united nations educational, scientific and cultural organization
international atomic energy agency
the abdus salam international centre for theoretical physics
40th anniversary 1964 2004

SMR.1587 - 9

SCHOOL AND WORKSHOP ON
QUANTUM ENTANGLEMENT, DECOHERENCE, INFORMATION, AND GEOMETRICAL PHASES IN COMPLEX SYSTEMS
(1 November - 12 November 2004)

Propagation, collision and entanglement of electron wavepackets in quantum dot arrays

D. PETROSYAN

Institute of Electronic Structure & Laser (IESL)
Foundation of Research and Technology-Hellas (FORTH)
Heraklion 71110, Crete
Greece

These are preliminary lecture notes, intended only for distribution to participants

Propagation, collision and entanglement of electron wavepackets in quantum dot arrays

David Petrosyan

dap@iesl.forth.gr

Institute of Electronic Structure & Laser (IESL)

Foundation for Research and Technology – Hellas (FORTH)

Heraklion 71110, Crete

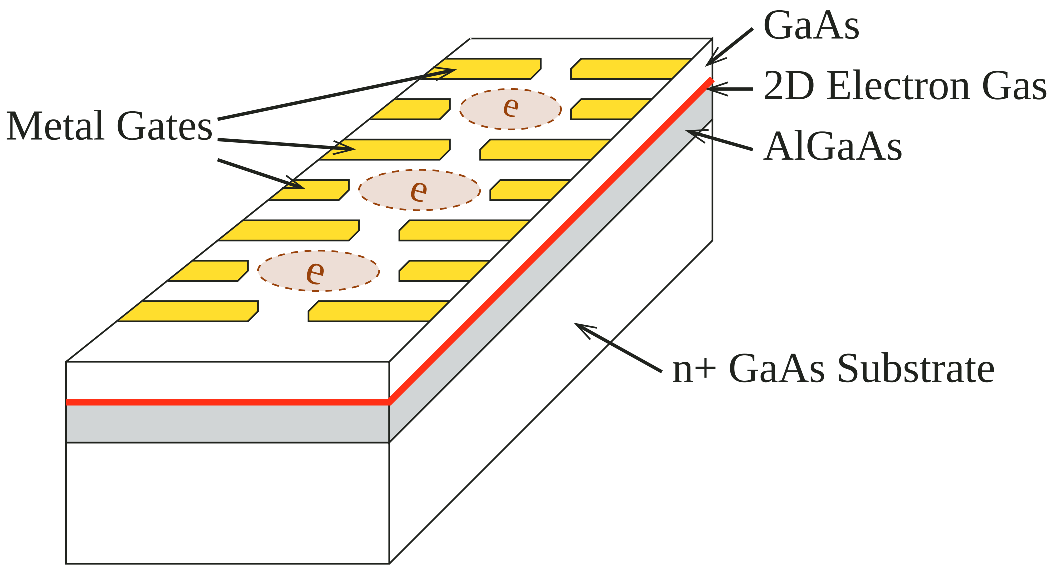
GREECE

Outline

- Design and Multidot Experiments
- Mathematical Formalism
  - Mott-Hubbard Hamiltonian
  - Parameters and Approximations
  - Quantum Monte-Carlo Stochastic Wavefunctions
- One Excess (mobile) Electron
- Two Excess Electrons, Bonding and Collisions
- Quantum Entanglement with Heisenberg Exchange Interaction
- Quantum Channel with QD Array
- Summary

Design of Quantum Dot Array

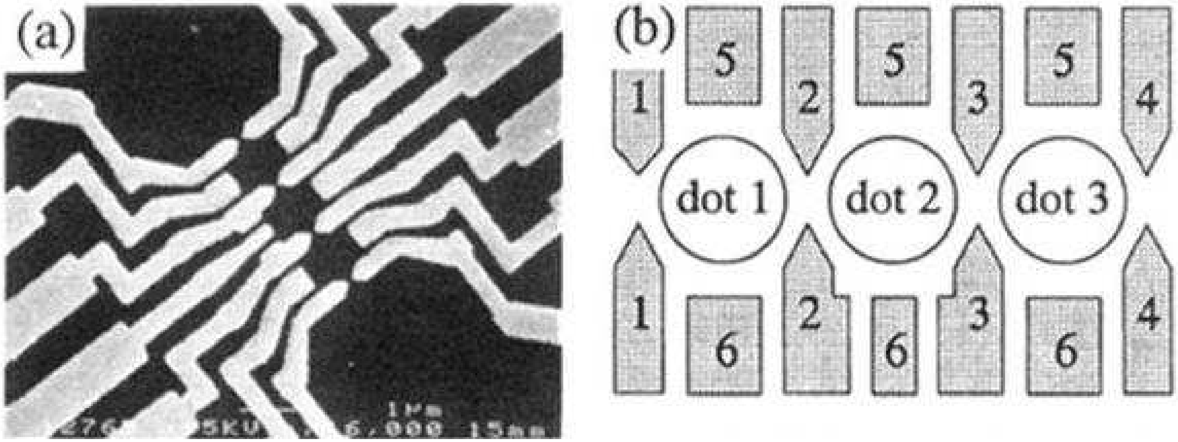
Schematic drawing of the Heterostructure

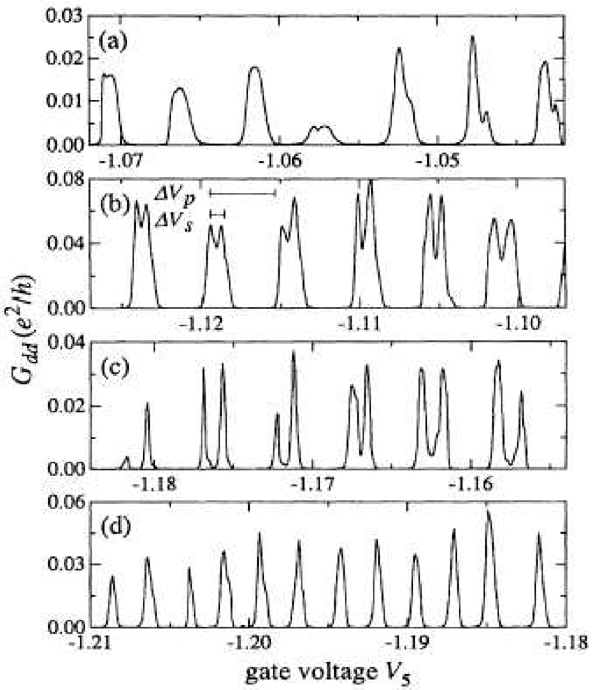


- ~ 5 nm thick AlGaAs layer is sandwiched between two GaAs layers $\Rightarrow$ 2DEG is formed at the top GaAs-AlGaAs interface
- Array of metallic gates on top of the structure (with externally controlled voltages) restricts the movement of electrons and forms a chain of 0D QDs.
- Resonant tunneling of electrons between the QDs mediates their coherent propagation in 1D

Multidot Experiments: Static Properties

SEM photograph and schematic view of a chain of 3 QDs





Conductance vs. gate voltage V_5
for double−dot system (QDs 2 and 3)

