The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific
and Cultural Organization

International Atomic
Energy Agency

SMR 1646 - 11

Conference on
Higher Dimensional Quantum Hall Effect, Chern-Simons Theory and
Non-Commutative Geometry in Condensed Matter Physics and Field Theory
1 - 4 March 2005

Intrinsic Spin Hall Effect

Shuichi MURAKAMI
Department of Applied Physics, University of Tokyo
Tokyo, Japan

These are preliminary lecture notes, intended only for distribution to participants.

Strada Costiera 11, 34014 Trieste, Italy - Tel. +39 040 2240 111; Fax +39 040 224 163 - sci_info@ictp.it, www.ictp.it

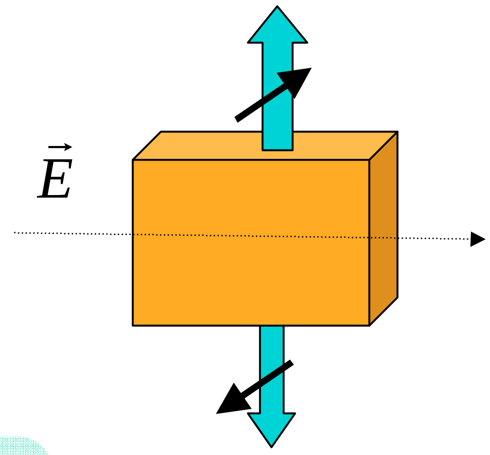
March 3,2005, ICTP, Trieste, Italy

Intrinsic Spin Hall Effect

Shuichi Murakami
(Department of Applied Physics, University of Tokyo)

Collaborators : Naoto Nagaosa (U.Tokyo)
Shoucheng Zhang (Stanford)
Masaru Onoda (AIST, Japan)

Spin Hall effect (SHE)

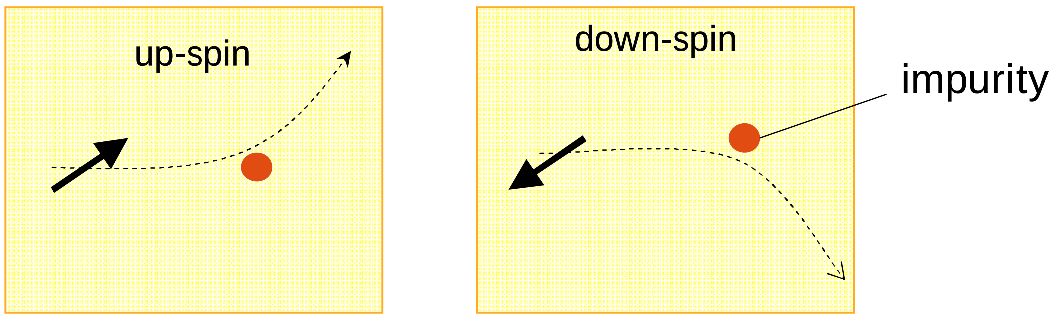


Electric field induces a transverse spin current.

- **Extrinsic spin Hall effect**

D'yakonov and Perel' (1971)
Hirsch (1999), Zhang (2000)

impurity scattering = spin dependent (skew-scattering)
↑
Spin-orbit couping



Cf. Mott scattering

- **Intrinsic spin Hall effect** **Berry phase in momentum space**

Independent of impurities !

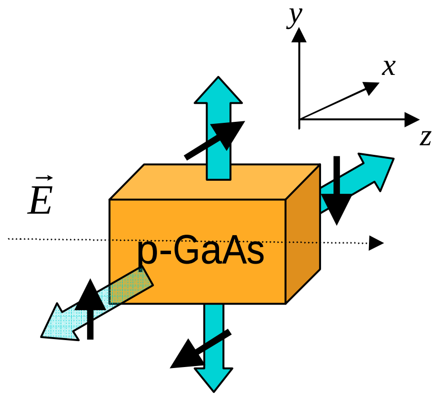
Intrinsic spin Hall effect

- p-type semiconductors (SM, Nagaosa, Zhang, Science (2003))

Luttinger model

$$H = \frac{\hbar^2}{2m}\left[\left(\gamma_1 + \frac{5}{2}\gamma_2\right)k^2 - 2\gamma_2\left(\vec{k}\cdot\vec{S}\right)^2\right]$$

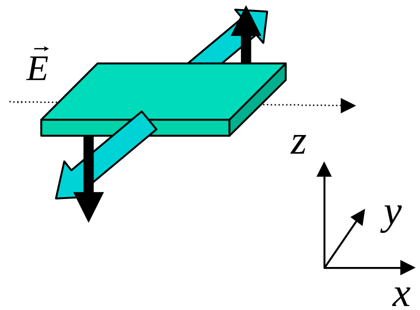
($\vec{S}$: spin-3/2 matrix)



- 2D n-type semiconductors in heterostructure
(Sinova, Culcer, Niu, Sinitsyn,Jungwirth, MacDonald, PRL (2003))

Rashba model

$$H = \frac{k^2}{2m} + \lambda\left(\vec{\sigma}\times\vec{k}\right)_z$$



Berry phase in momentum space (U(1) gauge field)

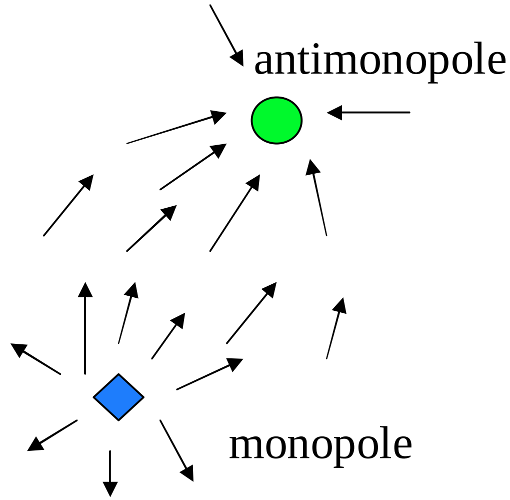
$$A_{ni}(\vec{k}) = -i\left\langle n\vec{k}\left|\frac{\partial}{\partial k_i}\right|n\vec{k}\right\rangle = -i\int_{\text{unit cell}} u^*_{n\vec{k}}\frac{\partial u_{n\vec{k}}}{\partial k_i}d^d x$$: Gauge field

($u_{n\vec{k}}$: periodic part of the Bloch wf.)

$$\psi_{n\vec{k}}(\vec{x}) = u_{n\vec{k}}(\vec{x})e^{i\vec{k}\cdot\vec{x}}$$

$$\vec{B}_n(\vec{k}) = \nabla_{\vec{k}} \times \vec{A}_n(\vec{k})$$: Field strength

(n : band index)



Intrinsic Hall conductivity

$$\sigma_{xy} = -\frac{e^2}{h}\sum_{n,\vec{k}} n_F\left(E_n(\vec{k})\right)B_{nz}(\vec{k})$$

Hall conductivity due to Berry phase
(intrinsic contribution)

$$\sigma_{xy} = -\frac{e^2}{h}\sum_{n,\vec{k}} n_F\left(E_n(\vec{k})\right)\underline{B_{nz}(\vec{k})}$$

Berry phase

Kubo formula (Thouless et al. (1982))

Semiclassical eq. of motion (Sundaram,Niu (1999))

$$\begin{cases} \dot{\vec{x}} = \dfrac{1}{\hbar}\dfrac{\partial E_n(\vec{k})}{\partial \vec{k}} + \dot{\vec{k}} \times \vec{B}_n(\vec{k}) \\ \dot{\vec{k}} = -e\vec{E} - e\dot{\vec{x}} \times \vec{B}(\vec{x}) \end{cases}$$

Anomalous velocity due to Berry phase

- Quantum Hall effect
- Anomalous Hall effect
- Spin Hall effect

$\vec{B}_n(\vec{k})$ =“magnetic field in k-space”

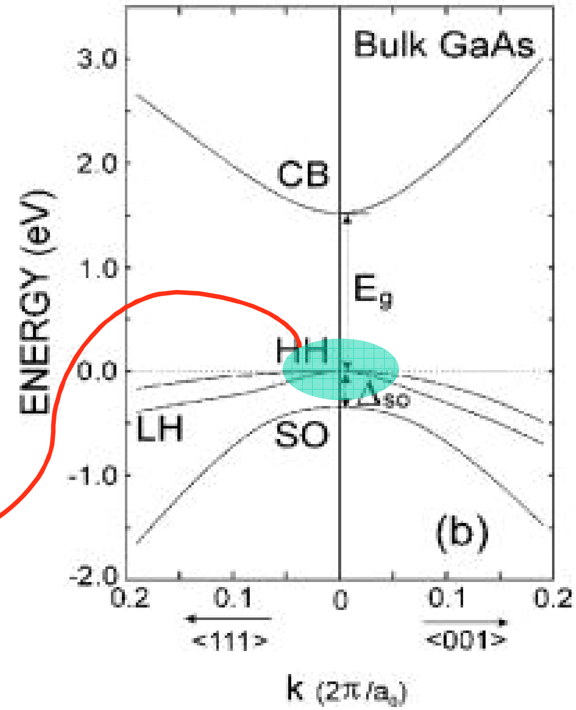
Valence band of semiconductors
(diamond (Si, Ge) or zincblende (GaAs))

p-orbit (x,y,z)× (↑,↓)

+ spin-orbit coupling

split-off band (SO)
heavy-hole band (HH)
light-hole band (LH)

doubly degenerate (Kramers)



Luttinger Hamiltonian (Luttinger(1956))

$$H = \frac{\hbar^2}{2m}\left[\left(\gamma_1 + \frac{5}{2}\gamma_2\right)k^2 - 2\gamma_2\left(\vec{k}\cdot\vec{S}\right)^2\right]$$

($\vec{S}$: spin-3/2 matrix)

Helicity $\lambda = \hat{k}\cdot\vec{S}$ is a good quantum number.

