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Non-perturbative phenomena in gauge theories (IV)

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Please note: These are preliminary notes intended for internal distribution only.

(III - 1)

Baryon-number non-conservation in high-energy COLLISIONS.

Collision of two particles at $E \gtrsim E_{sph}$.

Energetically it is possible to overcome the barrier between neighboring vacua WITHOUT TUNNELING

Is this possible dynamically?

- Want to pass "near" sphaleron.

  Sphaleron: ~~that~~ thick configuration,

  size $r_{sph} \sim m_W^{-1}$

  Wavelength of an incoming particle small:

$$\lambda \sim \frac{1}{E} \lesssim \frac{1}{E_{sph}} \sim \frac{\alpha_W}{m_W} \ll r_{sph}$$

Mismatch. Does it result in exponential suppression?

Also "technicality": semiclassical object, sphaleron; semiclassical transition process,

BUT two incoming particles are NOT semiclassical.

- Baryon-number violating cross section indeed grows with energy <u>at low energies</u>.

Perturbation theory about an instanton

* Forget about fermions (they are indeed subdominant).

* Omit all indices, etc.

* Consider process ~~WW → lots of W's~~ ~~Q=1~~

$$WW \longrightarrow N \quad W's$$

$$\xrightarrow{\qquad\qquad} Q=1$$

from one vacuum to its neighbor.

The amplitude can be extracted from Green's function via LSZ procedure. Schematically

$$A(p_1, p_2 \to q_1, q_2, \dots, q_N)$$

$$= (p_1^2 - m_W^2)(p_2^2 - m_W^2)(q_1^2 - m_W^2)\dots(q_N^2 - m_W^2)\, G(p_1, p_2, \dots, q_N)\Big|_{\text{ON SHELL}}$$

Here

$$G(p_1,\dots,q_N) = \langle A(p_1)\, A(p_2) A(q_1) \cdots A(q_N) \rangle$$

is the Green's function with $(N+2)$ legs.

In other words, ~~amplity~~ amplitude is Residue of $G(p_1,\dots,q_N)$ at multiple pole $p_1^2 = m_W^2$, $p_2^2 = m_W^2$, $\dots$, $q_N^2 = m_W^2$.

Path integral for $G(x_1, x_2 ; y_1 \cdots y_N)$

$$G = \int \mathcal{D}A\, \mathcal{D}\phi\, e^{-S[A,\phi]}\, A(x_1) A(x_2) A(y_1) \dots A(y_N)$$

<u>NB</u>: calculate Euclidean Green's function, then analytically continue to Minkowski

Instanton = local minimum of $S[A,\phi]$ (modulo complications due to Higgs field)

Saddle-point calculation: take $A(x_1) \dots A(y_N)$ at saddle point.

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Instanton position X is arbitray $\Rightarrow$ integrate over X

(ignore integration over instanton size for the sake of ~~press~~ presentation).

$\Downarrow$

$$G^{inst}(x_1, x_2, \dots y_N) = \int d^4X \, A^{inst}(x_1 - X) \, A^{inst}(x_2 - X) \times$$
$$\times A^{inst}(y_1 - X) \dots A^{inst}(y_N - X) \cdot \mathrm{Det} \cdot e^{-S_{inst}}$$

Here $S_{inst} = \frac{8\pi^2}{g^2}$ + Higgs contribution

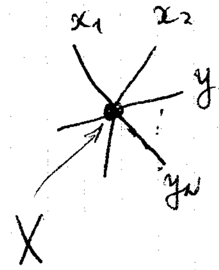
Det = integral over Gaussian fluctuations about instanton.

Immediate observation

G has structure of point-like Green's function.

For point-like interaction $\int \lambda \, A^{N+2}(x) \, d^4X$

the Green's function (at tree level) is



$$\int dX \lambda \, D_0(x_1 - X) \, D_0(x_2 - X) \, D_0(y_1 - X) \dots D_0(y_N - X)$$

Factorization up to integration over position of vertex (i.e. momentum conservation).

Same with G^{inst}

Point-like interaction $\Longleftrightarrow$ ~~inst~~ increasing cross-section.

