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SMR.1587 - 12

SCHOOL AND WORKSHOP ON
QUANTUM ENTANGLEMENT, DECOHERENCE, INFORMATION, AND GEOMETRICAL PHASES IN COMPLEX SYSTEMS
(1 November - 12 November 2004)

Non-orthogonal "resonances", Modified geometric phase and geometric dephasing (i.e. Weird dynamics of a non-isolated spin-1/2)

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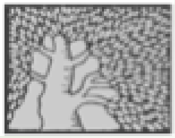
These are preliminary lecture notes, intended only for distribution to participants



Non-orthogonal "resonances", modified geometric phase and geometric dephasing

(i.e.Weird dynamics of a non-isolated spin-1/2)

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11th November 2004

For those readers who were not present during this seminar:

This talk divides into two independent pieces of work:

slides 4-18 : spin dynamics in a time-independent field

slides 20-33 : Berry phase experiment (slow time-dependent field)

The only connection between these two parts is made (rather briefly) on slide 31, it is not a central point of either piece of work

For those readers who were present during this seminar:

For pedagogical reasons, the two sections of the talk are in the reverse order to the way I presented them in Trieste.

Techical stuff:

A brief summary of the technique we use is given on slides 15-17.

This includes a brief presentation of the analogy between the diagrams and terms in a Lindblad-style Master equation. **Note:** This is an analogy not a mathematical equivalence, in-other-words I do not know of a proof of the equivalence.

16th November 2004

Outline

Static Hamiltonian

❶ Pictures of dynamics of dissipative spin–half
(spin+environment ➠ trace out environment)

❷ Choose **"resonance-states"**
≡ **density matrices** associated with **resonances**
(will be **non-orthogonal**)

❸ **Analytic results** : spin–dynamics (weak dissipation: Master eq+diagrams)

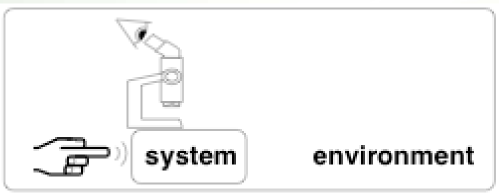
Slow time-dependent Hamiltonian: Geometric phase

❹ Environ. **modifies** Berry phase (monopole + complex **quadropole**)

❺ Geometric dephasing (can be of either sign: dephasing/**rephasing**?)

Definitions / philosophy

Universe =



Universe : Hamiltonian evolution
System : **dissipative** evolution

System's reduced density matrix : $\rho_{\text{system}} = \text{Tr}_{\text{env}}[\rho_{\text{univ}}]$

Define :

"SYSTEM" = **controlled/measured** degrees of freedom
"ENVIRONMENT" = all other degrees of freedom

➠ post–selection = measurement
No post–selection on **environment**

Models of system + environment

❶ Spin coupled to quantum environment

$$\mathcal{H}_{\text{univ}} = \mathcal{H}_{\text{syst}} + \mathcal{H}_{\text{interaction}} + \mathcal{H}_{\text{env}}$$

$$\mathcal{H}_{\text{syst}} = -\tfrac{1}{2}\mathbf{B}(t).\boldsymbol{\sigma} \qquad \mathcal{H}_{\text{interaction}} = -\sum_n C_n(a_n^\dagger + a_n)\sigma_z$$

where $a_n^\dagger, a_n$ create/annihilate nth environment mode

Example : **spin-boson model** Leggett *et al* (1986,87)

Environment = oscillators with smooth spectral distrib.

❷ Spin coupled to classical coloured noise

$$\mathcal{H}_{\text{syst}} = -\tfrac{1}{2}(\mathbf{B}(t) + \mathbf{K}(t)).\boldsymbol{\sigma} \quad \text{where } \langle \mathbf{K}(\tau)\mathbf{K}(0)\rangle = C^2 f(\tau)$$

Equiv. to quantum for some quantities: T_1, T_2 , Lamb shift, etc

Caldeira–Leggett(1983), Whitney–Makhlin–Shnirman–Gefen (2004)

