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SUMMER SCHOOL IN COSMOLOGY AND ASTROPARTICLE PHYSICS

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Dark energy or modified gravity? (II & III)

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A model of modified gravity:

Modified Einstein's Equation:

$$G_{\mu\nu} + m_c \left(K_{\mu\nu} - g_{\mu\nu} K \right) = 8\pi G_N T_{\mu\nu}$$

Extrinsic curvature of the surface.

$$S = -\frac{M_P^2}{2} \int d^4x \sqrt{-g}\, R - \frac{M_*^3}{2} \int d^4x\, dy \sqrt{-\tilde{g}}\, R_5(\tilde{g})$$

$$\tilde{g}_{\mu\nu}(x, y=0) = g_{\mu\nu}(x)$$

$$m_c \equiv 2M_*^3 / M_P^2 \sim H_0 \sim 10^{-42}\ \text{GeV}.$$

$N_\mu \equiv g_{\mu 5} \qquad g_{55} = N^2 + g_{\mu\nu} N^\mu N^\nu$

$$K_{\mu\nu} = \frac{1}{N}\left(\partial_y g_{\mu\nu} - D_\mu N_\nu - D_\nu N_\mu\right)$$

① $\sqrt{g}\, G_{\mu\nu}\, \delta^\mu_A \delta^\nu_B\, \delta(y) + \frac{1}{2} m_c\, G^{(5)}_{AB} \sqrt{\tilde{g}} = 8\pi G_N \sqrt{g}\, T_{\mu\nu}\, \delta(y)$

② $\mu\nu$: $\quad \sqrt{g}\, G_{\mu\nu}\, \delta(y) + \frac{1}{2} m_c \sqrt{\tilde{g}}\, G^{(5)}_{\mu\nu} = 8\pi G_N \sqrt{g}\, T_{\mu\nu}\, \delta(y)$

$$G^{(5)}_{yA} = 0.$$

③ $y=0 \quad G_{\mu\nu} + m_c\left(K_{\mu\nu} - g_{\mu\nu} K\right) = 8\pi G_N T_{\mu\nu}$

$y \neq 0 \quad \sqrt{\tilde{g}}\, G^{(5)}_{\mu\nu} = 0$

$$G^{(5)}_{\mu A} = 0.$$

1) What does the linearized theory do?

3) What are the effects of nonlinearities?

2) Cosmology.

$$\tilde{g}_{AB} = \eta_{AB} + h_{AB}$$

$$G^{(5)}_{AB} = \frac{1}{2}\Big(\Box_5 h_{AB} - \partial_A \partial_C h^C_B - \partial_B \partial_C h^C_A + \partial_A \partial_B h^C_C - \eta_{AB} \Box h^C_C + \eta_{AB} \partial_C \partial_D h^{CD}\Big)$$

Harmonic gauge:

$$\partial^A h_{AB} = \frac{1}{2} \partial_B h^C_C$$

$$G^{(5)}_{AB} = \frac{1}{2}\Big(\Box_5 h_{AB} - \frac{1}{2} \eta_{AB} \Box_5 h^C_C\Big)$$

$\mu 5$ equation:

$$\Box_5 h_{\mu 5} = 0 \qquad h_{\mu 5} = 0 \text{ is a solution.}$$

55 equation

$$\Box_5 h_{55} + \frac{1}{2} \Box_5 (h^\mu{}_\mu + h^5{}_5) = 0$$

$$\frac{1}{2} \Box_5 (h^\mu{}_\mu - h^5{}_5) = 0 \qquad h^\mu{}_\mu = h^5{}_5 \text{ is } \underline{\text{a solution}}$$

$(\mu\nu)$ eq. From $h_{\mu 5} = 0$, $h^\mu{}_\mu = h^5{}_5$ & gauge condition

$$\Rightarrow \quad \partial^\alpha h_{\alpha\mu} = \partial_\mu h^\beta{}_\beta .$$

$$2\, G_{\mu\nu} = \Box_4 h_{\mu\nu} - \partial_\mu \partial_\alpha h^\alpha{}_\nu - \cancel{\partial_\nu \partial_\alpha h^\alpha{}_\mu} + \cancel{\partial_\mu \partial_\alpha h^\alpha{}_\alpha}$$

$$- \eta_{\mu\nu} \cancel{\Box_4 h^\alpha{}_\alpha} + \eta_{\mu\nu} \cancel{\partial_\alpha \partial_\beta h^{\alpha\beta}}$$

$$= \Box_4 h_{\mu\nu} - \partial_\mu \partial_\nu h^\alpha{}_\alpha$$

$$G_{\mu\nu}\,\delta(y) + \frac{1}{2} m_c\, G_{\mu\nu} = 8\pi G_N T_{\mu\nu}\,\delta(y) \equiv \frac{1}{2}\tau_{\mu\nu}\,\delta(y)$$

