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SMR.1587 - 21

SCHOOL AND WORKSHOP ON
QUANTUM ENTANGLEMENT, DECOHERENCE, INFORMATION, AND GEOMETRICAL PHASES IN COMPLEX SYSTEMS
(1 November - 12 November 2004)

Wave-particle duality and quantum erasure in atom interferometry

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These are preliminary lecture notes, intended only for distribution to participants

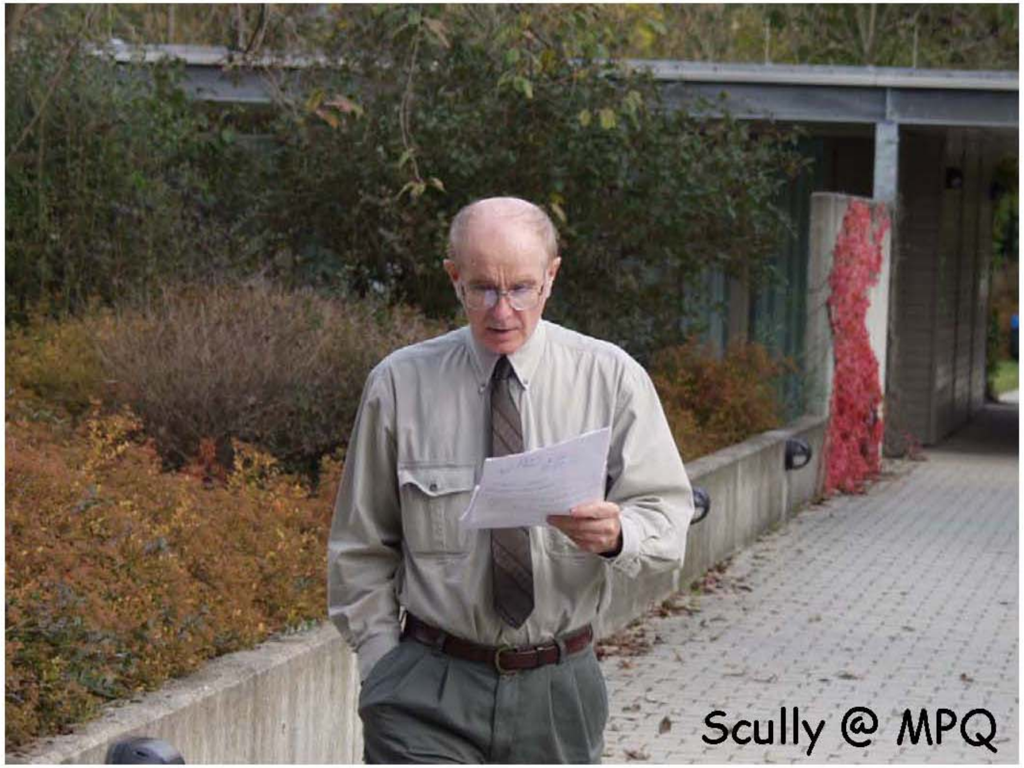
Quantum Dynamics Group
Max-Planck Institute for Quantum Optics

quantum gases
slow molecules
cavity quantum electrodynamics
quantum information processing

Complementarity and Quantum Erasure in Atom Interferometry

Stephan Dürr, Thomas Nonn, GR

University of Konstanz
Germany



Scully @ MPQ

Italian double slit

complementarity

principle of complementarity:
quantum systems possess properties
that are equally real but mutually exclusive

Bohr, Nature **121** (1928) 580

for all practical purposes:
no matter how the system is prepared, there is always
a measurement whose outcome is utterly unpredictable

Scully, Englert, Walther, Nature **351** (1991) 111

wave-particle duality:
depending on the experimental situation,
a quantum system behaves either like a particle or like a wave

Englert, Phys. Rev. Lett. **77** (1996) 2154

complementarity

we are not dealing with contradictory,
but with complementary pictures of the phenomena

Bohr, Nature **121** (1928) 580

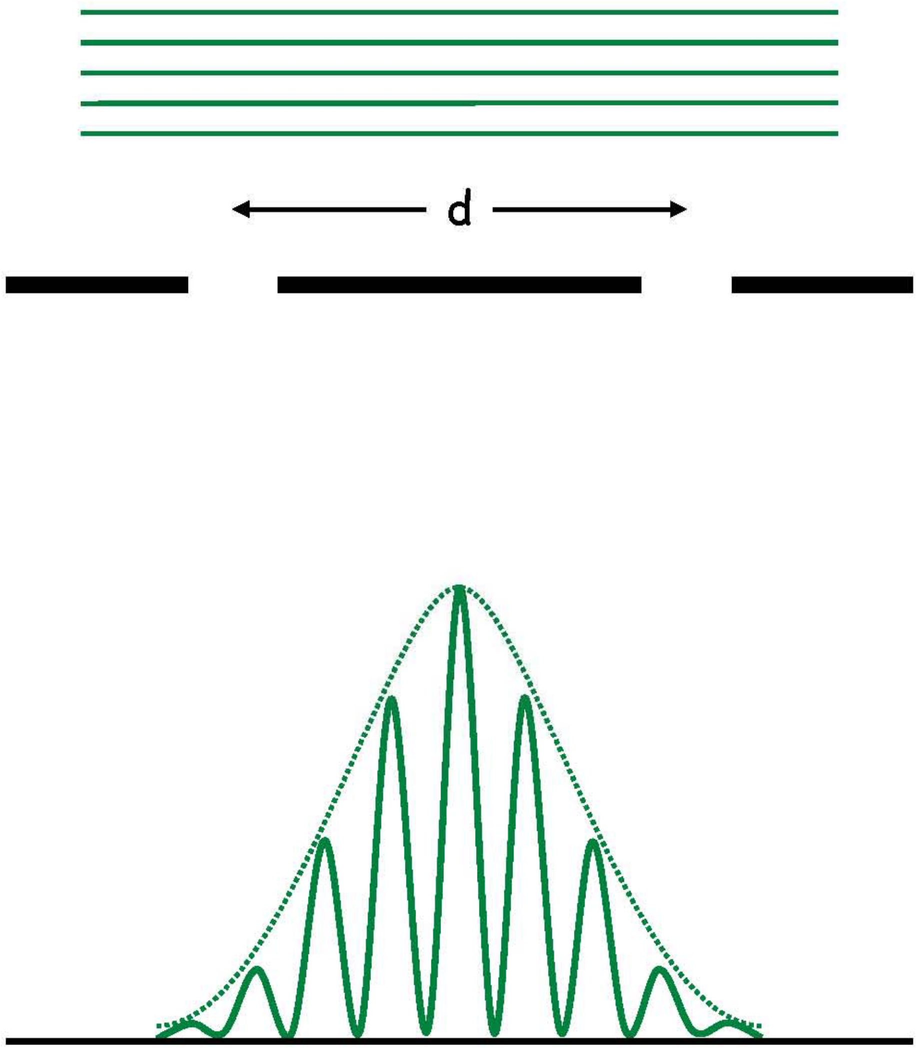
any given application of classical concepts precludes
the simultaneous use of other classical concepts
which in a different connection are equally necessary
for the elucidation of the phenomena

Bohr, Atomic Theory and the Description of Nature (1934)

evidence obtained under different experimental conditions
cannot be comprehended within a single picture, but must
be regarded as complementary in the sense that only the
totality of the phenomena exhausts the possible information
about the objects

Bohr, Discussion with Einstein (1949)

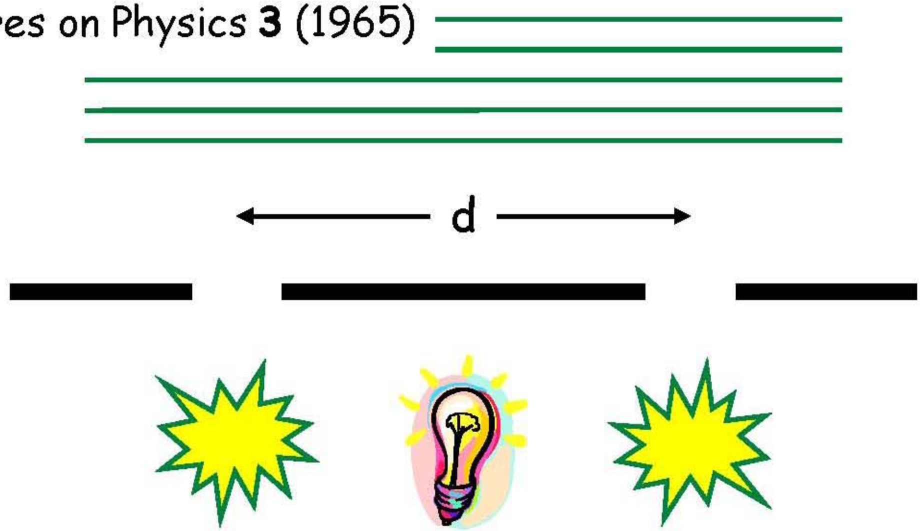
double-slit experiment



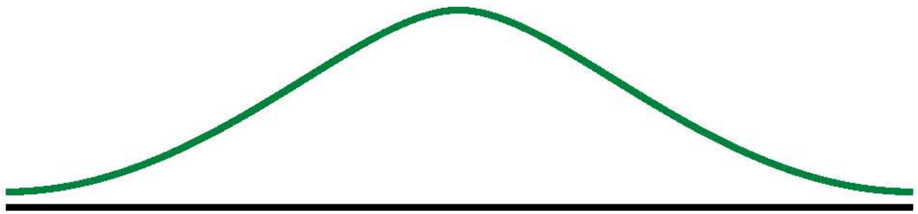
fringe period $\delta p = \hbar/d$

Feynman's gedanken experiment

Feynman et al., Lectures on Physics **3** (1965)



1.) position uncertainty $\Delta x < d$
2.) uncertainty relation $\Delta x \, \Delta p > \hbar$
3.) momentum uncertainty $\Delta p > \hbar/d > \delta p$



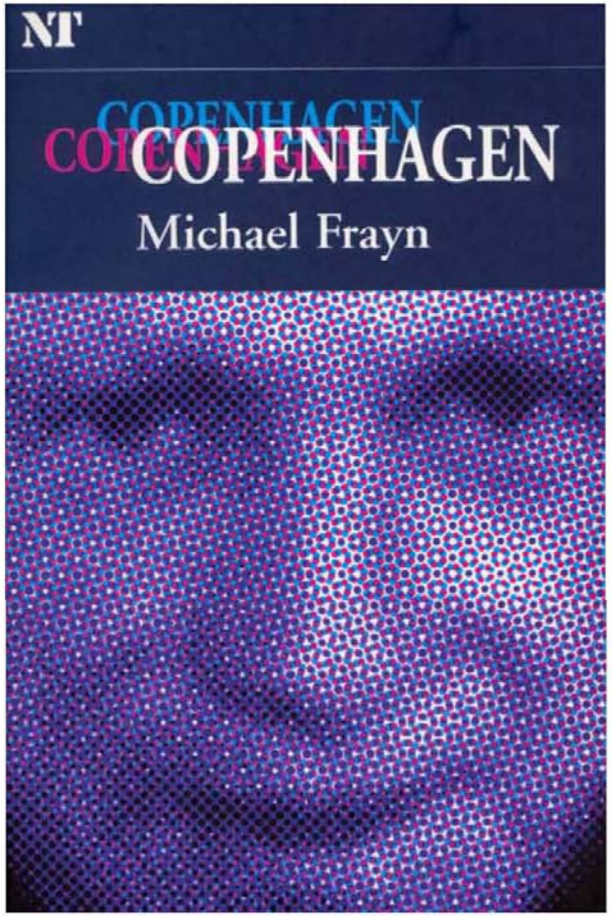
interference vanishes, envelope broadens

uncertainty and complementarity

uncertainty relation enforces complementarity:

