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Unified approach to dynamical control of decoherence

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Qubit decoherence

Decoherence and relaxation of a quantum system result from coupling to the environment (reservoir, bath), described by

$$H = H_S + H_R + V$$

For 2-level system (qubit) $H_S = \hbar\omega_a |e\rangle\langle e|,$

H_R – reservoir, $V = V_r + V_p$ – system-reservoir interaction,

$$V_r = R_r|e\rangle\langle g| + R_r^\dagger|g\rangle\langle e|, \;\; V_p = R_p|e\rangle\langle e|.$$

$|e\rangle, \; |g\rangle$ excited and ground states; $R_r, \; R_p$: reservoir operators;

V_r: off-diagonal coupling, causing population relaxation (PR) of $\rho_{ee}, \; \rho_{gg}$ (decay, excitation) and (life-time) relaxation of the coherence ρ_{eg} (decoherence);

V_p: diagonal coupling (proper dephasing – PD), causing decoherence.

Dynamical control of decay and decoherence

A. G. Kofman and G. Kurizki, Nature **405**, 546 (2000); PRL **87**, 270405 (2001); IEEE Trans. Nanotechnology (in press).

Our purpose is to **control** (suppress or, sometimes, enhance) decoherence and relaxation by external perturbations. Typically, population relaxation and proper dephasing are due to different reservoirs and their rates are significantly different.
Then it is sufficient to control only the fastest of them.

Consider **off-diagonal coupling to zero-temperature reservoir in the rotating-wave approximation (RWA):**

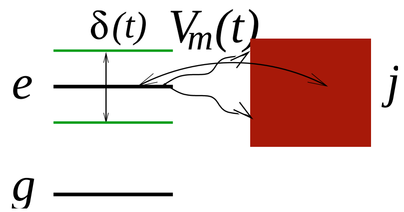
$$V = \hbar \sum_j V_{je} |g, j\rangle\langle e, \text{vac}| + \text{h.c.}.$$

Reservoir consists of oscillators (modes) or two-level systems (spins),
$|\text{vac}\rangle$ is its ground state,
$|j\rangle$ is an excited mode of the reservoir with energy $\hbar\omega_j$.

External perturbation – 2 types

Kofman and Kurizki, PRL **87**, 270405 (2001);

Barone, Kurizki and Kofman, PRL **92**, 200403 (2004).



$$H(t) = H_0 + V_m(t) + H_1(t),$$

$$H_0 = H_S + H_R \equiv \hbar\omega_a|e\rangle\langle e| + \hbar\sum_j \omega_j|j\rangle\langle j|,$$

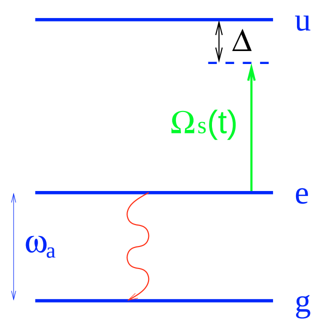
$$V_m(t) = \hbar\tilde{\epsilon}(t)\sum_j V_{je}|g,j\rangle\langle e,\text{vac}| + \text{h.c.}$$ – modulated coupling,

$$\hat{H}_1(t) = \hbar\delta_a(t)|e\rangle\langle e| + \hbar\delta_f(t)\sum_j |j\rangle\langle j|$$ – time-dependent energy shifts.

One can modulate:

(a) the coupling amplitude/phase, as, e.g., in photoionization;

(b) the energies/phases of the levels, e.g., by Stark shifts with an off-resonant field. 2π-pulses at adjacent transition change the level phase by π (Agarwal, Scully, Walter, 2001).



Wave-function approach

For off-diagonal coupling to $T = 0$ reservoir the qubit and reservoir are described by the wave function. In the interaction representation it is given by

$$|\Psi(t)\rangle = \alpha(t)e^{-i\omega_a t - i\int_0^t \delta_a(t')dt'}|e, \text{vac}\rangle + \sum_j \beta_j(t)e^{-i\omega_j t - i\int_0^t \delta_f(t')dt'}|g, j\rangle,$$

the initial condition being $|\Psi(0)\rangle = |e\rangle$. From the Schroedinger equation

$$i\hbar\frac{\partial}{\partial t}|\Psi\rangle = H(t)|\Psi\rangle$$

we obtain the equations

$$\dot{\alpha} = \sum_j \epsilon^*(t)V_{ej}e^{i(\omega_a-\omega_j)t}\beta_j, \quad \dot{\beta}_j = \epsilon(t)V_{je}e^{-i(\omega_a-\omega_j)t}\alpha,$$
$$\text{where } \epsilon(t) = \tilde{\epsilon}(t)\exp\left[-i\int_0^t[\delta_a(t_1) - \delta_f(t_1)]dt_1\right].$$

Both above types of modulation are accounted for by a **single** function $\epsilon(t)$.