* Fourier transform $A^{inst} \Rightarrow \tilde{A}^{inst}(p)$
* Residue of $\tilde{A}^{inst}(p)$ on shell

$$\text{Res } \tilde{A}^{inst}(p) \propto \vec{p}$$

$$\Downarrow$$

$$\text{Amplitude } (2 \to N) = \text{const}(N)\, e^{-\frac{8\pi^2}{g^2}} |\vec{p}_1||\vec{p}_2| \cdot |q_1| \cdots |q_N|$$

Point-like.

$$\Downarrow$$

Cross section, integrated over $\sqrt{\text{final}}$ momenta of W's

$$\sigma^{inst}_{2 \to N}(E) \propto e^{-\frac{16\pi^2}{g}} E^{4N}$$

Increases fast with energy.

Inclusive instanton cross section (sum over N) grows exponentially,

$$\sigma^{inst}_{tot}(E) \propto e^{-\frac{16\pi^2}{g^2}} \cdot e^{\frac{16\pi^2}{g^2} \cdot \frac{9}{8}\left(\frac{E}{E_0}\right)^{4/3}}$$

where $E_0 = \sqrt{6}\,\dfrac{\pi m_w}{\alpha_w} \sim E_{sph}$.

This is valid only for $E \ll E_0$. Series in $\left(\frac{E}{E_0}\right)^{2/3}$

in EXPONENT!

How to calculate σ^{inst} at $E \gtrsim E_{sph}$?

- σ^{inst}_{tot} has semiclassically looking form

$$\sigma^{inst}_{tot} = e^{-\frac{16\pi^2}{g^2} F\left(\frac{E}{E_0}\right)} = e^{-\frac{4\pi}{\alpha_W} F\left(\frac{E}{E_0}\right)}$$

[Perturbation theory about instanton:

$$F\left(\frac{E}{E_0}\right) = \left[1 - \frac{9}{8}\left(\frac{E}{E_0}\right)^{4/3} + \frac{9}{16}\left(\frac{E}{E_0}\right)^2 + \dots\right]$$

]

But incoming state is not semiclassical.

WAY TO DEAL WITH THIS:

- Consider inintial state WITH NUMBER of particles $N = \frac{\nu}{\alpha_W}$

  At finite ν IT is semiclassical

  $\nu \equiv \alpha_W N$: free parameter as yet.

- Fix energy and number of incoming particles, sum over all such initial states, and sum over all final states

PROBLEM is EXPLICITLY SEMICLASSICAL.

Calculate, in semiclassical way, exponential factor

$$\sigma^{inst}_{N\to ANY} = (\ ) \, e^{-\frac{4\pi}{\alpha_W} F\left(\frac{E}{E_0},\ \alpha_W N\right)}$$

$$\uparrow$$
$$\nu$$

- Send $\nu \equiv \alpha_W N \to 0$ $\Rightarrow$ Get $\sigma^{inst}_{few\to ANY} = (\ ) e^{-\frac{4\pi}{\alpha_W} F_{HG}(E}$

Claim: exponent for (few → ~~ma~~ ANY) is a limit of exponent for multiparticle incoming state:

$$F_{HG}(E) = \lim_{\nu \equiv \alpha_W N \to 0} F(E, \nu) .$$

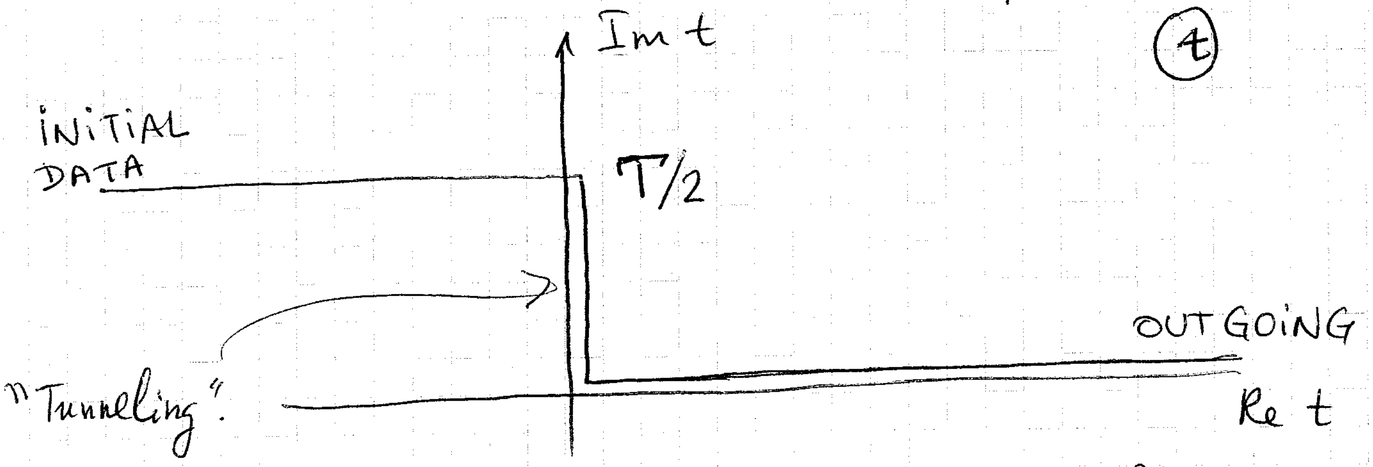
- Procedure explicitly checked in quantum mechanics of two degrees of freedom.
- DIFFICULT BUT DUABLE Numerically in field theory (at least in $O(3)$-symmetric set up; s-wave scattering).

Ingredients of the calculation of

$F(E, N)$

in semiclassical manner:

* Contour in time plane



* Field equation on this contour

* Boundary conditions:

- complex in the past
- Reality in the future

* Solution is complexified!
Every real field has to be treated as complex.

Still the problem is classical.

Today: tractable in $O(3)$-symmetric case (S-wave scattering).

E.g. Higgs field

$$\phi = \left(\mu(r,t) + i\nu(r,t) \cdot \vec{\tau}\vec{n}\right)\begin{pmatrix} 0 \\ v \end{pmatrix}$$

Define $\chi = (\mu + i\nu)(r,t)$

Phase of this field reflects topology.

Fig. 1.
Example of classical solution.

Imaginary time part of contour is inclined.

Fig. 2
Solutions at different energy (and hence T) and number of incoming particles

Fig. 3
Set of solutions in (E, N)-plan

New regime beyond the blue line:
Left of it: tunneling
Right: rearrangement, climb on the sphaleron.

~~Vat~~ Occurs in all systems studied so far (quantum mechanics, ~~fals~~ induced false vacuum decay, soliton pair creation by highly energetic particles).

Fig. 4.

Extrapolation down to $N\alpha_w \to 0$.

NB: $\frac{E\alpha_w}{M_w} = 8 \Rightarrow E \simeq 20\,\text{TeV}$
(need 100 TeV hadron machine to reach)

Still suppression is very strong.

Fig. 5

Bound on exponent of suppression.

Pessimistic conclusion: EW baryon # non-conservation ~~not~~ unlikely to observe in high-energy collisions.

INTERESTING DYNAMICS
— BUT UNOBSERVABLE EXPERIMENTALLY.