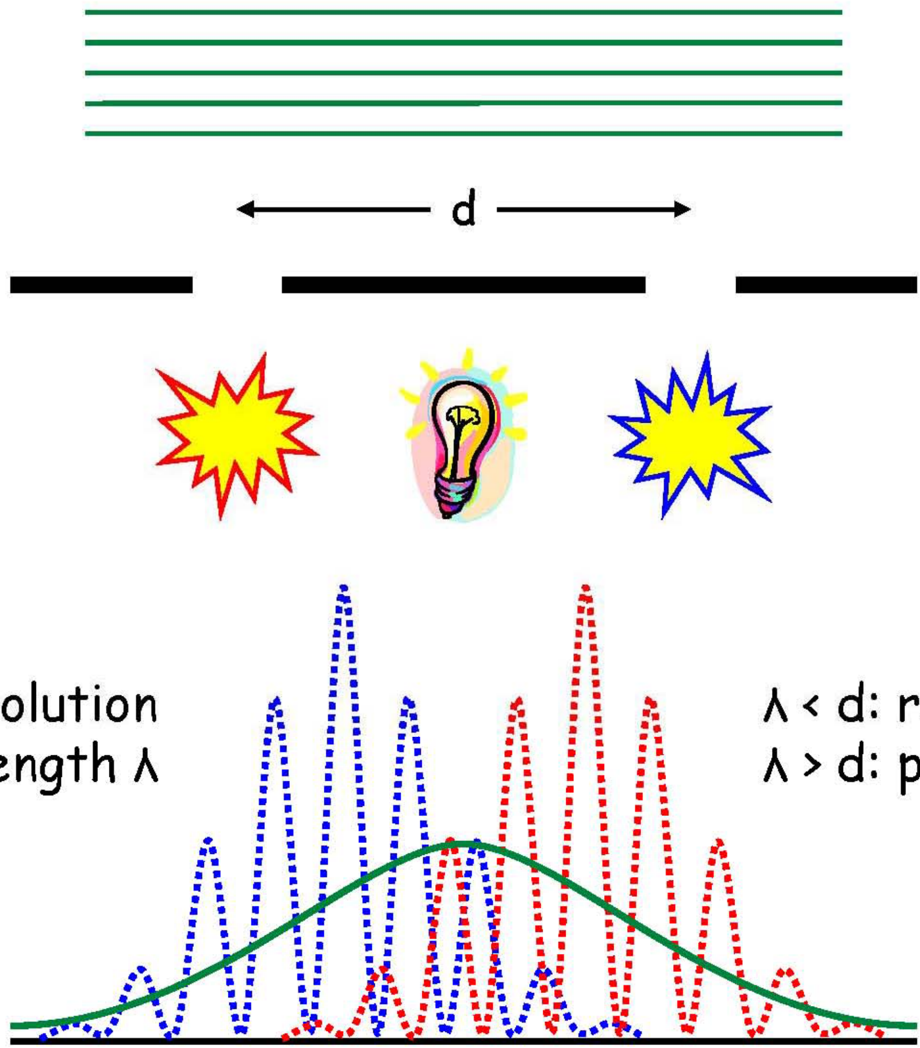
if an apparatus is capable of determining
which hole the electron goes through,
it cannot be so delicate that it does not
disturb the pattern. No one has ever
found (or even thought of) a way around
the uncertainty principle

Feynman et al., Lectures on Physics III, 1965



Bohr: "We ended up with a treaty.
Uncertainty and complementarity
became the two central tenets of
of the Copenhagen Interpretation
of Quantum Mechanics."

intuitive interpretation: random momentum kicks



position resolution
$\Delta x \approx$ wavelength λ

λ < d: random recoil kicks
λ > d: position unresolved

random fringe position, broadening of the envelope

new ideas

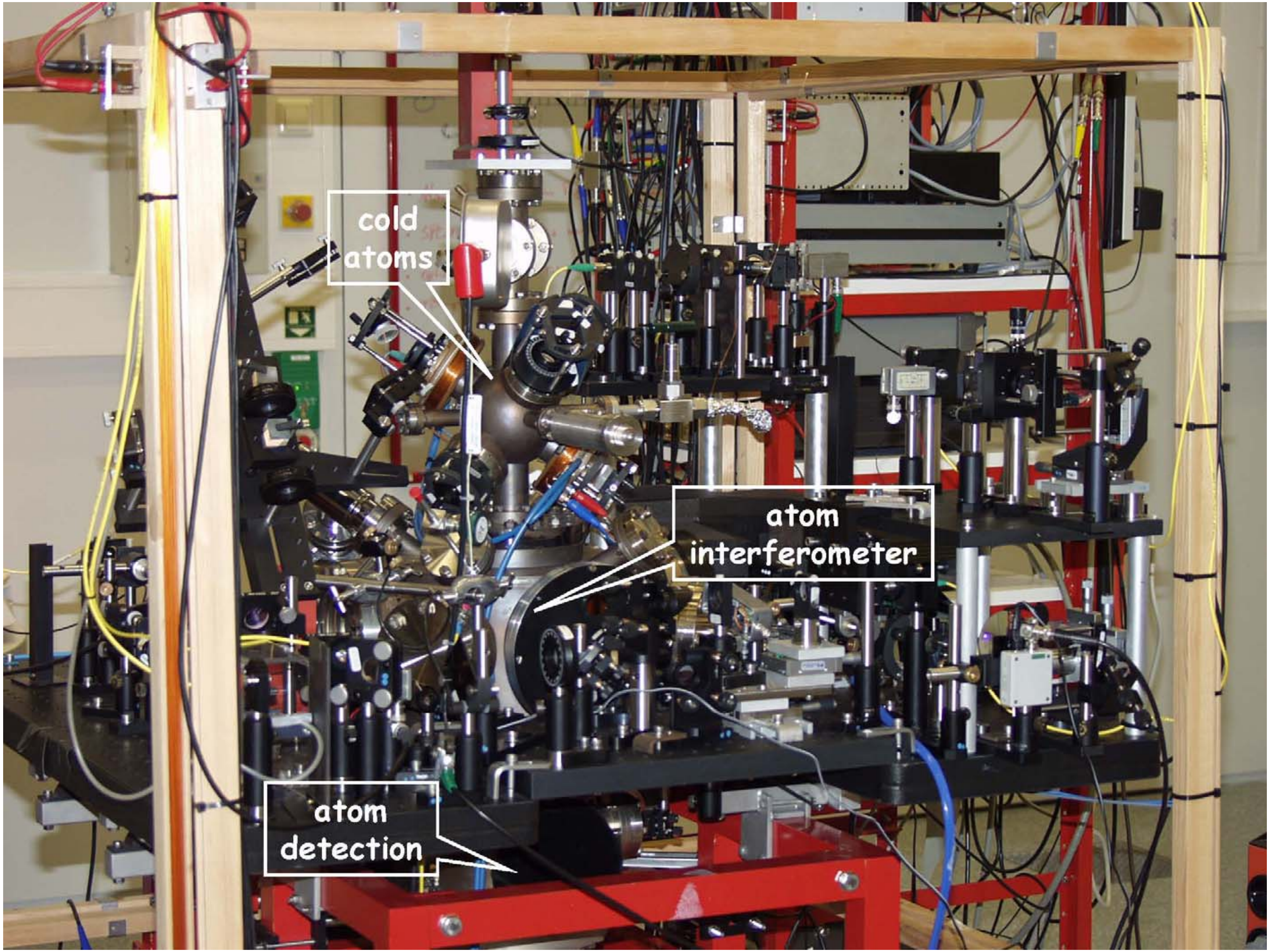
proposal:
new detectors provide a way and allow the investigation of other mechanisms that enforce complementarity

Scully, Englert, Walther, Nature **351** (1991) 111

rebuttal:
in interference experiments with a screen in the far-field, the pattern on the screen is the distribution of outgoing momentum. Any process that changes one pattern into another must involve momentum transfer

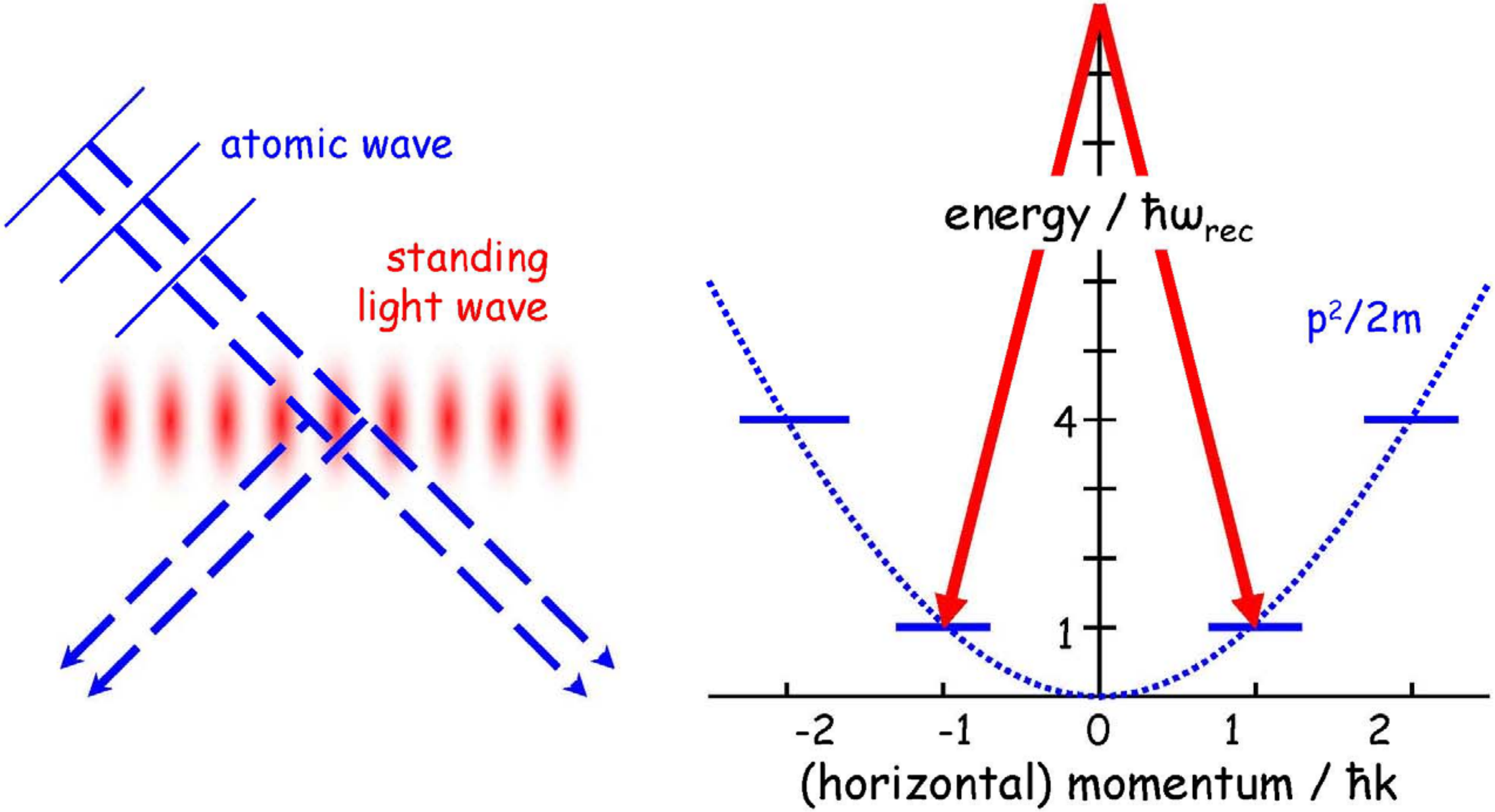
Storey, Tan, Collett, Walls, Nature **367** (1994) 626

in any path detection scheme involving a fixed double slit the amount of momentum transferred to the particle by a which-way detector is related to the slit separation in accordance with the uncertainty relation


cold atoms
atom interferometer
atom detection

Bragg diffraction: principle

Dürr et al., Quant. Semiclass. Opt. 8 (1996) 531

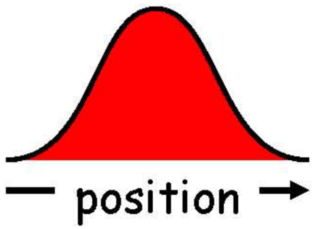


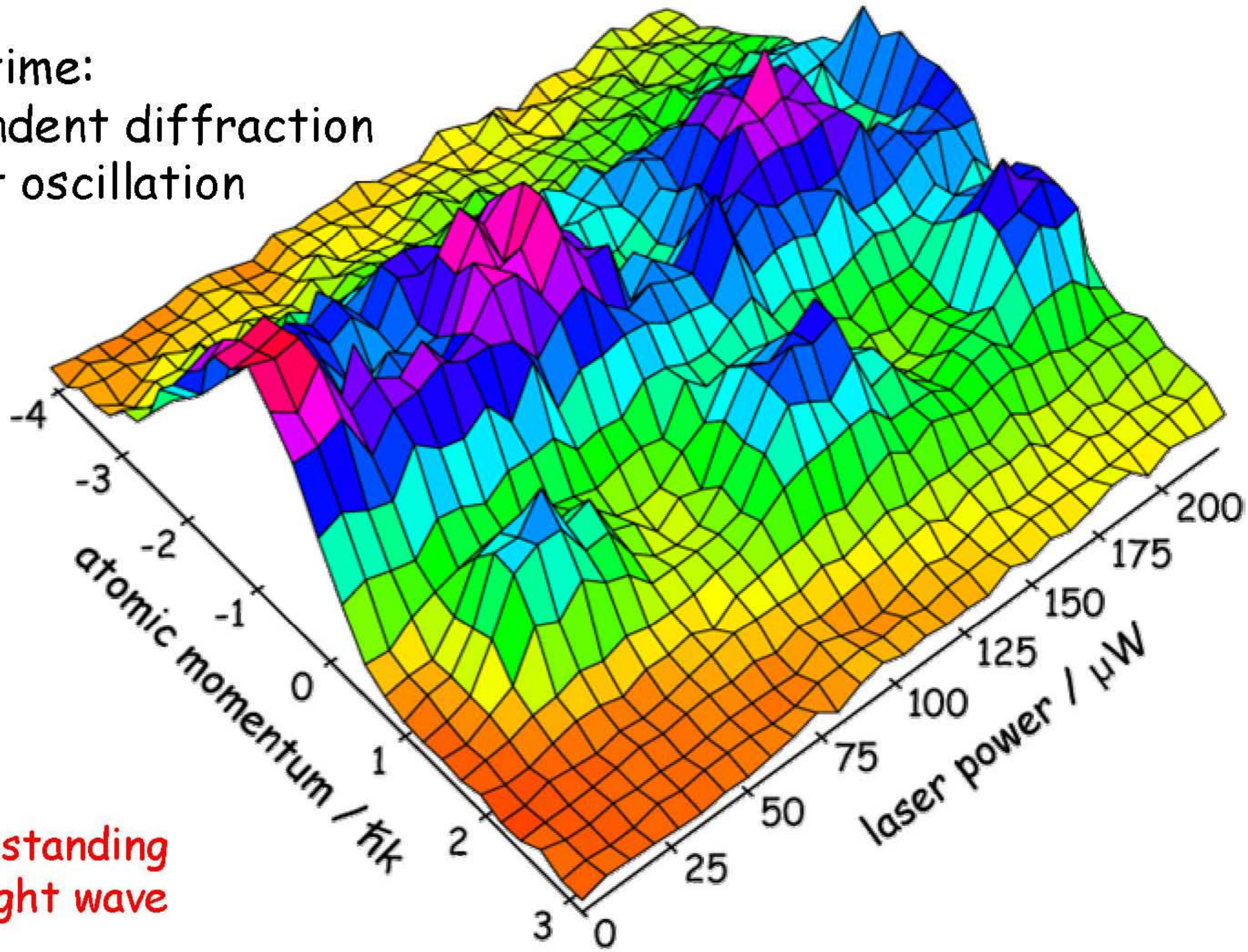
two-photon Rabi (Pendellösung) oscillation
between equal-energy momentum states