External perturbations can provide phase, amplitude or amplitude-phase modulation.

$$\dot{\alpha} = \sum_j \epsilon^*(t) V_{ej} e^{i(\omega_a - \omega_j)t} \beta_j, \quad \dot{\beta}_j = \epsilon(t) V_{je} e^{-i(\omega_a - \omega_j)t} \alpha,$$

where $\epsilon(t) = \tilde{\epsilon}(t) \exp\left[-i \int_0^t [\delta_a(t_1) - \delta_f(t_1)] dt_1\right]$.

Integrating equation for β_j, we get $\beta_j(t) = V_{je} \int_0^t dt' \epsilon(t') e^{-i(\omega_a - \omega_j)t'} \alpha(t').$

Inserting this into the equation for α, yields the **integro-differential equation**

$$\dot{\alpha} = -\int_0^t dt' e^{i\omega_a(t-t')} \epsilon^*(t) \epsilon(t') \Phi(t-t') \alpha(t'),$$

where $\Phi(t-t') = \sum_j |V_{ej}|^2 e^{-i\omega_j(t-t')}$ is the **reservoir memory** function.

If the reservoir is sufficiently broad and smooth, $\Phi(t)$ decays **faster** than $\alpha(t) \Longrightarrow \alpha(t') \approx \alpha(t)$. This simplification yields the solution

$$\alpha(t) = \exp\left[-\int_0^t dt' \epsilon^*(t') \int_0^{t'} dt'' \epsilon(t'') \Phi(t' - t'') e^{i\omega_a (t' - t'')}\right]. \qquad (1)$$

Expression (1) can be made more transparent by introducing the Fourier transform in the "window" $(0, t)$,

$$\epsilon_t(\omega) = \frac{1}{\sqrt{2\pi}} \int_0^t \epsilon(t_1) e^{i\omega t_1} dt_1. \qquad (2)$$

From the definition of $\Phi(t)$ we obtain that $\int_0^\infty dt \Phi(t) e^{i\omega t} = \pi G(\omega) - i\chi(\omega)$ (3)

Here $G(\omega)$ is the coupling spectrum, a weighted density of modes of the reservoir,

$$G(\omega) = \frac{1}{\hbar^2} \sum_j |\mu_{ej}|^2 \delta(\omega - \omega_j),$$

$\chi(\omega)$ is the generalized susceptibility, satisfying the Kramers-Kronig relation

$\chi(\omega) = \mathcal{P} \int \frac{G(\omega') d\omega'}{\omega' - \omega}$, where $\mathcal{P}$ is the principal value of the integral.

Universal formulas

Using Eq. (3) and the definition (2), one can transform (1) into

$$\alpha(t) = e^{-[R(t)/2+i\Delta_a(t)]Q(t)},$$

where $Q(t) = \int_0^t d\tau |\epsilon(\tau)|^2$ is the fluence. The density-matrix elements are

$$\rho_{ee}(t) = \rho_{ee}(0)|\alpha(t)|^2 = \rho_{ee}(0)e^{-R(t)Q(t)},$$

$$\rho_{gg}(t) = 1 - \rho_{ee}(t), \quad \rho_{eg}(t) = \rho_{eg}(0)\alpha(t).$$

$R(t)$ and $\Delta_a(t)$ are the average decay rate and shift of level $|e\rangle$, respectively, in the interval $(0, t)$. They are **modified by external perturbation.**

The modified decay rate obeys the **universal formula:**

$$R(t) = 2\pi \int_{-\infty}^{\infty} d\omega G(\omega + \omega_{\mathrm{a}}) F_t(\omega), \tag{4}$$

overlap of reservoir coupling spectrum $G(\omega)$ and the normalized spectral intensity of modulation

$$F_t(\omega) = \frac{|\epsilon_t(\omega)|^2}{Q(t)}; \quad \int_{-\infty}^{\infty} d\omega F_t(\omega) = 1.$$

This overlap determines suppressed or enhanced coupling to environment.