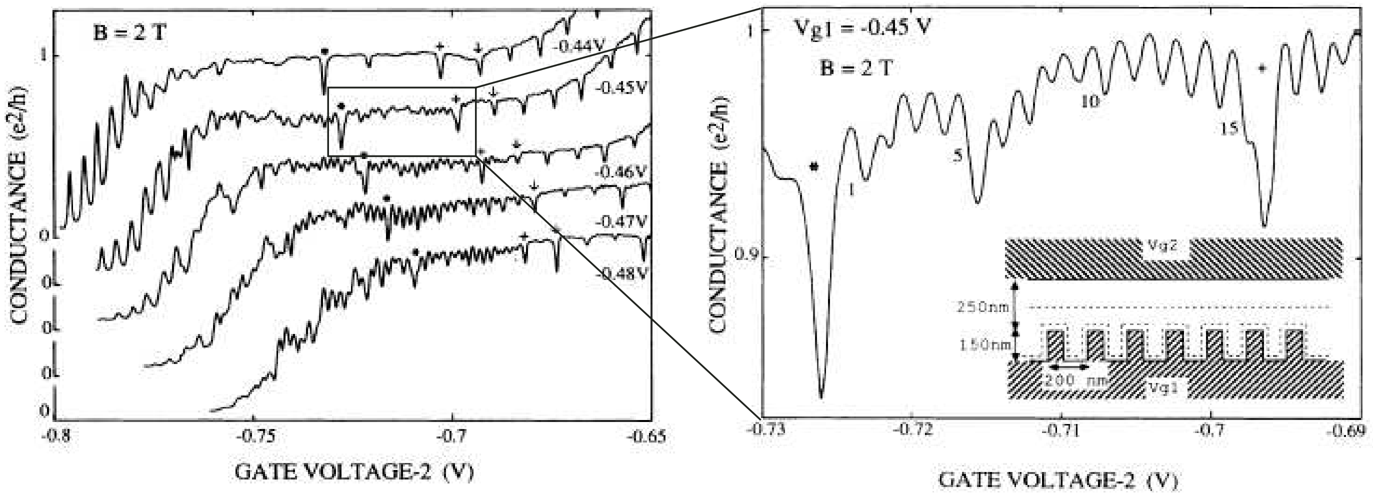
t_{23} = 0.03 (a)
= 0.88 (b)
= 1.37 (c)
= 1.94 (d)

X e^2/h

F. Waugh *et al.*, PRL **75**, 705 (1995)

Multidot Experiments: Static Properties

Conductance vs. gate voltage V_{g2} for a chain of 15 QDs



L. Kouwenhoven *et al.*, PRL **65**, 361 (1990)

Mott-Hubbard (SQ) Hamiltonian

$$H = \sum_{j,\alpha} \varepsilon_{j\alpha} a^\dagger_{j\alpha} a_{j\alpha} + \frac{1}{2}\sum_j U n_j(n_j - 1) + \sum_{j=i\pm 1,\alpha} t_{ij,\alpha} a^\dagger_{i\alpha} a_{j\alpha} + \sum_{i<j} V_{ij} n_i n_j$$

- $a^\dagger_{j\alpha}$ ($a_{j\alpha}$) creation (annihilation) operator for electron in state $|\alpha\rangle$ with energy $\varepsilon_{j\alpha}$ and electronic orbital $\psi_j(\mathbf{r})$
- $n_j = \sum_\alpha a^\dagger_{j\alpha} a_{j\alpha}$ electron number operator
- $U = \frac{e^2}{8\pi\epsilon_r\epsilon_0} \int \mathrm{d}\mathbf{r}\, \mathrm{d}\mathbf{r}' \, \frac{|\psi_j(\mathbf{r})|^2 |\psi_j(\mathbf{r}')|^2}{|\mathbf{r}-\mathbf{r}'|} \simeq \frac{e^2}{C_g}$ On-site Coulomb repulsion
  $C_g \simeq 8\epsilon_r\epsilon_0 R$ self-capacitance for 2D disk-shaped QD ($\epsilon_r \simeq 13$ for GaAs)
- $t_{ij} = \frac{\hbar^2}{2m^*} \int \mathrm{d}\mathbf{r}\, \psi^*_i(\mathbf{r}) \nabla^2 \psi_j(\mathbf{r})$ Interdot tunneling rate
  m^* electron effective mass ($m^* \simeq 0.067 m_e$ in GaAs)
- $V_{ij} \simeq U \left(\frac{C}{C_g}\right)^{|i-j|}$ Interdot Coulomb repulsion partially screened by image charges (interdot capacitance $C \ll C_g$)

Mott-Hubbard (SQ) Hamiltonian

Assumptions

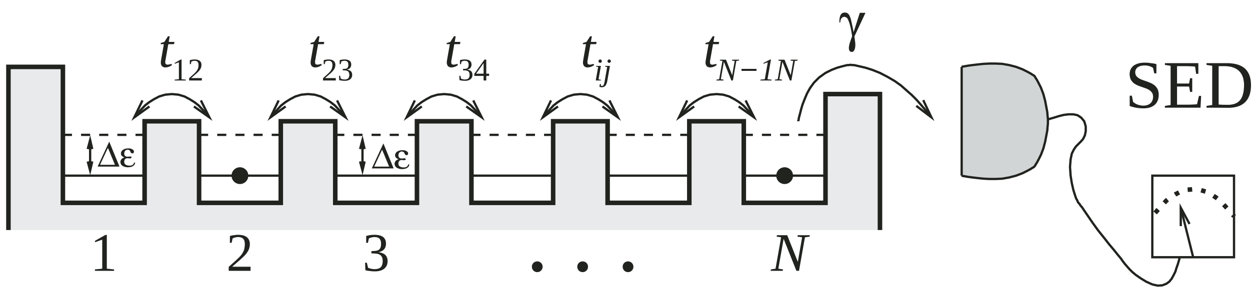
- Near-neighbor tunnel t_{ij} and electrostatic $V_{ij} = V$ interactions
  $\Rightarrow t_{ij}, V_{ij} \neq 0$, for $i = j \pm 1$
- Single-particle level spacing $\Delta\varepsilon > t_{ij}$ ($\Delta\varepsilon \simeq \frac{\hbar^2 \pi}{m^* R^2}$ in 2D potential)
  $\Rightarrow$ one double- (spin-) degenerate level per QD $(\alpha \in \{\uparrow, \downarrow\})$
- Coulomb Blockade regime $U \gg \Delta\varepsilon > t_{ij}$
  $\Rightarrow$ at most one electron per QD

Typical experimental parameters

for $30 - 50$ nm size GaAs/AlGaAs QDs, separated by ~ 100 nm

- Tunneling rates $t_{ij} \sim 0.05$ meV
- Single-particle level spacing $\Delta\varepsilon \sim 1.0$ meV
- On-site Coulomb repulsion $U \sim 15$ meV
- Thermal energy at $T \sim 2 - 10$ mK is $k_{\mathrm{B}}T \sim 0.2 - 1$ μeV

