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SCHOOL AND WORKSHOP ON
QUANTUM ENTANGLEMENT, DECOHERENCE, INFORMATION, AND GEOMETRICAL PHASES IN COMPLEX SYSTEMS
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Decoherence and decoherence suppression in collective quantum memories for photons

M. FLEISCHHAUER

Fachbereich Physik
Technische Universität Kaiserslautern
Postfach 3040
Erwin-Shrödinger Strasse Gbd. 46
67653 Kaiserslautern
Germany

Decoherence and decoherence suppression in collective quantum memories for photons

Michael Fleischhauer

Razmik Unanyan

Claudia Mewes

Fachbereich Physik, Techn. Univ. Kaiserslautern

$$ EU TMR network QUACS, DFG

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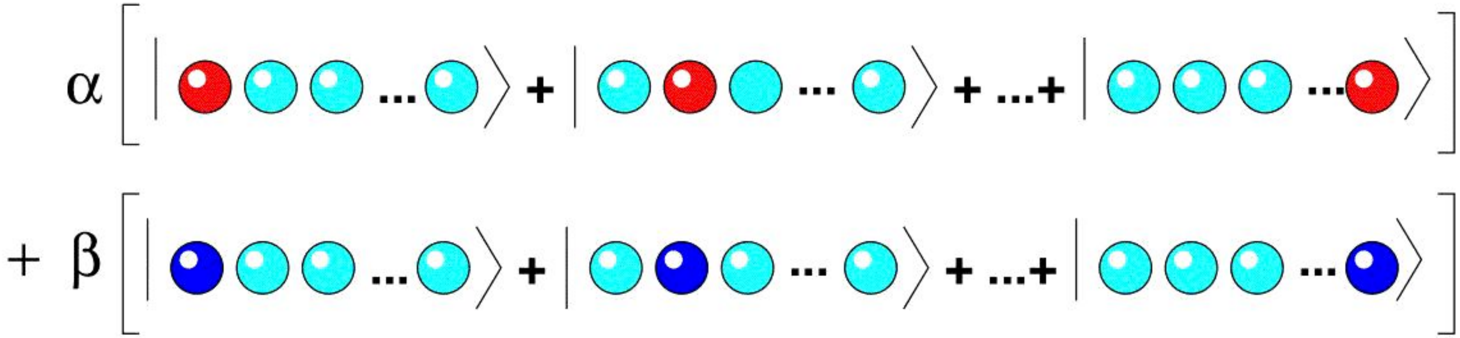
0. qubits as delocalized collective states

we usually consider qubits as states of *localized* spins

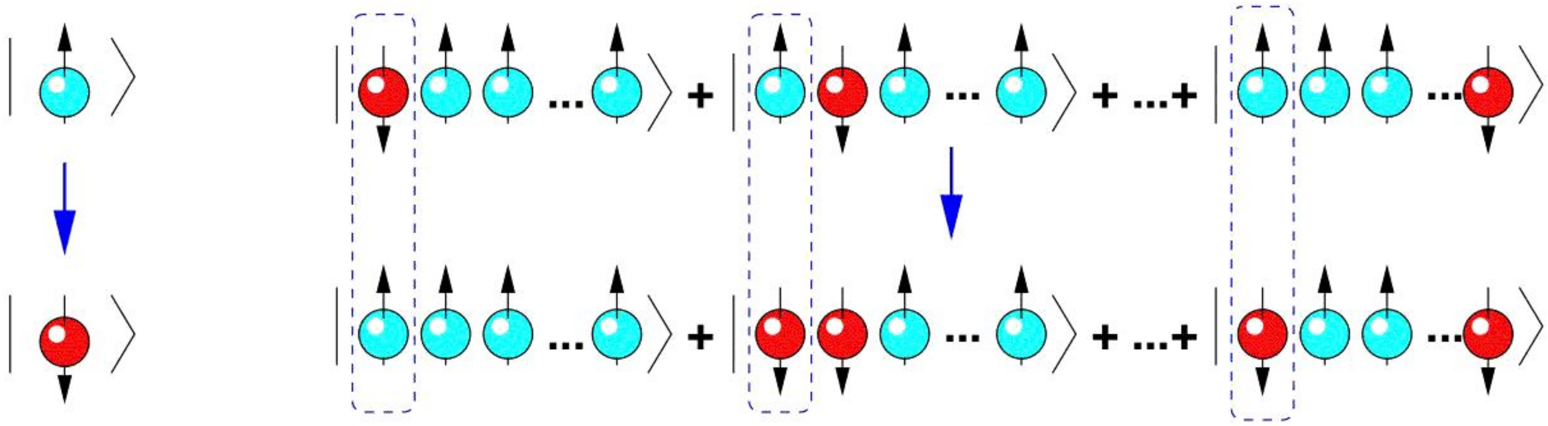
is there any advantage of using *delocalized collective excitations*?



these states are entangled states !