Helicity

$\lambda = \hat{k}\cdot\vec{S} = \pm\frac{3}{2} \Rightarrow E = \frac{\gamma_1 - 2\gamma_2}{2m}\hbar^2 k^2$: heavy hole (HH)

$\lambda = \hat{k}\cdot\vec{S} = \pm\frac{1}{2} \Rightarrow E = \frac{\gamma_1 + 2\gamma_2}{2m}\hbar^2 k^2$: light hole (LH)

Semiclassical Equation of motion

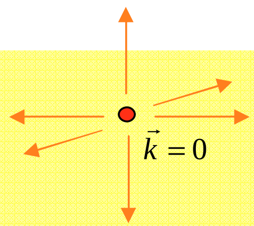
$$\hbar\dot{\vec{k}} = e\vec{E}, \quad \dot{\vec{x}} = \frac{\hbar\vec{k}}{m_\lambda} + \frac{e}{\hbar}\vec{E}\times\vec{B}^{(\lambda)}(\vec{k})$$

(λ: helicity= band index)

Drift velocity $= \dfrac{\partial E}{\partial k_i}$

Anomalous velocity
(due to Berry phase)

Two bands touch at k=0
→ monopole at k=0



$$\vec{B}^{(\lambda)}(\vec{k}) = \lambda\left(2\lambda^2 - \frac{7}{2}\right)\frac{\vec{k}}{k^3}$$

$$\lambda = \pm\frac{3}{2}: \mathrm{HH}, \quad \lambda = \pm\frac{1}{2}: \mathrm{LH}$$

A hole obtains a velocity perpendicular to both k and E.

Real-space trajectory

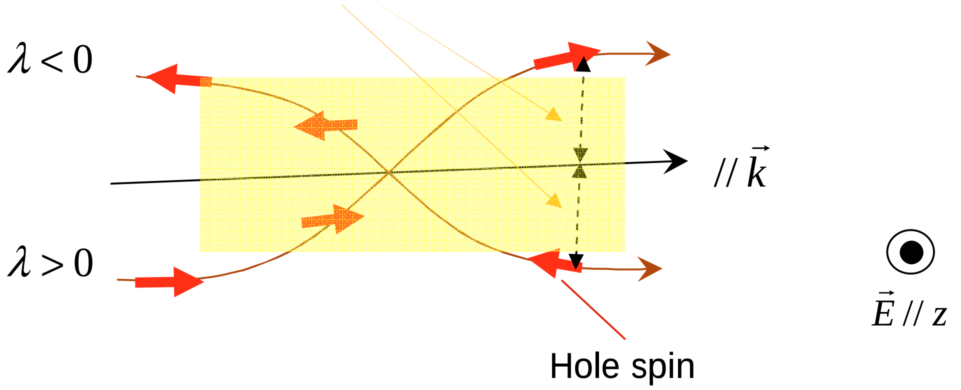
$$\hbar\dot{\vec{k}} = e\vec{E}, \quad \dot{\vec{x}} = \frac{\hbar\vec{k}}{m_\lambda} + \frac{e}{\hbar}\vec{E}\times\vec{B}^{(\lambda)}(\vec{k})$$

$$\vec{B}^{(\lambda)}(\vec{k}) = \lambda\left(2\lambda^2 - \frac{7}{2}\right)\frac{\vec{k}}{k^3}$$

$$\lambda = \pm\frac{3}{2}:\mathrm{HH}, \quad \lambda = \pm\frac{1}{2}:\mathrm{LH}$$

$$\lambda = \hat{k}\cdot\vec{S}$$

Anomalous velocity (perpendicular to $\vec{S}$ and $\vec{E}$)



Spin current (spin//x, velocity//y)

$$j_{yx}^{H} = \frac{\hbar}{3}\sum_{\lambda=\pm\frac{3}{2},\vec{k}} \dot{y} S_x n^{\lambda}(\vec{k}) = \frac{E_z k_F^H}{4\pi^2},$$

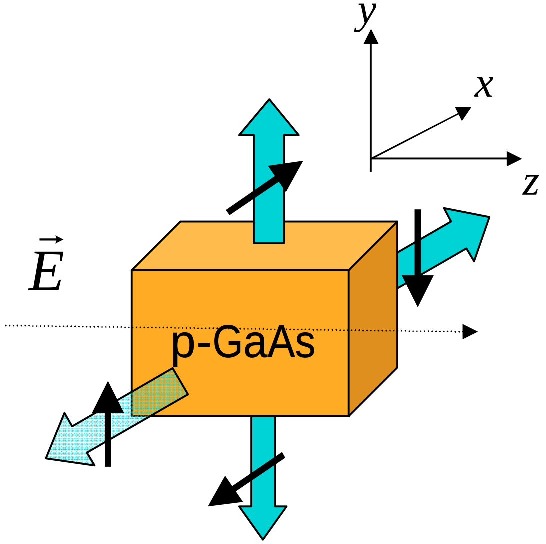
$$j_{yx}^{L} = \frac{\hbar}{3}\sum_{\lambda=\pm\frac{1}{2},\vec{k}} \dot{y} S_x n^{\lambda}(\vec{k}) = -\frac{E_z k_F^L}{12\pi^2},$$

$$\sigma_s = \frac{e}{12\pi^2}(3k_F^H - k_F^L)$$

Intrinsic spin Hall effect in p-type semiconductors

In p-type semiconductors (Si, Ge, GaAs,...), spin current is induced by the external electric field.

(SM, Nagaosa, Zhang, Science (2003))



$$j_j^i = \sigma_s \varepsilon_{ijk} E_k$$

- i: spin direction
- j: current direction
- k: electric field

σ_s : even under time reversal = reactive response (dissipationless)

- Nonzero in nonmagnetic materials.

Cf. Ohm's law: $j = \sigma E$

σ : odd under time reversal = dissipative response

- topological origin (Berry phase in momentum space)
- dissipationless
- All occupied state contribute.

Spin analog of the quantum Hall effect

Order estimate (at room temperature) : GaAs

$$j_y^x = \frac{eE_z}{12\pi^2}\left(3k_F^H - k_F^L\right) \equiv \frac{\hbar}{2e}\sigma_s E_z \longrightarrow$$ $\sigma_s\ (\Omega^{-1}\mathrm{cm}^{-1})$: Unit of conductivity

carrier density $n\ (\mathrm{cm}^{-3})$	mobility $\mu\ (\mathrm{cm}^2/\mathrm{Vs})$	Charge conductivity $\sigma\ (\Omega^{-1}\mathrm{cm}^{-1})$	Spin (Hall) conductivity $\sigma_s\ (\Omega^{-1}\mathrm{cm}^{-1})$
10^{19}	50	80	73
10^{18}	150	24	34
10^{17}	350	5.6	16
10^{16}	400	0.64	7.3

$$\sigma = en\mu$$

$$\sigma_S \propto k_F \propto n^{1/3}$$

As the hole density decreases, both σ and σ_S decrease.
σ decreases faster than σ_S

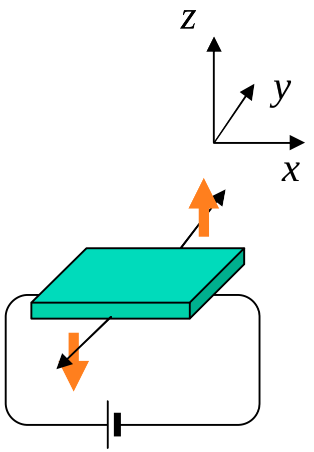
Intrinsic spin Hall effect for 2D n-type semiconductors in heterostructure

(Sinova, Culcer, Niu, Sinitsyn, Jungwirth, MacDonald, PRL(2003))

Rashba Hamiltonian

$$H = \frac{k^2}{2m} + \lambda\left(\vec{\sigma}\times\vec{k}\right)_z = \begin{pmatrix} \frac{k^2}{2m} & \lambda(k_y + ik_x) \\ \lambda(k_y - ik_x) & \frac{k^2}{2m} \end{pmatrix}$$

Effective electric field along z



2D heterostructure

Kubo formula : $\left\langle J_x J_y^{S_z} \right\rangle \longrightarrow \sigma_s = \frac{e}{8\pi}$

$J_y^{S_z} = \frac{1}{2}\{J_y, S_z\}$

independent of λ

Note: σ_S is not small even when the spin splitting λ is small.

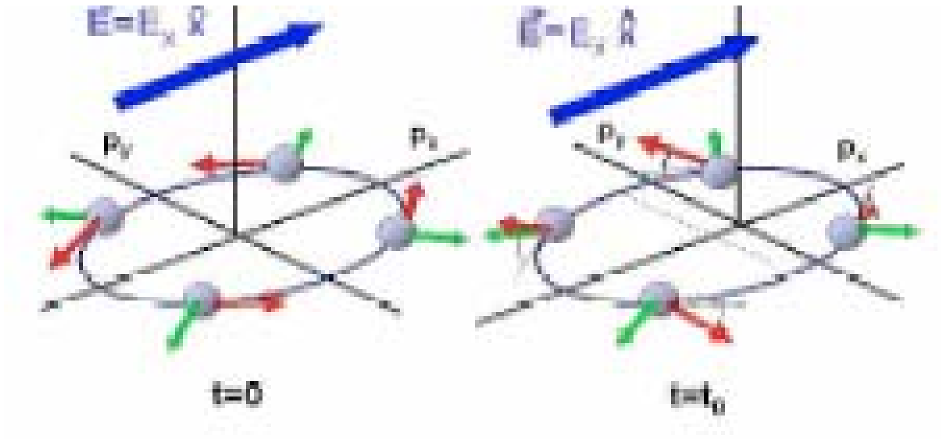
interband effect

spin Hall effect in the Rashba model

≈ Spin precession by "k-dependent Zeeman field"

$$H = \frac{k^2}{2m} + \lambda\left(\vec{\sigma}\times\vec{k}\right)_z$$

$$\vec{B}_{\mathrm{int}} = \lambda(\hat{z}\times\vec{k})$$



- **Semiclassical theory** **Culcer et al., PRL(2004)**
- **Rashba + Dresselhaus**

  - **Sinitsyn et al., PRB(2004)**
  - **Shen, PRB(2004)**

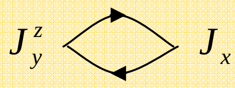
Disorder effect, edge effect

Green's function method

Rashba model: $H = \frac{k^2}{2m} + \lambda(\sigma_x k_y - \sigma_y k_x)$ + spinless impurities (δ-function pot.)

