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Dark energy or modified gravity? (I)

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Dark Energy or Modified Gravity?

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Plan

- Motivation: accelerated expansion of the Universe
- Brief introduction to General Relativity and basic cosmology.
- Acceleration due to cosmological constant and/or Quintessence
- Modified gravity and the accelerated Universe

Brief reminder of General Relativity.

Notations & conventions:

$$\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1) \qquad \hbar = c = 1$$

Gravitational field: $g_{\mu\nu}(x)$

$$ds^2 = g_{\mu\nu}(x)\, dx^\mu dx^\nu \qquad \text{Interval}$$

$$x^0 = t, \quad x^1 = x, \quad x^2 = y, \quad x^3 = z$$

Dynamics is described by the Einstein-Hilbert action:

$$S_{EH} = -\frac{1}{16\pi G_N} \int d^4x \sqrt{-g}\, R$$

$$\frac{1}{16\pi G_N} = \frac{M_P^2}{2} \qquad M_P \sim 10^{19}\,\mathrm{GeV}$$

G_N – Newton's constant

$g \equiv \det g_{\mu\nu}$

$R \equiv g^{\mu\nu} R_{\mu\nu}$ $\quad$ R - Ricci scalar; scalar curvature
$R_{\mu\nu}$ - Ricci tensor.

$$R_{\mu\nu} = \partial_\alpha \Gamma^\alpha_{\mu\nu} - \partial_\nu \Gamma^\alpha_{\mu\alpha} + \Gamma^\alpha_{\mu\nu}\Gamma^\beta_{\alpha\beta} - \Gamma^\alpha_{\mu\beta}\Gamma^\beta_{\nu\alpha}$$

$$\Gamma^\alpha_{\mu\nu} = \frac{1}{2} g^{\alpha\rho}\left(\partial_\mu g_{\rho\nu} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu}\right)$$

$\hookrightarrow$ connection (is not a tensor!)

The Einstein Equation:

Variation of $(S_{EH} + S_m)$ w.r.t. $g^{\mu\nu}$:

$$* \quad \underbrace{R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R}_{\substack{G_{\mu\nu} \\ \text{Einstein tensor}}} = 8\pi G_N T_{\mu\nu}$$

Matter energy-momentum tensor

Basic cosmology:

- Cosmological Principle: the Universe should look the same to all observers.
- Homogeneous & Isotropic Universe, described by Friedmann-Robertson-Walker cosmology

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 \underbrace{d\Omega^2}_{d\theta^2+\sin^2\theta d\varphi^2}\right]$$

k is a number

$k = 0$ flat universe $r \in [0, \infty)$

$k = -1$ open universe $r \in [0, \infty)$

$k = 1$ closed universe $r \in [0, \underbrace{1/\sqrt{k}}_{1}]$

$$R_{(3)} = \frac{6k}{a^2(t)} \quad \text{3d curvature.}$$

(r, θ, φ) are called coordinates that are COMOVING with the expansion, i.e., "COMOVING COORDINATES".

If no physical forces are acting on a probe object, it will maintain constant comoving coordinates.

$a(t)$ is called the "scale factor".

$$\left[\text{Physical distance}\right] = a(t) \left[\text{comoving distance}\right]$$

Matter and radiation in the Universe is modeled as a perfect fluid with energy density ρ and pressure p

$$T^{\mu}{}_{\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & -p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix}$$

Einstein eqs for cosmology (Friedmann)

(00 eq): $$\frac{\dot{a}^2}{a^2} = \frac{8\pi G_N}{3}\rho - \frac{k}{a^2} \qquad (1)$$

(ij eqs): $$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}(\rho + 3p) \qquad (2)$$

Conservation equation: $$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0 \qquad (3)$$

Note 1: (1), (2) & (3) are not independent

(1) + (2) $\Rightarrow$ (3) or (1) + (3) $\Rightarrow$ (2).

One usually takes (1) + (3).

Note 2: (3) is the First Law of Thermodynamics

$$dE + p\,dV = T\,dS \quad \text{with } dS = 0 \qquad (4)$$

$E = \rho a^3 V$ is energy, T temperature, S entropy of fixed comoving volume V. From (4) $\Rightarrow$ (3).

Also $dS = 0$ $S = \text{const}$. $S \sim T^3 a^3 V = \text{const} \Rightarrow$

$$\Rightarrow \quad a \sim 1/T$$

Consider two probe objects separated by a comoving distance r

(i.e. there two objects do not feel any other forces and do not participate in any other motion except the expansion of the Universe).