The problem with many-particle entanglement

spin flip of one atom generates orthogonal state



$P_{error} = p$

$P_{error} = 1 - (1 - p)^N \sim p\,N$

many-particle entanglement is highly susceptible to decoherence

I want to show here:

I. collective qubits allow easy interfacing to photons as information carrier

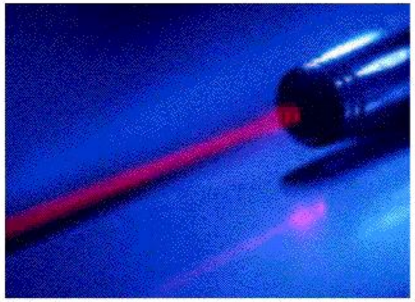
II. collective qubits can be as decoherence robust as single-particle qubits

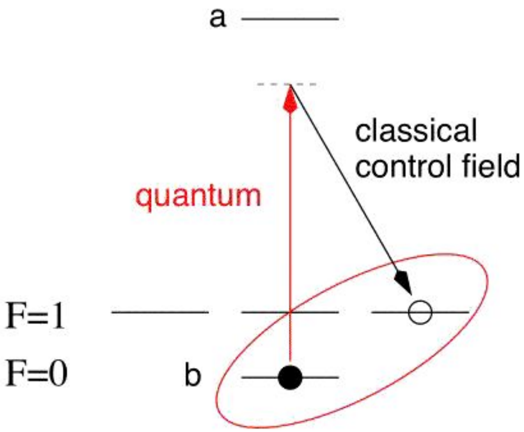
III. collective qubits allow for effective decoherence protection

- decoherence-free subspaces even for individual reservoir couplings
- dephasing protection by "bang-bang" control

I. interfacing collective qubits with photons

information carrier: qubit storage and processor:





photons individual atoms

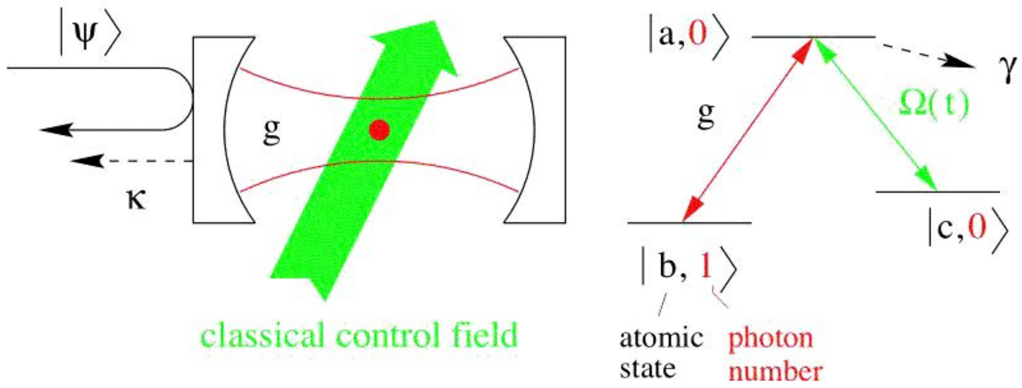
requirements: reversible (no dissipation)

unidirectional (explicit time-dependent H(t))

theoretical efficiency $\rightarrow 1 - 10^{-5}$

solution: **Raman adiabatic passage**

Raman adiabatic passage & cavity QED



- **dark-state** of cavity + atom system:

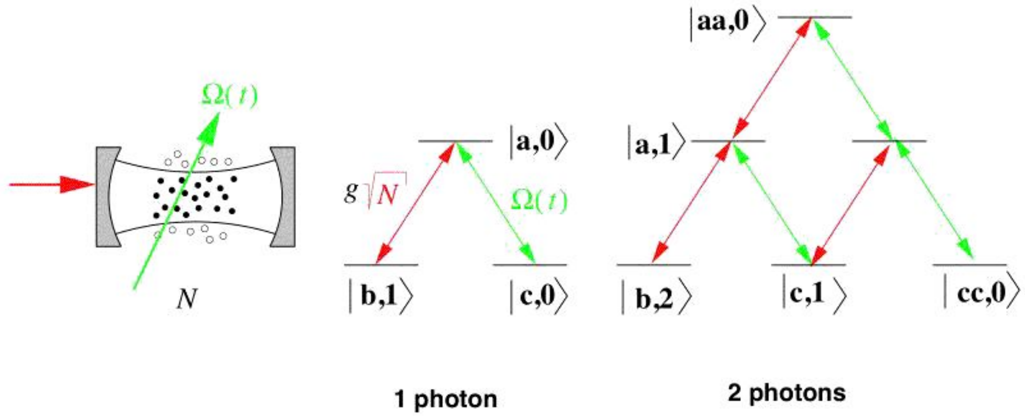
$$|\mathbf{D}\rangle = \cos\theta(t)\,|b,1\rangle - \sin\theta(t)\,|c,0\rangle \qquad \tan\theta(t) = \frac{g}{\Omega(t)}$$

adiabatic rotation of $\theta(t) = 0 \rightarrow \pi/2$: photon transfer to atom

- **problem:** small cross section strong coupling required: $\boxed{g^2 \gg \kappa\gamma}$

collective quantum memory: basics

- Raman coupling of radiation mode to collective spin $g^2 N \gg \kappa\gamma$



- Hamiltonian couples ground state to symmetric states states:

$$
\begin{aligned}
|\mathbf{b}\rangle &\equiv |bbb\ldots\rangle, \\
|\mathbf{c^1}\rangle &\equiv \frac{1}{\sqrt{N}}\Big[|cbb\ldots\rangle + |bcb\ldots\rangle + \cdots\Big], \\
|\mathbf{c^2}\rangle &\equiv \frac{1}{\sqrt{N(N-1)}}\Big[|ccb\ldots\rangle + |cbc\ldots\rangle + \cdots\Big]
\end{aligned}
$$