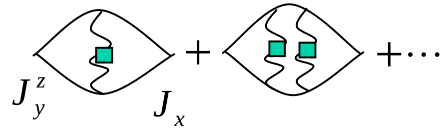
Intrinsic spin Hall conductivity (Sinova et al.(2003))

$$\sigma_S = \frac{e}{8\pi}$$



+ Vertex correction in the clean limit
(Inoue, Bauer, Molenkamp(2003))

$$\sigma_S^{\text{vertex}} = -\frac{e}{8\pi}$$



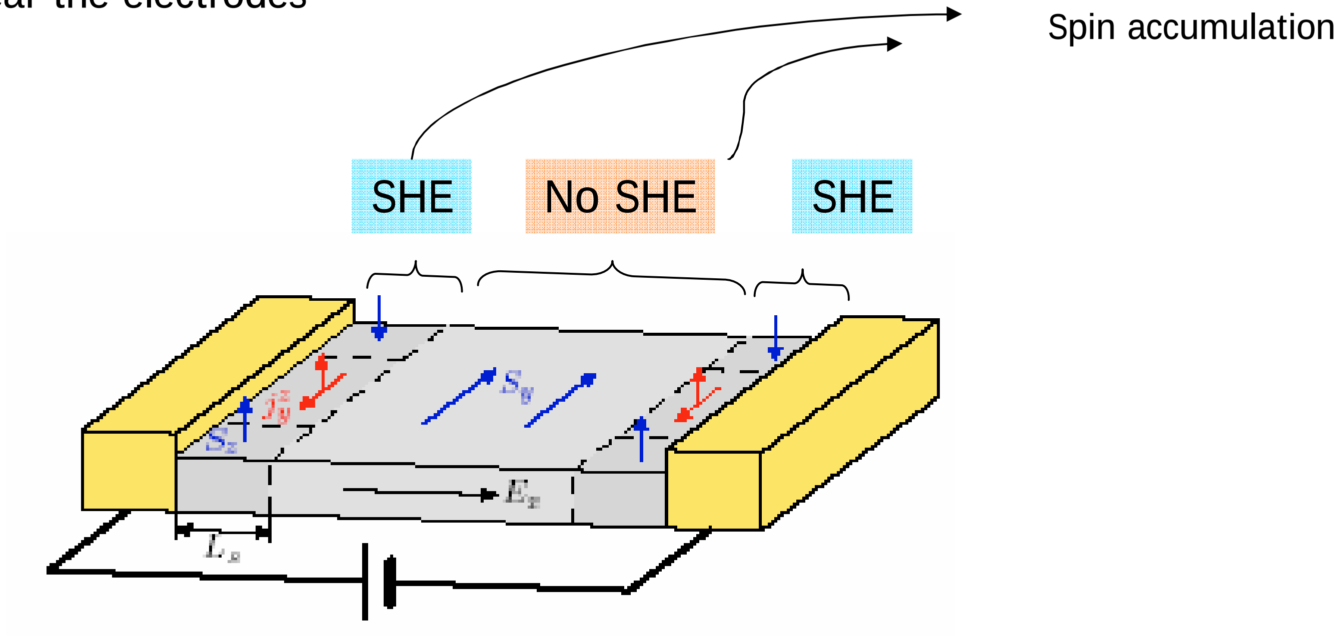
$$\sigma_S = 0$$

- Inoue, Bauer, Molenkamp (2004)
- Rashba (2004)
- Raimondi, Schwab (2004)
- Dimitrova (2004)

- **Calculation by Keldysh formalism** (Mishchenko, Shytov, Halperin (2004))

  Spin Hall current does not flow at the bulk – consistent with $\sigma_s = 0$

  Spin current only flows near the electrodes



- $\sigma_S(\omega=0)=0$ in Rashba model in the clean limit
- (several papers incorrectly claiming $\sigma_S(\omega=0)\neq 0$ in general.)

Question

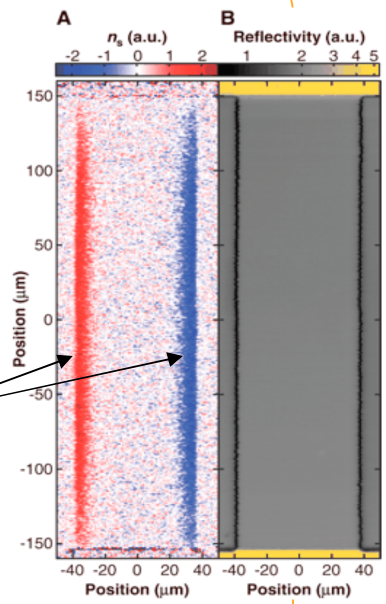
Intrinsic spin Hall effect is so fragile to impurities that it vanishes at the bulk?

Answer

NO! Not in general.

- Rashba model is an exception.
  there are models with nonzero $\sigma_S(\omega=0)$

- two experimental reports
  - **2D electron gas, spin accumulation at the edges**
    Y.K.Kato et al., Science (2004)
  - **2D hole gas, spin LED**
    J. Wunderlich et al., cond-mat (2004), to appear in PRL.



Intrinsic spin Hall effect is nonzero in general.

- If $\hat{H}(\vec{k}) = \hat{H}(-\vec{k})$ e.g. Luttinger model (p-type semicond.)

  → in the clean limit the vertex correction is ZERO. (SM (2004))
  SHE is finite.

- For models with $H(\vec{k}) = E_0(\vec{k}) + \sigma_x d_y(\vec{k}) - \sigma_y d_x(\vec{k})$: n-type semicond. in heterostructure

  (e.g. Rashba model: $E_0(\vec{k}) = \frac{k^2}{2m}, \quad \vec{d} = \lambda \vec{k}$)

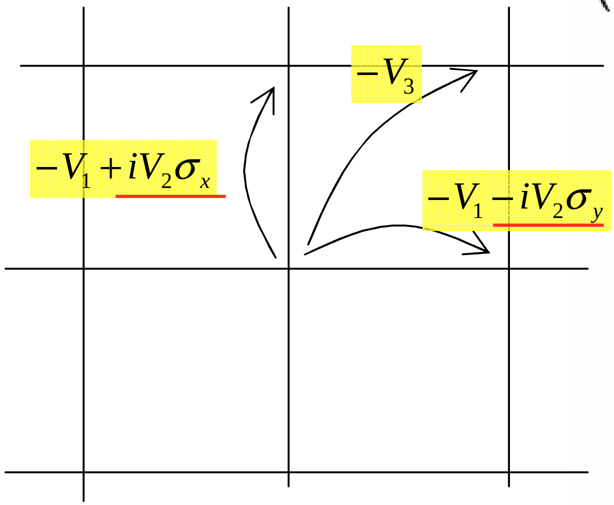
  ➡ (a) If $\frac{\partial E_0}{\partial \vec{k}} = A\vec{d}$ for constant A SHE vanishes.

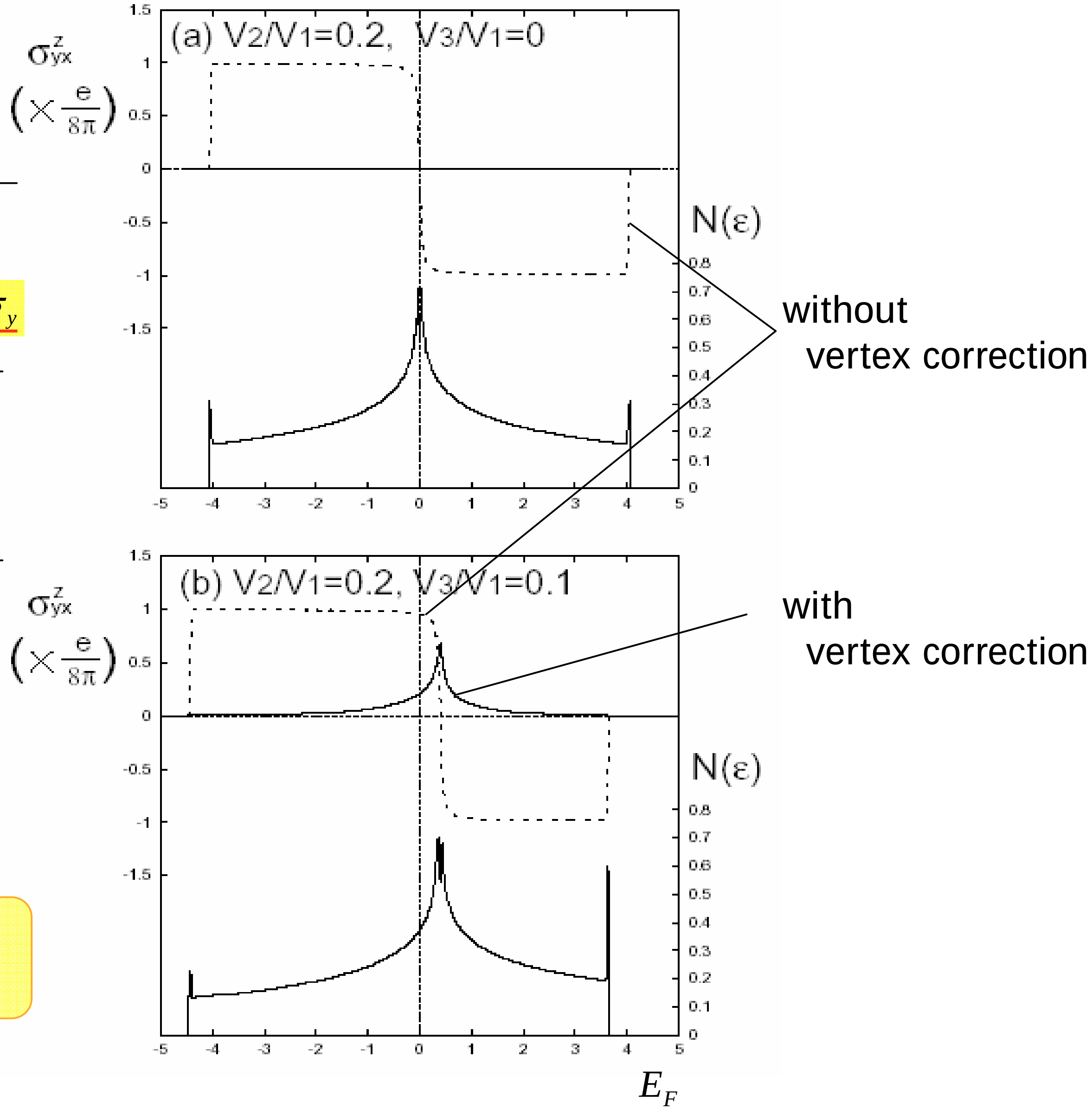
  (b) Otherwise, SHE is finite in general.

Rashba model $\lambda(\vec{\sigma} \times \vec{k})_z$ → (a)
Dresselhaus model $\beta(\sigma_x k_x - \sigma_y k_y)$ → (b)

In real materials SHE is finite in general.

