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Non-perturbative phenomena in gauge theories (I)

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NON-PERTURBATIVE PHENOMENA IN GAUGE THEORIES - I.

I. Preliminaries.

- Early Universe: arena of quantum field theories at high temperatures.

  Relevant temperature scales = mass scales inherent in microscopic theories

E.g. - strong interactions

$$T \gtrsim 100\ \mathrm{MeV} \div 1\ \mathrm{GeV}$$

- Electroweak interactions:

$$T \gtrsim 100\ \mathrm{GeV}$$

- Grand Unified interactions

$$T \gtrsim 10^{15}\ \mathrm{GeV}$$

(problematic in inflation theory).

- These temperatures are "low" in the following sense: particle interaction rates are typically higher than the expansion rate of the Universe.

$\Gamma \equiv$ Particle interaction rate ~ coupling constant α $\times T$

$$\Gamma = \alpha T$$

Expansion rate $H = \dfrac{T^2}{M_{Pl}^*}$ $\left[= \left(\frac{8\pi}{3} G\rho\right)^{1/2} \right]$

where $M_{Pl}^* = \left(1.66\sqrt{g_*}\right)^{-1} M_{Pl}$

g_* = number of degrees of freedom ~ 100

I - 2

$$M_{Pl}^* \sim 10^{18}\ \text{GeV}$$

$$\Downarrow$$

$$\Gamma \gg H \implies \alpha T \gg \frac{T^2}{M_{Pl}^*} \quad \text{i.e. } T \ll M_{Pl}^* \cdot \alpha$$

For $\alpha = 10^{-2}$ (sometimes 10^{-4}, depending on process)

$$\Gamma \gg H \quad \text{at} \quad T \gg 10^{16}\ (\text{or } 10^{14})\ \text{GeV}$$

$$\Downarrow$$

Thermal equilibrium at $T \gg 10^{14}$ GeV, unless something special happens.

* Thermal equilibrium: not very interesting apparently.

~~Most~~ But even for thermal equilibrium one has to understand

WHICH QUANTUM NUMBERS ARE CONSERVED.

Physically most interesting: baryon and lepton numbers

* Most interesting are

OUT-OF-EQUILIBRIUM PHENOMENA.

An example: phase transitions.

(Non-) Conservation of quantum numbers $\Rightarrow$ Non-perturbative aspects of quantum field theories IMPORTANT

Phase transitions $\nearrow$

In vacuo: non-perturbative phenomena relevant in strongly coupled theories only
Physical example: QCD

High T: Also in weakly coupled theories, in particular, in EW theory.

These lectures:

I) ELECTROWEAK BARYON NUMBER NON-CONSERVATION AT HIGH TEMPERATURES (+ HIGH ENERGY COLLISIONS)

II) ELECTROWEAK PHASE TRANSITION

COSMOLOGICAL APPLICATION:
BARYON ASYMMETRY OF THE UNIVERSE.

In a SENSE, introduction to course by T. Hambye.

TOPOLOGY OF GAUGE THEORIES AND NON-CONSERVATION OF FERMION QUANTUM NUMBERS.

Ingredients:

- Topology of gauge theories; vacua
- Classical solutions in gauge-Higgs theories (bosonic sector)
- Mechanism of non-conservation of fermion quantum numbers.
- Selection rules.

All these are most explicit in (1+1)-dimensional Abelian Higgs model.

Let us work everything out in this model, and then consider physical case of Electroweak Theory.

- Abelian Higgs model in (1+1) dimensions.

Coordinates $x^0 \equiv t$, $x^1 \equiv x$

NB: To be infrared safe, assume space to be large circle of ~~rad~~ length L.

→ Periodic boundary conditions

$$\text{Field}\left(x=-\frac{L}{2}\right) = \text{Field}\left(x=+\frac{L}{2}\right).$$

(Comment on why this is unimportant: later on).

Fields: (1) A_μ, $\mu = 0, 1$

$U(1)$ gauge field, "electromagnetism"

NB: The only non-zero field strength F_{01}:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$F_{01} = \partial_0 A_1 - \partial_1 A_0 \quad ; \quad F_{10} = -F_{01}.$$

(2) Higgs scalar $\Phi(x)$.

Gauge transformation:

$$A_\mu(x) \Rightarrow A_\mu(x) + \frac{1}{e}\partial_\mu \alpha(x)$$

$$\Phi(x) \Rightarrow e^{i\alpha(x)}\Phi(x).$$

$\alpha(x)$ = arbitrary real gauge function

Shall include fermions later on.