1D Array of N Quantum Dots



- The array is initially doped with $n = 1, 2...$ ($n \ll N$) electrons
  - Lower tunnel barriers and raise the confining potentials
    $\Rightarrow$ deplete the array
  - Lower confining potentials and open and close the tunnel barriers
    $\Rightarrow$ dope preselected QDs with single electrons
- Nth QD is dissipatively coupled to a SED with $\gamma \ll t_{ij}$
  $\Rightarrow$ detector monitors the evolution

Quantum Monte Carlo Simulations

- Disorder due to
  - Structure imperfections & Gate voltage fluctuations
  - Electron-phonon interactions & Thermal fluctuations

  $\Rightarrow$ ε_{ij}, t_{ij}—Gaussian random numbers with mean ε_0, t_0 & FWHM $\delta\varepsilon = 0.1t_0$($\sim 5$ μeV), $\delta t = 0.05t_0$(~ 2.5 μeV)
- Detector signal with $\gamma = 0.2t_0$(~ 2.4 GHz)
  - Generate random r ($0 \leq r < 1$)
  - Propagate $|\Psi(\tau)\rangle$ with $H_{\text{eff}} = H - \frac{i}{2}\gamma n_N$ until $||\Psi(\tau)||^2 = r$

    $\Rightarrow$ Quantum Jump $|\Psi\rangle \rightarrow \sum_\alpha \frac{a_{N\alpha}|\Psi\rangle}{\sqrt{\langle\Psi| a^\dagger_{N\alpha} a_{N\alpha} |\Psi\rangle}}$
  - Continue (with new r) until $|\Psi(\tau)\rangle = 0$

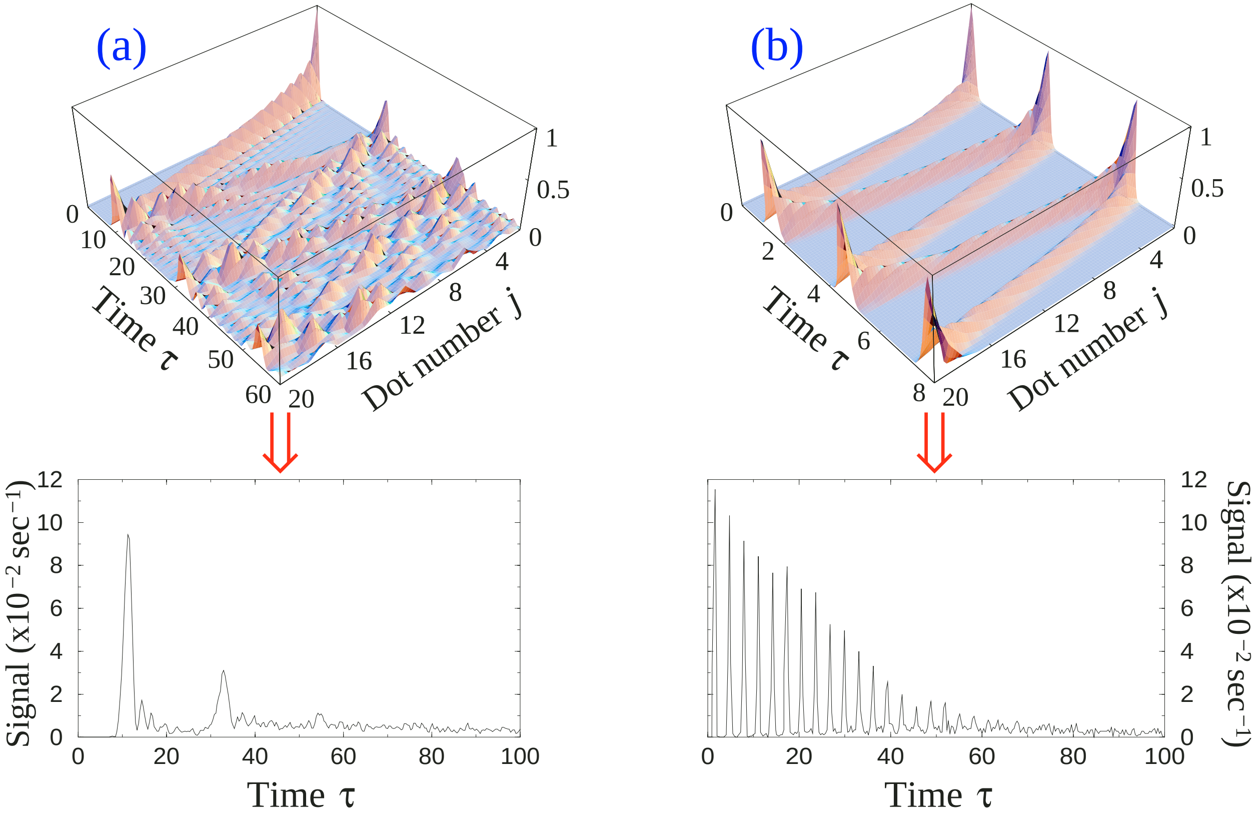
Average over many (5000) trajectories

$\Downarrow$

$$\Gamma_{\text{coherence}} \simeq \delta\varepsilon + \delta t \quad \Gamma_{\text{population}} \simeq \gamma/N$$

One Mobile Electron

$$|\psi_1(0)\rangle = |1_\alpha\rangle$$



(a) Equal Tunneling Rates $t_{jj+1} = t_0$ $(\pm\delta t)$ (Equal Coupling – EC)