$$\left(\Box_4 h_{\mu\nu} - \partial_\mu \partial_\nu h^\alpha_\alpha\right)\delta(y) + \frac{m_c}{2}\left(\Box_5 h_{\mu\nu} - \eta_{\mu\nu}\Box_5 h^\alpha_\alpha\right) = \tau_{\mu\nu}\,\delta(y)$$

taking trace:

$$-3 m_c \Box_5 h^\alpha_\alpha = 2\tau^\alpha_\alpha\,\delta(y)$$

Fourier transform to momentum space:

$$x_\mu \to p_\mu \qquad y \to y$$

$$\partial_\mu \to i p_\mu \qquad \Box_4 \to -p_\mu^2 \equiv p^2 \to \text{Euclidean momentum square.}$$

$$\Box_5 = \Box_4 - \partial_y^2 \Rightarrow p^2 - \partial_y^2$$

$$\begin{cases} \left(p^2 h_{\mu\nu} + p_\mu p_\nu h^\alpha_\alpha\right)\delta(y) + \frac{m_c}{2}\left((p^2-\partial_y^2)h_{\mu\nu} - \eta_{\mu\nu}(p^2-\partial_y^2)h^\alpha_\alpha\right) = \tau_{\mu\nu}\,\delta(y) \\ -3 m_c (p^2 - \partial_y^2) h^\alpha_\alpha = 2\tau^\alpha_\alpha\,\delta(y) \end{cases}$$

$$h_{\mu\nu}(p,y) = \chi_{\mu\nu}(p)\, e^{-\sqrt{p^2}\,|y|}$$

$$(p^2 - \partial_y^2)\, h_{\mu\nu} = 2p\,\delta(y)\,\chi_{\mu\nu} \qquad p \equiv \sqrt{p^2}$$

$$\left(p^2 \chi_{\mu\nu} + p_\mu p_\nu \chi\right)\delta(y) + m_c\left(\cancel{2}p\,\delta(y)\,\chi_{\mu\nu} - \eta_{\mu\nu}\cancel{2}p\,\delta(y)\,\chi\right)$$

$$= \tau_{\mu\nu}\,\delta(y)$$

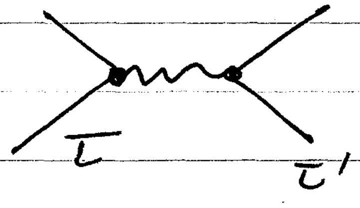
$$-3\, m_c\, 2p\,\delta(y)\,\chi = 2\tau\,\delta(y)$$

$$\chi = -\frac{\tau}{3 m_c p}$$

$$\left(p^2 + m_c p\right)\chi_{\mu\nu} = \tau_{\mu\nu} - \frac{1}{3}\eta_{\mu\nu}\tau + \frac{p_\mu p_\nu}{3 m_c p}\tau$$

$$h_{\mu\nu}(p,y) = \frac{\left(\tau_{\mu\nu} - \frac{1}{3}\eta_{\mu\nu}\tau + \frac{p_\mu p_\nu}{3 m_c p}\right)}{p^2 + m_c p}\, e^{-p|y|}$$

Amplitude



$$A \sim \frac{\tau_{\mu\nu}\tau'^{\mu\nu} - \frac{1}{3}\tau\tau'}{p^2 + m_c p}$$

$p^2 \gg m_c p \qquad A \sim \frac{1}{p^2} \qquad V(r) \sim \frac{1}{r}$

$p^2 \ll m_c p \qquad A \sim \frac{1}{p} \qquad V(r) \sim \frac{1}{r^2}$

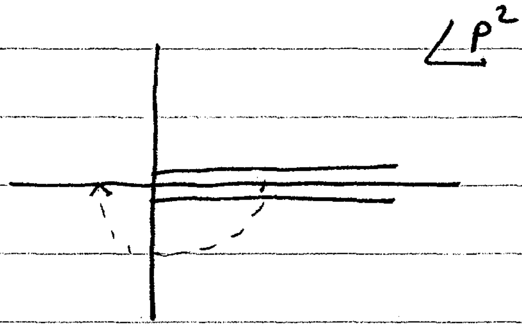
Critical distance $\quad r = r_c = m_c^{-1} \sim H_0^{-1}$

$\sim 10^{28}\,\mathrm{cm}.$

More careful studies of the pole structure:

I

$$\text{den} = p^2 + m_c\sqrt{p^2}$$



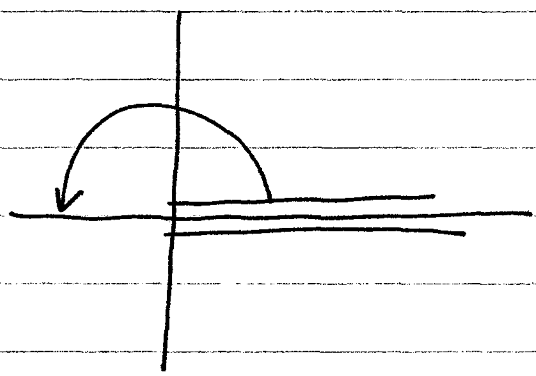
$$p^2 = e^{-i\pi} p_\mu^2$$

$$p_\mu^2 = m_c^2\, e^{-i\pi}$$

Metastable graviton.

II Second branch

$$\text{den} = p^2 - m_c\sqrt{p^2}$$



$$p_\mu^2 = e^{i\pi} m_c^2$$

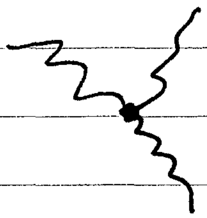
pole on a physical sheet at $p_\mu^2 < 0$

„tachyon"

Instability of Minkowski space

Good news!

Effects of nonlinearities:



$$\sim \frac{1}{m_c^2}$$

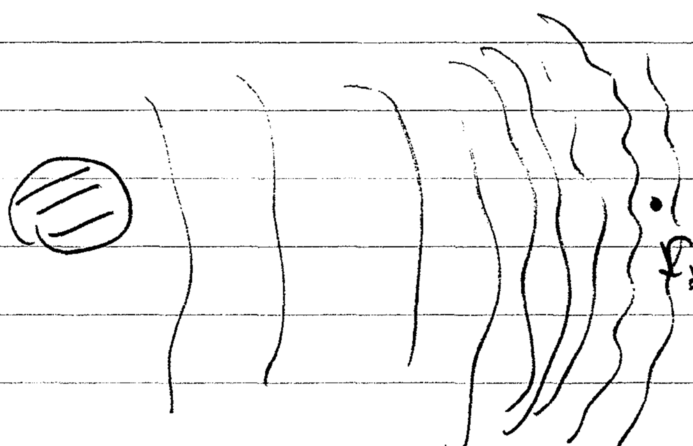
Nonlinear interactions become important

at $\quad r_* \sim \left(r_M \cdot r_c^2 \right)^{1/3}$

$$r_M = 2 G_N M \qquad r_c = m_c^{-1}$$

Sun: $\quad r_M \sim 3\ \mathrm{km} \qquad r_* \sim 10^{20}\ \mathrm{cm}$

Self shielding $\quad r \lesssim r_*$ we get nonzero 4D curvature.



$R_4 \neq 0$ shields itself from strong coupling regime.

$$S_{grav} = -\frac{1}{16\pi G_N}\left[\int d^4x\sqrt{-g}\,R + \frac{m_c}{2}\int d^4x\,dy\sqrt{-\tilde{g}}\,R_5(\tilde{g})\right]$$

$\tilde{g}_{AB}(x,y)$: $A, B = 0, 1, 2, 3, 5$ $\qquad$ $m_c \sim H_0 \sim 10^{-42}$ GeV

$$\tilde{g}_{\mu\nu}(x, y=0) = g_{\mu\nu}(x)$$

Looking for a solution:

$$ds^2 = +N^2(t,y)\,dt^2 - A^2(t,y)\,dx_i dx^i - B^2(t,y)\,dy^2$$

Matter

$$S_m = \int d^4x\sqrt{-g}\,\mathcal{L}_m(g, \text{matter fields})$$

$$\Downarrow$$

$$\dot{\rho} + 3H(\rho + p) = 0$$

$$N(t,y) = 1 + \epsilon |y| \ddot{a}/a$$

$$A(t,y) = a(t) + \epsilon |y| \dot{a}$$

$$B(t,y) = 1$$

$$N(t, y=0) = 1 \qquad A(t, y=0) = a$$

$$ds^2\Big|_{\text{Brane}} = dt^2 - a^2(t)\, dx_i dx^i$$

Hence, $a(t)$ is our scale factor.

