united nations educational, scientific and cultural organization

international atomic energy agency

the
abdus salam
international centre for theoretical physics

40th anniversary 1964 2004

SMR.1587 - 4

***SCHOOL AND WORKSHOP ON
QUANTUM ENTANGLEMENT, DECOHERENCE,
INFORMATION, AND
GEOMETRICAL PHASES IN COMPLEX SYSTEMS
(1 November - 12 November 2004)***

Interference, Entanglement and Correlations in Higher Dimensions

**Wolfgang Peter Schleich
Abteilung für Quantenphysik
Universität Ulm
Albert Einstein Allee 11
D-89069 Ulm
Germany**

These are preliminary lecture notes, intended only for distribution to participants

INTERFERENCE

ENTANGLEMENT AND CORRELATIONS

IN

HIGHER DIMENSIONS

J. P. DAHL [*]

H. MACK [†]

W. P. SCHLEICH [†]

S. VARRO [§]

A. WOLF [†]

[*] TECHNICAL UNIVERSITY OF DENMARK, LYNGBY

[†] UNIVERSITÄT ULM

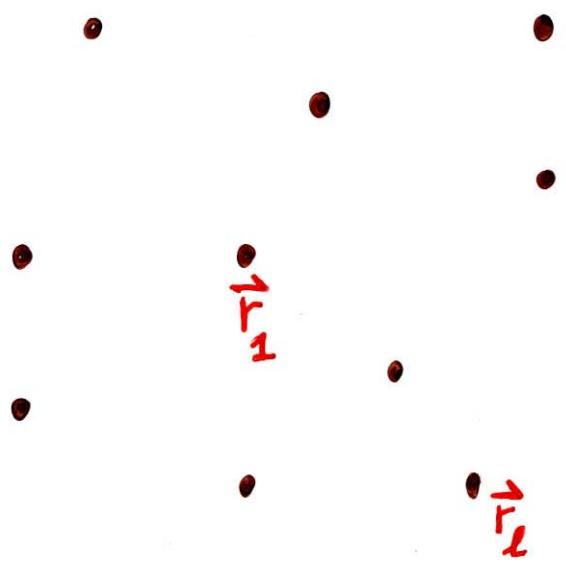
[§] RESEARCH INSTITUTE FOR SOLID STATE & OPTICS BUDAPEST

€ LAND BADEN-WÜRTTEMBERG

ALEXANDER VON HUMBOLDT-STIFTUNG

DAAD

N NON-INTERACTING ATOMS



$$\vec{r}_\ell = \left(x_1^{(\ell)}, \dots \; x_d^{(\ell)} \right)$$

$$r \equiv \sqrt{\vec{r}_1^{\,2} + \vec{r}_2^{\,2} + \dots + \vec{r}_N^{\,2}}$$

$$\Psi = \Psi(r) = \Psi(x_1, x_2, \dots, x_D)$$

$$D \equiv \begin{pmatrix} \text{NUMBER} \\ \text{OF} \\ \text{PARTICLES} \end{pmatrix} \cdot \begin{pmatrix} \text{NUMBER} \\ \text{OF} \\ \text{SPACE DIMENSIONS} \end{pmatrix} \equiv N \cdot d$$

OVERVIEW AND SUMMARY

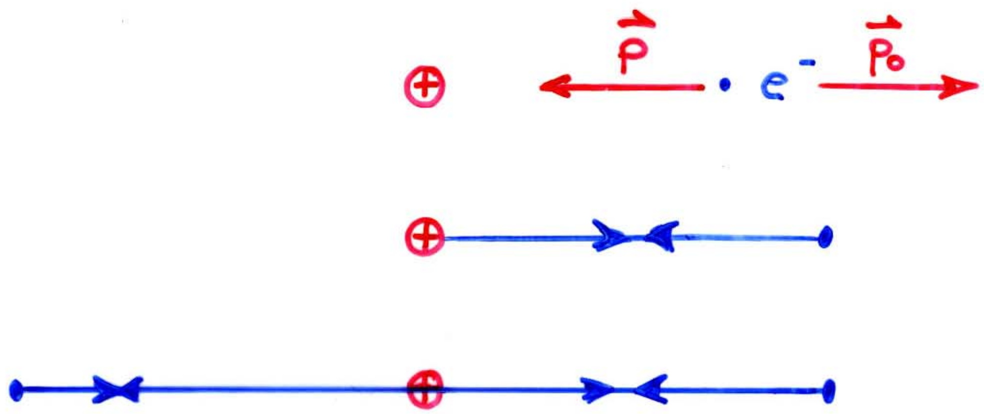
- FORMULATION OF PROBLEM
- SOMMERFELD'S PENDELBAHN
- CORRELATIONS IN PHASE SPACE
- ENTANGLEMENT AND NEGATIVE PARTS OF WIGNER FUNCTION
- FICTITIOUS POTENTIALS
- SPIN FROM DIMENSIONS ?


99 788

SOMMERFELD'S PENDELBAHN

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\vec{L} \equiv 0 \Longrightarrow \qquad \vec{r} \upuparrows \vec{p}$$