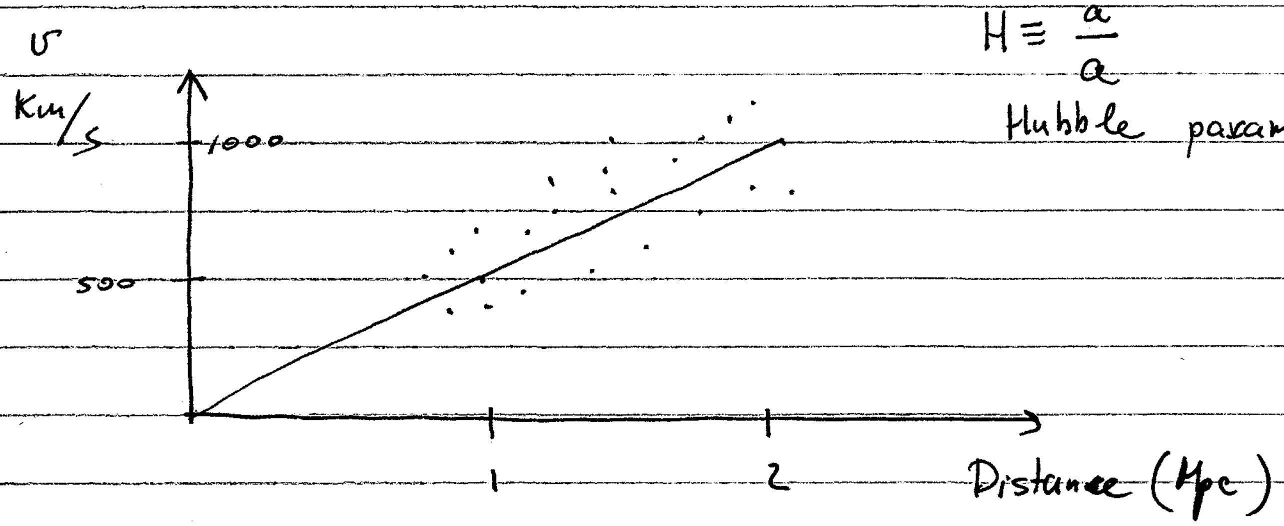
$r_{phys}(t) = a(t)\, r$ $\qquad$ $v = \frac{d}{dt} r_{phys}(t)$

$$v = \dot{a}(t) \cdot r = \frac{\dot{a}}{a} \cdot a r = \frac{\dot{a}}{a} r_{phys} \Rightarrow$$

$\Rightarrow$ $\boxed{v = H\, r_{phys}.}$ Hubble's law.

$H \equiv \frac{\dot{a}}{a}$

Hubble parameter



Below $k=0$.

(i) Matter dominated Universe (MD Univ.)
(dust).

Equation of state $p=0$.

Hubble parameter $H \equiv \frac{\dot{a}}{a}$

$$H^2 = \frac{8\pi}{3} G_N \rho \qquad (*)$$

$$\dot{\rho} + 3H\rho = 0 \qquad (**)$$

From $(**) \Rightarrow d\ln\rho = -3\, d\ln a \Rightarrow \rho = \frac{\rho_0}{a^3}$

From $(*) \Rightarrow \dot{a}^2/a^2 \sim 1/a^3 \Rightarrow a \sim t^{2/3}$

Summary of MD Univ: $\rho \sim \frac{1}{a^3}$; $a \sim t^{2/3}$; $\rho \sim \frac{1}{t^2}$

physical volume expands
as $a^3 \Rightarrow \rho \sim 1/a^3$

Radiation Dominated Universe; (RD) Univ.

Radiation $T^{\mu}_{\ \mu} = 0 \Rightarrow \rho - 3p = 0 \Rightarrow$

$\Rightarrow$ equation of state $\boxed{p = \frac{1}{3}\rho}$

$$H^2 = \frac{8\pi}{3} G_N \rho$$

$$\dot{\rho} + 3H\left(\rho + \frac{1}{3}\rho\right) = 0 \Rightarrow \rho \sim 1/a^4$$

Hence, $H^2 \sim 1/a^4 \Rightarrow a \sim t^{1/2}$

Summary of RD: $\rho \sim 1/a^4$; $a \sim t^{1/2}$ $\rho \sim 1/t^2$

dilution due to expansion $\sim a^3$ and also due to relativistic redshift effect $\Rightarrow \rho \sim 1/a^4$

Note: for a general equation of state $p = w\rho$

$$\rho \sim \frac{1}{a^{3(1+w)}} ; \quad a \sim t^{2/3(1+w)}$$

If we have many component "fluid"

with $\rho_\alpha = \rho_\alpha^\circ / a^{3(1+w_\alpha)}$ $\quad p_\alpha = w_\alpha \rho_\alpha$

$\hookrightarrow$ constants.

