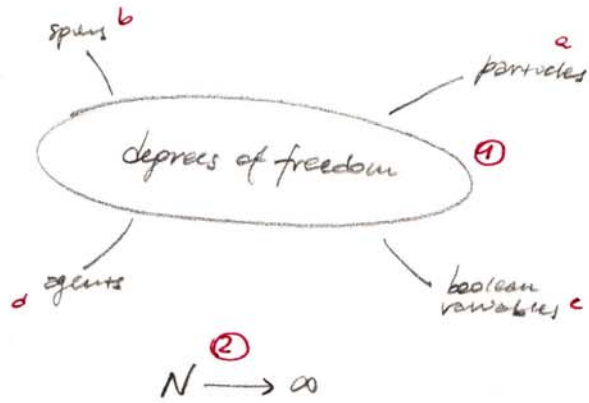
*" Structural glasses and complexity: an overview "*

presented by:

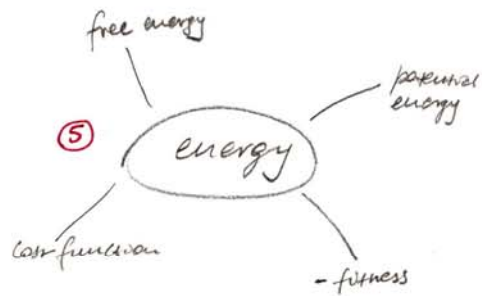
**A. Cavagna**

Università degli Studi "La Sapienza"  
Roma, Italy

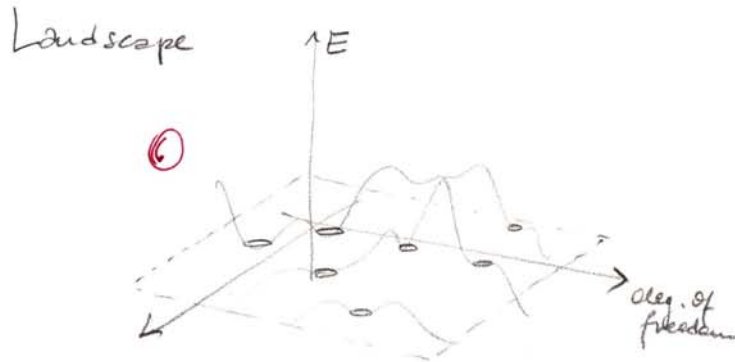
1



(4)  $CP \sim e^{N\Sigma}$   
 ↓  
 COMPLEXITY

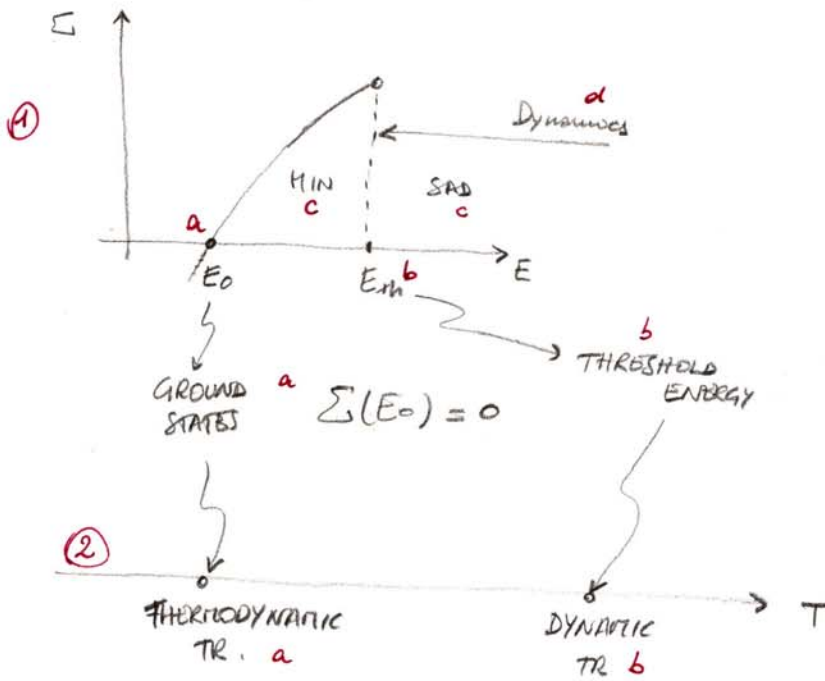


E: density!