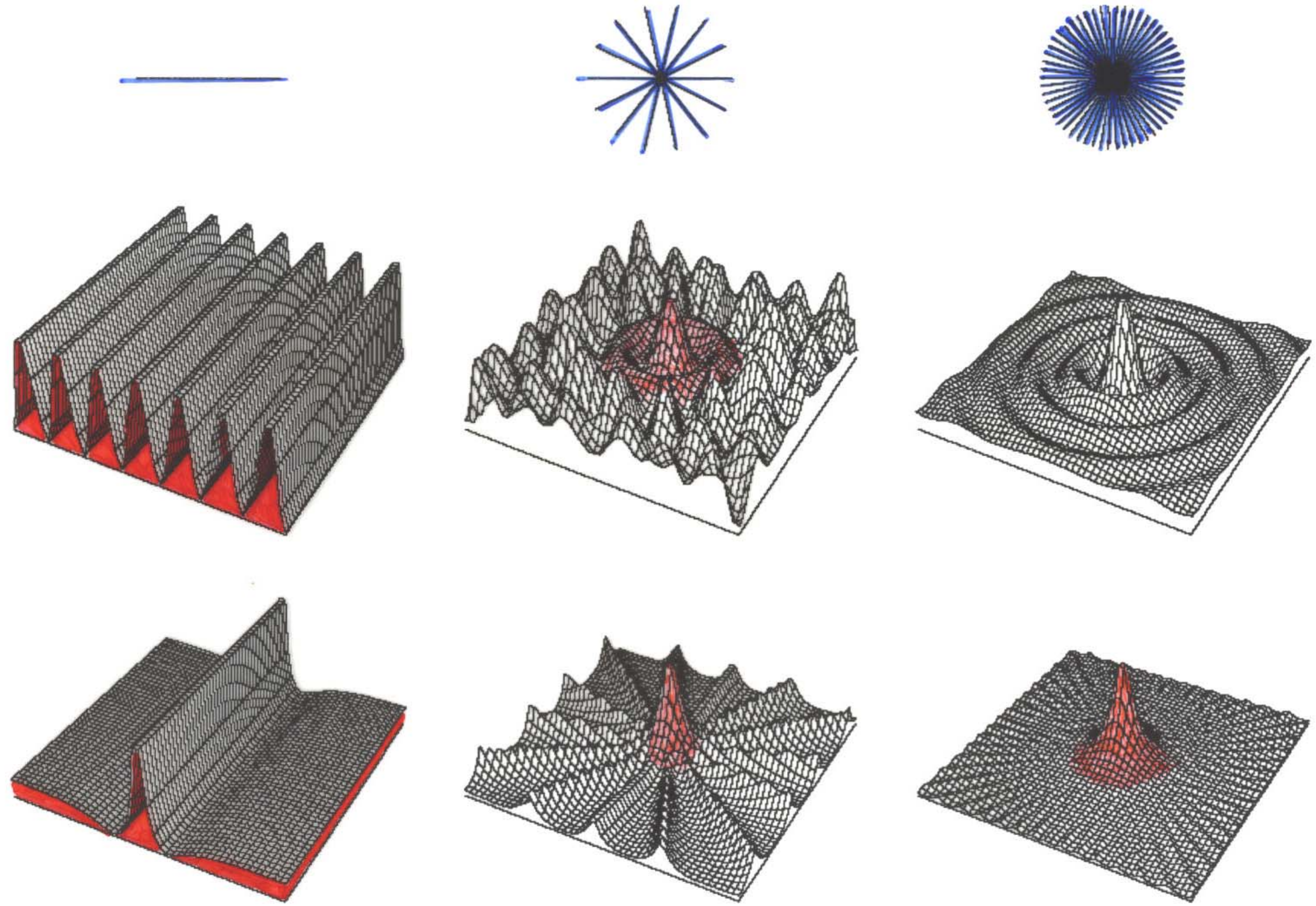


INTERNAL WAVES OF S-WAVE

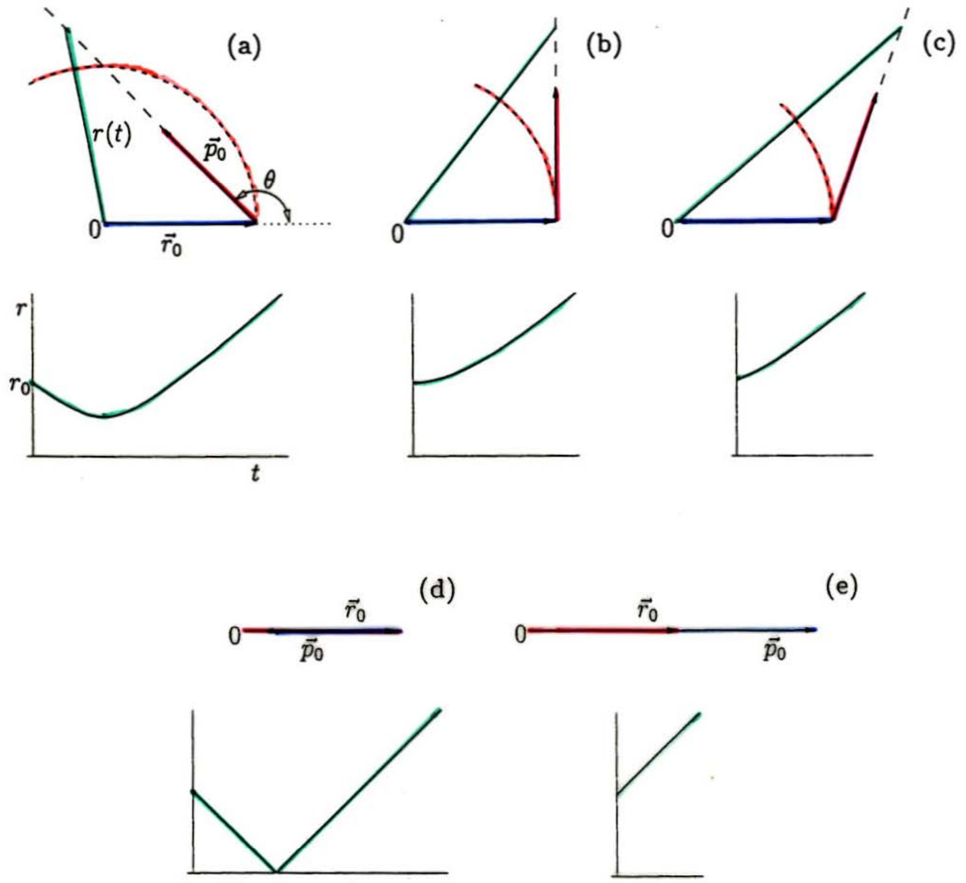
$$\psi(r) = \int d\Omega_{\vec{k}} \, \Sigma_{\vec{k}}(\vec{r})$$

$$\Sigma_{\vec{k}}(\vec{r}) = \frac{1}{2\pi} \frac{\partial}{\partial z} \left(z \, \psi(z) \right) \Big|_{z = \vec{e}_k \cdot \vec{r}}$$