The same formula determines quantum Zeno and anti-Zeno effects

(Kofman and Kurizki, Nature, 2000).

The modulation-modified level shift is given by a similar spectral overlap,

$$\Delta_a(t) = -\int_{-\infty}^{\infty} d\omega \chi(\omega + \omega_a) F_t(\omega). \tag{5}$$

If modulation is sufficiently slow, $F_t(\omega)$ is narrow and one can set $F_t(\omega) \approx \delta(\omega)$ in Eqs. (4) and (5), yielding unmodified results

$$R = 2\pi G(\omega_a) \text{ – Golden Rule,} \quad \Delta_a = -\chi(\omega_a).$$

Quasiperiodic modulation

Coherent amplitude and phase modulation (APM): $\epsilon(t) = \sum_k \epsilon_k e^{-i\omega_k t}$.

Here ω_k ($k = 0, \pm 1, \ldots$) are arbitrary discrete frequencies with the minimum spectral distance Ω. Now we obtain

$$Q(t) = \epsilon_c^2 t + \epsilon_c^2 \sum_{k \neq l} \lambda_k \lambda_l^* \frac{e^{i(\omega_l - \omega_k)t} - 1}{i(\omega_l - \omega_k)}, \tag{6}$$

$$|\epsilon_t(\omega)|^2 = \epsilon_c^2 t \sum_k |\lambda_k|^2 S(\eta_k, t) + \epsilon_c^2 \sum_{k \neq l} \lambda_k \lambda_l^* \frac{1 + e^{i(\eta_k - \eta_l)t} - e^{i\eta_k t} - e^{-i\eta_l t}}{2\pi \eta_k \eta_l}. \tag{7}$$

Here $\epsilon_c^2 = \sum_k |\epsilon_k|^2$ equals the average of $|\epsilon(t)|^2$ over a period of the order of $1/\Omega$, $\lambda_k = \epsilon_k/\epsilon_c$, $\eta_k = \omega - \omega_k$, and $S(\eta_k, t)$ is a bell-like function of η_k normalized to 1,

$$S(\eta_k, t) = \frac{2\sin^2(\eta_k t/2)}{\pi t \eta_k^2}.$$

For $t \gg \Omega^{-1}$ the first term in the expression for $|\epsilon_t(\omega)|^2$ is a sum of peaks, whose spacings are much greater than their width $2/t$.

The fast oscillating second term is also peaked at $\omega = \omega_k$, but we then find that the ratio of the first to the second terms, and that of their counterparts in (6), is $\sim (\Omega t)^{-1} \ll 1$. In the long-time limit, we then neglect these fast oscillating terms and obtain the decay probability

$$P(t) \equiv |\alpha(t)|^2 = \exp[-R(t)\epsilon_c^2 t], \quad (8)$$

whereas the universal formula for $R(t)$ now involves

$$F_t(\omega) \approx \sum_k |\lambda_k|^2 S(\eta_k, t).$$

For a sufficiently long time, the function $S(\eta_k, t)$ becomes narrower than the respective characteristic width $\xi(\omega_a + \omega_k)$ of $G(\omega)$ around $\omega_a + \omega_k$, and one can set

$$S(\eta_k, t) \approx \delta(\eta_k) \;\; (t \gg 1/\xi(\omega_a + \omega_k)).$$

Thus, when

$$t \gg \Omega^{-1}, \quad t \gg t_c \equiv \max_k \{1/\xi(\omega_a + \omega_k)\},$$

where t_c is the effective *correlation (memory) time* of the reservoir, the formula for $R(t)$ is reduced to

$$R = 2\pi \sum_k |\lambda_k|^2 G(\omega_a + \omega_k). \tag{9}$$

For the validity of (9) it is also necessary that

$$\epsilon_c^2 R t_c \ll 1. \tag{10}$$

This condition is well satisfied in the regime of interest, i.e., weak coupling to essentially any reservoir, unless (for some k) $\omega_a + \omega_k$ is extremely close to a sharp feature in $G(\omega)$, e.g., a band edge (Kofman, Kurizki and Sherman, JMO, 1994). Since R and t_c depend on the modulation, this criterion may be achieved using a suitable modulation, even if in the absence of modulation the coupling is strong(!).

