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SMR.1587 - 11

SCHOOL AND WORKSHOP ON
QUANTUM ENTANGLEMENT, DECOHERENCE, INFORMATION, AND GEOMETRICAL PHASES IN COMPLEX SYSTEMS
(1 November - 12 November 2004)

Collision and entanglement in cold condensed matter

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Collisions and entanglement in cold condensed matter

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Plan of the talk

- Review of the theory of weakly non-ideal Bose-condensed gases
- Quasi-1D BECs with laser-induced dipole-dipole interactions
- Off-resonant Raman and Bragg light scattering in BECs

Review of the theory of weakly non-ideal Bose-condensed gases

Review papers:

- L.P. Pitaevskii, Phys.-Uspekhi, **41**, 569 (1998)
- F. Dalfovo, S. Giorgini, L.P. Pitaevskii, S. Stringari, Rev. Mod. Phys. **71**, 463 (1999)
- A.J. Leggett, Rev. Mod. Phys. **73**, 307 (2001)
- G. Kurizki, I.E. Mazets, D.H.J. O'Dell, W.P. Schleich, Int. J. Mod. Phys. B, **18**, 961 (2004)

Bose-condensed ideal gas

$$n_k = \{\exp[\,-(\epsilon_k - \mu)/T\,] - 1\,\}^{-1}$$

$$T < T_{crit} = 3.31\,\frac{\hbar^2}{m}\left(\frac{N}{V}\right)^{2/3}$$

$$n_{k=0} = N\left[1 - (T/T_{crit})^{3/2}\right]$$

For an ideal Bose-gas at $T < T_{crit}$: $\mu = 0$.

Weakly non-ideal Bose gas

r_0 – van der Waals interaction range; $n = N/V$ – number density

$$r_0 \ll n^{-1/3}$$

Recall scattering theory:

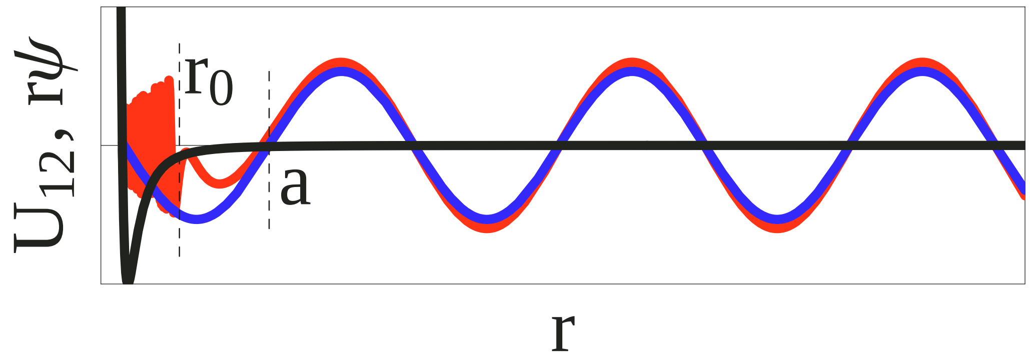
$$\psi = \sum_{l=0}^{\infty} \frac{1}{2k}(2l+1)i^l e^{i\delta_l} P_l(\cos\theta) R_{kl}(r) \approx e^{ikz} + \frac{f(\theta)}{r} e^{ikr},$$

$r \gg r_0$; $\qquad R_{kl} \approx \frac{2}{r} \sin\left(kr - l\pi/2 + \delta_l\right)$

$\delta_l \sim (kr_0)^{2l+1}$ for $kr_0 \lesssim 1$. Bose statistics: $l = 0, 2, 4, 6, \ldots$

Ultracold atoms ($T < 1$ mK): $kr_0 \ll 1 \Rightarrow s$-wave scattering only.

Cold atomic collisions



$$-\frac{\hbar^2}{2m_{12}}\frac{\partial^2}{\partial r^2}r\psi(r)+U_{12}(r)r\psi(r)=\frac{\hbar^2k^2}{2m_{12}}r\psi(r)$$

$m_{12}=m_1m_2/(m_1+m_2)$. Identical atoms: $m_{12}=m/2$

s-wave scattering length: $a=-\lim_{k\to 0}\frac{\delta_0}{k}$

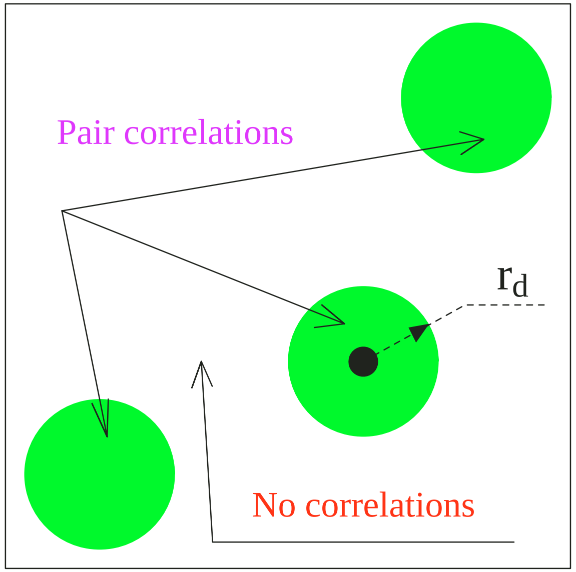
$\psi(r)\approx\frac{const}{r}\sin[k(r-a)]\Rightarrow$ Taking $U_{12}(r)$ into account can be substituted by the boundary condition: $\frac{1}{r\psi}\frac{\partial(r\psi)}{\partial r}\Big|_{r\to 0}=-\frac{1}{a}$

Interactions in weakly non-ideal BEC

Field theory: S.T. Beliaev, Sov. Phys. JETP **34(7)**, 299 (1958).