Naïve expectation for dissipative spin dynamics

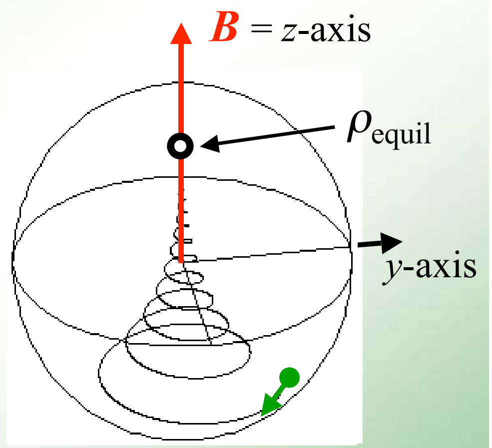
Density matrix, $\rho(t)$,

➠ position in sphere at time t

$<\sigma_x>$, $<\sigma_y>$, $<\sigma_z>$ give coordinate in x, y, z directions

$$\rho = \frac{1}{2}\begin{pmatrix} 1+\langle\sigma_z\rangle & \langle\sigma_x\rangle + \mathrm{i}\langle\sigma_y\rangle \\ \langle\sigma_x\rangle + \mathrm{i}\langle\sigma_y\rangle & 1-\langle\sigma_z\rangle \end{pmatrix}$$

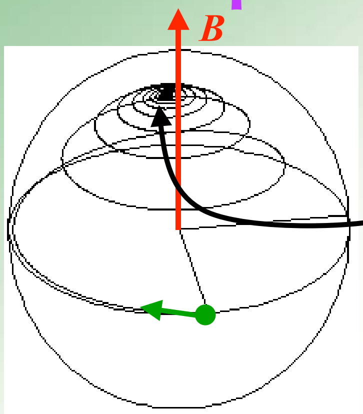
✪ **Pure** states are **on** sphere
✪ **Mixed** states are **inside** sphere



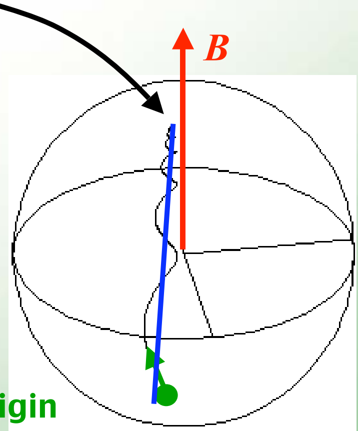
Expected spin behaviour

❶ **thermalise** along z-axis; T_1
❷ **decohere** in x-y plane; T_2

"True" dissipative dynamics: zero temperature



does NOT decay to $|\uparrow\rangle$ relative to B
decays to pure superposition



Spin precesses
about BLUE axis NOT the B-axis

- ★ BLUE axis not || to B-axis
- ★ BLUE axis doesn't go through origin

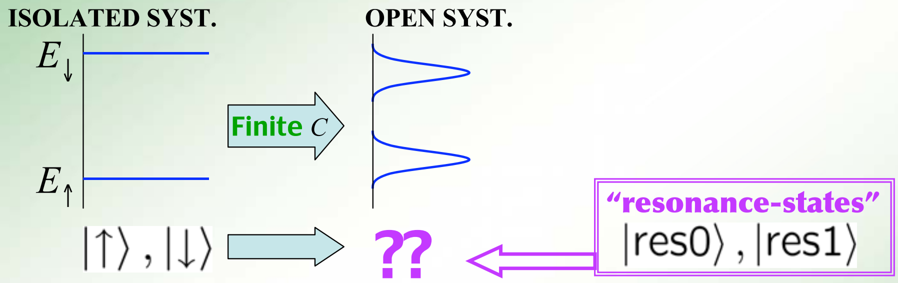
"True" dissipative dynamics: large temperature

For high temperatures; typically $T_2 << T_1$

Timescale separation – dephase much faster than thermalise

draw pictures on blackboard

Eigenstates ➠ "Resonance-states"



EIGENSTATES of ISOLATED nxn density matrix ➠ n eigenstates

★ pure states ★ time–independent (superpositions precess)

Desired properties of "resonance-states" :

❶ There should be n resonance–states

❷ Each should deform smoothly onto an eigenstate when $C\to 0$

Do Zurek's pointer states satisfy ❶? Only when $T_2/T_1 \to 0$.

A definition of a resonance?

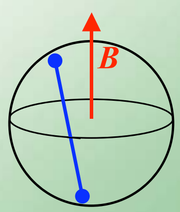
What about the following definition??

