united nations educational, scientific and cultural organization

international atomic energy agency

the abdus salam international centre for theoretical physics

40th anniversary 1964 2004

SMR.1587 - 22

SCHOOL AND WORKSHOP ON
QUANTUM ENTANGLEMENT, DECOHERENCE, INFORMATION, AND GEOMETRICAL PHASES IN COMPLEX SYSTEMS
(1 November - 12 November 2004)

Spin control and relaxation in mesoscopic quantum transport

K. RICHTER

Institut für Theoretische Physik
Universität Regensburg
Universitätsstrasse 31
D-93040 Regensburg
Germany

Spin Control and Relaxation in Mesoscopic Quantum Transport

Oleg Zaitsev, Diego Frustaglia, Markus Popp, KR

Institut für Theoretische Physik - Universität Regensburg

Scuola Normale Superiore, Pisa

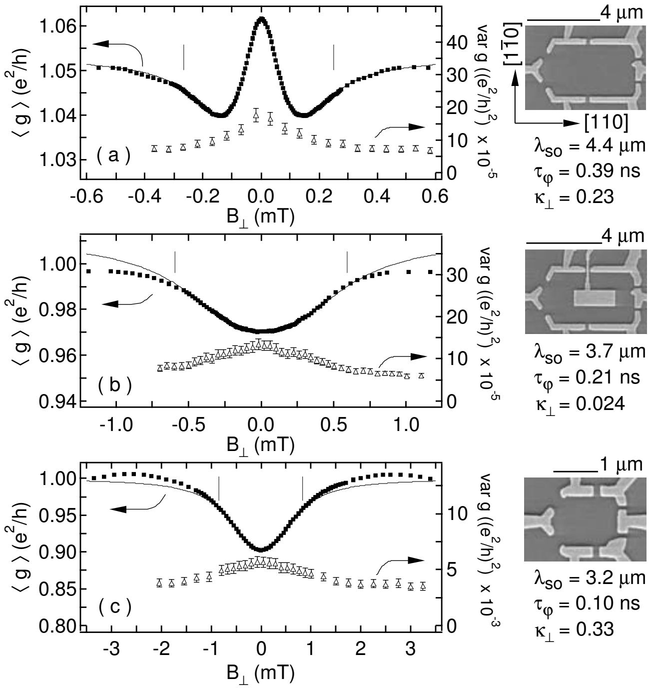
Scope

Spin dynamics of confined electrons in non-uniform magnetic fields

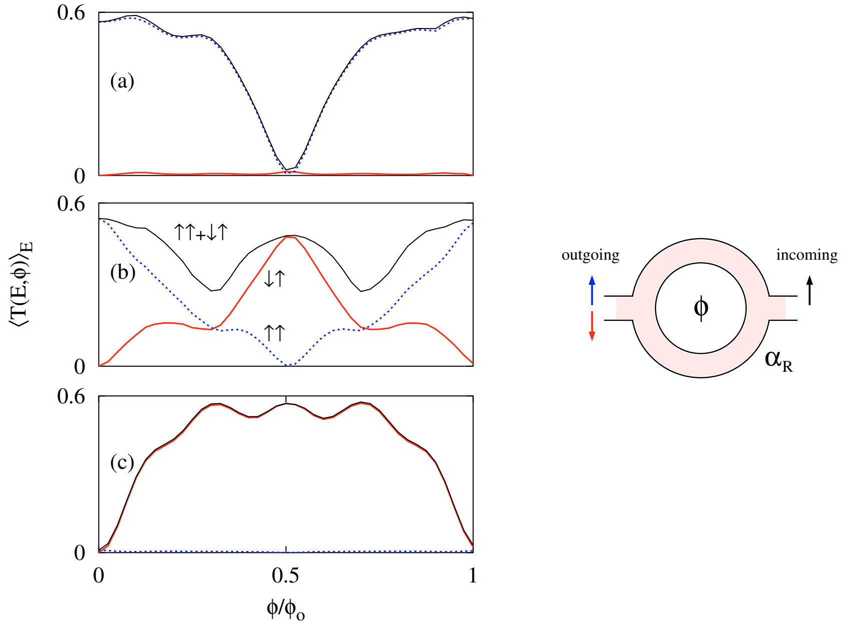
- intrinsically: spin-orbit interaction ⇒ effective B-field ⇒ spin relaxation
- externally: imposed inhomogeneous B-fields ⇒ spin control

Topics:

Conductance of quantum dots with spin-orbit interaction



Aharonov-Bohm physics with spin: mesoscopic rings as spin switches



Berry-phases in quantum transport in ballistic and disordered conductors

Spin-orbit effects in DEGs

revival of spin-orbit (SO) effects owing to their role for:

- spin interference devices
- spin rotators, filters, pumps, . . .
- spin-based quantum computation

here:

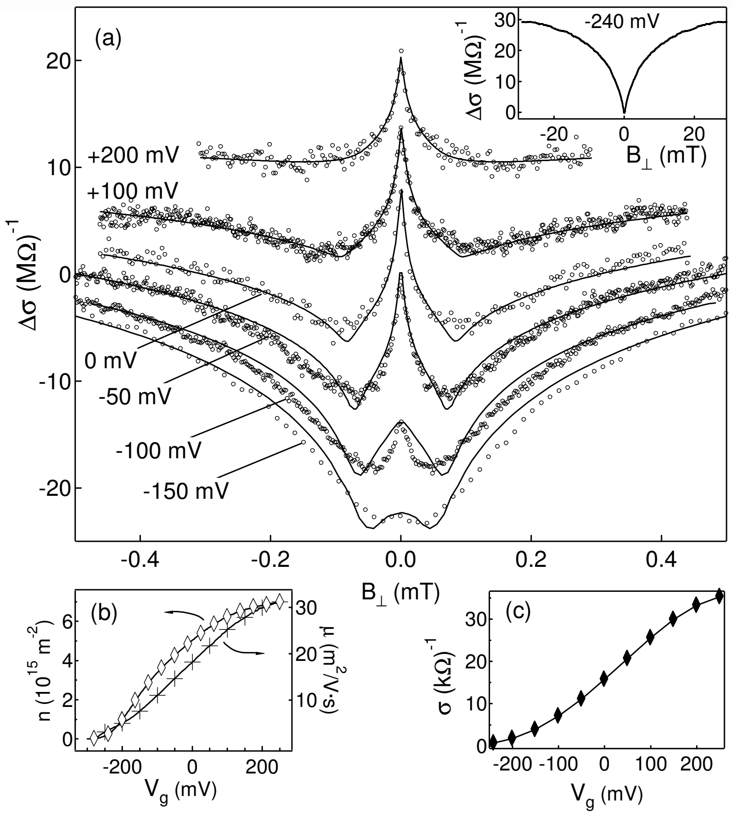
- SO-induced spin relaxation in confined nanosystems
- effect of orbital motion on spin evolution
- SO-controlled spin switching

experimental probes for SO interaction in quantum transport:

- beating patterns in Shubnikov-de Haas oscillations in 2DEGs with tunable SO coupling
- weak (anti-)localization

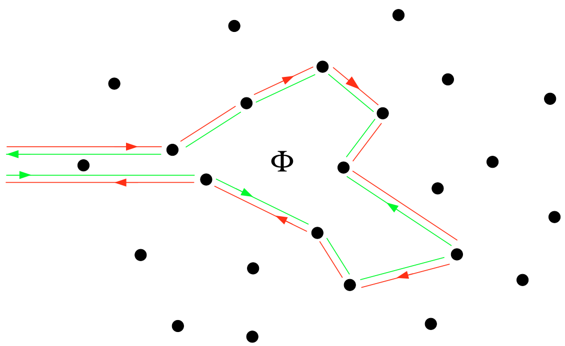
Weak antilocalization in 2DEGs

gate-tunable crossover from weak localization (WL) to antilocalization (AL)



experiments (bulk):

- J.B. Miller et al., PRL (2003)
- Ch. Schierholz et al., phys. stat. sol. (2002)
- F.E. Meijer et al., cond-mat (2004)



early theory on WL and AL in disordered systems:

S. Hikami, A.L. Larkin, Y. Nagaoka, Prog. Theor. Phys. (1980)

Weak antilocalization in quantum dots

interplay between SO coupling
and confinement?

- role of orbital dynamics:
  diffusive vs. chaotic
  chaotic vs. regular
- effect of additional
  Zeeman interaction

related theoretical work:

- A.V. Khaetskii, Y.V. Nazarov, PRB (2000, 2001)
- I.L Aleiner, V.I. Falko, PRL (2001)
- J.-H. Cremers et al., PRB (2003)

Weak antilocalization in quantum dots

interplay between SO coupling and confinement?