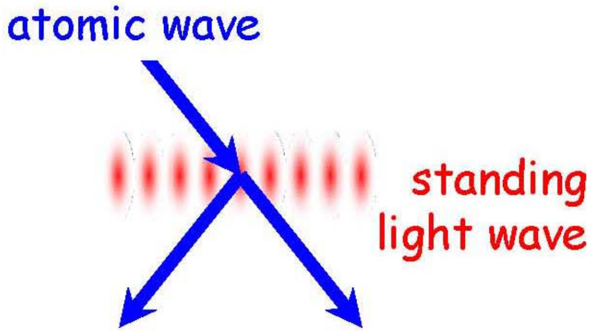
Bragg diffraction: continuous-wave

Kunze et al., Europhys. Lett. **34** (1996) 343

fixed interaction time:
- momentum-dependent diffraction
- power-dependent oscillation



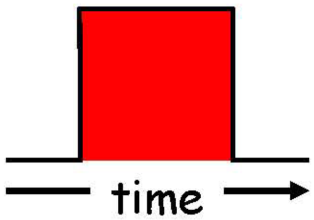


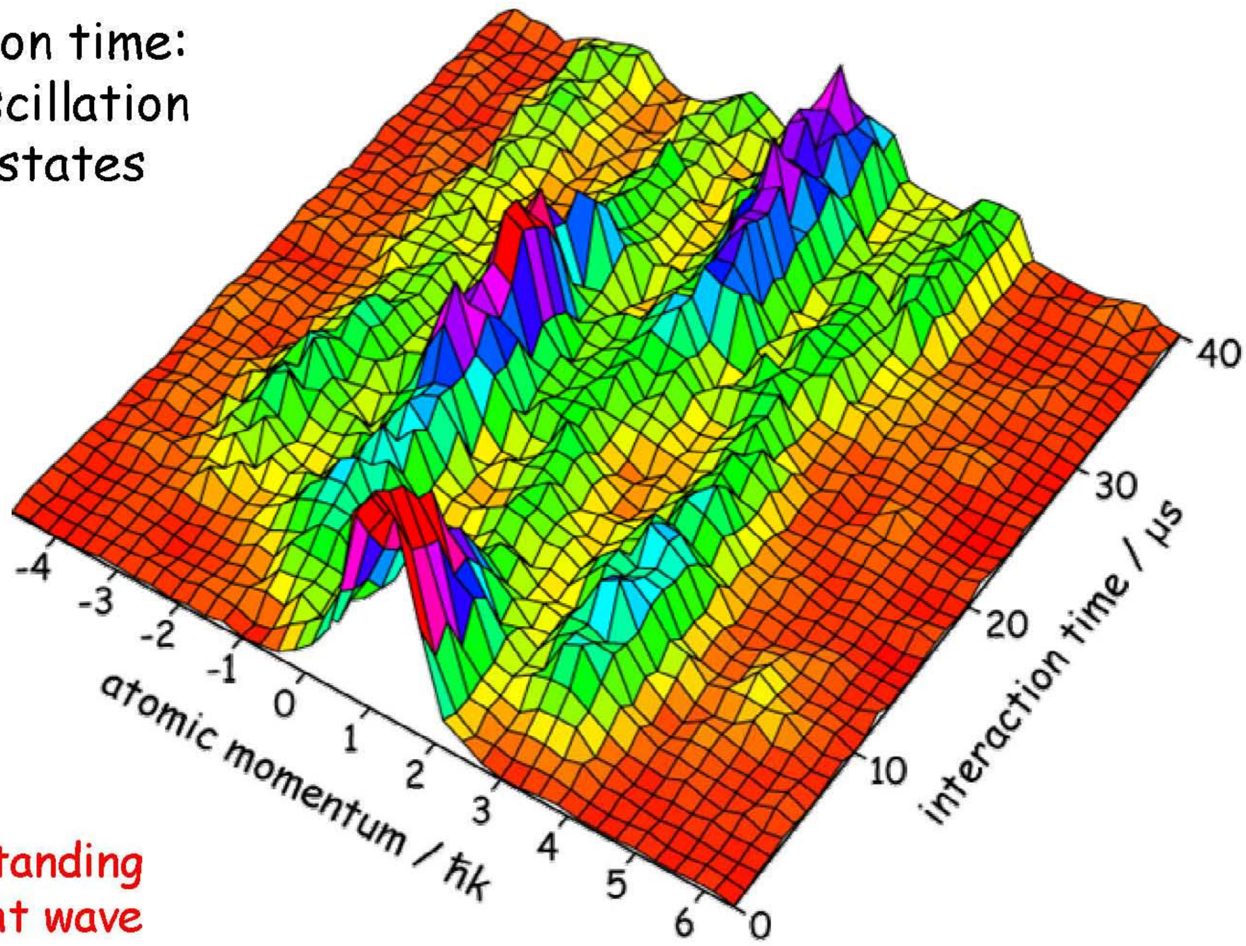


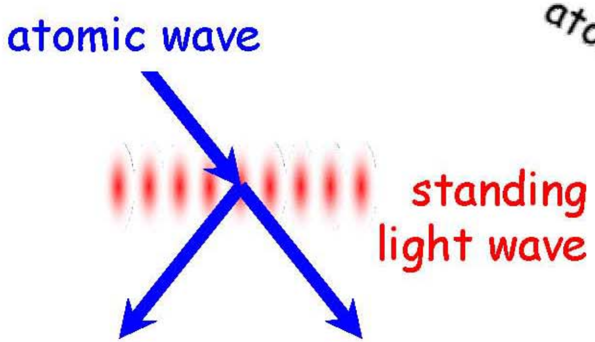
Bragg diffraction: instantaneous switching

Keller et al., Appl. Phys. B **69** (1999) 303

adjustable interaction time:
- time-dependent oscillation
- higher momentum states