- **family of dark-states:** $(|\mathbf{c}^0\rangle \equiv |\mathbf{b}\rangle)$

$$|D,n\rangle = \sum_{k=0}^{n} \binom{n}{k}^{1/2} \left(-\sin\theta\right)^k \left(\cos\theta\right)^{n-k} |\mathbf{c}^k, n-k\rangle, \qquad \tan\theta = \frac{g\sqrt{N}}{\Omega}$$

- **storage of arbitrary photon states** with $n \ll N$

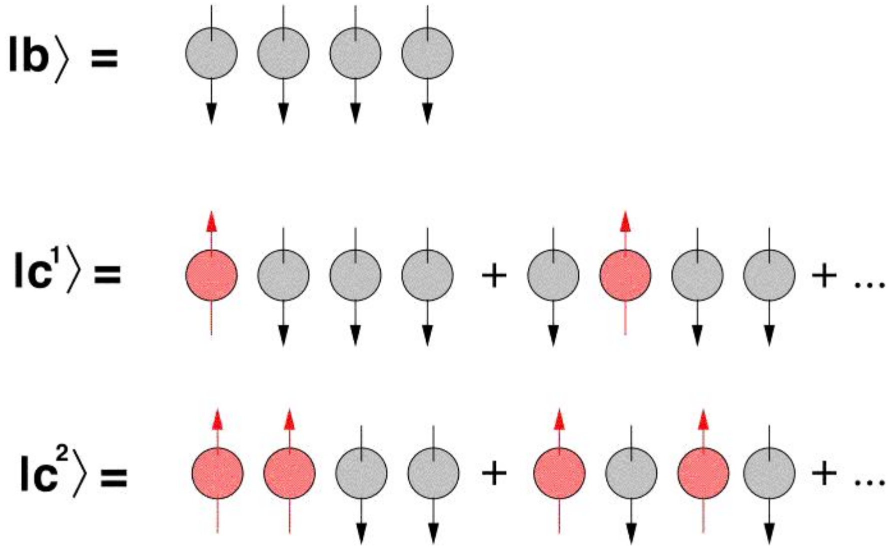
$\theta = 0$	$\theta = \pi/2$
$\Omega \gg g\sqrt{N}$	$\Omega \ll g\sqrt{N}$
$\vert D,n\rangle = \vert\mathbf{b},n\rangle$	$\vert D,n\rangle = -\vert\mathbf{c}^n,0\rangle$

all dark states rotate in the same way, no relative phase

$$\sum_{nm}^{n_{\max}} \rho_{nm} |n\rangle\langle m| \otimes |\mathbf{b}\rangle\langle\mathbf{b}| \leftrightarrow |0\rangle\langle 0| \otimes \sum_{nm}^{n_{\max}} \rho_{nm} |\mathbf{3}^n\rangle\langle\mathbf{3}^m|$$

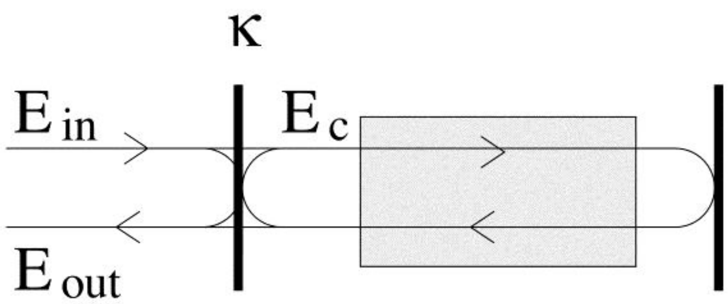
- **state transfer:**

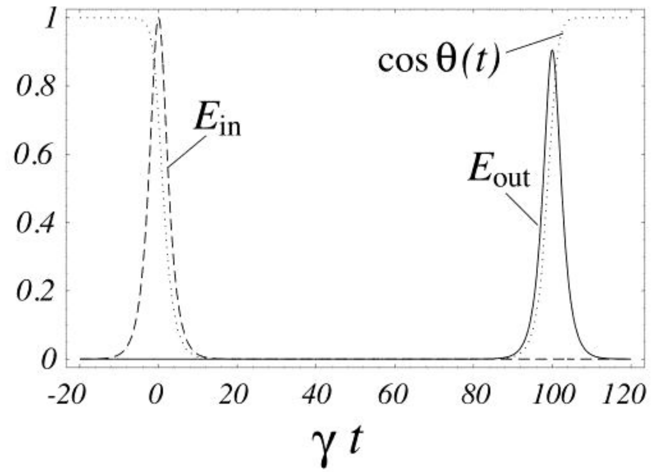
$$|n\rangle_{\mathrm{ph}}\,|\mathbf{b}\rangle_{\mathrm{at}} \qquad \leftrightarrow \qquad |0\rangle_{\mathrm{ph}}\,|\mathbf{c}^{n}\rangle_{\mathrm{at}}$$



photon Fock states $\leftrightarrow$ symmetric collective excitations

- **input-output:**





proper tuning of mixing angle $\theta(t)$ guarantees complete transfer into and from cavity system (quantum impedance matching)

$$\cos^2 \theta(t) = \frac{|E_{\rm in}(t)|^2}{\kappa \int_{-\infty}^{t} {\rm d}\tau \, |E_{\rm in}(\tau)|^2}$$

