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SMR.1587 - 17

SCHOOL AND WORKSHOP ON QUANTUM ENTANGLEMENT, DECOHERENCE, INFORMATION, AND GEOMETRICAL PHASES IN COMPLEX SYSTEMS (1 November - 12 November 2004)

Control of decoherence: Comparison of different strategies

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Control of decoherence:
comparison of different strategies

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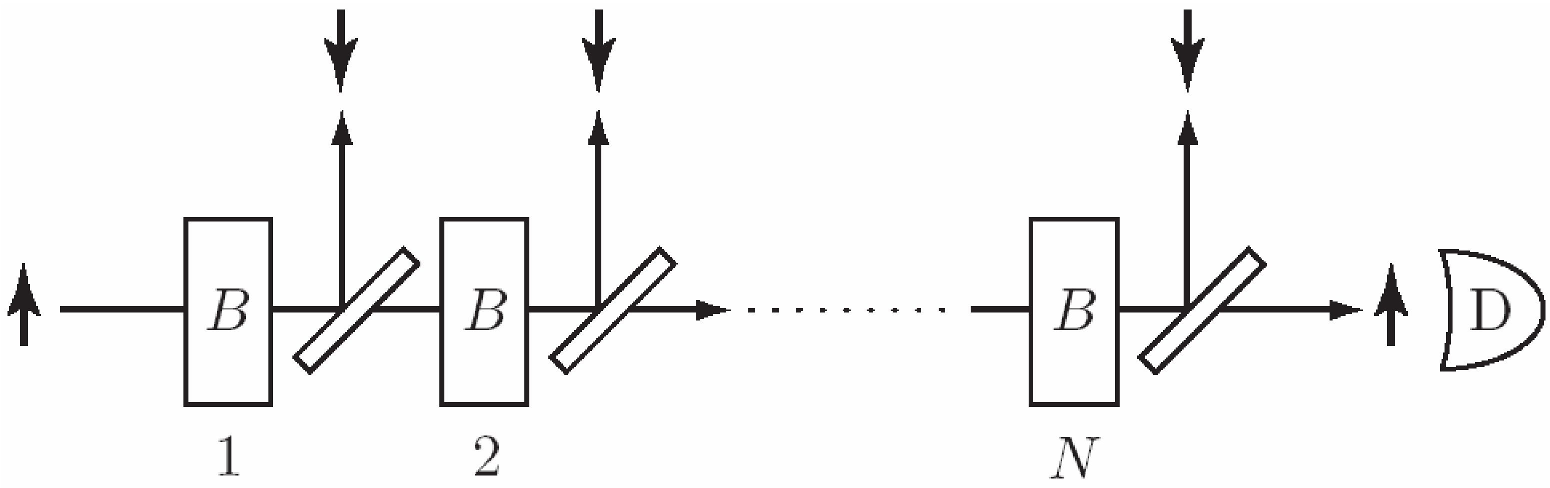
J. Rehacek (Olomouc)
S. Tasaki (Waseda)

Trieste,
9 november 2004

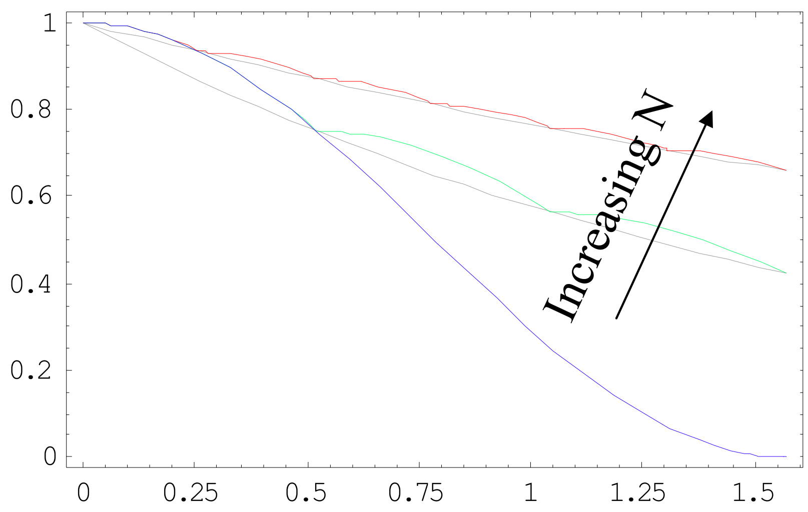
Example: neutron spin



$$p(t) = \cos\left(\frac{t}{\tau_Z}\right)^2$$

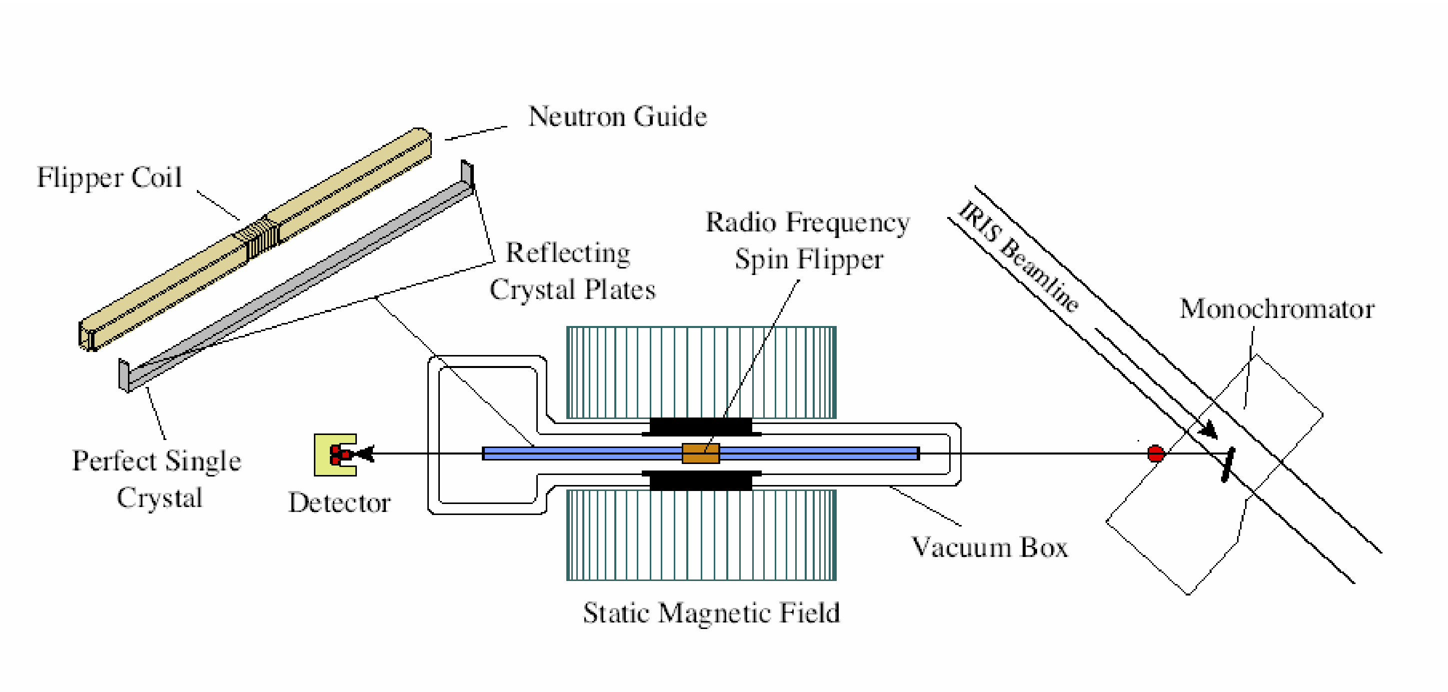
$$p^{(N)}(t) = p\left(\frac{t}{N}\right)^N = \cos\left(\frac{t}{N\tau_Z}\right)^{2N}$$

$$p^{(N)}(t) \xrightarrow{N\to\infty} 1$$



Pascazio, Namiki, Badurek, Rauch, Phys. Lett. A **169**, 155 (1993)

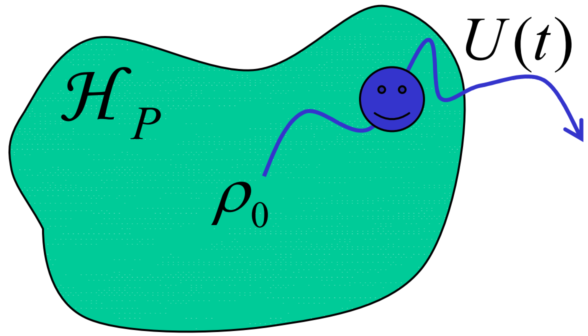
VESTA II @ ISIS



Jericha, Schwab, Jakel, Carlile, Rauch, Physica B 283, 414 (2000);

Rauch, Physica B 297, 299 (2001).