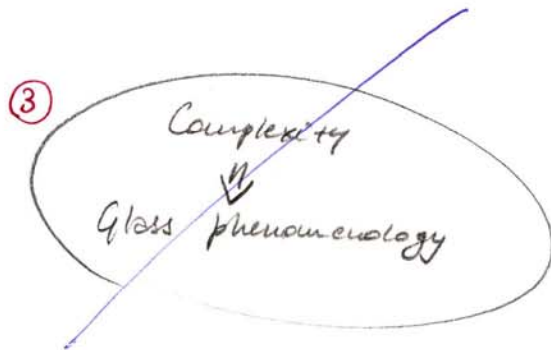
(7)  $CP(E) \sim e^{N\Sigma(E)}$

Complexity



3 def. of threshold

- topology
- MAX
- DYNAMICS (not too smart)





4

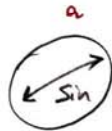


M. Goldstein  
 J. Chem. Phys. 51 3728  
 (1969)

(3) AT LOW ENOUGH  $T \Rightarrow$  ACTIVATION RULES DYNAMICS

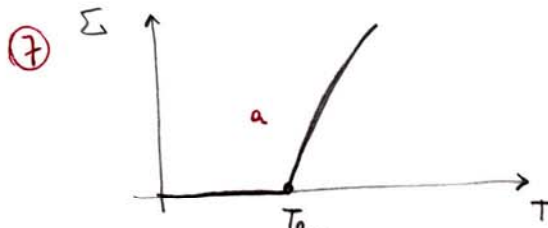
Entropy:

(4)  $S_{LO} = S_{in} + \Sigma$

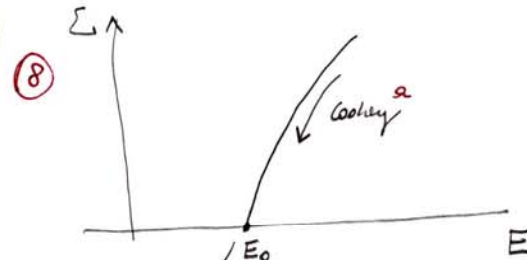


(5) low  $T$ :  $S_{in} \approx S_{LO} \Rightarrow \Sigma \approx S_{LO} - S_{LO}$  (back to plot  $\rightarrow$  mark the difference)

(6)  $\Sigma(T) \rightarrow 0$  for  $T \rightarrow T_0$



Why function at  $T_0$ ?

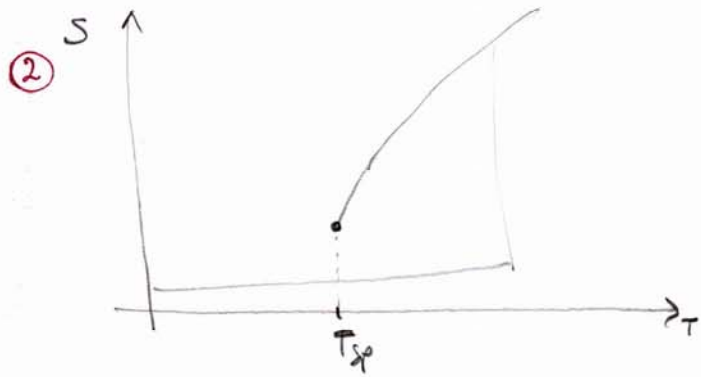
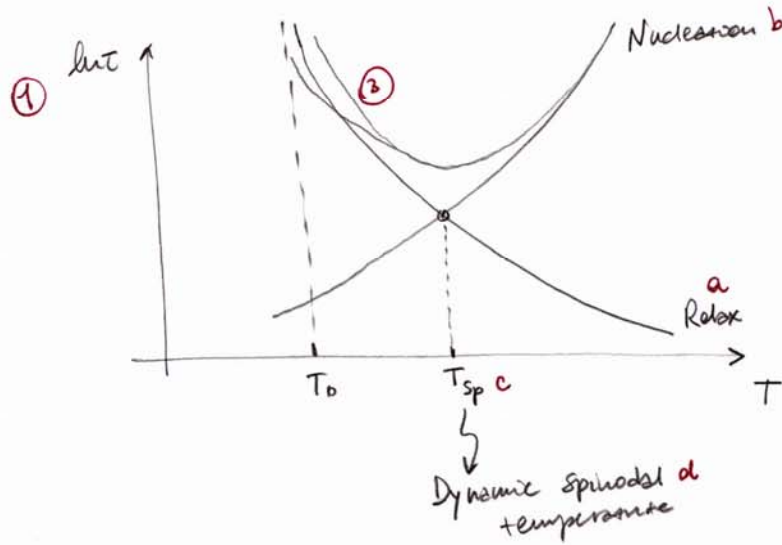