LOCV approach: S. Cowell et al., PRL **88**, 210403 (2001).

$$E_0 = \frac{N(N-1)}{2} E_{pair} \approx \frac{N^2}{2} E_{pair}$$



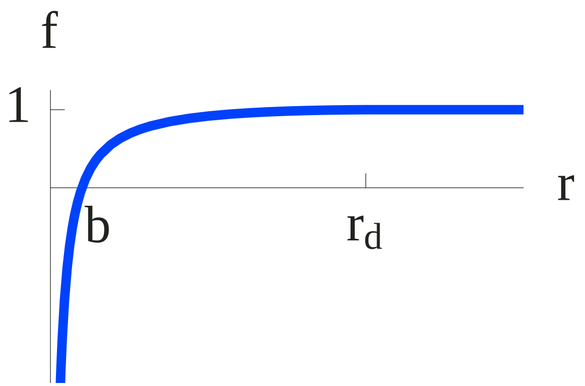
$n^{-1/3} \gg a \gtrsim r_0 \Rightarrow$ Jastrow wave function

$\Psi = \prod_i \psi(\mathbf{r}_i) \prod_{j>i} f(\mathbf{r}_j - \mathbf{r}_i)$, $\psi(\mathbf{R}) = \frac{1}{\sqrt{V}}$

$f(r) \equiv 1$ for $r > r_d \sim n^{-1/3}$ (loss of correlation) $\partial f/(\partial r)|_{r=r_d} = 0$

$(rf)^{-1}\partial(rf)/(\partial r) = -1/a$

$-\frac{\hbar^2}{m}\frac{\partial^2}{\partial r^2} rf(r) = \frac{\hbar^2 k_d^2}{m} rf(r)$



Solution for $r < r_d$:

$$f(r) = \frac{r_d}{r} \frac{\sin[\,k_d(r-b)\,]}{\sin[\,k_d(r_d-b)\,]}$$

$$b \approx a, \qquad k_d^2 \approx 3ar_d^{-3}$$

$$E_{pair} = \frac{\int d^3\mathbf{r}\, f\left(-\frac{\hbar^2}{m}\nabla^2\right) f}{\int d^3\mathbf{r}\, f^2} = \frac{4\pi\int_0^{r_d} dr\, r^2 f^2 \hbar^2 k_d^2/m}{V} \approx \frac{4\pi\hbar^2 a}{mV}$$

Ground-state energy of BEC: $E_0 = \frac{2\pi\hbar^2 a N^2}{mV}$

Chemical potential: $\mu = (\partial E_0/\partial N)_V = \frac{4\pi\hbar^2 a n}{m}, \quad n \equiv \frac{N}{V}$

Interactions are reduced to pseudopotential $\frac{4\pi\hbar^2 a}{m}\delta(\mathbf{r}) \equiv g\delta(\mathbf{r})$

Secondary-quantized Hamiltonian

$$\hat{H} = \int d^3\mathbf{r} \left\{ \hat{\Psi}^\dagger(\mathbf{r}) \left[-\frac{\hbar^2}{2m}\nabla^2 + U_{trap}(\mathbf{r}) \right] \hat{\Psi}(\mathbf{r}) + \frac{g}{2} \int d^3\mathbf{r}' \, \hat{\Psi}^\dagger(\mathbf{r}')\hat{\Psi}^\dagger(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}')\hat{\Psi}(\mathbf{r})\hat{\Psi}(\mathbf{r}') \right\}$$

Homogeneous BEC ($U_{trap}(\mathbf{r}) = 0$) $\Rightarrow$ plane wave basis

$$\hat{\Psi}(\mathbf{r}) = \sum_{\mathbf{k}} \hat{a}_{\mathbf{k}} \frac{\exp(i\mathbf{k}\mathbf{r})}{\sqrt{V}}, \quad \sum_{\mathbf{k}} = V \int \frac{d^3\mathbf{k}}{(2\pi)^3}$$

$$\hat{H} = \sum_{\mathbf{k}} \frac{\hbar^2 k^2}{2m} \hat{a}^\dagger_{\mathbf{k}} \hat{a}_{\mathbf{k}} + \frac{g}{2V} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} \hat{a}^\dagger_{\mathbf{k}+\mathbf{q}} \hat{a}^\dagger_{\mathbf{k}'-\mathbf{q}} \hat{a}_{\mathbf{k}'} \hat{a}_{\mathbf{k}}$$

The Bogoliubov transformation

The number of condensed atoms $N_c = N - \sum_{\mathbf{k}\neq 0} \hat{a}^\dagger_{\mathbf{k}} \hat{a}_{\mathbf{k}} \approx N \gg 1$

$\Rightarrow$ We may regard $\hat{a}_0$ and $\hat{a}^\dagger_0$ as c-numbers $= \sqrt{N_c}$ $\Rightarrow$

$$\hat{H} = \frac{gN^2}{2V} + \sum_{\mathbf{k}\neq 0} \left[\frac{\hbar^2 k^2}{2m} \hat{a}^\dagger_{\mathbf{k}} \hat{a}_{\mathbf{k}} + \frac{gN}{2V} (\hat{a}_{\mathbf{k}} \hat{a}_{-\mathbf{k}} + \hat{a}^\dagger_{\mathbf{k}} \hat{a}^\dagger_{-\mathbf{k}} + 2\hat{a}^\dagger_{\mathbf{k}} \hat{a}_{\mathbf{k}}) \right]$$

Diagonalization of $\hat{H}$ by the Bogoliubov transformation

$$\hat{a}_{\mathbf{k}} = u_k \hat{b}_{\mathbf{k}} - v_k \hat{b}^\dagger_{-\mathbf{k}}, \quad \hat{a}^\dagger_{\mathbf{k}} = u_k \hat{b}^\dagger_{\mathbf{k}} - v_k \hat{b}_{-\mathbf{k}}$$