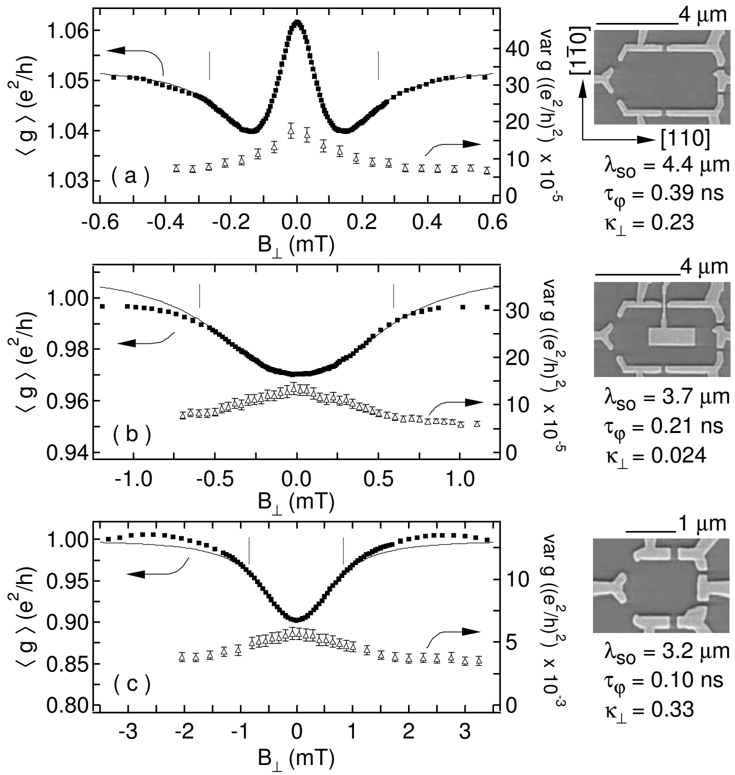
- role of orbital dynamics: diffusive vs. chaotic chaotic vs. regular
- effect of additional Zeeman interaction

related theoretical work:

- A.V. Khaetskii, Y.V. Nazarov, PRB (2000, 2001)
- I.L Aleiner, V.I. Falko, PRL (2001)
- J.-H. Cremers et al., PRB (2003)

suppression of AL in small dots !

(8.0 μm^2 — 5.8 μm^2 — 1.2 μm^2)



Zumbühl et al., PRL (2002)

Hamiltonian

- $\hat{H} = \hat{H}_0\,(\hat{\mathbf{p}},\hat{\mathbf{q}}) + \hbar\,\hat{\mathbf{s}}\cdot\hat{\mathbf{C}}\,(\hat{\mathbf{p}},\hat{\mathbf{q}})$
  - includes: Zeeman term
    SO interaction
- Example: Rashba SO coupling:

$$\hat{\mathbf{C}} = \frac{2\alpha_R}{\hbar^2}\left(\hat{\mathbf{p}} - \frac{e}{c}\mathbf{A}(\hat{\mathbf{q}})\right) \times \mathbf{e}_z$$

  - $\hat{C}$ acts as an effective magnetic field
  - measure for SO-strength α_R:
    Rashba spin precession length: $L_R = 2\pi|\mathbf{v}|/C$
- devise quantum theory accounting for different types of orbital dynamics:

$\rightarrow$ Semiclassics with spin

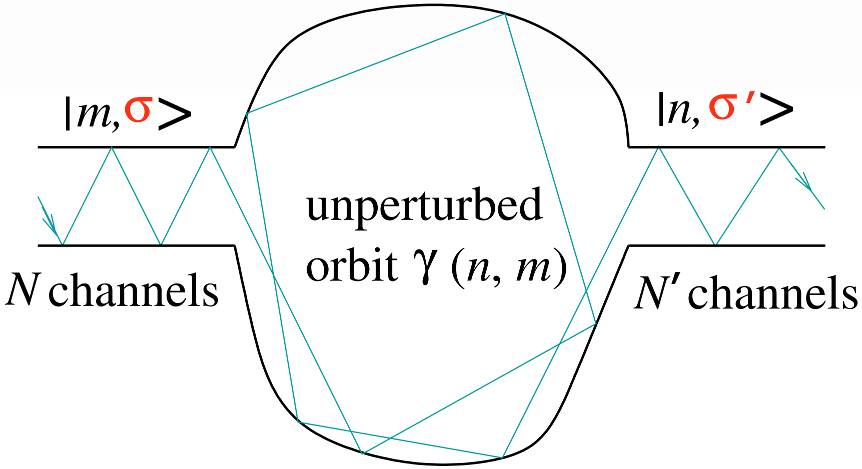
Spin semiclassics

- Assumptions:
  - semiclassical approach to Feynman path integral

    for density of states:

    Bolte, Keppeler (1998), Brack, Pletyukhov, Zaitsev (2002,2003)
  - semiclassics for orbital part $\hat{H}_0$:
    classical action $S_0 \gg \hbar$ i.e. $k_F L_b \gg 1$ (L_b: system size)
  - moderate SO coupling:
    $\hbar s |\mathbf{C}| \ll H_0$ and $L_b / L_R \lesssim 1$
- Consequences:
  - orbital motion is not affected by spin
  - paths of H_0 generate local field acting on spin:
    $\hat{H}_{\text{spin}}(t) = \hbar \hat{\mathbf{s}} \cdot \mathbf{C}(\mathbf{p}_0(t), \mathbf{q}_0(t))$

Spin-dependent quantum transport



quantum transport:

$G = (e^2/h)\mathcal{T}$ with

$$\mathcal{T} = \sum_{n=1}^{N'} \sum_{m=1}^{N} \sum_{\sigma,\sigma'=-s}^{s} |t_{n\sigma',m\sigma}|^2$$

$$t_{n\sigma',m\sigma} \simeq \sum_{\gamma(n,m)} (\hat{K}_\gamma)_{\sigma'\sigma} A_\gamma \exp\left[\frac{i}{\hbar} S_\gamma\right] \quad ; \quad \hat{K}_\gamma = T\left\{\exp\left[-\frac{i}{\hbar}\int_0^{T_\gamma} \mathrm{d}t' \hat{H}_{\mathrm{spin}}(t')\right]\right\}$$

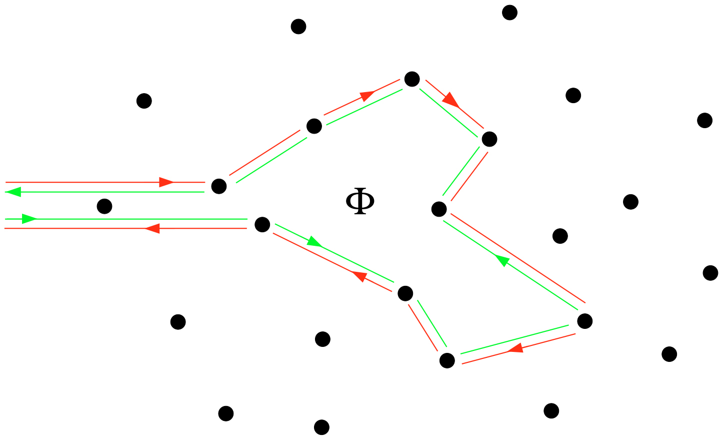
$$\mathcal{T} \simeq \sum_{nm} \sum_{\substack{\gamma(n,m) \\ \gamma'(n,m)}} \mathcal{M}_{\gamma,\gamma'} \mathcal{A}_\gamma \mathcal{A}^*_{\gamma'} \exp\left[\frac{i}{\hbar}(S_\gamma - S_{\gamma'})\right]$$

- semiclassical Landauer formula for spin-dependent magneto transport
- spin modulation factor $\mathcal{M}_{\gamma,\gamma'} = \mathrm{Tr}\,(\hat{K}_\gamma\, \hat{K}^\dagger_{\gamma'})$

Coherent backscattering

diffusive systems:

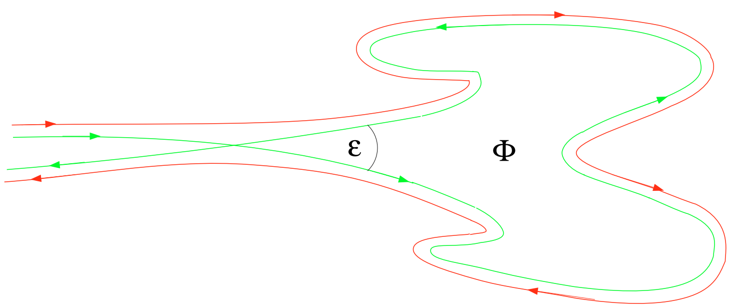
disorder average
$\rightarrow$ diagonal contribution:



time reversed paths

ballistic systems:

energy average
$\rightarrow$ diagonal $+$ loop contribution:



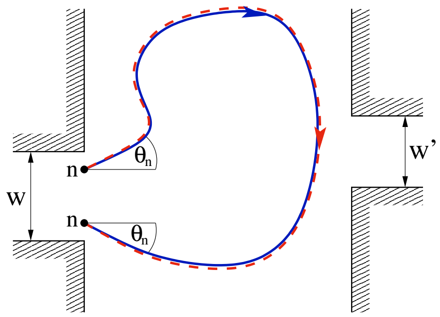
"entangled or-bits"
KR and M Sieber, PRL (2002)

Spin-dependent reflection

contributions to (energy-)averaged reflection:

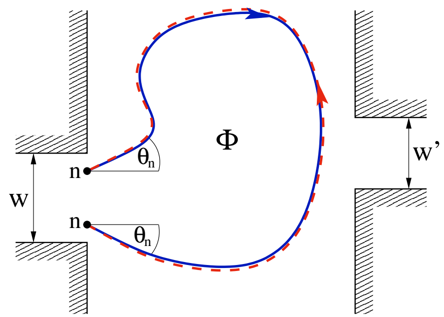
classical | quantum

$\gamma' = \gamma$



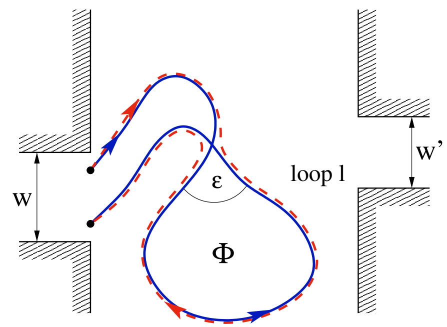
$\mathcal{M}_{\gamma,\gamma'} = 2s + 1$

diagonal $\gamma' = \gamma^{-1}$



$\mathcal{M}_{\gamma,\gamma'} = \mathrm{Tr}\,(\hat{K}_\gamma^2)$

loop $l' = l^{-1}$



$\mathcal{M}_{\gamma,\gamma'} = \mathrm{Tr}\,(\hat{K}_l^2)$

give rise to weak (anti-)localization

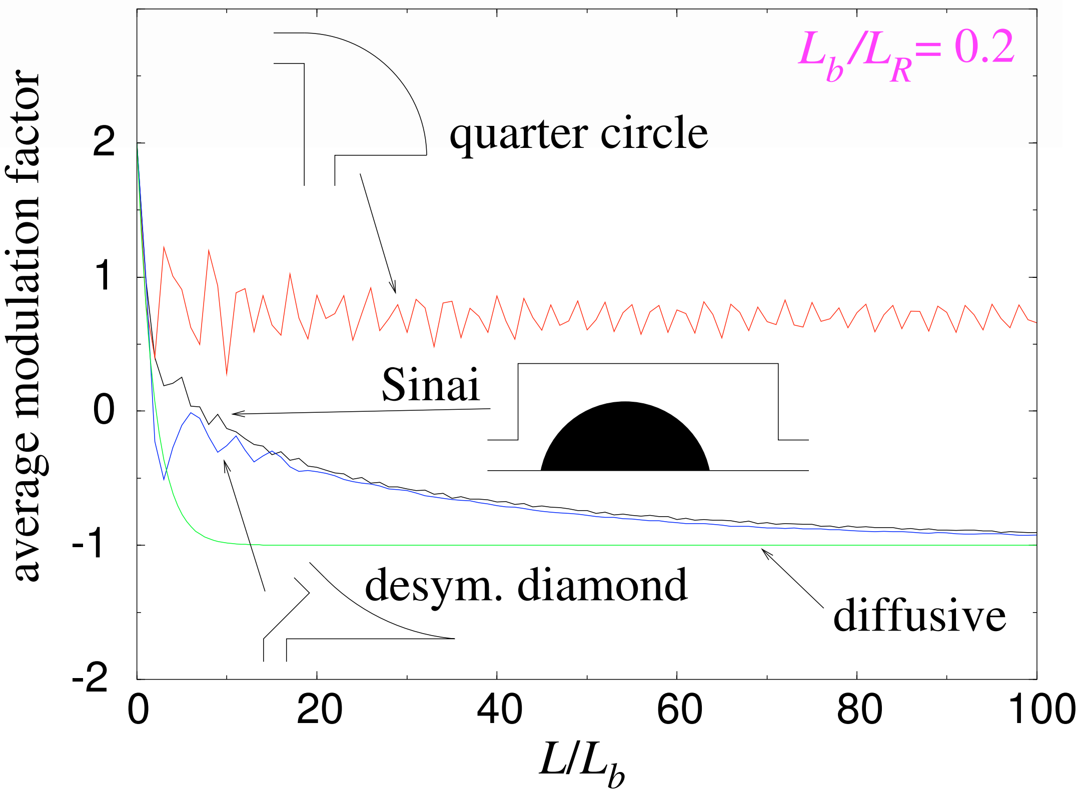
Quantum correction to reflection

$$\frac{\delta\mathcal{R}}{\delta\mathcal{R}^{(0)}} = \int_0^\infty \mathrm{d}L\, P(L)\, \langle\mathcal{M}\varphi\rangle(L;\mathbf{B})$$

- $\delta\mathcal{R}^{(0)}$: weak-localization correction without spin, $\mathbf{B} = 0$.
- (initial) polarization: $\langle\mathcal{M}\rangle = 2s + 1 = 2$ for spin 1/2
  complete relaxation: $\langle\mathcal{M}\rangle = (-1)^{2s} = -1$ for spin 1/2
- Aharonov-Bohm phase factor $\varphi = \exp(4\pi \mathrm{i} A B_z/\Phi_0)$
- orbit-length distribution $P(L)$:
  - $P(L) = L_{\text{esc}}^{-1} \exp(-L/L_{\text{esc}})$ for chaotic systems.
  - power law for integrable systems.

→ quantum spin evolution determined from classical numerical simulations !

Spin-relaxation $\langle \mathcal{M} = \mathrm{Tr}(\hat{K}_\gamma^2) \rangle$



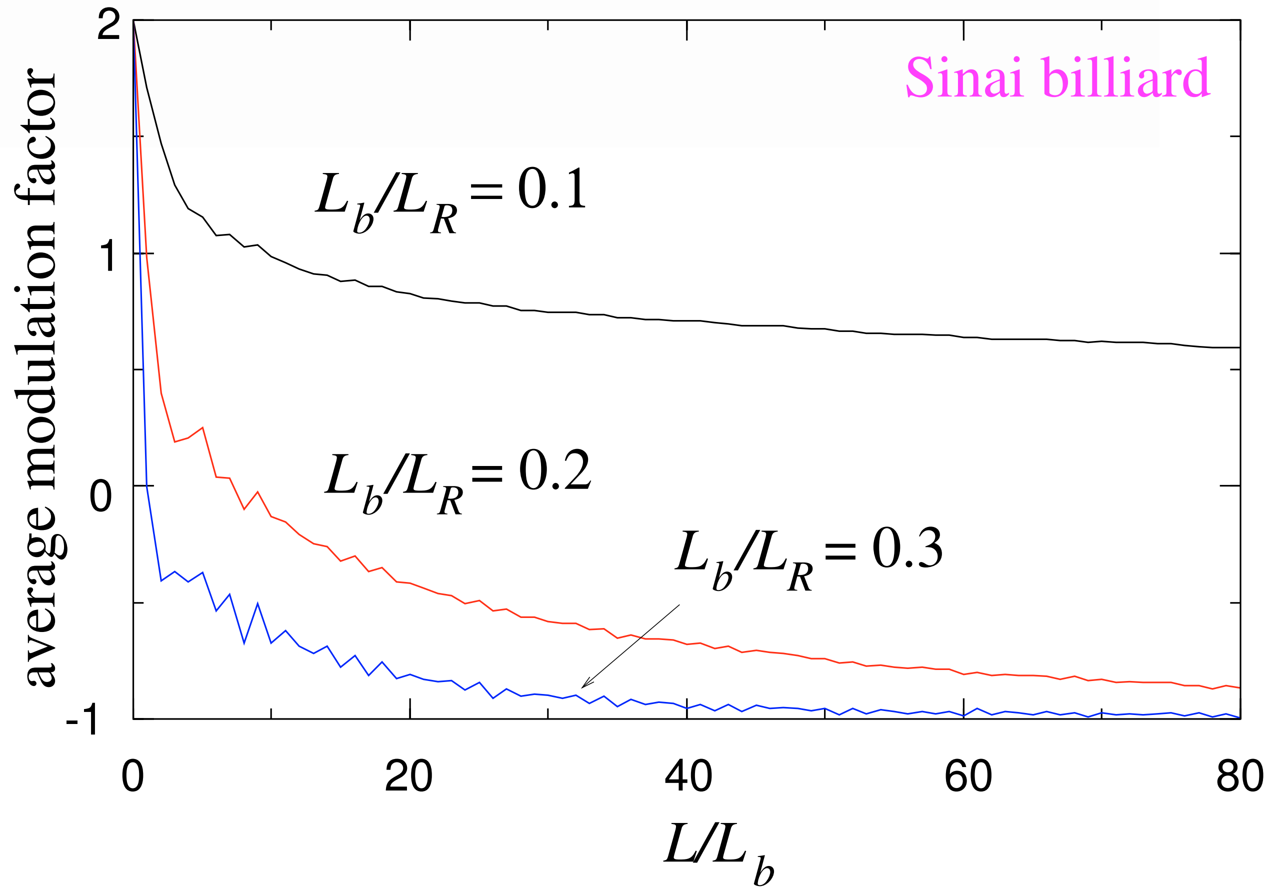
L_b: bounce length in quantum dot, mean free path for diffusive systems

⇒ fast relaxation in diffusive systems: $\langle \mathcal{M} \rangle (L) \simeq 3 \exp\left[-\frac{1}{3} \left(2\pi \frac{L_b}{L_R} \right)^2 \frac{L}{L_b} \right] - 1$

⇒ universal features for chaotic systems !

⇒ saturation for integrable systems !