M.F., S. Yelin, M. Lukin, Opt.Comm. (2000); M. Lukin, S. Yelin, M.F. PRL (2000);

- **travelling fields:** use of resonator is not required

M.F. and M.D. Lukin, PRL (2000); PRA (2002)

- **dark-state polaritons**   dark states are Fock-states of bosonic quasi-particle

$$|D,n\rangle = \frac{1}{\sqrt{n!}}\left(\hat{\Psi}^\dagger\right)^n |\mathbf{b}\rangle|0\rangle_{\text{field}}$$

$$\hat{\Psi} = \cos\theta(t)\,\hat{a} - \sin\theta(t)\frac{1}{\sqrt{N}}\sum_j \hat{\sigma}_{bc}^j$$

- **storage = adiabatic rotation of dark polariton**

$$\hat{\Psi} = \hat{a} \quad \longleftrightarrow \quad \hat{\Psi} = \frac{1}{\sqrt{N}}\sum_j \hat{\sigma}_{bc}^j$$

- **bright polaritons** $N+1$ degrees of freedom, need also

$$\hat{\Phi}_0 = \sin\theta(t)\,\hat{a} + \cos\theta(t)\frac{1}{\sqrt{N}}\sum_j \hat{\sigma}_{bc}^j$$

$$\hat{\Phi}_l = \frac{1}{\sqrt{N}}\sum_j \mathrm{e}^{i\phi_{jl}}\,\hat{\sigma}_{bc}^j \qquad \text{for} \quad l \in \{1, 2, \ldots, N-1\}$$

$\Phi_l^\dagger$ excite non-symmetric states:

$$\Phi_l^\dagger|\mathbf{b}\rangle = \frac{1}{\sqrt{N}}\sum_{j=1}^{N} \mathrm{e}^{i\phi_{jl}}|b_1 \ldots c_j \ldots b_N\rangle$$

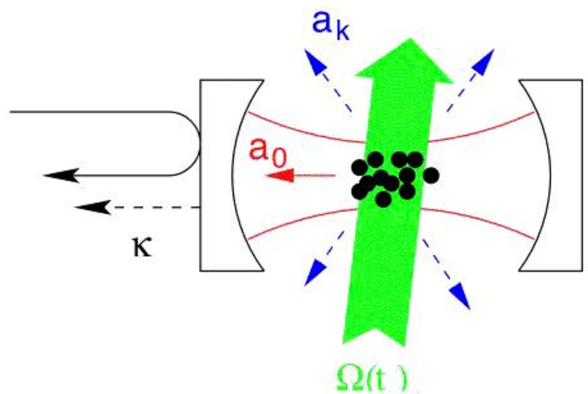
II. decoherence of collective qubit states

symmetric collective excitations = N-atom entangled states

sensitivity to decoherence ??

- **mapping:**

$\theta = 0$				$\theta = \pi/2$
$\hat{a}$	$\longleftarrow$	$\hat{\Psi}$	$\longrightarrow$	$\hat{\sigma}_{\mathrm{spin}}$
$\hat{\sigma}_{\mathrm{spin}}$	$\longleftarrow$	$\hat{\Phi}_0$	$\longrightarrow$	$\hat{a}$
$\hat{\sigma}'_{\mathrm{spin}}$	$\longleftarrow$	$\hat{\Phi}_l$	$\longrightarrow$	$\hat{\sigma}'_{\mathrm{spin}}$



only excitations in Ψ matter after readout

$$\varrho = Tr_{\Phi_0,\Phi_1,\ldots,\Phi_{N-1}}\Big(W\Big)$$

equivalence classes

all density operators W' that are generated out of a perfect storage state W by the completely positive maps

$$W' = \sum_i E_i \, W \, E_i^\dagger \qquad \sum_i E_i^\dagger E_i = \mathbf{1}$$

with Kraus operators

$$E_i = E_i\left[\left\{\Phi_l^\dagger, \Phi_l\right\}\right]$$

are **equivalent** with respect to the stored quantum state

- **single-atom spin-flip**

$$\hat{\sigma}_j^+ = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \qquad \hat{\sigma}_j^+ = \frac{1}{\sqrt{N}} \sum_{l=1}^{N-1} \mathrm{e}^{-i\phi_{jl}} \, \hat{\Phi}_l^\dagger + \frac{1}{\sqrt{N}} \, \hat{\Psi}^\dagger$$

has no consequence "bad" part

assume:

- individual uncorrelated reservoirs
- decoherence affects all atoms with the same probability:
  $\sum_{jl} \mathrm{e}^{-i\phi_{jl}} \Phi_l^\dagger W \Psi \to 0$

$$\mathcal{L} = \gamma \sum_{j=1}^{N} \mathcal{L}_j \qquad \mathcal{L}_j = \gamma \Big\{ 2\hat{\sigma}_j^+ \rho \hat{\sigma}_j^- - \hat{\sigma}_j^- \hat{\sigma}_j^+ \rho - \rho \hat{\sigma}_j^- \hat{\sigma}_j^+ \Big\}$$

$$\longrightarrow \qquad \mathcal{L} = \gamma \sum_{l=1}^{N-1} \mathcal{L}_{\Phi_l} + \gamma\, \mathcal{L}_\Psi$$

- similar result holds for reduced dynamics if W is diagonal in Φ's:
  $\mathrm{Tr}_\Phi \Big\{ \Phi_l^\dagger W \Psi \Big\} = 0$

$\Rightarrow$ **sensitivity to decoherence not enhanced compared to single atom**

- **atom loss**

$$|D,n\rangle_N \equiv |D,n\rangle_{N-k} \qquad \text{if } n \ll N-k$$

fidelity:

$$f = 1 - \frac{1}{N}\sum_l \rho_{ll} l + \frac{1}{N}\sum_l \sum_s \rho_{ll-1}\rho_{s-1s}\sqrt{ls} + \mathcal{O}(1/N^2)$$

- **atomic motion**

assume diffusive motion with diffusion constant D

fidelity:

$$f_{\mathsf{Fock}} = \mathrm{e}^{-nDt} + \mathcal{O}(1/N^2)$$

III. quasi-decoherence free subspaces

can we suppress the effects of decoherence even below the level of single-spin qubits?

decoherence-free subspaces (DSP)

see e.g.: Lidar, Whaley, quant-ph/0301032

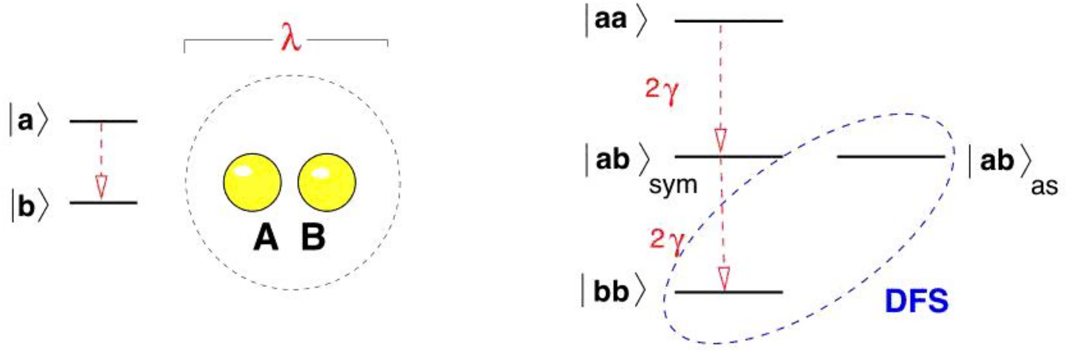
dissipative dynamics described by Liouville operator $\mathcal{L}$

$$\dot{\rho} = \mathcal{L}\,\rho$$

DFS $\mathcal{H}_0$: reduced dynamics in $\mathcal{H}_0$ unitary

- one state of this kind always exists $\rightarrow$ **stationary state**
- need however subspace $\mathcal{H}_0 \in \mathcal{H}$ with $d =$dim$[\mathcal{H}_0] \geq 2$

example with $d = 2$: Dicke-subradiance



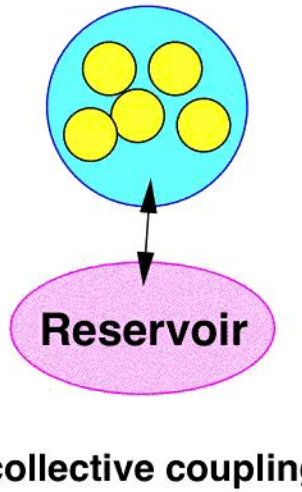
system - reservoir interaction:

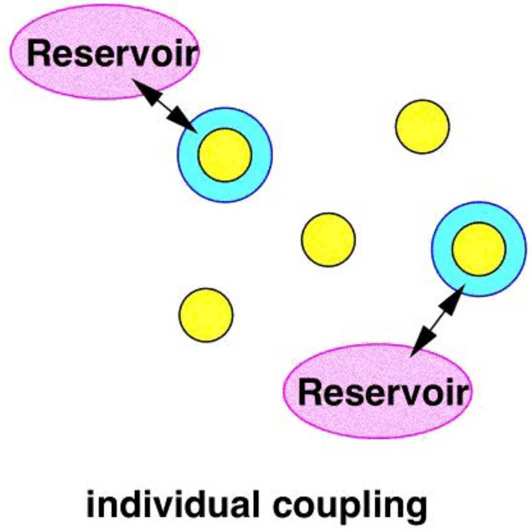
$$H_{\mathrm{SR}} = \sum_k g_k \left[\hat{\sigma}_A + \hat{\sigma}_B\right] \hat{a}_k + h.a. \qquad \sigma = |a\rangle\langle b|$$

general property of all systems that have DFS with $d \geq 2$:

high symmetry in system-reservoir coupling

often individual reservoir coupling more physical:



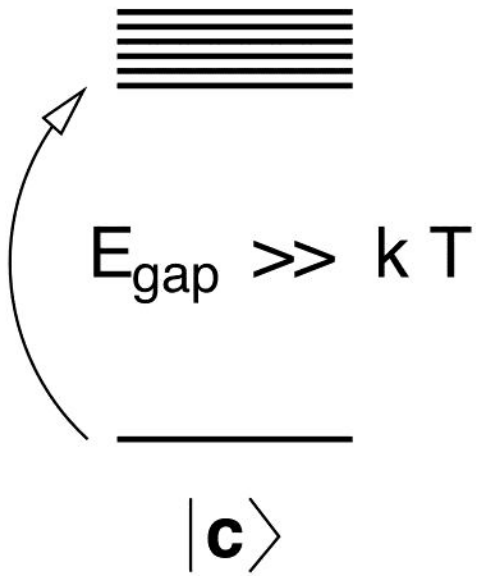


decoherence suppression for individual reservoir interaction ??

$$H_{\mathrm{SR}} = \sum_{kl} g_k \, \hat{a}_{kl} \hat{\sigma}_l + h.a. \qquad \sigma_l = |a\rangle_{ll}\langle b|$$

quasi-DFS by energetic splitting

a single state can be protected effectively by energy splitting

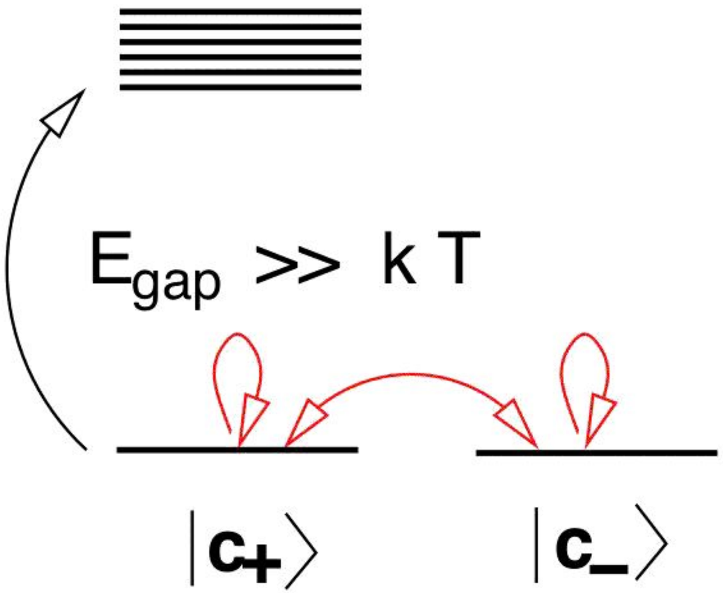


only relevant decoherence process

$$|c\rangle \rightarrow |\phi\rangle$$

exponentially suppressed by energy gap

- how to extend this to $d \geq 2$??



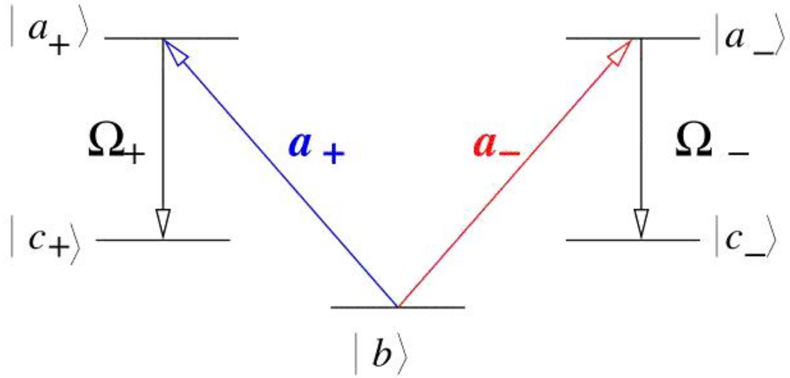
no protection against

$$|c_+\rangle \leftrightarrow |c_-\rangle$$

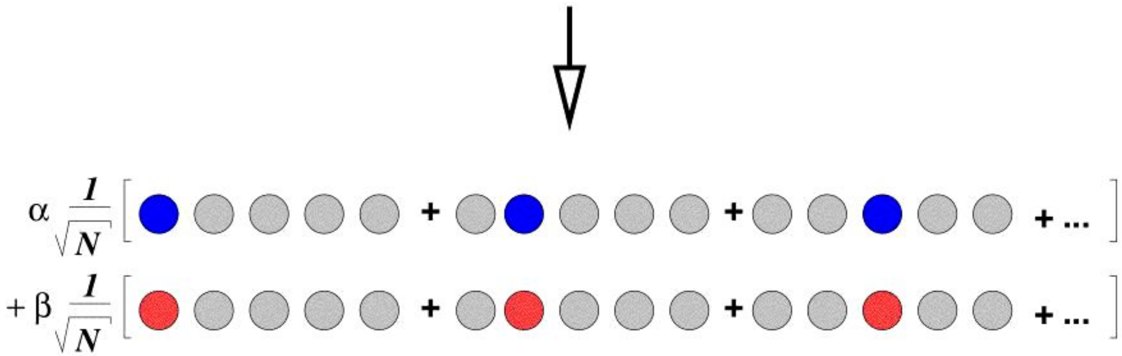
$$|c_\pm\rangle \rightarrow \mathrm{e}^{i\phi_\pm}|c_\pm\rangle$$

