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# Surface Waves in Laterally Inhomogeneous Media

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## 1. Introduction

Surface waves form the longest and strongest parts of seismic records excited by explosions and shallow earthquakes. Traversing areas with diverse geological structures they 'absorb' information on the properties of these areas. This information is best reflected in dispersion, the dependence of velocity on frequency. The other properties of these waves such as polarization, frequency content, attenuation, azimuthal variation of the amplitude and phase, are also controlled by the medium between the source and receiver. Some of these are affected by the properties of the source itself and by the conditions around it.

The information about the Earth structure and the seismic source carried out by surface waves may be extracted from seismic records and used for solving numerous scientific and practical problems. We should mention some of them:

- determination of regional crustal, lithospheric, and upper mantle structure,
- reconnaissance of sedimentary basins on land and at seas,
- survey of loose sediments and evaluation of statical corrections for seismic prospecting goals, especially in multicomponent surveys, using PS reflections.
- determination of the structure and elastic/nonelastic properties of ground in various civil engineering, archeological, and environmental studies,
- source characterization, including determination of magnitude, moment tensor, source dynamic parameters (size of the rupture zone, direction and speed of rupturing),
- discrimination between underground nuclear tests and other seismic events (natural or man-made).

Some insight into possible applications, corresponding frequency ranges and depth penetration can be obtained from the Table 1.

The theory which describes the surface wave propagation is reasonably well developed. There are textbooks, monographs, and numerous papers in geophysical journals describing the subject. You can find some information about this literature in the reference list which is far from to be complete. There are also numerous computer codes for solving forward and inverse problems of the surface wave seismology for different types of the Earth and source models. At least some of them are easily available. In what is following I will present a short discussion of surface wave propagation in laterally homogeneous media and the extension of this theory to some types of laterally inhomogeneous media. I will also discuss briefly the existing techniques of surface wave analysis. The subject of the next lecture will be the methodology of surface wave measurements and phenomenology of surface wave propagation in the real Earth. The applications of this methodology in global, regional and local tomographic studies carried out by the Center for Imaging the Earth's Interior at the University of Colorado at Boulder, U.S.A. will be presented by Professor Ritzwoller. Some aspects of this work will be also discussed in my third lecture.

**Table 1. Surface Waves in the Earth's studies**

| frequency (Hz) | period (sec) | wavelength (km) | phase velocity (km/sec) | depth of penetration (km) | application |
|---|---|---|---|---|---|
| 10-50 | 0.02-0.1 | 0.002-0.05 | 0.1-0.5 | 0.020 | Static Corrections, Civil Engineering, Archeology |
| 1-5 | 0.2-1 | 0.15-1.5 | 0.1-1.5 | 0.2 | Static Corrections, Upper Sediments Studies |
| 0.1-0.2 | 5-10 | 7-30 | 2-3 | 5 | Sedimentary Basins Studies |
| 0.03-0.1 | 10-35 | 30-100 | 3.0-3.5 | 40 | Crustal Studies |
| 0.003-0.03 | 35-350 | 200-1000 | 4-5 | 300 | Upper Mantle Studies |

## 2. Surface waves in laterally homogeneous media

The theoretical studies of surface waves were started by Lord Rayleigh (1887) and by Love (1911). Further contributions by Stoneley (1924), Pekeris (1948), Ewing *et al.* (1957) were extremely important. The list of names of later contributors is too long to be presented here. The modern status of the theory may be found in Aki & Richards (1980), Levshin *et al.*(1989), Kennet (2001).

For surface waves to exist at least one of two following conditions should be fulfilled:

**(1) presence of a discontinuity** :

- the free surface of an elastic body - *Rayleigh wave*
- the discontinuity between elastic and liquid half-spaces - *Stoneley wave*; in seismic prospecting literature it is often called *Scholte wave* (Scholte, 1958)
- the discontinuity between two elastic half-spaces (with some significant constraints) - *Stoneley wave.*

Waves of this type may be called the *boundary waves*. Boundary waves propagate without dispersion: their phase velocity $C$ does not depend on frequency $f$. They may be considered

