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SCHOOL AND WORKSHOP ON
QUANTUM ENTANGLEMENT, DECOHERENCE, INFORMATION, AND GEOMETRICAL PHASES IN COMPLEX SYSTEMS
(1 November - 12 November 2004)

Dynamical control of decoherence via active decoupling techniques: Random dynamical decoupling

L. VIOLA

Department of Physics and Astronomy
6127 Wilder Laboratory
Darmouth College
Hanover, NH 03755-3528
United States of America

Dynamical control of decoherence via active decoupling techniques: *Random dynamical decoupling*

Lorenza Viola

Lorenza.Viola@Dartmouth.edu

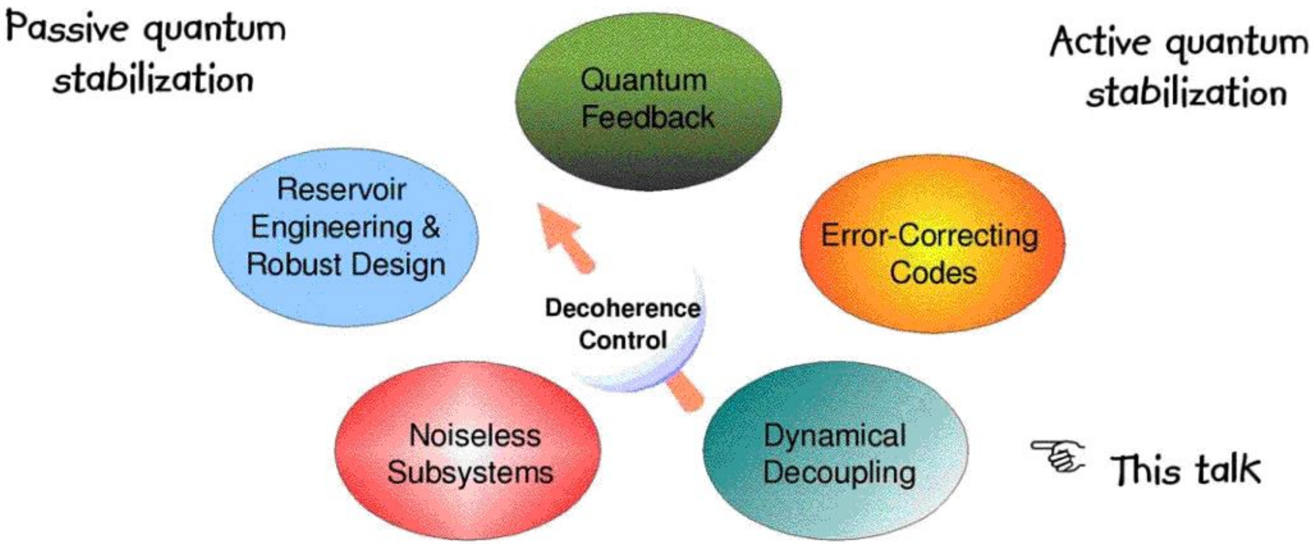
Lea F. Dos Santos (Dartmouth)
Manny Knill (NIST Boulder)

Why decoherence control?

Goal: Maintain & *coherently* manipulate *long-lived* quantum states under realistic *open-system* dynamics.

- **Practical motivation:**
  - ➞ Challenge to practically realize quantum information processing.
  - ➞ Experimental progress in nanoscale design and fabrication.
  - ➞ Precision measurements, quantum-limited systems.
- **Conceptual motivation:**
  - ➞ Develop control-theoretic framework for open quantum systems.

Control toolbox:



Outline

From: Coherent averaging techniques

Coherent control of nuclear spin Hamiltonians in HR multi-pulse NMR spectroscopy.

E.L. Hahn, PR 80, 580 (1950);
U. Haeberlen & J.S. Waugh, PR 175, 453 (1968).

To: Dynamical decoupling techniques

Open-loop dynamical control schemes relying on the repeated application of pulsed or switched controls drawn* from a finite control set.

*deterministically or...

- Dynamical decoupling with *unlimited* control resources

→ Bang-bang decoupling [arbitrary strength, arbitrarily fast, perfect control]

Problem: How do we design *efficient* and *robust* decoupling schemes operating under realistic control assumptions?

- Dynamical decoupling with *limited* control resources

→ Eulerian decoupling [bounded strength, still arbitrarily fast, faulty control]

*... stochastically!

→ Random decoupling [arbitrary strength, not necessarily ultrafast?, perfect control]

Quantum bang-bang control

L. Viola & S. Lloyd, PRA **58**, 2733 (1998).

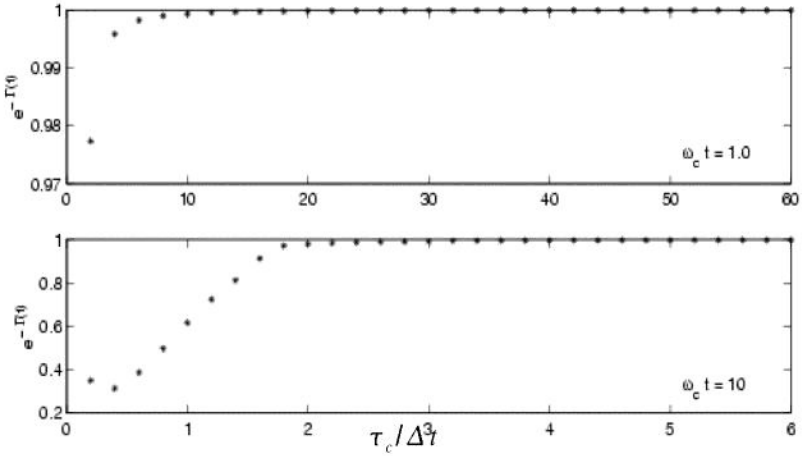
Dephasing spin-boson model:

$$H_0 = \nu_0 \sigma_z + \sum_k \omega_k b_k^\dagger b_k + \sigma_z \sum_k g_k \left(b_k + b_k^\dagger \right)$$