Hence, the long-time limit of the general decay rate under the APM is a sum of the Golden-Rule rates, corresponding to the resonant frequencies shifted by ω_k, with the weights $|\lambda_k|^2$. Formula (9) provides a *simple general recipe* for manipulating the decay rate by APM.

Impulsive phase modulation

Let the phase of the coupling amplitude jump by an amount ϕ at times $\tau, 2\tau, \ldots$. Such modulation can be achieved by a train of identical, equidistant, narrow pulses of nonresonant radiation, which produce pulsed frequency shifts $\delta_{af}(t)$. Now

$$\epsilon(t) = e^{i[t/\tau]\phi},$$

where $[\ldots]$ is the integer part. One then obtains that

$$Q(t) = t, \;\; \epsilon_c = 1, \;\; F_{n\tau}(\omega) = \frac{2\sin^2(\omega\tau/2)\sin^2[n(\phi+\omega\tau)/2]}{\pi n\tau\omega^2\sin^2[(\phi+\omega\tau)/2]}.$$

The decay, according to Eq. (8), has then the form (at $t = n\tau$)

$$P(n\tau) = \exp[-R(n\tau)n\tau],$$

where $R(n\tau)$ is defined by the universal formula with the above $F_{n\tau}(\omega)$.

For sufficiently long times one can use Eq. (9). The poles and residues of

$$\hat{\epsilon}(s) = \frac{1-e^{-s\tau}}{s(1-e^{i\phi-s\tau})} \implies \omega_k = \frac{2k\pi}{\tau} - \frac{\phi}{\tau}, \;\; |\lambda_k|^2 = \frac{4\sin^2(\phi/2)}{(2k\pi-\phi)^2}$$

For *small phase shifts*, $\phi \ll 1$, the $k = 0$ peak dominates,

$$|\lambda_0|^2 \approx 1 - \frac{\phi^2}{12},$$

whereas

$$|\lambda_k|^2 \approx \frac{\phi^2}{4\pi^2 k^2} \ \ (k \neq 0).$$

In this case one can retain only the $k = 0$ term in Eq. (9) [unless $G(\omega)$ is changing very fast]. Then the modulation acts as a constant shift

$$\Delta \equiv \omega_0 = -\phi/\tau.$$

With the increase of $|\phi|$, the difference between the $k = 0$ and $k = 1$ peak heights diminishes *vanishing* for $\phi = \pm\pi$.

Thus for $\phi = \pi$

$$|\lambda_0|^2 = |\lambda_1|^2 = 4/\pi^2, \quad \omega_1 = -\omega_0 = \pi/\tau,$$

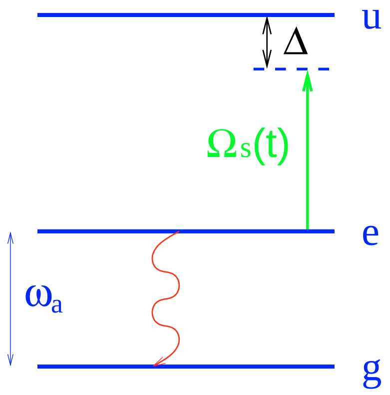
i.e., $F_t(\omega)$ for $\phi = \pm\pi$ contains *two identical peaks symmetrically shifted in opposite directions* [the other peaks $|\lambda_k|^2$ decrease with k as $(2k-1)^{-2}$, totaling 0.19].

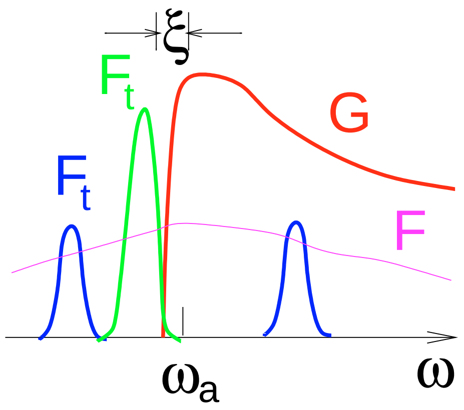
The above features allow one to adjust the modulation parameters for a given scenario to obtain an optimal decrease or increase of R.