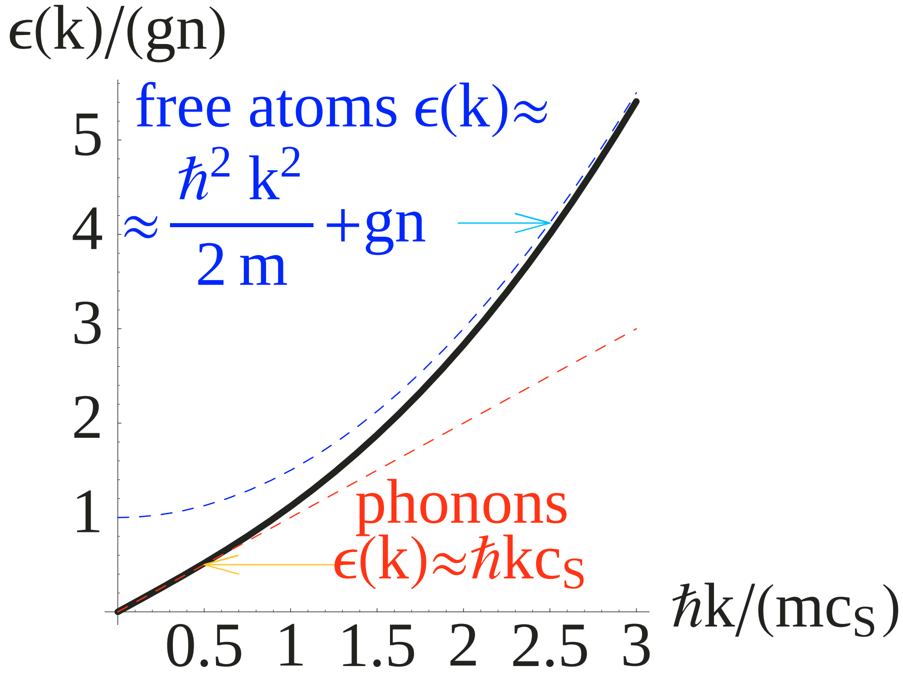
Unitarity ($[\hat{b}_{\mathbf{k}}, \hat{b}^\dagger_{\mathbf{k}'}] = \delta_{\mathbf{k}'\mathbf{k}}$, $[\hat{b}_{\mathbf{k}}, \hat{b}_{\mathbf{k}'}] = 0$) is ensured by:

$$u_k = \sqrt{\frac{\hbar^2 k^2/(2m) + gn}{2\epsilon(k)} + \frac{1}{2}}\,, \qquad v_k = -\sqrt{u_k^2 - 1}$$

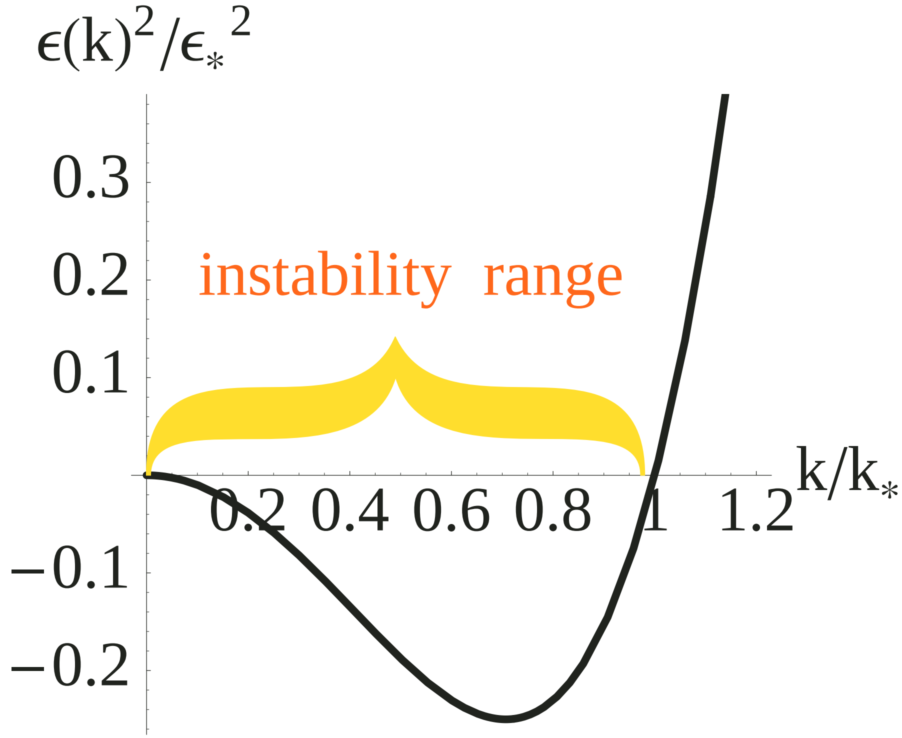
$$\epsilon(k) = \sqrt{\frac{\hbar^2 k^2}{2m}\left(\frac{\hbar^2 k^2}{2m} + 2gn\right)}\,, \quad \hat{H} = \tilde{E}_0 + \sum_{\mathbf{k}\neq 0} \epsilon(k) \hat{b}^\dagger_{\mathbf{k}} \hat{b}_{\mathbf{k}}$$

The Bogoliubov excitation spectrum

$a > 0$



$a < 0$



Speed of sound:

$$c_s = \sqrt{\frac{gn}{m}} = \frac{\hbar}{m}\sqrt{4\pi n a}$$

$$\epsilon_* = \hbar^2 k_*^2/(2m)$$

$$k_* = \sqrt{16\pi n|a|}$$

^{23}Na: $a \approx 3$ nm ^{87}Rb: $a \approx 5.3$ nm ^{85}Rb: $a \approx -200$ nm

Quantum depletion ($a > 0$)

"Old" vacuum: $\hat{a}_{\mathbf{k}}|0\rangle = 0$ "New" vacuum: $\hat{b}_{\mathbf{k}}|\tilde{0}\rangle = 0$ (for all $\mathbf{k}$)

Number of above-condensate *atoms* at $T = 0$:

$$\sum_{\mathbf{k}\neq 0} \langle\tilde{0}|\hat{a}^{\dagger}_{\mathbf{k}}\hat{a}_{\mathbf{k}}|\tilde{0}\rangle = \sum_{\mathbf{k}\neq 0} v_k^2 = \frac{8}{3\sqrt{\pi}} N\sqrt{na^3}$$