Control action:

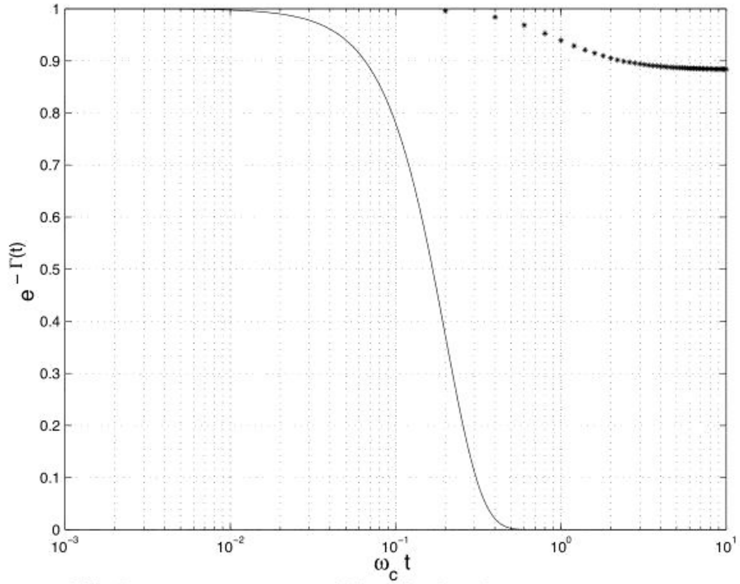
A train of identical, resonant π pulses, with separation Δt - arbitrarily strong and fast (BB).

Decoherence suppression if $\Delta t \ll \tau_c$



Low-temperature Ohmic bath

$$\frac{\omega_c}{T} = 100 \qquad \frac{\Delta t}{\tau_c} = 2N \frac{\tau_c}{t} \qquad N = 1, \ldots N_{max} = 30$$



High-temperature Ohmic bath

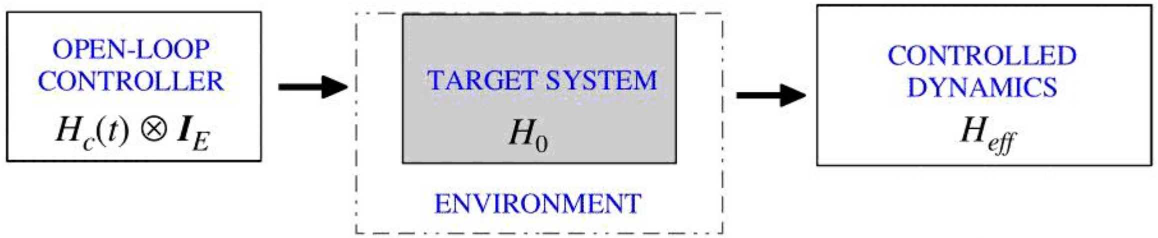
$$\frac{\omega_c}{T} = 0.01 \qquad \frac{\Delta t}{\tau_c} = 0.1 \qquad \omega_c t_N = 2N \left(\frac{\Delta t}{\tau_c} \right),$$

$$N = 1, \ldots, N_{max} = 50$$

Both suppression and acceleration possible…

Quantum interference effect mediated by non-Markovian response.

Control-theoretic framework



$\mathcal{H}_S \simeq \mathbb{C}^d$ for some d

- Target system S is coupled to *uncontrollable environment* E:Total drift

$$H_0 = H_S \otimes \boldsymbol{I}_E + \boldsymbol{I}_S \otimes H_E + \sum_a S_a \otimes B_a \,, \quad S_a \in \mathrm{End}(\mathcal{H}_S), \text{ traceless}$$

- Adjoin controller acting on S *alone*, $H_0 = H_0 + H_c(t) \otimes \boldsymbol{I}_E$

Design object: $U_c(t) = T\exp\{-i\int_0^t dx\ H_c(x)\}$ **Control propagator**

Controlled evolution: $U(t) = U_c(t)\tilde{U}(t)$

$$\tilde{U}(t) = T\exp\{-i\int_0^t dx\ U_c^{\dagger}(x) H_0 U_c(x)\}$$ **Logical propagator**

→ The logical propagator $\tilde{U}(t)$ describes the evolution of the system in a *logical frame* that follows the applied control.

Average-Hamiltonian theory

- Assume that controller operates cyclically: $U_c(t + T_c) = U_c(t)$ for $T_c > 0$.
  - ➞ Logical and physical frames coincide at times $t_M = M\,T_c$.

    Stroboscopic controlled evolution: $U(t_M = M\,T_c) = \tilde{U}(t_M = M\,T_c)$.