Quantum Zeno effect: fundamentals



Consider a quantum system Q, whose states are in $\mathcal{H}$

Time evolution $U(t) = \exp(-iHt)$.

P a projection operator $[P, H] \neq 0$, and $P\mathcal{H} = \mathcal{H}_P$ its eigenspace.

Initial density matrix ρ_0 in $\mathcal{H}_P$: $\quad \rho_0 = P\rho_0 P, \qquad \mathrm{Tr}[\rho_0 P] = 1$

Perform a measurement at time τ, in order to check whether Q has "survived".

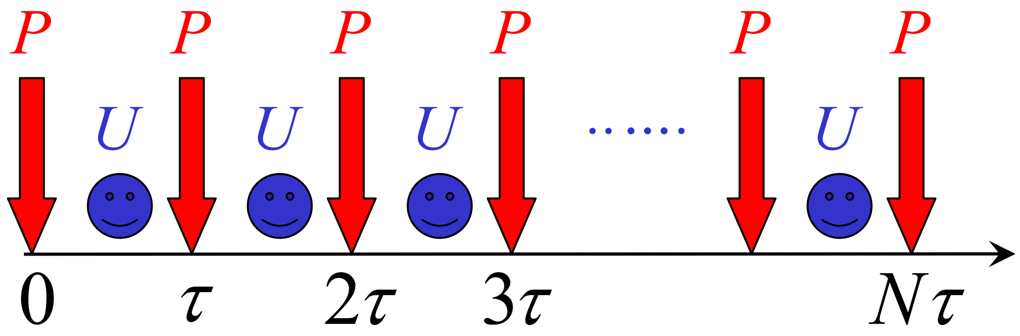
$$\rho_0 \longrightarrow \rho(\tau) = PU(\tau)\rho_0 U(\tau)^\dagger P,$$

with probability

$$p(\tau) = \mathrm{Tr}[U(\tau)\rho_0 U(\tau)^\dagger P] = \mathrm{Tr}[V(\tau)\rho_0 V(\tau)^\dagger], \qquad V(\tau) = PU(\tau)P$$

QZE: fundamentals

Prepare Q in the initial state ρ_0 at time 0 and perform a series of P-observations at times $\tau_j = jt/N = j\tau, \quad (j = 1, \ldots, N)$.

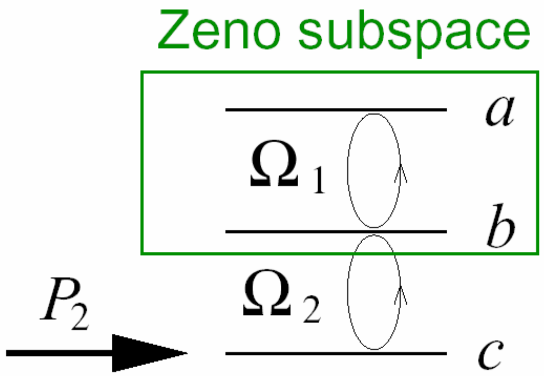


$$\rho^{(N)}(t) = V_N(t)\rho_0 V_N(t)^\dagger, \qquad V_N(t) = [PU(t/N)P]^N$$

The probability to find the system in $\mathcal{H}_P$ reads

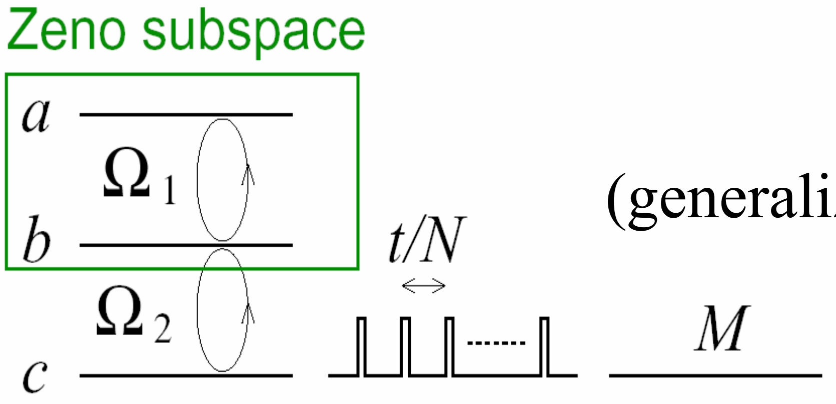
$$p^{(N)}(t) = \mathrm{Tr}[V_N(t)\rho_0 V_N(t)^\dagger] \xrightarrow{N\to\infty} \mathrm{Tr}[P\rho_0] = 1$$

Quantum Zeno effect: repeated observation in succession inhibit transitions outside $\mathcal{H}_P$



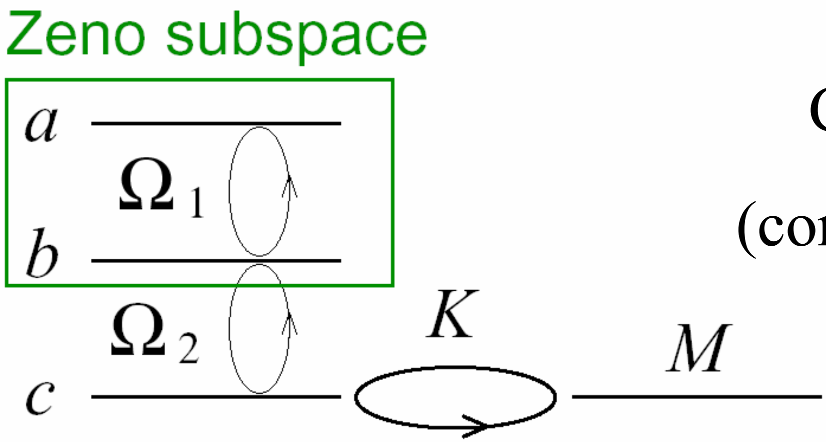
Measurements *à la* von Neumann

(projections)



Unitary "kicks"

(generalized spectral decompositions)



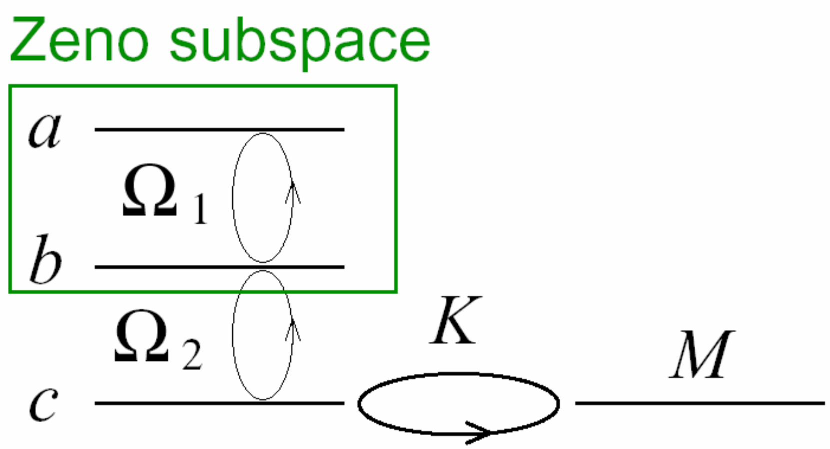
Continuous coupling

(continuous measurement)

Continuous coupling

Simonius 1978

Harris & Stodolsky 1982

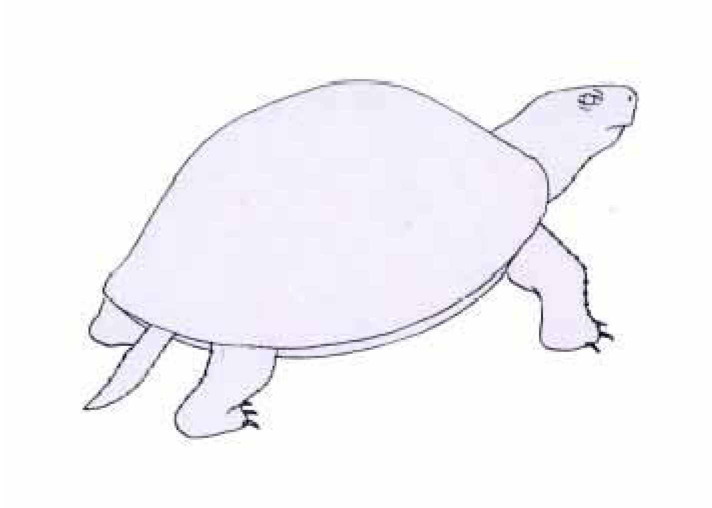


$$K H_c = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & K \\ 0 & 0 & K & 0 \end{pmatrix}$$