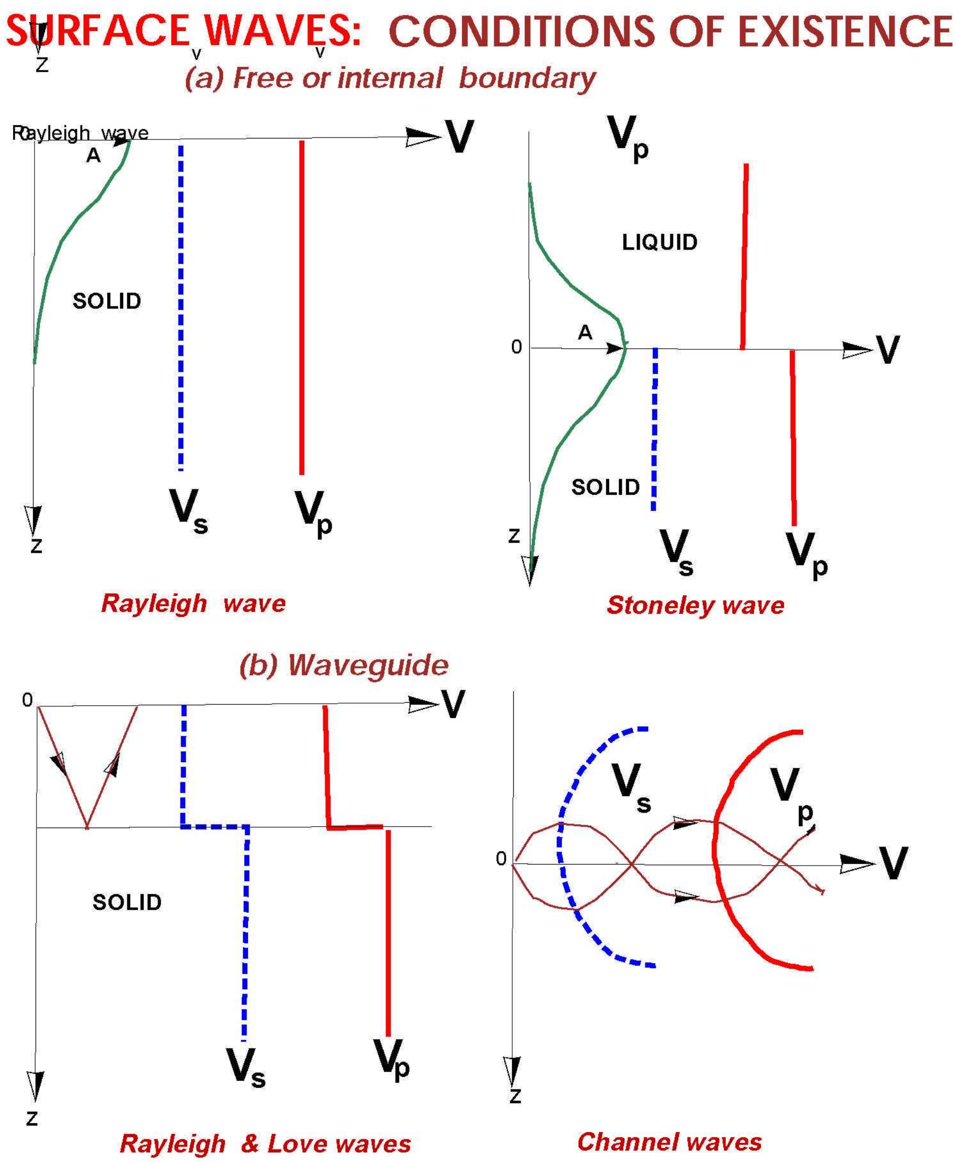


**Figure 1. When surface or channel waves may exist.**

as a combination of inhomogeneous P and S waves propagating along the boundary with the same velocity which is smaller than the lowest shear/longitudinal velocity in the medium. They are polarized in a plane of propagation and have an elliptical particle motion. Their energy exponentially decays with the distance $z$ from the boundary. The rate of decay is proportional to the ratio $z/\lambda$ where $\lambda = C/f$ is the wavelength. Only a single boundary wave of each type may exist (no higher modes!).

**(2) presence of the waveguide** (Figure 1b):

- Elastic layer laying on an elastic half-space with higher shear wave velocity - *Love waves*
- Liquid layer laying on an elastic half-space with the speed of sound less than the speed of longitudinal waves in solid
- Zone of lower shear velocities inside some homogeneous or inhomogeneous medium - *channel waves.*

These waves may be called the *interferential waves.* This type of waves exhibits dispersion, i.e. their phase velocity depends on frequency. They are presented by the infinite number of modes with different dispersion laws and energy-depth profiles. These profiles are characterized by the exponential decay with distance from the waveguide axis outside the waveguide itself. These waves may be elliptically polarized in the plane of propagation (combining P-SV motions) or linearly polarized in a direction perpendicular to this plane (SH motion). They are called, respectively, generalized Rayleigh and Love waves.

Both types of conditions exist in a real Earth (Figure 2). As a result we observe surface waves which are really a combination of boundary and interferential waves.

## Properties of surface waves

There are two types of surface waves propagating along the laterally homogeneous isotropic elastic half-space, namely Rayleigh (R) waves and Love (L) waves. Each of them is presented by the infinite number of modes: the fundamental mode with index $k = 0$ and overtones $k = 1, 2, ....$

Let us consider the propagation of the harmonic (sinusoidal) surface waves. For a given type of wave (R or L) and a given mode index $k$ the displacement carried out by the harmonic surface wave in the half-space $0 < r < +\infty$, $0 < \phi < 2\pi$, $0 < z < +\infty$ may be described by this way:

$$\mathbf{V}(r, \phi, z, t, \omega) = (V_z, V_r, V_\phi) = \mathbf{f}(z, \omega)\frac{\exp[i\omega(t - r/C(\omega)]}{\sqrt{r}}B(\omega, \phi, h)I(\omega) \quad (1)$$

Here $t$ is time, $\omega$ is a circular frequency, $h$ is a source depth (under the assumption of a point source), $C$ is a phase velocity, $B$ is a complex source/medium-dependent factor, $I$ is a medium-dependent real amplitude factor. The functions $\mathbf{f}, C, B, I$ are different for different wave types