- Assume that drift Hamiltonian is *time-independent*, with $\|H_0\|_2 \leq K = Max\,|eig(H_0)|$
  - ➞ A time-independent average Hamiltonian exists s.t.

$$U(t_M = M\,T_c) = \exp\{-i\,\overline{H}\,t_M\} = \exp\{-i(\overline{H}^{(0)} + \overline{H}^{(1)} + \ldots)t_M\}$$

Magnus series

Convergent for $KT_c < 1$.

- First-order decoupling: Generate $\overline{H}$ to lowest order in T_c,

$$\overline{H}_0 = \frac{1}{T_c}\int_0^{T_c} dx\; U_c^\dagger(x) H_0 U_c(x)$$

$\overline{H}_0$ approaches $\overline{H}$ in the fast control limit, $T_c = T/M,\; M \to \infty$.

- Physical requirement for manipulation:

Coupling must remains coherent over manipulation time scale, $T_c \ll \tau_c = \min_i \{\tau_i^{corr}\}$

Decoherence contributions from $\overline{H}^{(k)}$ scale as $O(KT_c)^{2k} = O(T_c/\tau_c)^{2k}$, $k \geq 1$. Focus on $\overline{H}_0$.

Bang-bang symmetrization

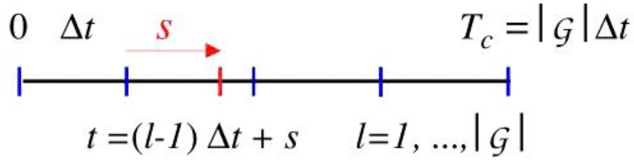
L. Viola, E. Knill, S. Lloyd, PRL **82**, 2417 (1999).

Keyword: Map time-average into group-theoretical average.

- Decoupling group: $\mathcal{G} = \{g_j\}$, $j = 0,\ldots, |\mathcal{G}| - 1$. $\mathbb{C}\mathcal{G}$ = decoupling algebra.
  $\mathcal{G}$ acts on state space $\mathcal{H}_S$ via a faithful, unitary, projective representation,
  $\mu(g_j) = \hat{g}_j \in \mathrm{Mat}_d(\mathbb{C})$, $\quad \widehat{g_j g_k} = \hat{g}_j \hat{g}_k$ up to phase, $\hat{g}_0 = \boldsymbol{I}$.

- BB protocol: Let $T_c = |\mathcal{G}|\Delta t$ and assign $U_c(t)$ over T_c as

$$U_c\big((l-1)\Delta t + s\big) = \hat{g}_{l-1}$$



i.e.

$$U_c(t) = \begin{cases} \hat{g}_0 & t = s \\ \hat{g}_1 & t = \Delta t + s \\ \cdots & \cdots \\ \hat{g}_{|\mathcal{G}|-1} & t = (|\mathcal{G}|-1)\Delta t + s \end{cases} \qquad s \in [0, \Delta t)$$

Instantaneously change control propagator at the end of each control subinterval.

Quantum error suppression

- Lowest-order BB effective Hamiltonian:

$$\overline{H}_0 = \frac{1}{|\mathcal{G}|}\sum_j \hat{g}_j^\dagger H_0 \hat{g}_j = \Pi_{\hat{\mathcal{G}}}(H_0),$$

$\mathcal{G}$-symmetrization

where $\Pi_{\hat{\mathcal{G}}}$ is the projector onto the commutant $\widehat{\mathbb{C}\mathcal{G}}'$.

- Noise suppression on entire state space iff $\Pi_{\hat{\mathcal{G}}}(S_a)=0 \ \forall a.$

$$\Pi_{\hat{\mathcal{G}}}(H_0) = \Pi_{\hat{\mathcal{G}}}(H_S)\otimes \boldsymbol{I}_E + \boldsymbol{I}_S \otimes H_E + \sum_a \Pi_{\hat{\mathcal{G}}}(S_a)\otimes B_a$$

(1) Maximal averaging: $\widehat{\mathbb{C}\mathcal{G}}' = \mathbb{C}\boldsymbol{I}$

➞ $\mathcal{G}$ acts *irreducibly* over $\mathcal{H}_S$. Averaging over a unitary error basis.

➞ Required when unwanted interaction is arbitrary/unknown.

(2) Selective averaging: $\widehat{\mathbb{C}\mathcal{G}}' \neq \mathbb{C}\boldsymbol{I}$

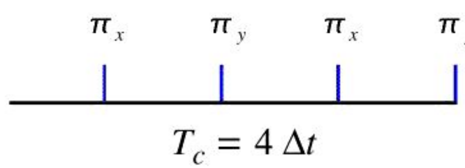
➞ $\mathcal{G}$ acts *reducibly* over $\mathcal{H}_S$.

➞ $\mathcal{G}$ may be determined from available knowledge of unwanted interaction.

- Examples: Single qubit

➞ Suppression of *pure dephasing* via $\mathcal{Z}_2$ symmetrization.

➞ Suppression of *arbitrary noise* via $\mathcal{Z}_2 \times \mathcal{Z}_2$ symmetrization.



Control of decoupled dynamics

L. Viola, S. Lloyd, E. Knill, PRL **83**, 4888 (1999).

Problem: Manipulations of effective dynamics are constrained in symmetry and timing.

