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SMR.1587 - 16

SCHOOL AND WORKSHOP ON QUANTUM ENTANGLEMENT, DECOHERENCE, INFORMATION, AND GEOMETRICAL PHASES IN COMPLEX SYSTEMS
(1 November - 12 November 2004)

Josephson Qubits: Manipulation and Decoherence

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Josephson Qubits:
Manipulations and Decoherence

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- Physics of Josephson junction qubits, experiments
- Decoherence, classification, models, results
- Decoherence at symmetry points
- Low-T decoherence, preparation, renormalization

Quantum computation:

- store information in qubit (spin, …)
- program: manipulate qubits by controlling Hamiltonian
- model Hamiltonian:

$$H(t) = -\sum_{i=1}^{N}\left[B_x^i(t)\,\sigma_x^i + B_z^i(t)\,\sigma_z^i\right] + \sum_{i<j} J^{ij}(t)\,\sigma_+^i\sigma_-^j + h.c. + H_{\mathrm{meas}}(t) + H_{\mathrm{diss}}$$

single-bit logic gates, NOT, $\sqrt{\mathrm{NOT}}$
superpositions

two-bit gates, CNOT
entanglement

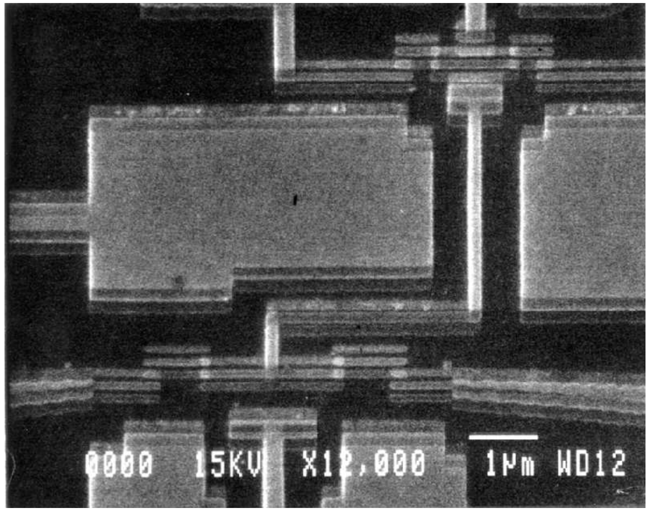
dephasing, relaxation

Requirements (DiVincenzo criteria)

1. N well defined qubits, scalable to large *N*
2. preparation of well-defined initial state
3. all single-bit gates and some two-bit gates, forming universal set
4. long phase coherence time $\tau_\varphi/\tau_{op} \geq 10^4$
5. read-out

1. Josephson Junction Qubits

Single-electron effects

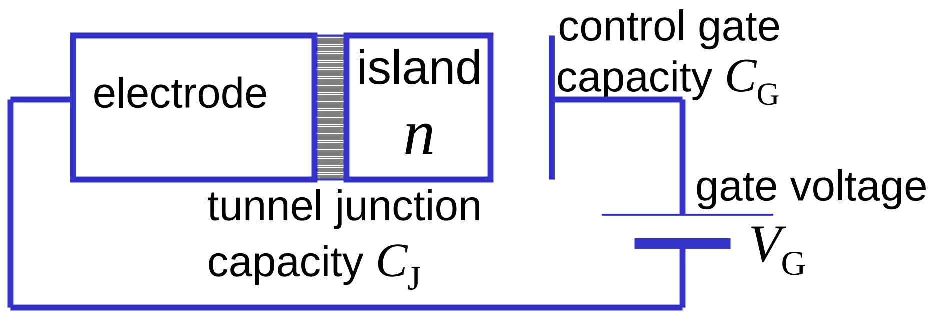


V. Bouchiat et al. (1996)

small-area tunnel junctions

typical area	100nm x 100nm
typical capacitance	$C \approx 10^{-15}$ F
charging energy scale	$E_C = e^2/2C \approx 1\mathrm{K}/k_B$

Single-electron box



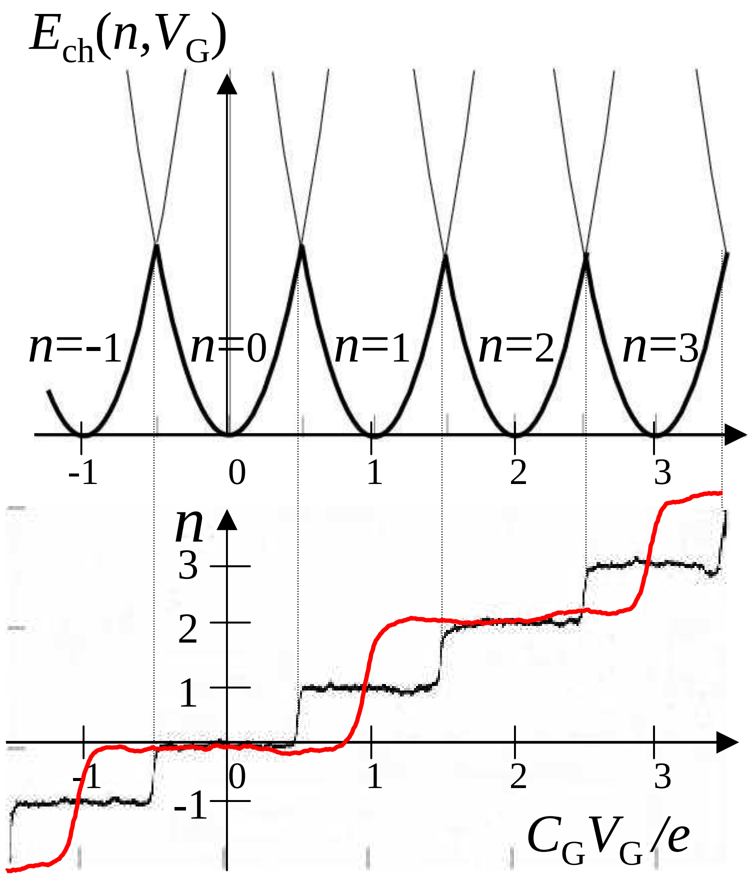
Charging energy

$$E_{ch}(n, V_G) = \frac{(ne - C_G V_G)^2}{2C}$$

$C = C_J + C_G$

n = ..., -2, -1, 0, 1, 2, ...

# of excess electrons on the island



Devoret et al. (90's)

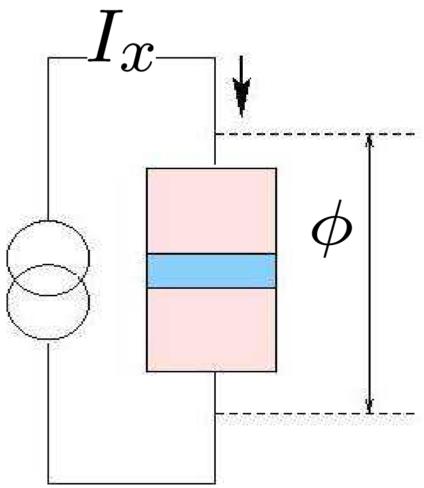
Superconducting Single-charge Box: $E_C \ll \Delta$

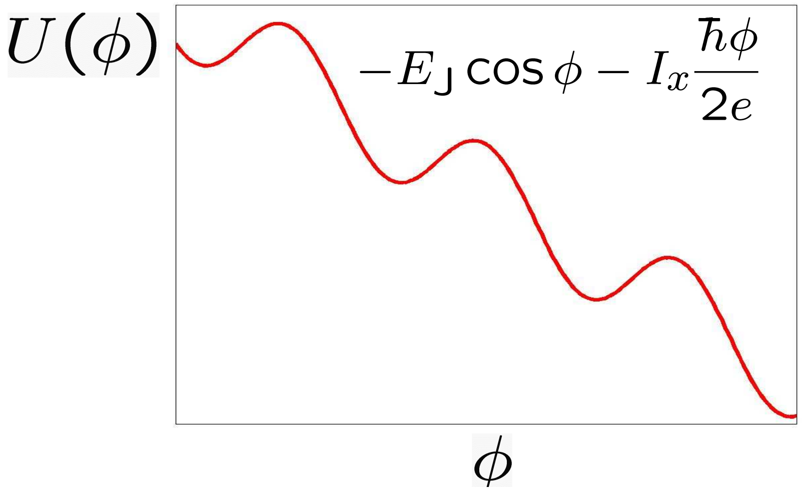
complete suppression of quasiparticle tunneling

coherent tunneling of Copper pair charge $2e$

Josephson Junctions

classical and quantum description





tilted washboard potential
energy scale:

$$E_{\mathrm{J}} = \frac{\hbar}{2e} I_{\mathrm{C}}$$

Josephson relations $\quad I_{\mathrm{S}} = I_{\mathrm{C}} \sin\phi \quad ; \quad 2eV = \hbar\dot{\phi} \quad ; \quad I_{\mathrm{C}} \propto \frac{\Delta}{R}$

Balance of currents $\quad \frac{\hbar}{2e} C\ddot{\phi} + \frac{1}{R}\frac{\hbar}{2e}\dot{\phi} + I_c \sin\phi = I_x$

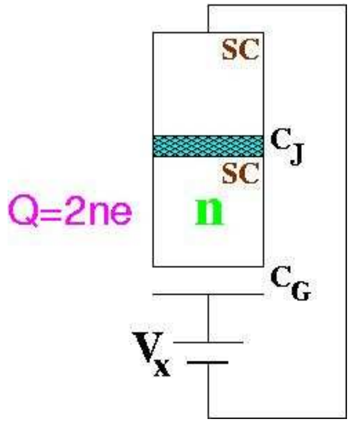
Hamiltonian

$$H = \frac{Q^2}{2C} - E_{\mathrm{J}}\cos\phi - I_x \frac{\hbar\phi}{2e} + H_{\mathrm{diss}}(\phi)$$

charge and phase
are conjugate variables

$$Q = \frac{\hbar}{i}\frac{d}{d\hbar\phi/2e}$$

Josephson charge qubit



- charging energy (Cooper-pairs)