s-waves: free particle and hydrogen



FREE PARTICLE : CLASSICAL MECHANICS



FREE PARTICLE : CLASSICAL MECHANICS

$$r(t) = \left| \vec{r}_0 + \frac{1}{M} \vec{p}_0 t \right|$$

$$= r_0 + \frac{1}{M} p_r t + \frac{1}{2} a_T t^2 + \ldots$$

$$r_0 \equiv |\vec{r}_0|$$

$$p_r \equiv |\vec{p}_0| \cos\theta$$

$$a_T \equiv \frac{1}{M^2} \frac{|\vec{p}_0|^2}{|\vec{r}_0|} \sin^2\theta$$

FREE PARTICLE : CLASSICAL ENSEMBLE

$$\langle r(t)\rangle = \langle r_0 \rangle + \frac{1}{M}\langle p_r \rangle t + \frac{1}{2}\langle a_r \rangle t^2 + \ldots$$

$$\langle r_0 \rangle = \int d^D r \int d^D p \, |\vec{r}| \, W_0(\vec{r}, \vec{p})$$

$$\langle p_r \rangle = \int d^D r \int d^D p \, |\vec{p}| \cos\theta \, W_0(\vec{r}, \vec{p})$$

$$\langle a_r \rangle = \frac{1}{M^2} \int d^D r \int d^D p \, \frac{|\vec{p}|^2}{|\vec{r}|} \sin^2\theta \, W_0(\vec{r}, \vec{p})$$

On the Quantum Correction For Thermodynamic Equilibrium

By E. WIGNER
Department of Physics, Princeton University

(Received March 14, 1932)

The probability of a configuration is given in classical theory by the Boltzmann formula $\exp[-V/hT]$ where V is the potential energy of this configuration. For high temperatures this of course also holds in quantum theory. For lower temperatures, however, a correction term has to be introduced, which can be developed into a power series of h. The formula is developed for this correction by means of a probability function and the result discussed.

If a wave function $\psi(x_1 \cdots x_n)$ is given one may build the following expression[2]

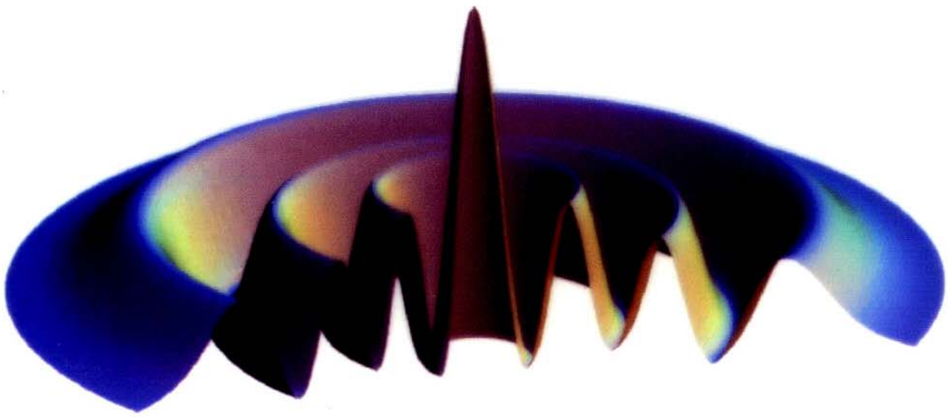
$$P(x_1, \cdots, x_n; p_1, \cdots, p_n) = \left(\frac{1}{h\pi}\right)^n \int_{-\infty}^{\infty} \cdots \int dy_1 \cdots dy_n \psi(x_1 + y_1 \cdots x_n + y_n)^* \psi(x_1 - y_1 \cdots x_n - y_n) e^{2i(p_1 y_1 + \cdots + p_n y_n)/h} \quad (5)$$

and call it the probability-function of the simultaneous values of $x_1 \cdots x_n$ for the coordinates and $p_1 \cdots p_n$ for the momenta.

[2] This expression was found by L. Szilard and the present author some years ago for another purpose.

Wolfgang P. Schleich

Quantum Optics in Phase Space



WILEY-VCH

S-STATE : WIGNER FUNCTION

$$W(\vec{r},\vec{k}) \sim \int d^D\xi \, e^{-i\vec{k}\cdot\vec{\xi}} \, \Psi^*(\vec{r}-\tfrac{1}{2}\vec{\xi}) \, \Psi(\vec{r}+\tfrac{1}{2}\vec{\xi})$$

$$\Psi(\vec{r}) = \Psi(r) \qquad r \equiv |\vec{r}|$$

s-STATE: WIGNER FUNCTION*

$$W(\vec{r},\vec{k}) = W(r,k,\sphericalangle(\vec{r},\vec{k}) \equiv \theta)$$

$$= \mathcal{N}_D \int_0^\infty d\xi\, \xi^{D-1} \int_0^\pi d\vartheta\, \sin^{D-2}\vartheta\, \frac{J_{\frac{D-3}{2}}(k\xi\sin\theta\sin\vartheta)}{(k\xi\sin\theta\sin\vartheta)^{\frac{D-3}{2}}}\, e^{ik\xi\cos\theta\sin\vartheta}\psi^*(r_-)\psi(r_+)$$

$$r_\pm \equiv \left(\vec{r}^2 + \frac{1}{4}\vec{\xi}^2 \pm r\xi\cos\theta\right)^{\frac{1}{2}}$$