A resonance is a pure state which may decay but does not precess

★ Spectrum tells us that resonances (usually) decays

★ Precession is characteristic of superpositions

★ Why pure state? Intuitive to sum pure states to get mixed states (although opposite is also legitimate)



❶ There are n resonance–states

❷ Each deforms smoothly onto an eigenstate when $C\to 0$

Coincides with pointer states when $T_1/T_2 \to 0$

Isolated spin 1/2

$d\rho/dt = -i[\mathcal{H}, \rho]$

Non-isolated spin 1/2

$\rho_{system} = \mathrm{Tr}_{env}[\rho_{univ}]$

(i) extreme Markov approx

$t_{mem} \to 0$ (quantum jumps)

(ii) weak Markov approx

$t_{mem} \ll T_1, T_2, \ldots$

Bloch-Redfield (1957)

Schoeller-Schon (1994) ⇦ **diagrams**

$$\frac{d}{dt}\begin{pmatrix}\rho_{11}\\ \rho_{12}\\ \rho_{21}\\ \rho_{22}\end{pmatrix} = \begin{pmatrix} X_{11} & X_{12} & X_{12}^* & -X_{44}\\ X_{31}^* & X_{22} & X_{32}^* & X_{34}^*\\ X_{31} & X_{32}^* & X_{22}^* & X_{34}\\ -X_{11} & -X_{12} & -X_{12}^* & X_{44}\end{pmatrix}\begin{pmatrix}\rho_{11}\\ \rho_{12}\\ \rho_{21}\\ \rho_{22}\end{pmatrix}$$

nearly Lindblad form (1976)

Preserves: trace ☑ **Hermiticity** ☑ **positivity** ☒

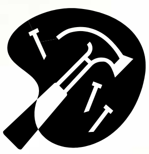
Weak coupling to environment

➡ Go to eigenbasis of $\mathcal{H}_{system}$

$$\frac{d}{dt}\begin{pmatrix}\rho_{11}\\ \rho_{12}\\ \rho_{21}\\ \rho_{22}\end{pmatrix} = \begin{pmatrix} X_{11} & X_{12} & X_{12}^* & -X_{44}\\ X_{31}^* & X_{22} & X_{32}^* & X_{34}^*\\ X_{31} & X_{32}^* & X_{22}^* & X_{34}\\ -X_{11} & -X_{12} & -X_{12}^* & X_{44}\end{pmatrix}\begin{pmatrix}\rho_{11}\\ \rho_{12}\\ \rho_{21}\\ \rho_{22}\end{pmatrix}$$

Matrix, **X**, is nearly diagonal : Red terms $\propto |B|$

Black terms $\propto |B| \times C^2$



Diagonalise **X**:

❶ diagonalise 4 corner elements exactly

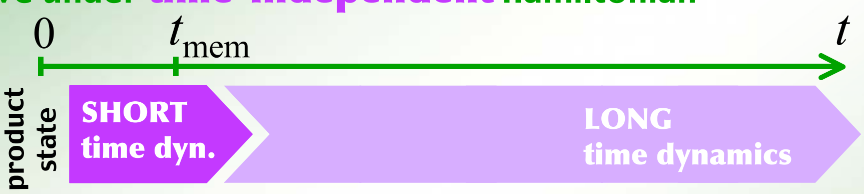
❷ diagonalise rest to order C^2

Time dependence of the X_{ij}s

Prepare spin by **measuring** it

⇒ Universe starts in **product** state, $|\psi_{\text{univ}}\rangle = |\sigma_{\text{syst}}\rangle\,|\sigma_{\text{env}}\rangle$

Evolve under **time-independent** Hamiltonian



short times: environment "remembers" $t = 0$

all elements of X_{ij} are **time-dependent**

long times: environment has "forgotten" $t = 0$

all elements of X_{ij} are **time-independent**

For **time-dependent** $\mathcal{H}_{\text{syst}}(t)$ both regimes are time–dependent

Eigenvalues/vectors of $\mathbf{X} = \mathcal{U}\mathbf{X}_{\text{D}}\mathcal{U}^{-1}$

$$\mathbf{X}_{\text{D}} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & X_{22} & 0 & 0 \\ 0 & 0 & X_{22}^{*} & 0 \\ 0 & 0 & 0 & X_{44} \end{pmatrix}$$

$$X_{22} = \text{i}(B + \delta B) - T_2^{-1}$$
$$X_{44} = -T_1^{-1}$$

Thermalising and dephasing times; T_1, T_2

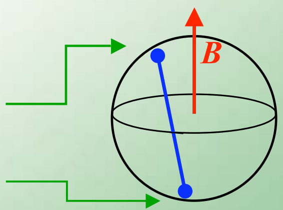
Lamb shift of energy gap; δB

$$\mathcal{U}^{-1} = \begin{pmatrix} \frac{(X_{11}^2 + X_{44}^2)^{1/2}}{X_{11}+X_{44}} & 0 & 0 & \frac{(X_{11}^2 + X_{44}^2)^{1/2}}{X_{11}+X_{44}} \\ -B^{-1}X_{31}^{*} & 1 & (2B)^{-1}X_{32}^{*} & -B^{-1}X_{34}^{*} \\ -B^{-1}X_{31} & (2B)^{-1}X_{32} & 1 & -B^{-1}X_{34} \\ \frac{-X_{11}}{X_{11}+X_{44}} & 0 & 0 & \frac{X_{44}}{X_{11}+X_{44}} \end{pmatrix}$$