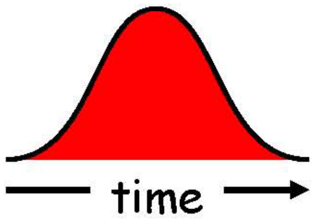


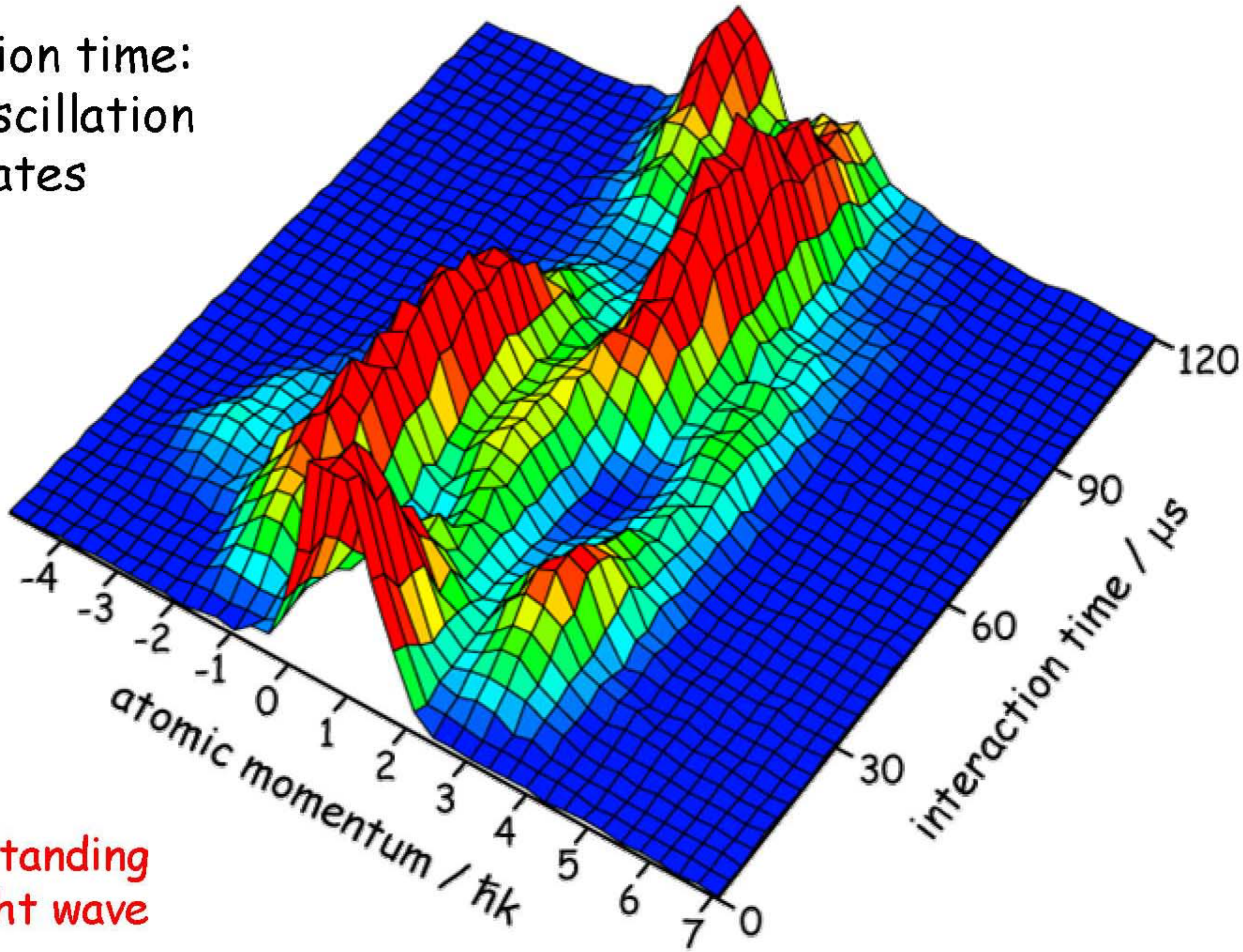


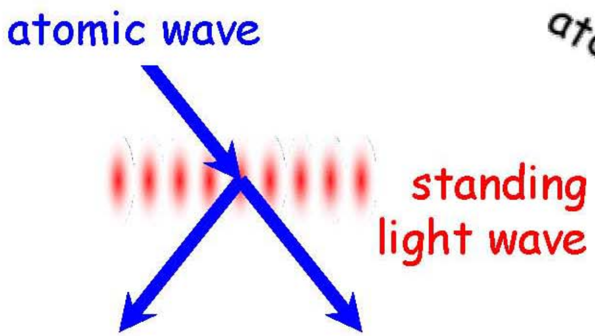
Bragg diffraction: soft switching

Keller et al., Appl. Phys. B **69** (1999) 303

adjustable interaction time:
- time-dependent oscillation
- two momentum states

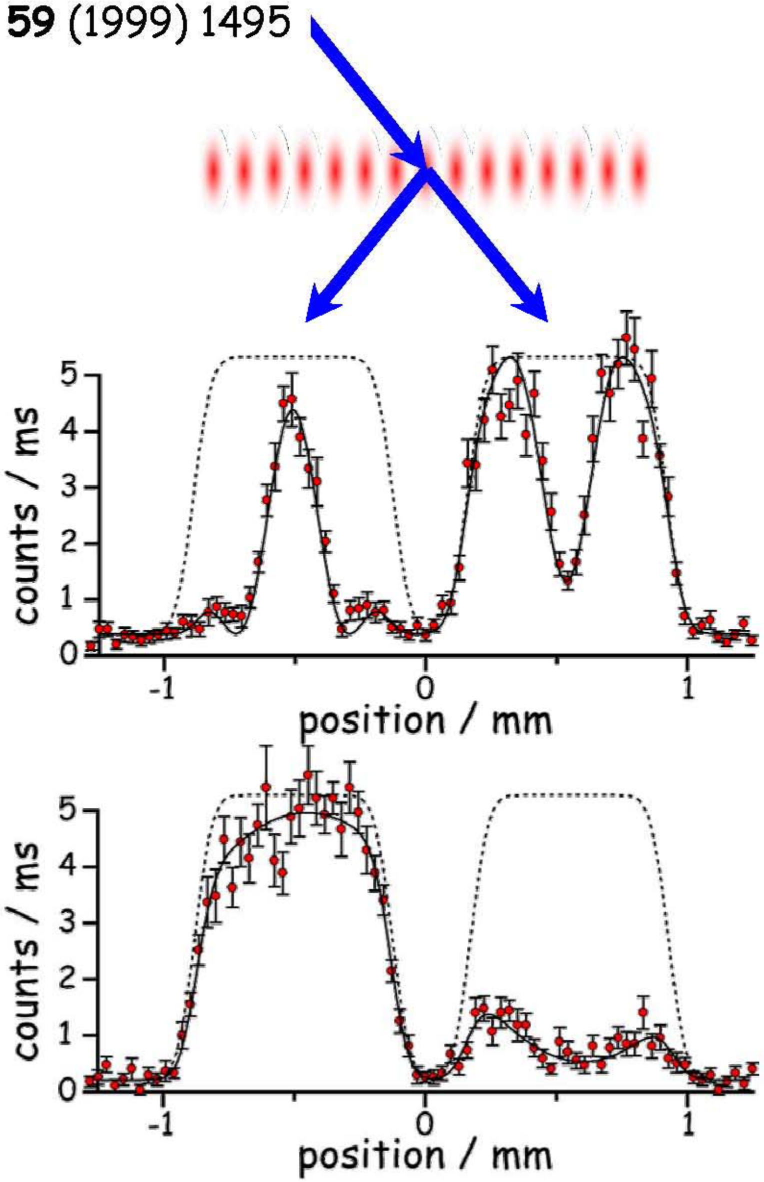






Bragg diffraction: acceptance angle

Dürr et al., Phys. Rev. A **59** (1999) 1495



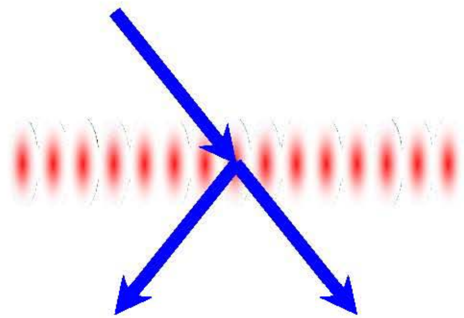
small
Stark shift
$X/\omega_{rec} = 0.15$