$$\mathcal{N}_D \equiv \ldots$$

* J.P. Dahl, S. Varro, A. Wolf, and W.P. Schleich, to be published

SHELL WAVE FUNCTION

$$\psi(r) = \mathcal{N}\, r^2 e^{-r^2/2}$$

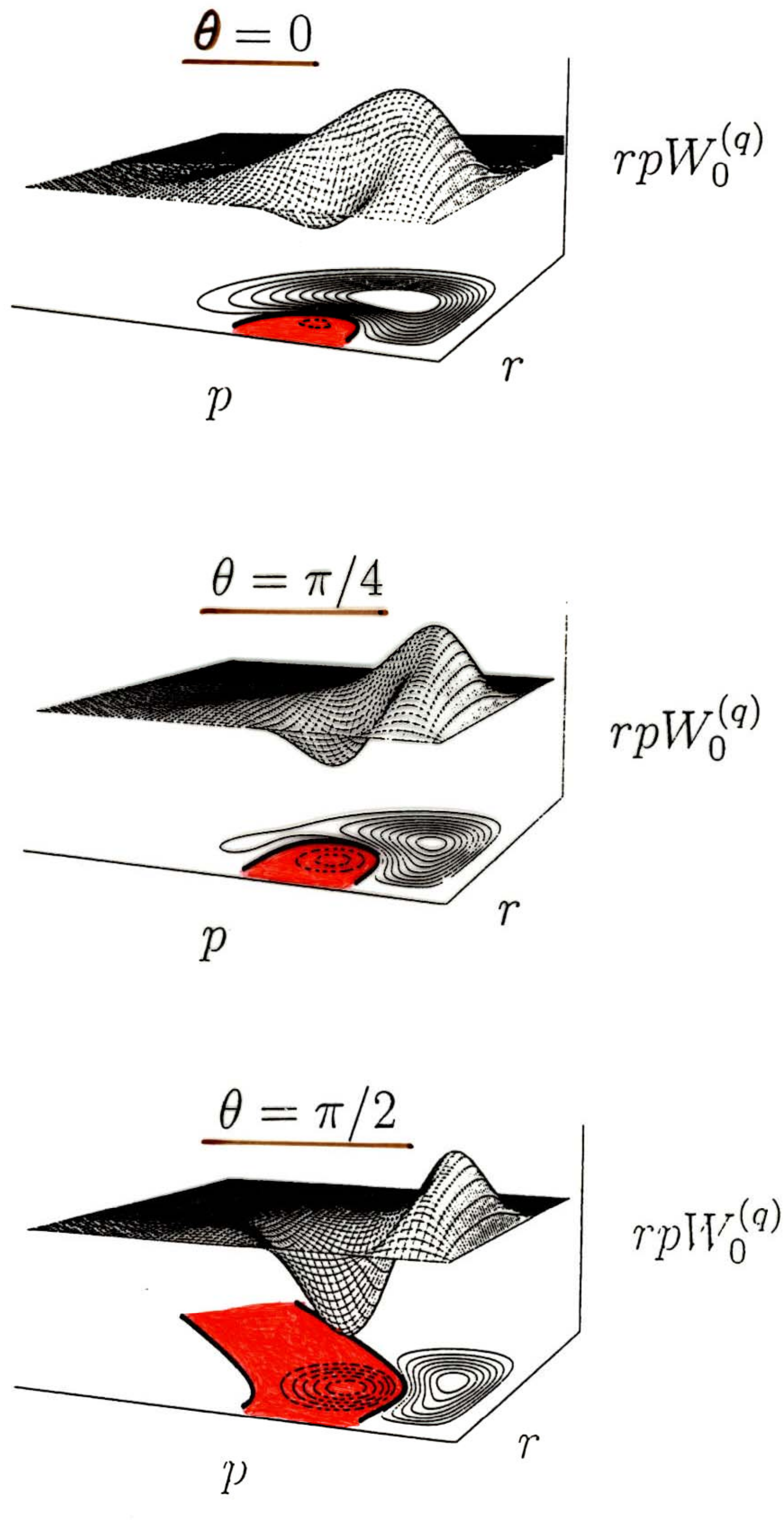
$$r \equiv \left[x_1^2 + x_2^2 + \dots + x_D^2 \right]^{1/2}$$

Fortschr. Phys. **52**, No. 11–12, 1118–1133 (2004) / **DOI** 10.1002/prop.200410182

Interference acceleration of a free particle*

J. P. Dahl[1,2], A. Wolf[1], and W. P. Schleich[1,]**

[1] Abteilung für Quantenphysik, Universität Ulm, Albert-Einstein Allee 11, 89069 Ulm, Germany
[2] Chemical Physics, Department of Chemistry,Technical University of Denmark, DTU 207, 2800 Kgs. Lyngby, Denmark



In- and Outbound Spreading of a Free-Particle s-Wave

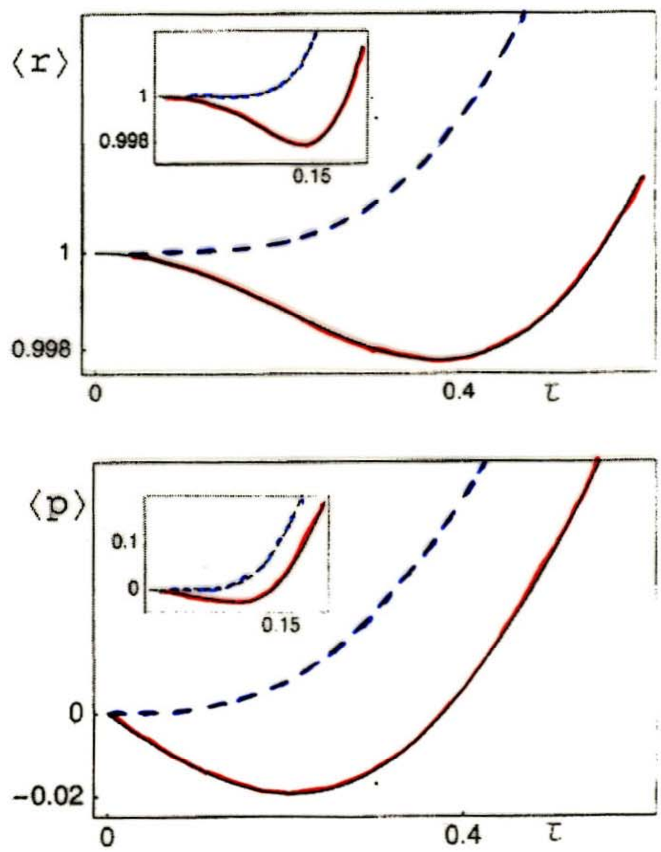
I. Białynicki-Birula,[1,2] M. A. Cirone,[2] J. P. Dahl,[2,3] M. Fedorov,[2,4] and W. P. Schleich[2]
[1]Center for Theoretical Physics, Polish Academy of Sciences, Aleja Lotników 32/46, 02-668, Warsaw, Poland
[2]Abteilung für Quantenphysik, Universität Ulm, D-89069 Ulm, Germany
[3]Chemical Physics, Department of Chemistry, Technical University of Denmark, DTU 207, DK-2800 Lyngby, Denmark
[4]General Physics Institute, Russian Academy of Sciences, 38 Vavilov Street, Moscow 117942 Russia
(Received 17 October 2001; published 23 July 2002)