Dot size and SO coupling strength

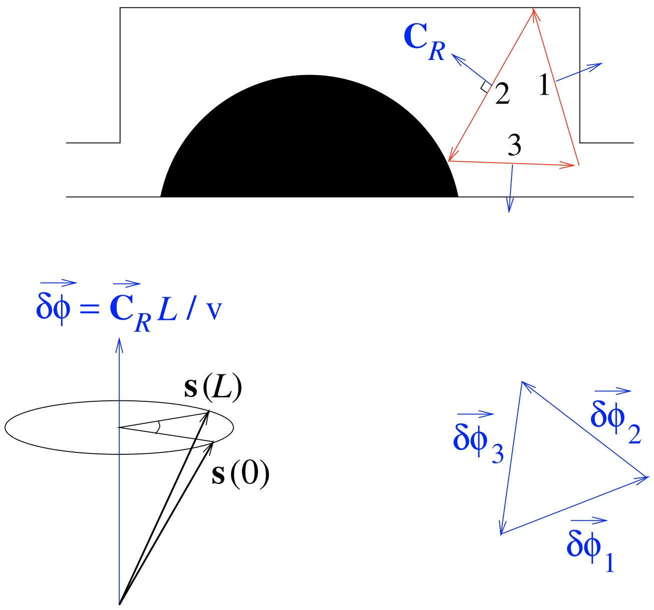


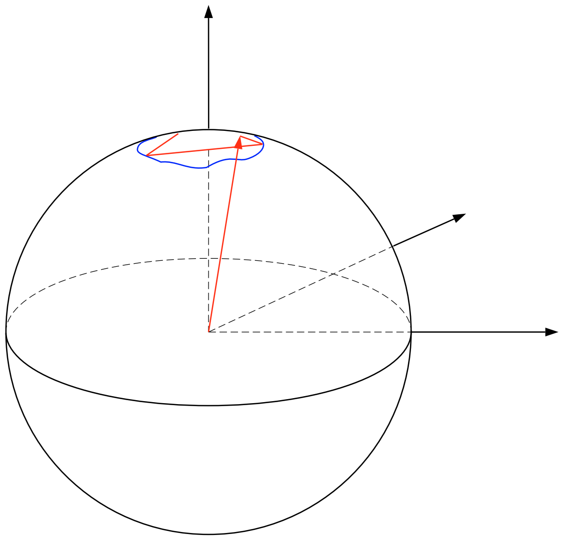
⇒ confinement suppresses spin relaxation !

for small SO coupling see also: Khaetskii, Nazarov (2000)

Spin evolution for small SO coupling

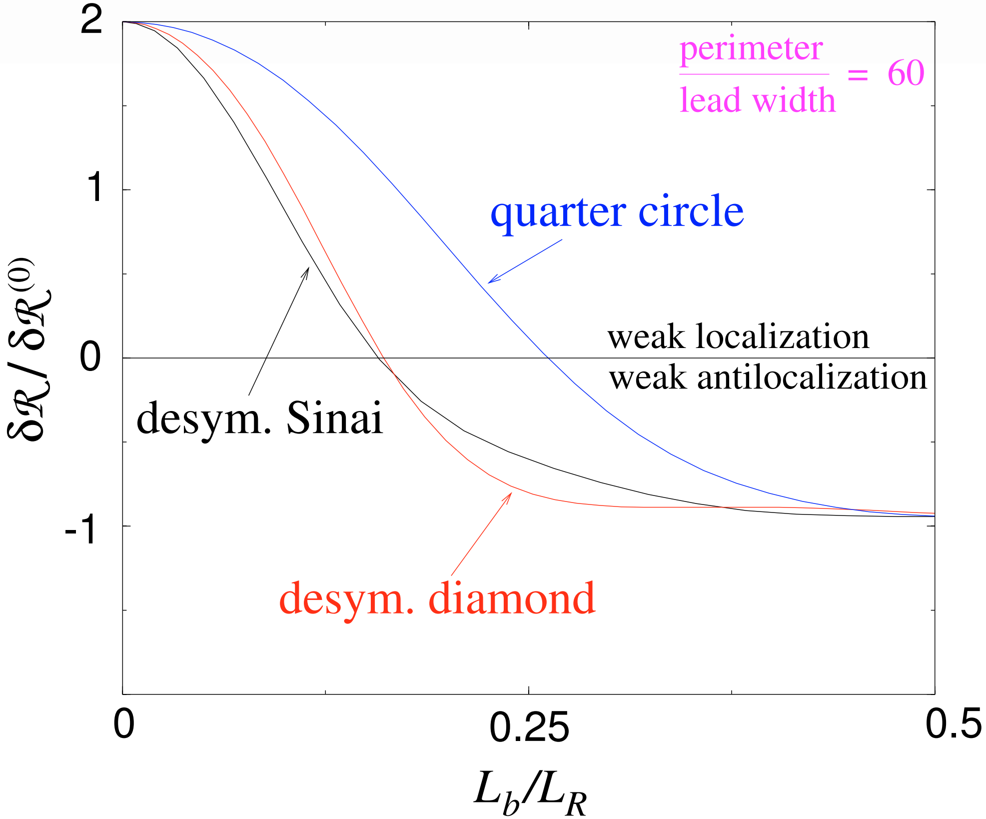
- for $L_b/L_R \ll 1$:
- spin evolution mimics orbital dynamics:





Weak (anti-)localization

$$\frac{\delta\mathcal{R}}{\delta\mathcal{R}^{(0)}} = \int_0^\infty \mathrm{d}L \, P(L) \, \langle\mathcal{M}\rangle(L)$$



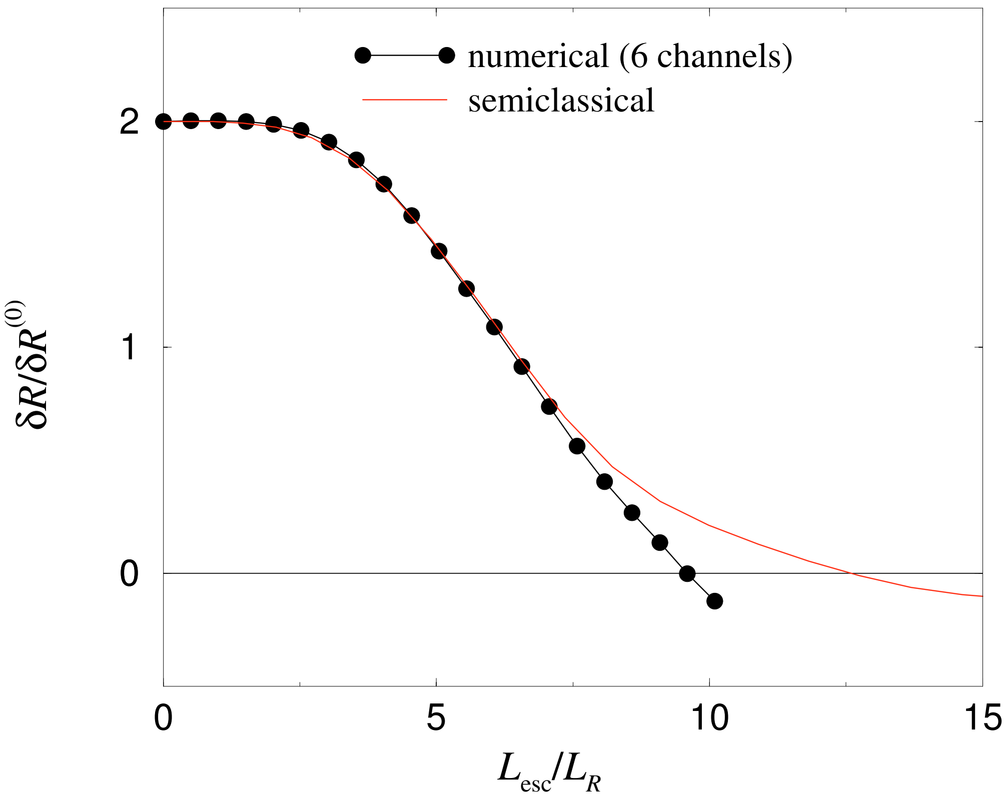
⇒ Suppression of AL in small dots (as in experiments)

⇒ Similarity among chaotic quantum dots

⇒ Suppression of AL in integrable systems

Numerical quantum calculations

- numerical quantum mechanical calculations for spin-dependent conductance (recursive Green functions technique)
- compare with semiclassical results for Sinai-billiard confinement

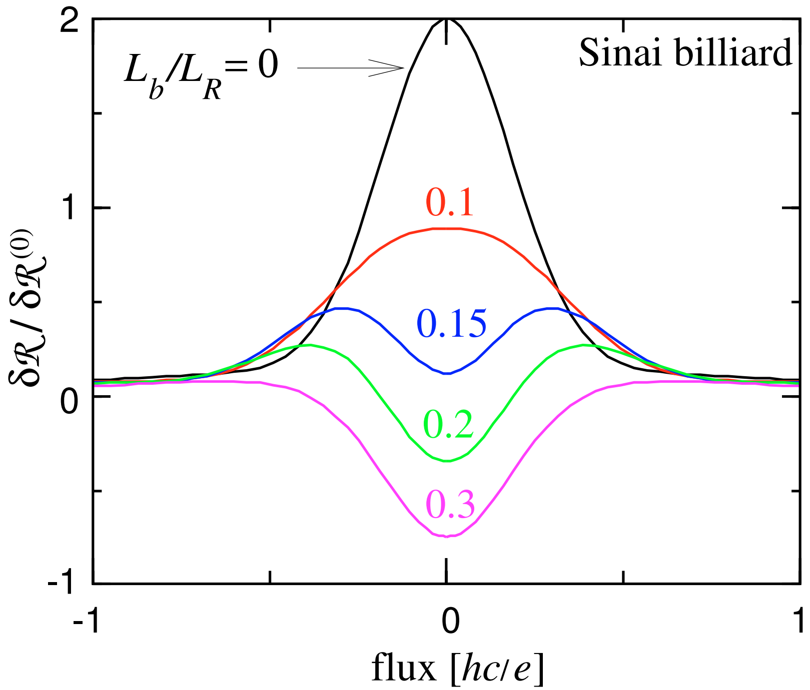


⇒ excellent agreement

⇒ no free parameters, in contrast to random matrix approaches

Magnetic flux dependence

$$\frac{\delta\mathcal{R}}{\delta\mathcal{R}^{(0)}} = \int_0^\infty \mathrm{d}L\, P(L)\, \langle\mathcal{M}\varphi\rangle(L)$$