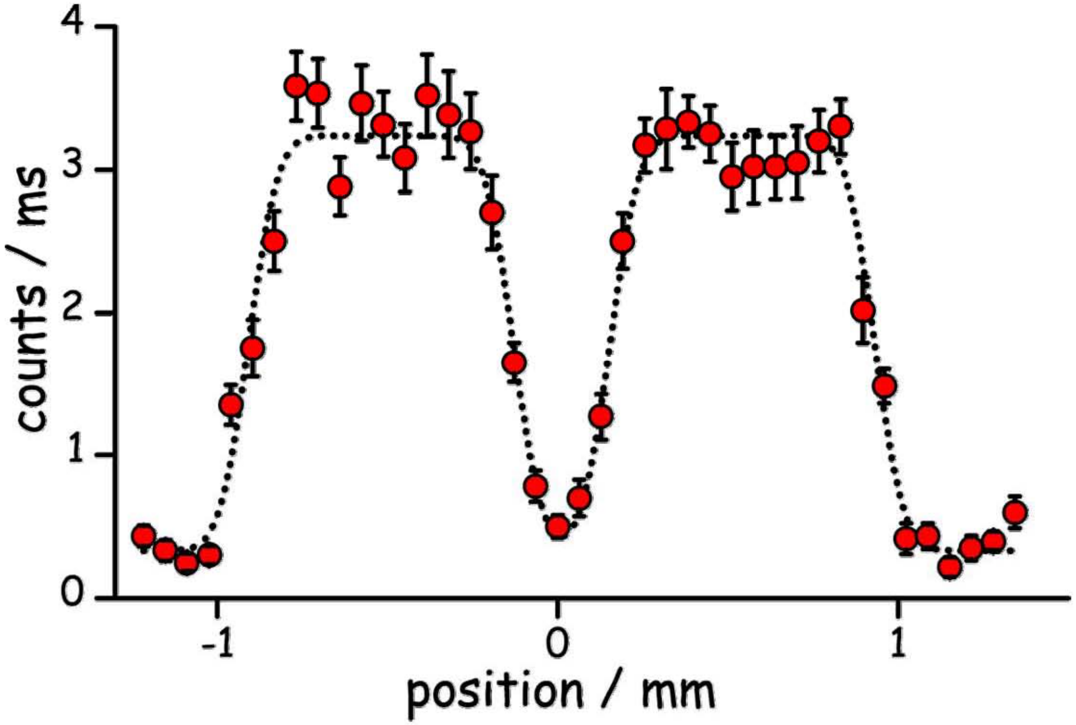
long
interaction time
$\tau = \pi/(2X)$

large
Stark shift
$X/\omega_{rec} = 1.0$

short
interaction time
$\tau = \pi/(2X)$

Bragg diffraction: beam splitter

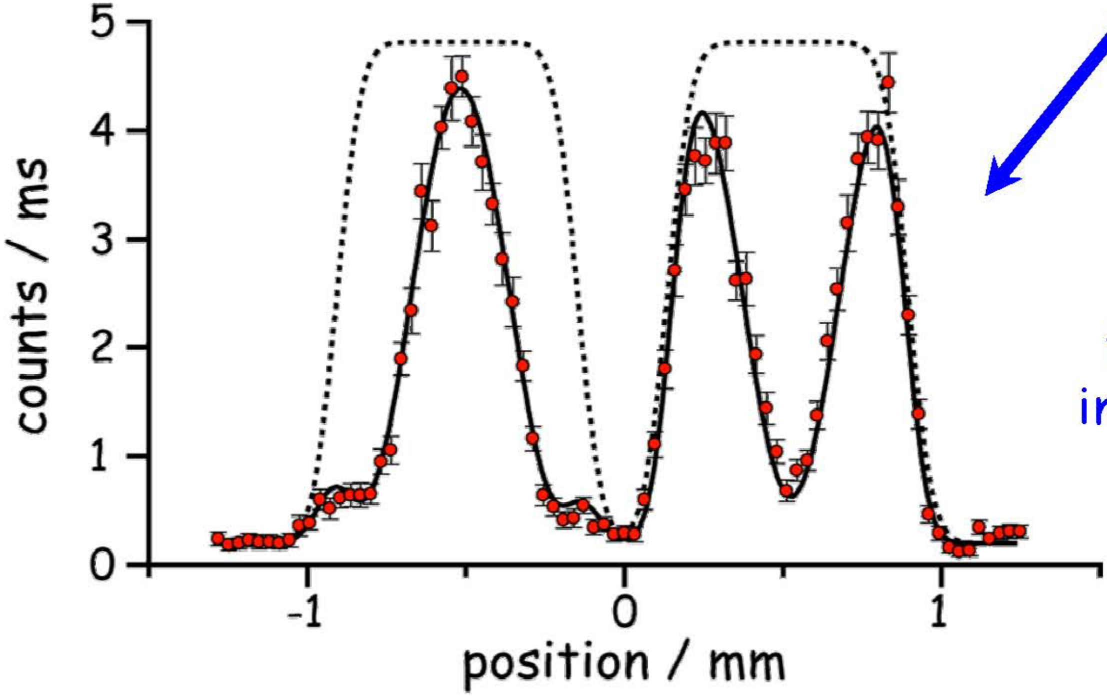


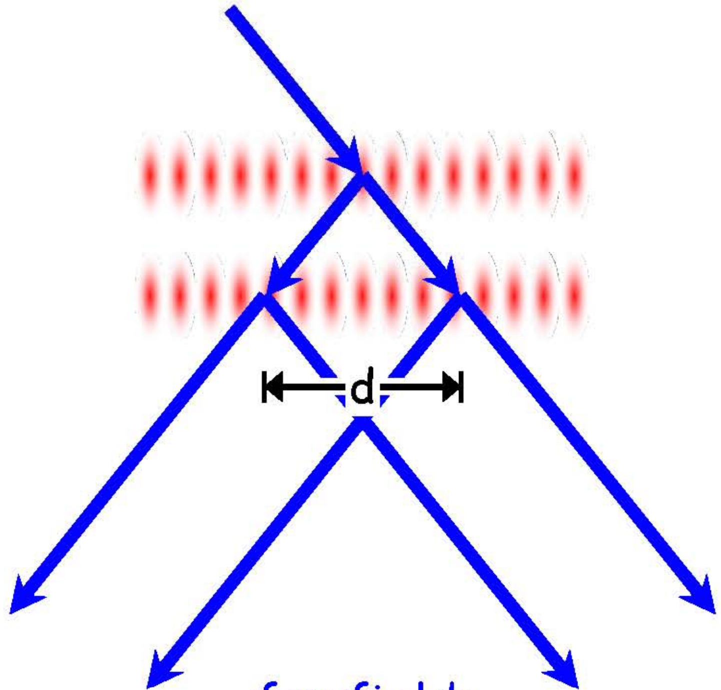


atom interferometer

Dürr et al., Nature **395** (1998) 33

pulse separation = 105 μs
==>
slit distance d = 1.3 μm





far field:
two non-overlapping
interference patterns

visibility = 75%

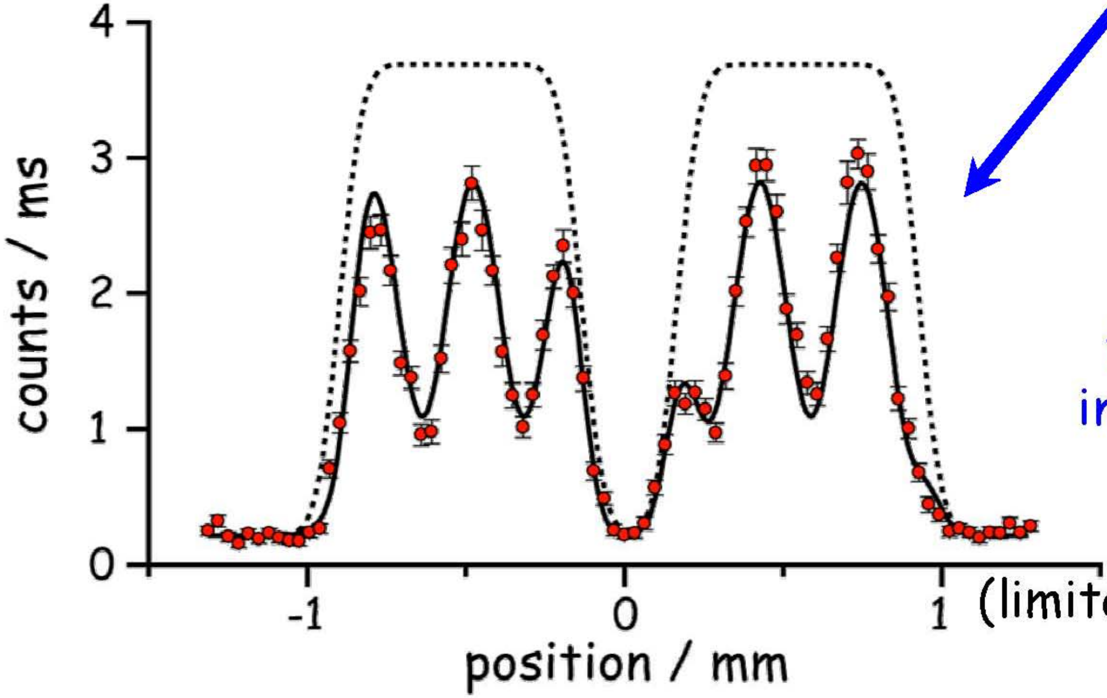
atom interferometer

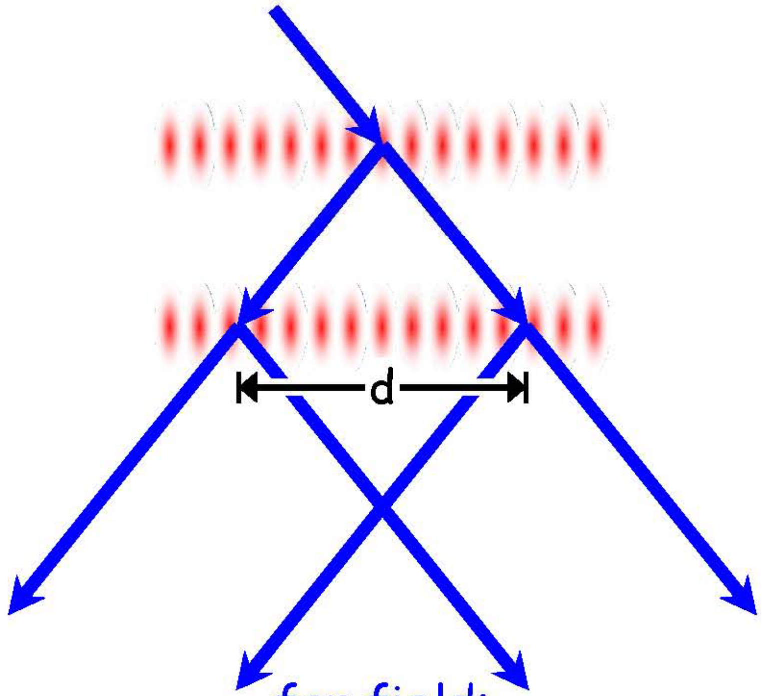
Dürr et al., Nature **395** (1998) 33

pulse separation = 255 μs

==>

slit distance d = 3.1 μm

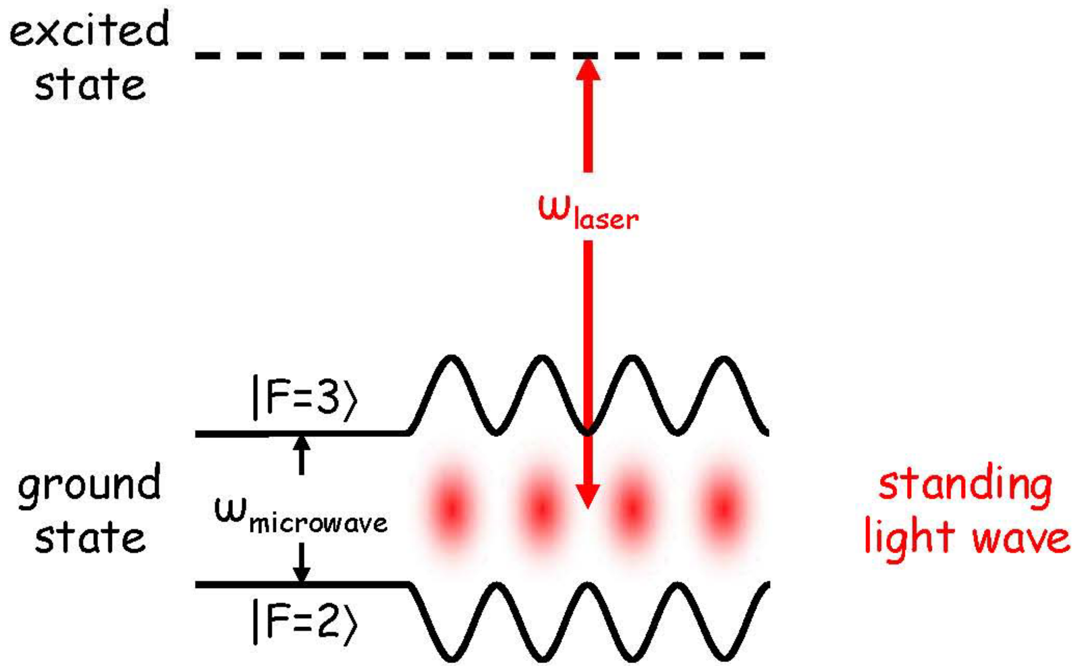




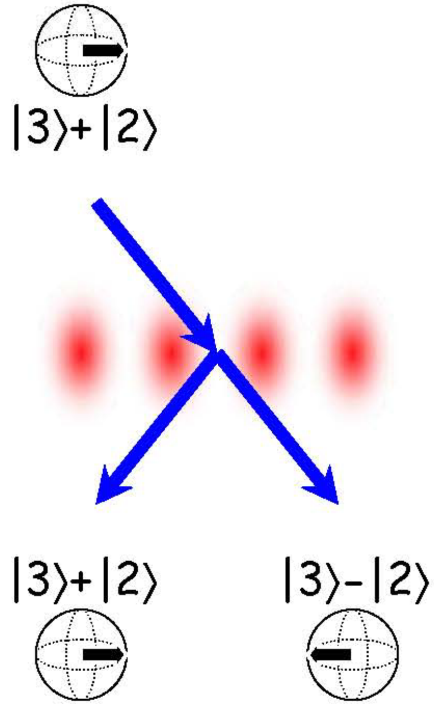
far field:
two non-overlapping
interference patterns

visibility = 44%
(limited by position resolution)

encoding which-way information

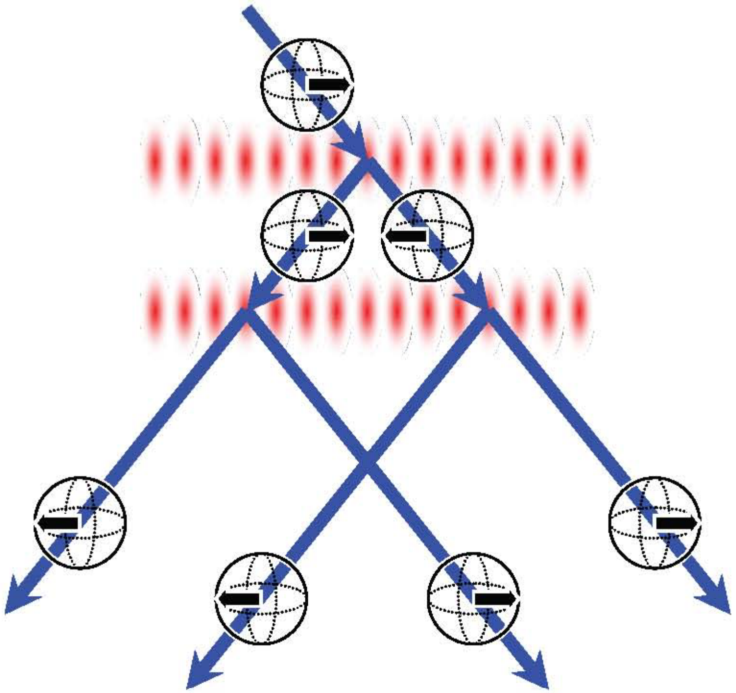


ground states experience
differential light shift
(~ intensity/detuning)
==>
differential phase shift

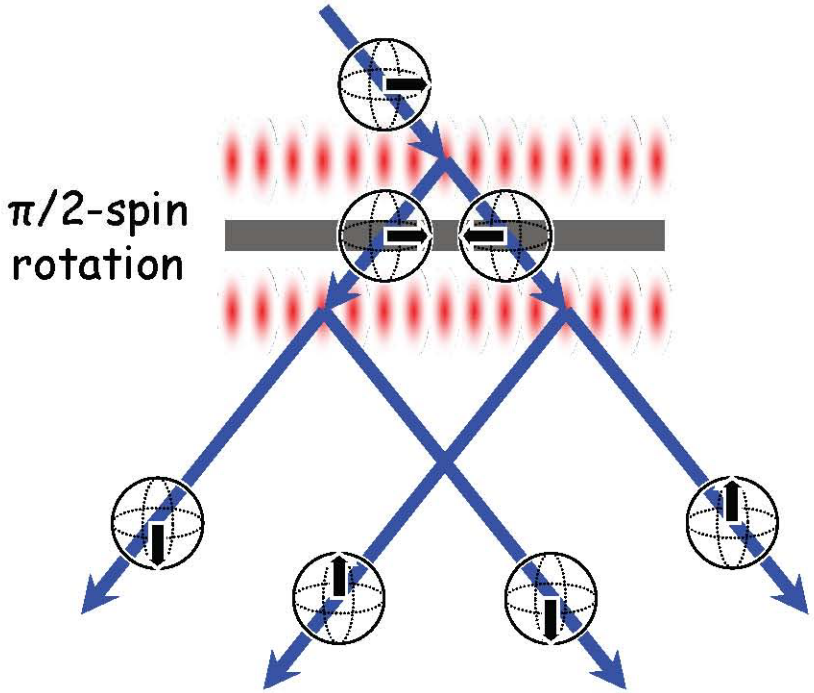


π-phase shift for
transmitted $|F=2\rangle$

atom interferometer with which-way information



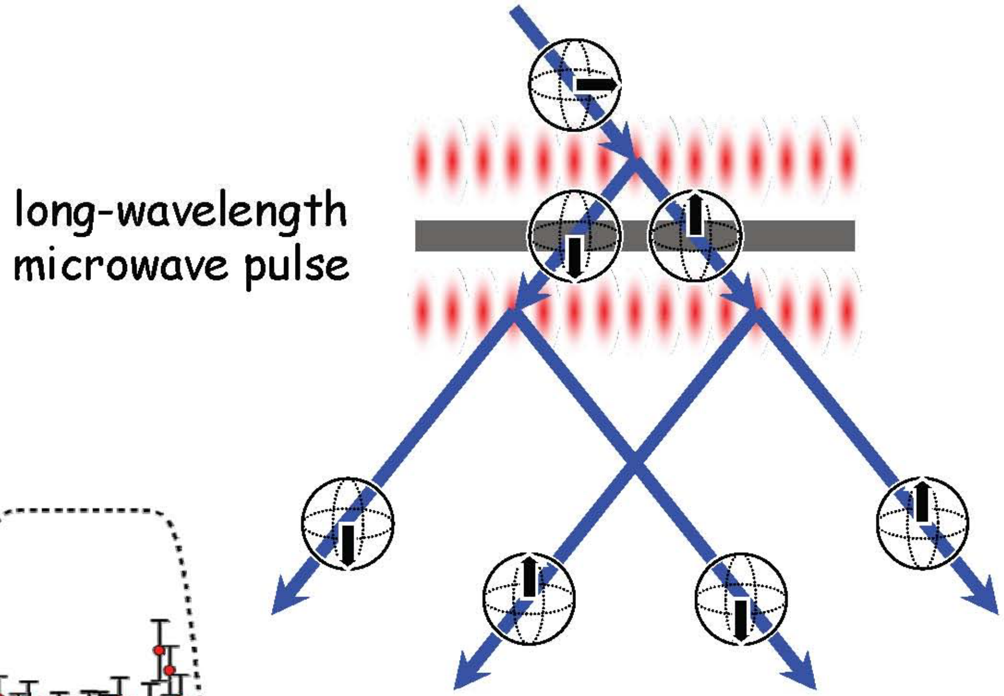
which-way information
encoded after 1th beam splitter,
but lost after 2nd beam splitter



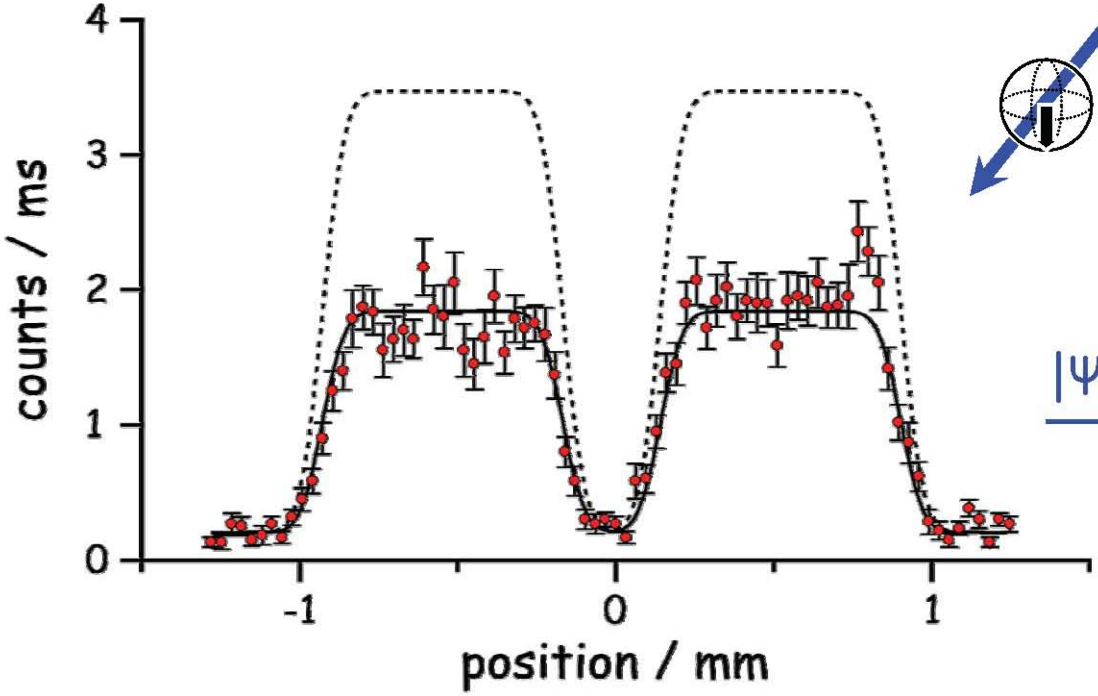
which-way information
permanently stored in
long-lived internal states