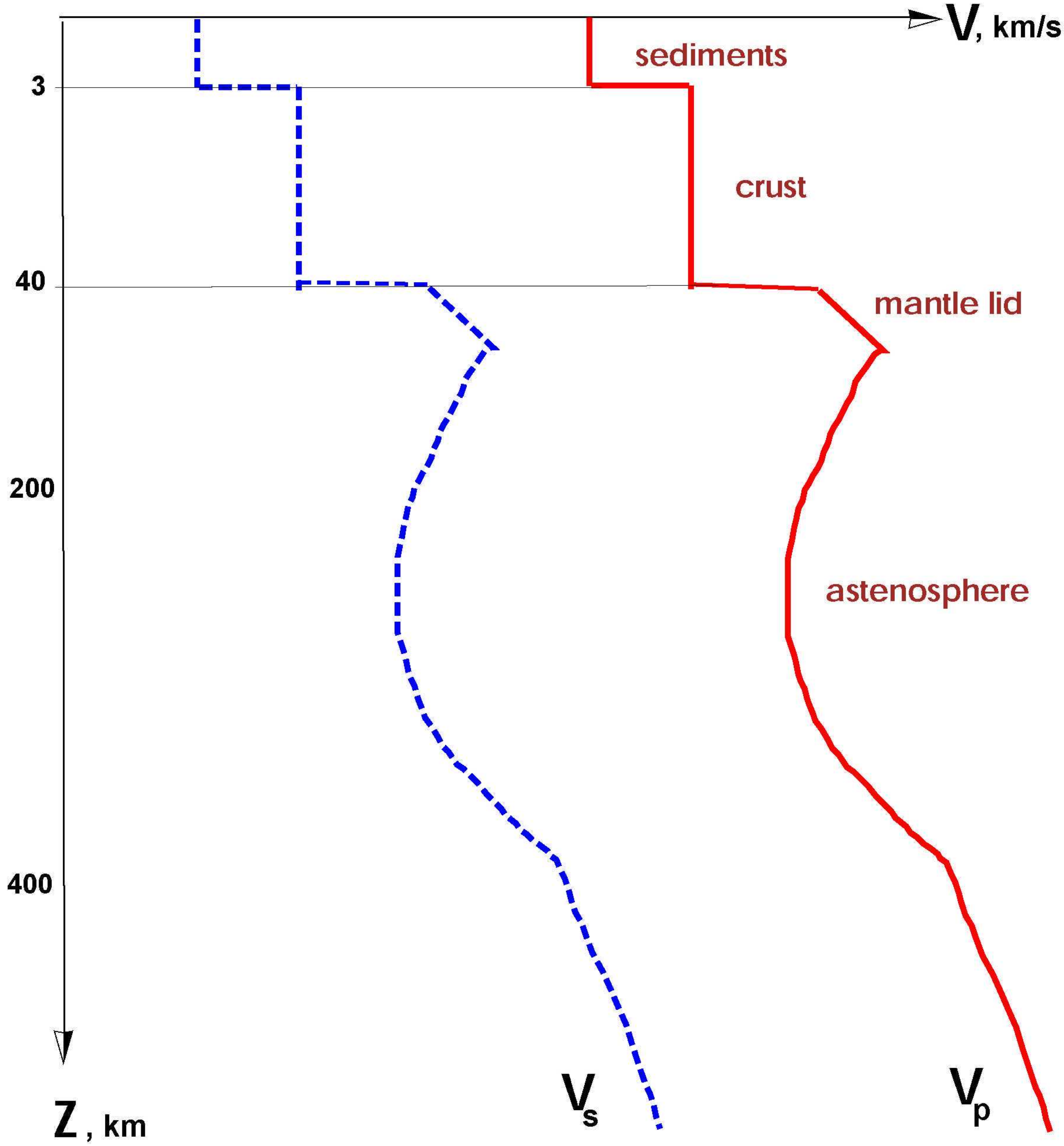


**Figure 2. Typical cross-section of the Earth's lithosphere**

and mode's orders.

For Rayleigh wave the complex vector $\mathbf{f}$ has two non-zero components:

$$\mathbf{f}(r, \phi, z, t, \omega) = (f_z, f_r, 0), \tag{2}$$

and

$$f_r = -i f_z \chi(z, \omega), \tag{3}$$

which indicates the elliptical particle motion in the plane of the wave propagation, and $\chi(z, \omega)$ is an ellipticity factor. For Love wave the vector $\mathbf{f}$ has only one non-zero component:

$$\mathbf{f}(r, \phi, z, t, \omega) = (0, 0, i f_\phi), \tag{4}$$

which indicates the linear (transverse) particle motion perpendicularly to the plane of the wave propagation. Dependence of phase velocity $C(w)$ on frequency determines dispersion characteristics of a given wave. The function

$$U(\omega) = \left[ \frac{d\left(\frac{\omega}{C(\omega)}\right)}{d\omega} \right]^{-1} = \frac{C(\omega)}{1 - \frac{\omega}{C(\omega)} \frac{dC(\omega)}{d\omega}} \tag{5}$$

is called the group velocity. Phase and group velocities $C(\omega)$, $U(\omega)$, an ellipticity factor $\chi(z, \omega)$, the factor $I(\omega)$ depend only on the structure parameters, namely the longitudinal and shear velocity profile $v_P(z)$, $v_S(z)$, and the density profile $\rho(z)$ (Rayleigh waves) or $v_S(z)$, $\rho(z)$ (Love waves). In general surface wave charateristics are most sensitive to the shear velocity profile $v_S(z)$.

From equation (1) it may be seen that each surface wave is a traveling wave along the radial coordinate $r$ and a standing wave along the vertical coordinate $z$. The phase velocity characterizes the speed of propagation of a harmonic (sinusoidal) surface wave along the free surface. The group velocity characterizes the propagation of a nonstationary surface wave package. The behavior of phase and group velocities in a laterally homogeneous continental lithospheric model is shown in Figure 3.

The behavior of corresponding components of the vector-function $\mathbf{f}(z)$ for several values of period $T = 2\pi/\omega$ is shown in Figure 4a.

When the period increases the depth of penetration of a surface wave increases. As the shear velocity in the medium usually increases with depth the phase velocity normally also increases with period. The behavior of group velocity dispersion curve is more complex, and such curve may have several maxima and minima. The sensitivity of phase and group velocities of the fundamental Rayleigh mode propagating across a continental platform to perturbations of shear velocities in the Earth is shown in Figure 4b. periods and

The synthetic seismograms of fundamental Rayleigh wave for the earthquake-type source at distances 1000, 2000 and 3000 km are shown in Figure 5.

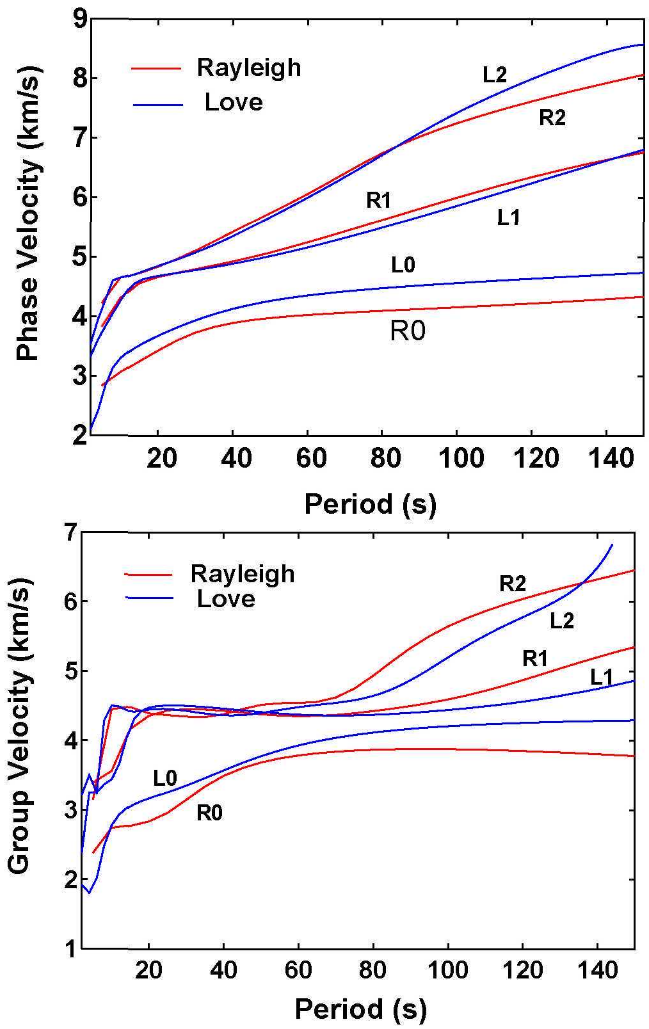


**Figure 3. Phase and group velocities of the three first modes of Rayleigh and Love waves at the continental model EUS**