Methods applicable to solid state/quantum chemistry/atomic physics

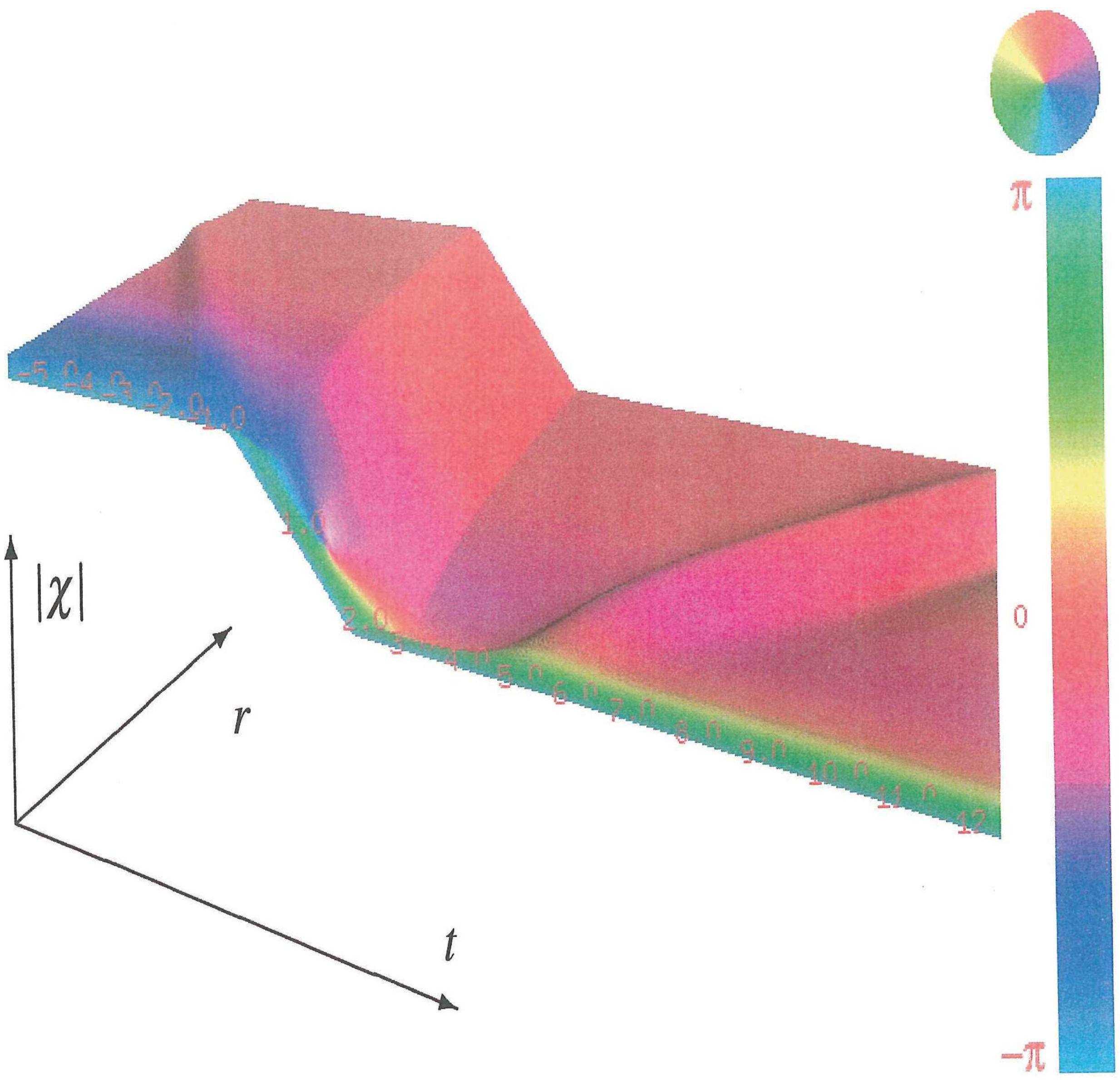


Fig. 1

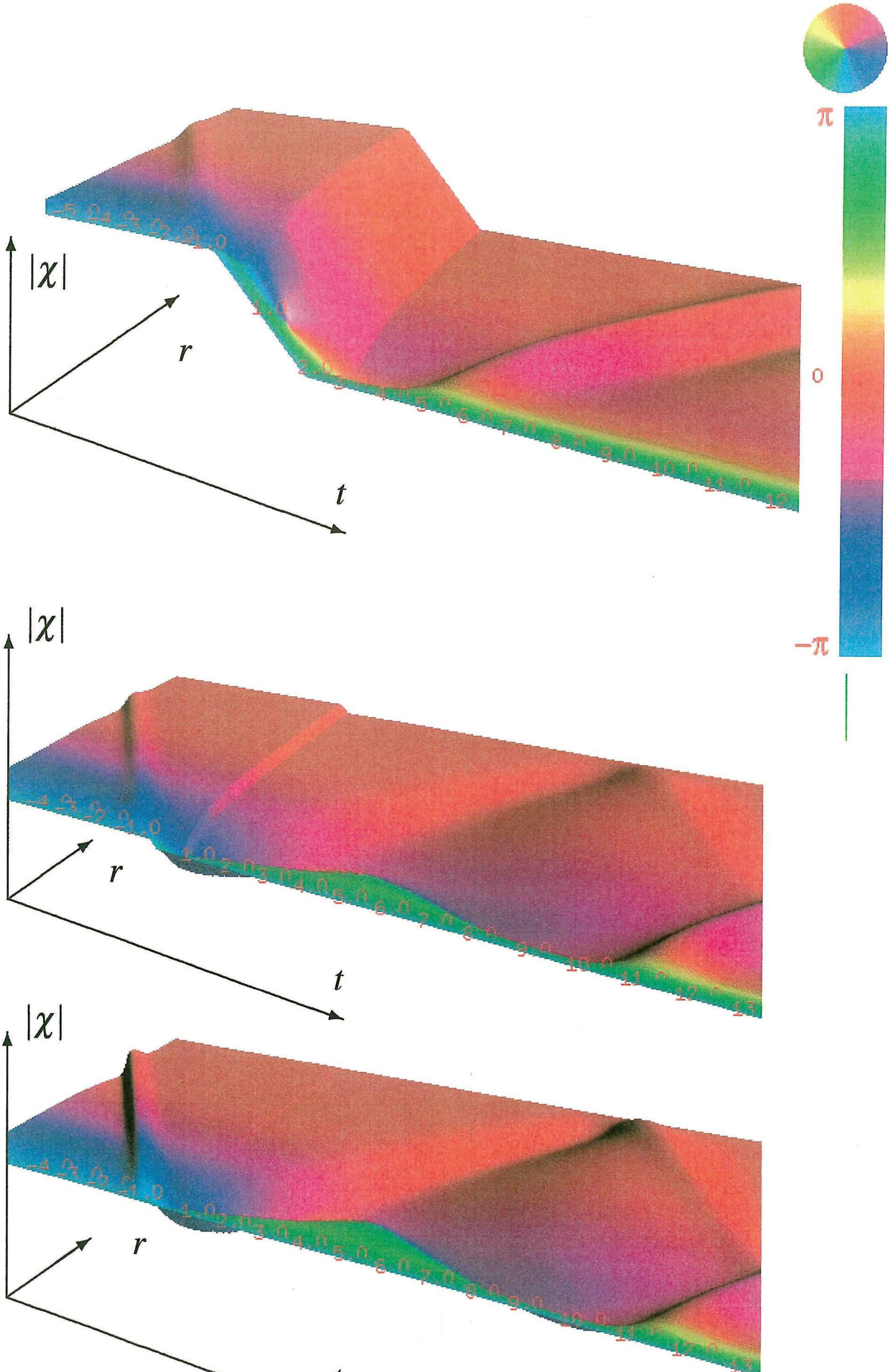