Then, it is customary to rewrite the Friedman equation as follows:

$$H^2(z) = H_0^2 \left\{ \Omega_M (1+z)^3 + \sum_\alpha{}' \Omega_\alpha (1+z)^{3(1+w_\alpha)} \right\}$$

Where $H_0 = H(t = t_0 = \text{today}) \simeq 71 \frac{\text{Km}}{\text{S} \cdot \text{Mpc}}$

Redshift: z: $\quad 1+z = \frac{a_0}{a}$

z today $= 0$ $\quad a(\text{today}) \equiv a_0$.

$$\Omega_\alpha = \frac{\rho_\alpha}{\rho_c^\circ} = \frac{\rho_\alpha}{3H_0^2/8\pi G_N}$$

Where $\rho_c \equiv \frac{3H^2}{8\pi G_N}$: $\rho_c(t = t_0) \equiv \rho_c^\circ \simeq 10^{-29} \text{g/cm}$

„today" $\quad t = t_0 \quad z = 0 \quad t_0 \simeq 13.7$ Gyr

$H = H_0$

$$\underline{1} = \Omega_M + \sum_\alpha{}' \Omega_\alpha \equiv \Omega_M + \underline{\underline{\Omega_X}}$$

Data:

$\Omega_M \simeq 0.27 \pm 0.04$

$\Omega_X \simeq 0.73 \pm 0.04$

$\Omega_{total} = 1.02 \pm 0.02$

acceleration

Ω_X : $\quad w_X \lesssim -1/3$!!! $\boxed{\ddot{a} > 0}$

↑ This is not matter ($w = 0$)

~~This is not radiation~~ ~~($w = 1/3$)~~

What is this?

<u>Dark Energy</u>

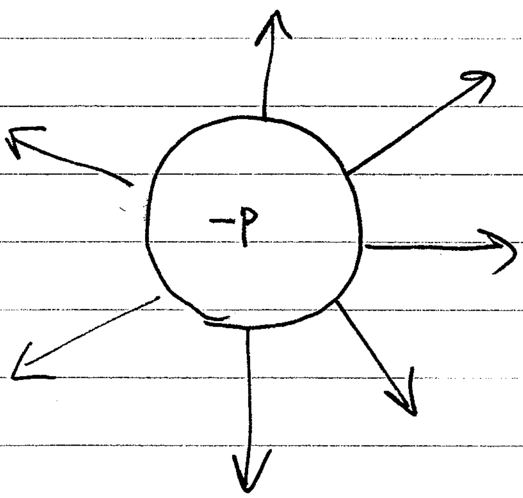
Candidates for Ω_x (dark energy):

- Cosmological constant
- Time dependent cosmological constant (Quintessence field)
- Modified gravity (modified Friedmann equation)

Instead of Dark Energy.

$$p = -\rho \qquad T^{\mu}{}_{\nu} = \begin{pmatrix} \rho & & & \\ & \rho & & \\ & & \rho & \\ & & & \rho \end{pmatrix}$$

$$T_{\mu\nu} = \begin{pmatrix} \rho & & & \\ & -\rho & & \\ & & -\rho & \\ & & & -\rho \end{pmatrix}$$



$$\dot{\rho} + 3H(p+\rho) = 0$$

$$\Downarrow$$

$$\dot{\rho} = 0$$

$$\boxed{\rho = \text{const.}}$$

$a(t)$ grows $\Rightarrow$ volume grows $\rho = \text{const}$

$\Rightarrow$ energy $E \sim \rho a^3$ grows (?)

This is because negative pressure does positive work on expansion.