atom interferometer with which-way information

Dürr et al., Nature **395** (1998) 33



$$\frac{|\Psi_{left}, F=3\rangle - |\Psi_{right}, F=2\rangle}{2^{1/2}}$$

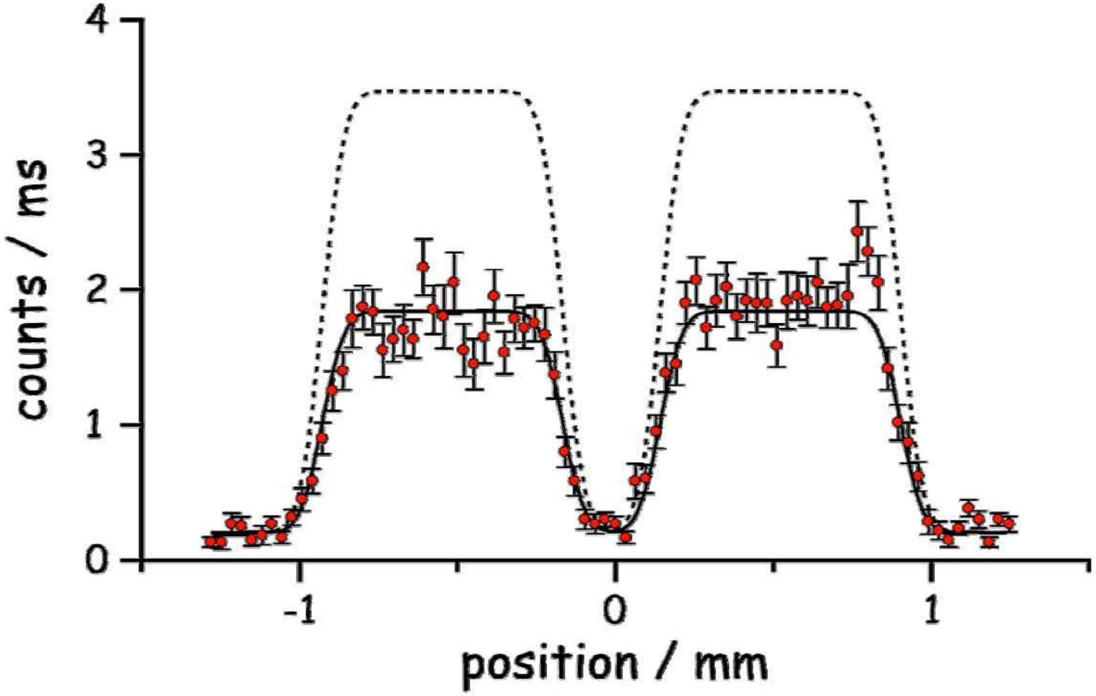


atom interferometer with which-way information

Dürr et al., Nature **395** (1998) 33

which-way information stored in internal atomic states,
but not read out by the fluorescence detector

no observable broadening of the envelope
==> no random momentum kicks



loss of interference

negligible recoil from microwave photons

storing which-way information in internal atomic states

$|\Psi_{out}\rangle = |\Psi_{left}, F{=}3\rangle - |\Psi_{right}, F{=}2\rangle$

destroys the interference pattern

$I(x) \sim |\Psi_{left}(x)|^2 + |\Psi_{right}(x)|^2$

$\quad - (\Psi_{left}(x)^* \Psi_{right}(x) \langle F{=}3|F{=}2\rangle + c.c.)$

no position-momentum uncertainty relation

position-momentum uncertainty relation

question:

why does the uncertainty relation apply
in case of the double slit, but not in our scheme ?

double-slit and our scheme have in common:

interference pattern in momentum space (p)
is observed in the far field in position space (x)

precise measurement of one variable has an
action onto the complementary observable

back-action:	double slit	atom interferometer
ways separated in:	x	p
back-action onto:	p	x
back-action destroys:	interference	nothing

interference versus which-way information

the observation of an
interference pattern
and the acquisition of
which-way information
are mutually exclusive

Englert
Phys. Rev. Lett. **77** (1996) 2154

quantum eraser: idea

conclusion: storing which-way information

$$|\Psi_{out}\rangle = |\Psi_{left}, F=3\rangle - |\Psi_{right}, F=2\rangle$$

destroys the interference (in the whole ensemble)

question: is the coherence destroyed ?

answer: erasing which-way information by a measurement

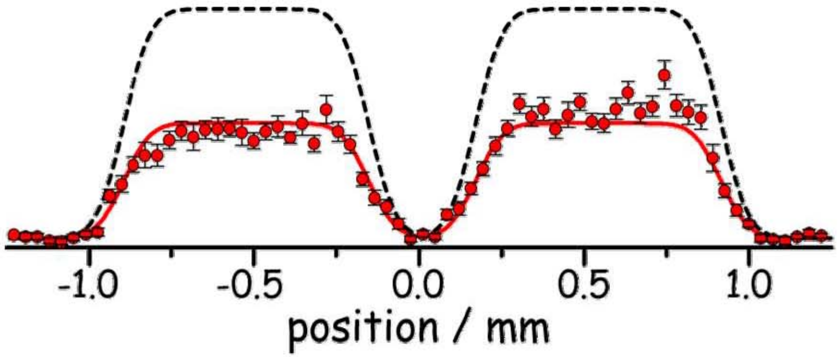
$$(\langle 3| + \langle 2|)\, |\Psi_{out}\rangle = |\Psi_{left}\rangle - |\Psi_{right}\rangle$$
$$(\langle 3| - \langle 2|)\, |\Psi_{out}\rangle = |\Psi_{left}\rangle + |\Psi_{right}\rangle$$

restores the interference (in a sub-ensemble)

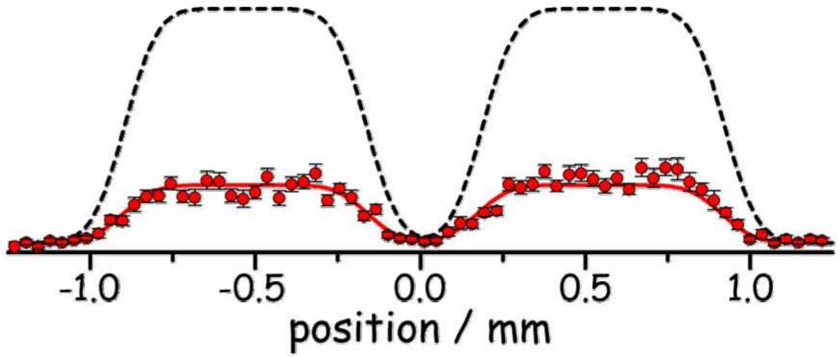
quantum eraser: experiment

Dürr et al., Opt. Commun. **179** (2000) 323

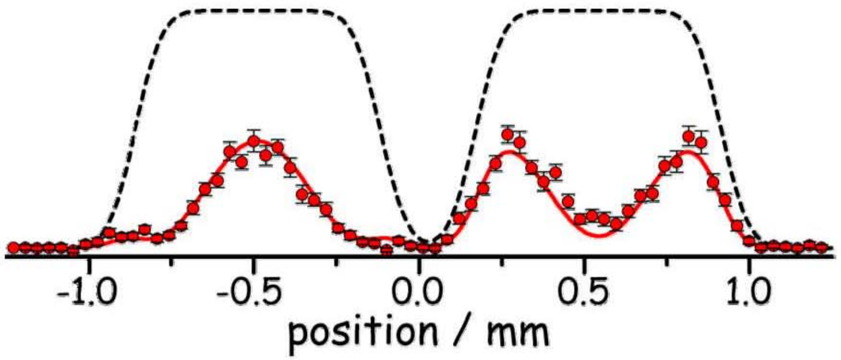
all atoms:



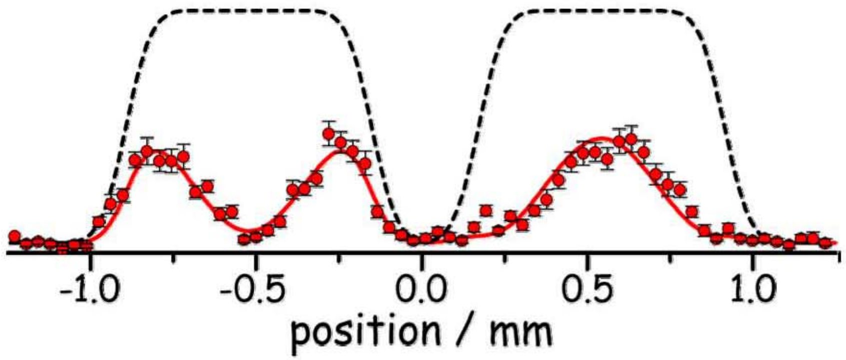
atoms in state |F=2⟩ or |F=3⟩:



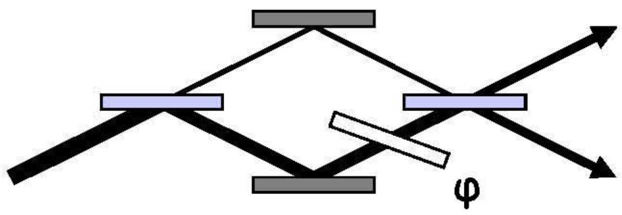
atoms in state |F=2⟩ + |F=3⟩:



atoms in state |F=2⟩ - |F=3⟩:



which-way information in asymmetric interferometers



asymmetric interferometer

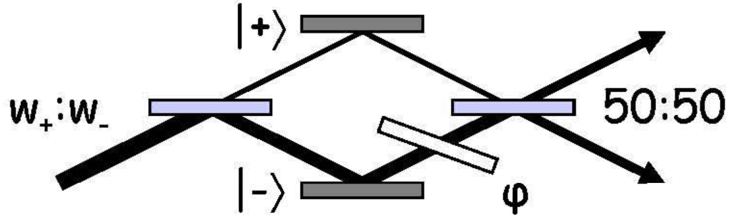
wave nature:
spatial interference pattern with (reduced) fringe visibility V

particle nature:
which-way information is a-priori available:
predictability of the ways P

predictability and visibility

Dürr et al., Am. J. Phys. **68** (2000) 1024

$\rho = \begin{pmatrix} w_+ & \rho_\pm \\ \rho_\pm & w_- \end{pmatrix}$



$|\psi_{out}\rangle = \dfrac{|+\rangle \pm e^{i\varphi}|-\rangle}{2^{1/2}}$

intensity: $I(\varphi) \sim \langle\psi_{out}|\rho|\psi_{out}\rangle \sim \frac{1}{2}(1 \pm 2\rho_\pm \cos\varphi)$

visibility: $V := (I_{max}-I_{min})/(I_{max}+I_{min}) = 2\rho_\pm$

predictability: $P := w_+ - w_-$ (with $w_+ > w_-$)

P-V connection:

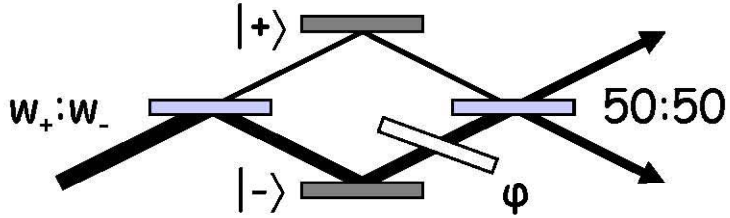
$$\begin{aligned} tr(\rho^2) &= w_+^2 + w_-^2 + 2\rho_\pm^2 \\ &= (w_+^2 - 2w_+w_- + w_-^2 + w_+^2 + 2w_+w_- + w_-^2 + 4\rho_\pm^2)/2 \\ &= (P^2 + (w_+ + w_-)^2 + V^2))/2 \\ &= (P^2 + 1 + V^2)/2 \\ &\leq 1 \end{aligned}$$

$P^2 + V^2 \leq 1$

Heisenberg-Robertson uncertainty relation

Dürr et al., Am. J. Phys. **68** (2000) 1024

$$\rho = \begin{pmatrix} w_+ & \rho_\pm \\ \rho_\pm & w_- \end{pmatrix}$$



Pauli spin operators σ_x, σ_y and σ_z:

$$[\sigma_j,\sigma_k] = 2i\Sigma_l \varepsilon_{jkl}\sigma_l$$

$\langle\sigma_x\rangle = tr(\rho\sigma_x) = 2\rho_\pm = V$ $\quad \Delta\sigma_x = (1-V^2)^{1/2}$ wave nature

$\langle\sigma_y\rangle = tr(\rho\sigma_y) = 0$ $\quad \Delta\sigma_y = 1$

$\langle\sigma_z\rangle = tr(\rho\sigma_z) = w_+-w_- = P$ $\quad \Delta\sigma_z = (1-P^2)^{1/2}$ particle nature

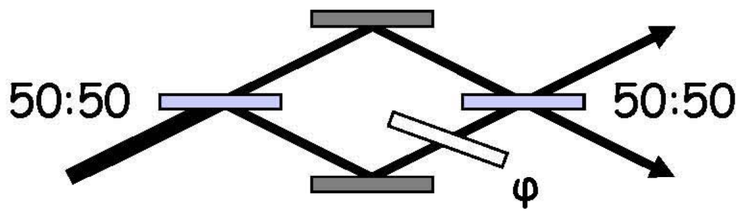
$$(1-V^2)^{1/2} = \Delta\sigma_x\Delta\sigma_y \geq \langle[\sigma_x,\sigma_y]\rangle/2i = \langle\sigma_z\rangle = P \implies P^2+V^2 \leq 1$$

$$(1-P^2)^{1/2} = \Delta\sigma_y\Delta\sigma_z \geq \langle[\sigma_y,\sigma_z]\rangle/2i = \langle\sigma_x\rangle = V \implies P^2+V^2 \leq 1$$

$$\Delta\sigma_z\Delta\sigma_x \geq \langle[\sigma_z,\sigma_x]\rangle/2i = \langle\sigma_y\rangle = 0$$