- Decoupled control schemes:

(1) Arbitrary (strength)/fast (switching) control (BB).

➡ Use BB pulses available beside BB decoupling pulses. Must be synchronized with cycle time.

(2) Weak (strength)/slow (switching) control.

➡ Any Hamiltonian $A \in \widehat{\mathbb{C}\mathcal{G}}'$ can be applied in parallel with controller for times $\Delta T \gg T_c$.

(3) Weak (strength)/fast (switching) control.

➡ A Hamiltonian $B \notin \widehat{\mathbb{C}\mathcal{G}}'$ can be obtained by applying a modulated Hamiltonian $B_j = g_j B g_j^\dagger$ during the *j*th sub-interval of each control cycle: $\widetilde{H}_{eff} = \Pi_{\hat{\mathcal{G}}}(H_0) + B$.

- Universal decoupled control over $\mathcal{H}_S$:

Achievable by combining control according to different decoupled schemes, *e.g.*

➡ Alternate slow control from *two* distinct commutants (if two controllers available);

➡ Alternate slow control from $\widehat{\mathbb{C}\mathcal{G}}'$ with weak/fast control outside.

➡ ...

Noise-tolerant universal quantum computation based solely on unitary means possible in principle.

How far away from reality?

Required control resources *unrealistic* for three main reasons:

- Amplitude requirement for BB control design: Arbitrarily strong control

  Instantaneous control pulses imply *unbounded* control strengths even for finite T_c.

  Additional drawbacks:

  → Poor spectral *selectivity*.

  → Inappropriate for including drift during pulses.

- Timescale requirement for complete decoherence suppression: Arbitrarily fast control

  Coupling must be coherent over manipulation time scale, $T_c \ll \tau_c = \min_i \{\tau_i^{corr}\}$

  → Estimated control rates may be *prohibitively high* if τ_c is short.

  → Averaging is more and more *inefficient* with growing group size $|\mathcal{G}|$.

  → What about interactions which *fluctuate* on time scale short wrto T_c ?

- Ideal control resources: Arbitrarily accurate control

  → How tolerant of *systematic* control faults?

  → How tolerant of *random* imperfection/jitter?

Can we decouple under more realistic assumptions?...

Eulerian control design

L. Viola & E. Knill, PRL **90**, 037901 (2003).

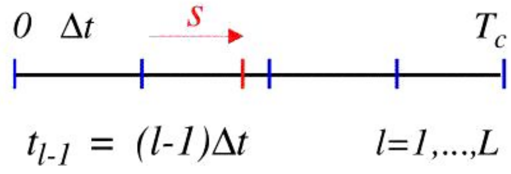
Keyword: Design continuous $U_c(t)$ according to Eulerian cycle.

- Control resources: Assume ability to implement group generators,

$$\hat{\gamma}_\lambda = T\exp\{-i\int_0^{\Delta t} ds\ h_\lambda(s)\} \equiv u_\lambda(\Delta t)\ ,\quad \lambda = 1,\ldots,|\Gamma|.$$

- Eulerian protocol: Choose Eulerian cycle $\mathcal{P}_E = (\gamma_{\lambda 1}, \gamma_{\lambda 2}, \ldots, \gamma_{\lambda L})$ on $G(\mathcal{G}, \Gamma)$. Let $T_c = L\,\Delta t$ and assign $U_c(t)$ over T_c as

$$U_c(t_{l-1}+s) = u_l(s)\,U_c(t_{l-1})$$



$$U_c(t) = \begin{cases} u_1(s) & t = s & s\in[0,\Delta t) \\ u_2(s)\,\hat{\gamma}_{\lambda_1} & t = \Delta t + s \\ u_3(s)\,\hat{\gamma}_{\lambda_2}\hat{\gamma}_{\lambda_1} & t = 2\Delta t + s \\ \ldots & \ldots \\ u_L(s)\,\hat{\gamma}_{\lambda_{L-1}} & t = (L-1)\Delta t + s \end{cases}$$

During the l-th interval, use as a control Hamiltonian the one that implements the generator $\hat{\gamma}_{\lambda_l}$, with γ_{λ_l} colouring the l-th edge in $\mathcal{P}_E$.

$\hat{\gamma}_{\lambda_L}\cdots\hat{\gamma}_{\lambda_2}\hat{\gamma}_{\lambda_1} = \boldsymbol{I}$ by cyclicity.

$\mathcal{G} = \mathscr{Z}_2 = \{ 0 , 1 \}$, additive (mod 2), abelian

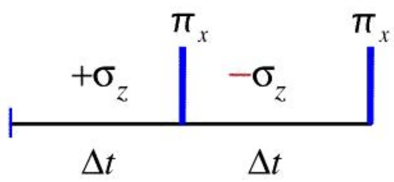
Generating set $\Gamma = \{ \gamma_1 \} = \{ 1 \}$, $\mathcal{P}_E = (\gamma_1, \gamma_1)$

- Applications: Elimination of unwanted σ_z dynamics in a single qubit.

$\mathcal{H}_S \simeq \mathbb{C}^2$, represent $\mathcal{G}$ in $\mathcal{U}(\mathcal{H}_S)$ as $\widehat{\mathcal{G}} = \{ \boldsymbol{I}, \sigma_x \}$, with $\Pi_{\hat{\mathcal{G}}}(\sigma_z) = 0$.