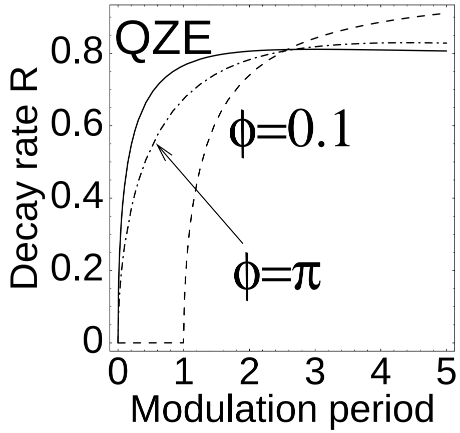
AC Stark modulation: $\delta(t) \simeq \Omega_s^2(t)/\Delta$.

Periodic PM ($\phi \ll 1$) – most effective near band edge.

On the other hand, if ω_a is near a *symmetric* peak of $G(\omega)$, periodic PM with $\phi = \pi$ (Agarwal, Scully, Walther, 2001) more effectively reduce R, since the main peaks of $F_t(\omega)$ at ω_0 and ω_1 then shift stronger with τ^{-1} than the peak at $\omega_0 = -\phi/\tau$ for $\phi \ll 1$.







Amplitude modulation can be considered similarly: Fischer, Gutierrez-Medina and Raizen, PRL (2001) – experiment; Kofman and Kurizki, PRL (2001) – theory.

Master equations for the density matrix

This approach is applicable for the most general case: diagonal and off-diagonal coupling, thermal reservoirs (baths), and violation of the RWA.

The Hamiltonian in question is the sum of the system (S), reservoir bath (B) and system-bath interaction (I) terms,

$$H = H_S(t) + H_B + H_I(t).$$

Here $H_S(t)$ is the driven (and modulated) system Hamiltonian.

The combined state of the system and the bath is described by the density operator $\rho_{S+B}(t)$, the density operator of the system

$$\rho = \mathrm{Tr}_B \rho_{S+B}, \quad \rho_{S+B}(0) = \rho(0) \otimes \rho_B, \quad \rho_B = Z^{-1} \exp[-(\beta/\hbar) H_B]$$

the density operator of the bath in equilibrium, with Z as the normalization factor, $\beta = \hbar/k_B T$ the inverse temperature (in frequency units), and k_B the Boltzmann constant.

The Liouville equation $i\hbar\dot{\rho}_{S+B} = [H, \rho_{S+B}]$
is convenient to recast in terms of superoperators $\mathcal{L}_i$,

$$\dot{\rho}_{S+B} = [\mathcal{L}_S(t) + \mathcal{L}_B + \mathcal{L}_I(t)]\rho_{S+B}, \quad \mathcal{L}_i\rho_{S+B} = -(i/\hbar)[H_i, \rho_{S+B}].$$

We define the projection (P) and averaging (Q) operators by

$$P \ldots = \rho_B \otimes \text{Tr}_B \ldots, \qquad Q \ldots = \text{Tr}_B(\ldots \rho_B) \equiv \langle \ldots \rangle.$$

Then one can use the well known technique (Zwanzig, 1964; Argyres and Kelley, 1964; Agarwal, 1974) to obtain a non-Markovian master equation (NME), involving terms up to 2nd order in H_I (the Born approximation). We shall use the differential NME (DME),

$$\dot{\rho} = \left[\mathcal{L}_S(t) + Q\mathcal{L}_I + \int_0^t d\tau K(t, t-\tau)U_S^{-1}(t, t-\tau)\right]\rho.$$

Here $K(t, t')$ is the second cumulant of the coupling operator,

$$K(t,t') = Q[\mathcal{L}_I(t)U_S(t,t') \otimes U_B(t-t')\mathcal{L}_I(t')] - Q\mathcal{L}_I(t)U_S(t,t')Q\mathcal{L}_I(t')$$
$$U_B(t) = \exp\left[-\tfrac{i}{\hbar}H_B t\right], \;\; U_S(t,t') = T_+ \exp\left[-\tfrac{i}{\hbar}\int_{t'}^t H_S(\tau)d\tau\right],$$

T_+ being the time-ordering operator.

$$H_I(t) = \mathcal{S}(t)B(t),$$

where $\mathcal{S}(t)$ is a system operator and $B(t)$ is a bath operator, whose choice depends on the system-bath coupling (linear or quadratic, diagonal or off-diagonal). These operators vary with time due to external fields.