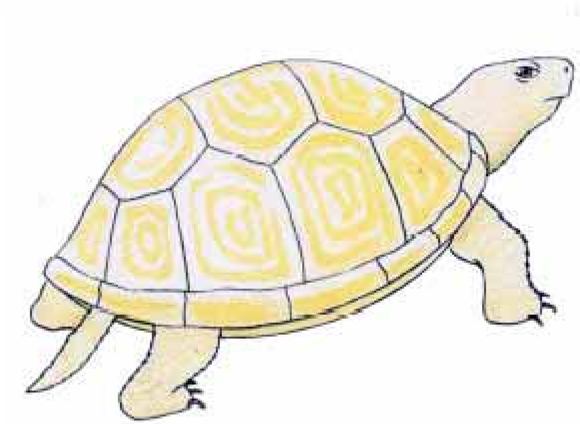
$$H = \begin{pmatrix} 0 & \Omega_1 & 0 & 0 \\ \Omega_1 & 0 & \Omega_2 & 0 \\ 0 & \Omega_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 0 & \Omega_1 & 0 & 0 \\ \Omega_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = H_Z$$

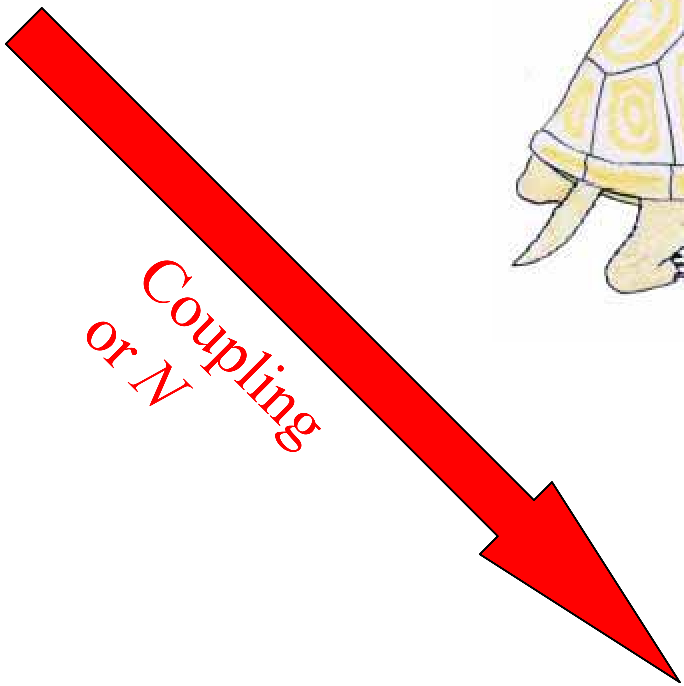
$\Uparrow$

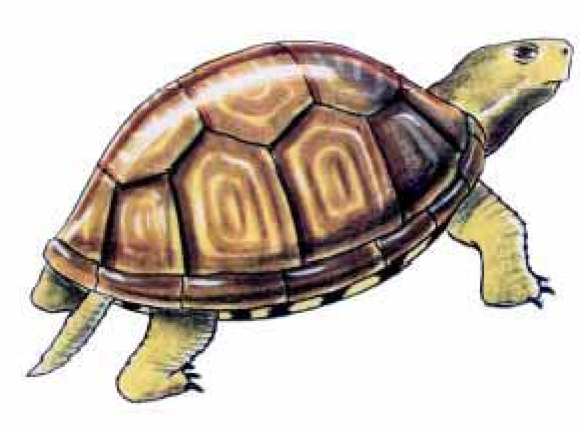
Zeno limit $(K \rightarrow \infty)$



Dynamical superselection sectors







Main objective: understand and suppress decoherence

Decoherence-free subspaces

Palma, Suominen and Ekert (1996)
Duan and Guo (1997)
Zanardi and Rasetti (1997)
Lidar, Chuang and Whaley (1998)
Viola, Knill and Lloyd (1999)
Vitali and Tombesi (1999, 2001)
Beige, Braun, Tregenna and Knight (2000)
…
Tasaki, Tokuse, Facchi, Pascazio (2002)
Facchi, Lidar and Pascazio (2003)
Fachi, Tasaki, Nakazato, Pascazio (2004)

Benenti, Casati, Montangero, Shepelyansky (2001)

Vitali, Tombesi, Milburn (1997, 1998)
Fortunato, Raimond, Tombesi, Vitali (1999)
Kofman, Kurizki (2001)
Agarwal, Scully, Walther (2001)

Calarco, Datta, Fedichev, Pazy, Zoller,
"Spin-based all-optical quantum computation with quantum dots: understanding and suppressing decoherence" (2003)

Falci (2003)
Falci, D'Arrigo, Mastellone, Paladino (2004)
Brion, Harel, Kebaili, Akulin, Dumer (2004)
Zhang, Zhou, Yu, Guo (2004)

Relevant timescales

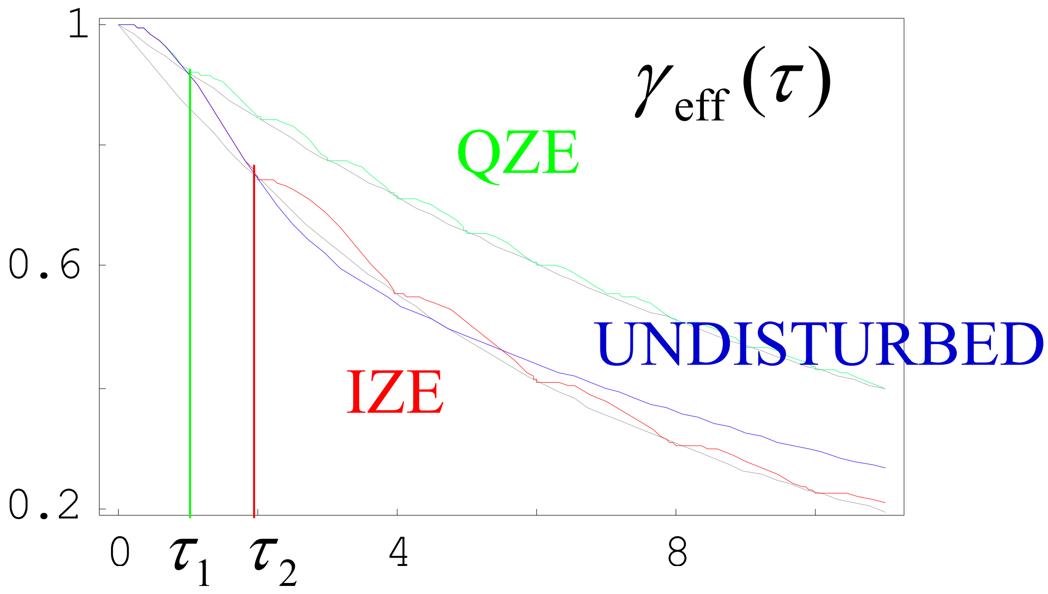
How frequent or strong must be the coupling?

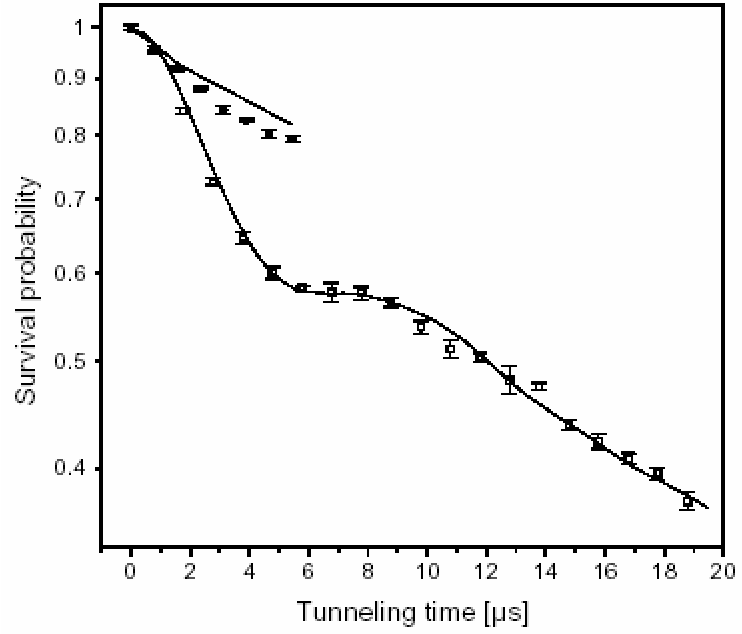
Facchi, Nakazato and Pascazio, Phys. Rev. Lett. **86**, 2699 (2001) [previous work by Kofman and Kurizki, Facchi and Pascazio]