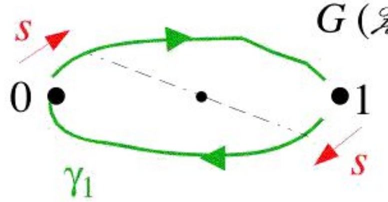
BB implementation:

$$\overline{H}_0 = \frac{1}{2\Delta t} \left[\int_0^{\Delta t} ds\, \boldsymbol{I}\, \sigma_z\, \boldsymbol{I} + \int_0^{\Delta t} ds\, \sigma_x \sigma_z \sigma_x \right] = \frac{1}{2} \left[\sigma_z + \sigma_x \sigma_z \sigma_x \right] = 0$$



Eulerian implementation:

$$u_x(t) = \exp\{ -i \int_0^t du\, h_x(u) \},$$
$$h_x(t) = f(t) \sigma_x, \quad u_x(\Delta t) = \sigma_x$$



$$\overline{H}_0 = \frac{1}{2\Delta t} \left[\int_0^{\Delta t} ds\, u_x^\dagger(s) \sigma_z u_x(s) + \int_0^{\Delta t} ds\, \sigma_x u_x^\dagger(s) \sigma_z u_x(s) \sigma_x \right]$$
$$= \frac{1}{2\Delta t} \left[\int_0^{\Delta t} ds\, u_x^\dagger(s) \sigma_z u_x(s) + \int_0^{\Delta t} ds\, u_x^\dagger(s) (-\sigma_z) u_x(s) \right] = 0$$

No time overhead wrto BB case.

Intrinsic robustness against any systematic error along z, y.

Eulerian symmetrization

Generalize to arbitrary *time-independent* systems and couplings:

- Under mild assumptions on the control Hamiltonians, $h_l(s) \in \widehat{\mathbb{C}\mathcal{G}} \quad \forall\, s \in [0, \Delta t],\ \forall\, l$,
  Eulerian design is able to retain the *same* $\mathcal{G}$-symmetrization of the BB limit:

$$\overline{H}_0 = \frac{1}{|\mathcal{G}|} \sum_j \hat{g}_j^\dagger H_0\, \hat{g}_j$$

→ For every *finite* control period, $0 < T_c \ll \tau_c$, control can be implemented using *bounded-strength Hamiltonians*.

- Stability analysis: Non-ideal control implementation due to systematic errors:

$$H'_c(t) = H_c(t) + \Delta H_c(t) = \text{ideal control} + \text{error component},$$
$$H_{tot}(t) = H_0 + H'_c(t) = [\, H_0 + \Delta H_c(t)\,] + H_c(t).$$

→ Eulerian decoupling is intrinsically *stable against faulty control implementations*:

> Residual errors are themselves symmetrized: $\Delta H_c(s) \rightarrow \overline{\Delta H_c(s)} \in \widehat{\mathbb{C}\mathcal{G}}'$

> They are either averaged out, or they can be compensated for by encoding into subsystems of S.

Improved amplitude requirements + added robustness

Recap

- Eulerian design:
  - ➞ Solves amplitude requirement;
  - ➞ Significantly mitigates ideality requirement.

- Disadvantage: Control cycle is *lengthened* by a (polynomial) factor $|\Gamma|$...
  - > Partial fix for *few-body* Hamiltonians/interactions via combinatorial design: *Eulerian orthogonal arrays*

    P. Wocjan, quant-ph/0410107.

  - > Complexity of decoupling (= required number of averaging subinterval in a cycle) still *quadratic* in the number n of qubits – still significant for large n.

- Enhanced error tolerance also achievable by concatenated dynamical decoupling:

  Nested cycles of pulses, with pulse interval shrinking between successive layers.

  K. Khodjasteh & D. A. Lidar, quant-ph/0408128.

- Disadvantage: *Exponential number of pulses* needed...
  - ➞ Timing requirements very stringent;
  - ➞ No general applicability to time-dependent interactions.

Try a different approach...

Random decoupling design

L. Viola & E. Knill, submitted (2004).

Keyword: Assign $U_c(t)$ according to a random control path.

Focus on switching off the evolution due to a *generic* $H_S(t)$ (*no* environment).

- Control resources: Available control in a discrete or continuous compact $\mathcal{G}$,

$$\mu(g_j) = \hat{g}_j \in \mathrm{Mat}_d(\mathbb{C}), \text{ projective.}$$

- Random protocol: Rotate state of S according to $\mathcal{G}$ randomly over time:
  - → Past control operations and control times: **Known.**
  - → Future control path: **Uniformly random.**

Depict the evolution of the system directly in the logical frame that follows the applied control.

Logical state: $\tilde{\rho}_S(t) = U_c^\dagger(t)\rho_S(t)U_c(t)$ Logical evolution: $\tilde{\rho}_S(t) = \tilde{U}(t)\tilde{\rho}_S(0)\tilde{U}^\dagger(t)$

$$\tilde{U}(t) = U_c^\dagger(t)U(t) = T\exp\{-i\int_0^t dx\ U_c^\dagger(x)H_0U_c(x)\}$$

→ Logical and physical frames almost never coincide: *Acyclic* decoupling.