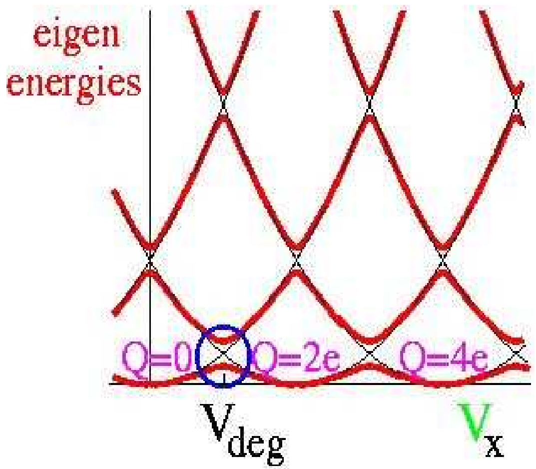
$$E_{\mathrm{ch}}(n, V_{\mathrm{x}}) = \frac{(2ne - C_{\mathrm{G}} V_{\mathrm{x}})^2}{2(C_{\mathrm{g}} + C_{\mathrm{J}})}$$

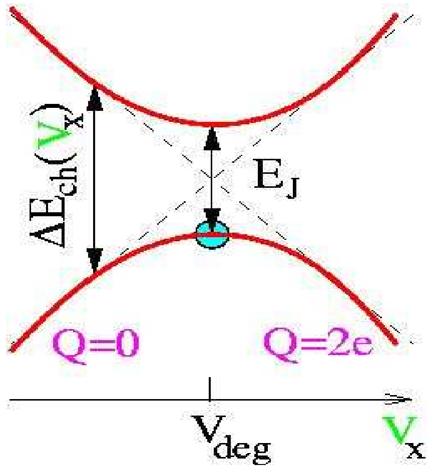
- Josephson coupling

$$E_{\mathrm{J}} \cos\phi \quad \rightarrow \quad \frac{E_{\mathrm{J}}}{2} |n\rangle \langle n \pm 1|$$

- Hamiltonian

$$H = \sum_n \left[E_{\mathrm{ch}}(n, V_{\mathrm{x}}) |n\rangle\langle n| + \frac{E_{\mathrm{J}}}{2} |n\rangle \langle n \pm 1| \right]$$





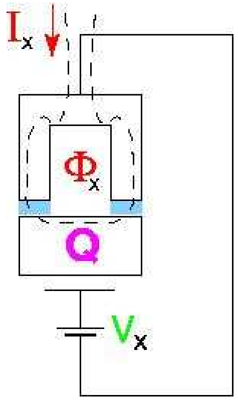
$$E_{\mathrm{C}} \gg E_{\mathrm{J}}, \quad C \approx 10^{-15}\mathrm{F}$$

2-state system
= qubit

Shnirman, G.S., Hermon ‘97

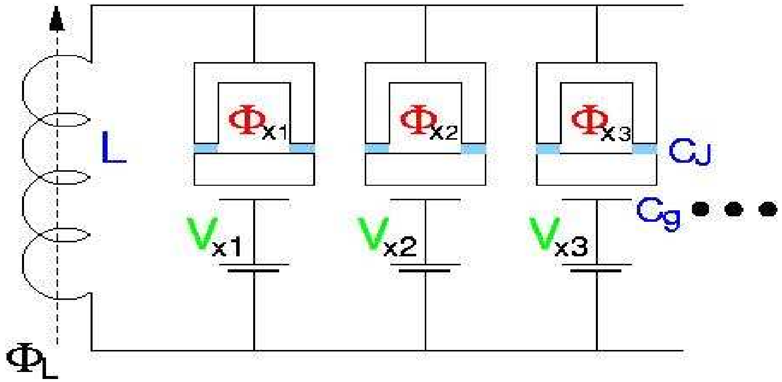
$$H = -\frac{1}{2} \Delta E_{\mathrm{ch}}(V_{\mathrm{x}})\, \sigma_z - \frac{1}{2} E_{\mathrm{J}}\, \sigma_x$$

Voltage- and flux-controlled qubit



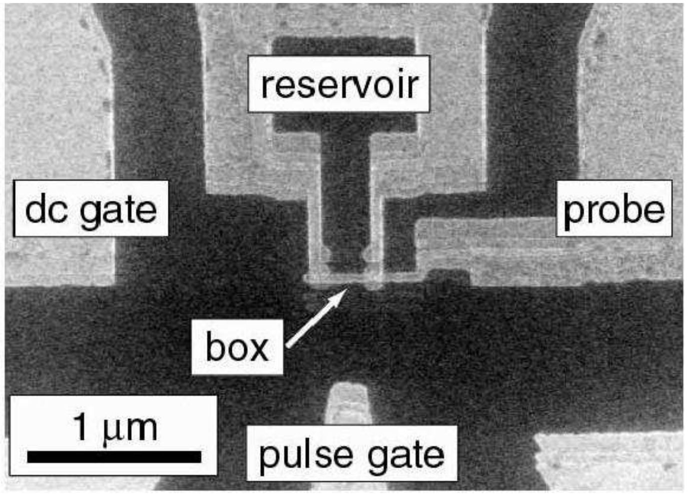
$$H = -\frac{1}{2}\Delta E_{\mathrm{ch}}(V_{\mathrm{x}})\,\sigma_z - \frac{1}{2}E_{\mathrm{J}}(\Phi_{\mathrm{x}})\,\sigma_x$$

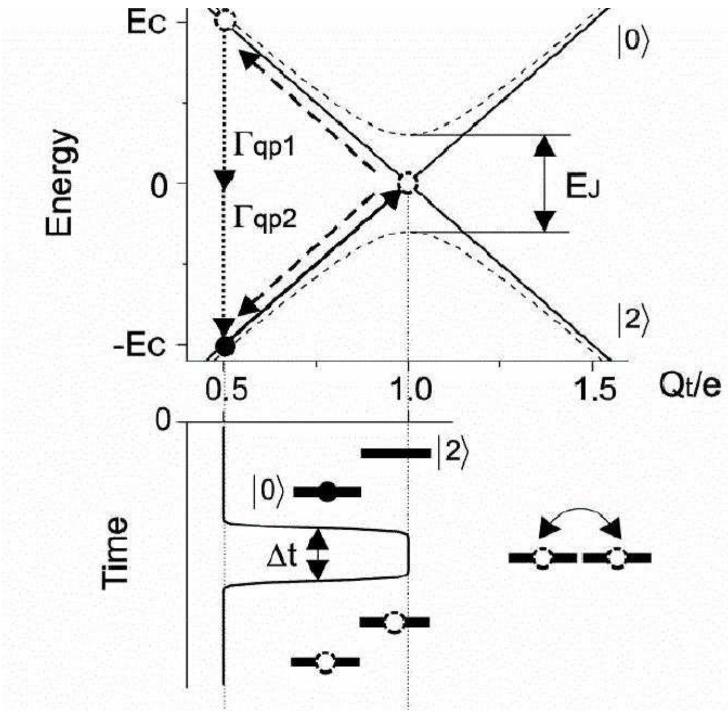
Qubits coupled by an *LC* – oscillator

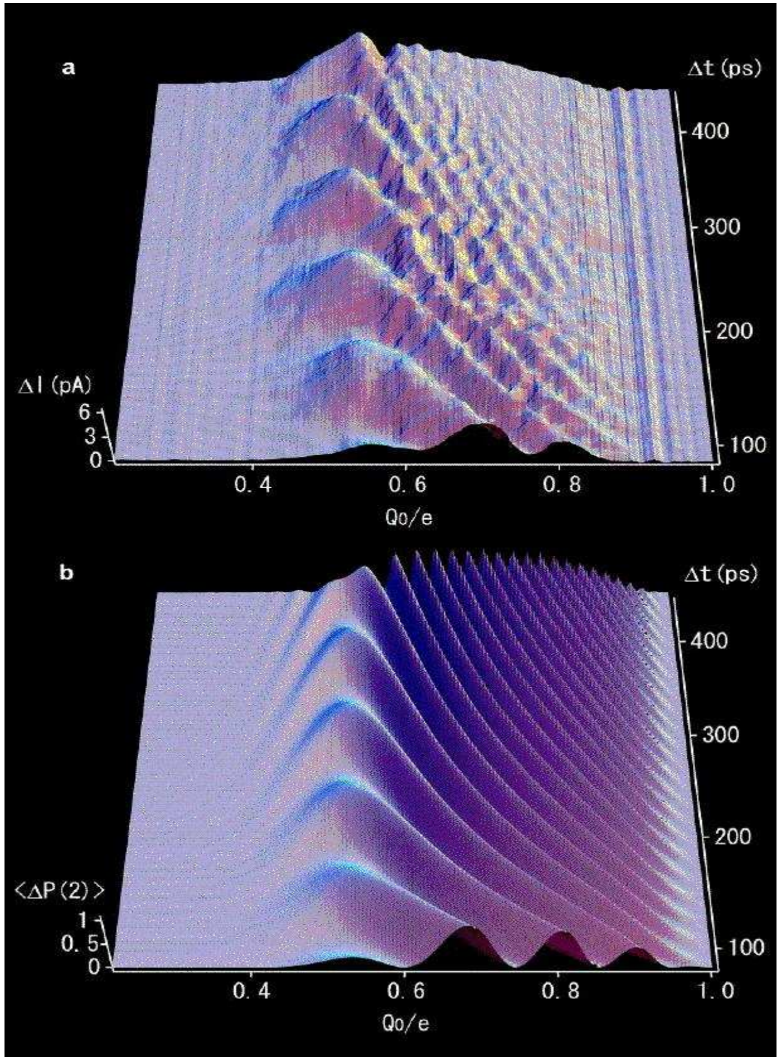


$$H = -\frac{1}{2}\sum_{i=1}^{N}\left[\Delta E_{\mathrm{ch}}(V_{\mathrm{x}i})\sigma_z^i + E_{\mathrm{J}}(\Phi_{\mathrm{x}i})\sigma_x^i\right] + \sum_{i<j}\pi^2\left(\frac{C_{\mathrm{g}}}{C_{\mathrm{J}}}\right)^2 \frac{E_{\mathrm{J}}(\Phi_{\mathrm{x}i})E_{\mathrm{J}}(\Phi_{\mathrm{x}j})}{\Phi_0^2/L}\sigma_y^i\sigma_y^j$$