Tight-binding model on a square lattice





SHE -- nonzero in general

Definition of spin current is not unique in the presence of spin-orbit coupling

- Spin-orbit coupling → spin is not conserved → no unique def. of spin current
  - Noether's theorem cannot be applied.
  - $$\frac{\partial S_i}{\partial t} + \nabla \cdot J_i^{(\mathrm{spin})} = 0 \Leftarrow \quad 0 = \frac{\partial}{\partial t}\int S_i d^d r = i\left[H, \int S_i d^d r\right]$$

    Eq. of continuity requires conservation of spin, but the spin is not conserved in these models

- In some models, spin current is covariantly conserved. (Zhang)

$$H = \frac{1}{2m}\left(\vec{p} + \vec{A}\right)^2, \quad \vec{A} = A_j^k t^k$$: gauge field associated with the spin-orbit coupling (e.g.) Rashba model

⇒ $$D_j = \partial_j + iA_j$$: covariant derivative

$$J_j^i = \frac{i}{2m}\left[\left(D_j\psi\right)^+ t^i \psi - \psi^+ t^i \left(D_j\psi\right)\right]$$: spin current

$$\partial_t S^i + \left(D_j J_j\right)^i = 0$$

Criterion for nonzero spin Hall conductivity

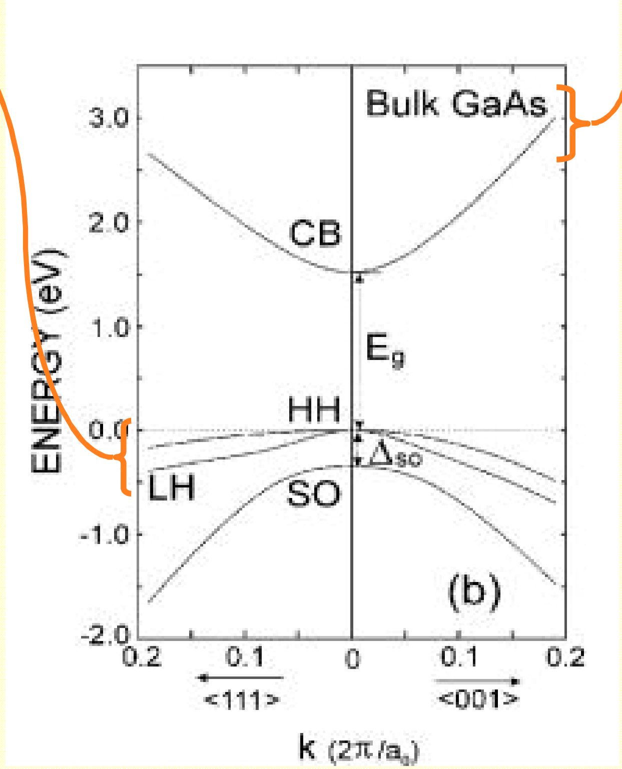
- Different filling for bands in the same multiplet of $\vec{J} = \vec{L} + \vec{S}$

  $\langle c | H_{\mathrm{s.o.}} | v \rangle \neq 0$

(Example) : GaAs

Valence band: J=3/2

Hole-doping gives a different filling for HH and LH bands.
→ **Spin Hall effect**



Conduction band: J=1/2

Electron-doping does not give rise to different filling for two conduction bands
→ **NO spin Hall effect**

In 2D heterostructure,
Rashba coupling lifts the degeneracy
→ **Spin Hall effect**

(Intrinsic) spin Hall effect should occur
in wide range of materials.

Experiments on spin Hall effect

- **2D electron gas, spin accumulation at the edges**
  Y.K.Kato, R.C.Myers, A.C.Gossard, D.D. Awschalom, Science 306, 1910 (2004)
- **2D hole gas, spin LED**
  J. Wunderlich, B. Kästner, J. Sinova, T. Jungwirth, cond-mat/0410295, to appear in PRL

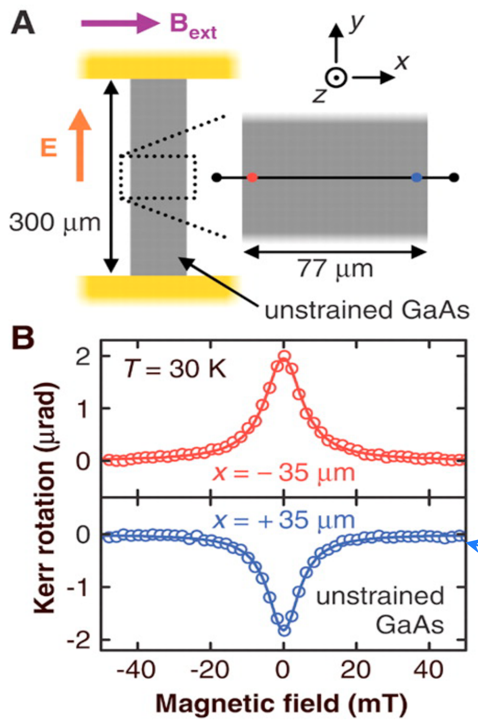
Experiment -- Spin Hall effect in a 2D electron gas --

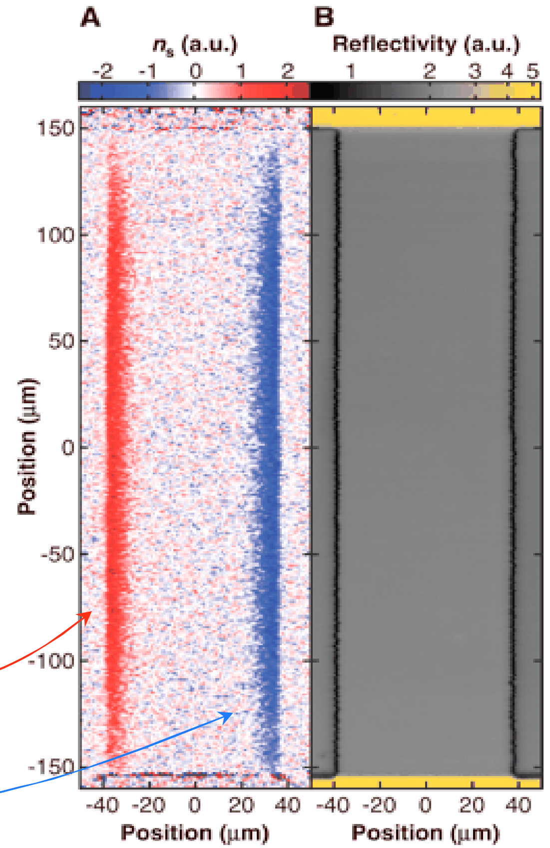
Y.K.Kato, R.C.Myers, A.C.Gossard, D.D. Awschalom, Science 306, 1910 (2004)

(i) Unstrained n-GaAs
(ii) Strained n-$In_{0.07}Ga_{0.93}As$

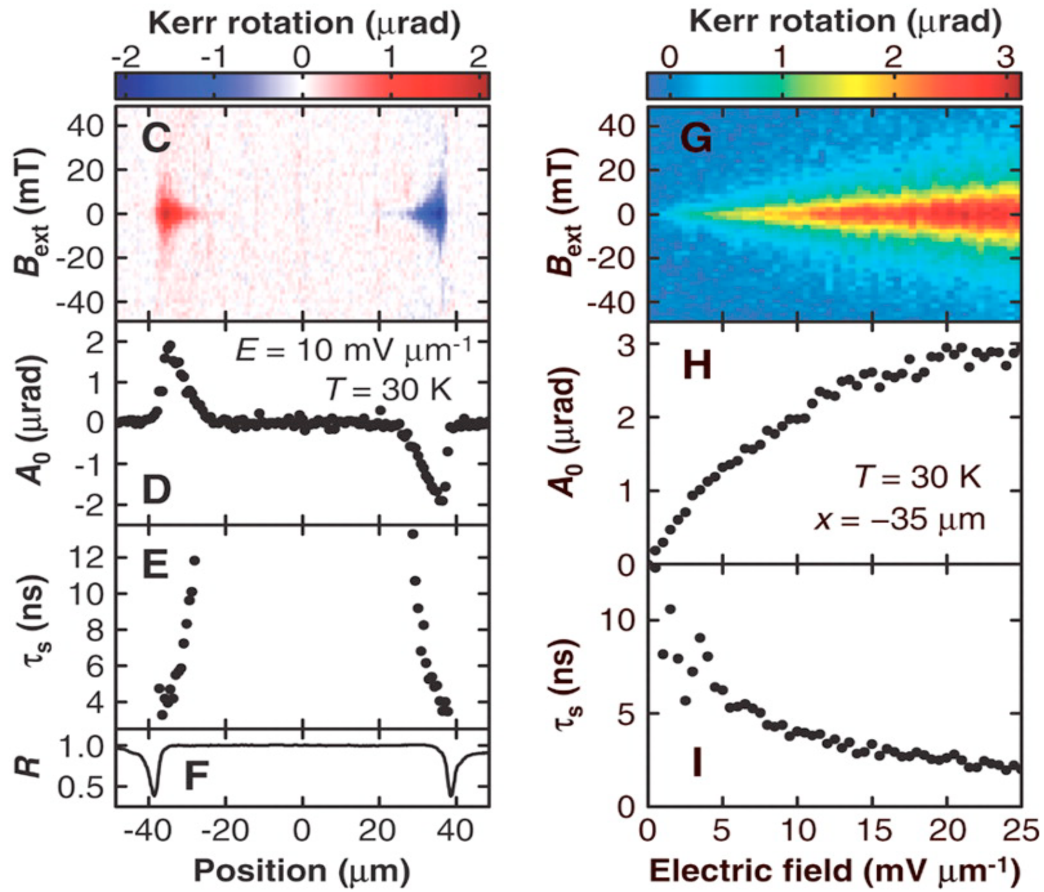
T=30K, Hole density: $3\times10^{16}\,\mathrm{cm}^{-3}$

S_z : measured by Kerr rotation





Spatial profile



Spin density maximum

$n_0 \approx 10 \mu m^{-3}$

↓

Spin current $j_s \approx 10 nA \mu m^{-2}$

Very small

(Charge current $j_c \approx 50 \mu A \mu m^{-2}$)

Y.K.Kato et al., Science (2004)

- unstrained GaAs -- (Dresselhaus) spin splitting negligibly small ($) \propto k^3$
- strained InGaAs -- no crystal orientation dependence

→ **It should be extrinsic!**

Bernevig, Zhang, cond-mat (2004)

- Dresselhaus term is relevant. **It should be intrinsic!**

- Dresselhaus term Δ is small, but induced SHE is not small.
  Rather, experimental value is 10^{-3} times smaller than theory.

- For Dresselhaus term the vertex correction is zero.