We show that a free quantum particle in two dimensions with zero angular momentum (s wave) in the form of a ring-shaped wave packet feels an attraction towards the center of the ring, leading first to a contraction followed by an expansion. An experiment to demonstrate this effect is also outlined.

$$\Psi(r) \sim r^2 e^{-\frac{1}{2}r^2}$$

— $r^2 = x^2 + y^2$

---- $r^2 = x^2 + y^2 + z^2$



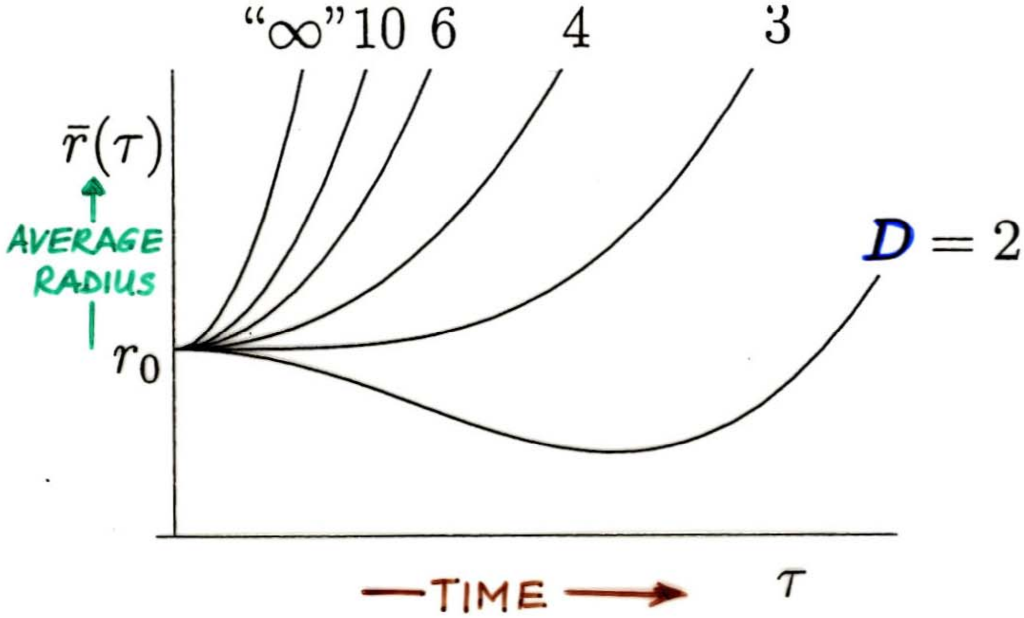
— TIME →

 © 2002 The American Physical Society

TIME DEPENDENCE OF $\bar{r}$

$$\psi(r) \sim r^{2} e^{-r^{2}/2}$$

$$r \equiv \left[x_1^2 + x_2^2 + \ldots + x_D^2 \right]^{1/2}$$



INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF OPTICS B: QUANTUM AND SEMICLASSICAL OPTICS

J. Opt. B: Quantum Semiclass. Opt. 4 (2002) S393–S396 PII: S1464-4266(02)37882-0

Attractive and repulsive quantum forces from dimensionality of space

I Białynicki-Birula[1,2], M A Cirone[1], J P Dahl[1,3], R F O'Connell[4] and W P Schleich[1]

[1] Abteilung für Quantenphysik, Universität Ulm, D-89069 Ulm, Germany
[2] Center for Theoretical Physics, Polish Academy of Sciences, Al. Lotnikow, 32/46, 02-668, Warsaw, Poland
[3] Chemical Physics, Department of Chemistry, Technical University of Denmark, DTU 207, DK-2800 Lyngby, Denmark
[4] Department of Physics, Louisiana State University, Baton Rouge, LA 70803-4001, USA

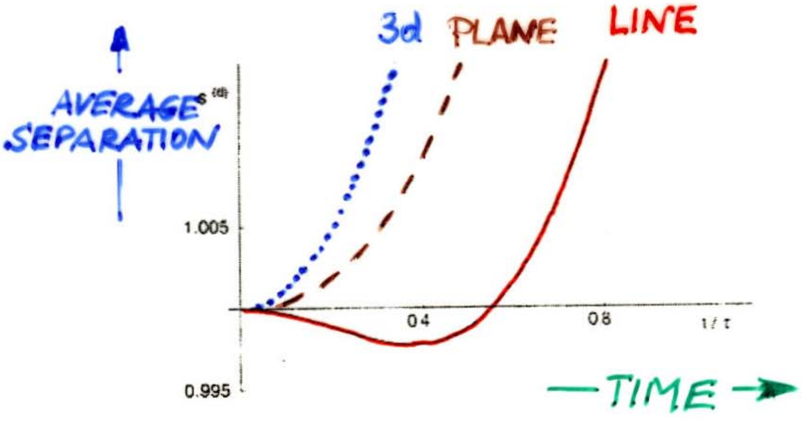
Received 19 January 2002
Published 29 July 2002
Online at stacks.iop.org/JOptB/4/S393

Abstract
Two particles of identical mass attract and repel each other even when there exist no classical external forces and their average relative momentum vanishes. This quantum force depends crucially on the number of dimensions of space.

$$\psi(\vec{r}_1, \vec{r}_2) \sim (\vec{r}_1^{\,2} + \vec{r}_2^{\,2}) \exp\left(-\frac{1}{2}(\vec{r}_1^{\,2} + \vec{r}_2^{\,2})\right)$$