Observation of coherent oscillations : Nakamura, Pashkin, and Tsai, ‘99



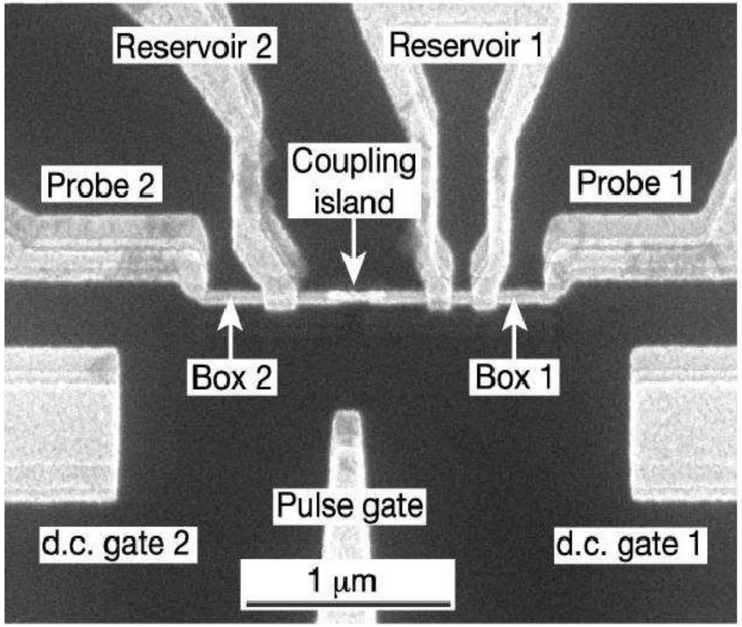


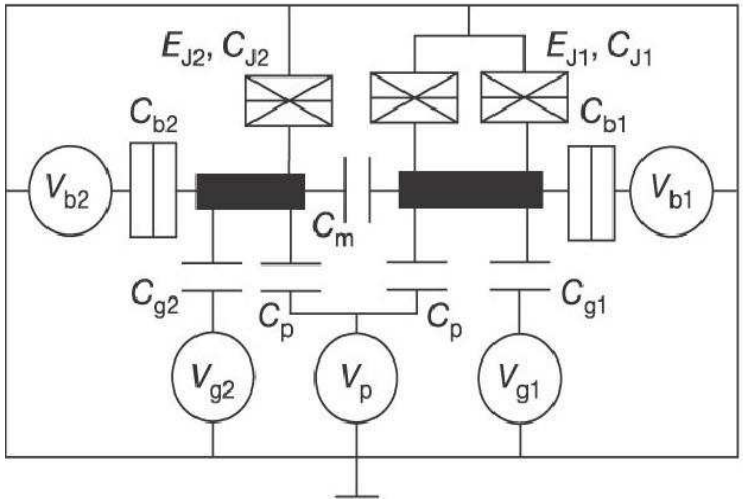


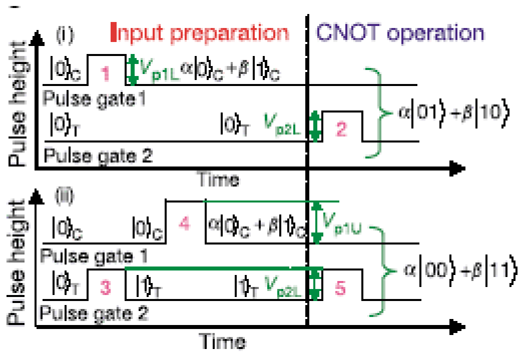
≈ 50 oscillations

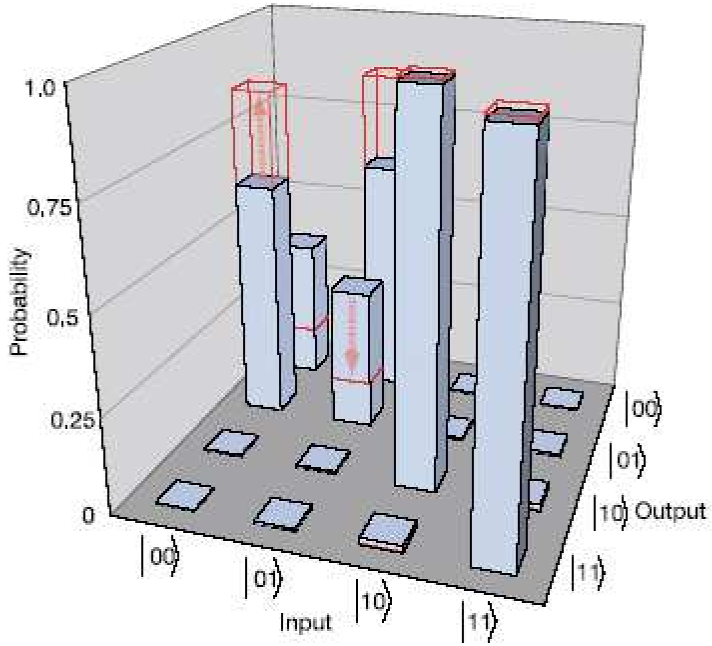
$\tau_{op} \approx 50 \dots 100$ psec, $\tau_{\varphi} \approx 5$ nsec

2 Coupled charge qubits/CNOT operation: Nakamuro et al. 02, 03





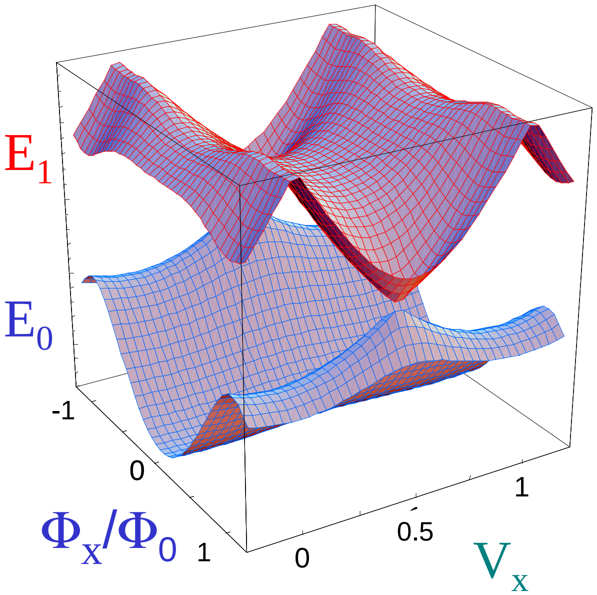


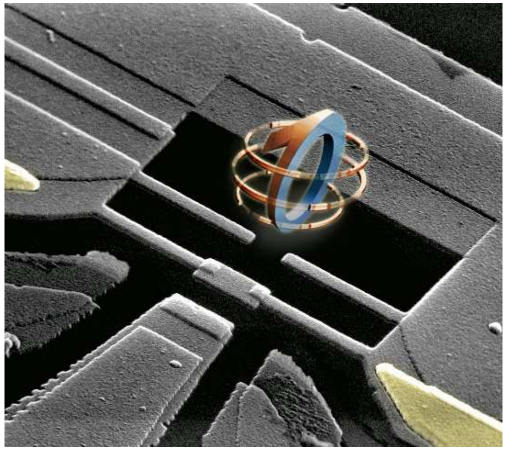


Charge-phase qubit

Devoret, Esteve, …(Saclay)

$$H = -\frac{1}{2} E_{ch}(V_x)\,\sigma_z - \frac{1}{2} E_J(\Phi_x)\,\sigma_x$$





Quantronium

$$E_C \approx E_J$$

less sensitive to charge noise

Operation at saddle point

$E_{ch}(V_{x0}) = 0$, $dE_J(\Phi_{x0})/d\Phi_x = 0$

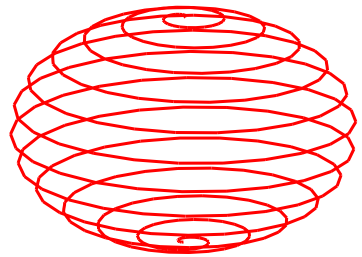
- minimizes noise effects
- voltage fluctuations couple transverse
- flux fluctuations couple quadratically

$$H = -\frac{1}{2} E_J(\Phi_{x0})\sigma_x - \frac{1}{2}\frac{\partial E_{ch}}{\partial V_x}\Big|_{V_{x0}} \delta V_x\, \sigma_z - \frac{1}{4}\frac{\partial^2 E_J}{\partial \Phi_x^2}\Big|_{\Phi_{x0}} \delta\Phi_x^2\, \sigma_x$$

Tool box:

Rabi oscillations

$$H = -\frac{1}{2}B_z\,\sigma_z - \frac{1}{2}\Omega_R(\cos\omega t\;\sigma_x + \sin\omega t\;\sigma_y)$$