$dA = -p\,dV > 0$ when $-p > 0$ and $dV > 0$

$$H^2 = \frac{8\pi G_N}{3}\rho$$ since $\rho = \text{const}$

$H = \text{const}$.

$$\Rightarrow \frac{\dot a}{a} = H$$

$$\boxed{a \sim e^{Ht}}$$

Can explain both: inflation in early Universe

if $\underline{\underline{H \gg 10^{-3}\,eV}}$

and the presend day acceleration

if $H \sim H_0 \sim \underline{\underline{10^{-42}\,GeV}}$

Cosmological constant in Einstein's equation:

$$G_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

Typical scales:

$$\Lambda \sim H_0^2 \sim \left(10^{-42} \text{GeV}\right)^2$$

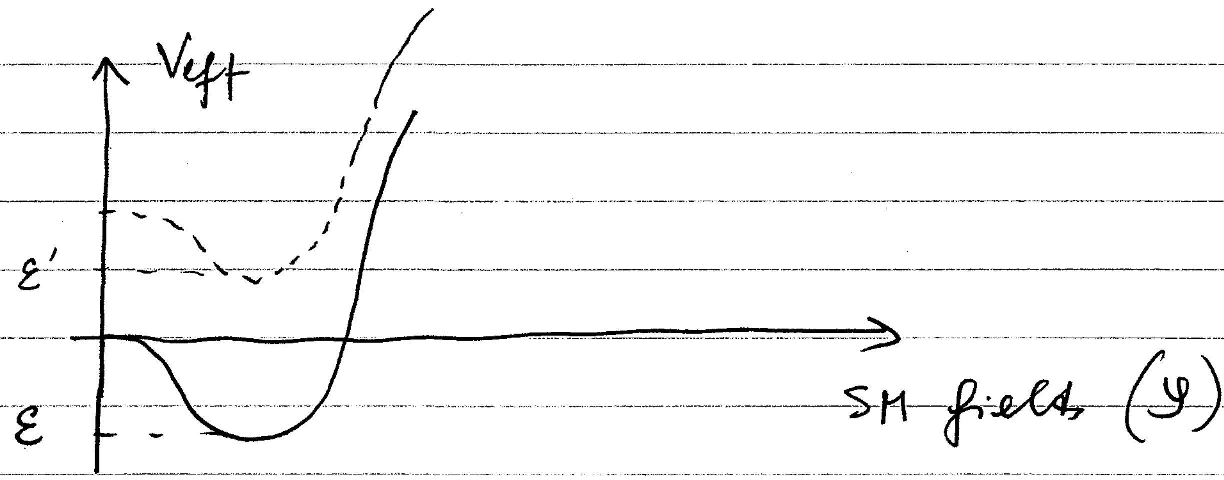
$$\rho_\Lambda \sim \frac{H_0^2}{8\pi G_N} \sim M_p^2 H_0^2 \sim \left(10^{-4} \text{eV}\right)^4$$

Hence, a new energy density scale $\sim 10^{-4}$ eV too low from the particle physics point of view

$$\rho_\Lambda^{\text{particles}} \gtrsim \left(10^{12} \text{eV}\right)^4$$

Cosmological constant problem: $\left(?\right)$

Digression to Particle Physics and the Vacuum Energy Problem:



Many different contributions to vacuum energy:

QCD $\sim (1\,\text{GeV})^4$

~~STP~~

Electroweak physics $\sim (100\,\text{GeV})^4$

Grand Unification $(10^{16}\,\text{GeV})^4$
~~SUSY~~ at GUT

~~They conspire to cancel to $(10^{-4}\,\text{eV})^4$~~

Quantum fluctuations.

$$\rho \sim \sum_i \int_0^{\mu_{UV}} d^3\vec{k}\ \sqrt{k^2+m_i^2}$$

$$\sim \mu_{UV}^4$$

μ_{UV} can be the scale of supersymmetry breaking

$$\mu_{UV} \gtrsim \text{TeV} \quad \Rightarrow$$

$$\rho \gtrsim \left(10^3\ \text{GeV}\right)^4$$

All these contributions should conspire to cancel down to $\left(10^{-4}\ \text{eV}\right)^4$!!!

Extreme fine tuning.

Quintessence

$$\frac{\ddot{a}}{a} = -\frac{4}{3}\pi G_N (\rho + 3p) \quad \Rightarrow$$

For $\ddot{a} > 0$ we need $\underline{p < -\frac{1}{3}\rho}$

Introduce a scalar field.