Error bounds

- Characterize decoupling performance by an appropriate error metric on pure states:

A-priori error probability:

$$\epsilon_T(P_S) = \boldsymbol{E}\{tr_S(P_S^{\perp}\tilde{\rho}_S(t))\} \qquad P_S = |\psi\rangle_S\langle\psi|$$

$$= \boldsymbol{E}\{tr_S(P_S^{\perp}\tilde{U}(T)P_S\tilde{U}(T)^{\dagger})\}$$

Worst-case pure-state error probability:

$$\epsilon_T = Max_{P_s}\{\epsilon_T(P_S)\}$$

- **Theorem (Random decoupling).** Suppose that the following conditions hold:

  (*i*) $\mathcal{G}$ acts *irreducibly* on the state space.
  (*ii*) $U_c(t)$ is *uniformly random* for every fixed t.
  (*iii*) For any $t, s > 0$, $U_c(t)$ and $U_c(t+s)$ are *independent* for $s > \Delta t$.
  (*iv*) $\|H_S(t)\|_2 \leq K$ *uniformly* in time. Then

$$\epsilon_T = O(T\,\Delta t\,K^2) \qquad T\,\Delta t\,K^2 \ll 1$$

Random averaging works!

- Simplest implementation setting:

  → *Discrete* control group $\mathcal{G}$. Enforce random walk on $\mathcal{G}$ through a *sequence of equally spaced BB pulses randomly drawn from* $\mathcal{G}$.

Random decoherence suppression

- Extend to a system S coupled to an environment E: Seek error metric depending only on reduced state of S in the logical frame:

A-priori error probability:

$$\epsilon_T(P_S) = \boldsymbol{E}\{tr_S(P_S^{\perp}\tilde{\rho}_S(t))\} \qquad P_S = |\psi\rangle_S\langle\psi|,\ P_E = |\phi\rangle_E\langle\phi|$$

$$= \boldsymbol{E}\{tr_{S,E}(P_S^{\perp}\otimes \boldsymbol{I}_E \tilde{U}'(T) P_S \otimes P_E \tilde{U}'(T)^{\dagger})\}$$

$$\tilde{U}'(t) = Texp\{-i\int_0^t dx\ U_c^{\dagger}(x)[H_S + H_{SE}]U_c(x)\}$$

Worst-case pure-state error probability:

$$\epsilon_T = Max_{P_S}\{\epsilon_T(P_S)\}$$

- **Theorem (Random decoherence control).** Suppose that the same irreducibility, uniformity, and independence assumptions hold as before, and that in addition

$(iv)\ \ \|H_S(t) + H_{SE}(t)\|_2 \leq K$ *uniformly* in time. Then

$$\epsilon_T = O(T\Delta t K^2) \qquad T\Delta t K^2 \ll 1$$

→ K is a measure of the overall noise strength, $1/K \sim \tau_c = \min_i\{\tau_i^{corr}\}$.

Arbitrarily accurate decoupling of S from E possible on average in the logical frame.

Random vs deterministic decoupling

How does this compare to standard cyclic decoupling?...

- **Theorem (Deterministic error bound).** Suppose that the following conditions hold:

  (i) $\mathcal{G}$ acts *irreducibly* on the state space.

  (ii) $U_c(t)$ is assigned according to a cyclic scheme with $M=|\mathcal{G}|$ intervals.

  (iii) $\|H_S\|_2 \leq K$, $KT_c < 1$. Then

$$\epsilon_T = O\left(\left(T T_c K^2\right)^2\right) \qquad T T_c K^2 \ll 1 - K T_c$$

- Meaning: $R = T \Delta t K^2 = (K \Delta t)^2 \dfrac{T}{\Delta t} =$ **Max error probability/step * No. of Steps**

$$\epsilon_T^{Random} = O(R) \qquad vs \qquad \epsilon_T^{Deterministic} = O\left(R\left(M^2 R\right)\right)$$

→ Superior performance of *cyclic* decoupling expected if:

1. Any time-dependence has time scale *longer* than $T_c = M\Delta t$;
2. $M^2 R << 1$

→ Superior performance of *random* decoupling expected if:

1. Time-dependencies have time scale *short* compared $T_c = M\Delta t$, long compared to Δt;
2. $M^2 R >> 1$

Randomly kicked decohering qubit

L. F. Dos Santos & L. Viola, submitted (2004).

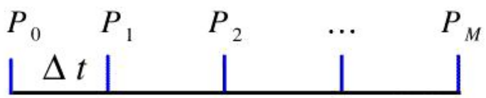
Keyword: Assign $U_c(t)$ according to a random control path on $\mathscr{Z}_2$.

- System: Single qubit decohering via coupling to a bosonic bath,

$$H_0 = \nu_0 \sigma_z + \sum_k \omega_k b_k^\dagger b_k + \sigma_z \sum_k g_k \left(b_k + b_k^\dagger\right)$$

- Control: A train of random BB pulses, with separation Δt,

$$Prob\{P_j = \boldsymbol{I}\} = 0.5 \qquad Prob\{P_j = \pi_x\} = 0.5$$



$T_M = M\Delta t$

→ Exact expression for decay of coherence element *in the logical frame*:

$$\tilde{\rho}_{01}(t_M) = Z_{01}(t_M)\,\tilde{\rho}_{01}(0) \qquad Z_{01}(t_M) = \exp\left\{-\frac{1}{2}\sum_k \left|\sum_{j=0}^{M-1} \Xi_j\, e^{\omega_k t_j}\, \xi_k(\Delta t)\right|^2 \coth\left(\frac{\omega_k}{2k_B T}\right)\right\}$$

$$Z_{01}(t_M) = \exp\{-\Gamma(t_M)\}$$

$$\Xi_j = (-1)^{\lambda_0 + \ldots \lambda_j} \qquad \xi_k(\Delta t) = \frac{2 g_k}{\omega_k}\left(1 - e^{i\,\omega_k \Delta t}\right)$$

Decoherence suppression in the fast control limit:

$$\boldsymbol{E}\{Z_{01}(t_M)\} \to 1$$

→ Quantitative study of *typical* performance in various regimes possible...