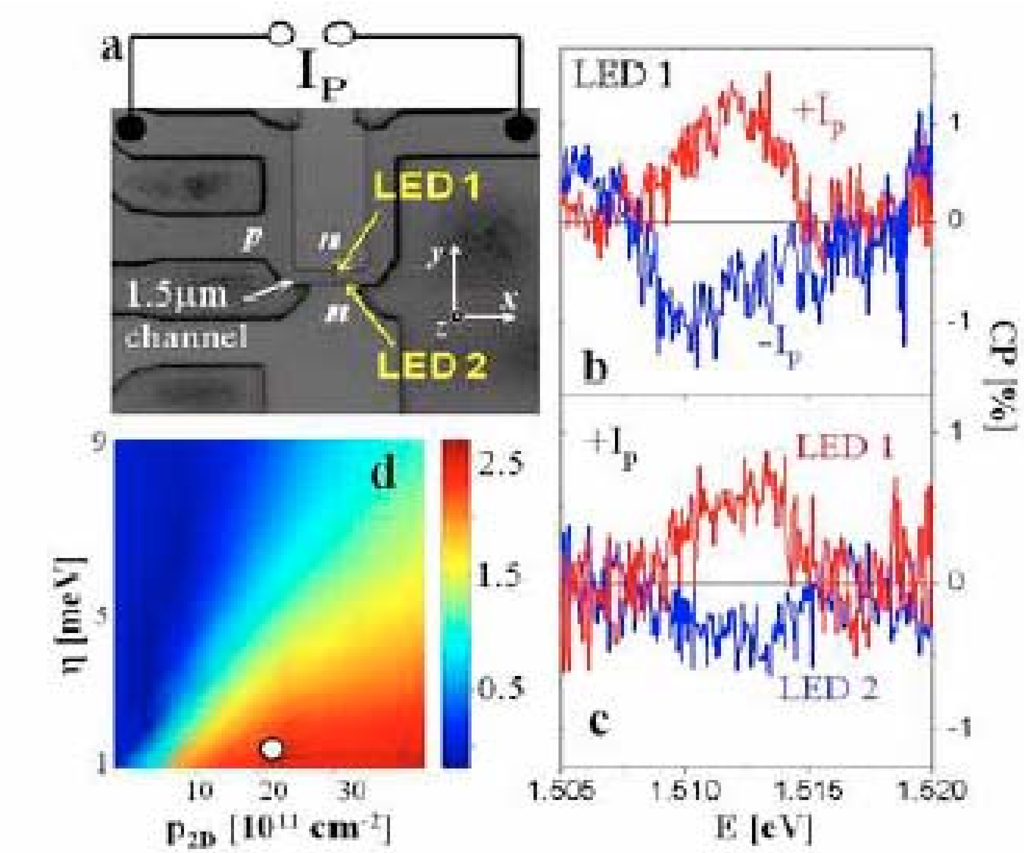
- Dirty limit : $\Delta \approx 0.025 meV, \quad \hbar/\tau \approx 1.6 meV$
  → SHE suppressed by some factor, which is larger than $\left(\frac{\Delta}{\hbar/\tau}\right)^2 \approx 10^{-4}$

Consistent with experiments

Experiment -- Spin Hall effect in a 2D hole gas --

J. Wunderlich, B. Kästner, J. Sinova, T. Jungwirth, PRL (2005)

- LED geometry



- Circular polarization $\approx 1\%$
- **Clean limit** :

  $\hbar / \tau \approx 1.2 meV$

  much smaller than spin splitting
- vertex correction =0
  (Bernevig, Zhang (2004))

It should be intrinsic!

Spin Hall insulator

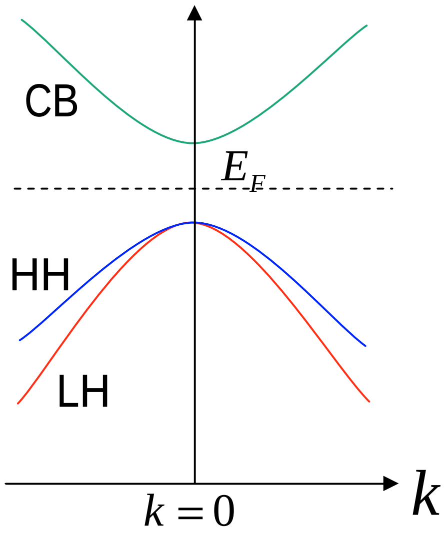
- Nonzero spin Hall effect in band insulators

  - SM, Nagaosa, Zhang,
    Phys. Rev. Lett.93, 156804 (2004)

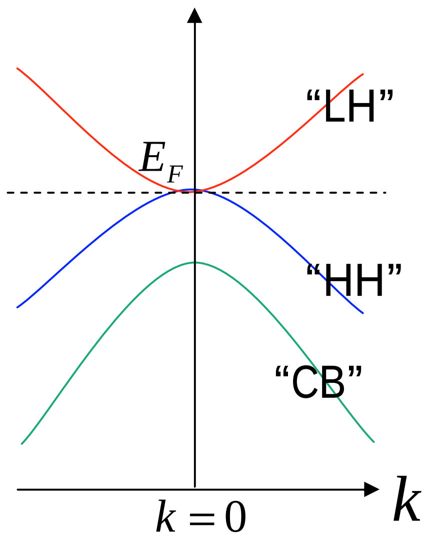
Nonzero spin Hall effect in band insulators

1) Zero-gap semiconductors : α-Sn, HgSe, HgTe, β-HgS…

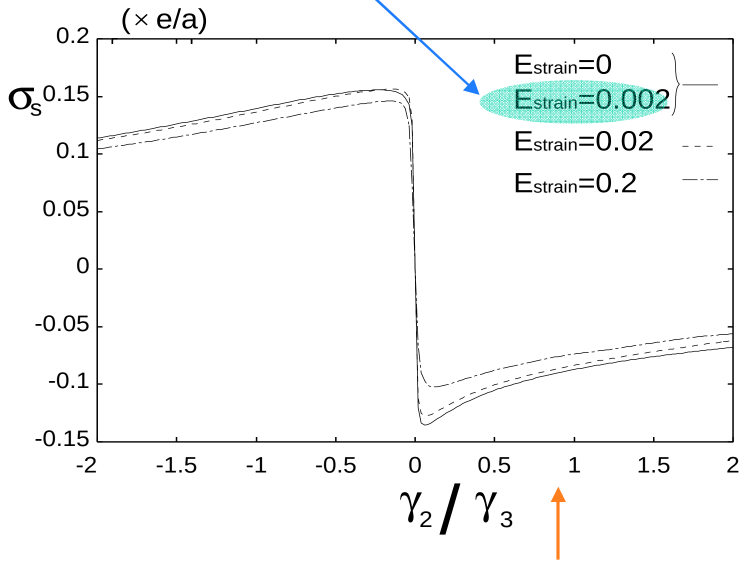
conventional



zero-gap



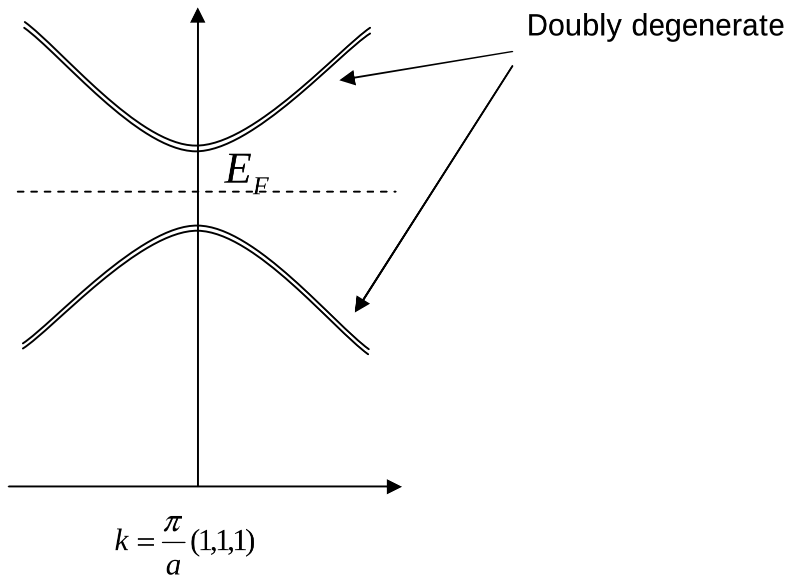
α-Sn under uniaxial stress = 3.2×10^9 dyn/cm^2
→ energy gap 44meV

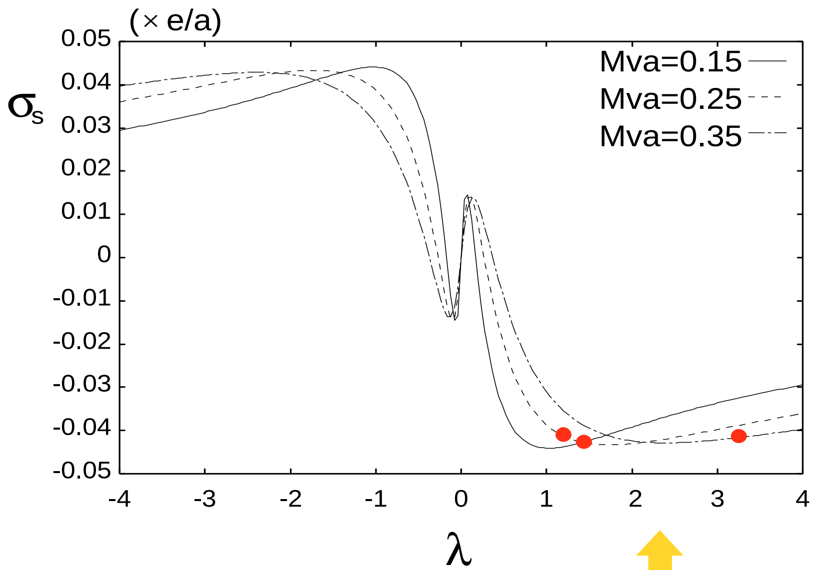


- Spin Hall effect is nonzero $(\approx 0.1e/a)$ in band insulators
- Uniaxial strain → finite gap at k=0

Spin Hall insulator

2) Narrow-gap semiconductors : PbS, PbSe, PbTe





Direct gap (0.15eV-0.3eV) at 4 equivalent L-points

$$H = v\vec{k}\cdot\hat{p}\,\tau_1 + \lambda v\vec{k}\cdot(\hat{p}\times\vec{\sigma})\tau_2 + Mv^2\tau_3$$