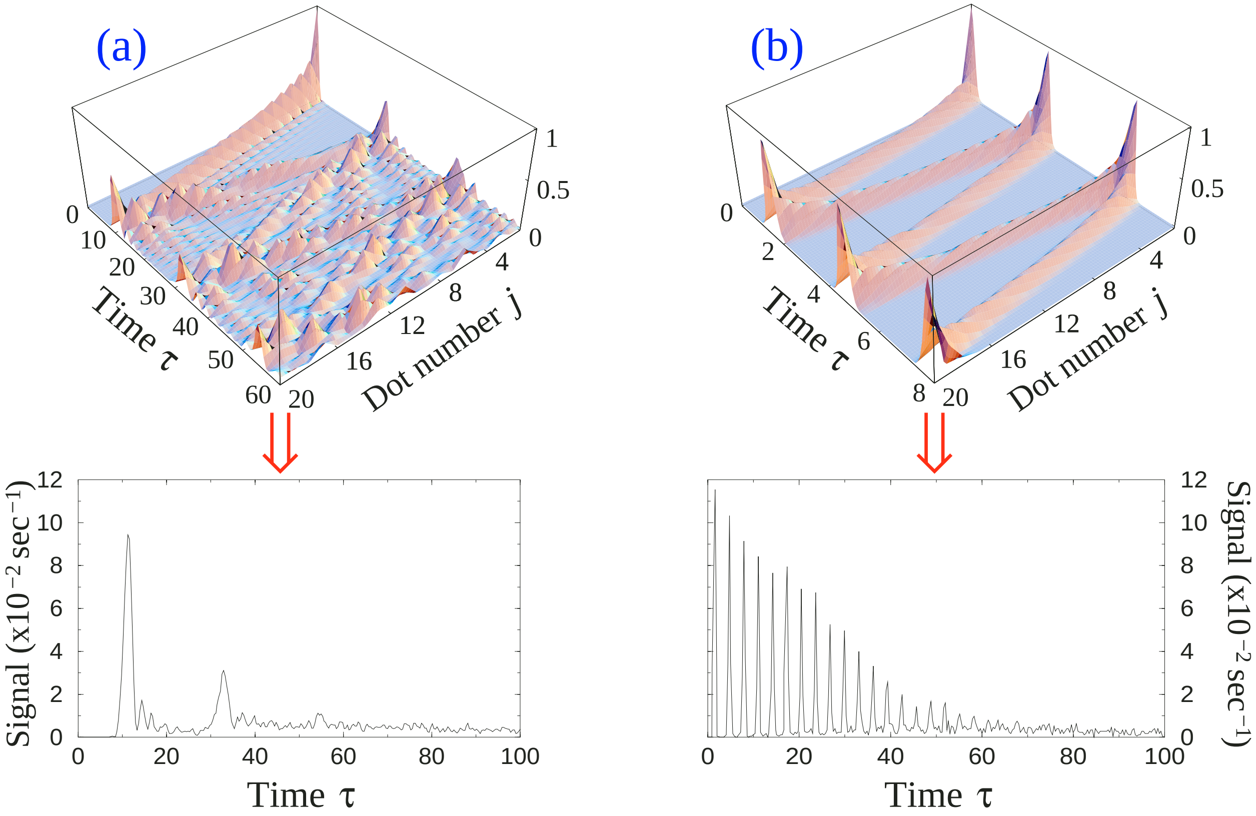
$\Rightarrow$ Incommensurate eigenenergies $\lambda_k = 2t_0 \cos\left(\frac{k\pi}{N+1}\right)$

Amplitudes $A_j^\alpha = \frac{2}{N+2} \sum_{k=1}^{N} \exp\left[-i2t_0\tau \cos\left(\frac{k\pi}{N+1}\right)\right] \sin\left(\frac{jk\pi}{N+1}\right) \sin\left(\frac{k\pi}{N+1}\right)$

WP spreading and delocalization – dispersion

One Mobile Electron

$$|\psi_1(0)\rangle = |1_\alpha\rangle$$



(b) Tunneling Rates $t_{jj+1} = t_0\sqrt{(N-j)j}$ $(\pm\delta t)$ (Optimal or SM Coupling – OC)

$\Rightarrow$ Commensurate eigenenergies $\lambda_k = t_0(2k - N - 1)$

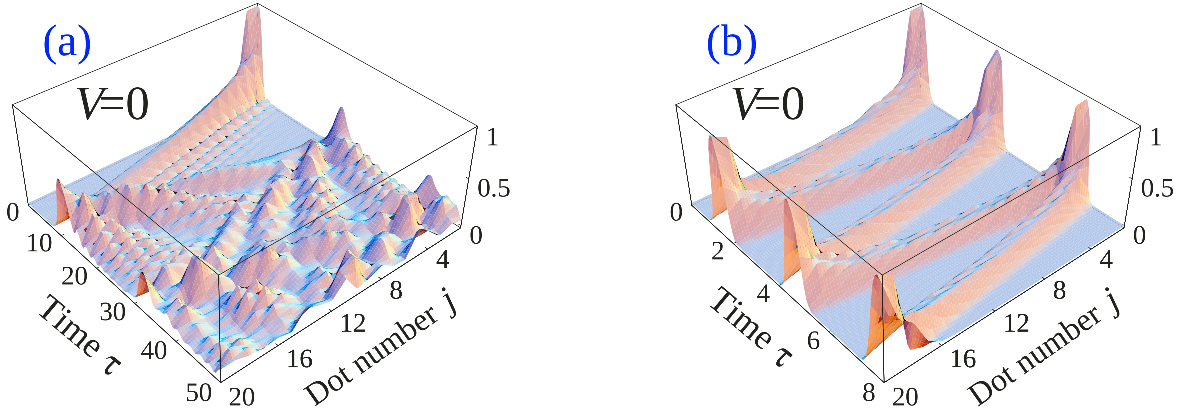
Amplitudes $A_j^\alpha = \begin{pmatrix} N-1 \\ j-1 \end{pmatrix}^{1/2} [-i\sin(t_0\tau)]^{(j-1)} \cos(t_0\tau)^{(N-j)}$

$|A_1^\alpha|^2 = \cos(t_0\tau)^{2(N-1)}$ $|A_N^\alpha|^2 = \sin(t_0\tau)^{2(N-1)}$: Revivals at $t_0\tau = \frac{m\pi}{2}$

Perfectly periodic behavior

Two Mobile Electrons

$$|\psi_2(0)\rangle = |1_\alpha, 2_\beta\rangle$$

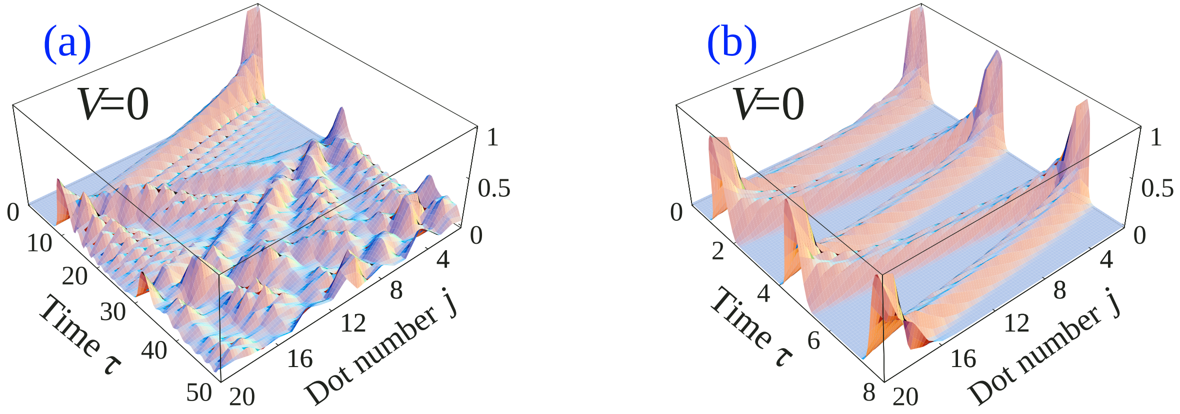


(a) EC $t_{jj+1} = t_0$ & $V = 0$ (No Repulsion)

WP dispersion as in 1e case

Two Mobile Electrons

$$|\psi_2(0)\rangle = |1_\alpha, 2_\beta\rangle$$



(b) OC $t_{jj+1} = t_0\sqrt{(N-j)j}$ & $V = 0$ (No Repulsion)

$\Rightarrow$ Commensurate ($2N-3$ distinct) eigenenergies $\lambda_k = t_0(2k - N + 2)$