"Naive" calculation of correction to $\tilde{E}_0$ gives divergent $\tilde{E}_0$

$\Rightarrow$ Renormalization of the coupling constant [Beliaev (1958)]

Simplified approach: $$g :\to \tilde{g} = g\left(1 + \frac{g}{V}\sum_{\mathbf{k}\neq 0}\frac{m}{\hbar^2 k^2}\right)$$

$$\tilde{E}_0 = \frac{4\pi\hbar^2 an}{m} N\left(1 + \frac{128}{15\sqrt{\pi}}\sqrt{na^3}\right)$$ T.D. Lee, C.N. Yang, Phys. Rev. **105**, 1119 (1957)

liquid ^{4}He: $\sqrt{na^3} \sim 1$; ^{87}Rb at $n = 10^{14}$ cm^{-3}: $\sqrt{na^3} \sim 0.004$

Quasi-1D BECs with laser-induced dipole-dipole interactions

Long-Range Interactions

- Decrease as r^{-3} or slower
- Cannot be reduced to a pseudopotential $g\delta(\mathbf{r}-\mathbf{r}')$

$$\hat{H} :\longrightarrow \hat{H}_{new} = \hat{H} + \hat{H}_{lr}$$

$$\hat{H}_{lr} = \frac{1}{2}\int d^3\mathbf{r} \int d^3\mathbf{r}'\, \hat{\Psi}^\dagger(\mathbf{r}')\hat{\Psi}^\dagger(\mathbf{r})U_{lr}(\mathbf{r}-\mathbf{r}')\hat{\Psi}(\mathbf{r})\hat{\Psi}(\mathbf{r}')$$

Laser-induced dipole-dipole interactions (LIDDI)

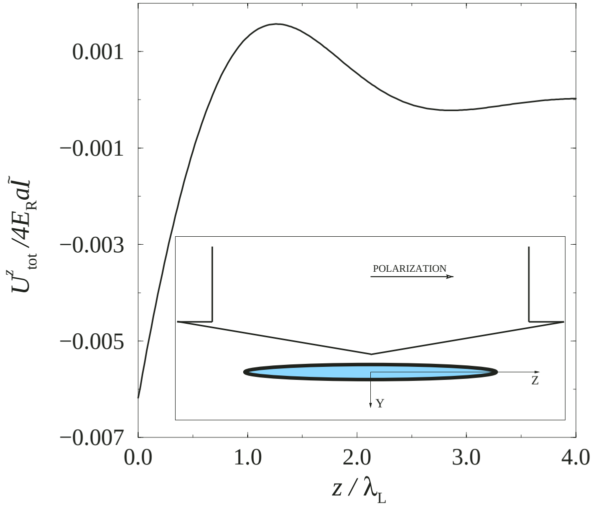
- Off-resonant laser radiation induces dipole moment on BEC atoms
- *Retarded* dipole-dipole interaction of atoms emerges

Optically-induced rotons

D.H.J. O'Dell, S. Giovanazzi, G. Kurizki, PRL **90**, 110402 (2003);
I.E. Mazets, D.H.J. O'Dell, G. Kurizki, N. Davidson, W.P. Schleich, J. Phys. B **37**,S155 (2004).

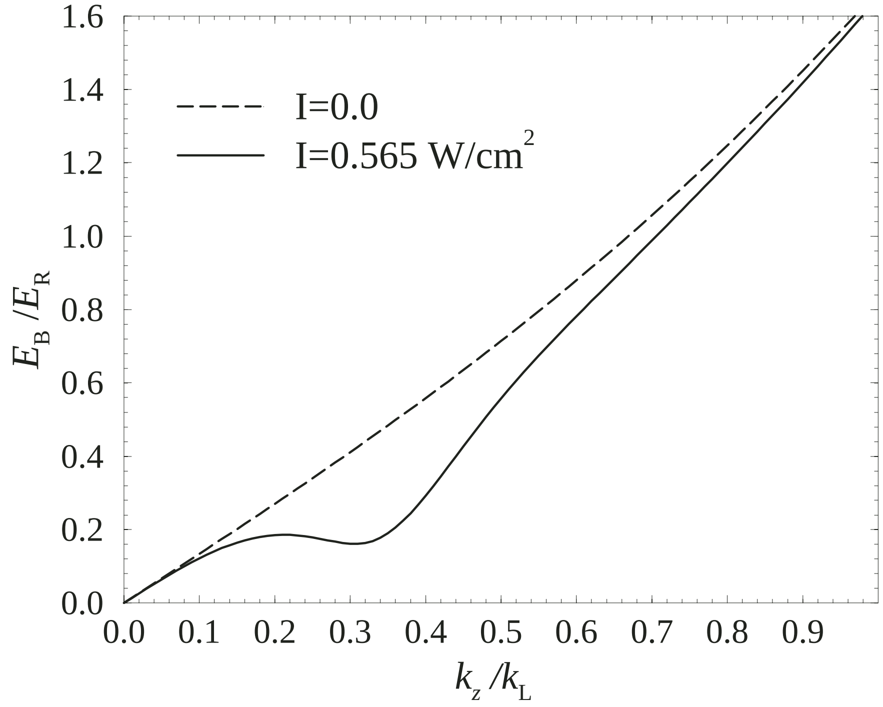
$$U_{\rm dd}(\mathbf{r}) = \frac{I\alpha^2(\omega)\,k_{\rm L}^3}{4\pi c\varepsilon_0^2}\frac{1}{k_{\rm L}^3 r^3}\Big[\big(1-3\cos^2\theta\big)\big(\cos k_{\rm L}r + k_{\rm L}r\sin k_{\rm L}r\big) - \sin^2\theta \;\; k_{\rm L}^2 r^2 \;\; \cos k_{\rm L}r\Big]\cos k_{\rm L}y\ , \qquad \text{Lin. polariz.}$$