$\rightarrow$  KAUFMANN TRANSITION

stuck at  $E_0$  for  $T \leq T_0$

THERMODYN. GLASS

5



④ if  $T_{sp}$  is there, there cannot be any  $T_0$  (Kauzmann 48)

HOWEVER ALL THIS QUITS MYSTIC,  
since cannot explain experimentally

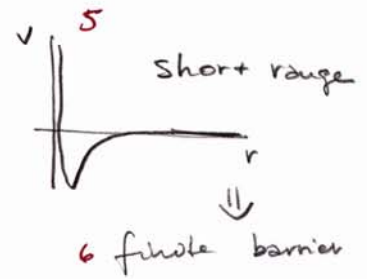
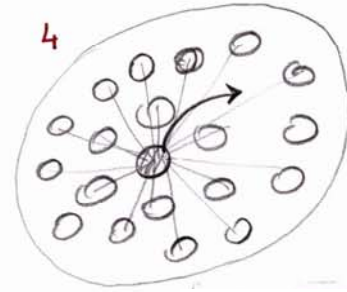
6

1 Sharp dynamical transition

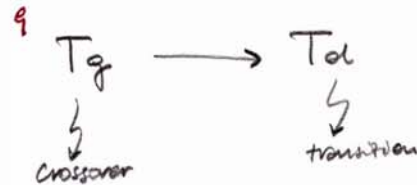
↑  
2 Kill activation

↑  
3 Make barrier infinite

↑  
7 Mean-Field ( $\infty$ -range)



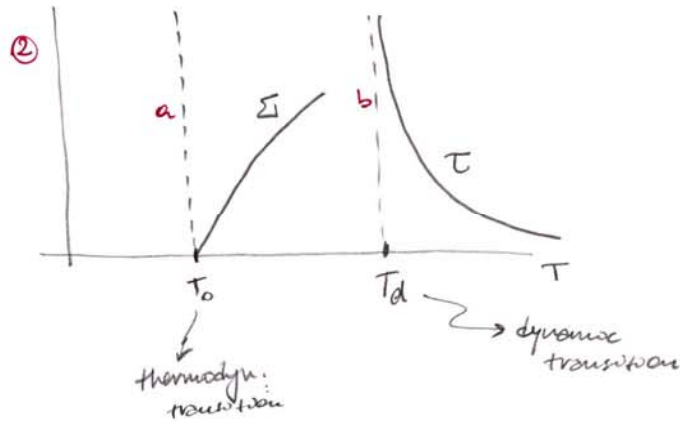
- 8
- p-spin spin-glass models (1RSB)
  - Mean Coupling Theories



Warning on  $\infty$  barriers in NCV-MF systems.

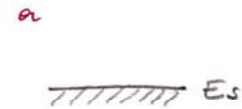
7

① EXACT SOLUTIONS



- ③ Dynamic :  $E(t) \xrightarrow{a} E_d$
- o State :  $b - \frac{\partial \ln Z}{\partial \beta} = E_s$

$c E_d > E_s$



$b \Rightarrow$  metastable states

⑤ Computing  $\Sigma$  :