Fig. 2

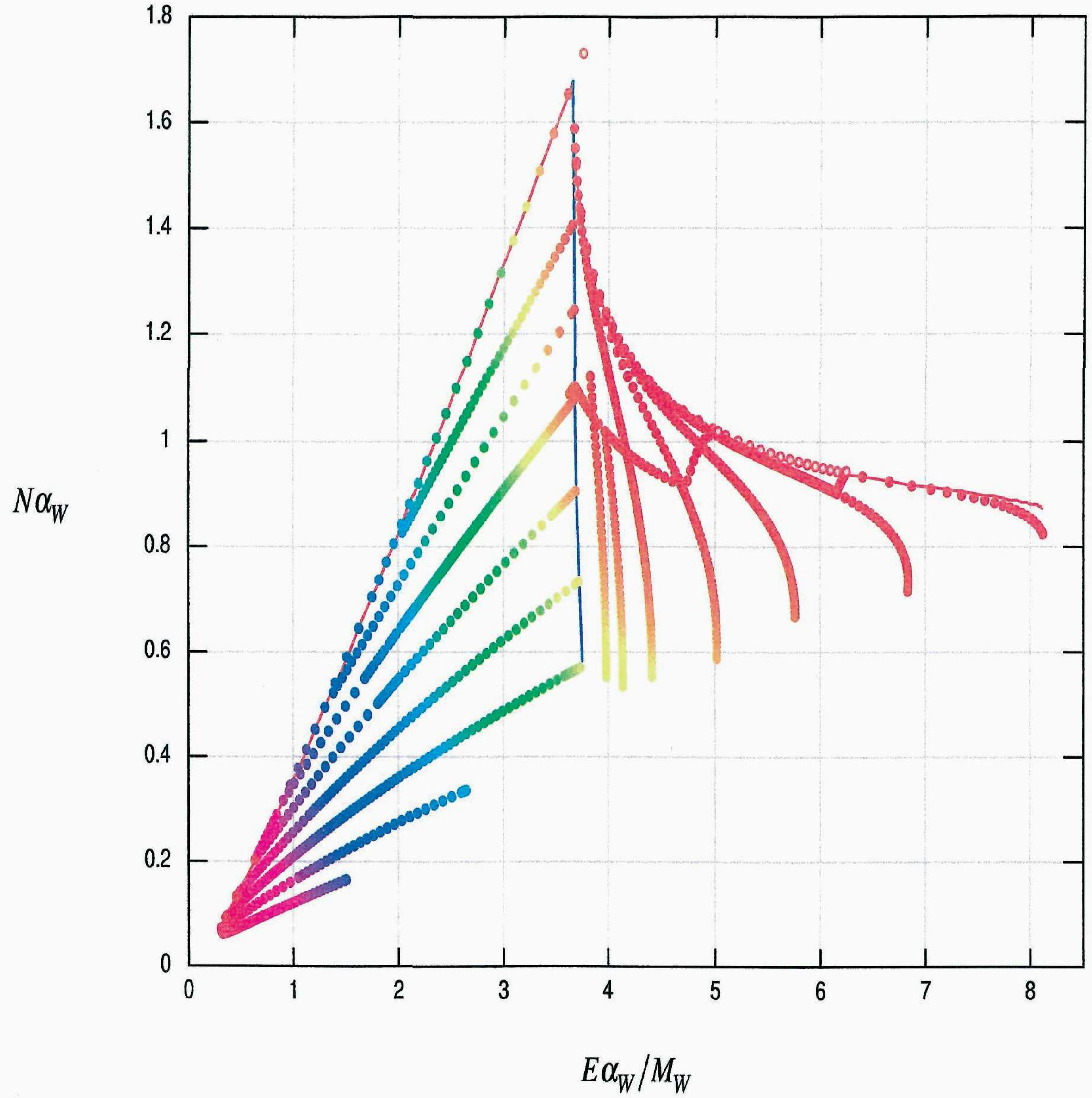


Fig. 3