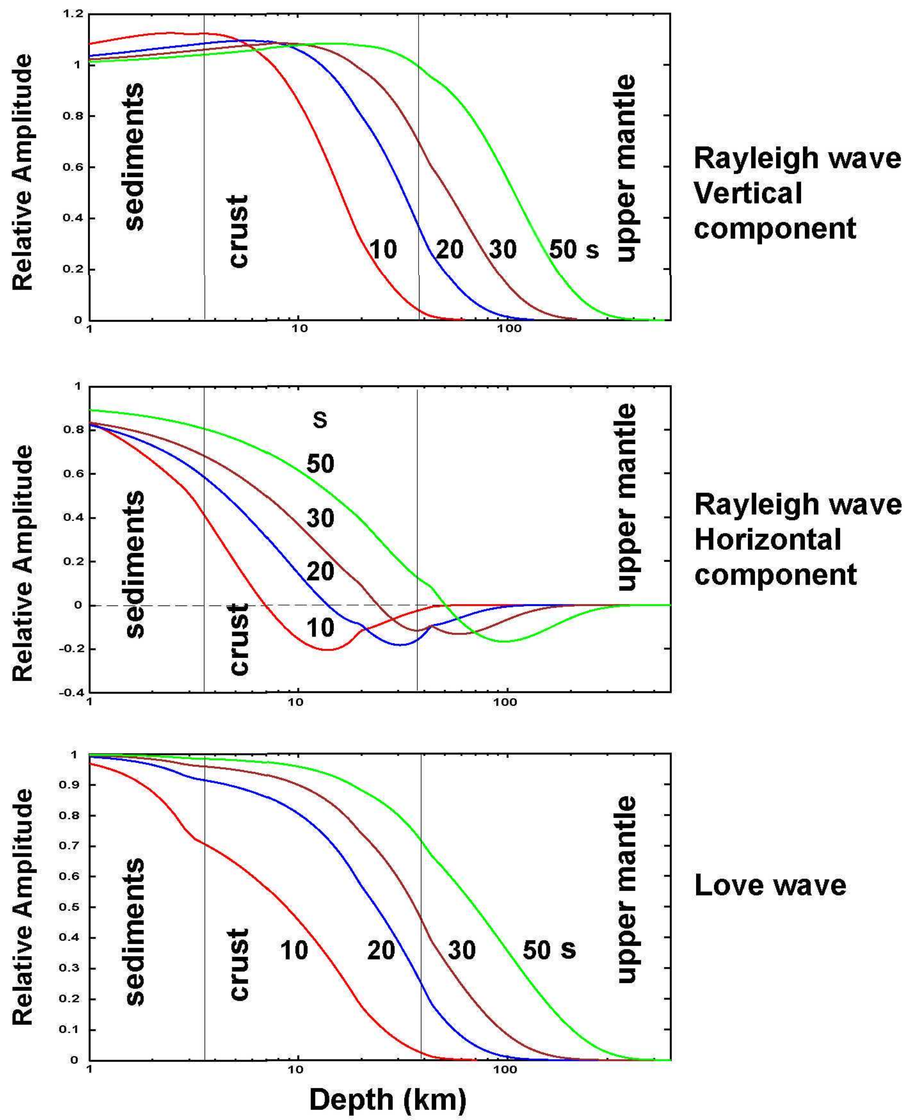


**Figure 4. Eigenfunctions of Rayleigh and Love fundamental modes in the continental model EUS for a set of periods**

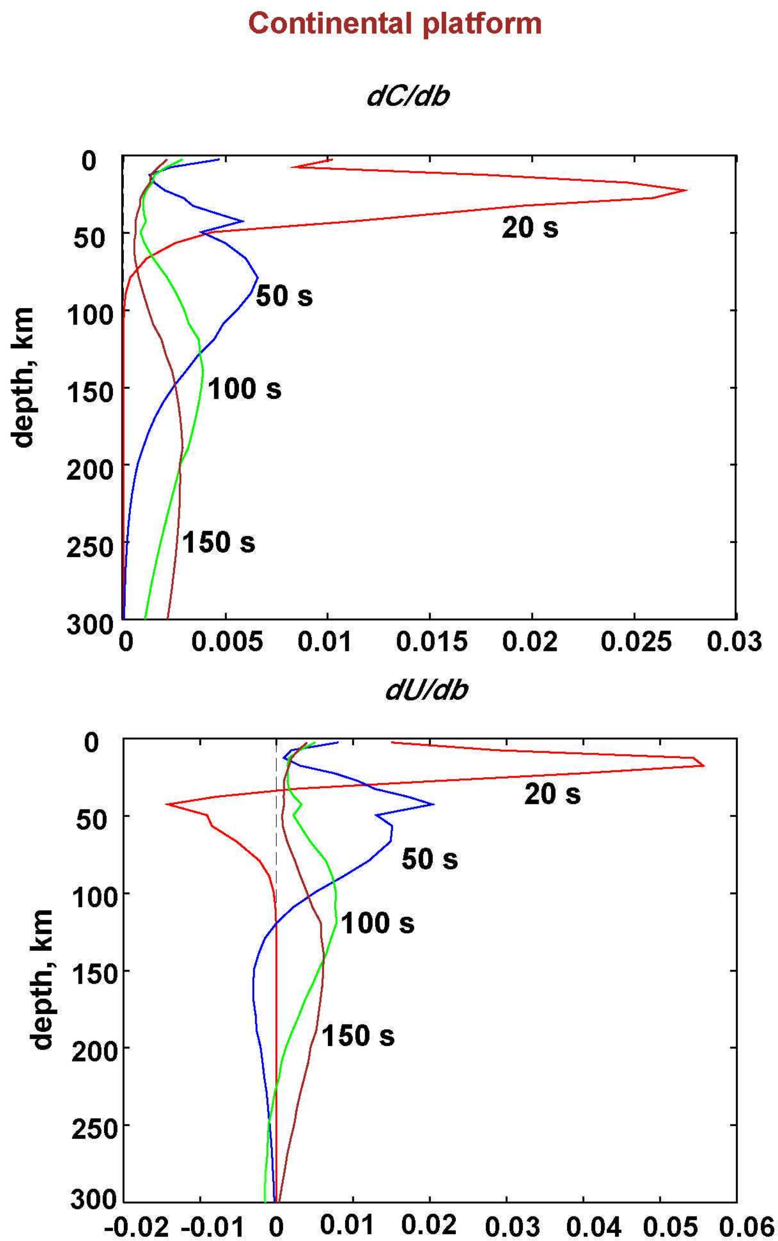


**Figure 4b. Partial derivatives of Rayleigh wave phase and group velocities by shear velocity**

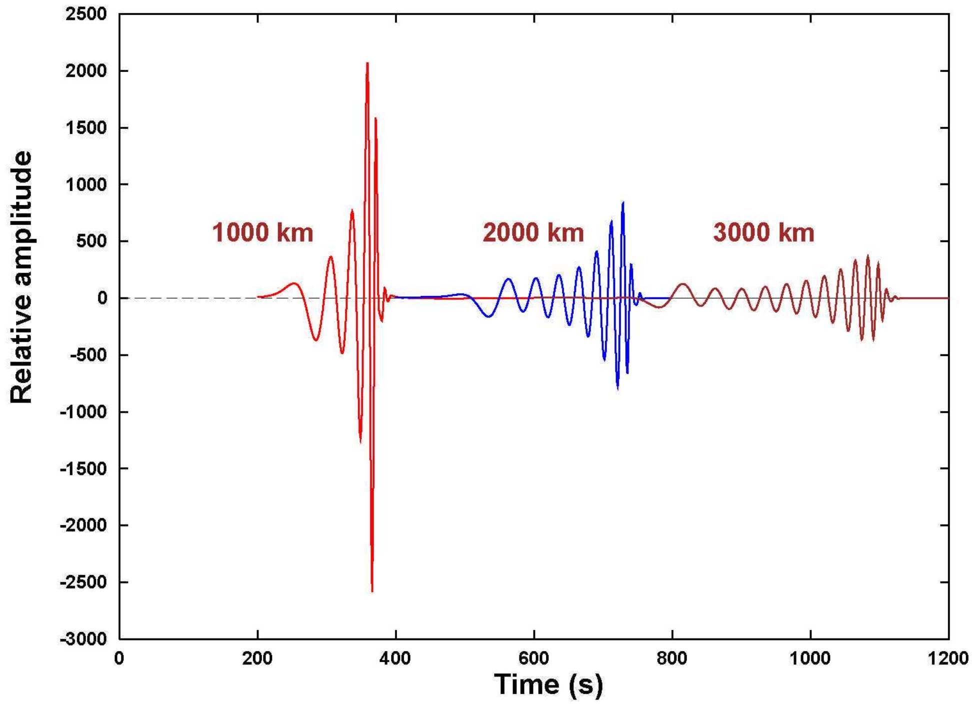


**Figure 5. Synthetic seismograms of the vertical component of the Rayleigh wave (model EUS)**

The complete expression for a spectral amplitude of a surface wave observed at the point $(r, \phi)$ of the surface $(z = 0)$ of a homogeneous medium and generated by a source at $(0, 0, H)$ may be presented in a following form:

$$V^q(\omega, r, \phi, h) = \frac{\exp(-i\pi/4)}{\sqrt{8\pi\omega}} \frac{\exp(-i\omega r/C(\omega))}{\sqrt{r}} \frac{\epsilon^q}{\sqrt{UI}} \frac{W(\omega, \phi, h)}{\sqrt{UIC}} \tag{6}$$

Here the component index $q$ is equal $r, \phi$ or $z$;
$I(\omega)$ is a kinetic energy integral;
$W(\omega, \phi, h)$ is a source dependent term.
For the source described by a moment tensor $m_{qs}(t)$
$W(\omega, \phi, h) = B_{qs}(\omega, \phi, h)\hat{m}_{qs}(\omega)$, and $B_{qs}$ is a strain tensor at the source.

For Rayleigh waves:
$\epsilon^r = -i\chi(\omega, 0)$, where $\chi(\omega, 0)$ is an ellipticity (aspect ratio) at the free surface,
$\epsilon^\phi = 0$, $\epsilon^z = 1$
For Love waves:
$\epsilon^r = 0$, $\epsilon^\phi = i$, $\epsilon^z = 0$
Remember that the functions $V_q, C, U, W, I, \chi$ are different for each mode and wave type. We see that this expression is a product of four factors. Each of them describes different effects.

- The first factor is a frequency dependent complex amplitude.
- The second factor expresses propagation effects: the phase delay (nominator) and the geometrical spreading (denominator). If a small anelasticity (attenuation) is present and described by intrinsic quality profiles $Q_P(\omega, z)$ and $Q_S(\omega, z)$, it may be taken into account by including into the second factor an additional term $\exp[-r/(\omega Q(\omega)U(\omega))]$. Here $Q$ is an apparent surface wave quality factor different for each mode order and wave type.