Action (bosonic part)

$$S_B = \int d^2x \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \phi)^* D^\mu \phi - V(\phi) \right]$$

NB: Minkowski signature, $ds^2 = (dx^0)^2 - (dx^1)^2$

$$\eta_{\mu\nu} = (-1, 1).$$

Gauge invariance $\Longleftrightarrow$ $D_\mu \phi = \partial_\mu \phi - ie A_\mu \phi$

Higgs potential

$$V(\phi) = \frac{\lambda}{2} \left(\phi^* \phi - v^2 \right)^2$$

Perturbation theory

Vacuum: $A_\mu = 0$

$\Phi = v$

Bosons: A_μ : massive ; $m_V = \sqrt{2}\, e v$

Higgs boson $|\Phi| - v$; $m_H = \sqrt{2\lambda}\, v$

Non - perturbative treatment.

Convenient gauge $A_0 = 0$.

$$\Downarrow$$

$$F_{01} = \partial_0 A_1$$

$$D_0 \phi = \partial_0 \phi$$

$$D_1 \phi = \partial_1 \phi - ie A_1 \phi$$

$$S_B = \int dx\, dt \left[+ \frac{1}{2} (\partial_0 A_1)^2 + (\partial_0 \phi)^* (\partial_0 \phi)^2 - |D_1 \phi|^2 - V(\phi) \right]$$

Canonical form: $S = \int dx dt$ ["Kinetic" - "Potential"]

- Energy $= \int dx \left[\frac{1}{2} (\partial_0 A_1)^2 + |\partial_0 \phi|^2 + |D_1 \phi|^2 + V(\phi) \right]$

This and any other theory:
First question to ask: what are VACUA?

Classical vacua $\equiv$ ground states of classical theory $\equiv$ minima of energy.

$A_1 = 0$, $\phi = v$ $\Rightarrow$ $E = 0$, minimum of E.

Are there other configurations with $E = 0$?

I - 8

$$E = 0$$
$$\Downarrow$$

* $\partial_0 A_1 = 0$; $A_1(x)$, independent of t.

* ~~Parametrize by $\alpha(x)$~~

$$\cancel{A_1 = \frac{1}{e} \partial_1 \alpha(x)}$$

* $V(\phi) = 0$; $|\phi| = v$; $\phi = e^{i\beta(x^1)} \cdot v$

* $\partial_0 \phi = 0 \Rightarrow \beta(x)$, independent of t

$$D_1 \phi \equiv \partial_1 \phi - ieA_1 \phi = 0$$
$$\Downarrow$$
$$i\,\partial_1 \beta(x) - ieA_1(x) = 0$$
$$\Downarrow$$
$$A_1 = \frac{1}{e} \partial_1 \beta(x)$$

Thus, $E = 0$

$$\Updownarrow$$

$$A_1 = \frac{1}{e} \partial_1 \beta(x)$$
$$\phi = e^{i\beta(x)} \cdot v$$

Gauge transformation of vacuum $A_1 = 0$, $\phi = v$.

Apparently triviality.

NOT QUITE!

I-9

Periodicity:

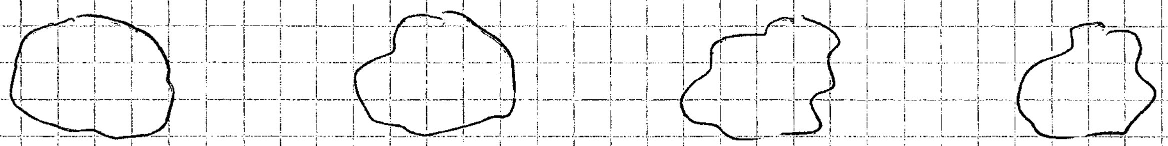
$$\phi\left(-\frac{L}{2}\right) = \phi\left(\frac{L}{2}\right)$$

$$\Downarrow$$

$$\beta\left(+\frac{L}{2}\right) - \beta\left(-\frac{L}{2}\right) = 2\pi n \quad , \; \underline{\underline{n = \text{integer.}}}$$

A_1 is then periodic.

Topological sectors in space of vacua



$n=-1$ $\quad$ $n=0$ $\quad$ $n=1$ $\quad$ $n=2$

Disjoint sectors labeled by n.

TRAVEL <u>WITH $E=0$</u> impossible from one sector to another!