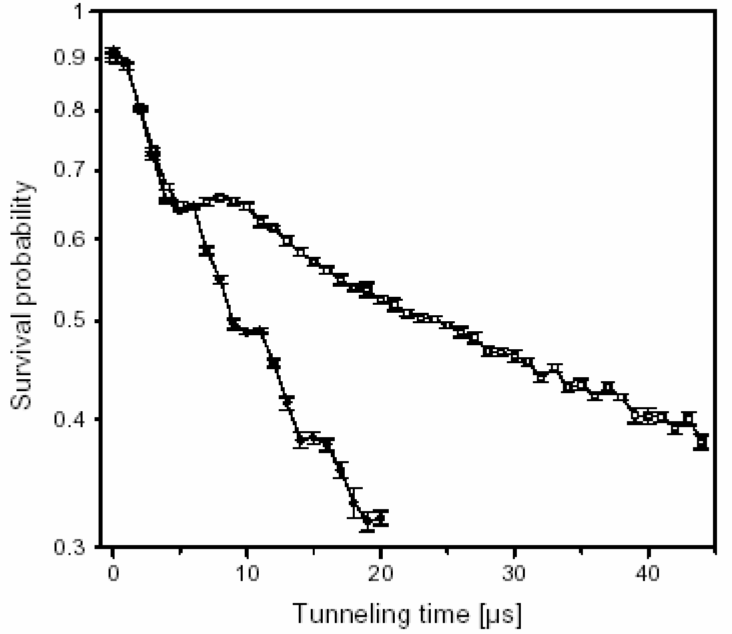
Experiment by Fischer, Gutièrrez-Medina and Raizen, Phys. Rev. Lett. **87**, 040402 (2001)

NOTICE:

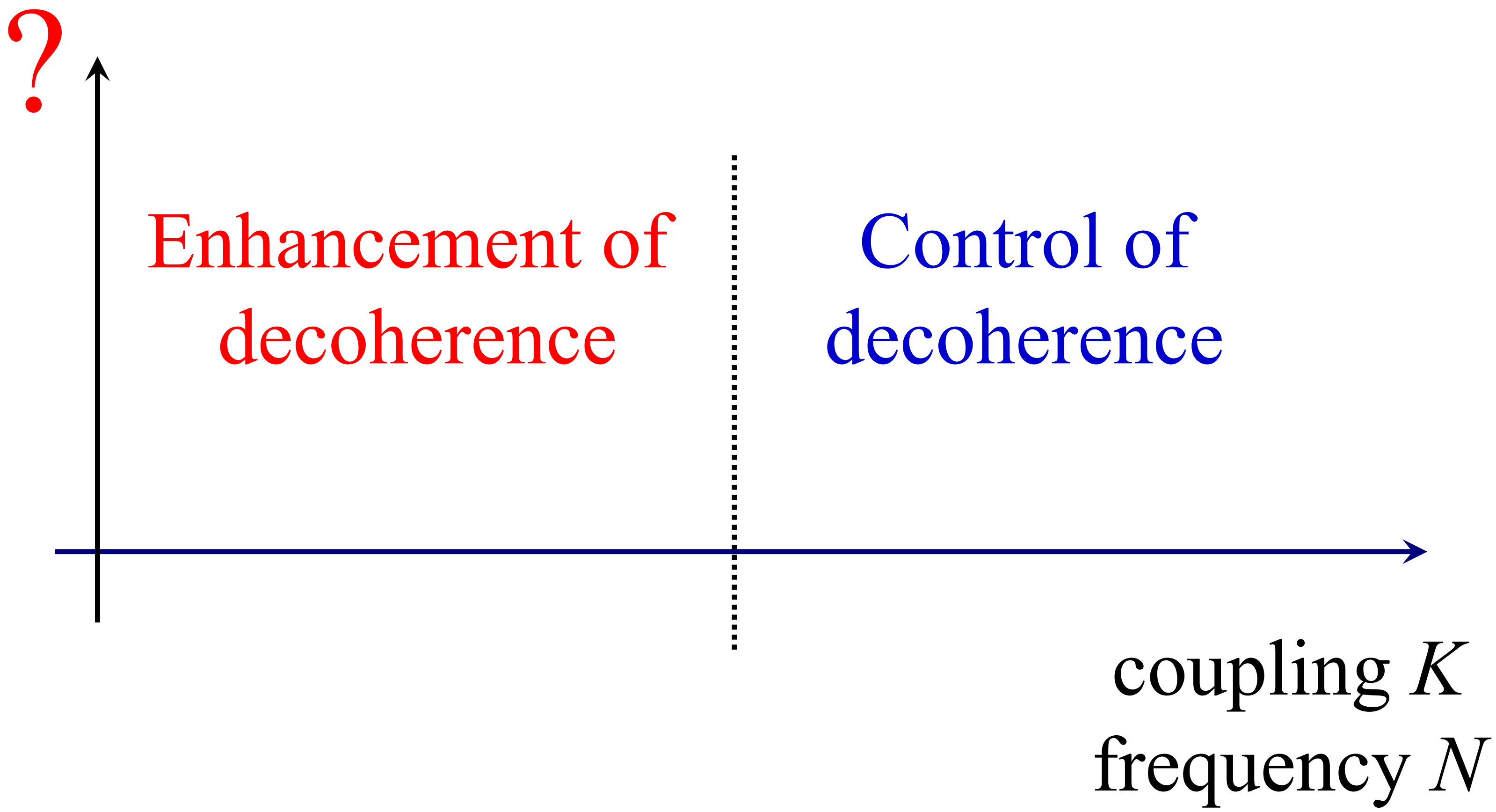
$\gamma_{\text{eff}}(\tau)$







Main idea


?
Enhancement of decoherence
Control of decoherence
coupling *K*
frequency *N*

Framework

$$H_{\text{tot}} = H_0 + H_{SB} = H_S \otimes 1_B + 1_S \otimes H_B + H_{SB}$$

decoherence

$$\mathrm{P}_{\text{tot}}\rho = -i[H_{\text{tot}}, \rho]$$

$$\mathrm{P}_{\text{tot}} = \mathrm{P}_0 + \mathrm{P}_{\text{SB}}$$

Liouvillian

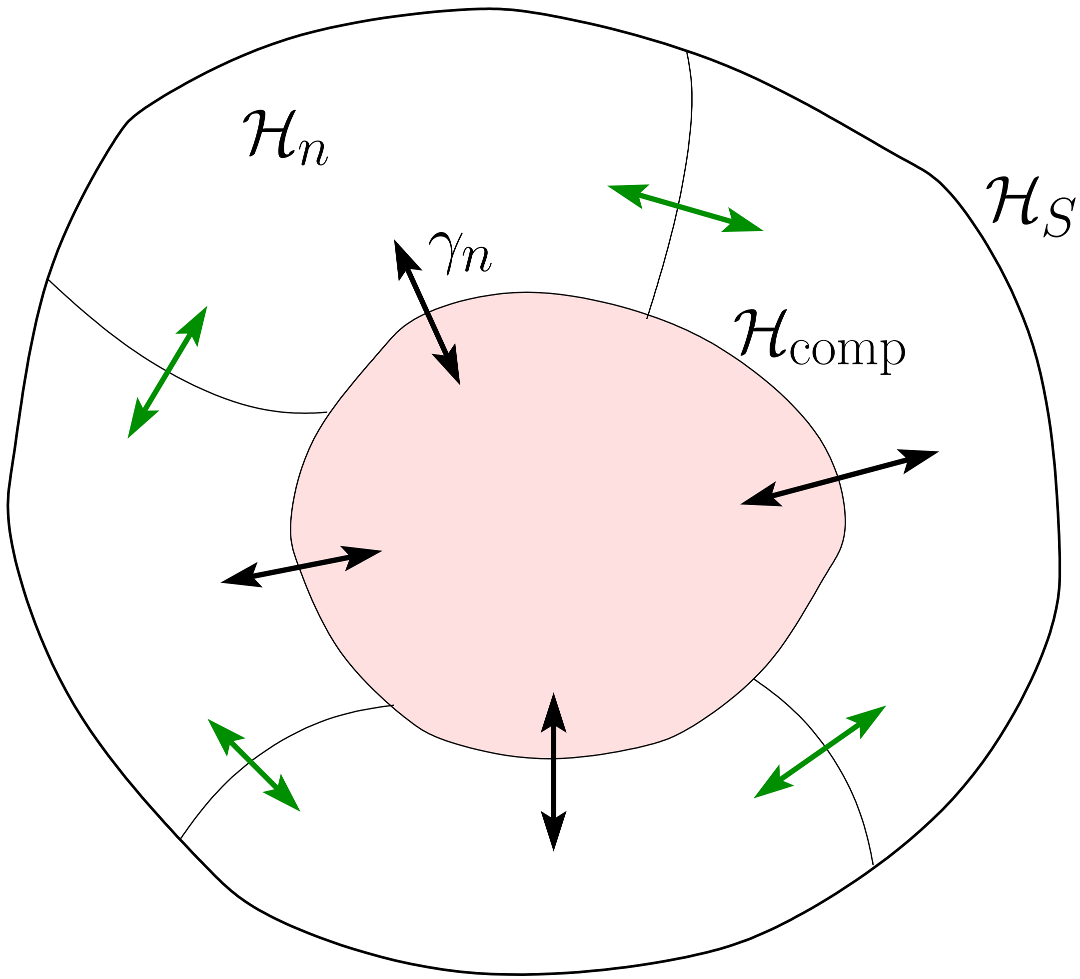
$$\mathrm{L}_{\text{tot}} = \mathrm{L}_S \otimes \mathrm{L}_B$$