Amplitudes for states $|i_\alpha, j_\beta\rangle$

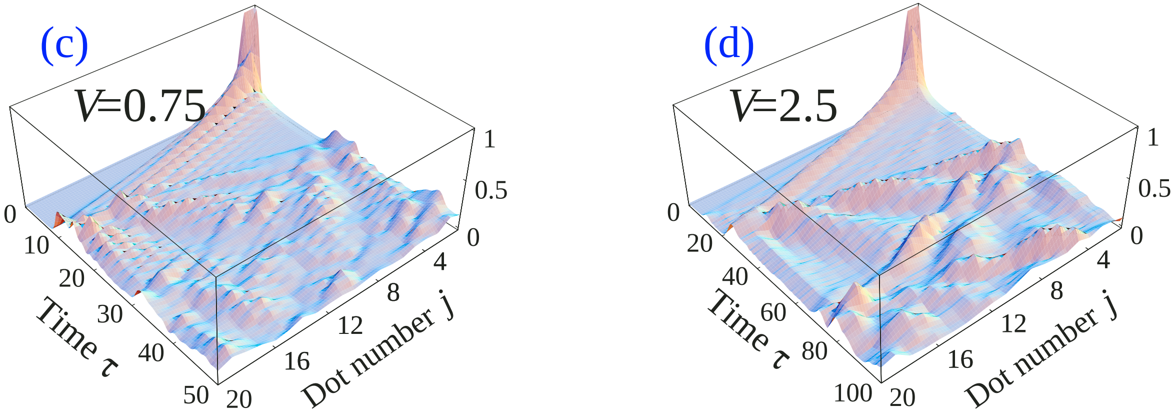
$$B_{ij}^{\alpha\beta} = \left[\frac{(j-i)^2(N-1)!(N-2)!}{(i-1)!(j-1)!(N-i)!(N-j)!}\right]^{1/2}[-i\sin(t_0\tau)]^{i+j-3}\cos(t_0\tau)^{2N-i-j-1}$$

$|B_{12}^{\alpha\beta}|^2 = \cos(t_0\tau)^{4N-8}$ $\quad |B_{N-1N}^{\alpha\beta}|^2 = \sin(t_0\tau)^{4N-8}$: Revivals at $t_0\tau = \frac{m\pi}{2}$

Perfectly periodic behavior

Two Mobile Electrons: Bonding

$$|\psi_2(0)\rangle = |1_\alpha, 2_\beta\rangle$$

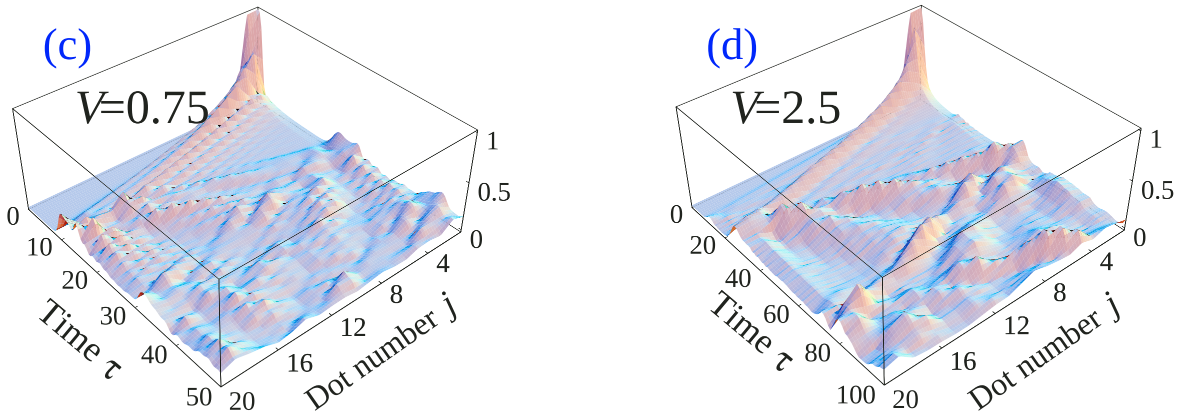


(c) EC $t_{jj+1} = t_0$ & $0 < V \leq t_0$ (Weak Repulsion)

Enhanced dispersion of WP due to inhomogeneity

Two Mobile Electrons: Bonding

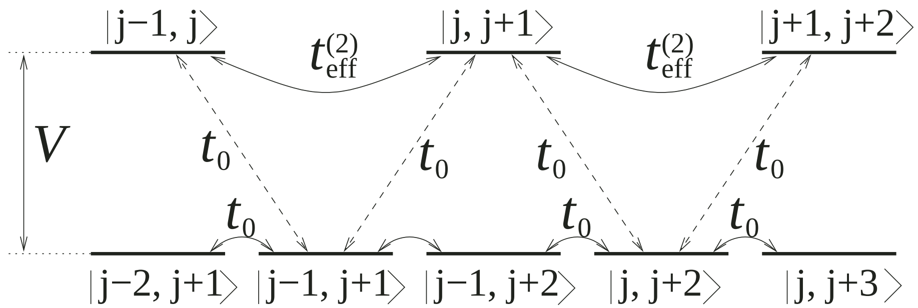
$$|\psi_2(0)\rangle = |1_\alpha, 2_\beta\rangle$$



(d) EC $t_{jj+1} = t_0$ & $V > t_0$ (Strong Repulsion)

Energy of $|j, j \pm 1\rangle$ is larger

than of $|j, j \pm 2\rangle$ etc., by $V > t_0$



⇒ transitions $|j, j \pm 1\rangle \rightarrow |j, j \pm 2\rangle$ are non-resonant

Effective tunneling rate for $|j, j+1\rangle \rightarrow |j+1, j+2\rangle$ is $t_{\rm eff}^{(2)} = \frac{t_0^2}{V} < t_0$