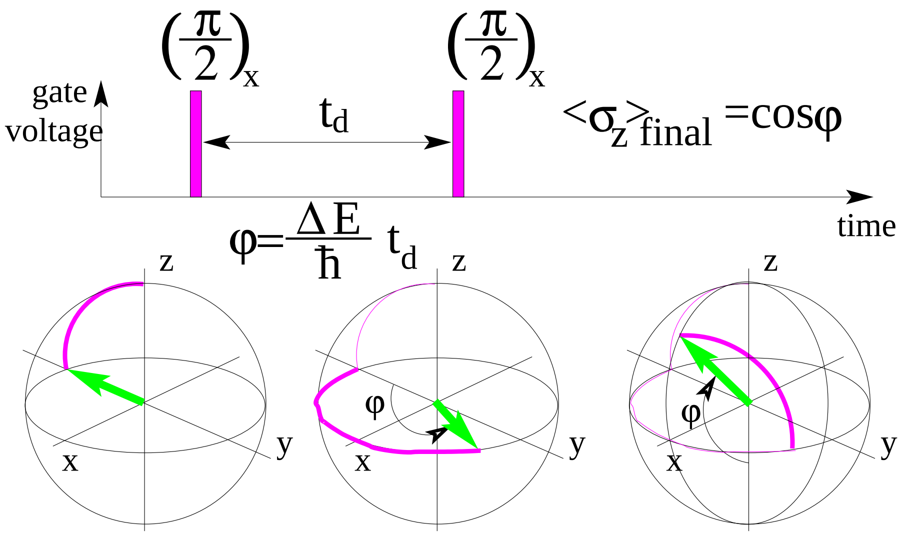


in lab frame

operate at resonance $\omega = B_z$

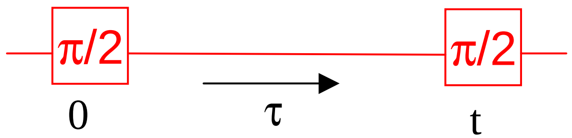
in rotating frame
(unitary transformation) $H' = -\frac{1}{2}\Omega_R\sigma_x$

Ramsey fringes

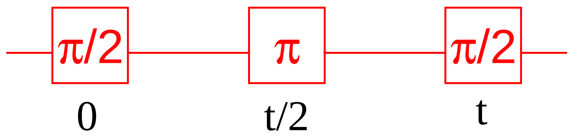


Echo experiment

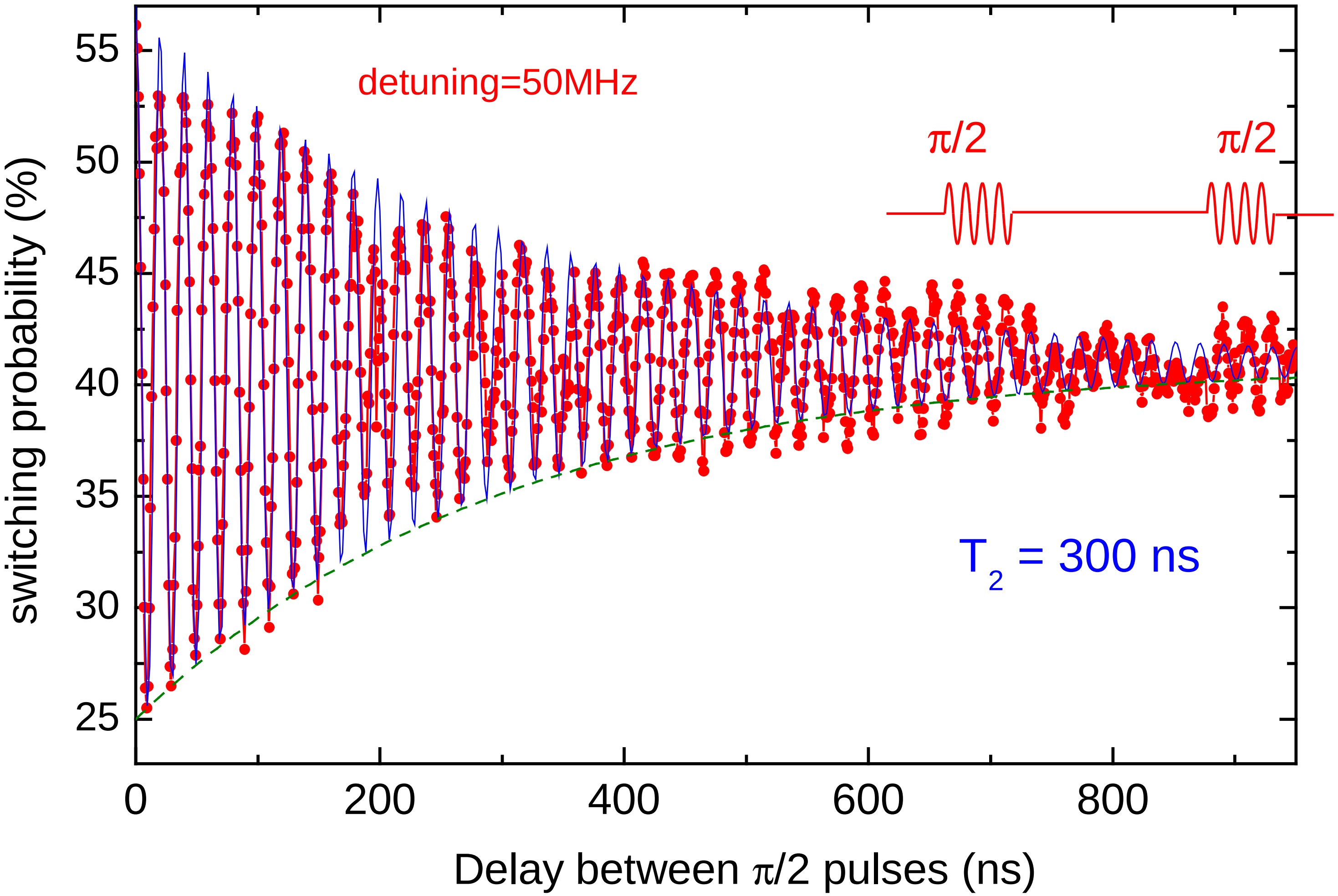
Free decay (Ramsey fringes)



Echo signal



Decay of Ramsey fringes at optimal point



Vion et al., Science 02, …

2. Coherence and Dephasing

Bloch equations, relaxation ($\Gamma_{\rm rel} = 1/T_1$) and dephasing ($\Gamma_\varphi = 1/\tau_\varphi = 1/T_2$)

$$\frac{d}{dt}\mathbf{M} = \mathbf{B}\times\mathbf{M} - \frac{1}{T_1}\left(M_z - M_0\right)\mathbf{e}_z - \frac{1}{T_2}\left(M_x\mathbf{e}_x + M_y\mathbf{e}_y\right)$$

Bloch (46,57)
Redfield (57)

For 2-level system (spin 1/2) **M** is determined by density matrix

$$\mathbf{M} = \langle\sigma\rangle = Tr[\sigma\rho]$$

$$\rho = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix}$$

$$\dot\rho_{00} = -\Gamma_\uparrow\rho_{00} + \Gamma_\downarrow\rho_{11}$$

$$\dot\rho_{11} = \Gamma_\uparrow\rho_{00} - \Gamma_\downarrow\rho_{11}$$

$$\dot\rho_{01} = -i\mathrm{B_z}\rho_{01} - \Gamma_\varphi\rho_{01}$$

$\Gamma_{\rm rel} = \Gamma_\uparrow + \Gamma_\downarrow$ and $M_0 = (\Gamma_\downarrow - \Gamma_\uparrow)/(\Gamma_\uparrow + \Gamma_\downarrow)$

Sources of noise

‘external’:
- electro smog *(reduce by shielding)*
- background charge fluctuations
- nuclear spin flips
- …

intrinsic:
- quasiparticle tunneling *(suppressed at low T)*
- fluctuations due to control circuit
- fluctuations due to measurement device *(need quantum switch)*
- …

Properties of noise

Gaussian or non-Gaussian
- spectrum: Ohmic (white), 1/f, ….
- different coupling

Models for noise and classification

$$H = -\frac{1}{2}\Delta E\,\sigma_z \;-\frac{1}{2}X\cos\eta\,\sigma_z - \frac{1}{2}X\sin\eta\,\sigma_x - \frac{1}{4}X^2\,\sigma_z + H_{bath}$$

longitudinal – transverse – quadratic (longitudinal) …

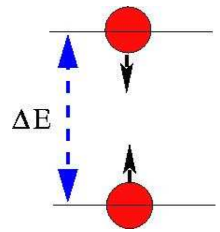
Power spectrum:

$$S_X(\omega) = \frac{1}{2}\int dt \left\langle \{X(t),X(0)\}_+ \right\rangle e^{i\omega t} \quad , \quad \chi'' = A_X(\omega) = \frac{1}{2}\int dt \left\langle [X(t),X(0)]_- \right\rangle e^{i\omega t}$$

model \ noise	Bosonic bath	Spin bath
	$X = \sum_j c_j(a_j + a_j^\dagger)\,,\ H_{bath} = \sum_j \omega_j\, a_j^\dagger a_j$ $J(\omega) \equiv \sum_j c_j^2\,\delta(\omega-\omega_j)$	$X = \sum_j c_j\sigma_{x,j}\,,\ H_{bath} = \sum_j \omega_j\,\sigma_{z,j}$ $J(\omega) \equiv \sum_j c_j^2\,\delta(\omega-\omega_j)$
Ohmic	$J(\omega) = \alpha\omega$ $S_X(\omega) = \alpha\omega\coth\frac{\omega}{2T}$	
1/f (Gaussian)	$J(\omega) = \alpha$ $S_X(\omega) = \alpha\coth\frac{\lvert\omega\rvert}{2T_{1/f}} \approx \frac{E_{1/f}^2}{\lvert\omega\rvert}$	$J(\omega) = E_{1/f}^2/\omega$ $S_X(\omega) = E_{1/f}^2/\lvert\omega\rvert$

Transverse coupling ⇒ relaxation

$$H = -\frac{1}{2}\Delta E\sigma_z - \frac{1}{2}X\sigma_x + H_{Bath}$$



Golden Rule:

$$\Gamma_\uparrow = \frac{2\pi}{\hbar}\frac{1}{4}\sum_{i,f}\rho_{ii}^{Bath}\left|\langle 0,i\,|\,X\sigma_x\,|1,f\rangle\right|^2\delta\left(E_i + \Delta E - E_f\right)$$ compare "P(E)-theory"

$$= \frac{2\pi}{\hbar}\frac{1}{4}\sum_{i,f}\rho_{ii}^{Bath}\langle i\,|\,X\,|\,f\rangle\langle f\,|\,X\,|\,i\rangle\frac{1}{2\pi\hbar}\int dt\exp\left[i\left(E_i + \Delta E - E_f\right)t/\hbar\right]$$

$$= \frac{1}{4\hbar^2}\int dt\sum_i\rho_{ii}^{Bath}\langle i\,|\,X(t)X(0)\,|\,i\rangle\exp\left[i\Delta Et/\hbar\right]$$

$$\Gamma_\uparrow = \frac{1}{4\hbar^2}\langle X(t)X(0)\rangle_{\omega=\Delta E/\hbar}$$

$$\Gamma_\downarrow = \frac{1}{4\hbar^2}\langle X(t)X(0)\rangle_{\omega=-\Delta E/\hbar}$$

$$\Rightarrow\quad \frac{1}{T_1} \equiv \Gamma_{\text{rel}} = \Gamma_\uparrow + \Gamma_\downarrow = \frac{1}{2\hbar^2}S_X(\omega = \Delta E/\hbar)$$