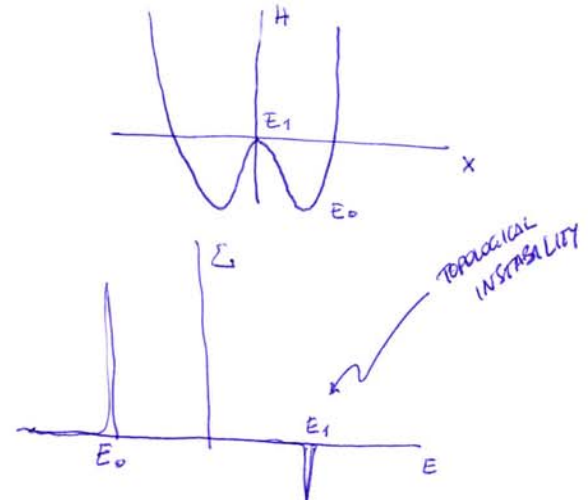
a minima of  $H(x) \quad \partial_x H = 0$

b  $CP(E) = \int Dx \delta(\mathcal{M}H) \det(\mathcal{M}H) \delta(H-E)$

$c \Sigma(E) = \frac{1}{N} \ln CP(E)$

- ⑥ \* all stat. points a
- \* missing modulus

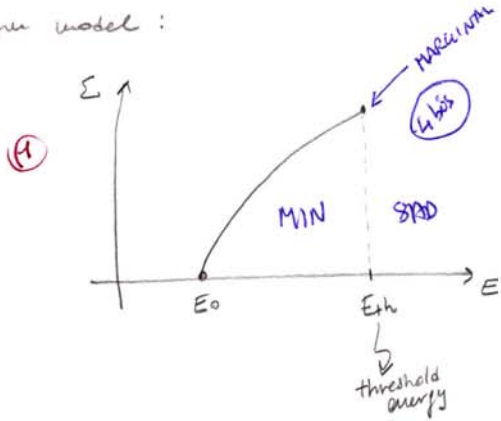
b if  $\det(\mathcal{M}H) > 0 \rightarrow CP > 0 \rightarrow \Sigma \in \mathbb{R}$



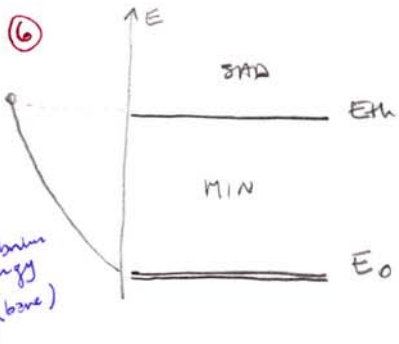


8

p-spin model:



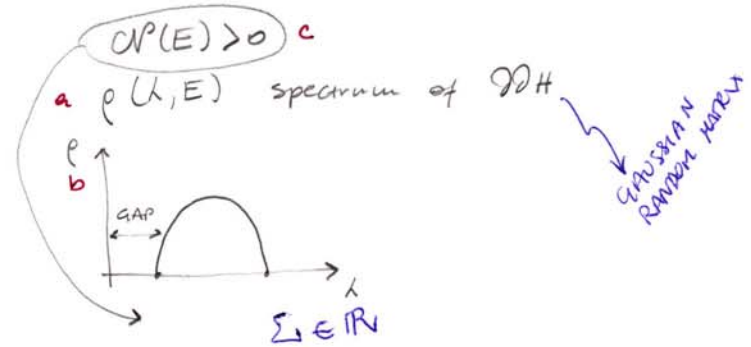
- ⑤ a Minima dominant for  $E < E_{th}$
- a Saddles " " "  $E > E_{th}$



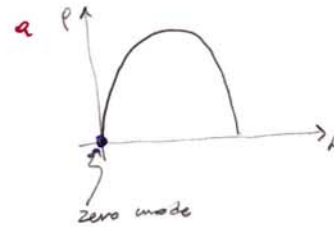
⑦  $E(T) \rightarrow E_{th}$  for  $T \rightarrow T_d$

⑧ MF

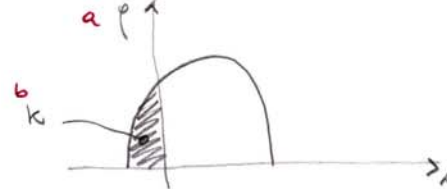
②  $E < E_{th}$



③  $E = E_{th}$



④  $E > E_{th}$



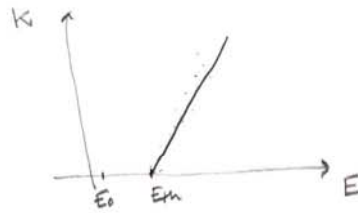
$CP(E) \sim (-1)^{N_k} \rightarrow \# \text{ of negative modes}$