$\vec{\sigma}$: spin $\vec{\tau}$:orbital

	Mva	λ
PbS	0.26	1.2
PbSe	0.16	1.4
PbTe	0.35	3.3

- Spin Hall effect is nonzero $(\approx 0.04e/a)$ in band insulators

Spin Hall insulator

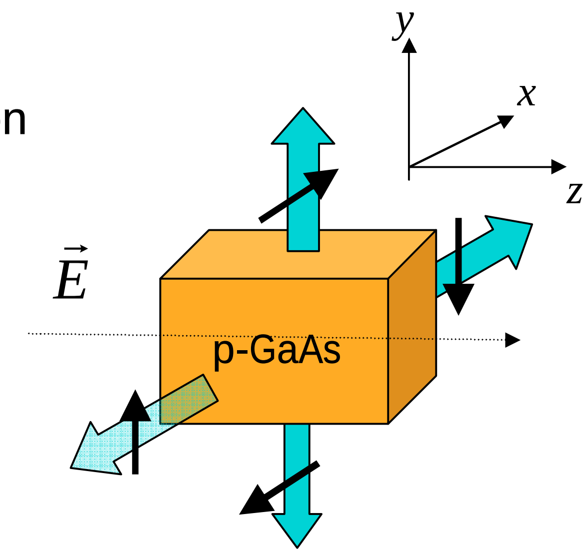
Conclusion

In semiconductors (Si, Ge, GaAs,...), spin current is induced by the external electric field.

$$j_y^x = \frac{eE_z}{12\pi^2}\left(3k_F^H - k_F^L\right)$$

: semiclassical result

x: spin direction
y: current direction
z: electric field



- Topological origin
- Dissipationless
- All occupied states contribute.
- Large even at room temperature

} Spin analog of the QHE

Spin Hall effect can be nonzero in band insulators.

(examples)

- Zero-gap semiconductors (α-Sn, HgTe, HgSe, β-HgS)
- Narrow-gap semiconductors (PbS, PbTe, PbSe)

Hall effect of light

- Anomalous velocity due to Berry phase

  → interference of waves

  Common for every wave phenomenon.

How about "light" ?

↓

YES!

Hall effect of light

Hall effect of light

Onoda, SM, Nagaosa, Phys. Rev. Lett. (2004)

-- Analog of the spin Hall effect --

Isotropic medium, slowly varying refractive index $n(\vec{r})$
→ pick up terms up to $O(\lambda \nabla \ln n)$

Semiclassical eq. of motion

$$\begin{cases} \dot{\vec{r}} = v(\vec{r})\hat{k} + \dot{\vec{k}} \times \langle z | \vec{\Omega}(\vec{k}) | z \rangle \\ \dot{\vec{k}} = -k \nabla v(\vec{r}) \\ |\dot{z}\rangle = -i\dot{\vec{k}} \cdot \vec{\Lambda}(\vec{k}) | z \rangle \end{cases}$$

Shift of a trajectory of light
"Hall effect of light"

Polarization change
- Chiao,Wu('86) : theory
- Tomita,Chiao('86) : experiment

$v(\vec{r}) = \dfrac{c}{n(\vec{r})}$: slowly varying

$|z\rangle$: polarization

$\vec{\Lambda}(\vec{k})_{ij} = -i\vec{e}_i^{\,+} \nabla_{\vec{k}} \vec{e}_j$: gauge field

$\vec{\Omega}(\vec{k}) = \nabla_{\vec{k}} \times \vec{\Lambda} + i\vec{\Lambda} \times \vec{\Lambda}$: curvature

In the vacuum

$$\vec{\Omega}(\vec{k}) = \frac{\vec{k}}{k^3}\begin{pmatrix} 1 & \\ & -1 \end{pmatrix} \begin{matrix} \longrightarrow \text{right} \\ \longrightarrow \text{left} \end{matrix}$$

in the basis of circular polarization

$$\dot{\vec{r}} = v(\vec{r})\hat{k} + \dot{\vec{k}} \times \langle z | \vec{\Omega}(\vec{k}) | z \rangle$$

$$\dot{\vec{k}} = -k \nabla v(\vec{r})$$

$$| \dot{z} \rangle = -i \dot{\vec{k}} \cdot \vec{\Lambda}(\vec{k}) | z \rangle$$

Onoda, SM, Nagaosa, PRL (2004)

equivalent

$$\dot{\vec{r}} = \frac{c}{n}\left[\vec{s} - \sigma \frac{c}{\omega n} \vec{s} \times \nabla \ln n \right]$$

$$\dot{\vec{s}} = \frac{c}{n}\left[\nabla \ln n - \vec{s}\left(\vec{s} \cdot \nabla \ln n \right) \right]$$

$$\dot{\vec{e}} = -\frac{c}{n} \vec{s}\left(\vec{e} \cdot \nabla \ln n \right)$$

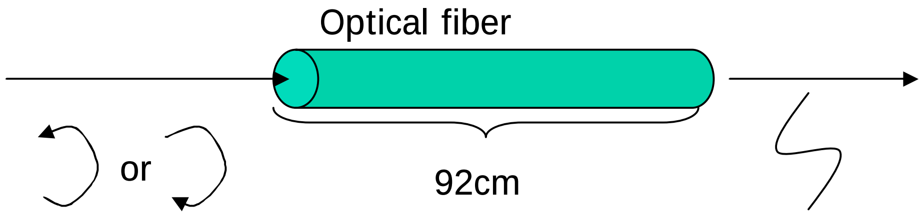
$\vec{s} = \vec{k} / k$ $\quad \vec{e}$: polarization

Zel'dovich, Liberman (1990)
Bliokh (2004)

“Optical magnus effect”

Experiment

Dugin, Zel'dovich, Kundikova, Liberman (1991)



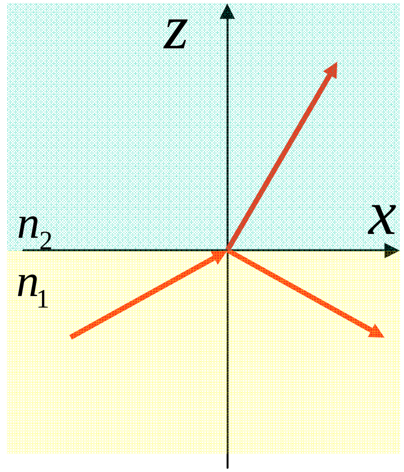
Beam profile
→ rotated by 3.2 degrees

Hall effect of light in interface refraction/reflection

$$\dot{\vec{r}} = v(\vec{r})\hat{k} + \dot{\vec{k}} \times \langle z|\vec{\Omega}(\vec{k})|z\rangle$$

$$\dot{\vec{k}} = -k\nabla v(\vec{r})$$

$$|\dot{z}\rangle = -i\dot{\vec{k}} \cdot \vec{\Lambda}(\vec{k})|z\rangle$$



$\leftarrow \vec{k}_\perp$ changes

At the interface, the refractive index changes

→ the trajectory is transversely shifted due to Berry phase (**to y-direction**)

Opposite for right & left circular polarization

$$\left(\vec{\Omega}(\vec{k}) = \pm \frac{\vec{k}}{k^3} \quad \text{for left (right) circular polarization} \right)$$

Imbert shift

Theory: Fedorov (1955)

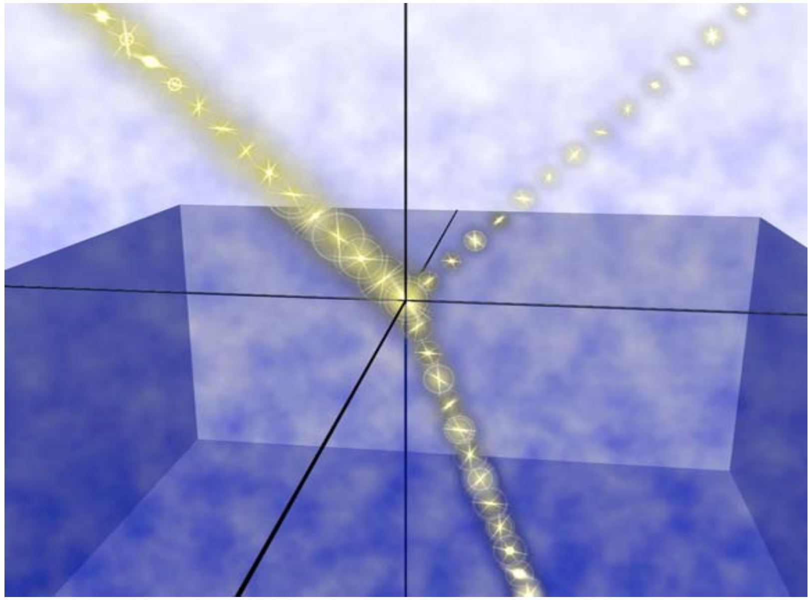
Experiment: Imbert(1972)

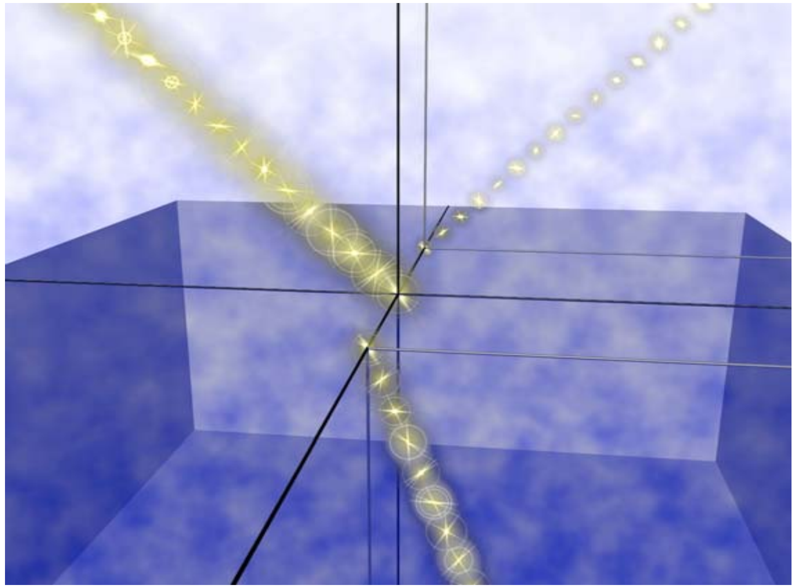
Cf. Conservation of total angular momentum