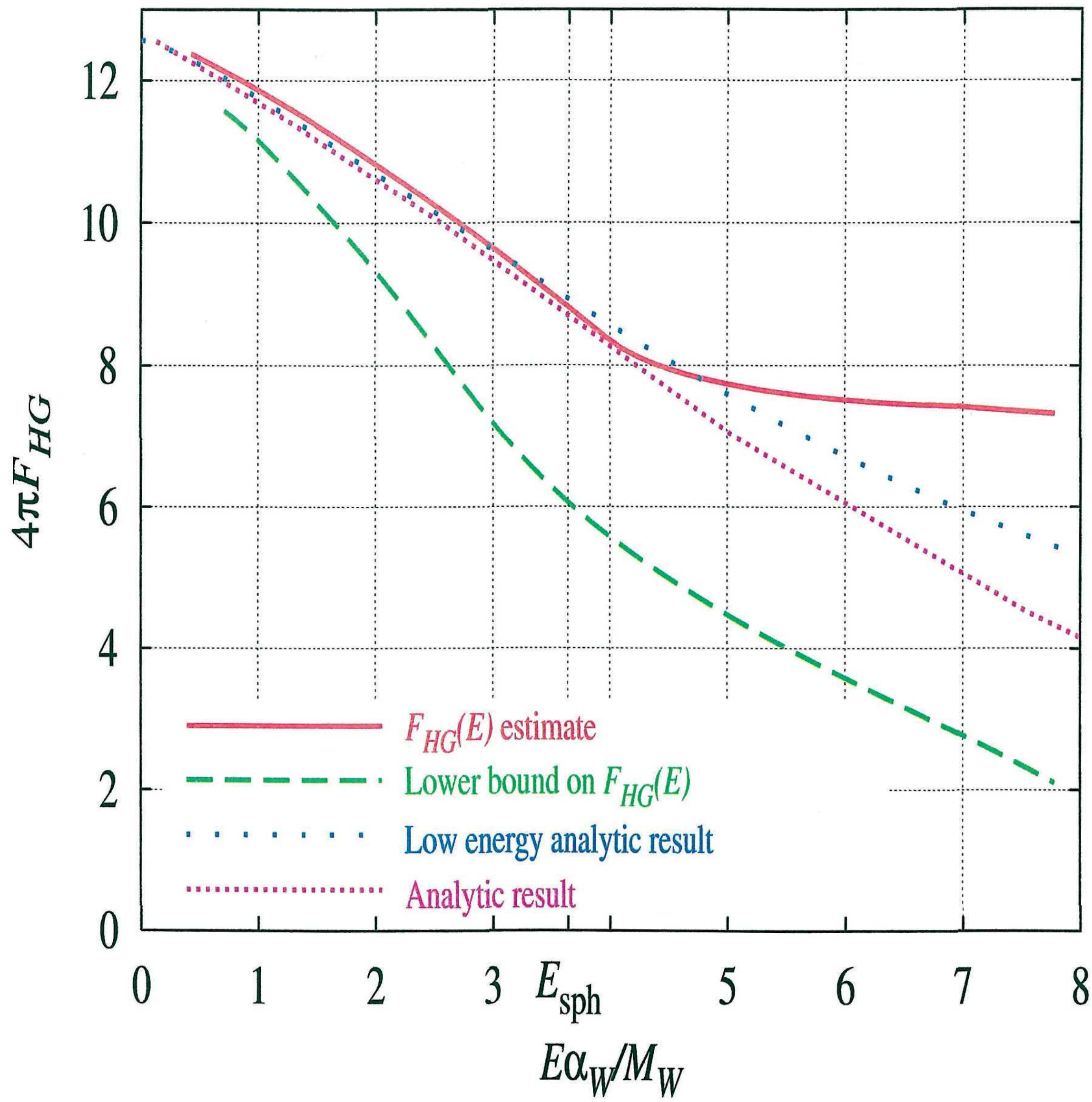
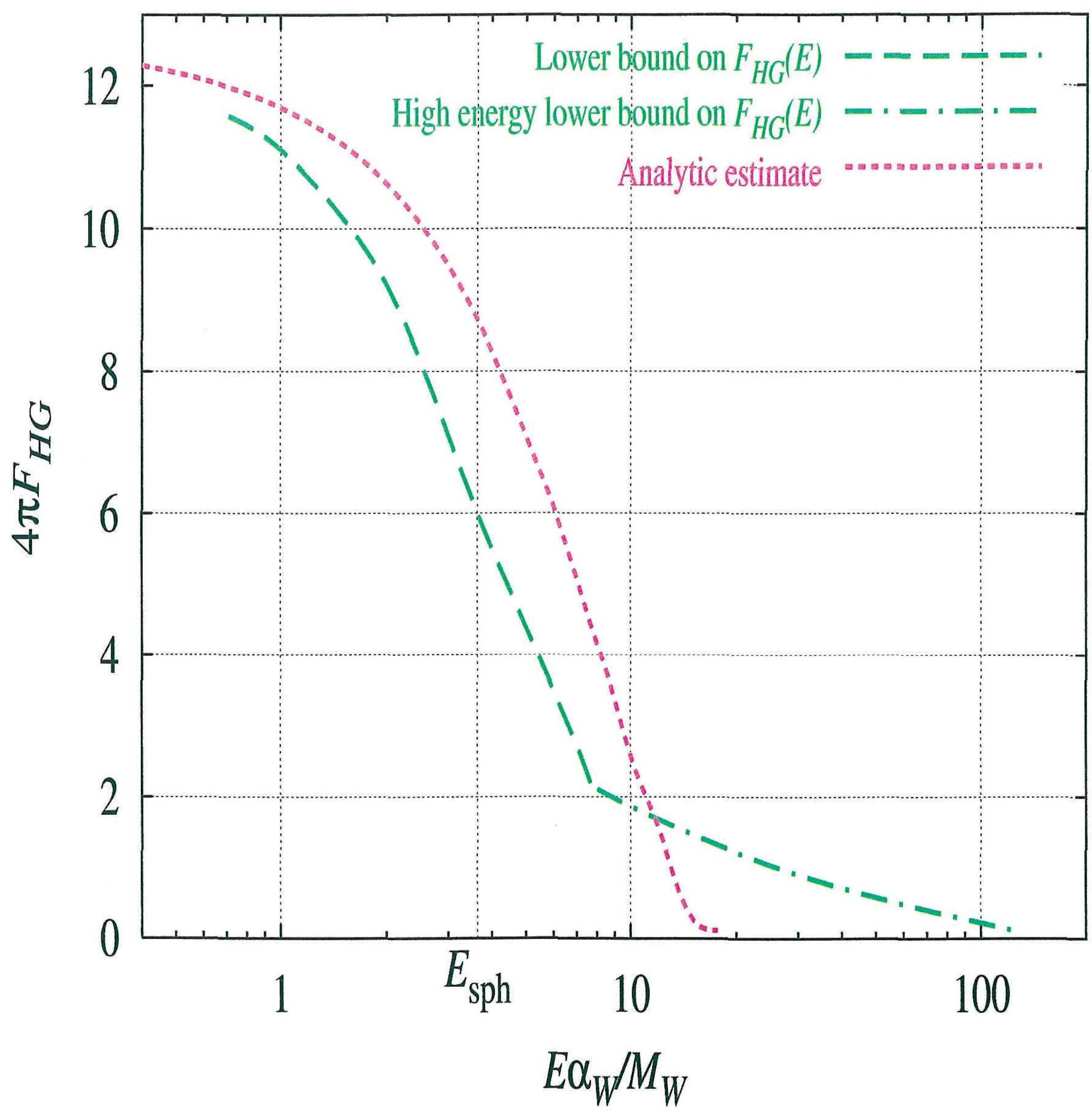