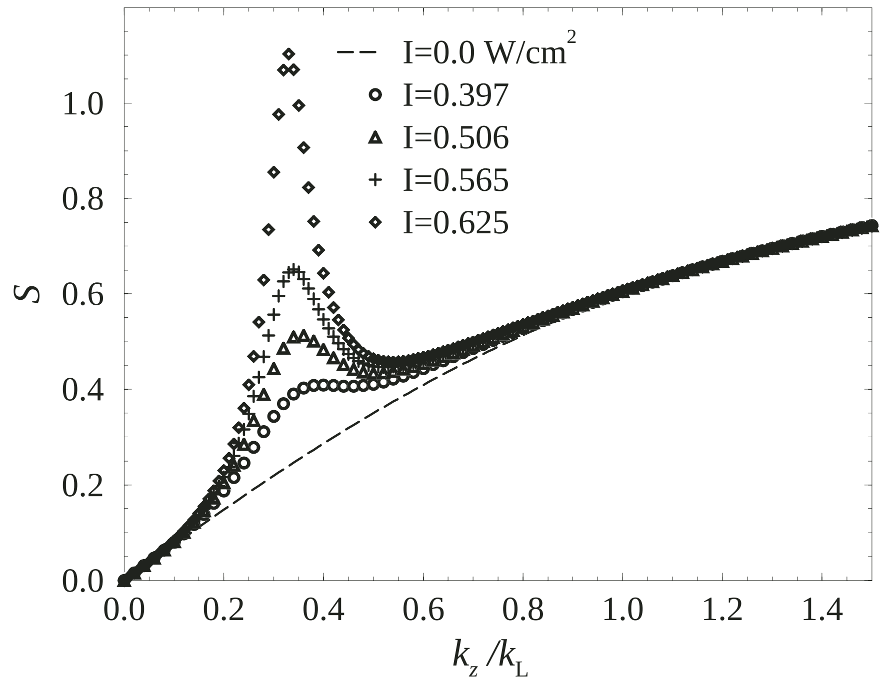
Quasi-1D BEC: Tight radial trapping. Averaging over the radial profile $\exp[-(x^2+y^2)/w_r^2]/(\pi w_r^2)$, $k_{\rm L}w_r \lesssim 1$, results in 1D Hamiltonian with $U_{\rm dd}^{(1D)}(z)$



Roton-like spectrum

^{87}Rb BEC peak density $= 10^{12}$ cm^{-3} $w_r = 3.5\lambda_L$





$E_R = \hbar^2 k_L^2/(2m)$

Static structure factor $S(k) = (u_k - v_k)^2 = \hbar^2 k_L^2/[2m\epsilon(k)]$

Enhancement of atomic pair correlation

For higher intensities the roton dip approaches 0

Onset of instability? Phase transition to a supersolid state?

Laser-induced “supersolid”

S. Giovanazzi, D. O’Dell, G. Kurizki, PRL **88**, 130402 (2003); M. Kalinski, I.E. Mazets, G. Kurizki, B.A. Malomed, K. Vogel, W.P. Schleich, cond-mat/0310480.

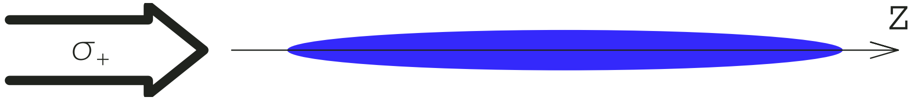
Supersolid: periodically modulated ground state structure AND superfluidity [A.J. Leggett, PRL **25**, 1543 (1970)].

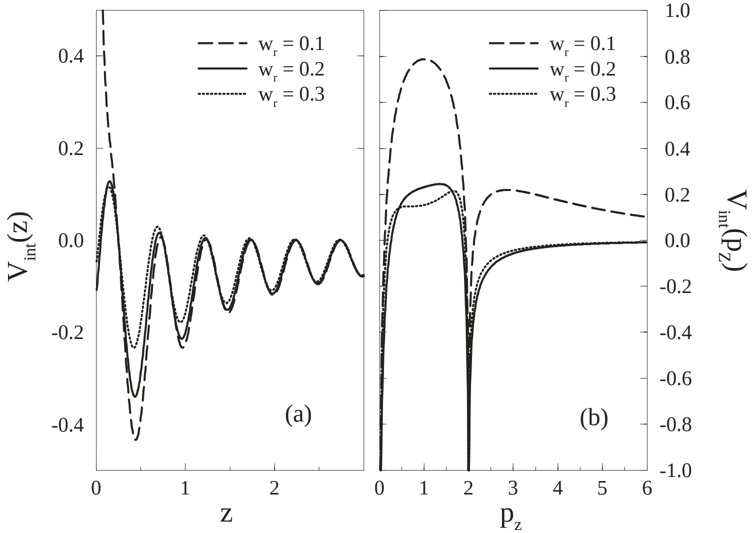
LIDDI in the case of *circular* polarization:

$$V_{\rm dd}(\mathbf{r}) = \frac{I\alpha^2(\omega)\,k_{\rm L}^3}{8\pi c\varepsilon_0^2}\Big[\frac{2z^2-x^2-y^2}{(k_{\rm L}r)^5}\big(\cos k_{\rm L}r + k_{\rm L}r\sin k_{\rm L}r\big) - \frac{2z^2+x^2+y^2}{(k_{\rm L}r)^3}\cos k_{\rm L}r\Big]\cos k_{\rm L}z$$

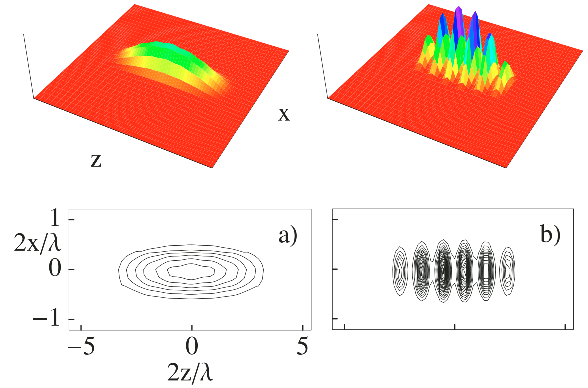
Spontaneous symmetry breaking. An optical lattice is formede by interference of the *incident* and *back-scattered* light.