$$\mathrm{L}_S = \mathrm{L}_{\text{comp}} \oplus \mathrm{L}_{\text{orth}}$$

$$\left(e.g.: \mathrm{L}_{\text{comp}} = C^2\right)$$

Hilbert spaces

Zeno subspaces



The general case (Gardiner & Zoller)

$$H_{SB} = \sum_m X_m \otimes A_m^+ + X_m^+ \otimes A_m$$

$$\mathrm{P}_S X_m = i\omega_m X_m$$

$$A_m = A(\kappa_m) = \int \kappa_m^*(\omega) a(\omega) d\omega$$

form factor

$$[\kappa_m(\omega) = 0, \text{ for } \omega < 0]$$

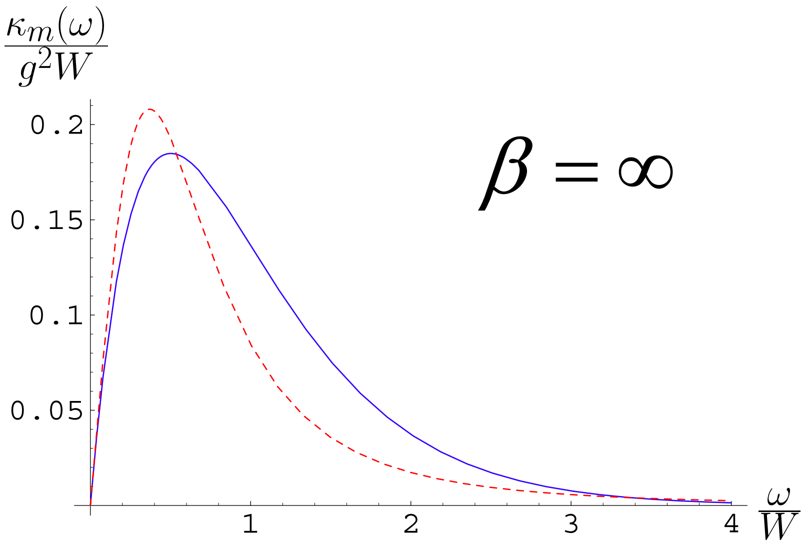
Polynomial and exponential case

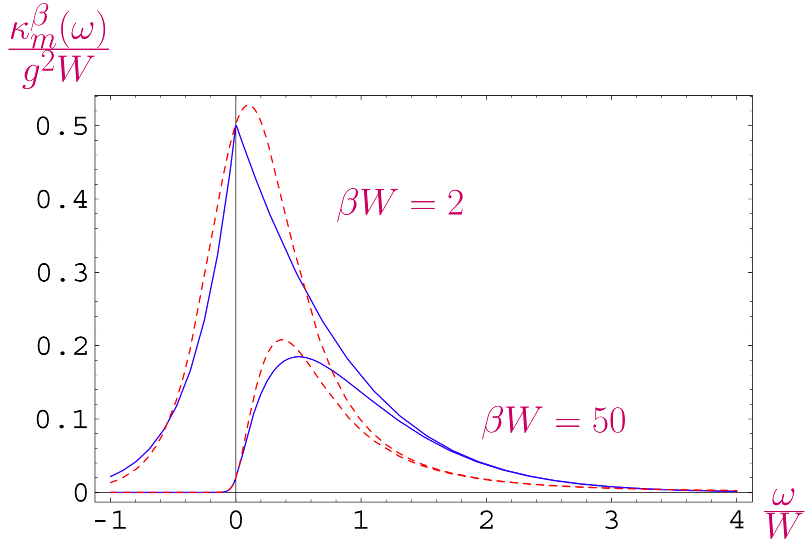
$$\kappa_m(\omega) = g^2 \omega \exp(-\omega/\Lambda)\theta(\omega) \quad \text{exponential}$$

$$\kappa_m(\omega) = g^2 \frac{\omega}{(1+(\omega/\Lambda)^2)^2}\theta(\omega) \quad \text{polynomial}$$

$$W = \tau_Z^2 \int_{-\infty}^{\infty} |\omega| \kappa_m(\omega) d\omega \quad \text{bandwidth}$$

Form factors





Full line: exponential; dashed line: polynomial form factor.

β inverse temperature

W bandwidth

Dynamics

$$\rho(t) = \left[\hat{P}\exp(\mathrm{P}_{\mathrm{tot}}(\tau))\,\hat{P}\right]^{t/\tau}\rho(0)\quad \text{Zeno (measurements)}$$

$$\rho(t) = \left[\exp(\mathrm{P}_{\mathrm{k}})\exp(\mathrm{P}_{\mathrm{tot}}(\tau))\right]^{t/\tau}\rho(0)\quad \text{kicks}$$

$$\rho(t) = \exp[(K\mathrm{P}_{\mathrm{c}} + \mathrm{P}_{\mathrm{tot}})t]\rho(0)\qquad \text{continuous coupling}$$

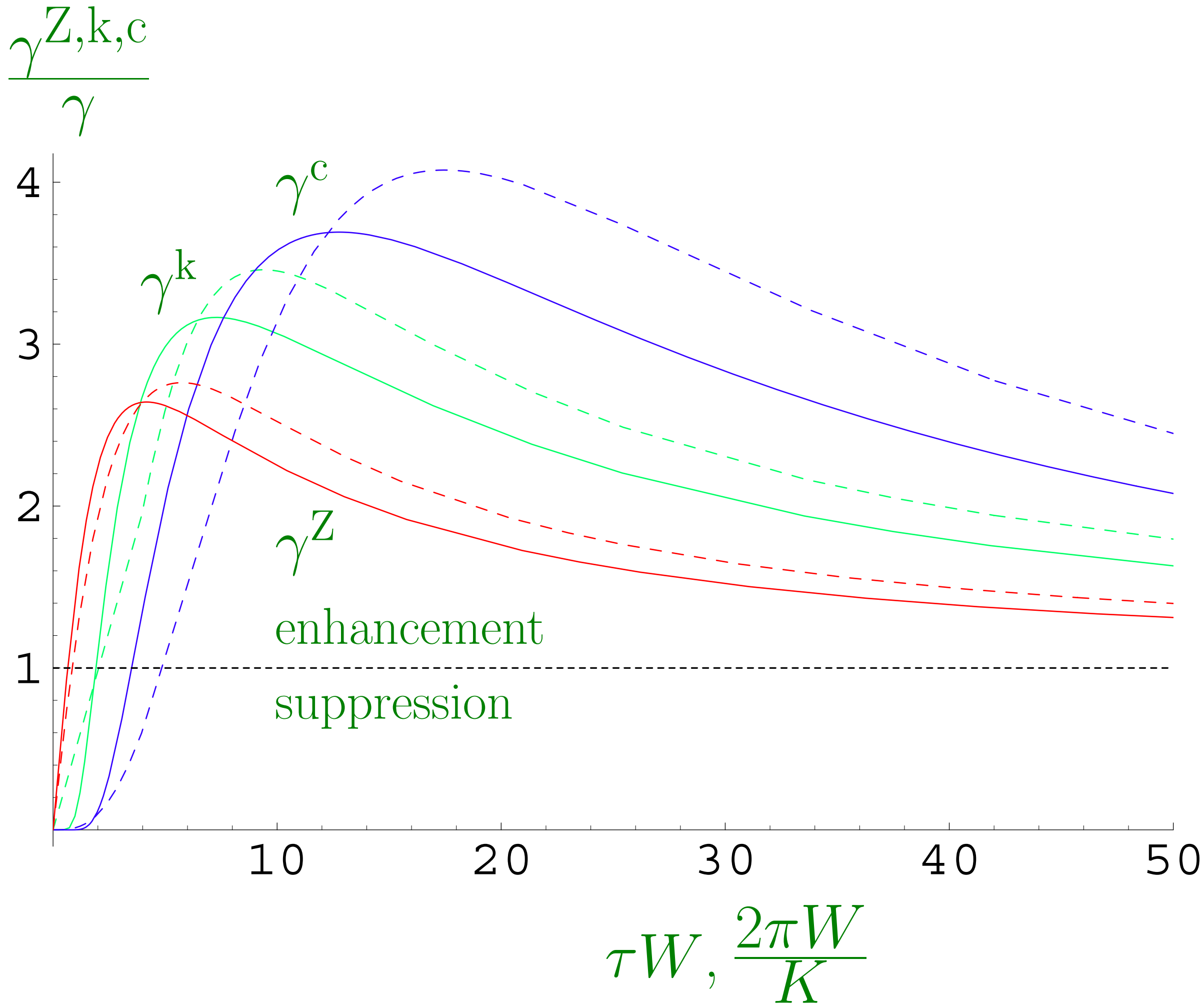
always look at the timescales γ^{-1} for dissipation

important !!