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \varphi)^2 - V(\varphi)$$

What ~~should~~ kind of potential can give $w < -1/3$?

$$T_{\mu\nu} = \partial_\mu \varphi \, \partial_\nu \varphi - g_{\mu\nu} \mathcal{L}$$

Assume time dependent, homogeneous $\varphi(t)$

$$T_{00} = \dot{\varphi}^2 - \frac{1}{2}\dot{\varphi}^2 + V = \frac{1}{2}\dot{\varphi}^2 + V$$

$$T_{ij} = -g_{ij}\left(\frac{1}{2}\dot{\varphi}^2 - V\right) = \delta_{ij}\left(\frac{1}{2}\dot{\varphi}^2 - V\right)$$

$$\rho = \frac{1}{2}\dot{\varphi}^2 + V$$

$$p = \frac{1}{2}\dot{\varphi}^2 - V \qquad \ddot{\phi} + 3H\dot{\phi} + V' = 0$$

if potential energy dominates over the kinetic energy, then $w = \frac{p}{\rho} \sim -1$!

$$w = \frac{\dot{\varphi}^2/2 - V}{\dot{\varphi}^2/2 + V}$$

$$\frac{w-1}{2}\dot{\varphi}^2 = -(1+w)V$$

$$\rho = \frac{\dot{\varphi}^2}{2} + V = \frac{\dot{\varphi}^2}{2} + \frac{\dot{\varphi}^2}{2}\frac{1-w}{1+w} = \dot{\varphi}^2\frac{1}{1+w}$$

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G_N}{3}\rho = \frac{8\pi G_N}{3(1+w)}\dot{\varphi}^2$$

$$d\ln a = \sqrt{\frac{8\pi G_N}{3(1+w)}}\, d\varphi$$

$$a = c\, e^{\sqrt{\frac{1}{3(1+w)}}\,\varphi/M_p}$$

$$\rho = \rho_0 \frac{1}{a^{3(1+w)}} \sim e^{-\sqrt{3(1+w)}\,\varphi/M_p}$$

$$\rho = \dot{\varphi}^2 \frac{1}{1+w} = \frac{1}{1+w} \cdot \frac{2(1+w)}{1-w} V$$

$$V = \frac{1-w}{2} \rho$$

$$\boxed{V = V_0 \, e^{-\sqrt{3(1+w)}\,\varphi/M_p}}$$

$\hookrightarrow$ this potential gives rize to the accelerated Universe.

$\varphi \ll M_p \quad \Rightarrow$

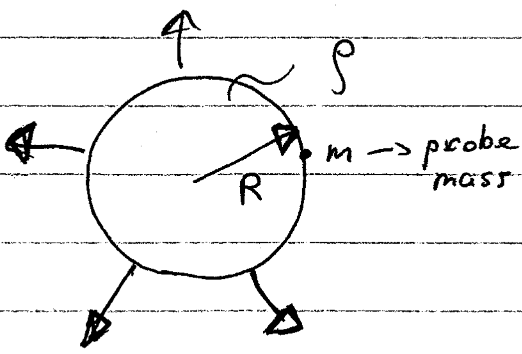
$V \sim V_0 \quad \Rightarrow$

$$V_0 \sim \rho_c^0 \sim \left(10^{-4}\,eV\right)^4$$

this type of potential is not easy to stabilize w.r.t. quantum loop corrections.

Newtonian Cosmology:

Gravitational Force Law determines the evolution.



$$\ddot{R} = \frac{F}{m} = -\frac{G_N M}{R^2}$$

$$M = \frac{4}{3}\pi R^3 \cdot \rho \qquad R = a(t)\, r_{\bullet}$$

↳ analog of comoving coordinate

Combining the above equations:

$$\frac{\ddot{a}}{a} = -\frac{4}{3}\pi G_N \rho(t)$$

Moreover, $E = \text{const}$.

$$E = M = \frac{4}{3}\pi R^3 \rho \implies \rho \sim 1/a^3$$

Solutions: $a \sim t^{2/3}$ $\qquad \ddot{a} < 0$

this is matter dominated universe.

Modified force:

Force modification.

$$\frac{F}{m} = -\frac{G_N M}{R^2} + m_c^2 R\, g(R)$$

$$\begin{cases} \ddot{R} = -\frac{4}{3}\pi G_N \rho R + m_c^2 g(R) R & \text{Newton's eq.} \\ d(\rho a^3) = 0 & \text{Conservation eq.} \end{cases}$$

For simplicity $g = 1$

(i) Early times $t \ll t_c$ so that

$G_N \rho \gg m_c^2$

$\ddot{R} \simeq -\frac{4}{3}\pi G_N \rho R$ Hence, $a \sim t^{2/3}$

Matter domination, no acceleration.

(ii) Late time $t \gtrsim t_c$ when $G_N \rho \lesssim m_c^2$

$\ddot{R} \simeq m_c^2 R \Rightarrow a \sim e^{m_c t}$ Accelerated expansion.

$\ddot{a} \sim m_c^2 > 0$