Longitudinal coupling $\Rightarrow$ pure dephasing

$$H = -\frac{1}{2} B_z \sigma_z - \frac{1}{2} X \sigma_z + H_{Bath}$$

X treated as classical, Gaussian random field

$$\left|\rho_{01}(t)\right| \propto \left\langle \exp\left(-\frac{i}{\hbar}\int_0^t X(\tau) d\tau \right) \right\rangle = \exp\left(-\frac{1}{2\hbar^2}\int_0^t d\tau_1 \int_0^t d\tau_2 \left\langle X(\tau_1) X(\tau_2) \right\rangle \right)$$

$$= \exp\left(-\frac{1}{2\hbar^2}\int \frac{d\omega}{2\pi} S_X(\omega) \frac{\sin^2(\omega t/2)}{(\omega/2)^2} \right) \approx \exp\left(-\frac{1}{2\hbar^2} S_X(\omega \approx 0) t \right)$$

$$\frac{\sin^2(\omega t/2)}{(\omega/2)^2} \approx 2\pi\delta(\omega) t$$

$\Rightarrow$

$$\Gamma_\varphi^* = \frac{1}{2\hbar^2} S_X(\omega \approx 0)$$

Linear coupling

$$H = -\frac{1}{2}\Delta E\,\sigma_z \;-\frac{1}{2}X\cos\eta\,\sigma_z - \frac{1}{2}X\;\sin\eta\;\sigma_x + H_{Bath}$$

Golden rule

exponential decay law $\propto e^{-\Gamma t}$

$$\frac{1}{T_1} = \Gamma_{\mathrm{rel}} = \frac{1}{2}S_X(\omega = \Delta E)\sin^2\eta$$

$$\frac{1}{T_2} = \Gamma_\varphi = \frac{1}{2}\frac{1}{T_1} + \underbrace{\frac{1}{2}S_X(\omega\approx 0)\cos^2\eta}_{\text{pure dephasing: } \Gamma_\varphi^*}$$

Example: Ohmic noise

$$S_X(\omega) = \alpha\,\omega\;\coth\frac{\omega}{2T}$$

$$\Rightarrow$$

$$\Gamma_{\mathrm{rel}} = \frac{\alpha}{2}\Delta E\;\coth\frac{\Delta E}{2T}\sin^2\eta$$

$$\Gamma_\varphi^* = \alpha T\cos^2\eta$$

Dephasing due to 1/f noise, T=0, nonlinear coupling, … ?

1/f noise,
longitudinal linear coupling

$$H = -\frac{1}{2}(\Delta E + X)\sigma_z + H_{Bath}$$

$$|\rho_{01}(t)| = \left\langle \exp\left(-i\int_0^t X(\tau)\,d\tau\right)\right\rangle = \exp\left(-\frac{1}{2}\int \frac{d\omega}{2\pi} S_X(\omega)\frac{\sin^2(\omega t/2)}{(\omega/2)^2}\right)$$

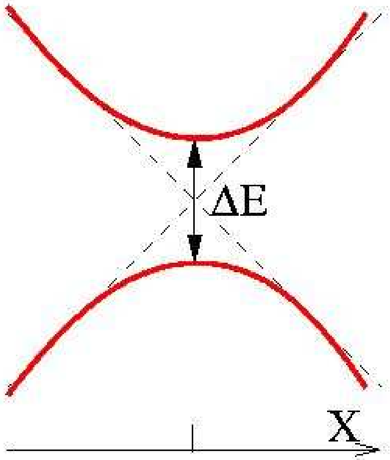
$$= \exp\left(-\frac{E_{1/f}^2}{2\pi} t^2 \ln|\omega_{ir} t|\right) \qquad \text{for} \quad S_X(\omega) = \frac{E_{1/f}^2}{\omega}$$

Cottet et al. (01)

⇨ Non-exponential decay of coherence

$S_X(\omega)$ regular $\Rightarrow \dfrac{\sin^2(\omega t/2)}{(\omega/2)^2} = 2\pi\,\delta(\omega)\,t \Rightarrow$ Golden rule, exponential decay

1/f noise, transverse coupling



$$H = -\frac{1}{2}\Delta E\,\sigma_z - \frac{1}{2} X\,\sigma_x + H_{\text{bath}}$$

Adiabatic approximation for $\omega \ll \Delta E$

$$\Rightarrow\; H = -\frac{1}{2}\Delta E(X(t))\;\sigma_z + H_{\text{bath}}$$

$$\approx -\frac{1}{2}\left(\Delta E + \frac{X^2(t)}{2\Delta E}\right)\sigma_z + H_{\text{bath}}$$

Linear transverse *1/f* noise $\Rightarrow$ quadratic longitudinal noise

- Spectrum of fluctuations of $X^2(t)$?
- Distribution of fluctuations of $X^2(t)$?

Quadratic longitudinal coupling:

$$H = -\frac{1}{2}\left(\Delta E + \lambda X^2\right)\sigma_z$$

Even if $X(t)$ is distributed Gaussian (central limit theorem), $X^2(t)$ is not!

As 1st step, let as **assume** that $X^2(t)$ is distributed Gaussian.

for reference; good or bad (?) approximation

- **Ohmic noise**

$$S_X(\omega) = \alpha\,\omega\ \coth\frac{\omega}{2T} \Rightarrow S_{X^2}(\omega) \equiv \frac{1}{2}\left\langle\left\{X^2(t), X^2(0)\right\}\right\rangle_\omega = \alpha^2\,\omega(\omega^2 + T^2)\ \coth\frac{\omega}{2T}$$

$$\Rightarrow \qquad \Gamma_\varphi^* = \alpha^2\lambda^2 T^3$$

if X^2 is Gaussian

good approx.

- **1/*f* noise**

$$S_X = \frac{E_{1/f}^2}{|\omega|} \Rightarrow \qquad S_{X^2} \approx \frac{E_{1/f}^4}{|\omega|}\ln\frac{\omega}{\omega_{ir}}$$

again 1/*f* noise with different scale

$$\Rightarrow \quad \left|\rho_{01}(t)\right| = \exp\left(-\lambda^2 \frac{E_{1/f}^4}{\pi} t^2 \ln^2|\omega_{ir} t|\right)$$

if X^2 is Gaussian

Good or bad?

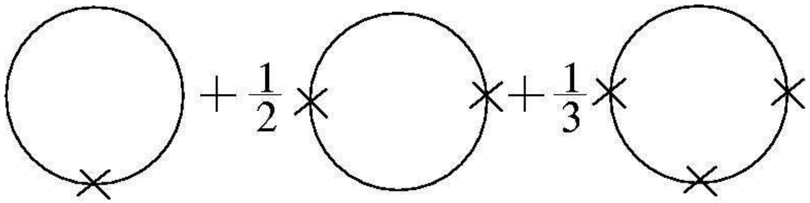
Quadratic longitudinal coupling: non-Gaussian effects

$$H = -\frac{1}{2}\left(\Delta E + \lambda X^2\right)\sigma_z$$

Yu. Makhlin, A. Shnirman, PRL 2004

$$\langle \rho_{01}(t)\rangle = e^{-i\Delta E t}\, P(t)$$ with $$P(t) = \left\langle \exp\left(i\lambda \int_0^t X^2(\tau)\,d\tau\right)\right\rangle$$

$$\ln P(t) = \sum \frac{1}{n} F_n =$$



linked cluster expansion

$$F_n = \frac{(2i\lambda)^n}{2}\int \frac{d\omega_1}{2\pi}\int \frac{d\omega_2}{2\pi}\ldots\int \frac{d\omega_n}{2\pi}\; f(\omega_1 - \omega_2)\, S_X(\omega_1)\, f(\omega_2 - \omega_3)\, S_X(\omega_3)\ \ldots$$