Two **pure** states which **do not precess**

$$|\text{res}0\rangle = |\uparrow\rangle + B^{-1}X_{31}|\downarrow\rangle$$

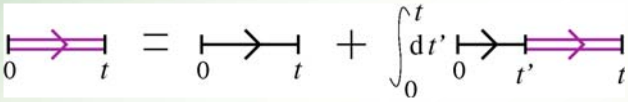
$$|\text{res}1\rangle = B^{-1}X_{34}^{*}|\uparrow\rangle + |\downarrow\rangle$$



Any classical mixture of these two states **does not precess**

Technicalities of Master eqn

Real time Dyson eqn :



exact Master equation for spin's density matrix

Schoeller-Schon(1994)

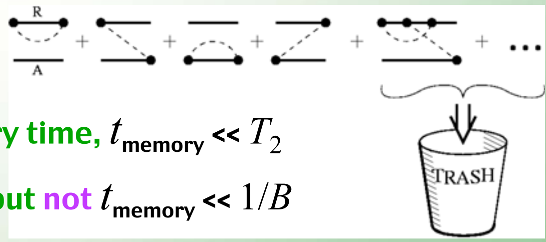
$$\frac{\mathrm{d}}{\mathrm{d}t}\rho_{ij}(t) = -\mathrm{i}\left[\mathcal{H}_{\mathrm{syst}}, \rho(t)\right]_{ij} + \int_0^t \mathrm{d}\tau \Sigma_{ij,i'j'}(\tau)\rho_{i'j'}(t-\tau)$$

Sum of irredicible "self-energy" diagrams

APPROXIMATIONS : systematic weak-coupling and Markov

Bloch-Redfield (1957)

$$\Sigma_{ij,i'j'}(\tau) =$$

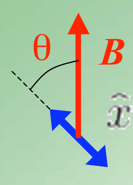


+ Assume environ. memory time, $t_{\mathrm{memory}} \ll T_2$

...but not $t_{\mathrm{memory}} \ll 1/B$

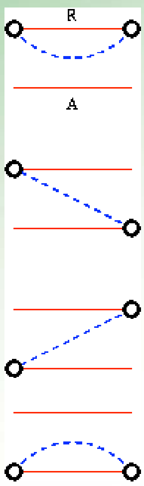
& no rotating wave approx

Irreducible diagrams



$$\hat{x} = \sum_n (\hat{a}_n^\dagger + \hat{a}_n)$$

it couples to blue axis



$$= C^2 \mathrm{tr_{env}}[\hat{x}(\tau)\hat{x}(0)\hat{\rho}_{\mathrm{env}}]\ \hat{\sigma}_\theta(\tau)\hat{\sigma}_\theta(0)\hat{\rho}_{\mathrm{syst}}$$
$$= C^2\langle\hat{x}(\tau)\hat{x}(0)\rangle\ \hat{\sigma}_\theta(\tau)\hat{\sigma}_\theta(0)\hat{\rho}_{\mathrm{syst}}$$

$$= C^2 \mathrm{tr_{env}}[\hat{x}(0)\hat{\rho}_{\mathrm{env}}\hat{x}(\tau)]\ \hat{\sigma}_\theta(0)\hat{\rho}_{\mathrm{syst}}\hat{\sigma}_\theta(\tau)$$
$$= C^2\langle\hat{x}(\tau)\hat{x}(0)\rangle\ \hat{\sigma}_\theta(0)\hat{\rho}_{\mathrm{syst}}\hat{\sigma}_\theta(\tau)$$

$$= C^2\langle\hat{x}(0)\hat{x}(\tau)\rangle\ \hat{\sigma}_\theta(\tau)\hat{\rho}_{\mathrm{syst}}\hat{\sigma}_\theta(0)$$

$$= C^2\langle\hat{x}(0)\hat{x}(\tau)\rangle\ \hat{\rho}_{\mathrm{syst}}\hat{\sigma}_\theta(0)\hat{\sigma}_\theta(\tau)$$