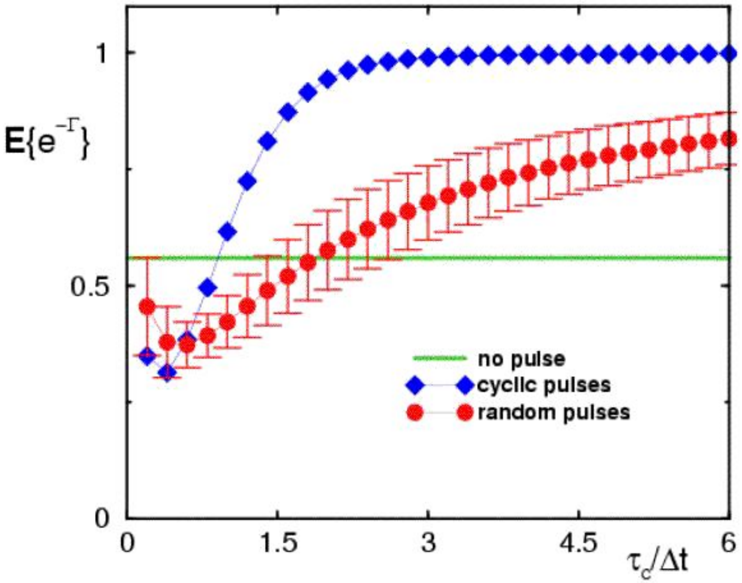
Results

Constant interactions-

$$H_{SE} = \sigma_z \sum_k g_k \left(b_k + b_k^\dagger \right)$$

Low-temperature Ohmic bath

$$\frac{\omega_c}{T} = 100 \qquad \omega_c t = 10$$



1. Randomization works.

2. No improvement for const couplings ($\mathcal{G}$ too small!) - yet...

3. Safer decoupling procedure for time-dep couplings.

Time-varying interactions-

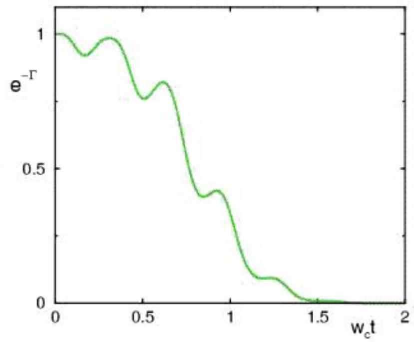
$$g_k \rightarrow g_k(t) = g_k \cos(c_1 t) * \sin(c_2 t)$$

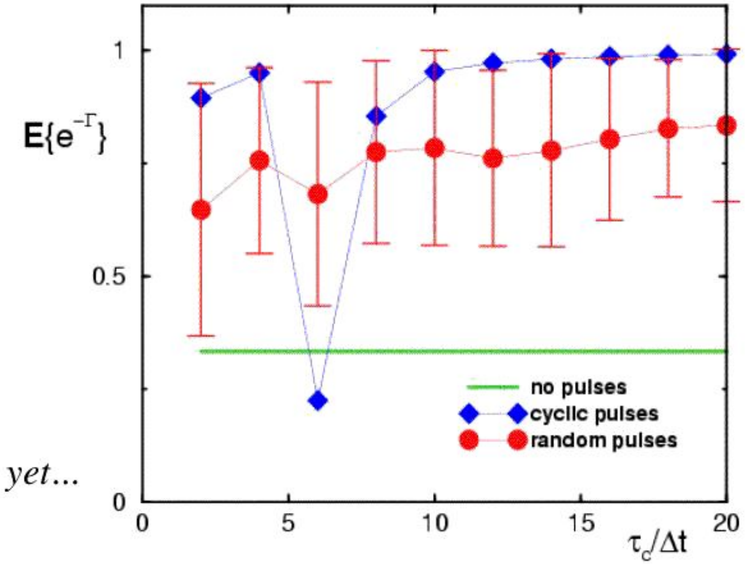
High-temperature Ohmic bath

$$\frac{\omega_c}{T} = 0.01 \qquad \omega_c t = 1$$

$$c_1 = 295\pi$$

$$c_2 = 325\pi$$

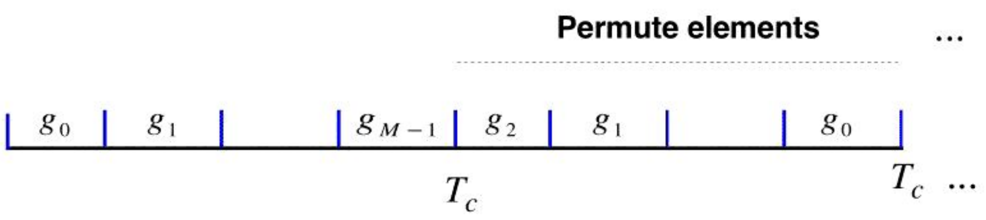




Generalizations and outlook

- Hybrid decoupling schemes: Merge cyclic with random control design
  - ➞ Randomize the cycles:

If $|\mathcal{G}|=M$, there are $M!$ paths to follow to traverse $\mathcal{G}$



$$T_c = M\Delta t$$

Retains stroboscopic overlap between physical and logical frame.