$$f(\omega) = \frac{\sin(\omega t/2)}{\omega/2}$$

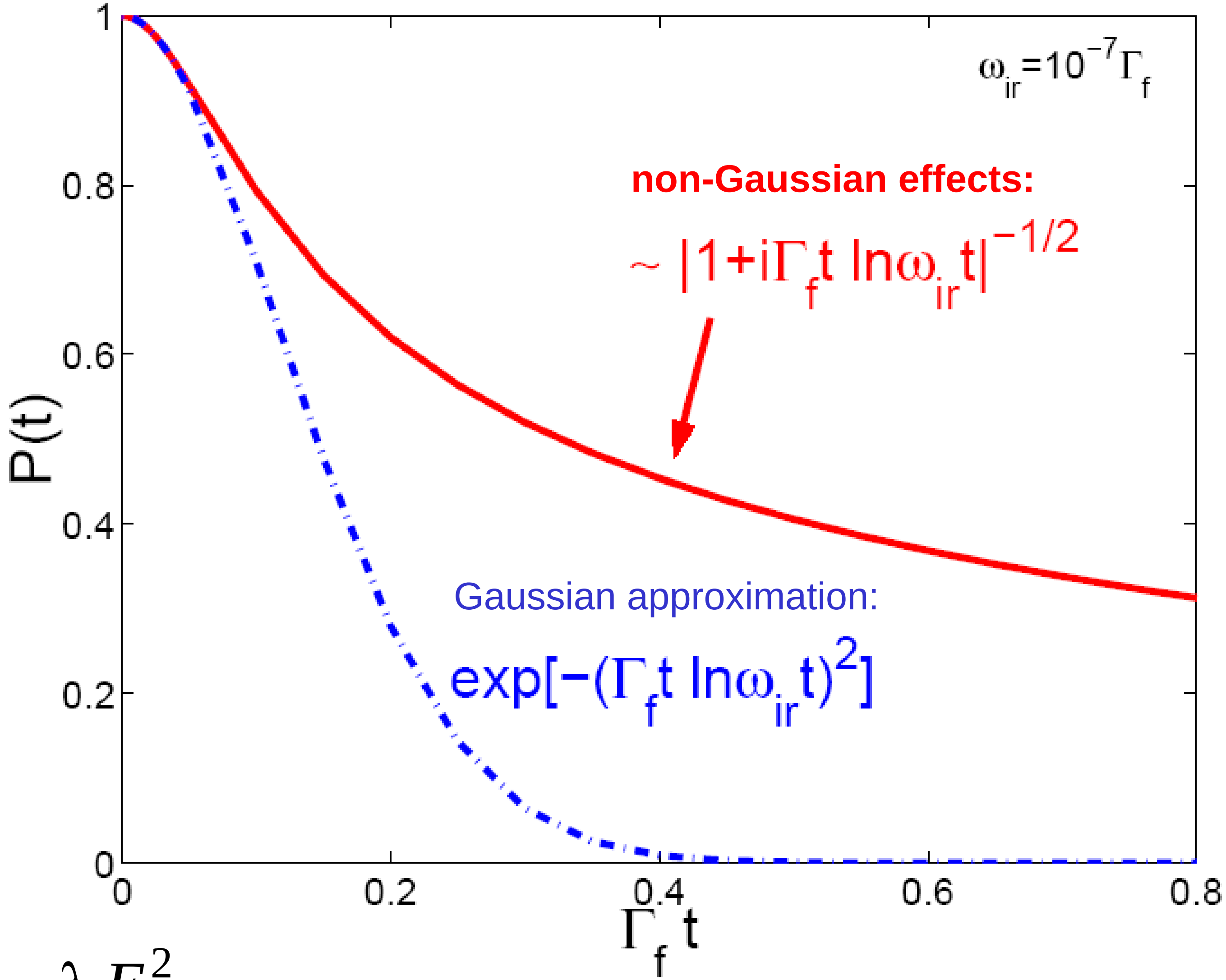
short times $$f(\omega) = \frac{\sin(\omega t/2)}{\omega/2} \approx t$$

long times $$f(\omega) = \frac{\sin(\omega t/2)}{\omega/2} \approx 2\pi\,\delta(\omega)$$

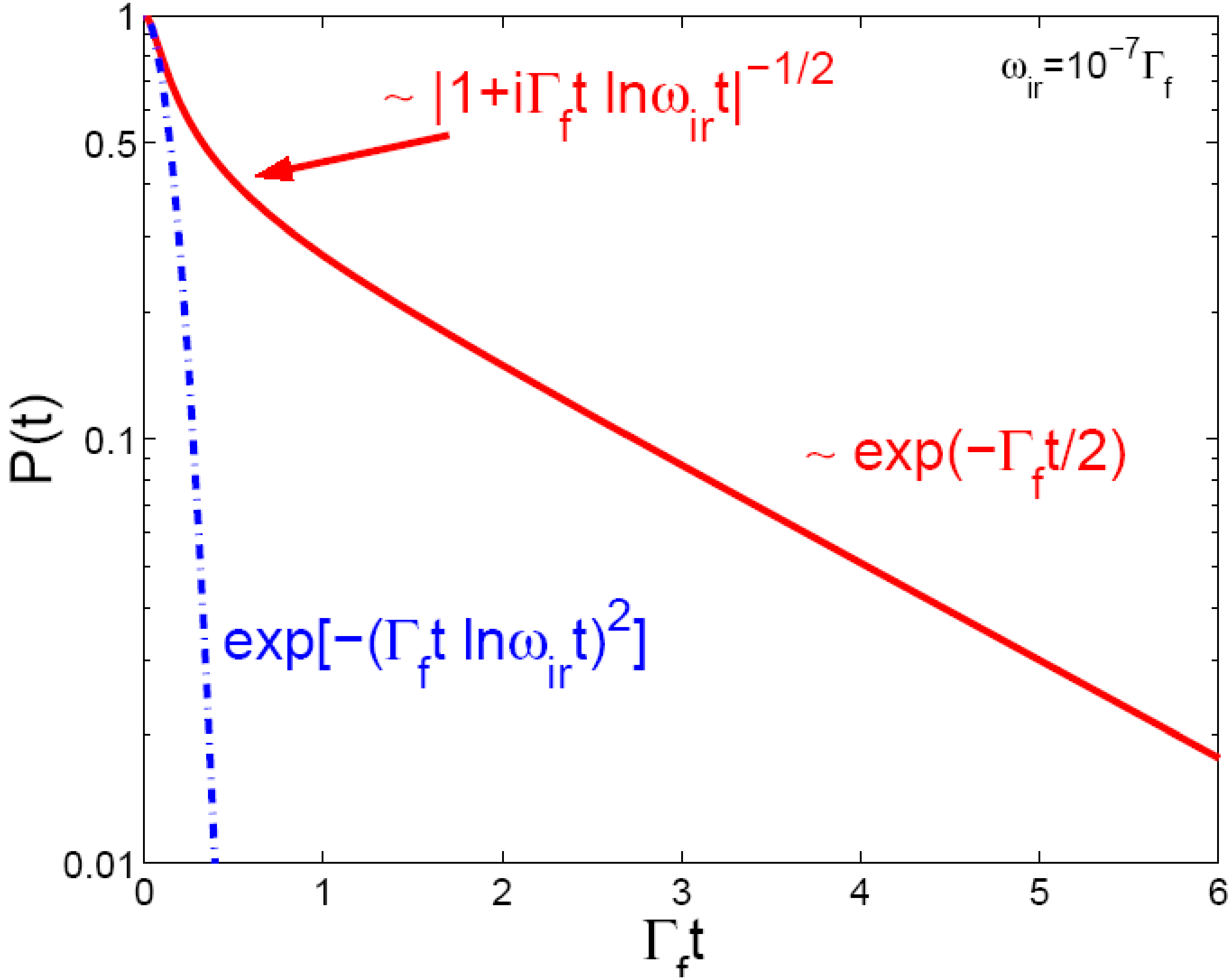
$$|P(t)| = \left[1 + \left(\frac{2}{\pi}\lambda E_{1/f}^2\, t \ln \omega_{ir} t\right)^2\right]^{-\frac{1}{4}}$$