with

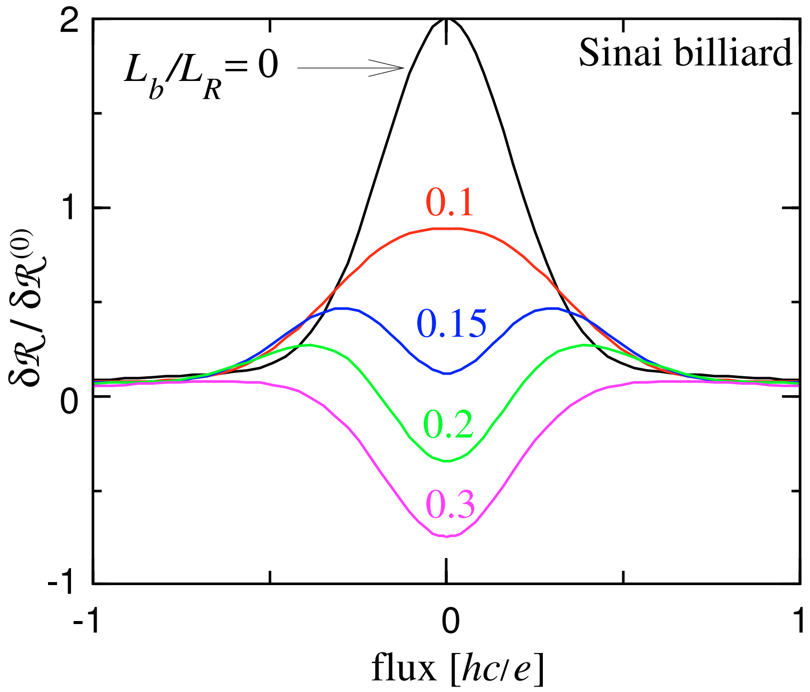
$$\langle\mathcal{M}\varphi\rangle(L) \simeq \sum_\pm \exp\left(-\gamma_\pm \frac{L}{L_b}\right)$$

and

$$\gamma_\pm = \left[\alpha B \pm \beta\,(L_b/L_R)^2\right]^2$$

Magnetic flux dependence

$$\frac{\delta\mathcal{R}}{\delta\mathcal{R}^{(0)}} = \int_0^\infty \mathrm{d}L\, P(L)\, \langle\mathcal{M}\varphi\rangle(L)$$



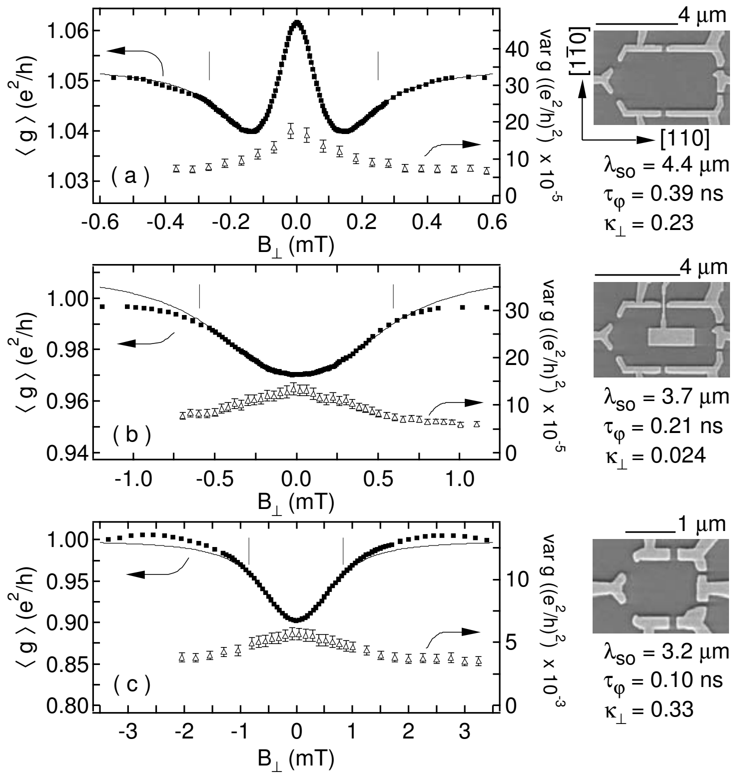
with

$$\langle\mathcal{M}\varphi\rangle(L) \simeq \sum_\pm \exp\left(-\gamma_\pm \frac{L}{L_b}\right)$$

and

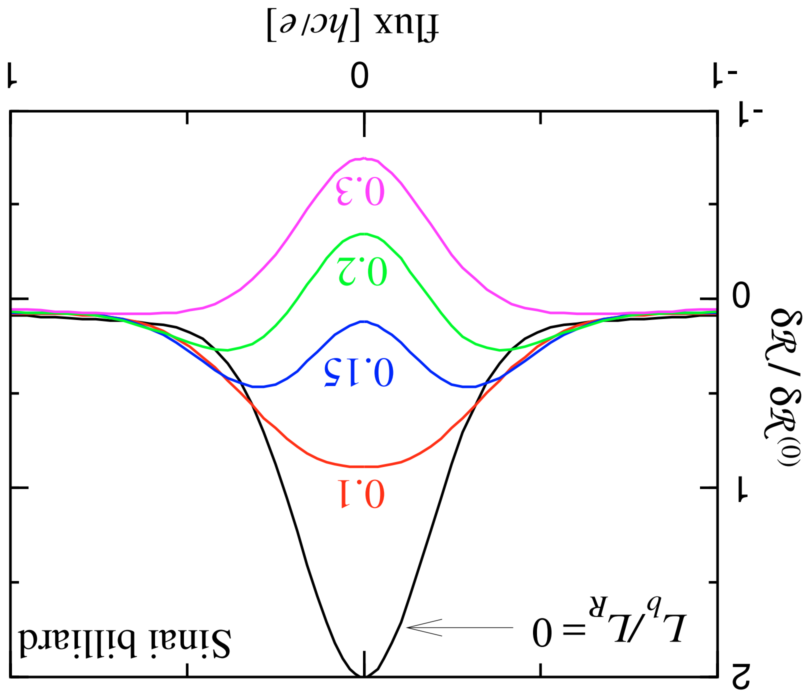
$$\gamma_\pm = \left[\alpha B \pm \beta\,(L_b/L_R)^2\right]^2$$

Experiment:

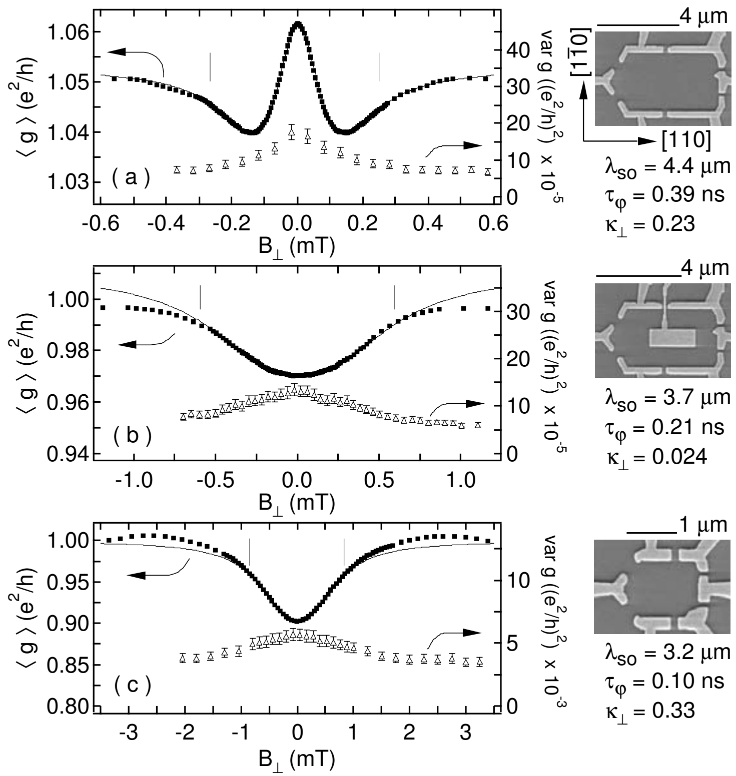


Magnetic flux dependence

$$\frac{\delta\mathcal{R}}{\delta\mathcal{R}^{(0)}} = \int_0^\infty \mathrm{d}L\, P(L)\, \langle\mathcal{M}\varphi\rangle(L)$$



Experiment:



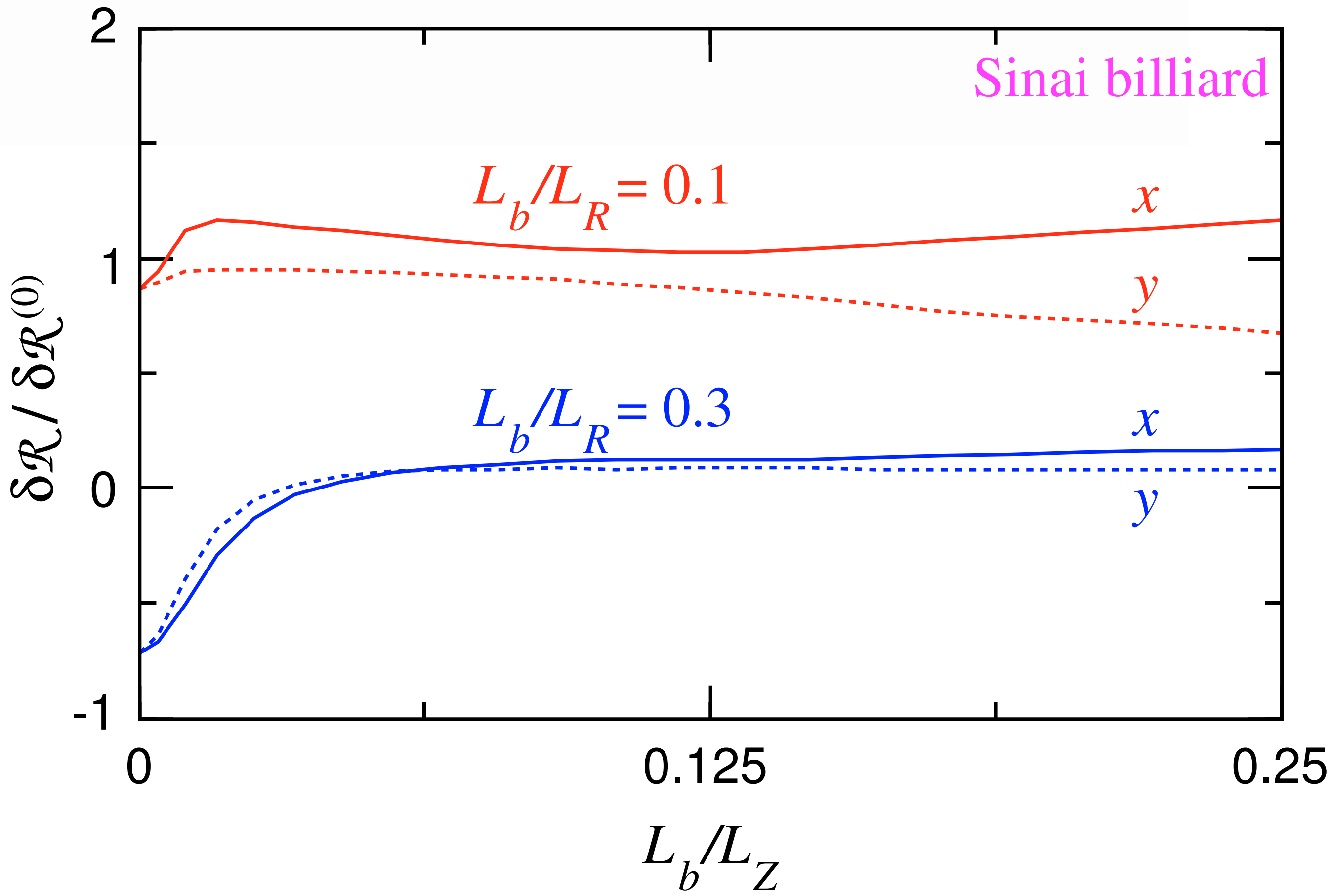
with

$$\langle\mathcal{M}\varphi\rangle(L) \simeq \sum_\pm \exp\left(-\gamma_\pm \frac{L}{L_b}\right)$$

and

$$\gamma_\pm = \left[\alpha B \pm \beta\,(L_b/L_R)^2\right]^2$$

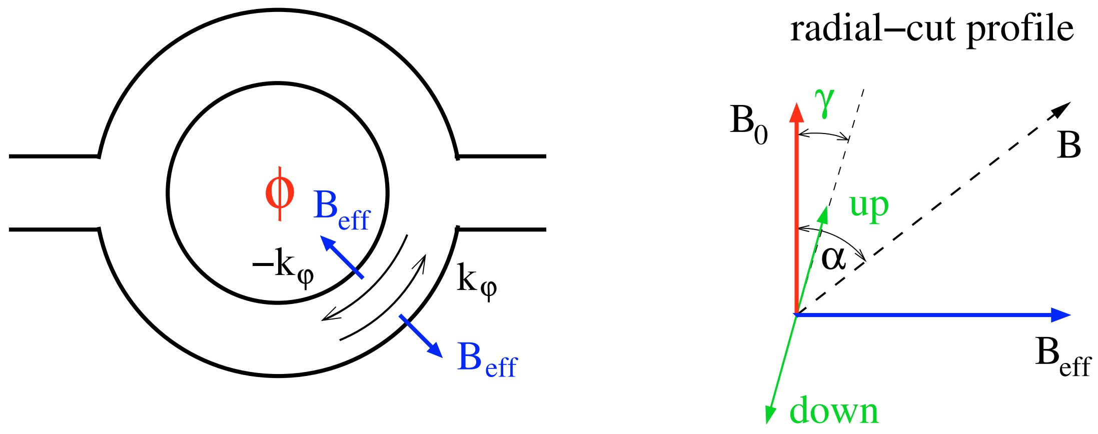
Effect of Zeeman interaction on AL



Zeeman interaction suppresses antilocalization

Aharonov-Bohm physics with spin

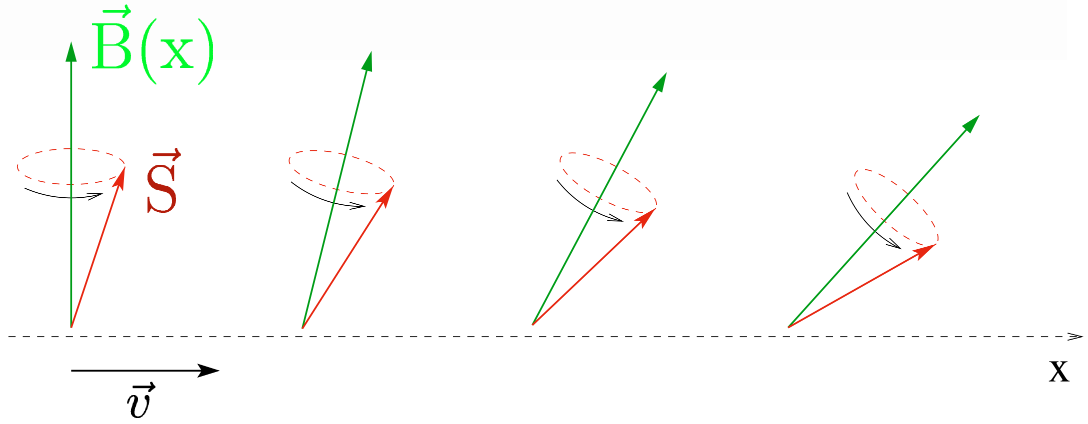
- how to use SO interaction to tune spin (directions) in a controlled way?
- consider ballistic AB rings with Rashba interaction (+ additional perpendicular field B_0)



- radial field: $B_{\rm eff} \sim \alpha_{\rm R}\, k_\varphi$
- experiments:
  Morpurgo et al., PRL (1998), Yau et al. PRL (2002)
  $\rightarrow$ signatures of spin-orbit induced Berry phases

Adiabatic spin transport

spin in an orientationally inhomogenous B-field:



adiabatic spin transport $\Leftrightarrow$ spin stays aligned with changing B-field direction

Relevant timescales:

- Larmor frequency $\omega_L = 2\mu B/\hbar$
- characteristic time $t_c = l_c/v$ on which B-field direction changes significantly

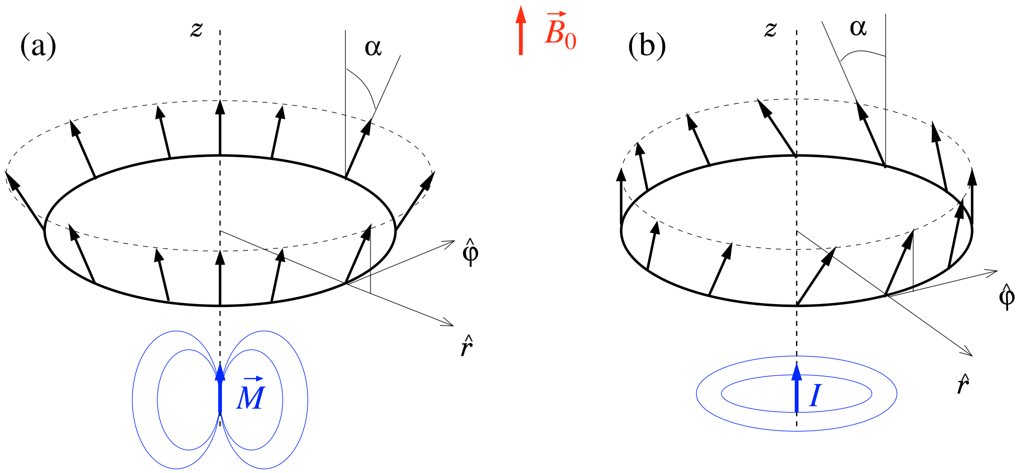
adiabaticity parameter: $Q \equiv \frac{\omega_L}{\omega_c} = \frac{\omega_L}{\frac{2\pi}{t_c}} \sim \omega_L t_c$