Assume that $QB(t) = \langle B(t)\rangle = 0 \implies Q\mathcal{L}_I = 0$.
Then the DME can be shown to reduce to

$$\dot{\rho} = -\frac{i}{\hbar}[H_S(t), \rho] + \int_0^t dt' \{\Phi_T(t,t')[\tilde{\mathcal{S}}(t',t)\rho, \mathcal{S}(t)] + \text{H.c.}\}. \quad (11)$$

Here

$$\Phi_T(t,t') = \langle U_B^\dagger(t-t')B(t)U_B(t-t')B(t')\rangle$$

is the bath "memory" (correlation) function (CF) and

$$\tilde{\mathcal{S}}(t',t) = U_S(t,t')\mathcal{S}(t')U_S^\dagger(t,t').$$

Eq. (11) *generalizes* previously known master equations to *arbitrary* time-dependent hamiltonians for the system and for system-bath coupling.

Qubit decoherence

Henceforth, we explicitly consider a driven qubit undergoing both population relaxation and proper dephasing. Qubit's resonant frequency and *dipolar* coupling to the reservoir are dynamically modulated, $B(t) = B$ is constant, so that

$$H_S(t) = \hbar[\omega_a + \delta_a(t) + \delta_d(t)]|e\rangle\langle e| + V(t)\sigma_x, \;\; H_I(t) = \mathcal{S}(t)B = \tilde{\epsilon}(t)\sigma_x B.$$

Here $\delta_a(t)$ is the dynamically imposed Stark shift, $\delta_d(t)$ is its *random* counterpart (proper dephasing),

$$V(t) = V_0(t)e^{-i\omega_c t} + \text{c.c.}$$

is the control (flipping) field with the nominal frequency ω_c, $V_0(t)$ being the Rabi frequency, $\sigma_x = |e\rangle\langle g| + |g\rangle\langle e|$ is the dipole-transition operator, whose time modulation is given by the *real* amplitude $\tilde{\epsilon}(t)$.

If the bath consists of oscillators and the coupling is linear, then

$$H_B = \sum_\lambda \hbar\omega_\lambda a_\lambda^\dagger a_\lambda, \;\; H_I(t) = \hbar\tilde{\epsilon}(t)\sigma_x \sum_\lambda (\kappa_\lambda a_\lambda + \kappa_\lambda^* a_\lambda^\dagger),$$

where ω_λ and a_λ are the frequency and annihilation operator, respectively, of the mode λ and κ_λ is the coupling amplitude. $H_I(t)$ does not involve the RWA.

Population relaxation

Consider first situations wherein population relaxation is dominant compared to the proper-dephasing rate [determined by $\delta_d(t)$], so that the latter may be neglected.

In the case $\delta_d(t) = 0$, we obtain from the DME our generalized Bloch equations for the components of the qubit density matrix

$$\dot{\rho}_{ee} = -\dot{\rho}_{gg} = iV(t)(\rho_{eg} - \rho_{ge}) - R_e(t)\rho_{ee} + R_g(t)\rho_{gg}, \qquad (12)$$

$$\dot{\rho}_{eg} = \dot{\rho}^*_{ge} = -\{R(t) + i[\tilde{\omega}_a(t) + \delta_a(t)]\}\rho_{eg} \\ +iV(t)(\rho_{ee} - \rho_{gg}) + [R(t) - i\Delta_a(t)]\rho_{ge}. \qquad (13)$$

Equations (12) and (13) account for the presence of *upward transitions* $|g\rangle \longrightarrow |e\rangle$ (caused by either temperature or anti-resonant effects) at a rate $R_g(t)$, in addition to *downward* decay $|e\rangle \longrightarrow |g\rangle$ at a rate $R_e(t)$.
Their half-sum $R(t) = [R_e(t) + R_g(t)]/2$ provides the decoherence rate.