$\Sigma(E)$  no longer defined

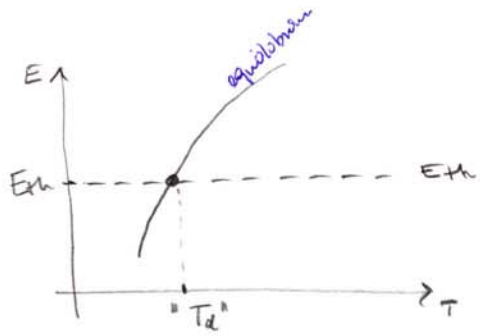
9

Non-mean field ?

Determine  $E_{th}$  :



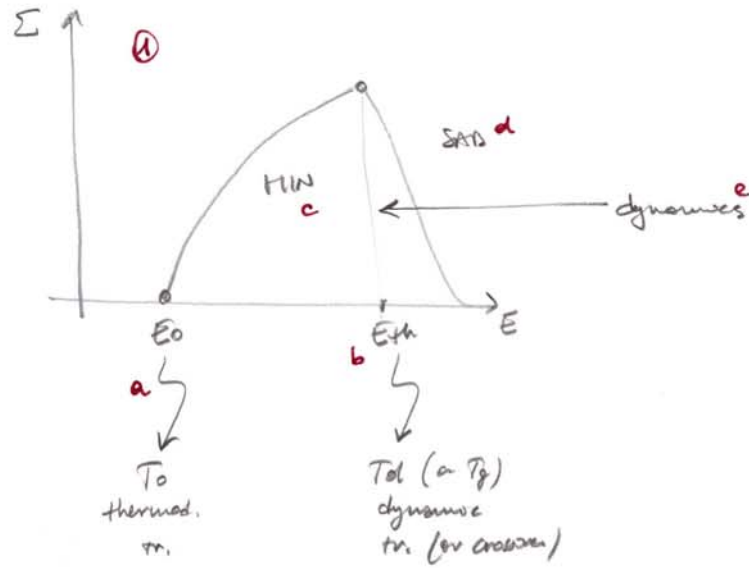
numerical !



numerical !

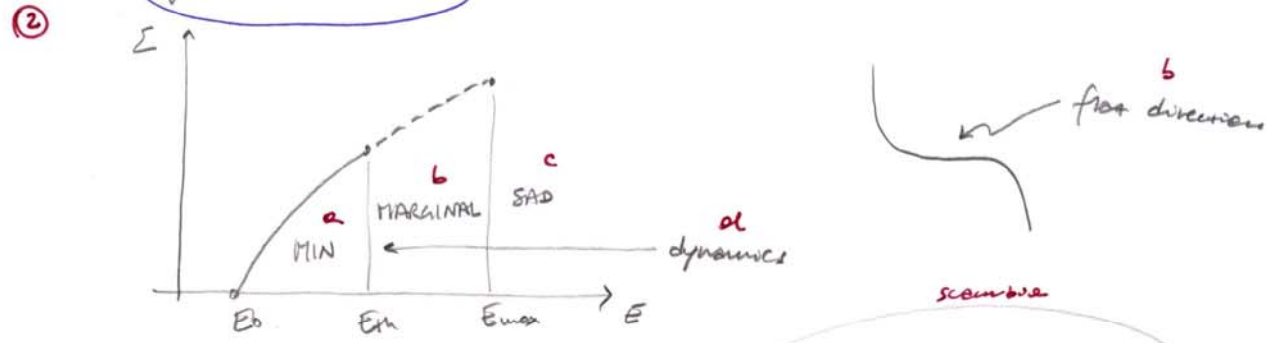
for fragile systems  $T_d \approx T_g$

10



1RSB-scenario f

General scenario



- ③ p-spin models :  $E_{th} = E_{max}$  (structural glass)  $\rightarrow T_0 < T_d$
- ④ SK model :  $E_0 = E_{th}$   $\rightarrow T_0 = T_d$