- The third factor depends on seismometer's orientation.
- The fourth one depends on the depth of source, the moment tensor, and the source-receiver geometry.

This factorization may seem to be artificial but it becomes more important when we will treat surface wave propagation in laterally inhomogeneous media.
As it was mentioned before there are several computational algorithms and corresponding computer codes which provide convenient means for calculation of all surface wave parameters like $C, U, I, B, W, Q, \chi$.
It should be remembered that the formula (6) is an asymptotic one and is accurate enough only in a far field when $\omega r/C(\omega) >> 1$, and $r >> h$.

# 2. Surface waves in laterally inhomogeneous media

When body wave velocities $v_P$, $v_S$, or density $\rho$ depend not only on vertical, but also on horizontal coordinates, the asymptotic theory considered above does not work, and exact analytical expressions for surface wave fields do not exist. Nevertheless, there are several approaches which permit to generalize the described formalism at least for several types of the Earth's models.

## 2.1 Surface waves in smoothly laterally inhomogeneous media

Let us consider now the medium in which both elastic parameters and density are weakly dependent on horizontal coordinates, i.e.

$v_P = v_P(\epsilon x, \epsilon y, z)$, $v_S = v_S(\epsilon x, \epsilon y, z)$, $\rho = \rho(\epsilon x, \epsilon y, z)$.

Here $\epsilon$ is a small parameter.

Let us also assume that discontinuities inside this medium and its free surface have only smooth undulations (Figure 6, top):

$Z_i = Z_i(\epsilon x, \epsilon y), i = 0, 1, 2, ..$; Here $Z_0$ corresponds to the free surface.