$Q \leq 1$ $\Leftrightarrow$ spin flips

$Q \gg 1$ $\Leftrightarrow$ adiabatic regime

Spin coupling to textured B-fields

$$H = \frac{1}{2m^*}\left(\vec{P} - \frac{e}{c}\vec{A}_{\rm em}\right)^2 + V(\vec{r}) + \mu\vec{B}(\vec{r})\cdot\vec{\sigma}$$



Stern (1992), Loss, Schöller, Goldbart (1993)

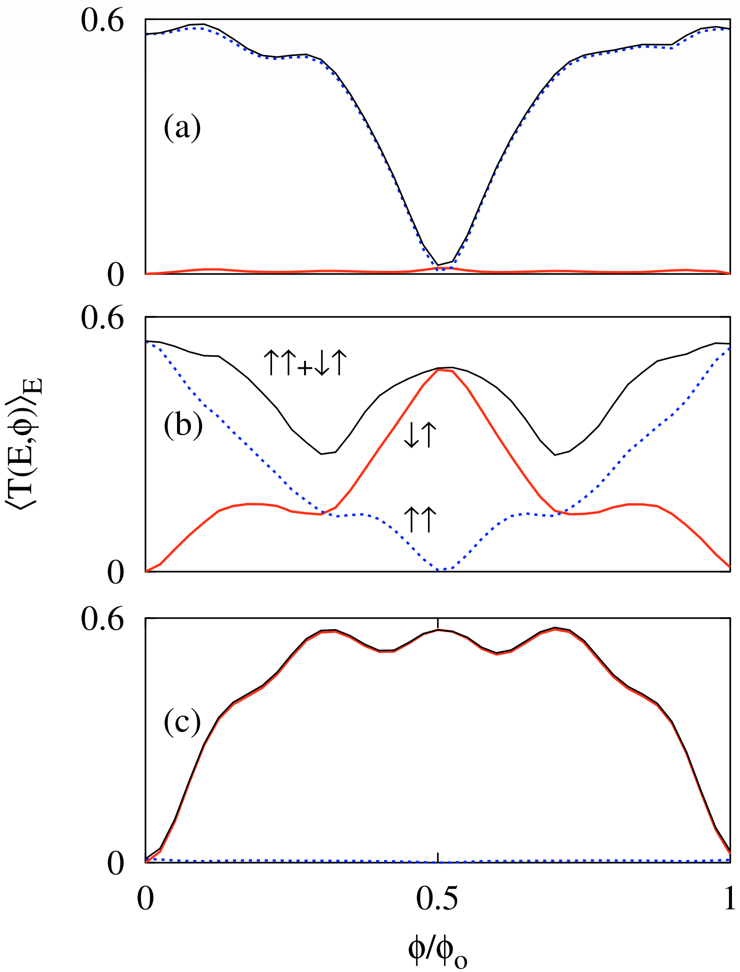
adiabatic regime: $Q \gg 1: \quad H = \frac{1}{2m^*}\left(\vec{P} - \frac{e}{c}\vec{A}_{\rm em} - \vec{A}_{\rm g}^{\uparrow\downarrow}\right)^2 + V(\vec{r})$

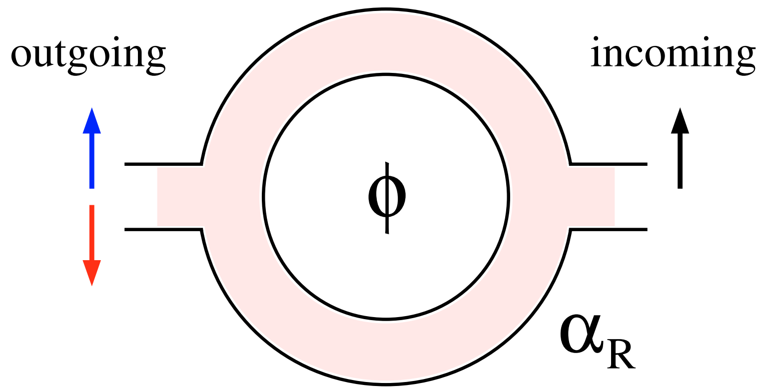
→ Berry phase: $\gamma^{\uparrow\downarrow} = \frac{1}{\hbar}\oint_\Gamma \vec{A}_{\rm g}^{\uparrow\downarrow}\cdot d\vec{\ell}$

→ effective flux: $\phi^{\uparrow\downarrow} = \phi_{\rm em} - [1 \pm \cos\alpha(B)]\phi_0/2$

Aharonov-Bohm ring as spin switch

Magneto conductance of spin-polarized ballistic currents:





a $Q_R \ll 1$: weak Rashba coupling

b,c $Q_R = 1$, $Q_R = 1.7$: moderate Rashba

D. Frustaglia, K.R., PRB (2004)

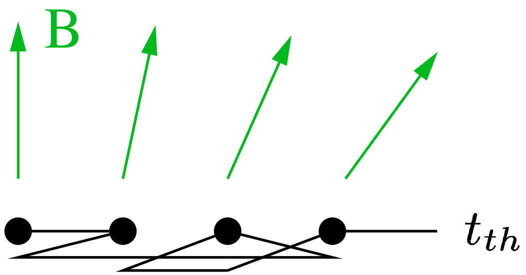
D. Frustaglia, M. Hentschel, K.R., PRL (2001)

- Control of the outgoing spin polarization by means of a small flux!
- Adiabatic spin transport is not required.

Berry phases in disordered systems?

What is the characteristic time t_c ? $(Q \sim \omega_L t_c)$

- Ballistic systems: $t_c \sim L/v$ ✓
- Disordered systems: two candidates:
  - ➢ Thouless time $t_{\rm th} = L^2/D \quad \rightarrow \quad Q > \ell/L$



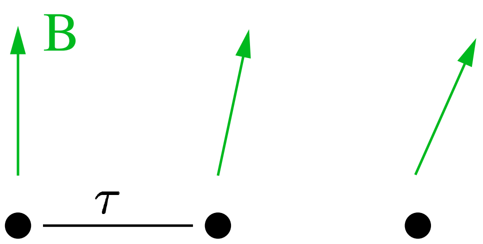
adiabaticity in metals:

B $\gtrsim$ 100 mT

for quantum corrections to the conductance

(Loss, Schöller, Goldbart (1999), Engel, Loss, (2000))

  - ➢ Mean elastic scattering time $\tau \quad \rightarrow \quad Q > L/\ell$



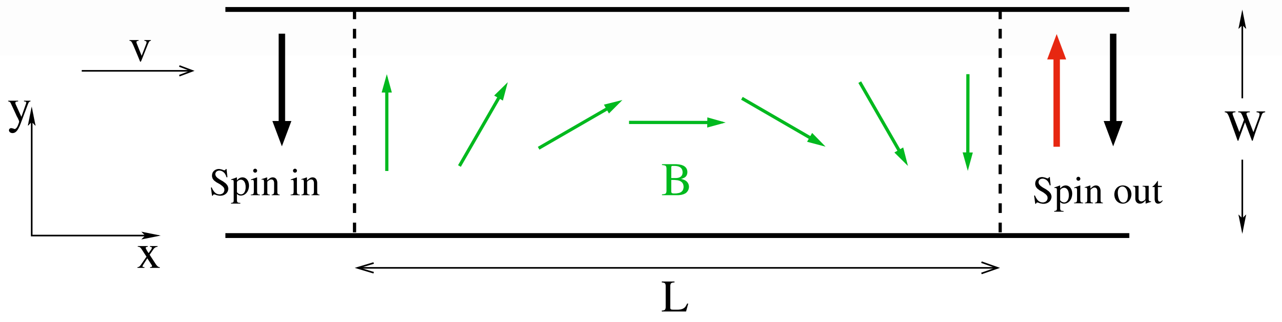
adiabaticity in metals:

B $\gtrsim$ 1000 T

(Stern (1992), van Langen, Knops, Paaschens, Beenakker (1999))

Case study: magnetic domain wall

ballistic strip:



Adiabaticity parameter for each transverse mode: $Q_i = \left(\frac{LWB}{\phi_0}\right)\left(\frac{m^*}{m_0}\right)\frac{g^*}{k_i W}$

Transmission $|\downarrow\rangle \to |\downarrow\rangle$ (transfer matrix method):

$$T_{\downarrow\downarrow} = \sum_{i=1}^{M} \frac{1}{1+Q_i^2}\sin^2\left(\frac{\pi}{2}\sqrt{1+Q_i^2}\right) \qquad (E_F \gg \mu B)$$

$$\Rightarrow T_{\downarrow\downarrow} \simeq \frac{1}{1+Q_{\rm bal}^2} \qquad Q_1 \le Q_{bal} \le Q_M$$

Introduce effective parameter $Q_{\rm bal} \simeq 1.4\, Q(k_F)$ (independent of M !)