12
10
8
6
4
2
0
$4\pi F_{HG}$
0
1
2
3
$E_{\rm sph}$
5
6
7
8
$E\alpha_W/M_W$
$F_{HG}(E)$ estimate
Lower bound on $F_{HG}(E)$
Low energy analytic result
Analytic result

Fig. 4



Lower bound on $F_{HG}(E)$
High energy lower bound on $F_{HG}(E)$
Analytic estimate
12
10
8
6
4
2
0
$4\pi F_{HG}$
1
$E_{\rm sph}$
10
100
$E\alpha_W/M_W$

fig. 5

L1

Notes on literature.

- Instanton solution found by Belavin, Polyakov, Schwarz, Tyupkin 1975
  The story begins.

- Non-conservation of fermion quantum numbers, including baryon number in EW:
  't Hooft '76

- Interpretation in terms of gauge theory vacua and tunneling between them:
  Jackiw, Rebbi '76
  Callan, Dashen, Gross '76.

- Sphaleron and height of barrier in electroweak theory
  Manton '83
  Klinkhamer, Manton '84

- Baryon number non-conservation at high temperature and relevance to baryogenesis
  Kuzmin, V.R., Shaposhnikov '85

- Growth of instanton cross section with energy
  Ringwald '90
  Espinosa '90

L2

- Exponential behaviour of total instanton cross section

  McLerran, Vainshtein, Voloshin '90

  Zakharov '90

  Khlebnikov, V.R., Tinyakov '90

  Porrati '90

  Yaffe '90

- Semiclassical technique for calculating the exponent for instanton-like cross sections:

  V.R., Son, Tinyakov '92

- Actual calculation

  Bezrukov, Levkov, Rebbi, V.R., Tinyakov '2003.

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Books: R. Rajaraman Solitons and Instantons (North-Holland, 1982)

V.R. Classical theory of gauge fields (Princeton U., 2002)