Heisenberg picture: $\hat{\sigma}_\theta(\tau) = \mathrm{e}^{\mathrm{i}B\tau\hat{\sigma}_z}\hat{\sigma}_\theta(0)\mathrm{e}^{-\mathrm{i}B\tau\hat{\sigma}_z}$

diagrams are $\mathcal{H}_{\mathrm{syst}}$ dependent

Microscopic calc. for quantum & classical

Everything written in terms of

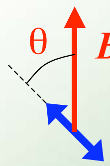
$$S_{\pm}(\Omega) = \text{Fourier trans.}\left[\, C^2\langle \hat{x}(t)\hat{x}(0) \pm \hat{x}(0)\hat{x}(t)\rangle \,\right]$$

$$X_{22} = iB - \frac{i}{2}\int \frac{d\Omega}{2\pi} S_{+}(\Omega)\left[\frac{2\cos^2\theta}{\Omega + i0^+} + \frac{\sin^2\theta}{\Omega - B + i0^+}\right]$$

$$X_{31} = -\frac{i}{2}\int \frac{d\Omega}{2\pi}\left[S_{-}(\Omega)\frac{\sin\theta(\cos\theta+1)}{\Omega+i0^+} - \left(S_{+}(\Omega)+S_{-}(\Omega)\right)\frac{\sin\theta\cos\theta}{\Omega-B+i0^+}\right]$$

$$X_{34} = -\frac{i}{2}\int \frac{d\Omega}{2\pi}\left[S_{-}(\Omega)\frac{\sin\theta(\cos\theta-1)}{\Omega+i0^+} + \left(S_{+}(\Omega)-S_{-}(\Omega)\right)\frac{\sin\theta\cos\theta}{\Omega-B+i0^+}\right]$$

$$X_{11} = \cdots$$
$$X_{44} = \cdots$$
$$X_{12} = \cdots$$
$$X_{32} = \cdots$$



$$\hat{x} = \sum_n (\hat{a}_n^\dagger + \hat{a}_n)$$

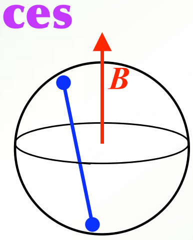
it couples to blue axis

If $\mathcal{H}_{\text{syst}} = -\frac{1}{2}B(t).\sigma$ **(i.e. classical noise)** ⇒ $S_{-}(\Omega) = 0$

Conclusion for resonances

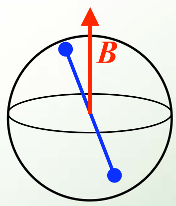
Dynamics with QUANTUM environ.

Coupling term does not commute with $\mathcal{H}_{\text{env}}$



Dynamics with CLASSICAL environ.

Coupling term commutes with $\mathcal{H}_{\text{env}}$



Suggested definition for resonance-states:

Pure states that may decay but do not precess

Resonance-states are the BLUE dots:

- ★ smoothly deform to eigenstates as $C \to 0$
- ★ defined for any T_2/T_1

Berry phase for (isolated) spin-half

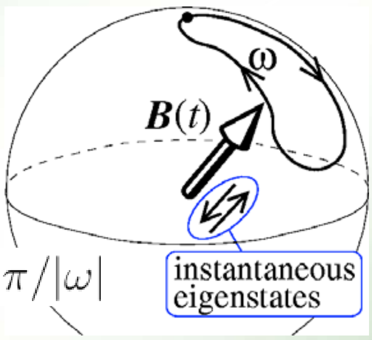
Berry (1984)

Slowly varying B-field : $|\omega| \ll |\boldsymbol{B}|$

$$P\big(\,|\!\uparrow\rangle \to |\!\downarrow\rangle\,\big) \sim (Bt)^{-1} \ll 1$$

i.e. Adiabatic evolution

$t \sim 2\pi/|\omega|$

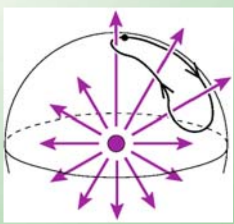


$$\Phi = \Phi_{\text{Dynamic}} + \Phi_{\text{Berry}} + \mathcal{O}[(Bt)^{-1}]$$

$$\Phi_{\text{Dynamic}} = \tfrac{1}{2}|\boldsymbol{B}|t$$

$\Phi_{\text{Berry}} = \frac{1}{2}$ **(enclosed solid angle)**

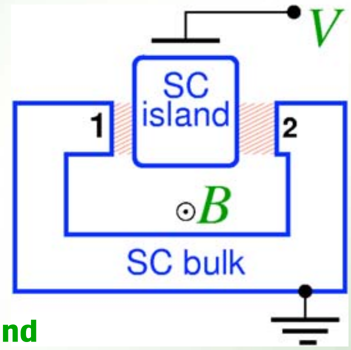
$= \frac{1}{2}$ **(flux of monopole thru loop)**



Berry phase in qubit systems?