$$\tau^{*} \cong \gamma\tau_{Z}^{2}$$

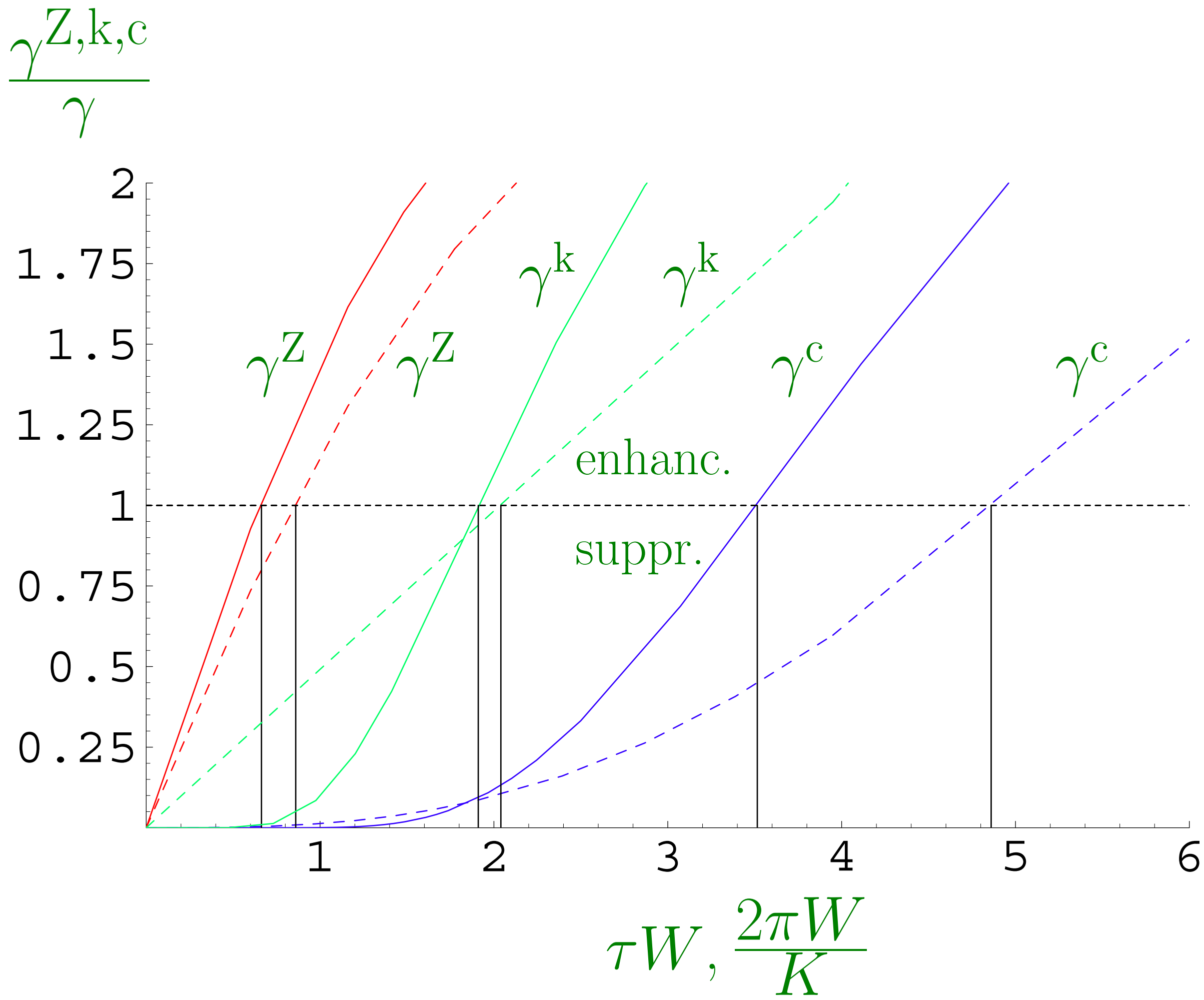
control (protection) ⟷ enhancement

Comparison



$\frac{\gamma^{\mathrm{Z,k,c}}}{\gamma}$
4
3
2
1
γ^{c}
γ^{k}
γ^{Z}
enhancement
suppression
10
20
30
40
50
$\tau W, \frac{2\pi W}{K}$

Comparison
(small times 1/*N* -- strong coupling *K*)



Remarkable differences

$$\gamma_{\text{eff}} \cong \frac{\tau}{\tau_Z^2} = \tau \int d\omega \kappa^{\beta}(\omega)$$

Zeno control
(non-unitary)

$$\gamma_{\text{eff}} \cong \frac{8}{\pi} \kappa^{\beta} \left(\frac{\pi}{\tau} \right)$$

"bang bang" control
(kicks) (unitary)

$$\gamma_{\text{eff}} \cong \pi \kappa^{\beta} (K)$$

control via
continuous coupling
(unitary)

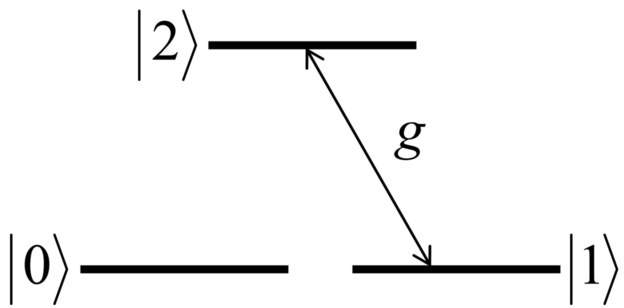
For unitary controls γ_{eff} feels the "tail" of the form factor

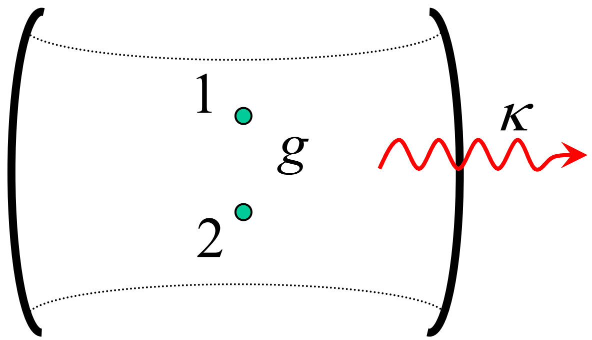
Example: countering decoherence

Quantum computation with two 3 - level atoms in a cavity

$$H_{\text{tot}} = H_{\text{comp}} + g\sum_{i=1}^{2}\left(b|2\rangle_{ii}\langle 1| + b^{\dagger}|1\rangle_{ii}\langle 2|\right) - i\kappa b^{\dagger}b$$

Beige, Braun, Tregenna and Knight, Phys. Rev. Lett. **85**, 1762 (2000)





For $\kappa, g \to \infty$, decoherence - free subspace

$$\mathcal{H}_{P_0} = \left\{|0,0\rangle, |01\rangle, |10\rangle, |11\rangle, \left(|21\rangle - |12\rangle\right)/\sqrt{2}\right\}$$

Zeno Hamiltonian $H_Z = P_0 H_{\text{comp}} P_0$

Facchi and Pascazio, quant-ph/0202174

(Solvay conference 2002)

Static vs dynamic imperfections

Benenti, Casati, Montangero, Shepelyansky 2001

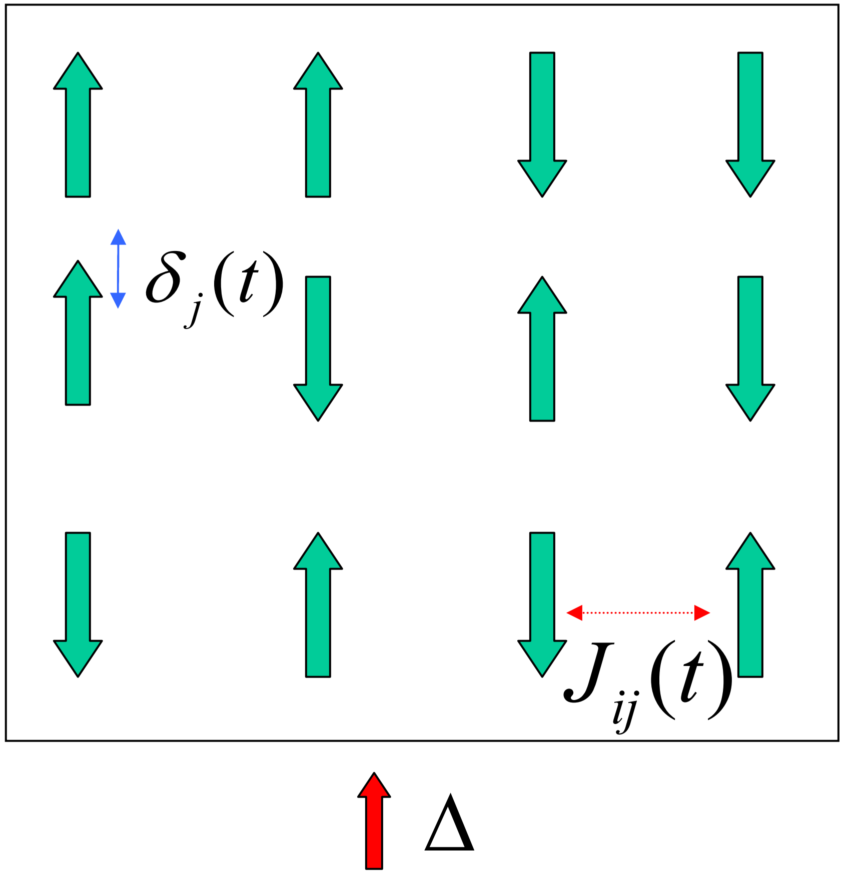
"… static imperfections […] are therefore more dangerous for quantum computation."