$$|P(t)| = \exp\left[-\frac{\lambda E_{1/f}^2}{2} t\right]$$

Decay law for Ramsey interference

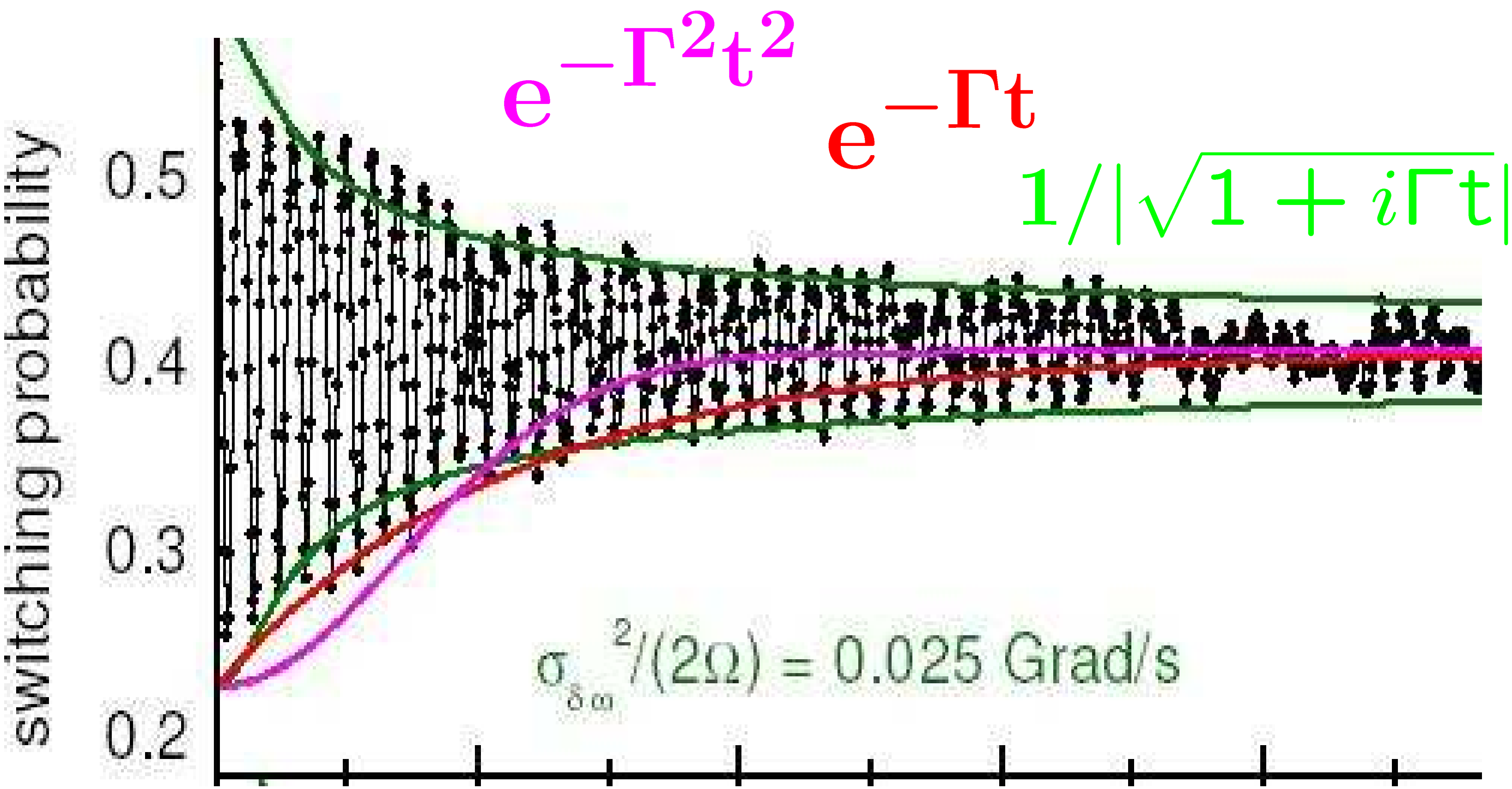


$$\Gamma_f = \lambda E_{1/f}^2$$

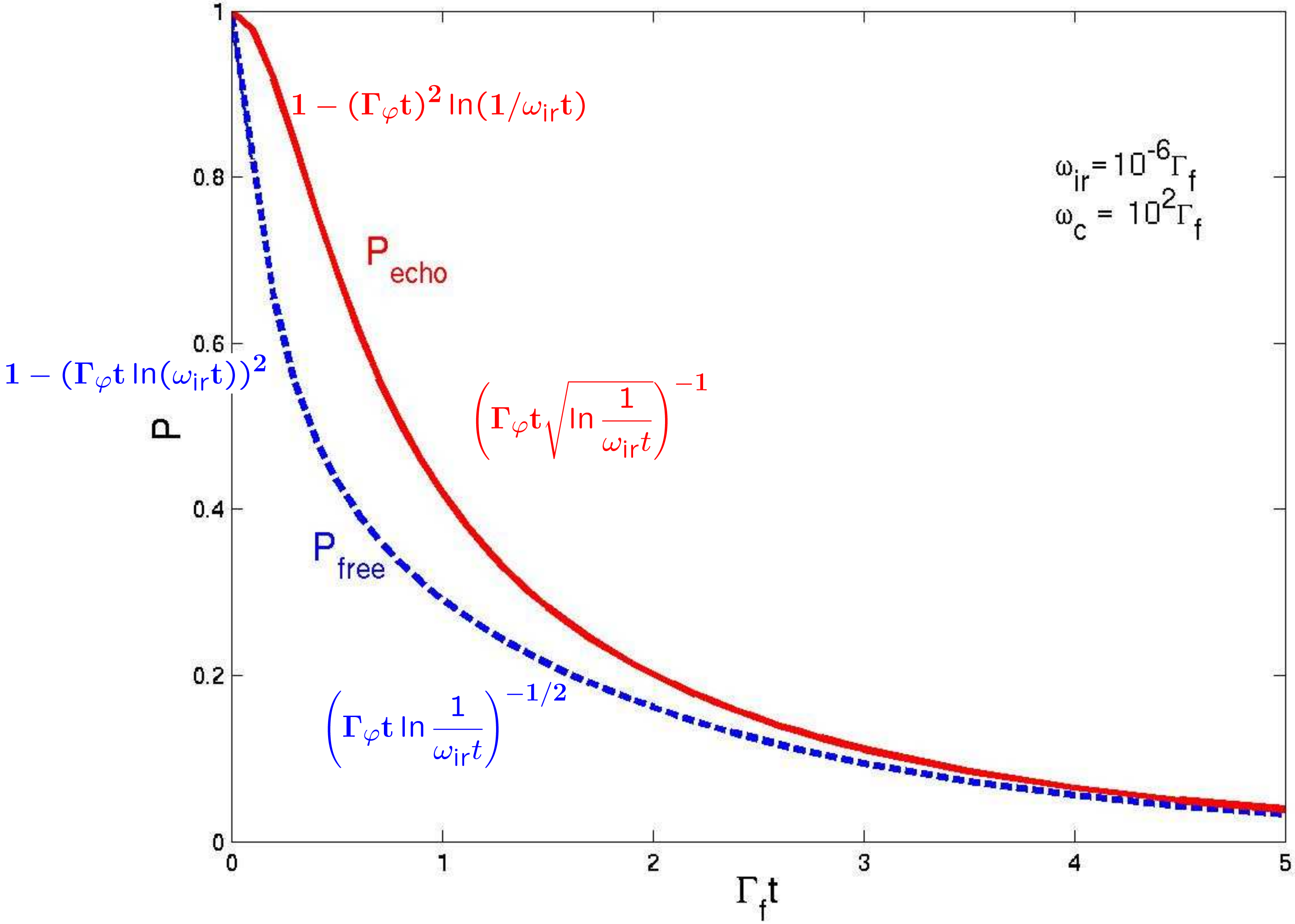


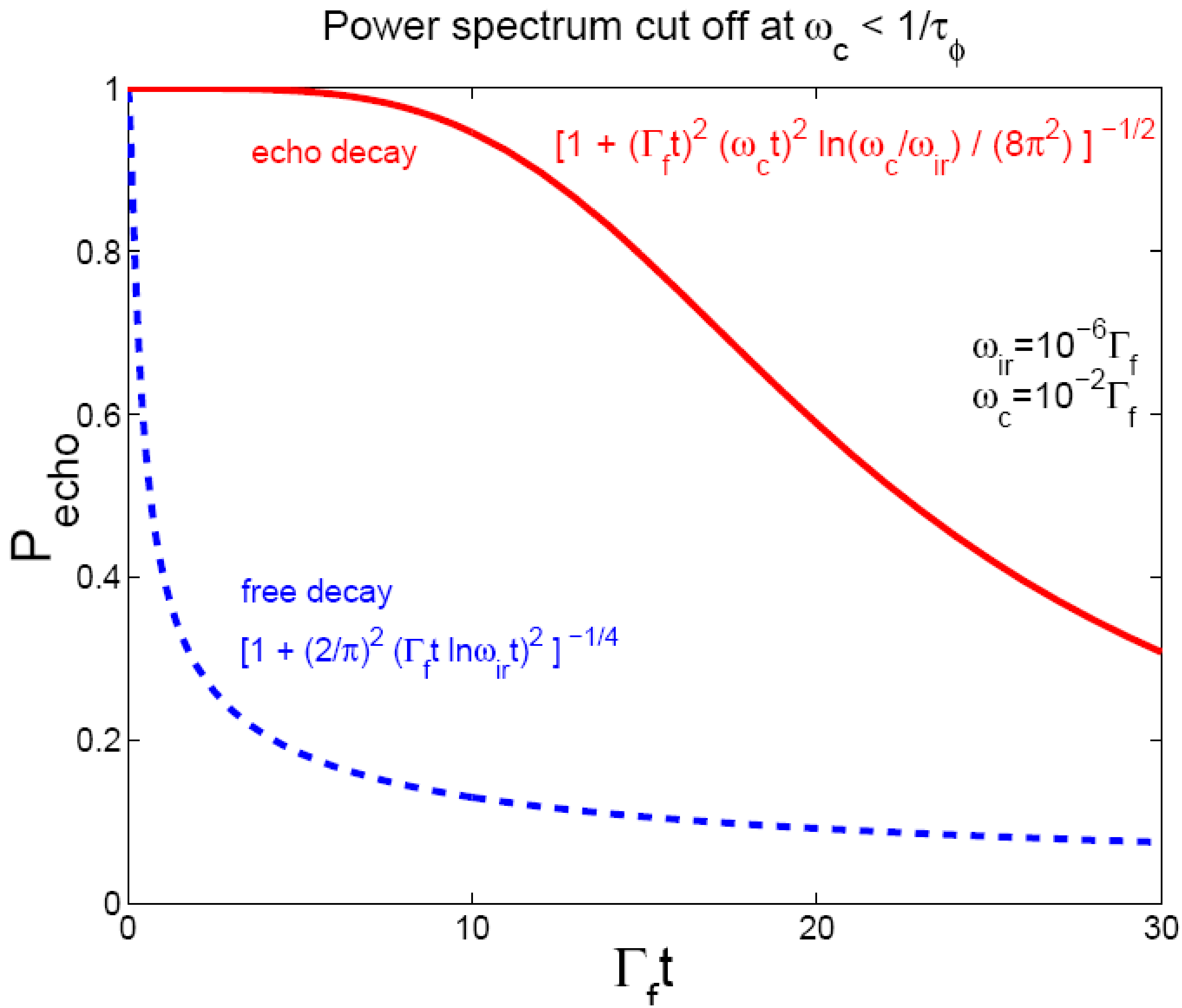
ω_ir=10^−7Γ_f
~ |1+iΓ_f t lnω_ir t|^−1/2
~ exp(−Γ_f t/2)
exp[−(Γ_f t lnω_ir t)^2]
P(t)
1
0.5
0.1
0.01
0
1
2
3
4
5
6
Γ_f t