$\vec{r}_1$ $\vec{r}_2$

$$s^{(d)} \equiv \langle |\vec{r}_1 - \vec{r}_2| \rangle$$



PHYSICAL REVIEW A, VOLUME 65, 052109

Dimensional enhancement of kinetic energies

W. P. Schleich[1] and J. P. Dahl[1,2,*]
[1]Abteilung für Quantenphysik, Universität Ulm, D-89069 Ulm, Germany
[2]Chemical Physics, Department of Chemistry, Technical University of Denmark, DTU 207, DK-2800 Lyngby, Denmark
(Received 27 December 2001; published 18 April 2002)

$$T \equiv -\frac{\hbar^2}{2M} \langle \Delta^{(D)} \rangle = T_r + T_V$$

$$T_r \equiv \int_0^\infty dr\, u^*(r) \left(-\frac{\hbar^2}{2M}\right) \frac{d^2}{dr^2} u(r)$$

$$T_V \equiv \int_0^\infty dr\, V_Q(r) \left| u(r) \right|^2$$

$$V_Q(r) \equiv \frac{\hbar^2}{2M} \frac{(D-1)(D-3)}{4r^2}$$

 ©2002 The American Physical Society

Collected papers on quantum philosophy

Speakable and unspeakable in quantum mechanics

J. S. BELL

CERN

The right of the University of Cambridge to print and sell all manner of books was granted by Henry VIII in 1534. The University has printed and published continuously since 1584.

CAMBRIDGE UNIVERSITY PRESS

Cambridge

New York New Rochelle Melbourne Sydney

21

EPR correlations and EPW distributions

Dedicated to Professor E. P. Wigner

When W happens to be initially nowhere negative, the classical evolution (7) preserves the non-negativity. The original EPR wave function[6]

$$\delta((q_1 + \tfrac{1}{2}q_0) - (q_2 - \tfrac{1}{2}q_0)), \tag{8}$$

assumed to hold at $t_1 = t_2 = 0$, gives

$$W(q_1, q_2, p_1, p_2, 0, 0) = \delta(q_1 - q_2 + q_0)2\pi\delta(p_1 + p_2) \tag{9}$$

This is nowhere negative, and the evolved function (7) has the same property. Thus in this case the EPR correlations are precisely those between two classical particles in independent free classical motion.

SCHMIDT DECOMPOSITION

$$D = 2d$$

$$\Psi^{(D)} = \Psi^{(D)}(\vec{s}_1, \vec{s}_2)$$

$$\rho(\vec{s}, \vec{s}') = \int dt\, \Psi^{(D)}(t, \vec{s})\, \Psi^{*(D)}(t, \vec{s}')$$

$$\int d\vec{s}'\, \rho(\vec{s}, \vec{s}')\, \psi_j(\vec{s}') = \lambda_j\, \psi_j(\vec{s})$$

$$\Psi^{(D)}(\vec{s}_1, \vec{s}_2) = \sum_j \lambda_j^{1/2}\, \psi_j(\vec{s}_1)\, \phi_j(\vec{s}_2)$$

ENTANGLEMENT OF FORMATION

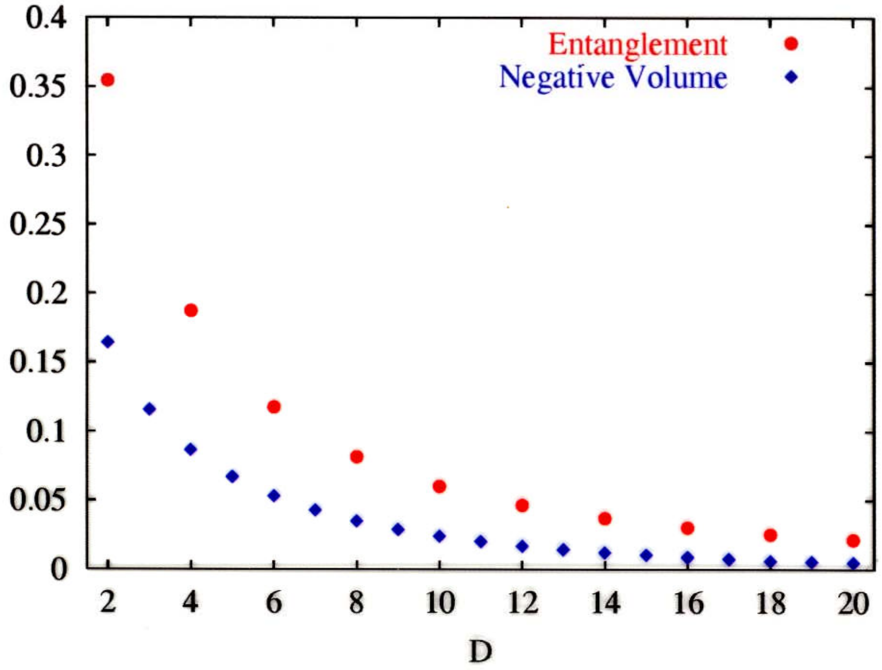
PARKER et al. PRA 61, 032305 (2000)

$$E(d) \equiv \sum_j \lambda_j \log_2 \lambda_j$$

ENTANGLEMENT VERSUS NEGATIVE WIGNER

$$\Psi^{(D)}(\vec{r}) \sim r^2 e^{-r^2/2}$$

$$\lambda_{\pm}(d) = \frac{1}{2} \pm \frac{\sqrt{d(d+2)}}{2(d+1)}$$



INTERFERENCE OF PLANE WAVES: D-DIMENSIONS

$$\psi \sim \sum_{j} e^{i\vec{k}_j\cdot\vec{r}} = \sum_{j} e^{ikr\,\vec{e}_j\cdot\vec{e}_r}$$