Montangero - Benasque (Spain) – Facchi

TIMESCALES ?

Facchi, Fazio, Montangero, Pascazio (2004)

Static vs dynamic imperfections (cont'd)



$$H(t) = \sum_j (\Delta + \delta_j(t))\sigma_z^{(j)} + \sum_{\langle i,j\rangle} J_{ij}(t)\sigma_x^{(i)}\sigma_x^{(i)}$$

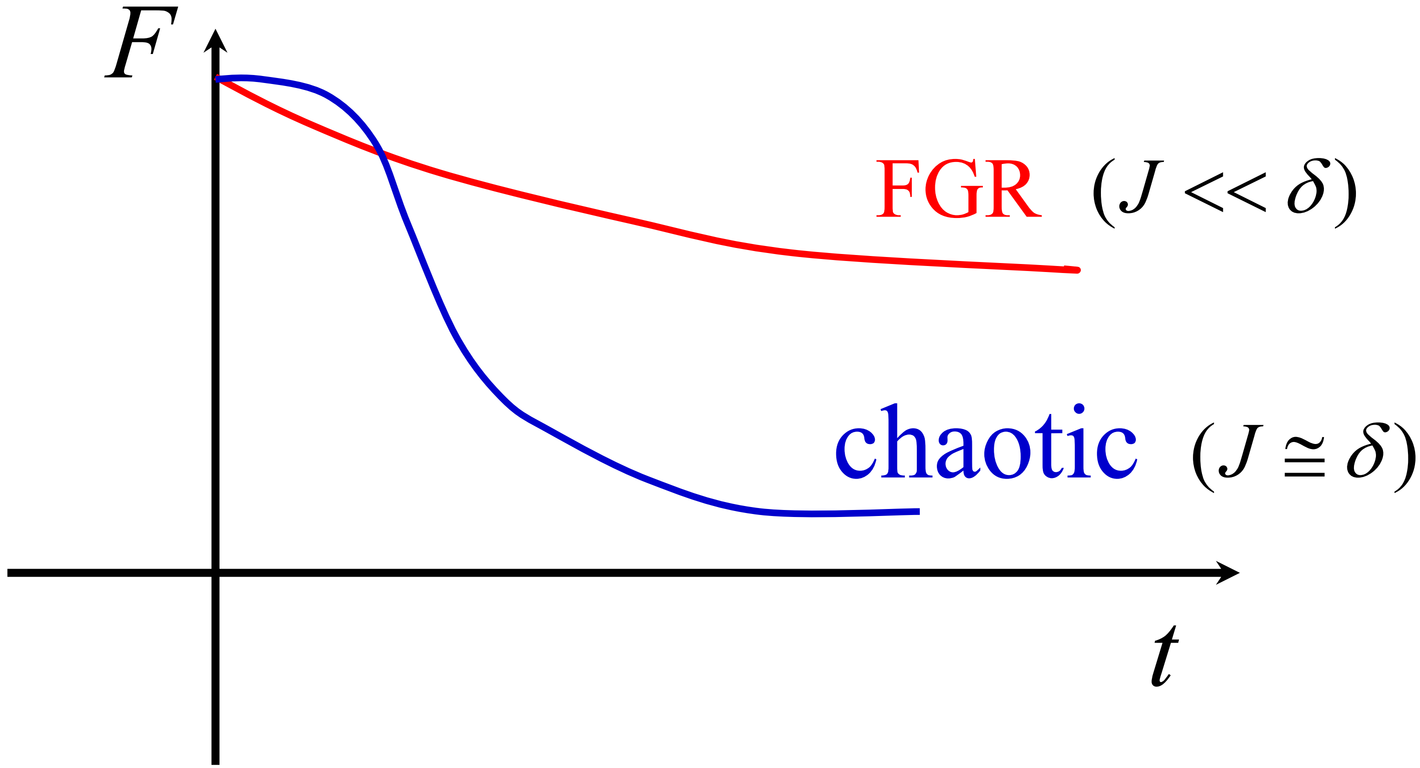
$$n_q = 10$$
$$\Delta = 1$$
$$\delta_i(t) \in [-\delta, \delta]$$
$$J_{ij}(t) \in [-J, J]$$

Initial state:

$$\sum_j \sigma_z^{(j)} |\Psi(t=0)\rangle = 0$$

(central band)

Static case (memorandum)



Dynamics: definitions

Fidelity

$$F \equiv \langle \Psi(t) | \Psi(t=0) \rangle$$

Error

$$E \equiv 1 - F$$

Total (final) time

$$T = N\tau = 25$$

contains static imperfections!

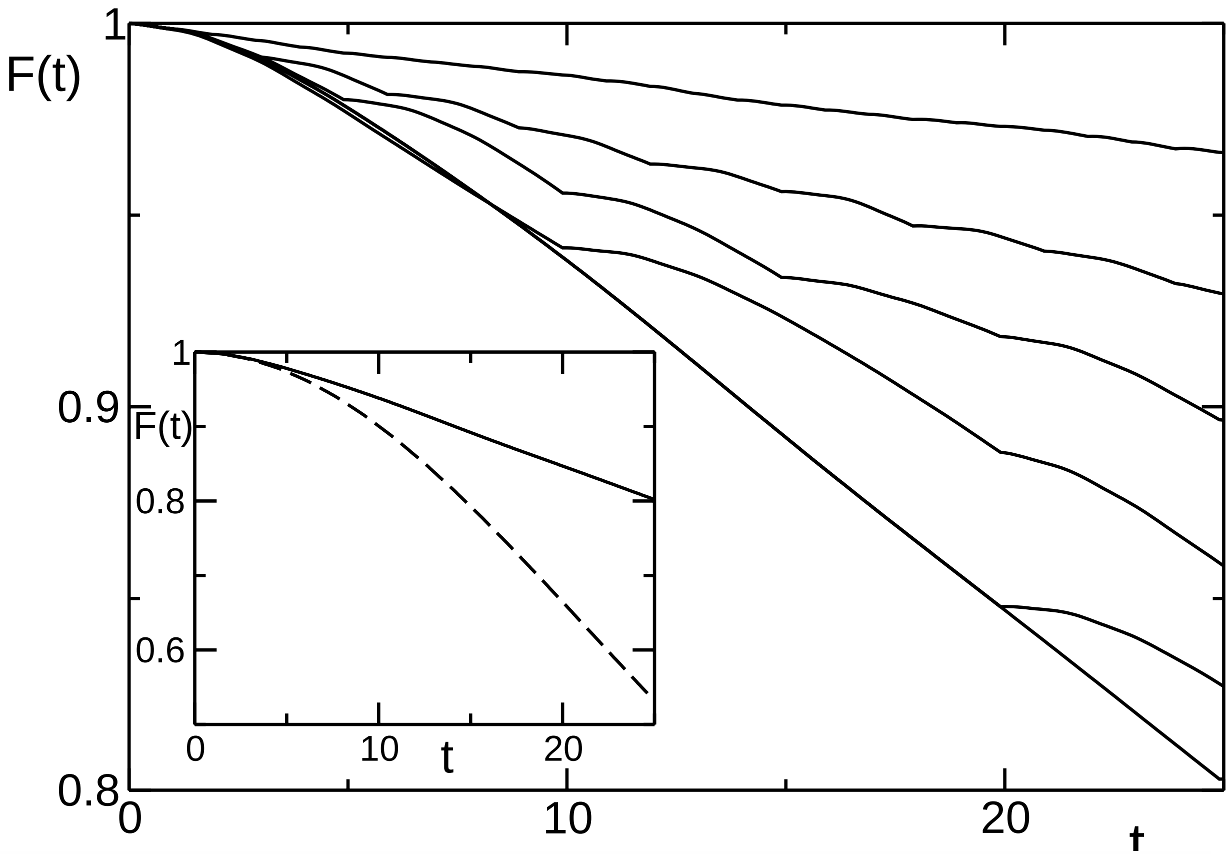
Average (over time or realizations)

$$H_0 = \langle H(t) \rangle$$

Evolution (theorem)!