The resonance frequency is dynamically shifted by
$\tilde{\omega}_a(t) - \omega_a = \Delta_a(t) = \Delta_e(t) - \Delta_g(t)$, where $\hbar\Delta_{e(g)}(t)$ is the Lamb shift of $|e\rangle$ ($|g\rangle$), caused by the dynamically modified coupling to the bath.

Since we are interested here in dynamical control of relaxation, we shall concentrate on the transition rates $R_{e(g)}(t)$ rather than the level shifts. One can show that the average rate of the $|e\rangle \to |g\rangle$ transition $R_e(t)$ and its $|g\rangle \to |e\rangle$ counterpart $R_g(t)$ are given by

$$R_{e(g)}(t) = 2\pi \int_{-\infty}^{\infty} d\omega F_t(\omega) G_T(\pm\omega). \tag{14}$$

Here the upper (lower) sign corresponds to the subscript e (g).

$$G_T(\omega) = (2\pi)^{-1} \int_{-\infty}^{\infty} \Phi_T(t) e^{i\omega t} dt$$

is the SD of the bath CF. For the oscillator bath one finds that

$$G_T(\omega) = [n(\omega) + 1] G_0(\omega) + n(-\omega) G_0(-\omega),$$

where $G_0(\omega) = \sum_\lambda |\kappa_\lambda|^2 \delta(\omega - \omega_\lambda)$ and $n(\omega) = (e^{\beta\omega} - 1)^{-1}$ is the average number of quanta in the bath mode with frequency ω.

We apply Eqs. (14) to the case of *coherent modulation of quasiperiodic* form. Without a limitation of the generality, we assume that $\sum_k |\epsilon_k|^2 = 1$. We then find, using Eq. (14), that the rates $R_{e(g)}(t)$ tend to the long-time limits

$$R_{e(g)} = 2\pi \sum_k |\epsilon_k|^2 G_T(\pm(\omega_a + \omega_k)). \tag{15}$$

The limits (15) are approached when $\Omega t \gg 1$ and $t \gg t_c$, as above.

Had we used the standard dipolar RWA hamiltonian in the case of an oscillator bath, dropping the antiresonant terms in $H_I(t)$, we would have arrived at the transition rates

$$R_{e(g)}^{\mathrm{RWA}} = 2\pi \int_0^\infty d\omega F(\omega) G_T(\pm\omega), \tag{16}$$

wherein the integration is performed from 0 to ∞, rather than from $-\infty$ to ∞, as in (14). This means that the RWA transition rates hold for a slow modulation, when $F(\omega) \simeq 0$ at $\omega < 0$, being peaked near ω_a. However, whenever the suppression of $R_{e(g)}$ requires modulation at a rate comparable to ω_a, the RWA is inadequate. For instance, at $T = 0$, the rate R_g^{RWA} vanishes identically, irrespective of $F(\omega)$, in contrast to the true upward-transition rate R_g in Eq. (15), which may be comparable to R_e for ultrafast modulation.

The difference between the RWA and non-RWA decay rates stems from the fact that the RWA implies that a downward (upward) transition is accompanied by emission (absorption) of a bath quantum, whereas the non-RWA (negative-frequency) contribution to $R_{e(g)}$ in Eq. (14) allows for just the opposite: downward (upward) transitions that are accompanied by absorption (emission). The latter processes are possible since the modulation may cause *level* $|e\rangle$ *to be shifted below* $|g\rangle$.

Dynamically modified proper dephasing

We turn now to proper dephasing when it dominates over decay.

$$H(t) = \hbar[\omega_a + \delta_d(t)]|e\rangle\langle e| + V(t)\sigma_x. \quad (17)$$

The random frequency fluctuations $\delta_d(t)$ are typically characterized by a (single) correlation time t_d, with ensemble mean $\bar{\delta}_d = 0$. When the field $V(t)$ is used only for gate operations, we assume that it does not affect proper dephasing.

The ensemble average over $\delta_d(t)$ results in an increase of the decoherence rate

$$R(t) \rightarrow R(t) + R_d(t),$$

with the dephasing rate

$$R_d(t) = \int_0^t dt' \Phi_d(t').$$

The dephasing CF $\Phi_d(t) = \overline{\delta_d(t)\delta_d(0)}$ is the counterpart of the bath CF $\Phi_T(t)$.