$$J_z = S_z + L_z$$

Imbert shift

Imbert shift for Left circular polarization



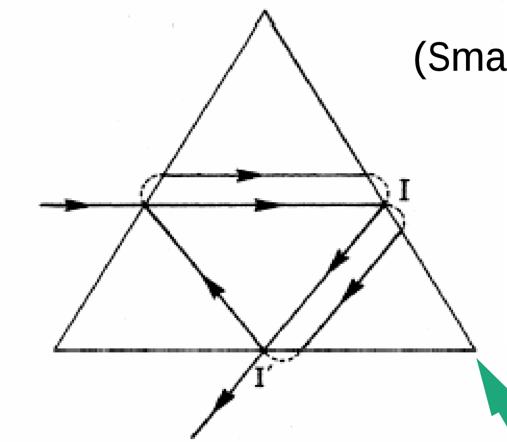


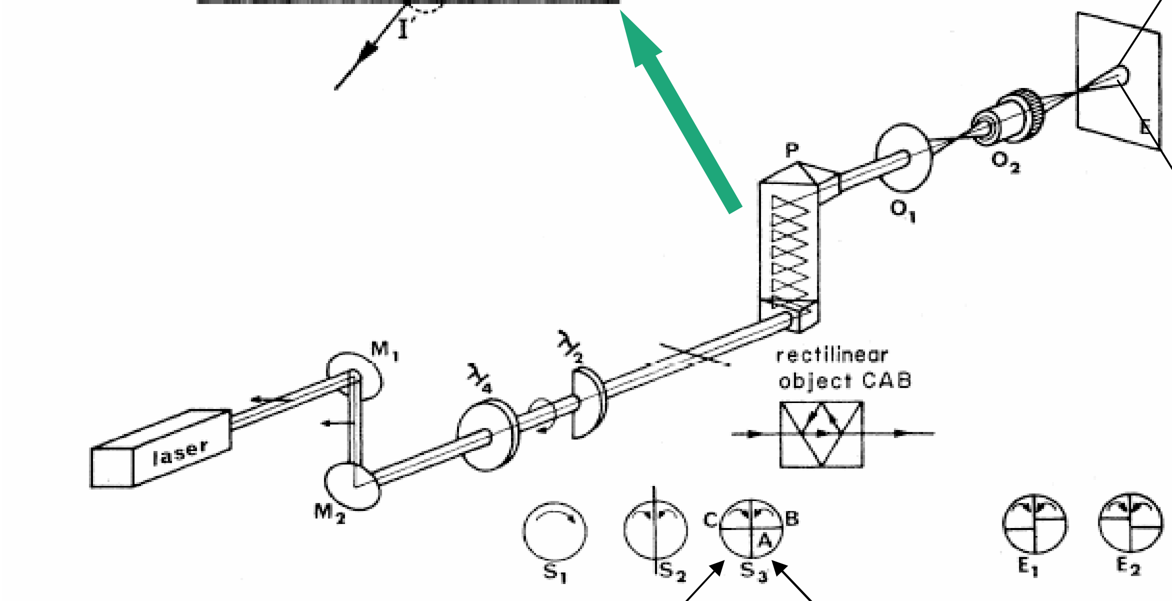
Magnitude of the shift $\approx \lambda$
Width of the beam is much larger → not easy to observe.

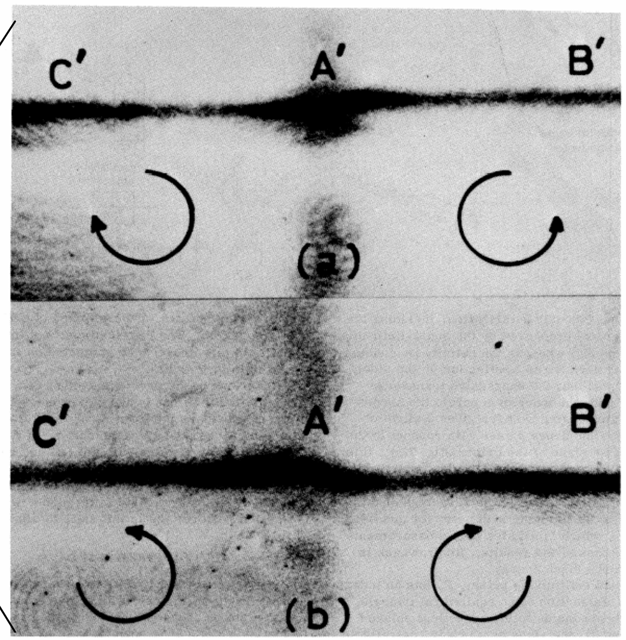
Experimental measurement of the Imbert shift

C. Imbert, PRD (1972)

(Small shift) * (28 successive reflection)







right circular pol.

left circular pol.

Shift is opposite for the two circular polarizations.

Photonic crystals and Berry phase

Electrons in condensed matter:

Periodic lattice enhances the Hall effect by some orders of magnitude

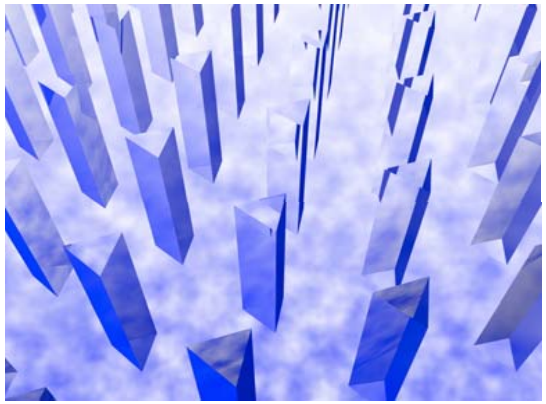
↓

Will the "Hall effect of light" enhanced in photonic crystals?
→ YES!

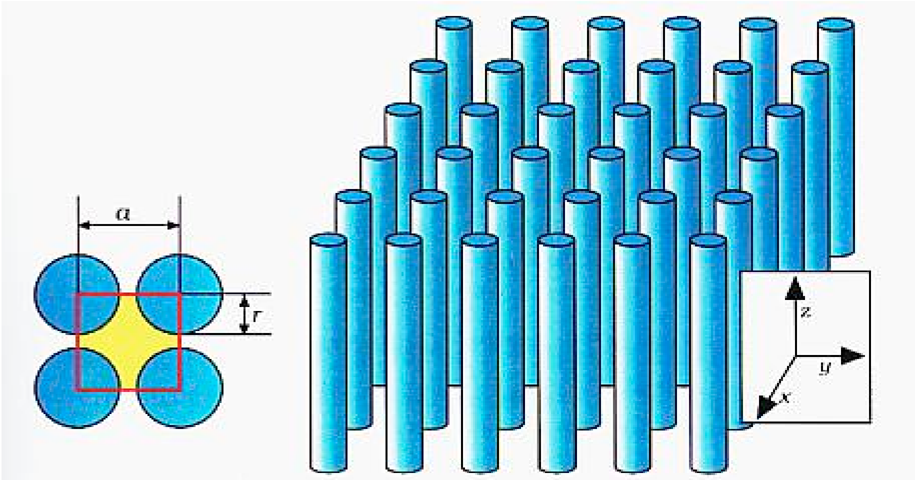
(Example) 2D photonic crystals (PC)

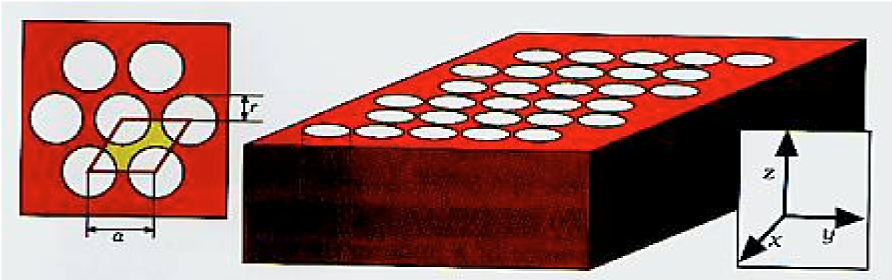
Caution :
Berry curvature is zero for 2D PC with inversion symmetry.

We use a 2D PC without inversion symmetry.



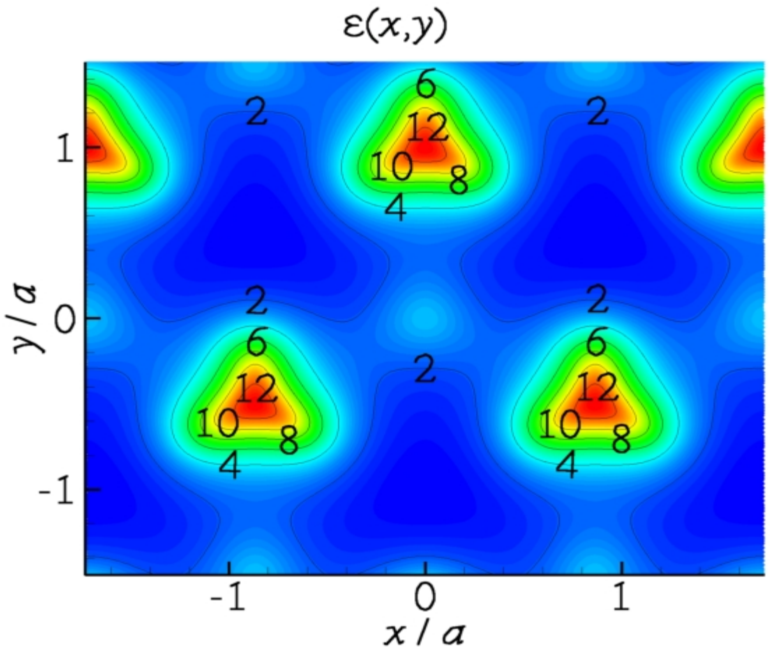
<u>2D photonic crystals</u>



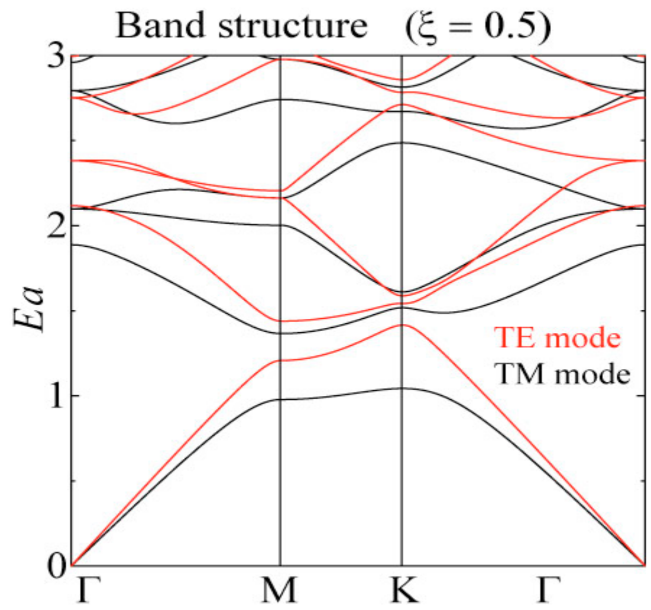


(“Photonic Crystals”, Joannopoulous et al.)