Potential solid-state realisation

Berry phase in Superconducting nanocircuit(qubit) Falci *et al* (2000)



$$\hat{\mathcal{H}} = E_C \left(\hat{n} - n_V\right)^2 - E_J(B) \cos\left[\hat{\theta} - \alpha_B\right]$$

Consider only lowest 2 charge-states of island

$$|\uparrow\rangle \equiv |n\rangle \quad \& \quad |\downarrow\rangle \equiv |n+1\rangle$$

Reduced Hamiltonian: $\hat{\mathcal{H}} = \begin{pmatrix} E_J \cos(\alpha_B) \\ E_J \sin(\alpha_B) \\ E_C(1-n_V) \end{pmatrix} \cdot \hat{\sigma}$

Environment? ♣ charge fluctuations couple via σ_z
♣ current fluctuations couple via σ_x, σ_y

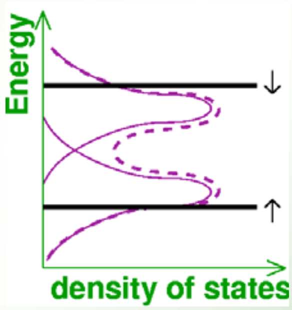
Berry phase with dephasing?

Environment induces level-broadening

$\Longrightarrow$ No Gap

$$P(|\uparrow\rangle \rightarrow |\downarrow\rangle) \rightarrow 1 \quad \text{as} \quad t \rightarrow \infty$$

No Adiabaticity $\Rightarrow$ No Berry phase



Whitney–Gefen, *PRL* (2003)

BUT : All real expts are non-isolated,
yet Berry phase is observed

Berry phase is observable whenever
adiabatic time << dephasing time

$$\hbar / E_{\text{gap}} \ll T_2$$

i.e. require small matrix elements for transitions not a true gap

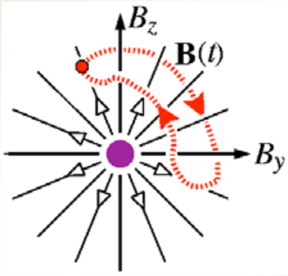
Env.-induced modification of the Berry phase

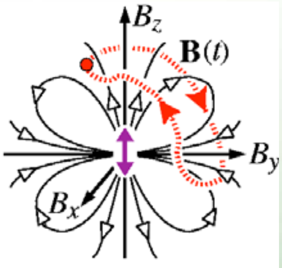
get phase as $\oint$ along path of $B(t)$ $\Rightarrow$ use Stokes' theorum

$\Rightarrow$ surface int. $\Phi_{\mathrm{Berry}} = \int \mathrm{d}\boldsymbol{S} \cdot (\boldsymbol{b} + \delta \boldsymbol{b})$

monopole pseudo-field $\boldsymbol{b}$

"quadrupole" pseudo-field $\delta \boldsymbol{b}$





Angular = $Y_{20}(\theta,\varphi)$
Radial ≠ B^{-4}

(non-zero curl)

Amplitude of monopole = 1/2

Amplitude of "quadrupole" = C^2 × complex function (env. spectrum)

Berry phase as derivative of gap

For isolate spin: $\Phi_{\mathrm{Berry}} = \int_0^t \mathrm{d}t' \omega_z \frac{\mathrm{d}|\boldsymbol{B}|}{\mathrm{d}B_z} = \oint \mathrm{d}\varphi \frac{\mathrm{d}}{\mathrm{d}B_z} B$

For NON-isolate spin: $B \longrightarrow [B + \delta B + \mathrm{i}\Gamma_2]$

where $\delta B =$ Lamb shift of energy level

$\Gamma_2 =$ dephasing rate $\Gamma_2 = T_2^{-1}$

Pretty result : $\Phi_{\mathrm{Berry}} = \oint \mathrm{d}\varphi \frac{\mathrm{d}}{\mathrm{d}B_z} [B + \delta B + \mathrm{i}\,\Gamma_2]$

Revisit this later

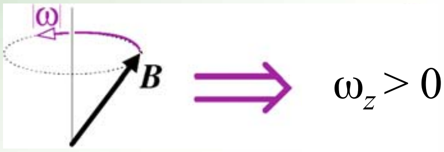
- ♣ Berry phase is complex if spin T_2 is B–dependent
- ♣ Real ("phase") part is modified if Lamb shift is B–dependent