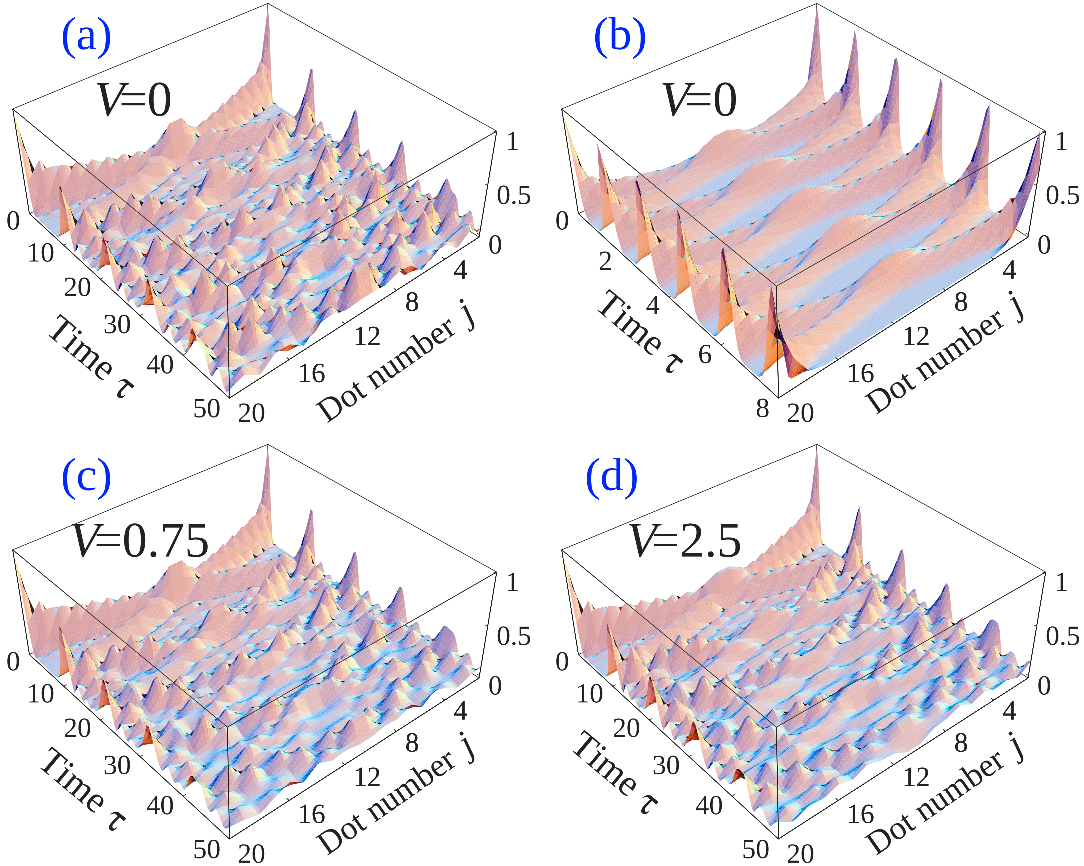
⇒ slow propagation

Two-electron bonding via Coulomb repulsion

($V > 0 \Rightarrow$ 2e bound state is unstable)

Two Mobile Electrons: Collisions

$$|\psi_2(0)\rangle = |1_\alpha, N_\beta\rangle$$



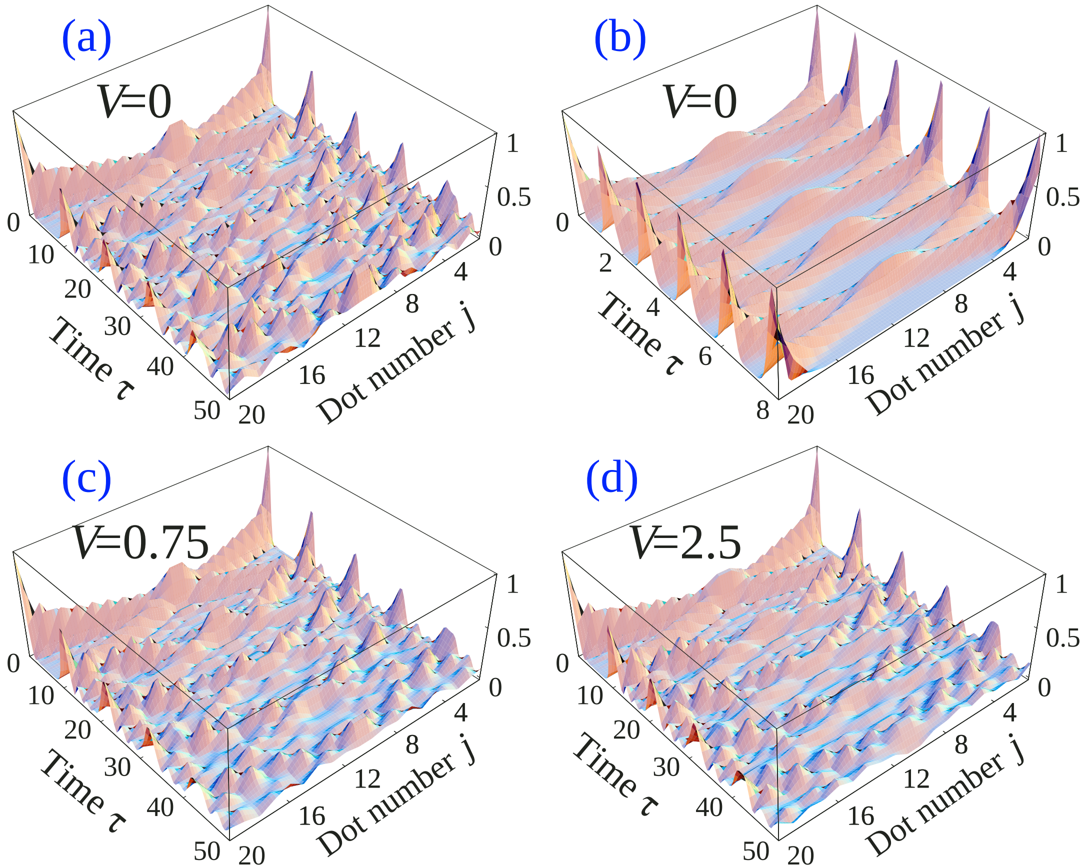
(a),(c),(d) Equal Coupling $t_{jj+1} = t_0$:

Electrons collide in the center ⇒ Each electron has $\frac{N}{2}$ accessible QDs

WP dispersion as in 1e case

Two Mobile Electrons: Collisions

$$|\psi_2(0)\rangle = |1_\alpha, N_\beta\rangle$$

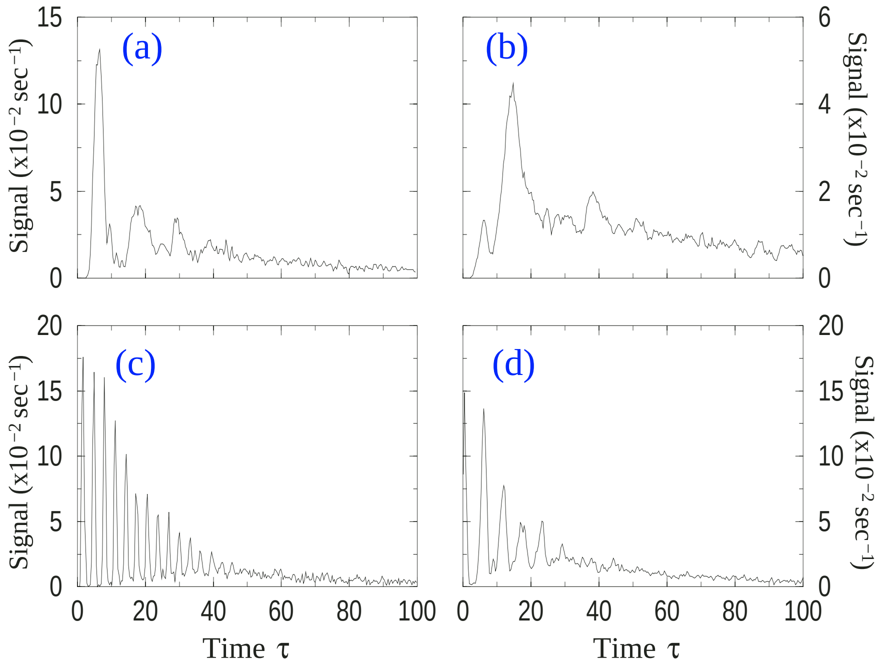


(b) Optimal Coupling $t_{jj+1} = t_0\sqrt{(N-j)j}$:

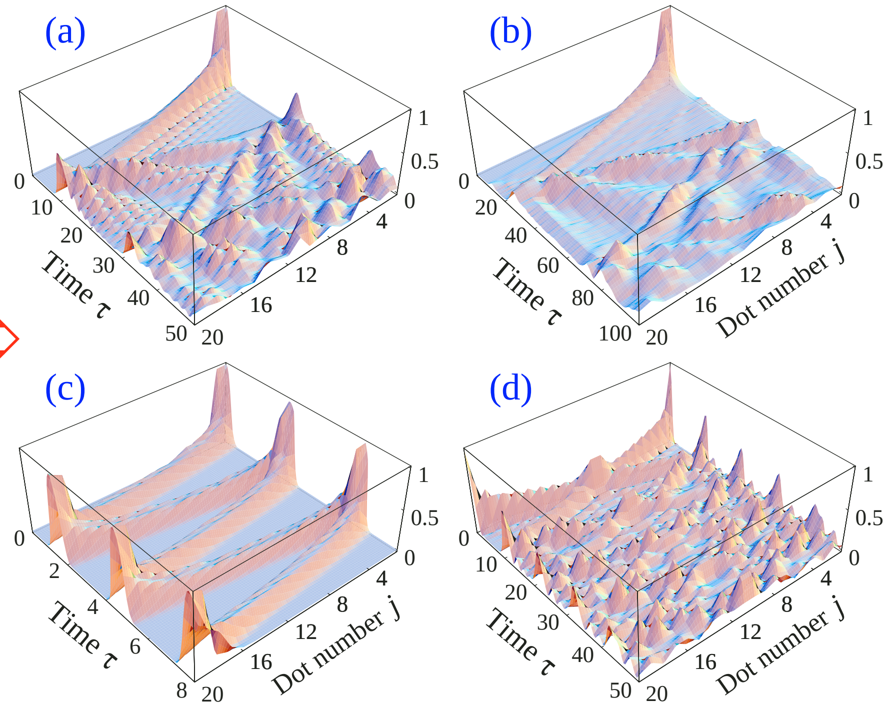
Collisions & revivals at $t_0\tau = \frac{m\pi}{4}$ $\Leftarrow$ Each electron has $\frac{N}{2}$ accessible QDs

Perfectly periodic behavior

Two Mobile Electrons: Monte-Carlo Smls.



⟺

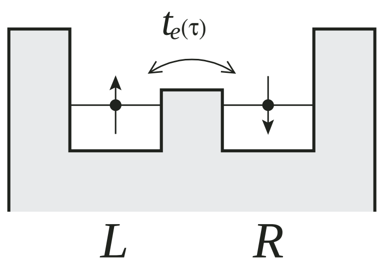


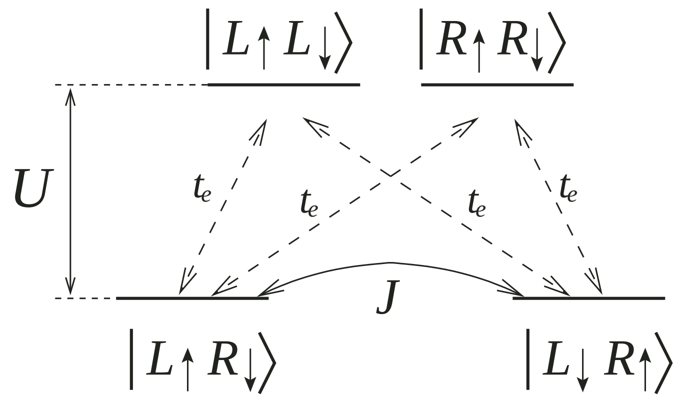
Decoherence & Decay

$$\Gamma_{\mathrm{coherence}} \simeq \delta\varepsilon + \delta t, \quad \Gamma_{\mathrm{population}} \simeq \gamma/N$$

Interdot Heisenberg Exchange Interaction

$$|\phi\rangle = |L_\uparrow R_\downarrow\rangle$$





$t_e \ll U$ – Adiabatic elimination of nonresonant (virtual) $|L_\uparrow L_\downarrow\rangle$, $|R_\uparrow R_\downarrow\rangle$
$\Rightarrow$ effective Heisenberg exchange interaction $|L_\uparrow R_\downarrow\rangle \leftrightarrow |L_\downarrow R_\uparrow\rangle$

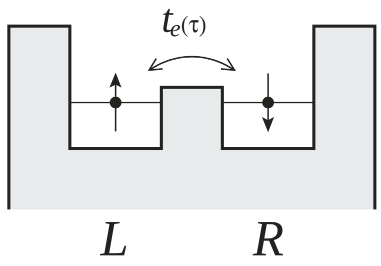
$$H_s(\tau) = J(\tau)\vec{S}_L \cdot \vec{S}_R, \quad J(\tau) = -\frac{4t_e^2(\tau)}{U}$$

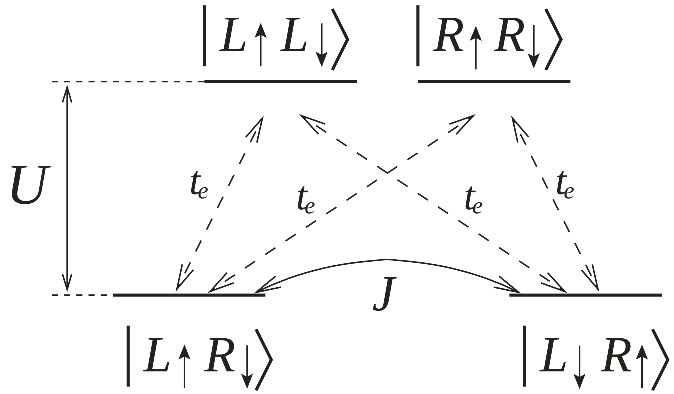
- For $\theta \equiv \int J(\tau)d\tau = \pi$

  $\Rightarrow$ SWAP $|L_\alpha R_\beta\rangle \to i\,|L_\beta R_\alpha\rangle$ $(\alpha, \beta \in \{\uparrow, \downarrow\})$

Interdot Heisenberg Exchange Interaction

$$|\phi\rangle = |L_\uparrow R_\downarrow\rangle$$





$t_e \ll U$ – Adiabatic elimination of nonresonant (virtual) $|L_\uparrow L_\downarrow\rangle$, $|R_\uparrow R_\downarrow\rangle$
$\Rightarrow$ effective Heisenberg exchange interaction $|L_\uparrow R_\downarrow\rangle \leftrightarrow |L_\downarrow R_\uparrow\rangle$

$$H_s(\tau) = J(\tau)\vec{S}_L \cdot \vec{S}_R, \quad J(\tau) = -\frac{4t_e^2(\tau)}{U}$$