Fitting the experiment



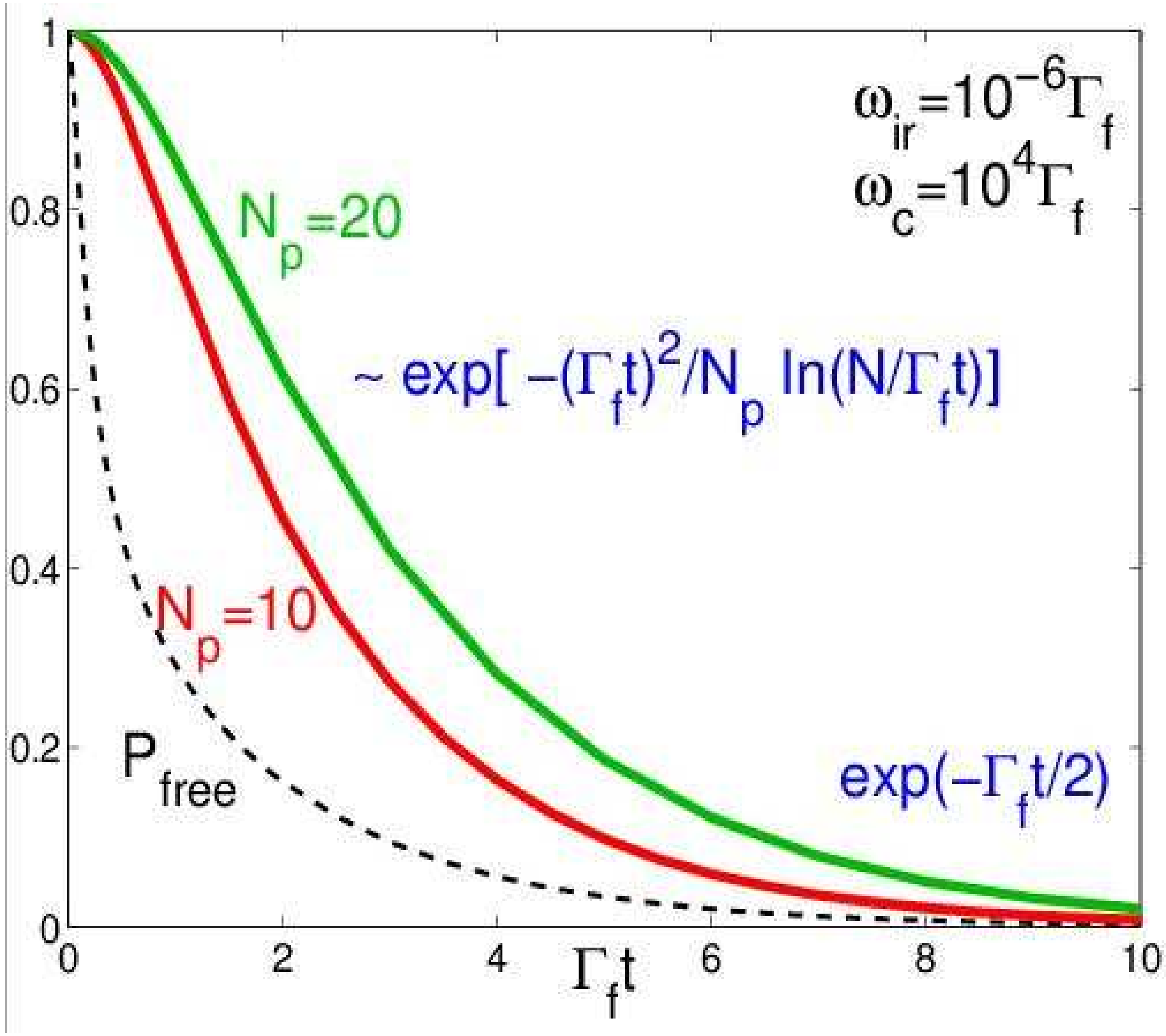
Decay law for echo signal




Power spectrum cut off at $\omega_c < 1/\tau_\phi$
echo decay
$[1 + (\Gamma_f t)^2 (\omega_c t)^2 \ln(\omega_c/\omega_{ir}) / (8\pi^2)]^{-1/2}$
$\omega_{ir}=10^{-6}\Gamma_f$
$\omega_c=10^{-2}\Gamma_f$
free decay
$[1 + (2/\pi)^2 (\Gamma_f t \ln\omega_{ir} t)^2]^{-1/4}$
P_{echo}
$\Gamma_f t$
0
0.2
0.4
0.6
0.8
1
10
20
30

Echo with many (N) pulses (Bang-Bang)

$$P^{\text{hf}}_{\text{echo}}(t) \simeq \exp\left(-\frac{1}{2N}\left(\frac{4}{\pi}\Gamma_f t\right)^2 \ln\frac{N}{\Gamma_f t}\right) \quad : \quad \Gamma_f t \ll N$$

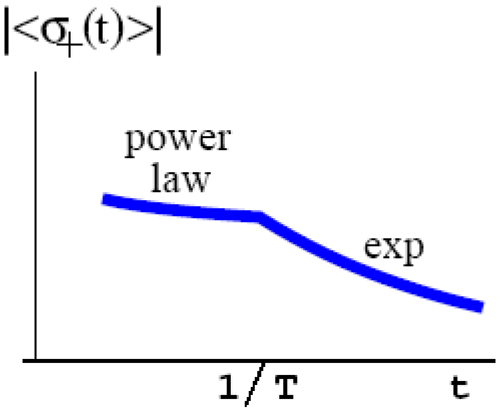


Low-Temperature Dephasing

- Ohmic: $J(\omega) = \frac{\pi}{2}\alpha\,\omega\,\Theta(\omega_c - \omega)$, cutoff ω_c

  for $\omega_c^{-1} \ll t \ll T^{-1}$: $|\langle\sigma_+(t)\rangle| = P_{\omega_c}(t) \approx (\omega_c t)^{-2\alpha}$

  for $T^{-1} \ll t$: $|\langle\sigma_+(t)\rangle| = P_{\omega_c}(t) \approx e^{-2\pi\alpha T t}$

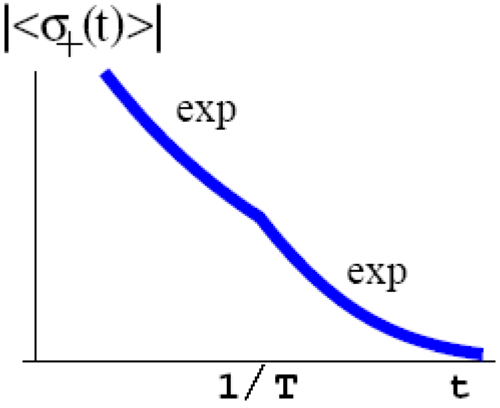


- Sub-Ohmic: $J(\omega) = \frac{\pi}{2}\alpha\,\omega_0^{1-s}\omega^s\,\Theta(\omega_c - \omega)$

for $0 < s < 1$

  for $\omega_c^{-1} \ll t \ll T^{-1}$: $|\langle\sigma_+(t)\rangle| = P_{\omega_c}(t) \approx e^{-\alpha(\omega_0 t)^{1-s}}$

  for $T^{-1} \ll t$: $|\langle\sigma_+(t)\rangle| = P_{\omega_c}(t) \approx e^{-\alpha T t(\omega_0 t)^{1-s}}$



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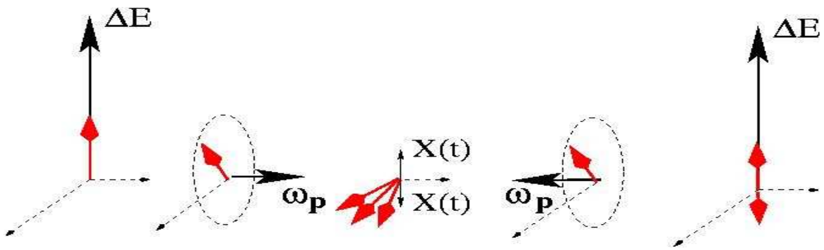
Preparation Effects

Introduce frequency scale ω_p

Slow modes $\omega \ll \omega_p$ **dephasing**, fast modes $\omega \gg \omega_p$ **renormalization**



a) Initially $|\uparrow\rangle\,|g_\uparrow\rangle$

$|g_\uparrow\rangle$ ground state of $H_\uparrow \equiv |\uparrow\rangle\langle\uparrow| \cdot \sum_j c_j(a_j + a_j^\dagger) + \sum_j \hbar\omega_j\, a_j^\dagger a_j$

b) $\frac{\pi}{2}$ pulse $|\uparrow\rangle \rightarrow \frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$

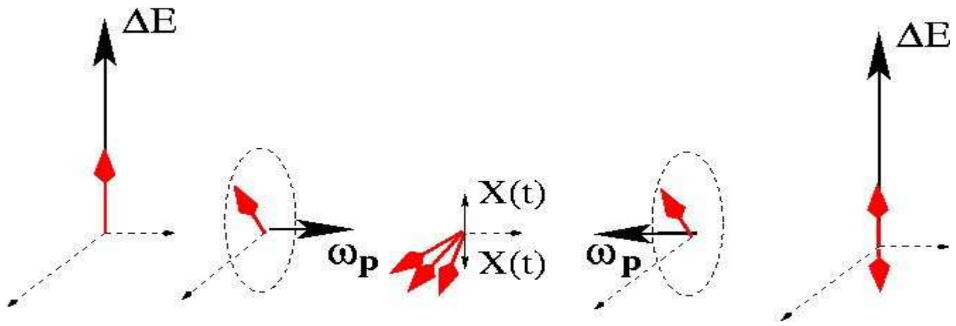
implemented as $H = -\frac{1}{2}\, Z^{-1}\, \omega_p \hat{\sigma}_x$ for $\tau = \frac{\pi\hbar}{2\omega_p}$

Slow oscillators do not react

Fast oscillators follow adiabatically

$$|\uparrow\rangle \rightarrow \frac{1}{\sqrt{2}}\left[|\uparrow\rangle\,|g_\uparrow^{\text{fast}}\rangle + |\downarrow\rangle\,|g_\downarrow^{\text{fast}}\rangle\right]\,|g_\uparrow^{\text{slow}}\rangle$$

$\hat{\rho}_{\uparrow,\downarrow} = Z \equiv \langle g_\uparrow^{\text{fast}}|g_\downarrow^{\text{fast}}\rangle$ **BUT** $\hat{\rho}_{\tilde{\uparrow},\tilde{\downarrow}} = 1$ where $|\tilde{\uparrow}\rangle \equiv |\uparrow\rangle|g_\uparrow^{\text{fast}}\rangle$, $|\tilde{\downarrow}\rangle \equiv |\downarrow\rangle|g_\downarrow^{\text{fast}}\rangle$



c) Free evolution, dephasing

$$\hat{\rho}_{\uparrow,\downarrow}(t) = Z\,P_{\omega_p}(t) \text{ and } \hat{\rho}_{\tilde{\uparrow},\tilde{\downarrow}} = P_{\omega_p}(t) = \langle g_{\uparrow}^{\text{slow}}|e^{-iH_{\downarrow}t}|g_{\uparrow}^{\text{slow}}\rangle$$

d) $-\frac{\pi}{2}$ pulse

e) Measurement of $\hat{\sigma}_z$, $\langle\hat{\sigma}_z\rangle = |P_{\omega_p}(t)|$

Slow oscillators ⇒ dephasing $P_{\omega_p}(t)$

Fast oscillators ⇒ renormalization Z

Appropriate basis: renormalized (dressed) spin $|\tilde{\uparrow}\rangle, |\tilde{\downarrow}\rangle$