$P^2 + V^2 \leq 1$ <==> uncertainty relation for spin components

which-way information in symmetric interferometers

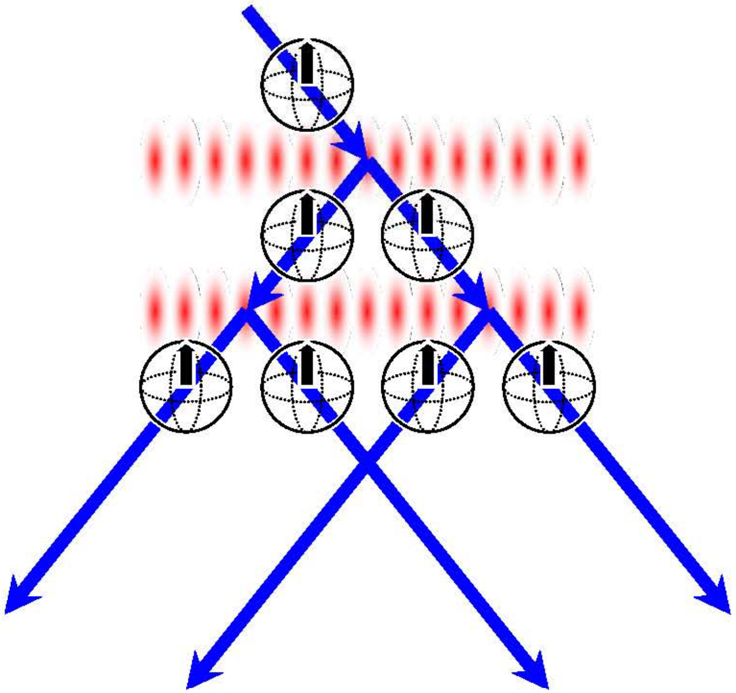


symmetric interferometer

wave nature:
spatial interference pattern with fringe visibility V

particle nature:
which-way information obtained from which-way marker:
distinguishability of the ways D

no which-way information stored

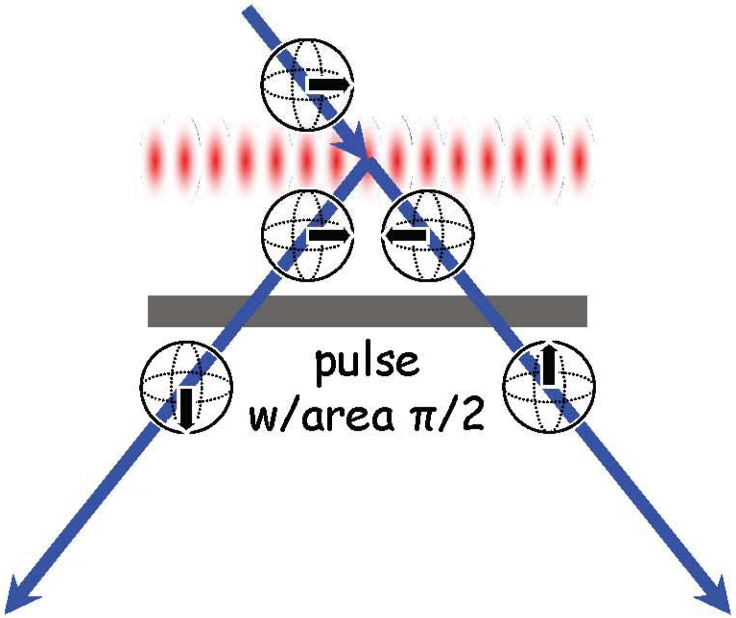


measurement basis	left way	right way
$\|F=2\rangle$	1/4	1/4
$\|F=3\rangle$	1/4	1/4

likelihood for guessing the way right
$L := 1/4 + 1/4 = 1/2$

acquired which-way knowledge
$K := 2L - 1 = 0$

complete which-way information stored



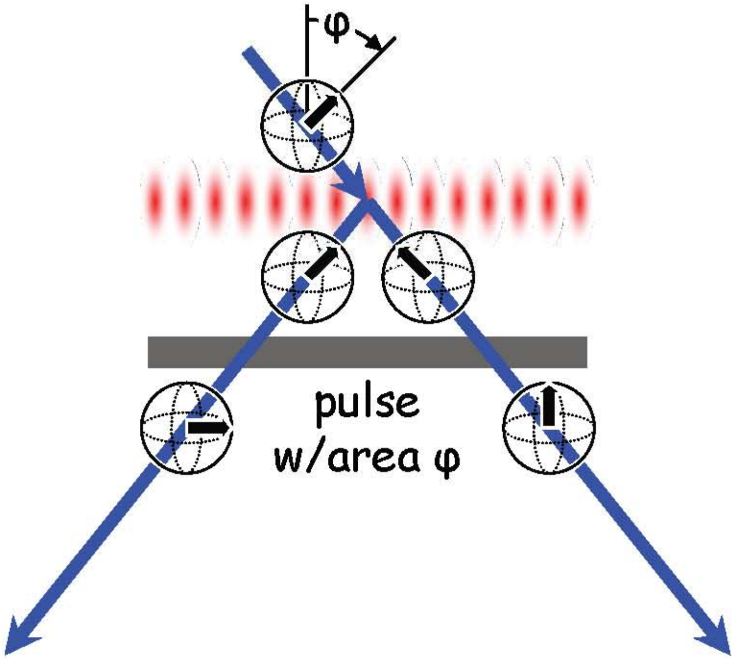
$$\frac{|\Psi_{left}, F=3\rangle - |\Psi_{right}, F=2\rangle}{2^{1/2}}$$

measurement basis	left way	right way
$\vert F=2\rangle$	0	1/2
$\vert F=3\rangle$	1/2	0

likelihood for guessing the way right
$L = 1/2 + 1/2 = 1$

acquired which-way knowledge
$K = 2L - 1 = 1$

incomplete which-way information stored



$$\frac{|\Psi_{left}\rangle(\cos\varphi|2\rangle + \sin\varphi|3\rangle) - |\Psi_{right},2\rangle}{2^{1/2}}$$

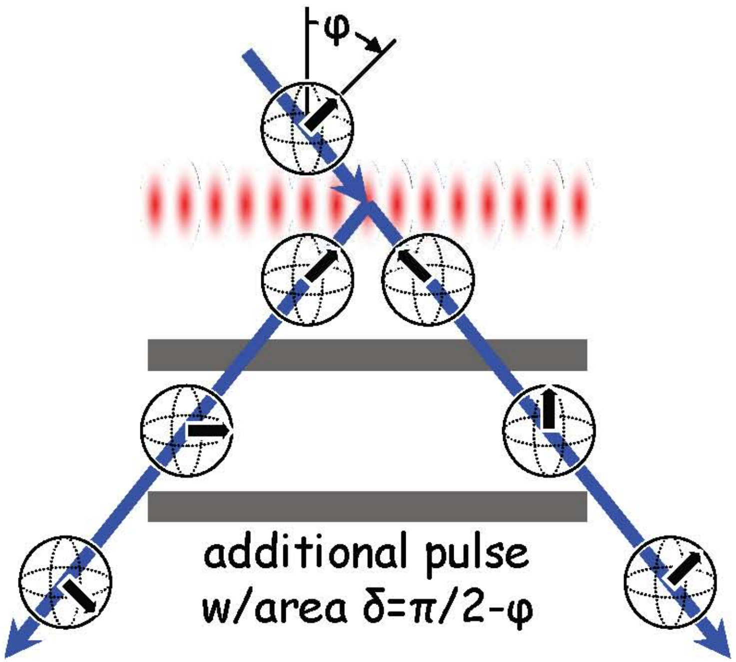
measurement basis	left way	right way
$\|F=2\rangle$	$\frac{1}{2}\cos^2\varphi$	$\frac{1}{2}$
$\|F=3\rangle$	$\frac{1}{2}\sin^2\varphi$	0

likelihood for guessing the way right
$L = \frac{1}{2} + \frac{1}{2}\sin^2\varphi$

acquired which-way knowledge
$K = 2L - 1 = \sin^2\varphi$

is this the maximum possible which-way information ?

distinguishability



measurement basis	left way	right way
$\cos\delta\|2\rangle - \sin\delta\|3\rangle$	$\frac{1}{2}\sin^2\delta$	$\frac{1}{2}\cos^2\delta$
$\sin\delta\|2\rangle + \cos\delta\|3\rangle$	$\frac{1}{2}\cos^2\delta$	$\frac{1}{2}\sin^2\delta$

likelihood for guessing the way right

$$L = \tfrac{1}{2}\cos^2\delta + \tfrac{1}{2}\cos^2\delta = \tfrac{1}{2}(1 + \sin\varphi)$$

$$\frac{|\Psi_{left}\rangle(\cos\varphi|2\rangle + \sin\varphi|3\rangle) - |\Psi_{right},2\rangle}{2^{1/2}}$$

acquired which-way knowledge

$$K = 2L - 1 = \sin\varphi$$

visibility of the interference

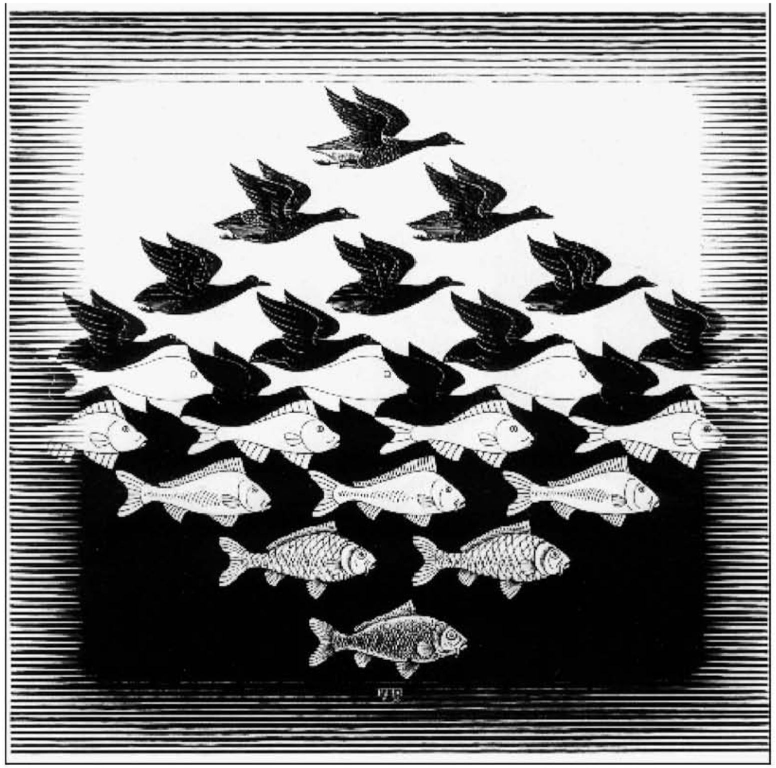
$$V = |\cos\varphi|$$

distinguishability of the ways

$$D := K_{max} = |\sin\varphi|$$

wave-particle duality made quantitatively

Jaeger et al., Phys. Rev. A **51** (1995) 54
Englert, Phys. Rev. Lett. **77** (1996) 2154