Statics and dynamics of supersolid formation

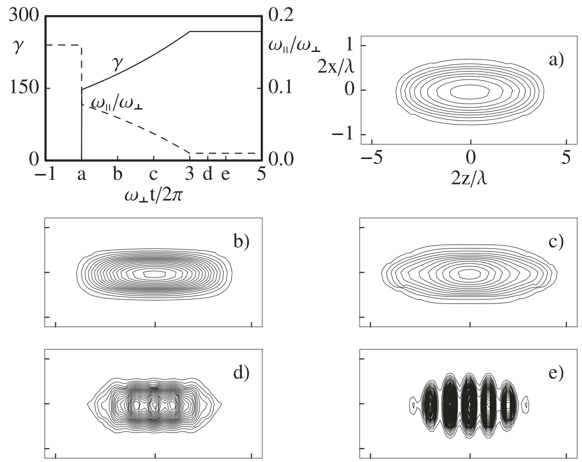




Static solution



Time behavior



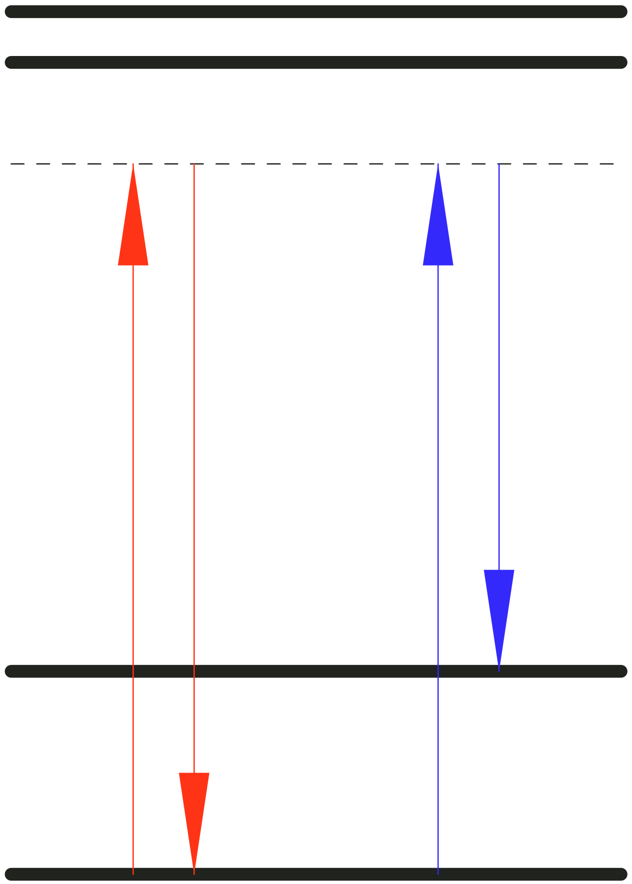
Intensity threshold:

$$\frac{I\alpha^2(\omega)n^{(1D)}m\Lambda}{16\pi c\hbar^2\varepsilon_0^2} > 1,$$

$$\Lambda = 2\log[\,\ell/(2w_r)\,]$$

(typically 10 – 100 mW/cm^2)

Off-resonant Raman and Bragg light scattering in BECs



Transferred momentum: $\hbar\mathbf{q} = \hbar(\mathbf{k}_1 - \mathbf{k}_2)$

Transferred energy:

Bragg: $\hbar\Delta = \hbar(\omega_1 - \omega_2)$

Raman: $\hbar\Delta = \hbar(\omega_1 - \omega_2) - E_D$

E_D – Zeeman/hyperfine splitting

Resonance:

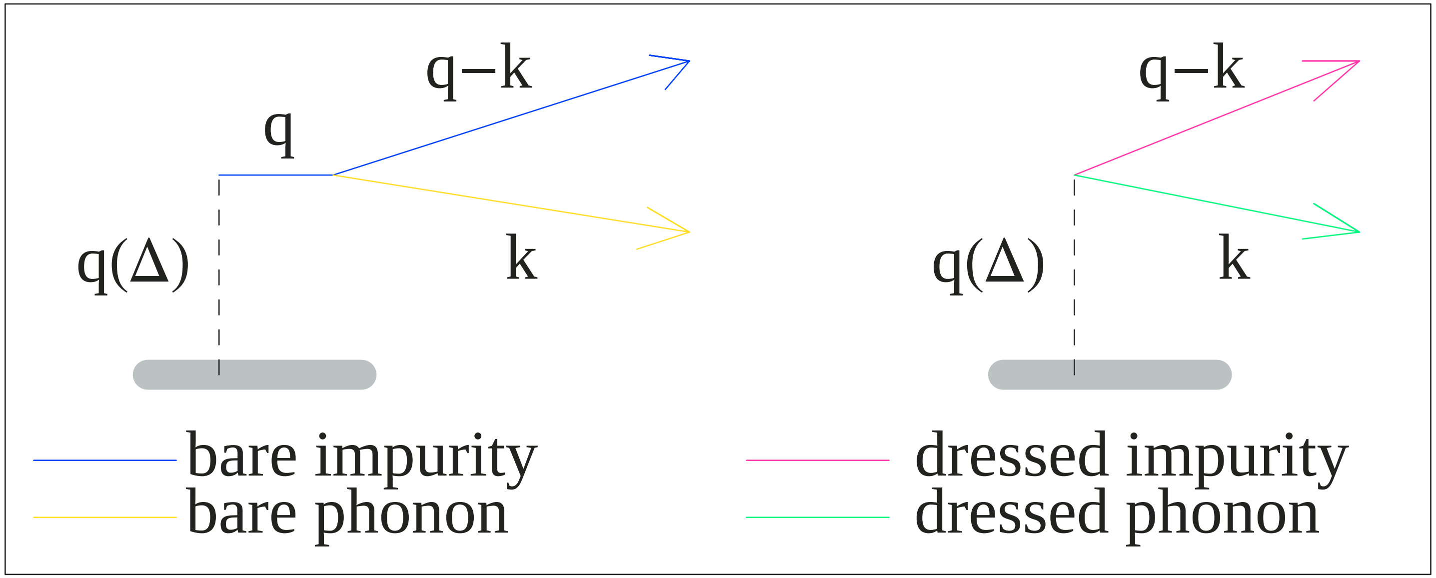
Bragg: $\hbar\Delta = \epsilon(\mathbf{q})$