The respective Friedmann equation:

$$\boxed{H^2 = \left(\sqrt{\frac{8\pi G_N}{3}\rho + \frac{m_c^2}{4}} + \epsilon \frac{m_c}{2}\right)^2}$$

Two possible branches:

I $\quad \epsilon = -1$

$t \ll t_c \sim m_c^{-1}$

$G_N \rho \gg m_c^2$

$$H^2 \simeq \frac{8\pi G_N}{3}\rho$$

Standard evolution (MD): $\rho \sim 1/a^3 \quad a \sim t^{2/3}$

$t \gg t_c$

$G_N \rho \ll m_c^2 \qquad H \sim \dfrac{G_N}{m_c}\rho$

(MD): $\rho \sim 1/a^3 \quad \Rightarrow a \sim t^{1/3}$ deceleration!

(RD): $\rho \sim 1/a^4 \quad \Rightarrow a \sim t^{1/4}$ deceleration!

need to combine with the cosmological constant or quintessence!

(II) $\epsilon = +1$.

$t \ll t_c \qquad G_N \rho \gg m_c^2$

$$H^2 \simeq \frac{8\pi G_N}{3}\rho \quad \Rightarrow \text{ the standard cosmology}$$

$t \gg t_c \qquad G_N \rho \ll m_c^2$

$H \simeq m_c$ de Sitter like expansion. with $H \sim H_0$!!

Can this explain the accelerated Universe?

$$H^2 = H_0^2 \left[\sqrt{\Omega_M (1+z)^3 + \Omega_{m_c}} + \Omega_{m_c} \right]^2$$

$$\Omega_{m_c} \equiv \frac{m_c^2}{4 H_0^2} \qquad \text{versus}$$

$$H^2 = H_0^2 \left[\Omega_M (1+z)^3 + \Omega_\alpha (1+z)^{3(1+w_\alpha)} - \right.$$

For $z = 0$

$$\sqrt{\Omega_M + \Omega_{m_c}} + \sqrt{\Omega_{m_c}} = 1$$

$$\Omega_{m_c} = \left(\frac{1 - \Omega_M}{2}\right)^2$$

$\Omega_M < 1$ $\Omega_{m_c} < 1$

Pre WMAP $\Omega_M = 0.18 \pm 0.07$

$m_c^{-1} = (1.21 \pm 0.09)\, H_0^{-1}$

Post WMAP $\Omega_M \simeq 0.21 \pm (?)$

still not done yet

Higher codimensions:

Example

6D:

Source $T\,\delta^{(2)}(y)$

$\uparrow$

tension of the brane is our cosmological constant.

$$ds^2 = \left(dt^2 - d\vec{x}^{\,2}\right) - \left[d\rho^2 + B\,\rho^2 d\theta^2\right]$$

$$B \sim \left(1 - T/M_6^4\right)^2$$

4D part is flat.

Cosmological constant affects the extra space but not the brane

CC is „off-loaded" into the bulk.

This would be a solution to the problem if one only could obtain 4D gravity on a brane.

Finite volume theories cannot do this since they flow to 4D GR in the Infrared.

One possibility is to look at the brane-induced gravity.

Technical difficulties = non-singular static solutions in $D \geq 6$ are not known.

(numerical work is needed).

$$ds^2 = A^2(\rho)\, \bar{g}_{\mu\nu}\, dx^\mu dx^\nu - B d\rho^2 - C\rho^2 d\Omega^2_{2+N}$$

Calculation of the spectrum is a challenge.

Can one model the effect

to see if it is viable?

Nonlocal interactions due to infinite tower of KK's.

$$(5D)\quad M_p^2\left(\Box + m_c\sqrt{+\Box}\right) h_{\mu\nu} = T_{\mu\nu} + \cdots$$

$$(D\geq 6)\quad M_p^2\left(\Box + \underbrace{m_c f(\Box)}_{\text{dominates in the IR}}\right) h_{\mu\nu} = T_{\mu\nu}$$

Moddeling this effect:

$$\left(1 + F(r_c^2\Box)\right) G_{\mu\nu} = T_{\mu\nu} + \cdots$$

$$F(\alpha) \cong 0 \qquad \alpha \gg 1$$

$$F(\alpha) \gg 1 \qquad \alpha \ll 1.$$

For uniformly distributed source such as the cosmological constant:

$$G_{\mu\nu} = -8_{\mu\nu} R/4 \qquad T_{\mu\nu} = \mathcal{E} g_{\mu\nu}$$

constant

$$F g_{\mu\nu} R = g_{\mu\nu} F R = g_{\mu\nu} F(0) R$$

$$R \sim \frac{\mathcal{E}}{M_p^2(1+F(0))} \longrightarrow 0$$

if $F(0) \to \infty$.

$$F(\alpha) \sim \frac{1}{\alpha}$$

$$\left(1 + F(r_c^2 \Box)\right) \left[\text{~~for~~ matter/radiation}\right]$$

$$\simeq [\text{matter/radiation}]$$

as long as typical time/distance scales are $\ll r_c$.