The approximate theory of surface wave propagation in such media was developed by Woodhouse, 1974; Babich and Chikhachev, 1975; Babich *et al.*, 1976. For further developments see Levshin, 1984; Yomogida, 1988; Levshin *et al.*, 1989; Tromp & Dahlen, 1992a,b; 1993a,b; 1998.

The main idea of this approximate theory is that energy of surface wave of a given frequency $\omega$ propagates along ray tubes which geometry is defined by the scalar field of phase velocity $C(\omega, x, y)$. This scalar field is defined by the following way: for each point (x,y) at the surface $Z_0(x, y)$ we have vertical profiles of velocities and densities. Then $C(\omega, x, y)$ is a phase velocity of this wave in a laterally homogeneous medium characterized by such profiles. Thus the phase velocity, as well as all other parameters of surface wave field mentioned above become local, i.e. depending on the position of the point (x,y). Ray tubes defined by the scalar field $C(\omega, x, y)$ are two-dimensional, and the third dimension of the tube depends on the depth of wave penetration, which is frequency-dependent. These assumptions lead to the following expression for the spectral amplitude $V_q(\omega, r, \phi, h)$ observed at the point M $(r, \phi, Z_0(r, \phi))$ on the free surface of the medium and generated at the point $M_o(0, 0, h - Z_o)$:

$$V^q(\omega, r, \phi, h)|_M = \frac{\exp(-i\pi/4)}{\sqrt{8\pi\omega}} \frac{\exp(-i\omega r/\int_l \frac{ds}{C(\omega,s)})}{\sqrt{J|_M}} \frac{\epsilon^q}{\sqrt{UI}}|_M \frac{W(\omega, \psi, h - Z_0)}{\sqrt{UIC}}|_{M_o} \tag{7}$$

Here $J$ is a geometrical spreading of the ray tube, angle $\psi$ is an azimuth of the ray leaving the source, and $l$ is the ray path . Now one can see the physical sense of each factor at (7): effects of propagation, seismometer's position and orientation, source position and orientation. To calculate spectral amplitudes using (7) it is necessary to define a scalar field $C(\omega, x, y)$ for a set of frequencies $\omega$, and trace the ray from $M_o$ to $M$ using 2-D tracing routines. The geometrical spreading $J$ should be also found as a result of ray tracing. Surface waves characteristics

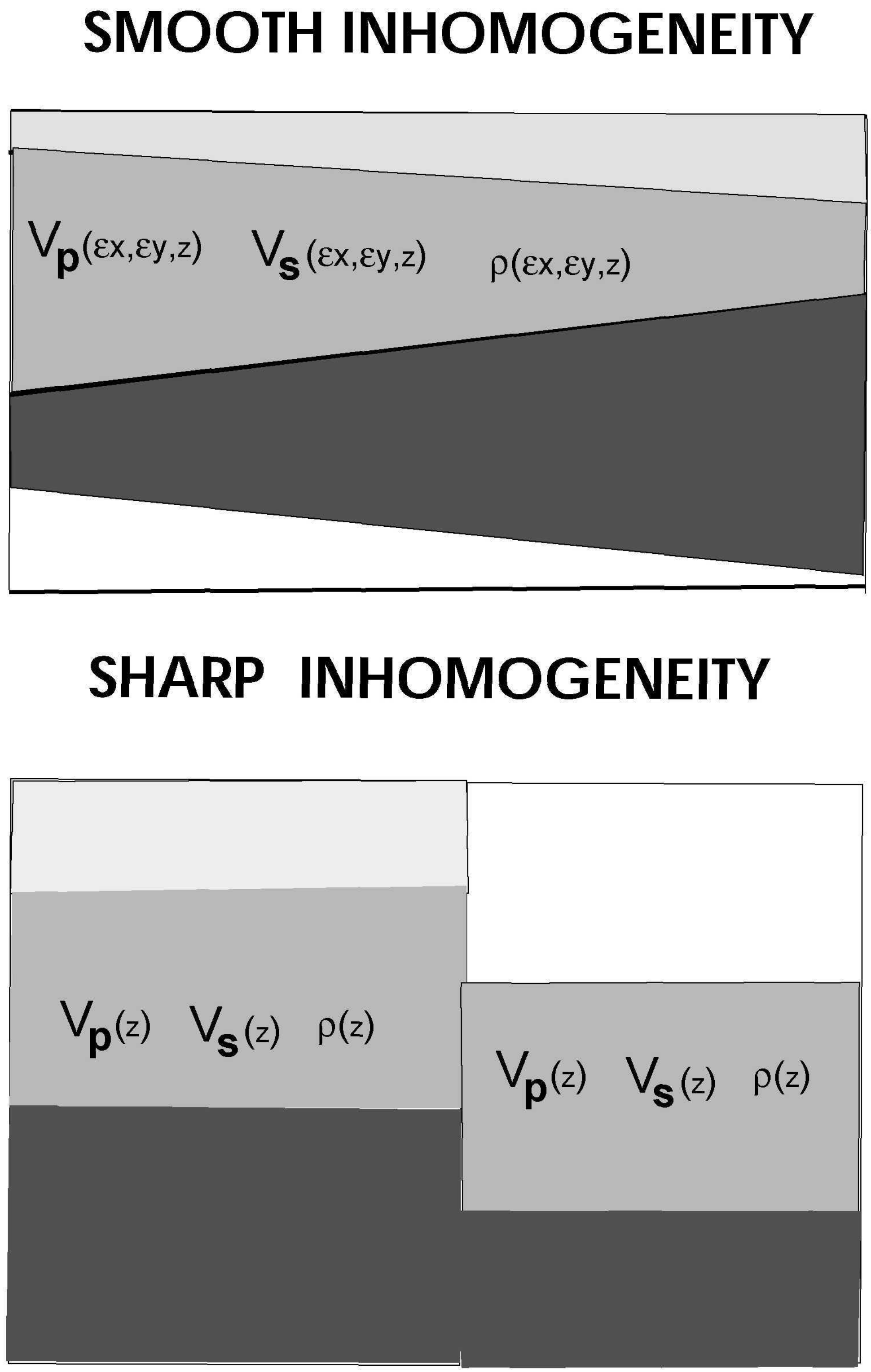
SMOOTH INHOMOGENEITY
Vp(εx,εy,z) Vs(εx,εy,z) ρ(εx,εy,z)
SHARP INHOMOGENEITY
Vp(z) Vs(z) ρ(z)
Vp(z) Vs(z) ρ(z)

**Figure 6**

$U, I, \chi$ near the receiver and $U, I, B$ near the source should be found by solving forward problems for the corresponding laterally homogeneous Earth's structure.