Geometric dephasing

Imaginary part of Berry phase ⇒ dephasing

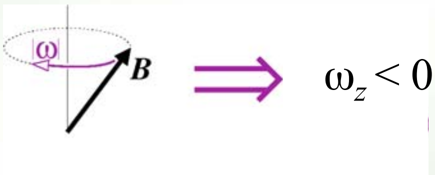
$$\mathrm{Im}[\Phi_{\mathrm{Berry}}] = \int_0^t \mathrm{d}t'\, \omega_z \frac{\mathrm{d}\Gamma_2}{\mathrm{d}B_z}$$

Can be either sign; depends of direction of winding



$\omega_z > 0$: geometric dephasing (positive)

⇒ increase total dephasing



$\omega_z < 0$: geometric REPHASING (negative)

⇒ REDUCES total dephasing

...but it is only a small modification of total dephasing

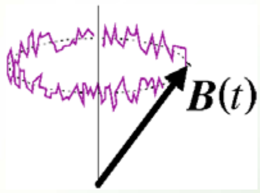
How do we get these results?

♣ **Toy problem : Noisy classical field**

see slides of Y. Gefen (seminar last Friday)

Whitney-Gefen, *Proc. Moriond* (2001)

Whitney-Makhlin-Shnirman-Gefen, *Proc. NATO-ARW* (2004)



♣ **Fully quantum problem :**

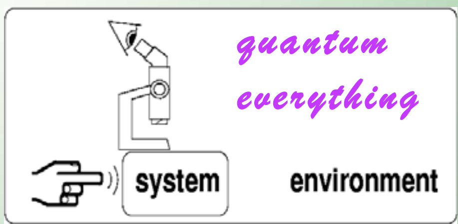
coupling many environ. modes, trace them out

Use rotating frame trick

Whitney-Gefen, *PRL* (2003)

Whitney-Makhlin-Shnirman-Gefen,

to be published (2004)

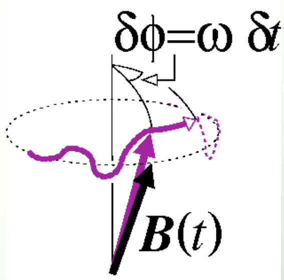


Noisy classical field

Toy problem : Gaussian white-noise Whitney-Gefen (2001)

$$\mathcal{H} = -\tfrac{1}{2}\ [\boldsymbol{B}(t) + \boldsymbol{K}(t)].\sigma$$

where $\langle K(\tau)K(0)\rangle_K = C^2\delta(\tau)$



Adiabatic evolution during one-time step

$$\left\langle \exp\left[\mathrm{i}|\mathbf{B}+\mathbf{K}|\delta t + \mathrm{i}\delta\phi\ \cos\theta_{(\mathbf{B}+\mathbf{K})}\right]\right\rangle_K$$

$$\langle\ldots\rangle_K = \int \mathrm{d}K(\ldots)\exp[-K^2\delta t/C^2]$$

$\langle\cos\theta_{(\mathbf{B}+\mathbf{K})}\rangle$ ⇒ **Modification of real (phase) geometric term**

cross-terms in completed squ.

⇒ **Imaginary part of geometric term**

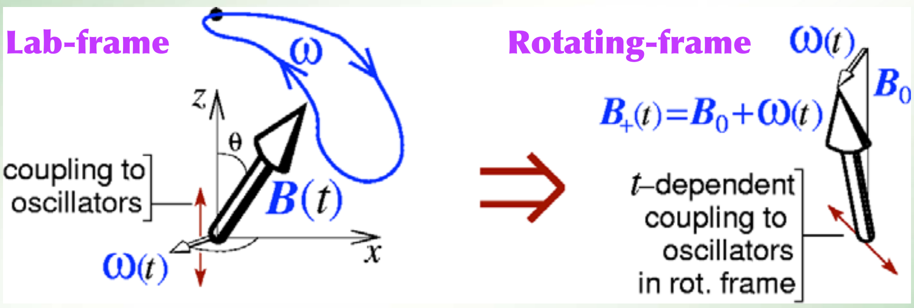
⇒ **geometric dephasing**

Phase in rotating frame ⇒ Berry phase

Berry (1987)