quasi-DFS with collective excitations and energy splitting

- encode qubits in two states with large "effective distance"



$$\alpha\ |1_+\rangle + \beta\ |1_-\rangle$$



$$\alpha \frac{1}{\sqrt{N}}\Big[|c_+ bb\ldots b\rangle + |b c_+ b\ldots b\rangle + \ldots + |bbb\ldots c_+\rangle\Big]$$

$$+ \quad \beta \frac{1}{\sqrt{N}}\Big[|c_- bb\ldots b\rangle + |b c_- b\ldots b\rangle + \ldots + |bbb\ldots c_-\rangle\Big]$$

- **spin-flip** $|c_\pm\rangle \to |b\rangle \quad \sim \{\Psi_\pm, \Phi_\pm\}$ causes de-excitation of polaritons

- **spin-flip** $|b\rangle \to |c_\pm\rangle \quad \sim \{\Psi^\dagger_\pm, \Phi^\dagger_\pm\}$ causes excitation of polaritons

- **spin-flip** $|c_-\rangle \leftrightarrow |c_+\rangle \quad \sim \{\Psi^\dagger_+\Psi_-; \Psi^\dagger_-\Psi_+\}$ or $\{\Phi^\dagger_+\Psi_-; \Phi^\dagger_-\Psi_+\}$ causes transfer of polaritons bewteen "+" and "−" modes

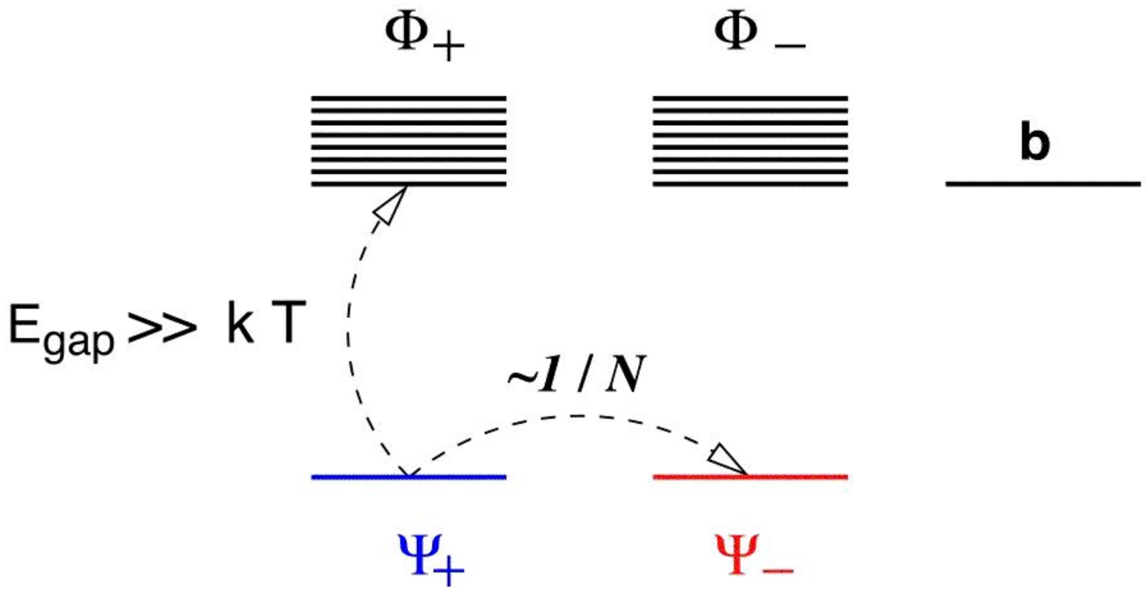
transition probability upon spin-flip of jth atom $p_j = 1/N^2$

individual independent reservoir interaction ⇒ total transition probability between storage states $\boxed{p \sim 1/N}$ suppressed!

- **dephasing** $|c_\pm\rangle \to |c_\pm\rangle e^{i\phi_\pm} \quad \sim \Psi_\pm \Phi^\dagger_{\pm,l}$ causes transitions from dark to bright polariton modes

- provide energy gap between two storage states and other states

$$H_{\mathrm{gap}} = -E_{\mathrm{gap}}\Big[|\Psi_+\rangle\langle\Psi_+| + |\Psi_-\rangle\langle\Psi_-|\Big]$$



- **spin-flip** $|c_\pm\rangle \rightarrow |b\rangle \quad \sim \{\Psi_\pm, \Phi_\pm\}$ causes deexcitation of polaritons

  exponentially suppressed by energy gap

- **spin-flip** $|b\rangle \rightarrow |c_\pm\rangle \quad \sim \{\Psi_\pm^\dagger, \Phi_\pm^\dagger\}$ causes excitation of polaritons

  exponentially suppressed by energy gap

- **spin-flip** $|c_-\rangle \leftrightarrow |c_+\rangle \quad \sim \{\Psi_+^\dagger\Psi_-; \Psi_-^\dagger\Psi_+\}$ causes transfer of polaritons bewteen "$+$" and "$-$" modes