Let us remember that this theory has the same limitations of far field approximation as one described in Section 1, plus additional limitations prescribing slow changes of the medium in horizontal directions at the length of a wavelength.

$$\frac{|grad_{\perp}(v_P)|}{v_P} << \frac{\omega}{C(\omega)}, \frac{|grad_{\perp}(v_S)|}{v_S} << \frac{\omega}{C(\omega)}, \frac{|grad_{\perp}(\rho)|}{\rho} << \frac{\omega}{C(\omega)}, \tag{8}$$

$$\frac{|grad_{\perp}(Z_i)|}{Z_i} << \frac{\omega}{C(\omega)}. \tag{9}$$

Here $grad_{\perp}$ is a horizontal projection of $grad$.
Further extension of this methodology is based on the Gaussian beam approximation (Yomogida & Aki, 1985; Lokshtanov, 1990; Levshin *et al.*, 1994). By using Gaussian beam approximation it is possible to avoid some problems of the ray theory approximation related to focusing, defocusing and shadow zones.

### 2.2 Surface waves in laterally inhomogeneous media with sharp vertical boundaries

Now we consider other type of a laterally inhomogeneous medium for which some approximate techniques for surface wave field calculations are known. Let us assume that two laterally homogeneous elastic quarter-spaces with different vertical velocity and density profiles are in the welded contact (Figure 6, bottom). Suppose that the surface waves propagates at one quarter-space in the direction to the vertical boundary. When this wave reaches the boundary it will partly propagate across the boundary, partly will be reflected back and scattered. There are several approximate techniques describing these phenomena and allowing us to calculate coefficients of reflection/refraction/conversion for surface waves (McGarr & Alsop, 1967; Gregersen, 1978, Its & Yanovskaya, 1985, Bukchin & Levshin, 1980; Levshin *et al.* 1989; Malischewsky, 1987; Kennet, 1984a, 1984b, 1984c; Maupin, 1988; Maupin & Kennet, 1987). Most of these techniques somehow neglect the effect of diffraction which converts surface wave energy into body waves. It means that the wave field near the vertical boundary may be poorly approximated by these techniques.

Using two techniques described above in a combination we can in principle construct synthetic surface wave fields in media consisting of big blocks with smooth horizontal variations inside each block, and vertical contacts between blocks (Figure 7):

# COMBINED MODEL

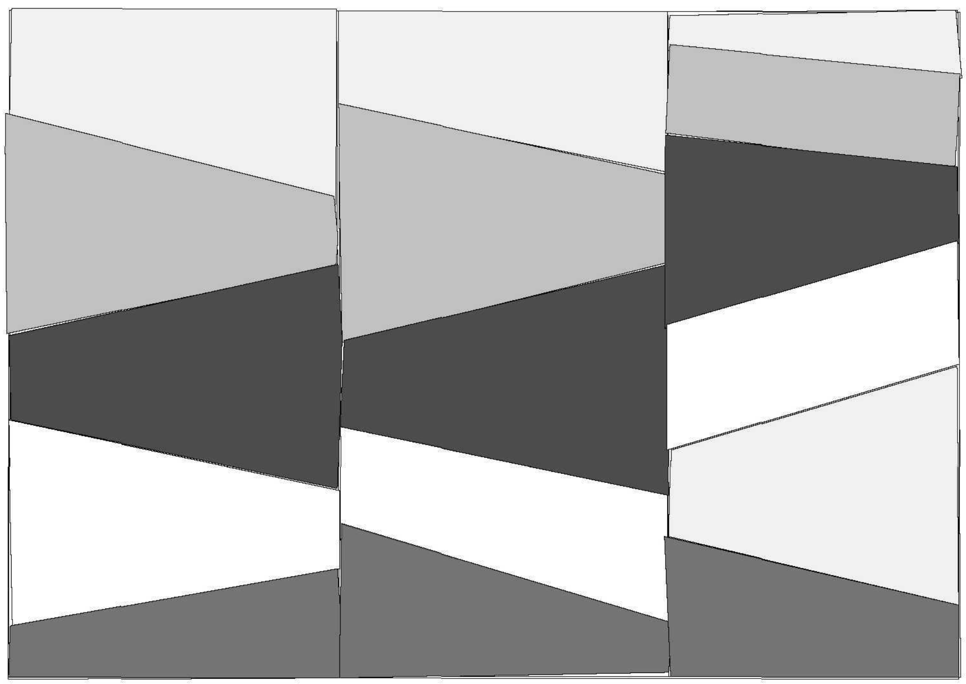

**Figure 7**

$$V^q(\omega, r, \phi, h)|_M = \frac{\exp(-i\pi/4)}{\sqrt{8\pi\omega}} \frac{\exp\left\{-i\omega\left[\sum_{j=1}^{N-1}\int\limits_{l_j}\frac{ds}{C(\omega, s)}\right]\right\}}{\sqrt{J|_M}} \frac{\epsilon^q}{\sqrt{UI}}|_M$$

$$\times \prod_{j=1}^{N-1}\left[,_{j,j+1}\sqrt{\frac{\cos\theta_j^-[UC]_{Oj}^-}{\cos\theta_j^+[UC]_{Oj}^+}}\right] \frac{W(\omega, \psi, h - Z_0)}{\sqrt{(UIC)|_{M_o}}} \tag{10}$$

Here $N$ is a number of blocks ($j = 1, 2, ..., N$) divided by vertical boundaries, $,_{j,j+1}$ is a transmission coefficient at the boundary between the $j$-th and $(j+1)$-th blocks, angles $\theta_j^+$, $\theta_j^-$ are angles of incidence and refraction of the ray at the point $O_j$ belonging to this boundary.

There are several other techniques for modeling surface wave fields in laterally heterogeneous media: (1) Born approximation (Snieder, 1986; Friederich, 1998); this approach is quite appropriate for treating smooth lateral heterogeneities but fails in presence of sharp boundaries; (2) purely numerical (finite element or finite difference) 2-D and 3-D schemes (Pedersen *et al.*, 1994; McLaughlin & Shkoller, 1996).
These purely numerical approaches need powerful computational facilities, and probably will become dominant in the coming years. They are out of the scope of this lecture.