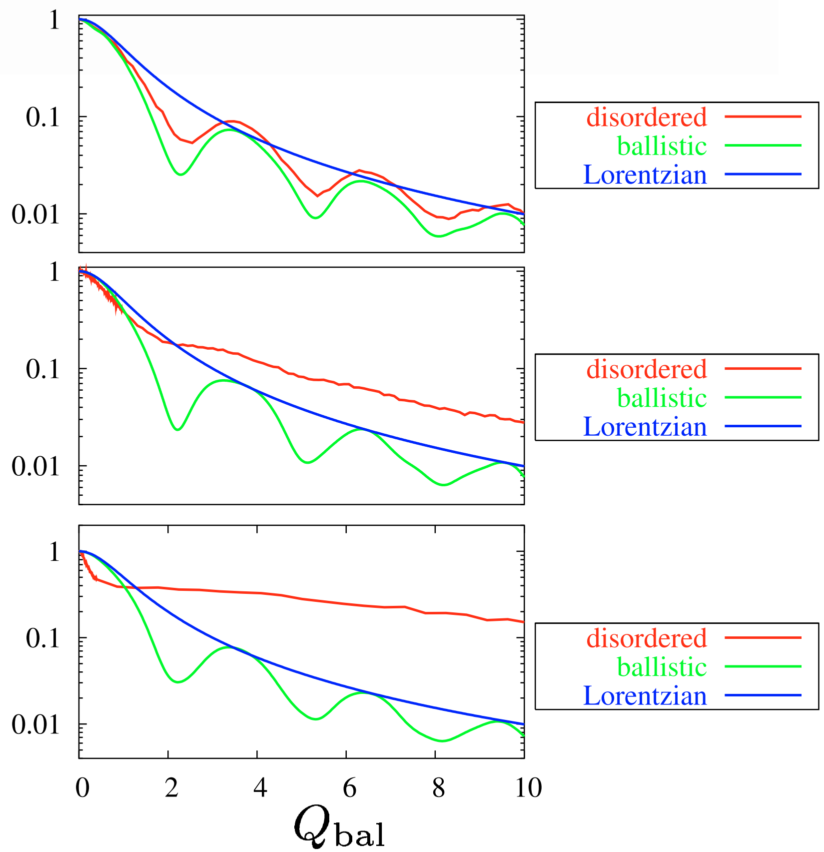
Spin transport in a disordered strip

Numerical quantum mechanical results for averaged transmission $\langle T_{\downarrow\downarrow}\rangle$:

$L/\ell = 0.5$

$L/\ell = 3$

$L/\ell = 10$



Regimes:

- Transmission plateau $\langle T_{\downarrow\downarrow}\rangle \approx \langle T_{\uparrow\downarrow}\rangle$ for $\ell/L \lesssim Q_{\mathrm{bal}} \lesssim L/\ell$
- $\langle T_{\downarrow\downarrow}\rangle > T^{bal}_{\downarrow\downarrow}$ for $Q_{bal} > L/\ell$ ➜ larger B-fields for adiabatic limit
- same adiabaticity condition for quantum corrections

Conclusions

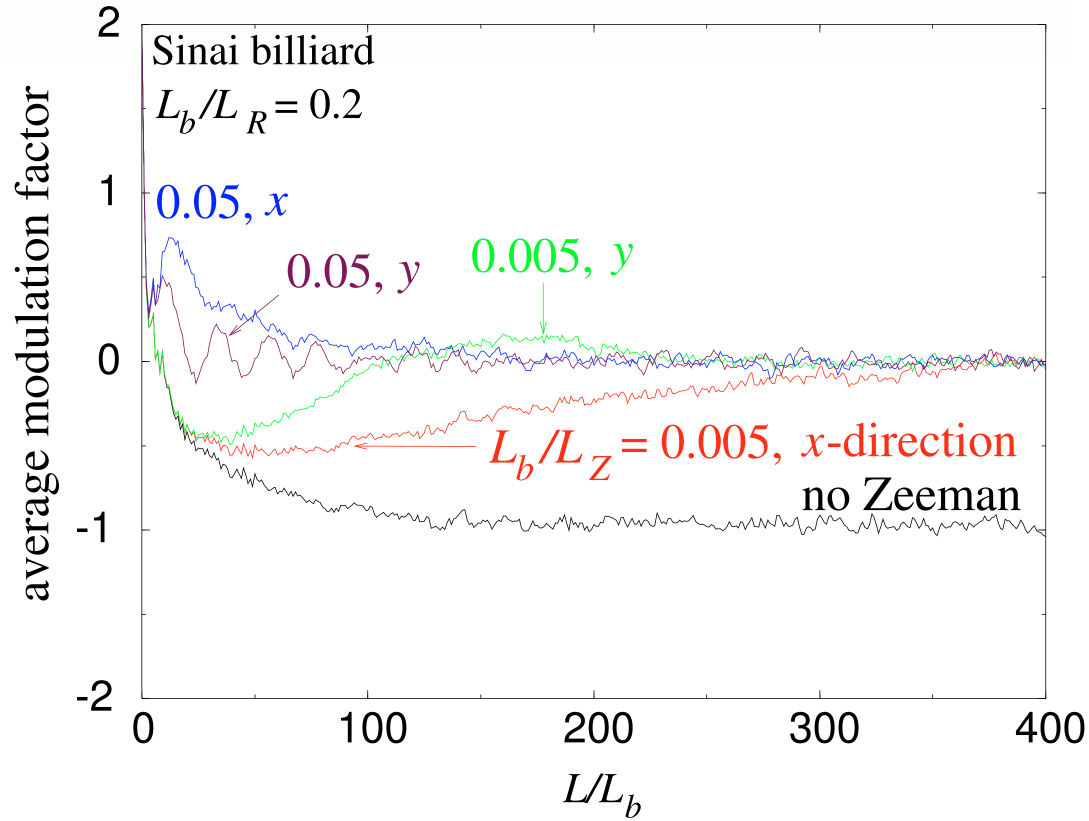
- SO interaction in quantum dots:
  - confinement reduces spin relaxation
  - orbital dynamics determines spin evolution:

    diffusive: fast exponential relaxation
    chaotic: universal decay features
    regular: saturation of relaxation
  - WL $\rightarrow$ AL transition upon deforming an integrable to a chaotic dot
- spin control: Aharonov-Bohm ring with SO interaction
- Berry phases: observation in diffusive conductors is unlikely
- References:

  - O. Zaitsev, D. Frustaglia, K.R., cond-mat/0405266 (2004)
  - D. Frustaglia and K.R. Phys. Rev. B **69**, 235310 (2004)
  - D. Frustaglia, M. Hentschel, and K.R., Phys. Rev. Lett. **87**, 256602 (2001)

Additional Zeeman interaction

$$\langle \mathcal{M} \rangle = \langle \mathrm{Tr}(\hat{K}_\gamma \hat{K}^\dagger_{\gamma^{-1}}) \rangle$$



- without Rashba: $\mathcal{M}(\mathbf{B}) \equiv \mathbf{2}$ in uniform field
- $\langle \mathcal{M} \rangle (L = \infty) = 0$: decrease of antilocalization
- Anisotropy in $\mathbf{B}$-direction

Very weak spin-orbit coupling

$L_{\mathsf{sys}}/L_R \ll 1$

- Position-dependent spin-frame rotation:

$$H \mapsto U^\dagger H U, U = \exp[ß \frac{\pi}{L_R} (x\sigma_y - y\sigma_x)]$$

$\Rightarrow$ Renormalized spin-orbit coupling:

$$\frac{\hbar}{2}\sigma \cdot \frac{2\pi}{L_R}\mathbf{v} \times \mathbf{e_z} \;\mapsto\; \frac{\hbar}{\mathbf{2}}\,\sigma_{\mathbf{z}}\,\frac{\mathbf{2}\pi^{\mathbf{2}}}{\mathbf{L_R^2}}(\mathbf{e_z} \times \mathbf{r}) \cdot \mathbf{v} + \mathcal{O}(\mathbf{L_R^{-3}})$$

Similar to uniform field $\frac{2\pi^2 c\hbar}{eL_R^2}$ times σ_z !

$\Rightarrow$ New modulation factor (chaotic):

$$\mathcal{M}_{\mathsf{new}}\,\bar{\varphi} \simeq e^{-\left[\alpha B+\beta\left(\frac{L_b}{L_R}\right)^2\right]^2 \frac{L}{L_b}} + e^{-\left[\alpha B-\beta\left(\frac{L_b}{L_R}\right)^2\right]^2 \frac{L}{L_b}}$$

— Double-peak structure in $\delta\mathcal{R}/\delta\mathcal{R}^{(0)}$;

— no antilocalization

Transformation of modulation factor

$$\mathcal{M}(t) \equiv \text{Tr}\left[\hat{K}_\gamma(t)\right]^2$$
$$= \text{Tr}\left[U\left(\mathbf{r}_\gamma(t)\right)\, \hat{K}_\gamma^{\text{new}}(t)\, U^\dagger\left(\mathbf{r}_\gamma(0)\right)\right]^2$$
$$\neq \text{Tr}\left[\hat{K}_\gamma^{\text{new}}(t)\right]^2 \equiv \mathcal{M}_{\text{new}}(t)$$

In the limit $\frac{L_b}{L_R},\ \left(\frac{L_b}{L_R}\right)^4 \frac{L}{L_b} \ll 1$

$$\bar{\mathcal{M}}(L) \simeq 2 - \left(\frac{2\pi}{L_R}\right)^2 \langle(\mathbf{r}(L) - \mathbf{r}(0))^2\rangle$$
$$- 2\beta^2 \left(\frac{L_b}{L_R}\right)^4 \frac{L}{L_b} + \mathcal{O}\left[\left(\frac{L_b}{L_R}\right)^4, \left(\frac{L_b}{L_R}\right)^6 \frac{L}{L_b}\right]$$