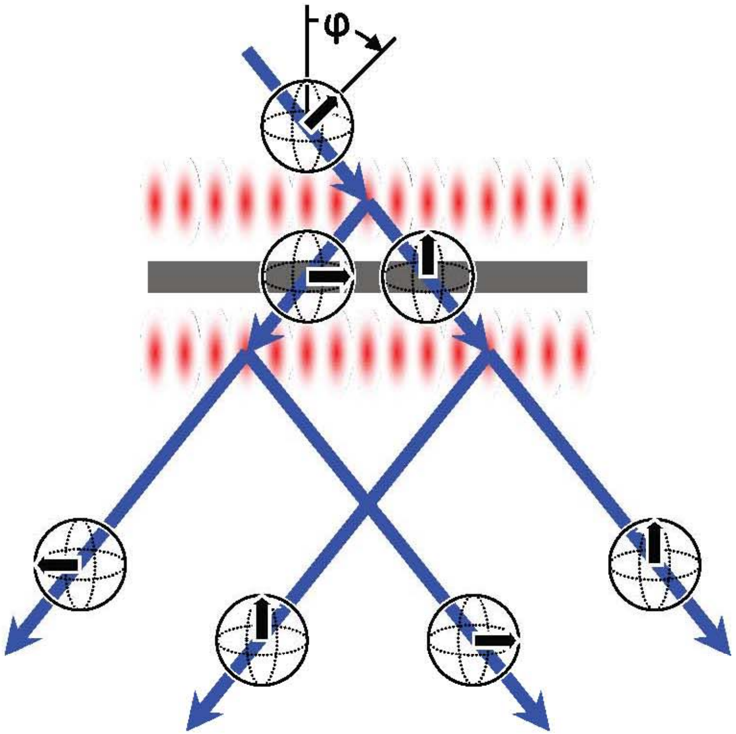
visibility of the interference
$V = |\cos\varphi|$

distinguishability of the ways
$D = |\sin\varphi|$

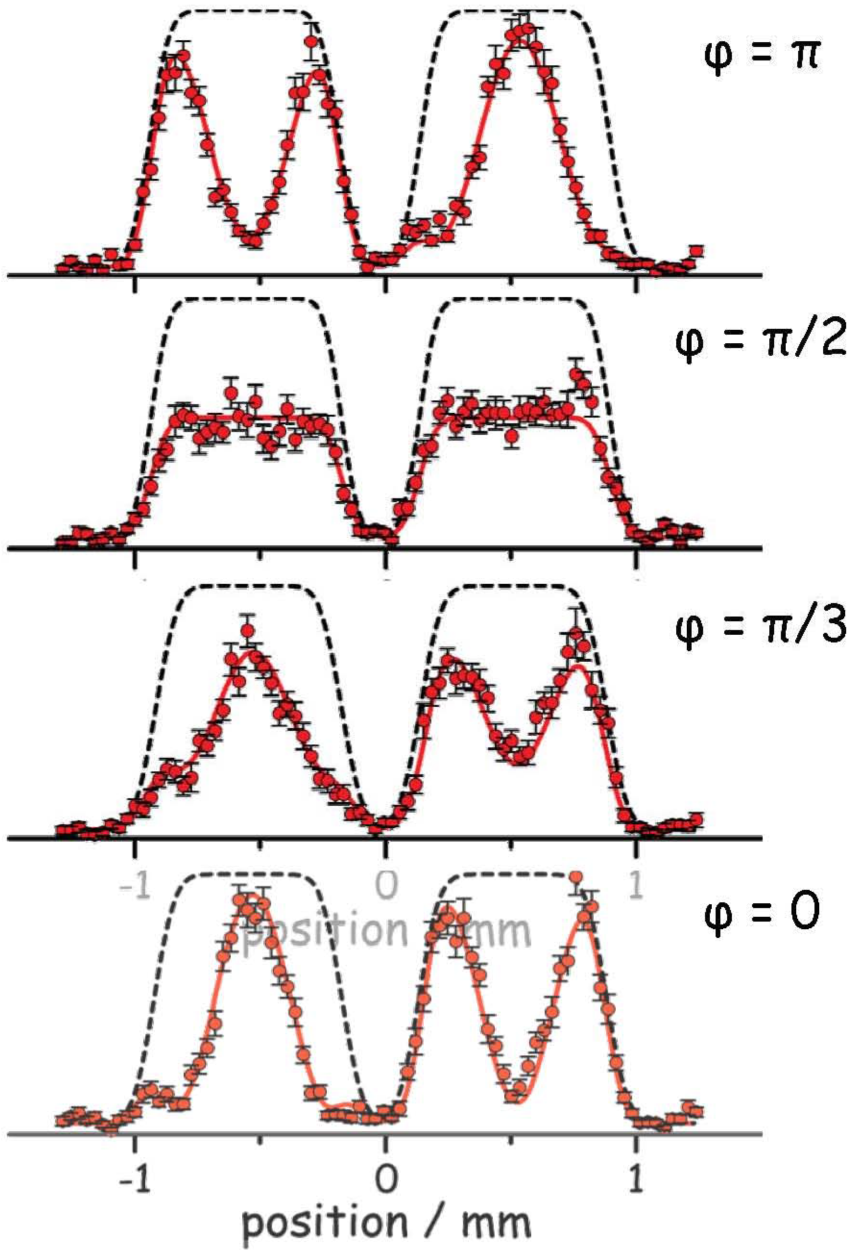
duality relation
$D^2 + V^2 \leq 1$

experimental observation

Dürr et al., Phys. Rev. Lett. **81** (1998) 5705

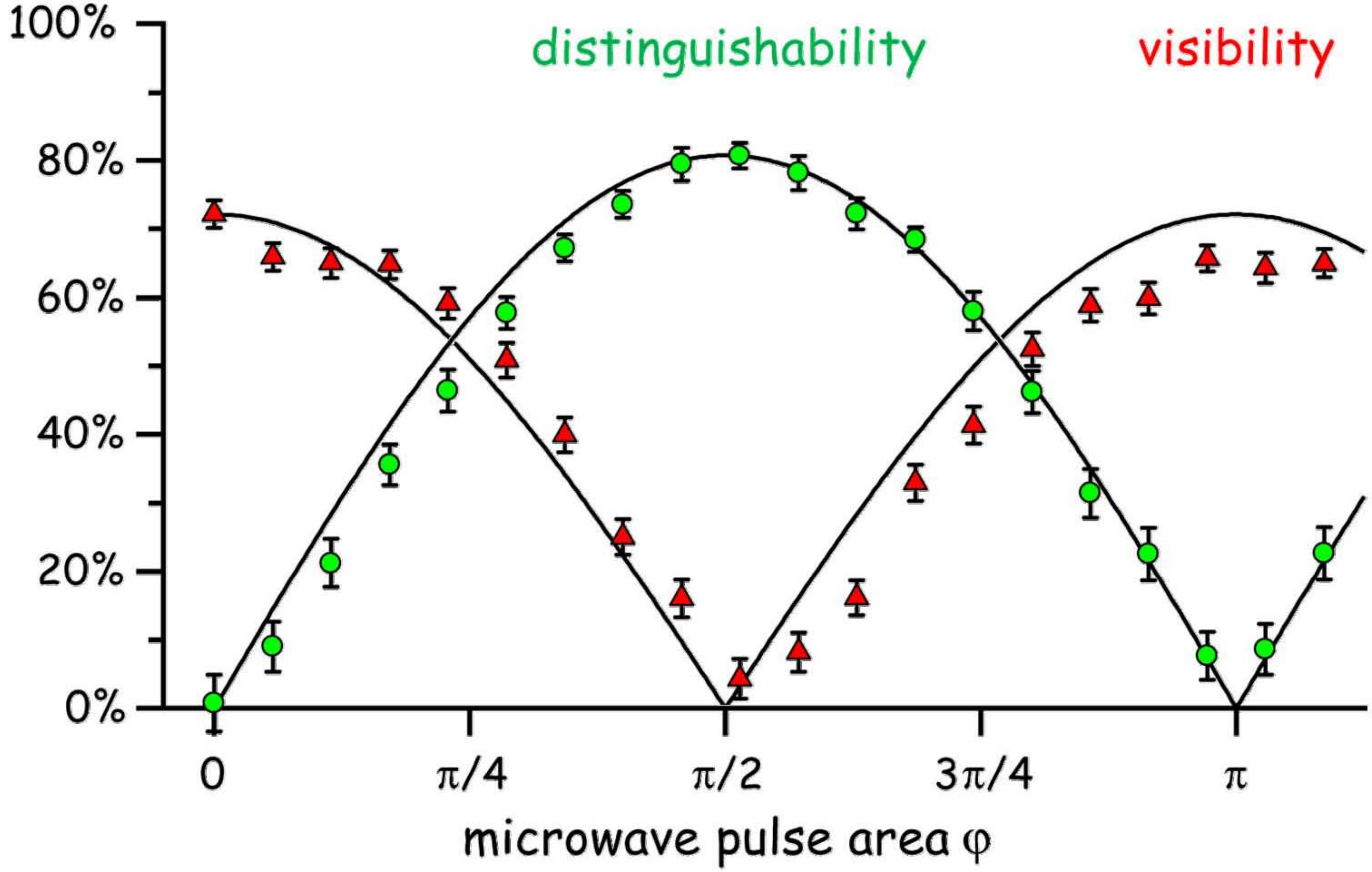


$V = |\cos\varphi|$



visibility and distinguishability

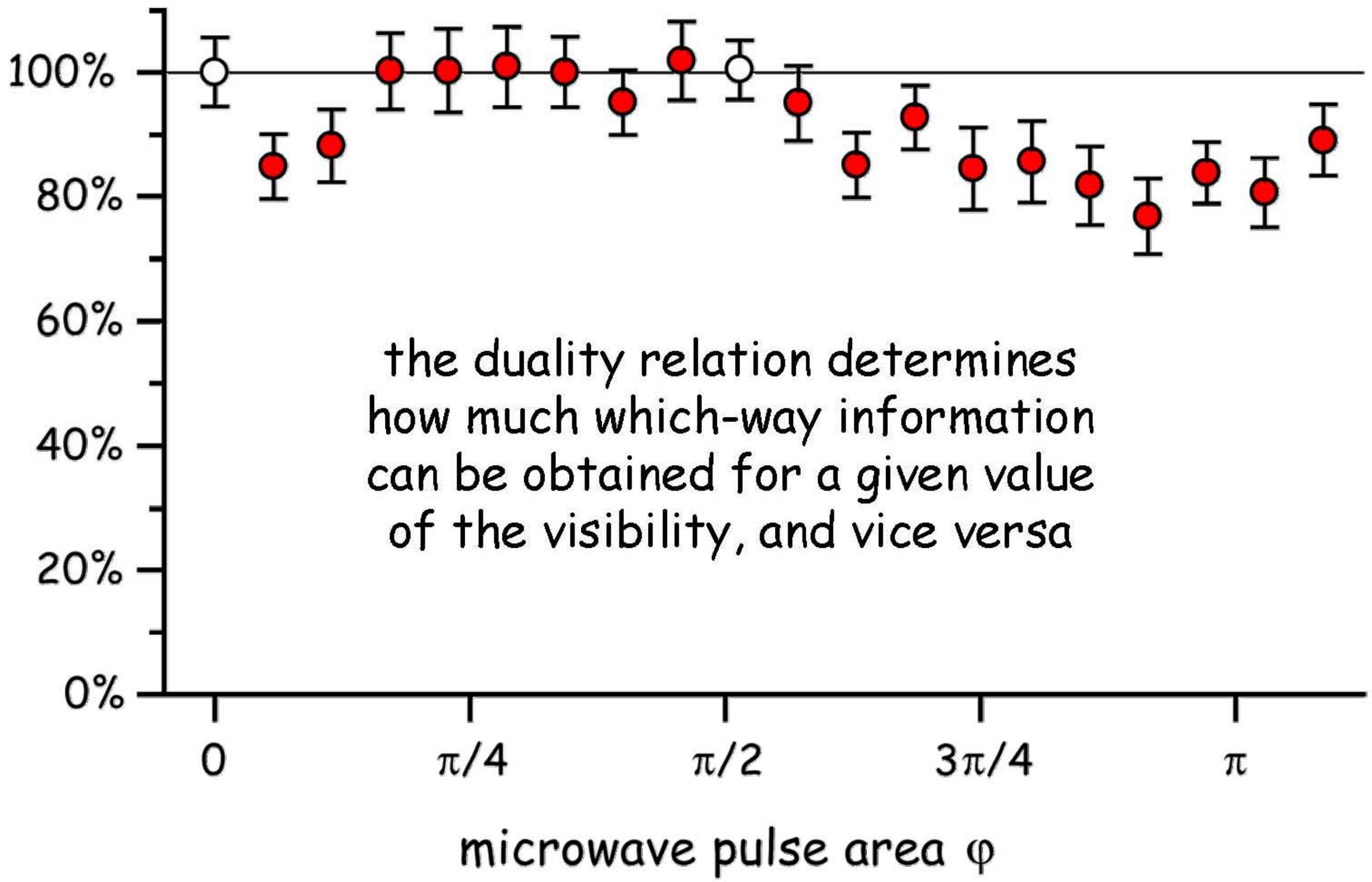
Dürr et al., Phys. Rev. Lett. **81** (1998) 5705



reduction from 100% due to technical imperfections

experimental test of the duality relation

Dürr et al., Phys. Rev. Lett. **81** (1998) 5705



the duality relation determines how much which-way information can be obtained for a given value of the visibility, and vice versa

see also Schwindt et al., Phys. Rev. A **60** (1999) 4285

what did we learn about wave-particle duality ?

- enforced by entanglement, not $\Delta x \Delta p$ uncertainty relation
- no "classical" description with random momentum kicks
- visibility and distinguishability limited by the duality relation
- interference pattern is restored by erasing which-way information

thank you for your attention

wave-particle duality:

Europhys. Lett. **27** (1994) 115
Phys. Rev. Lett. **78** (1997) 2038
Phys. Rev. Lett. **81** (1998) 5705
Nature **395** (1998) 33
Am. J. Phys. **68** (2000) 1024
Opt. Commun. **179** (2000) 323
Adv. AMO. Phys. **42** (2000) 29

atom interferometry:

Europhys. Lett. **34** (1996) 343
Qu. Semicl. Opt. **8** (1996) 531
J. Mod. Opt. **44** (1997) 1863
Phys. Rev. A **56** (1997) 2972
Phys. Rev. A **59** (1999) 1495
Appl. Phys. B **69** (1999) 303