Raman: $\hbar\Delta = \frac{\hbar^2 q^2}{2m} + \frac{4\pi\hbar^2 a_{imp} n}{m} - \mu$

Resonant Bragg spectroscopy (MIT, Weizmann Inst.) yields $\epsilon(\mathbf{q})$ (line position) and $S(\mathbf{q})$ (line intensity)

Blue-detuned Raman process

$$\hbar\Delta > \frac{\hbar^2 q^2}{2m} + \frac{4\pi\hbar^2 a_{imp} n}{m} - \mu$$



In what follows: $a_{imp} \equiv a$, $a_{BEC} \equiv a_0$

Diagonalization of the Hamiltonian

$$\hat{H} = \hat{H}_{at} + \hat{H}_{int},$$

$$\begin{aligned}\hat{H}_{at} &= \sum_{\mathbf{k}} \hbar\omega_k \hat{b}^{\dagger}_{\mathbf{k}} \hat{b}_{\mathbf{k}} + \sum_{\mathbf{k}} \frac{\hbar^2 k^2}{2m} \hat{\beta}^{\dagger}_{\mathbf{k}} \hat{\beta}_{\mathbf{k}} + \\ & \frac{4\pi\hbar^2 a\sqrt{n}}{m\sqrt{V}} \sum_{\mathbf{k}} \sqrt{S_k} \left(\hat{\varrho}_{-\mathbf{k}} \hat{b}^{\dagger}_{\mathbf{k}} + \hat{\varrho}_{\mathbf{k}} \hat{b}_{\mathbf{k}} \right), \\ \hat{H}_{int} &= \hbar\Omega\sqrt{N} \left(e^{-i\Delta t} \hat{\beta}^{\dagger}_{\mathbf{q}} + e^{i\Delta t} \hat{\beta}_{\mathbf{q}} \right).\end{aligned}$$

Ω – two-photon Rabi frequency; $\omega_k = k\sqrt{[\hbar k/(2m)]^2 + c_s^2}$;

$\hat{\varrho}_{\mathbf{k}}$ – momentum shift operator

Dressed (*polaronic*) states

$$|\mathbf{q}\rangle_d = \hat{\beta}^\dagger_{\mathbf{q}}|0\rangle - \frac{1}{\sqrt{V}}\sum_{\mathbf{k}'} f_{\mathbf{q},\,\mathbf{k}'}\hat{\beta}^\dagger_{\mathbf{q}-\mathbf{k}'}\hat{b}^\dagger_{\mathbf{k}'}|0\rangle,$$

$|0\rangle$ is the vacuum state of $\hat{H}_{at}$ (no phonons and no impurity atoms)

Amplitude of dressing the impurity atom with virtual BEC excitations:

$$f_{\mathbf{q},\,\mathbf{k}'} = \frac{4\pi\hbar a\sqrt{nS_{k'}}}{m[\,\omega_{k'} + \hbar k'^{\,2}/(2m) - \hbar\mathbf{q}\mathbf{k}'/m\,]}$$

Final state with energy $\epsilon_{|\mathbf{q}-\mathbf{k}|,\,k} = \hbar\omega_k + \hbar^2(\mathbf{q}-\mathbf{k})^2/(2m)$:

$$|\mathbf{q}-\mathbf{k},\,\mathbf{k}\rangle_d = \left(1 - \sum_{\mathbf{q}'}|\mathbf{q}'\rangle_d\,{}_d\langle\mathbf{q}'|\right)\hat{b}^\dagger_{\mathbf{k}}|\mathbf{q}-\mathbf{k}\rangle_d.$$

$$\Rightarrow \quad \hat{H}_{int} = \sum_{\mathbf{k}}\left[\frac{\hbar\Omega}{\sqrt{V}} f_{\mathbf{q},\,\mathbf{k}} e^{-i\Delta t}\hat{c}^\dagger_{\mathbf{q}-\mathbf{k},\,\mathbf{k}}\hat{c}_0 + \mathrm{H.}c.\right]$$

Rate of correlated pairs production

$$\Gamma(t) = 2\,\mathrm{Re}\,J(t)$$

$$J(t) = \int_0^t dt' \int \frac{d^3\mathbf{k}}{(2\pi)^3} \left(\Omega f_{\mathbf{q},\mathbf{k}}\right)^2 \times \exp\{i[\,\Delta - \omega_k - \hbar(\mathbf{q}-\mathbf{k})^2/(2m)\,]\,t'\}$$

The case of far-subcritical virtual impurity velocity ($\hbar q \ll m c_s$) admits analytic treatment within the Wigner-Weisskopf approach

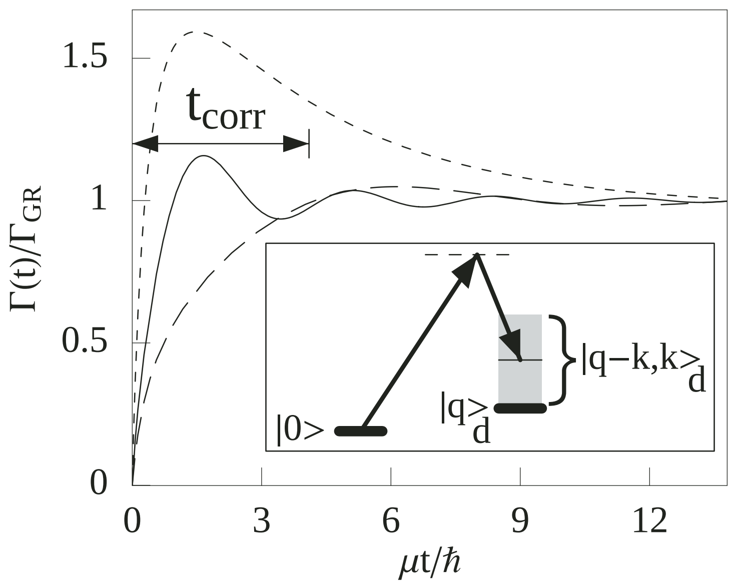
$$J(t) = \frac{\Gamma_*}{2} \int_0^t dt\, \frac{\mu}{\hbar} \Xi_{\Delta_q}\left(\frac{\mu t}{\hbar}\right), \quad \Gamma_* = \frac{8}{\sqrt{\pi}} \sqrt{\frac{a}{a_0}} \frac{\hbar\Omega^2}{\mu} \sqrt{n a^3},$$