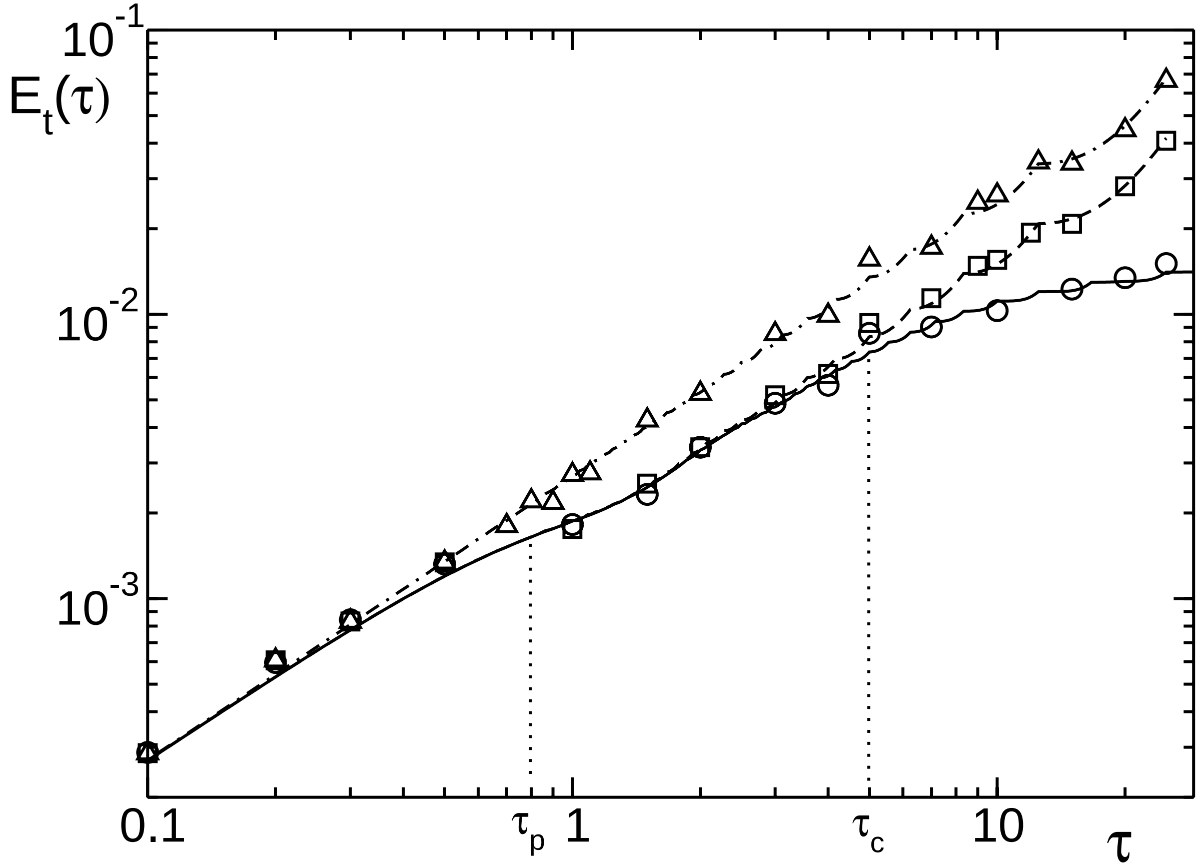
$$U_\tau(t) \xrightarrow{\tau \to 0} e^{-iH_o t}$$ (in probability)

Fidelity vs t



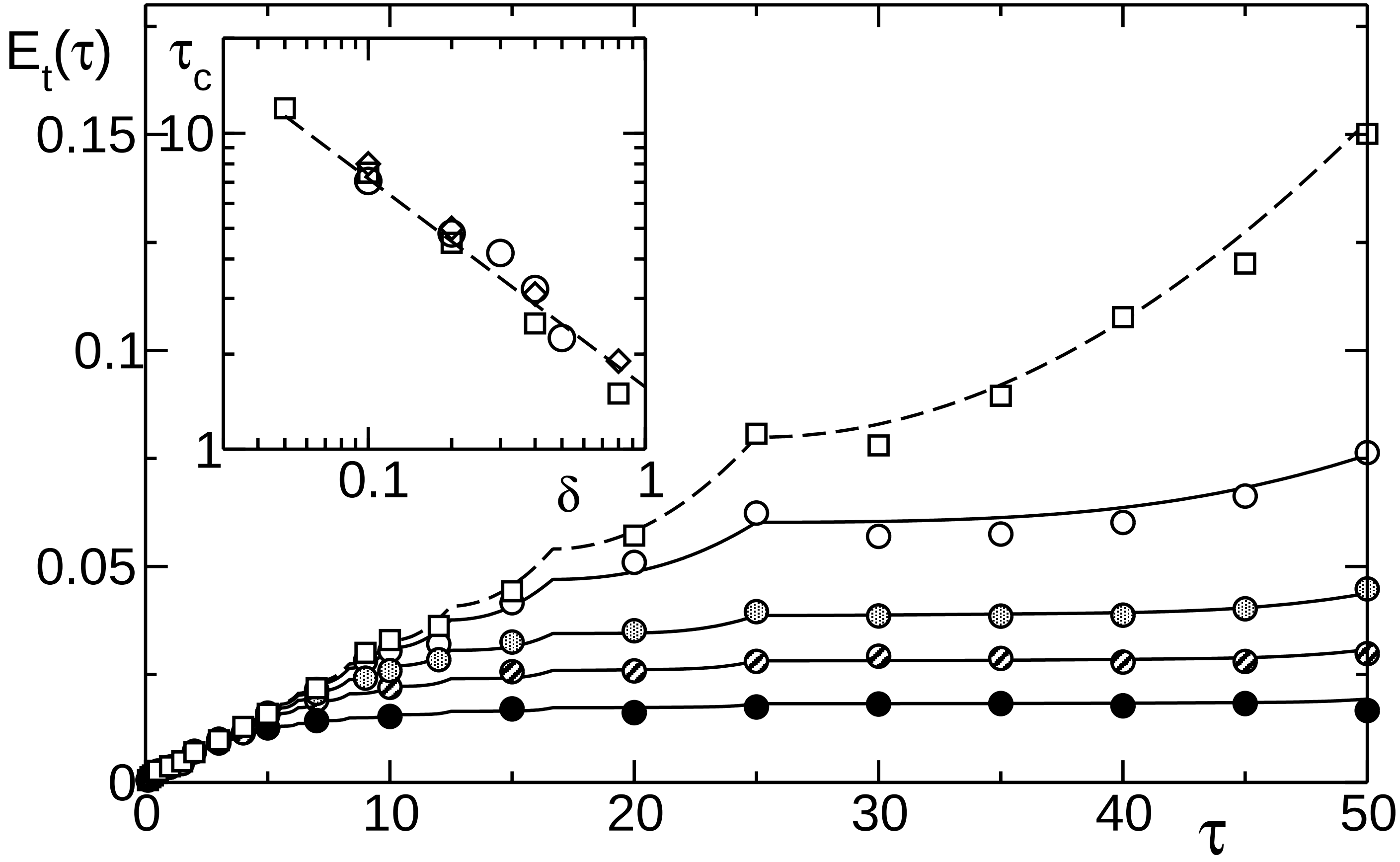
Fidelity as a function of time for $n = 14$ qubits in the FGR regime ($J = 2 \cdot 10^{-2}, \delta = 4 \cdot 10^{-1}$) and from top to bottom $\tau = 1, 3, 5, 10, 20, 25$ (static imperfections). Inset: Fidelity as a function of time in the ergodic ($J = \delta = 2 \cdot 10^{-2}$, dashed line), and in the FGR regime (full line): note the (common) short-time quadratic law.

Error vs time interval



Error E as a function of τ for $t = 25, n = 10, J = 5 \times 10^{-3}$, in the ergodic regime $\delta = 5 \times 10^{-3}$, $n_{\uparrow\downarrow} = 8$ (squares), $n_{\uparrow\downarrow} = 13$ (triangles) and in the FGR regime $\delta = 3 \times 10^{-1}$ (circles). The fits are given by the theoretical predictions with $\sigma^2 = 1/12$, $n_c = 13$, $\Delta_0 = 1$ and $(n_{\uparrow\downarrow}, n_{\uparrow\uparrow})$ equal to $(8, 5)$ (dashed), $(13, 0)$ (dot-dashed). The transition at τ_c is shown only in the former case. All the errors scale as J^2 (data not shown).

Error



Error at time $t = 50$, for $n = 10, J = 5 \cdot 10^{-3}$ and different δ values. The squares represent the ergodic regime $\delta = J$. The FGR regime is plotted for $\delta = 1, 2, 3, 5 \cdot 10^{-1}$ (empty, pointed, dashed, full circles respectively). Inset: τ_c as a function of δ for $n = 10, 12, 14$ (circles, squares and diamonds respectively). The dashed line is proportional to $\delta^{-2/3}$.