$$\longrightarrow \int d\Omega^{(D)}_{\vec{k}}\; e^{ikr\cos\theta_{\vec{k}}}$$

$$\psi \sim J_{\frac{D-2}{2}}(kr)$$

ODD - EVEN DIMENSIONS : S-WAVE

$$\psi \sim J_{\frac{D-2}{2}}$$

$D = 2l+1$: $\qquad J_{\frac{2l-1}{2}} \sim j(\sin, \cos)$

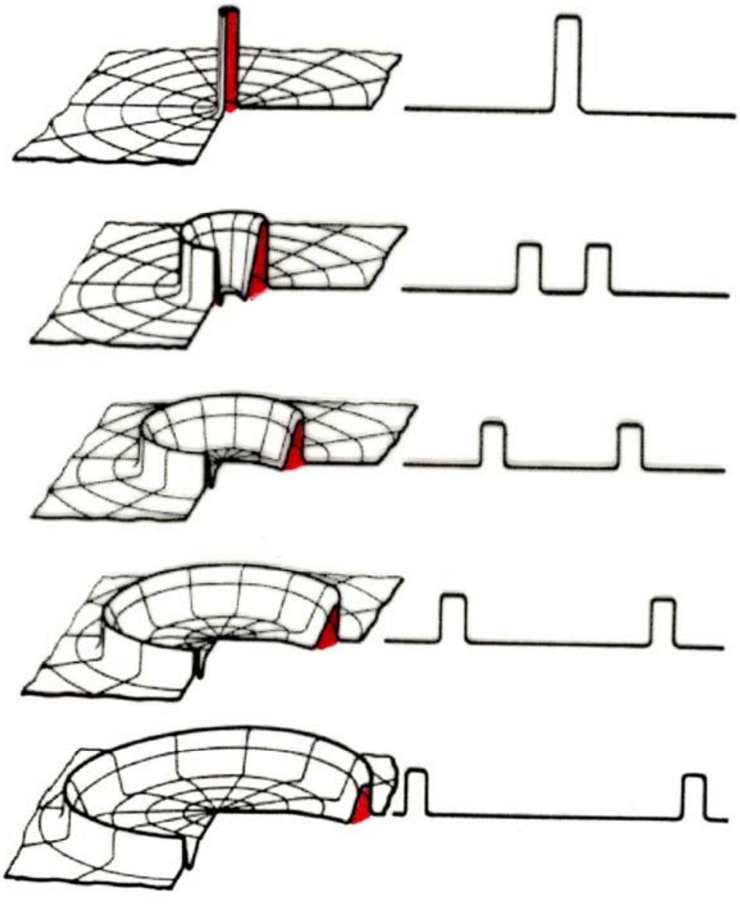
$D = 2k$: $\qquad J_k$

EM-WAVES : VIOLATION OF HUYGENS' PRINCIPLE IN EVEN DIMENSIONS

METHODS OF THEORETICAL PHYSICS

Philip M. Morse
PROFESSOR OF PHYSICS
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Herman Feshbach
ASSOCIATE PROFESSOR OF PHYSICS
MASSACHUSETTS INSTITUTE OF TECHNOLOGY



Ann. Phys. 61, 440 (1920)

2. *Welche Rolle spielt die Dreidimensionalität des Raumes in den Grundgesetzen der Physik?*[1]) *von P. Ehrenfest.*

„Warum hat unser Raum gerade drei Dimensionen?[1]) oder anders gefragt: „Welche *singulären*[2]) Vorkommnisse unterscheiden die Physik des R_3 vor der in den übrigen R_n?" — So gestellt, sind die Fragen vielleicht sinnlos, jedenfalls fordern sie zur Kritik heraus. Denn „ist" der Raum?, „ist" er dreidimensional? Und vollends die Frage nach dem „warum". Auch: was muß man unter „der" Physik des R_4 oder R_7 verstehen?

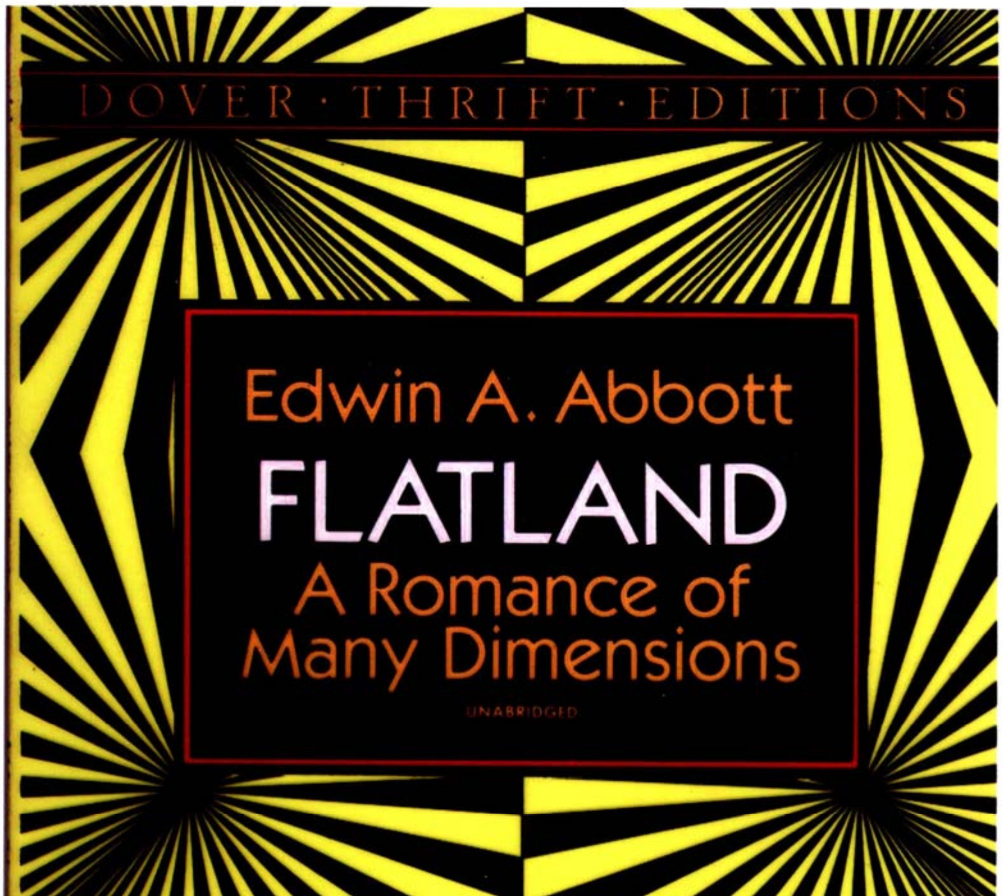
Ich werde nicht versuchen, diesen Fragen eine minder anstößige Form zu geben. Glückt es, nur erst mehr und mehr singuläre Eigenschaften des R_3 aufzufinden, dann wird schließlich von selber deutlich werden, welche „vernünftige" Frage sich zu der gefundenen Antwort konstruieren läßt.