⇒ **Rotating frame — rotates with B-field :**

Hamiltonian ≈ time-independent



Pseudo-forces/fields ⇒ Berry phase

Solve time-independent problem in rotating frame

⇒ **solution of time-dependent problem in Lab frame**

Understanding diff. eqn. for Berry phase

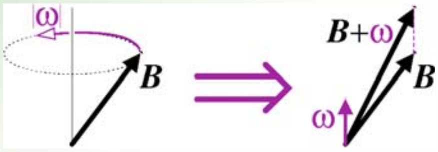
Revisiting : Berry phase as **differential of gap;**

$$\Phi_{\text{Berry}} = \oint \mathrm{d}\varphi \, \frac{\mathrm{d}}{\mathrm{d}B_z}[B + \delta B + \mathrm{i}\,\Gamma_2]$$

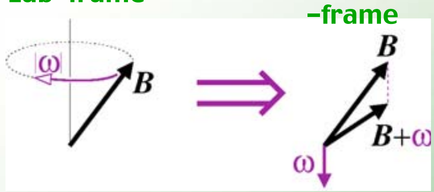
$\Gamma_2 = T_2^{-1}$

Equation is easily understood by going to **rotating frame**

Lab-frame ⇒ Rotating -frame



Lab-frame ⇒ Rotating -frame

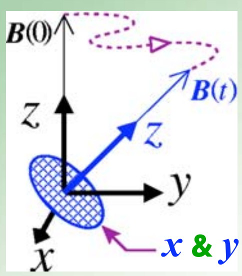


Lamb shift, δB
dephasing rate, Γ_2
} Functions of **rotating frame gap** $\propto |B+\omega|$

⇒ Taylor expand in ω

⇒ both have ω-terms ⇒ **Geometric terms**

Gauge-independence for open paths?



Ambiguity in choice of x & y axes
⇒ **gauge-dependence**
of Berry phase for **open paths**

x & y axes somewhere in this **plane**

Dephasing affects **magnitude** of **off-diag.** elements of density matrix

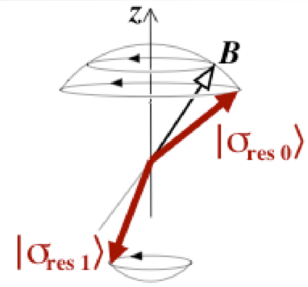
Off-diag. matrix element $= \langle \sigma_x \rangle \pm \mathrm{i} \langle \sigma_y \rangle$

Magnitude $= \sqrt{\langle \sigma_x \rangle^2 + \langle \sigma_y \rangle^2}$

Independent of choice of x & y axes ⇒ **gauge-independent**

Berry phase is not given by resonance-states

Resonance–states enclose different solid-angle to B–field



➡ Correction to solid angle is quadrupole-like

same angular dependence $\propto \sin^2\theta \cos\theta$

but wrong function of Environment spectrum

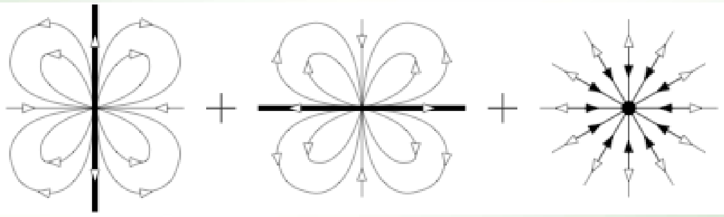
Extra solid–angle $\propto F(B, \{\Omega_n\})$ (function of env. spectrum)

but extra Berry phase $\propto F + a\, \mathrm{d}F/\mathrm{d}B$

Anisotropy is required

Consider isotropic coupling to environment

Isotropic $\equiv$ z-axis coupling + y-axis coupling + x-axis coupling



All three couplings equal

➡ all three "quadrupoles" have equal strength

"Quadrupoles" sum to ZERO

➡ Berry phase unmodified by environment

➡ No geometric dephasing

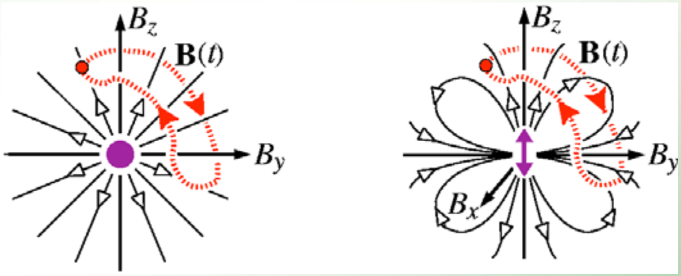
Summary of Berry phase

❶ Berry phase observable for **weak**-dissipation; $BT_2 >> 1$

i.e. small **matrix elements** for spin-flip

❷ Berry phase **modified** by **anisotropic** environment

➠ monopole + **complex quadrupole**



❸ **Geometric dephasing** : increases/**decreases** dephasing

♣ Well-defined (gauge-**independent**) for **open** paths