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SCHOOL AND WORKSHOP ON QUANTUM ENTANGLEMENT, DECOHERENCE, INFORMATION, AND GEOMETRICAL PHASES IN COMPLEX SYSTEMS (1 November - 12 November 2004)

Papers on nonhermitian geometric phases and degeneracies

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These are preliminary lecture notes, intended only for distribution to participants

Papers on nonhermitian geometric phases and degeneracies

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(Numbers refer to my publications list, available by clicking on the web: http://www.physics.bristol.ac.uk/staff/berry_mv.html)

[206] Berry, M V 1990 Geometric amplitude factors in adiabatic quantum transitions *Proc. Roy. Soc. Lond.* **A430** 405-411. These ‘phases’ are amplitude factors associated with complex-time (i.e. nonhermitian) level crossings. They were measured in an experiment by Zwanziger, J W, Rucker, S P & Chingas, G C 1991 Measuring the geometric component of the transition probability in a two-level system *Phys. Rev.* **A43** 3233-3240

[211] Berry, M V 1990 Budden and Smith's 'additional memory' and the geometric phase *Proc. Roy. Soc. Lond.* **A431** 531-537. Reinterpreting Budden and Smith’s calculation pf phases in general media, in terms of geometric phases for a general (i.e. nonhermitian) operator, using left and right eigenvectors.

[198] Berry, M V 1990 Quantum adiabatic anholonomy, In *Anomalies, phases, defects* (Eds,Bregola, U. M., G.Marmo and G.Morandi) Bibliopolis, Naples, pp. 125-181. See especially lecture 5, where in ‘the third generalization’ the “1/2” in the “n+1/2” for quantization in interpreted in terms of two $\pi/2$ phases associated with circuits round a nonhermitian degeneracy (each WKB turning point is a degeneracy).

[257] Berry, M V 1994 Pancharatnam, virtuoso of the Poincaré sphere: an appreciation *Current Science* **67** 220-223. At the heart of Pancharatnam’s discovery of a remarkable phenomenon in light propagation in absorbing crystals is the behaviour of eigenstates at a nonhermitian degeneracy. The degeneracies are called singular axes, and their birth when hermitian degeneracies split into two is well known in that subject (see Series, G W (Ed.) 1975 *Collected works of S.Pancharatnam* University Press, Oxford.).

[281] Berry, M V & Klein, S 1997 Transparent mirrors: rays, waves and localization *Eur. J. Phys.* **18** 222-228. (near equations 27-29) Stokes’s wrong analysis of propagation through a stack of transparent slabs, predicting that the transmission should be inversely proportional to the

number of slabs (instead of exponentially, as localization theory correctly implies), is interpreted in terms of a nonhermitian degeneracy of the product of intensity (ray) transfer matrices.

[293] Berry, M V & O'Dell, D H J 1998 Diffraction by volume gratings with imaginary potentials *J. Phys. A.* **31** 2093-2101. Atom optics interference phenomena observed by Zeilinger et al, interpreted as degeneracies of the nonhermitian potential icos(x).

[295] Berry, M V 1998 Lop-sided diffraction by absorbing crystals *J. Phys. A.* **31** 3493-3502. More atom optics interference phenomena observed by Zeilinger et al, interpreted as degeneracies of the nonhermitian potential exp(ix).

[350] Berry, M V 2003 Mode degeneracies and the Petermann excess-noise factor for unstable lasers *J. Mod. Opt.* **50** 63-81. Degeneracies, in the complex Fresnel-number plane, of the nonhermitian operator governing the modes of unstable lasers, are responsible for the enormous diffractive line-broadening. A range of characteristic mathematical properties of these degeneracies is involved.

[355] Berry, M V & Dennis, M R 2003 The optical singularities of birefringent dichroic chiral crystals *Proc. Roy. Soc. A.* **A459** 1261-1292. A large generalization of [257] above, where all essential phenomena in crystal optics are shown to be related to degeneracies, which are nonhermitian if the crystal is absorbing. The local model of section 6 describes the splitting as nonhermitian degeneracies are born (see especially figs 2 and 7), and the appendix gathers a number of eigenvector relations that are not widely known (though some must be ‘well known to those who know well’).

[362] Berry, M V & Dennis, M R 2004 Black polarization sandwiches are square roots of zero *J. Optics A* **6** S24-S25. Points out that the nonunitary 2x2 matrices describing arbitrary crystal plates between crossed polarizer and analyzer satisfy $M^2=0$, so all these sandwiches are physical embodiments of nontrivial square roots of zero.

[A] Berry, M V 2004 Physics of nonhermitian degeneracies, to appear in *Czech J. Phys.* A review and summary of some of the above papers.