§ 1. Schwerkraft und Planetenbewegung.

Was die Bewegung eines Planeten um einen Zentralkörper betrifft, so kann man feststellen, daß ein charakteristischer Unterschied zwischen R_3 und R_2 einerseits und allen höheren R_n anderseits besteht, was die Stabilität der Kreisbahn be-

1) Auszug aus der gleichnamigen Abhandlung in Versl. d. Ak. v. Wetensch. te Amsterdam **26.** S. 105. 1917 (Sitzung v. 26. V. 1917) = Proceedings **20.** S. 200. — Die interessante Bemerkung, die H. Weyl kürzlich über die Vierdimensionalität der Raum-Zeit-Welt gemacht hat (Gravitation u. Elektrizität. Sitzungsber. preuß. Ak. **26.** S. 474 oben. 1918; Neue Erweiterung der Relativtheorie. Ann. d. Phys. **59.** S. 133 Mitte. 1919), veranlaßt mich, hier noch einmal meine Bemerkungen zusammenzustellen, obwohl ich mir dessen bewußt bin, daß sie sehr elementar sind und einzeln genommen den meisten gut bekannt sein dürften. Aber ich hoffe eben, daß vielleicht andere dann einen Anlaß finden werden, reicheres und besseres Material zu dieser faszinierenden Frage vorzulegen.

2) Vgl. hierzu „Schlußbemerkung 1".


DOVER · THRIFT · EDITIONS
Edwin A. Abbott
FLATLAND
A Romance of
Many Dimensions
UNABRIDGED

RADIAL WAVE FUNCTION

$$\Psi = \Psi(r)$$

" s - WAVE "

$$\Psi \equiv \mathcal{N}_D \frac{u(r)}{r^{\frac{D-1}{2}}}$$

$$1 \equiv \int d^D x \, |\Psi|^2 = \int dr \, r^{D-1} \int \ldots$$

$$|u(r)|^2 dr \equiv \left(\begin{array}{l} \text{PROBABILITY TO} \\ \text{FIND PARTICLE} \\ \text{BETWEEN } r \text{ AND } r+dr \end{array} \right)$$

SCHRÖDINGER EQUATION : S-WAVE

D - DIMENSIONS

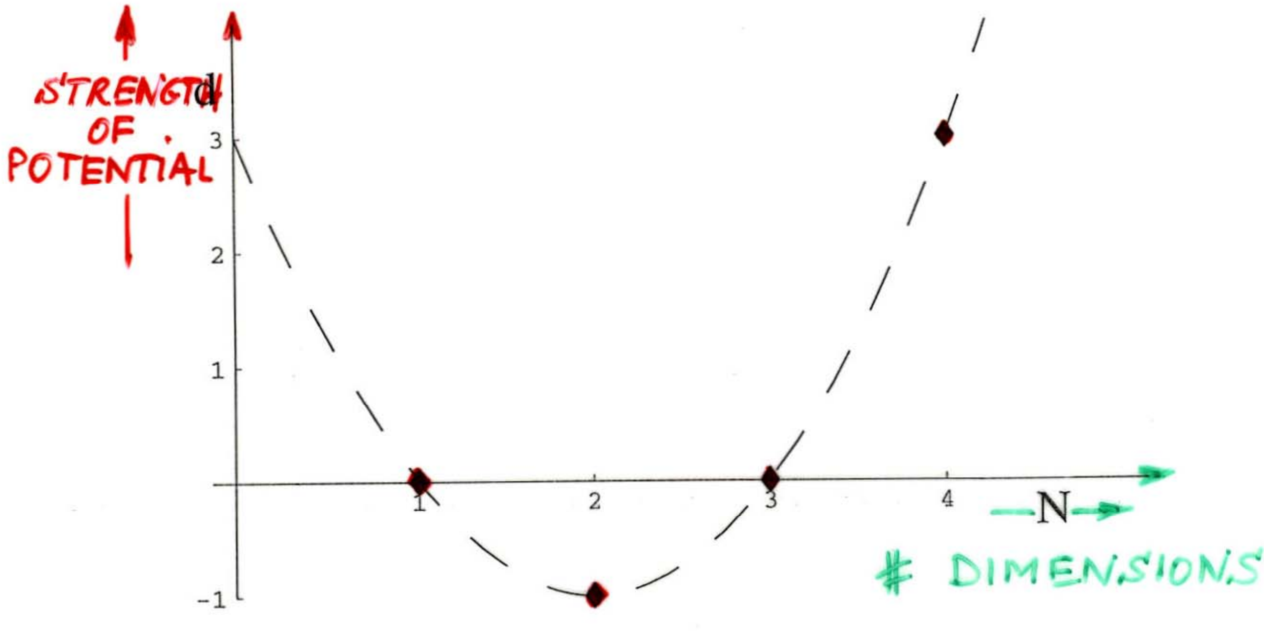
$$\left\{ \frac{d^2}{dr^2} + \frac{2M}{\hbar^2}\left[E - V(r) \right] \right\} u(r) = 0$$

$$V(r) \equiv \frac{\hbar^2}{2M} \frac{(D-3)(D-1)}{4r^2}$$

QUANTUM FICTITIOUS POTENTIAL

D - DIMENSIONS

$$V(r) \equiv \frac{\hbar^2}{2M} \frac{(D-3)(D-1)}{4r^2}$$



QUANTUM FICTITIOUS POTENTIAL CONNECTED TO SPIN ?

$$V_Q = \frac{\hbar^2}{2M} \frac{(D-3)(D-1)}{4r^2}$$

$$V_Q = \frac{\hbar^2}{2M} \frac{\lambda(\lambda+1)}{r^2}$$

$$\lambda \equiv \frac{1}{2}(D-3)$$

$D \mathrel{\hat{=}}$ ODD : λ INTEGER

$D \mathrel{\hat{=}}$ EVEN : λ HALF-INTEGER