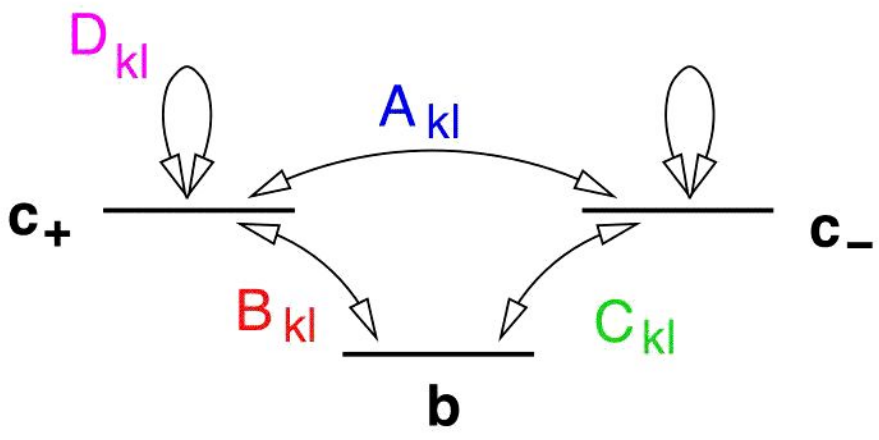
  suppressed by construction of states

- **dephasing** $|c_\pm\rangle \rightarrow |c_\pm\rangle e^{i\phi_\pm} \quad \sim \Psi_\pm\Phi_{\pm,l}^\dagger$ causes transitions from dark to bright polariton modes

  exponentially suppressed by energy gap

quasi-decoherence free subspace $|\Psi_+\rangle, |\Psi_-\rangle$

- **model**



A_{kl}, B_{kl}, C_{kl}, D_{kl} independent reservoirs for each atom

$H = H_0 + H_{\mathrm{gap}}$

reservoir interaction in Born-Markov approximation

density matrix equations $\Rightarrow$ confirmed decoherence suppression

- **how to create H_{gap}?**

$$H_{\mathrm{int}} = \alpha\Big(J_{z+} + J_{z-}\Big) + \beta\Big(J_{z+}^2 + J_{z-}^2\Big) - \gamma\Big(\mathbf{J}_+^{\mathbf{2}} + \mathbf{J}_-^{\mathbf{2}}\Big)$$

practically difficult, requires collective interaction

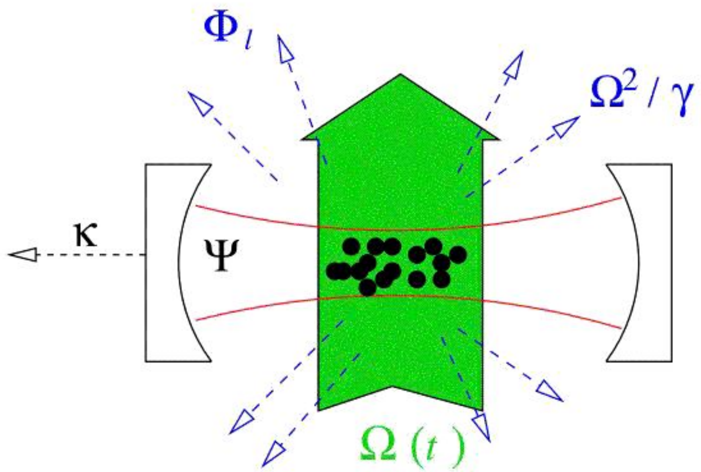
quasi-DFS without energy gap

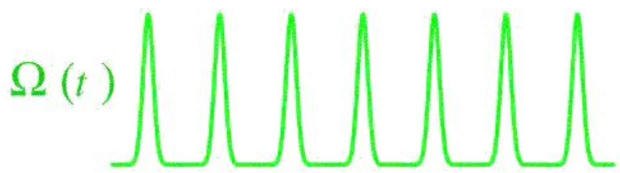
in solids dominant decoherence process: **dephasing**

$$|c_\pm\rangle \;\rightarrow\; |c_\pm\rangle\, \mathrm{e}^{i\phi_\pm} \qquad \sim \qquad \Psi_\pm \Phi^\dagger_{\pm,l}$$

can be suppressed by rapid monitoring of bright-polariton excitations

$$H_{\rm diss} = -i\hbar\kappa \cos^2\theta(t)\, \hat\Psi^\dagger\hat\Psi - i\hbar\frac{\Omega^2(t)}{\gamma}\sum_l \hat\Phi_l^\dagger\hat\Phi_l$$





Summary

- quantum memory stores photonic qubits in delocalized collective quasi-particle excitations (dark-state polaritons)
- no enhanced sensitivity to decoherence despite entanglement character due to **equivalence classes**
- storage of qubit $\alpha|1_+\rangle + \beta|1_-\rangle$ in states with large effective distance $\rightarrow$ direct transition probability $\sim 1/N$
- suppression of transition into other states by energy gap or "bang-bang" control
- **quasi-decoherence free subspace**

collective states can be useful
to suppress decoherence