Simulation : Dielectric constant and its band structure

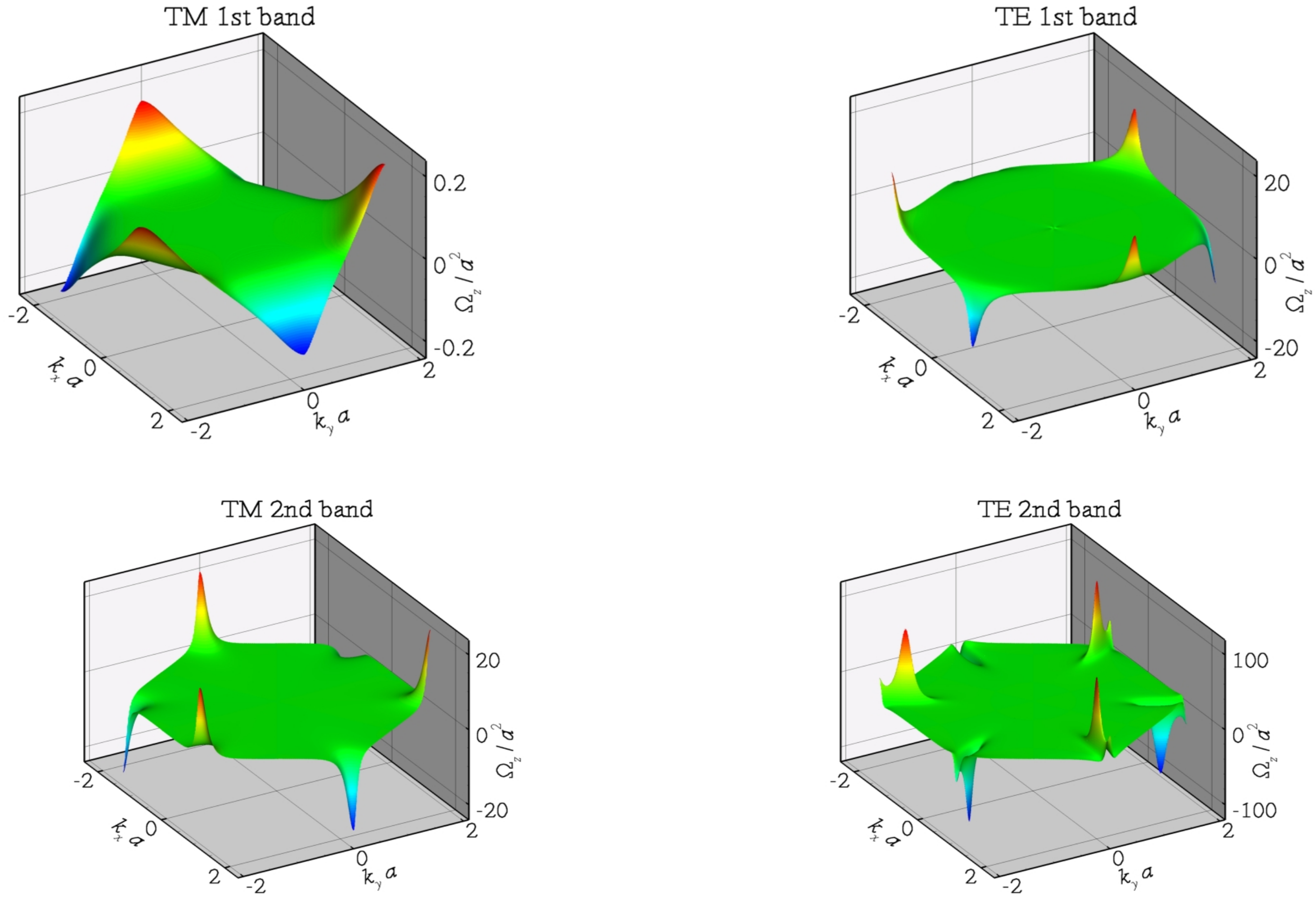


Dielectric constant



Photonic band structure

Berry phase in photonic crystals



Large Berry curvature when the band approach other bands in energy

Trajectory of light beam in photonic crystals

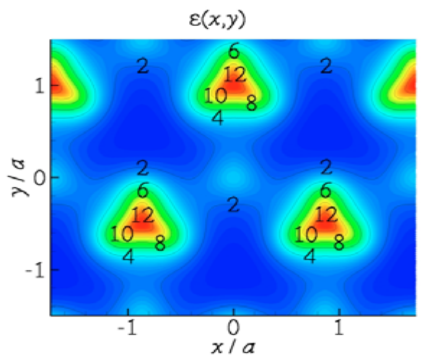
To see the anomalous velocity $\dot{\vec{k}} \times (z|\vec{\Omega}_{\vec{k}}|z)$
$\vec{k}$ should change in time.

(In addition to periodic modulation of $\varepsilon(\vec{r})$)
slow 1D modulation needed

slow 1D modulation near x=0

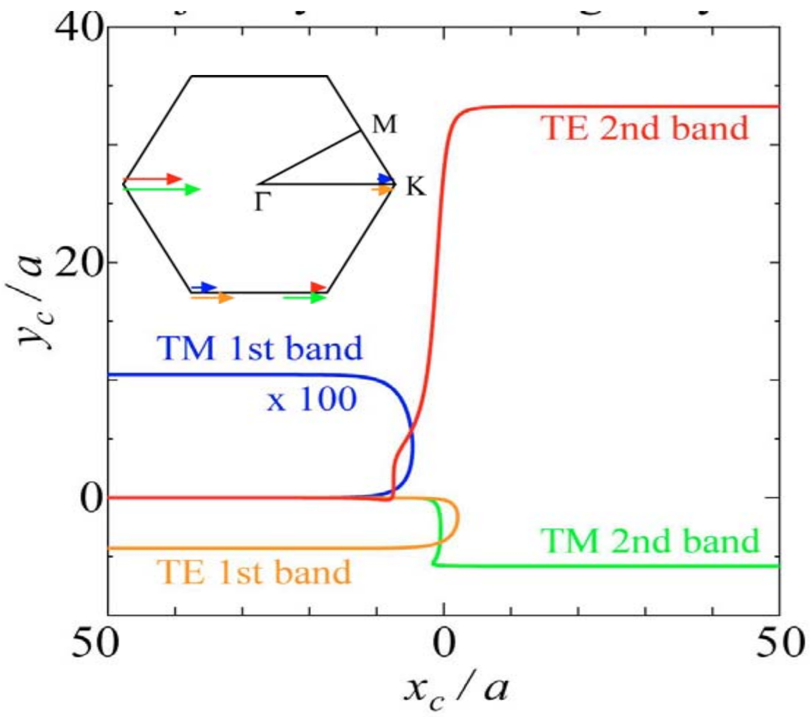
larger ε for larger x

-- Analog of the electric field in the SHE



× (envelope function linear in x)

Simulation



Large shift !!

Enhancement of Berry curvature in photonic crystals

- vacuum : $\left|\vec{\Omega}_{\vec{k}}\right| \approx \dfrac{1}{k^2}$ ⟶ Shift $\approx \lambda$ very small (e.g.: Imbert shift)

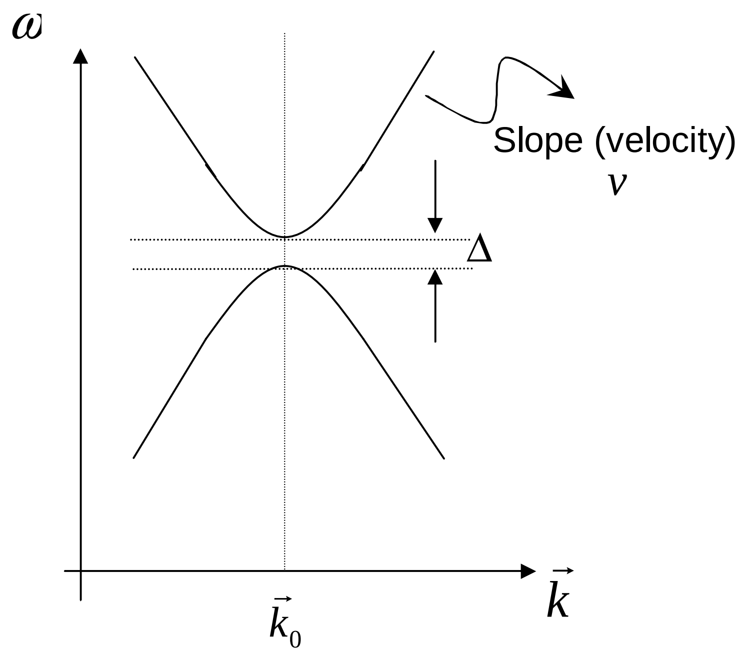
- photonic crystals:

$$\Omega_z \approx \frac{v^2 \Delta}{\left(\Delta^2 + v^2 (\vec{k} - \vec{k}_0)^2\right)^{3/2}}$$

⟶ maximum at the gap : $\Omega_z \approx \dfrac{v^2}{\Delta^2}$

Maximum shift of the beam $\dfrac{v}{\Delta}$

Bigger for smaller gap

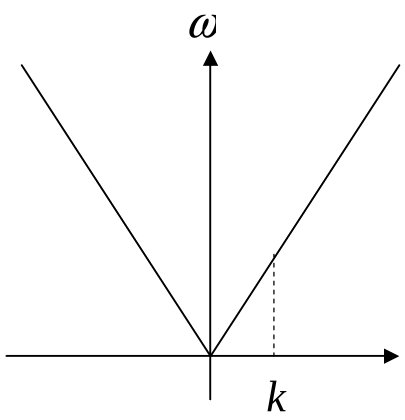


Shift of the light beam $\approx$ Distance from a monopole in k space

- Vacuum:

(shift) $\approx \lambda <<$ (width of the beam)

Small shift



- Photonic crystal:

(shift) $\approx \frac{v}{\Delta}$

(width of the beam) $>> \frac{1}{|\vec{k}|}$

Shift can be large
(for small gap)