Assuming, for simplicity, that the decay is neglected and the control field $V(t)$ is resonant ($\omega_c = \omega_a$) with real envelope $V_0(t)$, we derive the ME for the qubit density matrix averaged over the random fluctuations $\delta_d(t)$. To this end, we transform the system to the rotating frame, write the pseudospin vector in spherical coordinates,

$$Q \equiv (Q_{-1}, Q_0, Q_1) = (\rho_{ge}, (\rho_{gg} - \rho_{ee})/\sqrt{2}, -\rho_{eg}),$$

and tilt the frame to diagonalize the Hamiltonian of the TLS-field coupling [Eq. (17)] by the transformation

$$Q_m = \sum_{m'} Q'_{m'} d^{(1)}_{m'm}\left(-\frac{\pi}{2}\right),$$

where $d^{(1)}_{m'm}(-\frac{\pi}{2})$ is the finite-rotation matrix for spin 1. In the tilted frame, the master equation is

$$\dot{Q}'_{\pm 1} = \{\pm i[V_0(t) + \Delta_d(t)] - R_d(t)/2\}Q'_{\pm 1}, \quad \dot{Q}'_0 = -R_d(t)Q'_0, \tag{18}$$

where the dynamically affected decoherence rate and shift are given by the real and imaginary parts of

$$R_d(t) + 2i\Delta_d(t) = \int_0^t dt' \Phi_d(t-t') \exp\left(i\int_{t'}^t V_0(t'')dt''\right). \qquad (19)$$

At $t \gg t_d$ the decoherence rate and shift approach their asymptotic values

$$R_d = \lim_{t\to\infty} R_d(t), \;\; \Delta_d = \lim_{t\to\infty} \Delta_d(t).$$

For the validity of Eq. (18) it is necessary that $R_d, |\Delta_d| \ll 1/t_d$.

In Eq. (18) we have made the secular approximation, which holds if $V_0(t) \gg R_d, |\Delta_d|$.

Equation (19) reveals the *analogy of dynamically modified dephasing to dynamically modified relaxation*, both inferred from our unified treatment. One can obtain from Eq. (19) that

$$R_d(t) = \pi \int_{-\infty}^{\infty} d\omega F_t(\omega) G_d(\omega), \qquad (20)$$

where $F_t(\omega)$ is the spectrum of

$$\epsilon(t) = \exp\left[-i\int_0^t V(t')dt'\right], \;\; Q(t) = t, \;\; G_d(\omega) = \frac{1}{\pi}\int_0^\infty \Phi_d(t)\cos\omega t dt.$$

The proper dephasing rate associated with $\Phi_d(t) = Ae^{-t/t_d}$ is $R_d = At_d$. In the presence of a *constant* V_0 [cw $V(t)$], it is modified according to Eq. (19) into

$$R_d = \frac{At_d}{V_0^2 t_d^2 + 1}. \tag{21}$$

For a sufficiently strong field, the dephasing rate R_d can be *suppressed* by the factor $1/(V_0 t_d)^2 \ll 1$. This suppression reflects the ability of strong, near-resonant Rabi splitting to shift the system out of the bath bandwidth, or average its effects. By comparison, the “bang-bang” (BB) method involving τ-periodic π-pulses (Viola and Lloyd, 1998) is an analog of the above “parity kicks”. Using Eq. (20), such pulses can be shown to suppress R_d approximately according to Eq. (21) with $V_0 = \pi/\tau$. This BB method requires pulsed fields with Rabi frequencies $\gg 1/\tau$, i.e., *much stronger fields than the cw field* in our Eq. (21). Using $t_d \sim 10^{-7}$ s, cw Rabi frequencies exceeding 1 MHz achieve a significant dephasing suppression.

Conclusions

A. We discussed how to control qubit decoherence by external coherent perturbations.

B. Our simple **universal** formula results in general criteria for dynamical control of decoherence due to relaxation and proper dephasing.

C. Coherent (unitary) modulation of the coupling to the reservoir can emulate the QZE and AZE, but has advantages over frequent measurements:
(a) Coherent modulation does not destroy the coherence of the quantum system, in contrast to measurements.
(b) It can be designed for much more effective suppression of decoherence than QZE.

D. We took into account thermal and antiresonant effects.

E. We considered control of decay/decoherence in various systems: tunneling in optical lattices, Josephson junctions, entangled photon states.