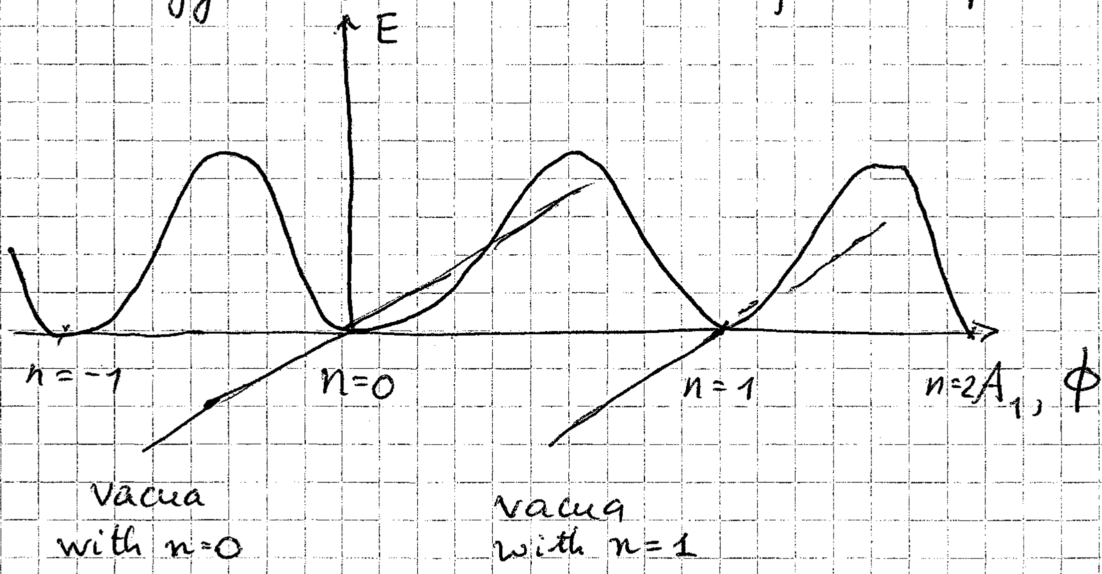
NB: Travel within each sector $\Leftrightarrow$ perturbation theory (gauge fixing, etc.)

Qn: Are there paths of <u>finite energy</u> between sectors with different n?

If there are, then
- Tunneling at low energies/temperatures
- "Motion" over barrier at high energies/temperatures.

If there are:

Energy functional in field space



Relevant quantities:

Low energies $\Rightarrow$ tunneling exponent

High energies $\Rightarrow$ height of barrier

Before addressing these:

TOPOLOGY

Vacuum: $\quad \omega(x) = e^{i\beta(x)} \in U(1)$

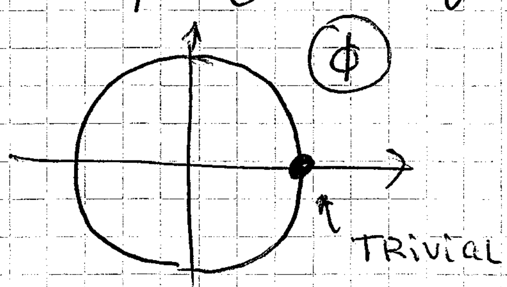
$$\omega: \ \text{Space} \rightarrow U(1)$$

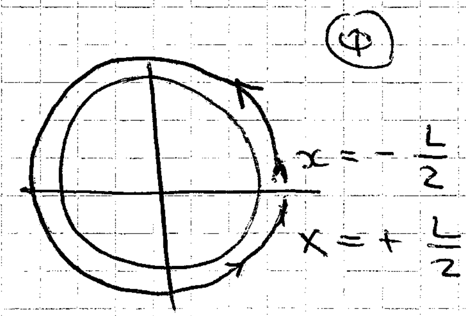
$$S^1_{\text{space}} \rightarrow U(1) \equiv S^1_{\text{gauge group}}$$

$n \equiv$ degree of mapping.

Another look:

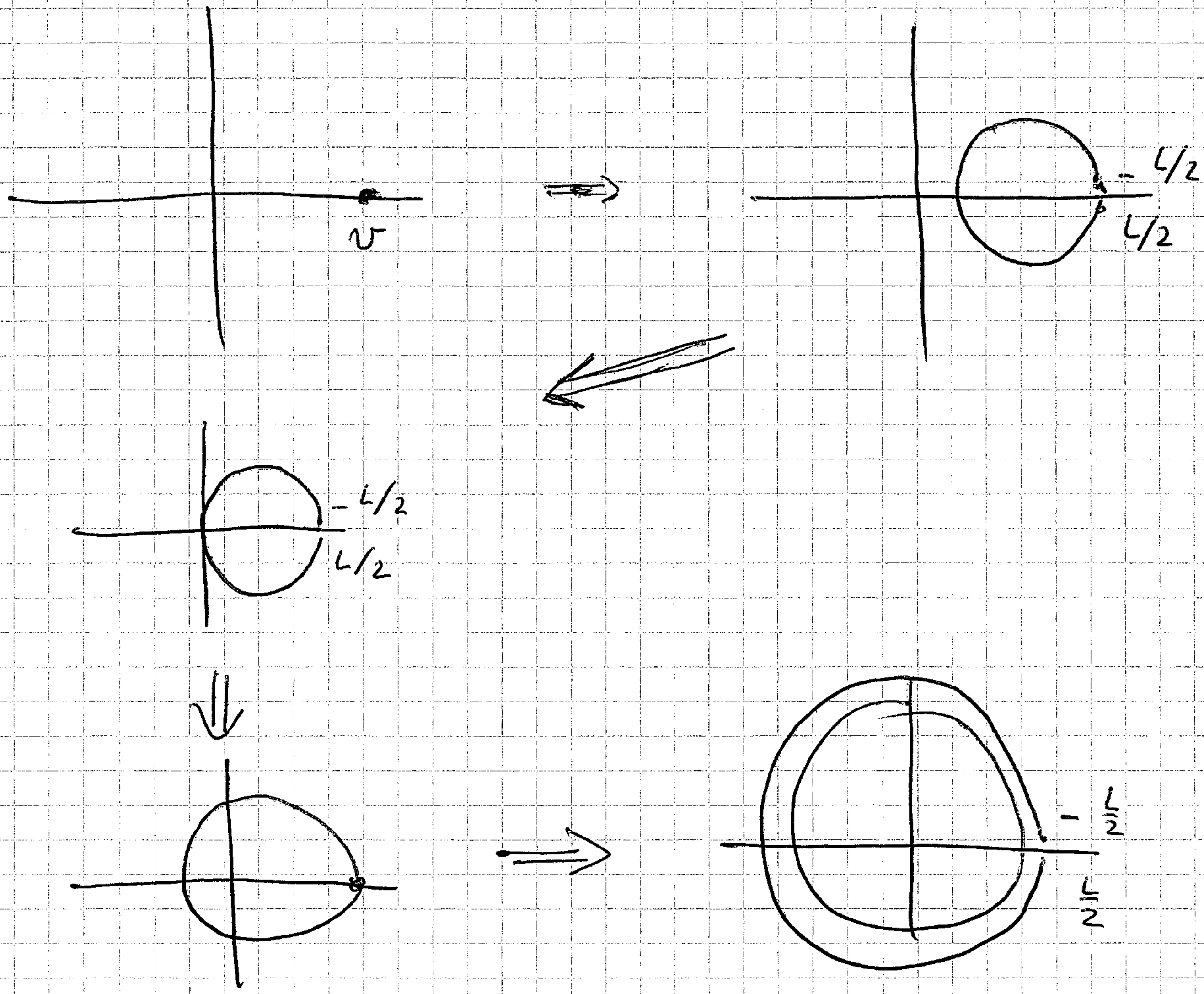
$\phi = e^{i\beta(x)} \cdot v \in S_1$





I-11

Path in field space from $n=0$ to $n=1$



Height of barrier:

Saddle point of
STATIC ENERGY Functional

$$E_{static} = \int dx \left[|D_1 \phi|^2 + V(\phi) \right]$$

Saddle point ≡ (unstable) solution to
static field equations

Name: sphaleron (given by
Klinkhamer, Manton)

sphaleros (Greek) = ready to fall.

(1+1) model:

E_{static} is gauge-invariant under

$$\phi \Rightarrow e^{i\gamma(x)}\phi \ ; \ A_1 \Rightarrow A_1 + \frac{1}{e}\partial_1 \gamma(x)$$

Static fields: can set $A_1 = 0$

$$E_{static} = \int dx \left(|\partial_1 \phi|^2 + \frac{\lambda}{2}\left(|\phi|^2 - v^2\right)^2 \right)$$

Solution: kink

$$\varphi = v \tanh\left(\sqrt{\frac{\lambda}{2}} \cdot x\right)$$

NB: Not periodic ⇒ make fields periodic by choice of gauge function $\gamma(x)$.

NB: $x = 0 \Leftrightarrow \phi = 0$, middle picture on page 11.

$$E_{sph} = \frac{4}{3}\sqrt{2\lambda} \cdot v^3 = \frac{4}{3} m_H v^2$$

NB: $E_{sph} \gg m_H, m_V$ at weak coupling $v^2 \gg 1$.

Problem:
check that there is exactly one linear independent perturbation about sphaleron (kink), along which static energy decreases.

Problem: consider theory on infinite line $x \in (-\infty, +\infty)$.

Show that any configuration whose energy is finite at all moments of time has $A_1 = 0$, $\phi = v$ at $x \to +\infty$ and $x \to -\infty$,

if initial configuration is $A_1 = 0$, $\phi = v$

(Energy may not be conserved; use gauge $A_0 = 0$). Thus, theory is effectively compactified on a circle of large radius: "points" $x = -\infty$ and $x = +\infty$ can be identified.