## 3. Effects of lateral inhomogeneities on surface wave propagation

Let us briefly discuss possible effects of lateral inhomogeneities on surface waves.

### 3.1 Lateral refraction

Presence of horizontal gradient of phase velocity $C(\omega, x, y)$ produces lateral refraction of surface wave rays. In result the wave path deviates from the great circle. Figure 8 demonstrates the path-wander of rays propagating across a laterally inhomogeneous medium. Figure 8a and 8b show minimum travel-time rays (solid lines) computed through the model of CUB2 (Shapiro & Ritzwoller, 2002) for the 20 s and 50 s Rayleigh wave compared with the great-circle (dashed lines) from source to receiver. Rays emanate from a source in Turkey and travel 70°. Maximum path-wander in kilometers is indicated outside the globe for each path. Figure 8c presents root-mean-square values of path-wander (in km) as a function of period, segregated by path-length, for the data set used in global tomography.

This effect may be found from studying the angle of approach of the wave front to the array of seismometers (Capon, 1970; Levshin & Berteussen, 1979; Cotte ét al, 1999) or from studying a wave polarization pattern using a single 3-component station (see, e.g., Levshin *et al.*, 1989; Levshin *et al.*, 1994; Laske, 1995, among others). These deviations are quite significant in some cases even for relatively long periods (up to $15^o$ and more at the 50 s period!). Measurements of the angle of approach at continents using a single station observations are sometime difficult, especially in the range of periods between 25-50 s. In this range group velocities of Love and

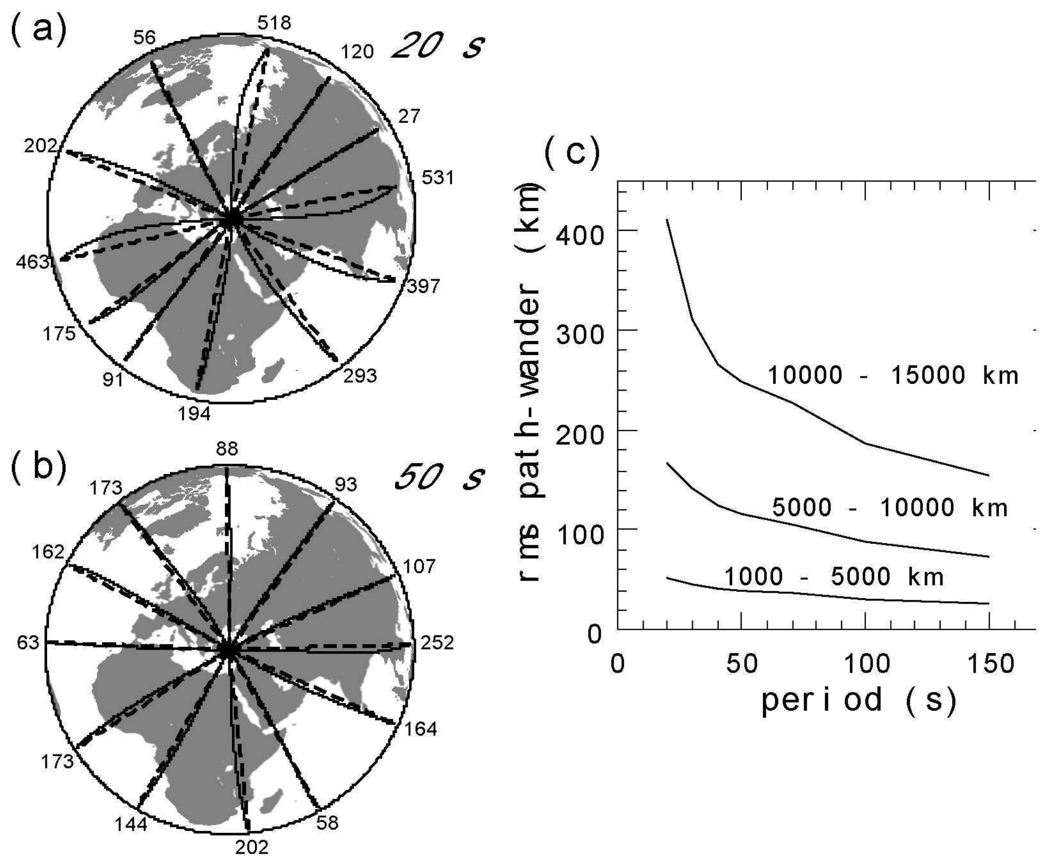


**Figure 8. Path-wander of geometrical rays. (a, b) Minimum travel-times rays through 3D model (solid lines) for Rayleigh waves of different period compared with the great-circle (dashed lines) from source to receiver. (c) Rms of path-wander as a function of period, segregated by path length.**

Rayleigh waves are very close or even cross each other, causing the strong interference of these waves at horizontal components. The reliable measurements of the ray deviation angle may be used together with travel time measurements in surface wave global and regional tomographic studies (Laske & Masters, 1996; Yanovskaya, 1996).

Other effects include the focusing and defocusing of surface waves. In result the significant amplitude anomalies may be observed which may distort determination of surface wave magnitude, seismic moment tensor and attenuation.

### 3.2 Multipathing

Due to the lateral heterogeneities surface waves may arrive to the receiver by different paths and at different time intervals. These phenomena which are quite typical for tectonic regions often complicate interpretation of wave fields especially in short period range (5-20 s). Sedimentary basins with a big thickness of sediments even if they are outside the great circle path often capture a significant part of the wave energy and tunnel it with a slow speed along the basin (Levsin & Ritzwoller, 1995).

The later surface wave arrivals may also be surface wave reflections from strong vertical inhomogeneities like the ocean-continent boundary, sharp boundaries of continental grabens and intra-continental rifts.

### 3.3 Coda

One of the most striking phenomena related to the surface wave propagation is the surface wave coda, i.e. the long train of high amplitude seismic waves trailing the direct arrivals. This train is usually too complicated to be split into separate arrivals which can be interpreted in a deterministic way. Such coda is a result of multiple scattering (reflection/refraction/conversion) of surface waves at numerous crustal and upper mantle heterogeneities met by waves on their path from the source to receivers. Professor Mitchell is presenting here his well developed methodology of studying and interpreting coda waves.

### 3.4 Measurements near antipode of the epicenter

At the epicentral distances $170^\circ - 190^\circ$ measurements become unreliable due to focusing of surface waves and interference of waves propagating along minor and major arcs.

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