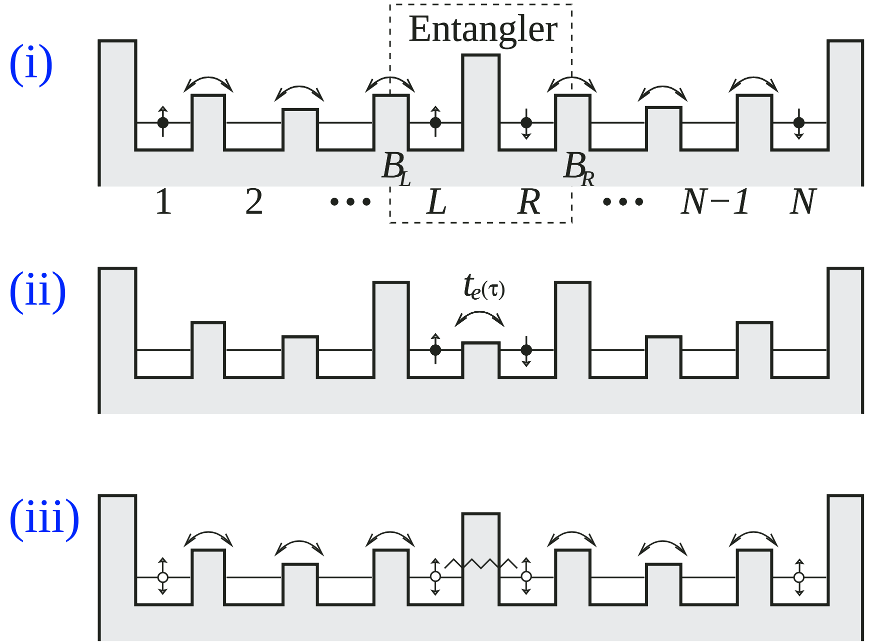
- For $\theta \equiv \int J(\tau)d\tau = \pi/2$

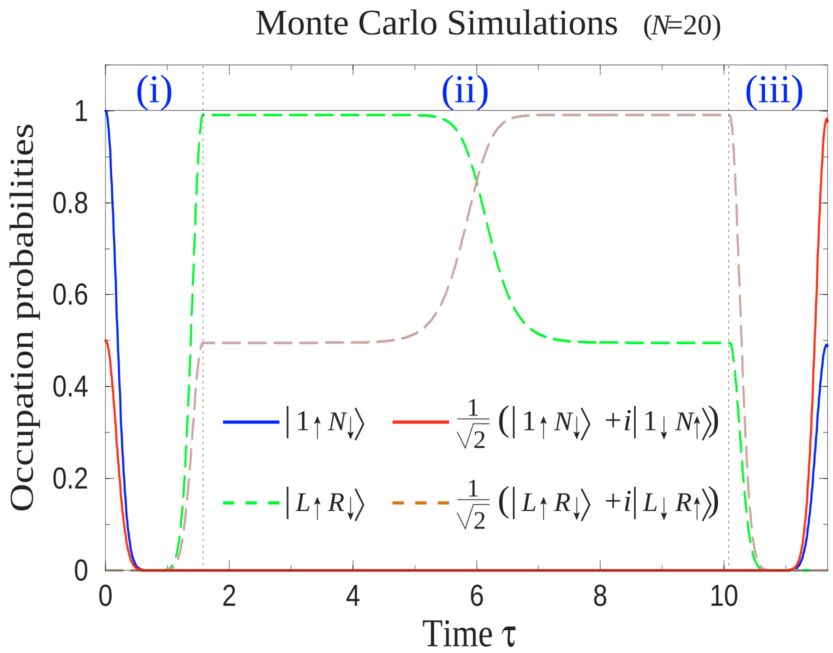
  $\Rightarrow \sqrt{\text{SWAP}}\ |\phi\rangle = |L_\uparrow R_\downarrow\rangle \rightarrow \frac{1}{\sqrt{2}}(|L_\uparrow R_\downarrow\rangle + i|L_\downarrow R_\uparrow\rangle)$

  For $\theta = \pi/2$ with $t_e(\tau) = t_e^{\max}\text{sech}[(\tau - \tau^{\max})/\Delta\tau] \Rightarrow (t_e^{\max})^2\Delta\tau = \pi U/16$

Loss, DiVincenzo (1998)

Quantum Channel with QD Array





(i) Coherent transport via OC & trapping $|1_\uparrow N_\downarrow\rangle \rightarrow |L_\uparrow R_\downarrow\rangle$

(ii) $\sqrt{\text{SWAP}}$ via Exchange interaction $|L_\uparrow R_\downarrow\rangle \rightarrow \frac{1}{\sqrt{2}}(|L_\uparrow R_\downarrow\rangle + i\,|L_\downarrow R_\uparrow\rangle)$

(iii) Reverse of (i) $\frac{1}{\sqrt{2}}(|L_\uparrow R_\downarrow\rangle + i\,|L_\downarrow R_\uparrow\rangle) \rightarrow \frac{1}{\sqrt{2}}(|1_\uparrow N_\downarrow\rangle + i\,|1_\downarrow N_\uparrow\rangle)$

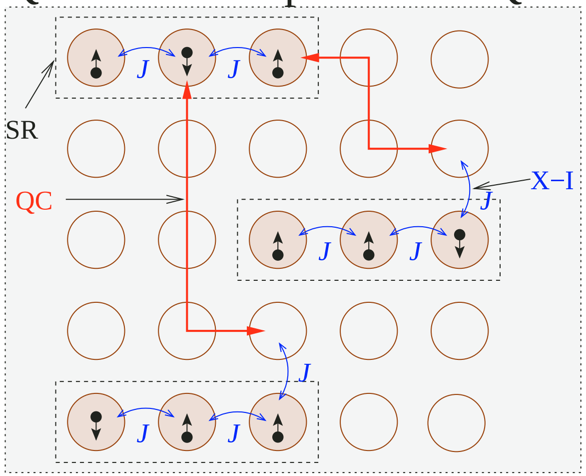
- Monte Carlo simulations $\Rightarrow$ Fidelity $\boxed{F = 0.98}$

  $N = 20$, $L = N/2$ with $U = 100t_0$, $t_e^{\max} = 6t_0$,
  Disorder prms. $\delta\varepsilon = 0.1t_0$, $\delta t = 0.05t_0$

Summary & Conclusions

- By manipulating the absolute values and relative magnitudes of tunneling rates between QDs in a 1D array, it is possible to
  - accelerate/decelerate electron wavepacket propagation dynamics
  - enhance/suppress wavepacket spreading and interference
- By manipulating the interdot Coulomb repulsion, it is possible to
  - form bonded multi-electron states
  - control electron collisions
- Possible applications for Quantum Computation and Information
  - Entanglement of 2 qubits (represented by spin states of QD electrons) via controlled spin-exchange collisions
  - Quantum Communication & Information Transport via Quantum Channel

Quantum Computer with QDs



Analogies with Other Systems

- Spin-wave propagation in spin chains
- EM wave propagation in periodic structures
(PBG materials & Waveguide lattices)
- Matter-wave (BEC) propagation in optical lattices