  - ➞ Significantly faster averaging expected for *large* control groups:

Examples, n qubits: Unitary error groups, $M=4^n$; Symmetric groups: $M=n!$...

- **Ongoing work:**
  - ➞ Develop more *physical* understanding: *How* is randomization working?
  - ➞ Error-bounds for hybrid schemes.
  - ➞ Typical performance of *discrete* randomly-controlled (*qudit*) systems.
  - ➞ Typical performance of *continuous* randomly-controlled (*harmonic*) systems.
  - ➞ Stability analysis: Concatenation and *fault-tolerance* properties.
  - ➞ Schemes for randomly controlled *universal quantum computation*.

...

Conclusions

- Dynamical decoupling techniques offer a well-defined conceptual framework to address a variety of open-loop coherent-control problems for quantum systems:
  - ➞ Decoherence control.
  - ➞ Quantum-dynamical engineering, physical and logical degrees of freedom.

- Eulerian design represents a significant advance over BB design in terms of both the required control resources (*bounded* vs unbounded-strengths) and the resulting control performance (intrinsic *robustness*).

- Randomization offers the potential for *faster convergence* and *relaxed timing requirements* compared to cyclic decoupling in relevant control scenarios.

➞ New prospects for efficient, robust dynamical decoupling schemes...
➞ Largely unexplored setting for quantum-dynamical control...

More soon!...

Further reading (A very incomplete list...)

1968	Coherent averaging in NMR.	Haeberlen & Waugh, PR **175**, 453
1998	BB control/Spin echo for single qubit.	Viola & Lloyd, PRA **58**, 2733; Ban, JMO **45**, 2315
1999	Error suppression/symmetrization.	Viola, Knill, Lloyd, PRL **82**, 2417; Zanardi, PLA **258**, 77
	Universal decoupled control.	Viola, Lloyd, Knill, PRL **83**, 4888; Duan & Guo, PLA **261**, 139
	Parity kicks for quantum oscillator.	Vitali & Tombesi, PRA **59**, 4178
2000	Dynamical generation of Nss/DFSs.	Viola, Knill, Lloyd, PRL **85**, 3520
	Algebraic framework.	Knill, Laflamme, Viola, PRL **84**, 2525; Zanardi, PRA **63**, 012301
	Collisional decoherence suppression.	Search & Berman, PRL **85**, 2272; PRA **62**, 053405
	Off-resonant effect suppression.	Tian & Lloyd, PRA **62**, 050301
2001	Exp. BB suppression of single-photon dephasing.	Berglund, quant-ph/0010001
	Inhibition of decay to continuum.	Agarwal, Scully, Walther, PRL **86**, 4271
	Encoded dynamical decoupling.	Lidar & Wu, PRL **88**, 017905; Viola, PRA **66**, 012307
2002	Exp. realization of encoded dynamical decoupling.	Fortunato, Viola, NJP **4**, 5.1
	Universal quantum simulation.	Wocjan et al, QIC **2**, 133; Lloyd & Viola, PRA **65**, 010101
	Heating/finite temperature reservoir.	Vitali & Tombesi, PRA **65**, 012305
	DFS dynamical generation.	Wu & Lidar, PRL **88**, 207902
	Solid-state QC design and decoherence.	Byrd & Lidar, PRL **89**, 047901
	Universal leakage suppression.	Wu, Byrd, Lidar, PRL **89**, 127901
	Empirical BB control.	Byrd & Lidar, quant-ph/0205156
	Non-linear reservoir couplings.	Uchiyama & Aihara, PRA **66**, 032313
2003	Robust bounded-strength design.	Viola & Knill, PRL **90**, 037901

(*over* →)

Further reading (continued)

2004	Connection with Quantum Zeno physics.	Facchi, Lidar, Pascazio, PRA **69**, 032314
	Application to 1/*f* spectral densities/power spectra.	Shiokawa & Lidar, PRA **69**, 030302
		Faoro & Viola, PRL **92**, 117905
		Falci et al, PRA **70**, 040101
	Connection with universal dynamical control.	Kofman & Kuritzki, PRL **93**, 130406
	Decoupling based on Hamilton cycles.	Roetteler, quant-ph/0408078
	Concatenated dynamical decoupling.	Khodjasteh & Lidar, quant-ph/0408128
	Control of entanglement.	Uchiyama & Aihara, quant-ph/0408139
	Equivalence with orthogonal arrays.	Roetteler & Wocjan, quant-ph/0409135
	Decoupling based on Eulerian orthogonal arrays.	Wocjan, quant-ph/0410107
	Random dynamical decoupling.	Viola & Knill, submitted
		Dos Santos & Viola, submitted
	Application to nanomechanical resonator cooling.	Zhang, Wang, Sun, quant-ph/0410149

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