$$\Xi_{\Delta_q}(z) = e^{i\hbar\Delta_q z/\mu} \left\{ \frac{2}{3}(1+iz) - e^{iz}\left(1+\frac{2iz}{3}\right)\sqrt{2\pi z} \times \left[\frac{1+i}{2} - S\left(\sqrt{\frac{2z}{\pi}}\right) - iC\left(\sqrt{\frac{2z}{\pi}}\right)\right]\right\}$$

Dynamics of the process

Golden Rule rate $\Gamma_{GR} = \Gamma_* [\,(\hbar\Delta_q/\mu) + 1\,]^{-5/2} \pi\hbar\Delta_q/(2\mu)$

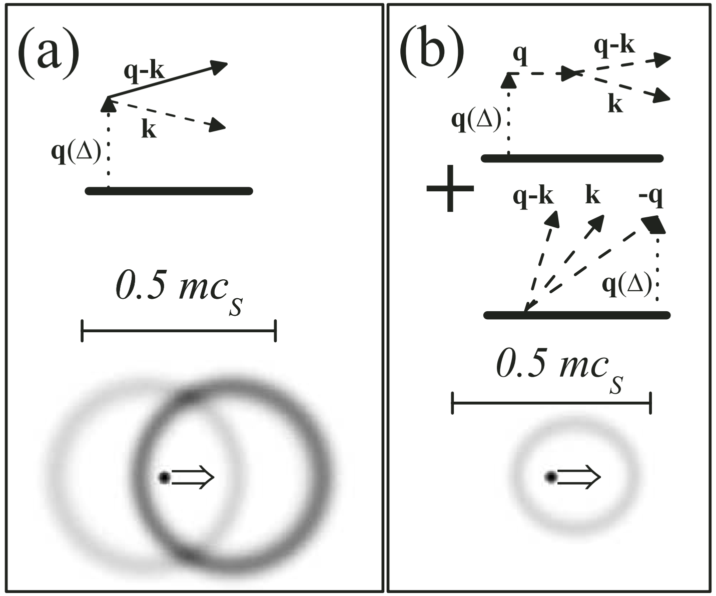
$\Delta_q = \Delta - \hbar q^2/(2m) \approx \Delta$



$\hbar\Delta_q/\mu = 2.0$ (solid line),

0.66 (long-dashed line),

and 0.07 (short-dashed line)



Momentum distribution:

(a) Raman, (b) Bragg

$\hbar\Delta = 0.66\mu$, $\hbar q = 0.14 mc_s$,

$t = 120\hbar/\mu$

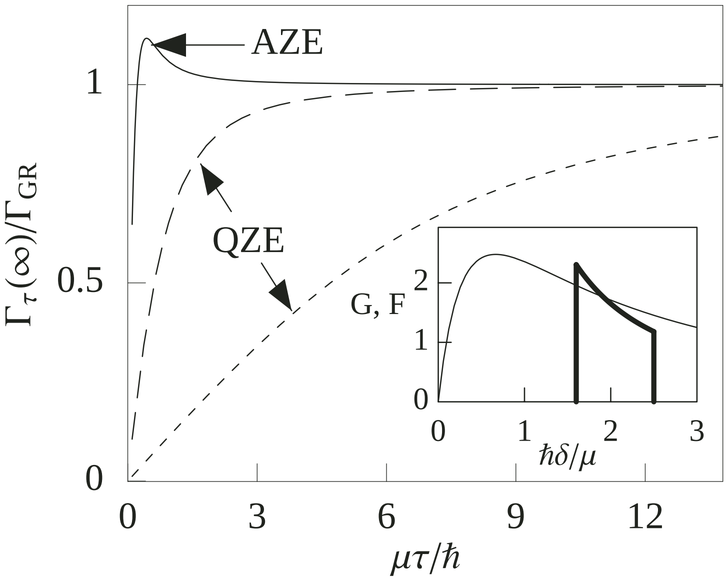
Probing the quasiparticle correlation time

The quantum Zeno and anti-Zeno effects

Well-defined Δq $:\longrightarrow$ spectrum $F(\delta)$

$F(\delta) = (2\sqrt{3})^{-1}\tau(\delta/\Delta_q)^{-3/2}$ for $\delta_1 < \delta < \delta_2$ and 0 otherwise

$\Delta_q = \sqrt{\delta_1\delta_2}$ – mean frequency $\quad \tau = \sqrt{3}/(\delta_2 - \delta_1)$ – dephasing time



$$\Gamma_\tau(\infty) = \Gamma_* \int_0^\infty d\delta\, G(\delta) F(\delta)$$

$G(\delta)$ – medium response function

A.G. Kofman and G. Kurizki, Nature **405**, 546 (2000); PRL **87**, 270405 (2001).

The characteristic time τ of the QZE (slowdown) and the AZE (speedup) induced by the dephasing is $\sim t_{corr}$.

Experimentally accessible!

Conclusions

- Oscillatory LIDDI potentials induce enhanced intrinsic correlations in quasi-1D BECs, giving rise to a roton-like dip in the Bogoliubov spectrum and/or phase transition to the supersolid state
- Time-resolved monitoring of off-resonant Raman/Bragg processes can reveal the Zeno or the anti-Zeno effects (slowdown or speedup of the decay rate compared to its Golden Rule value), thus serving as *unique probes of temporal correlations* of the BEC elementary excitations