The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific
and Cultural Organization

International Atomic
Energy Agency

SMR. 1649 - 6

*SPRING SCHOOL ON SUPERSTRING THEORY
AND RELATED TOPICS*

14 - 22 March 2005

Classifying Supergravity Solutions and Applications

PART II

J.P. GAUNTLETT
Department of Physics
Theoretical Physics Group
Imperial College London
South Kensington Campus
London SW7 2AZ
U.K.

BLACK RINGS

Jerome Gauntlett and Jan Gutowski

Introduction

Black hole uniqueness theorems in $D = 4$:
[Israel, Carter, Hawking, Robinson...]

Equilibrium black holes are specified by conserved charges M and J. Static $\rightarrow$ Schwarzschild. Stationary $\rightarrow$ Kerr. Black holes have no hair.

Violated in $D = 5$: vacuum gravity has rotating black holes with topology:
- S^3 [Myers, Perry]
- $S^1 \times S^2$ - "Black Rings" [Emparan, Reall]

with the same M and $J_1 = J_2$.

Recent discovery of charged supersymmetric black rings in $D = 5$ Elvang, Emparan, Mateos, Reall

Supersymmetry $\Rightarrow$ stability and can hope to understand quantum properties from string theory.

Solutions found using the classification of supersymmetric solutions of $D = 5$ supergravity

The classification provides powerful tools to find solutions and also can be used to obtain partial uniqueness theorems.

Uniqueness assumed in state counting calculations.

Plan

1. Classification of minimal supergravity. Black hole and black ring solutions.

2. Concentric black rings.

3. Multi-charged rings: have hair

4. Conclusions

Black Rings in Minimal Supergravity

Recall classification of $D = 5$ minimal SUGRA solutions

Timelike case:

$$ds^2 = -f^2(dt+\omega)^2 + f^{-1} ds^2_{HK}$$

$$F = \frac{\sqrt{3}}{2} d[f(dt+\omega)] - \frac{1}{\sqrt{3}} G^+$$

where ds^2_{HK} is an arbitrary hyper-Kähler metric and

$$\begin{aligned} G^+ &\equiv \frac{1}{2} f(d\omega + *d\omega) \\ dG^+ &= 0 \\ \Delta f^{-1} &= \frac{4}{9}(G^+)^2 \end{aligned}$$

Bena and Warner discussed a way to solve these for flat base space.

Special case: ds^2_{HK} is Gibbons-Hawking

Tri-holomorphic Killing-vector K:

$$\mathcal{L}_K J^{(a)} = 0$$

Locally, $K = \partial_\psi$:

$$ds^2_{HK} = H[dx^i dx^i] + H^{-1}(d\psi + \chi_i dx^i)^2$$

where

$$\nabla \times \chi = \nabla H$$

Clearly H is harmonic on $\mathbb{R}^3$.
If $\mathcal{L}_K g = \mathcal{L}_K F = 0$, then the most general solution is specified by three further harmonic functions, K, L and M on $\mathbb{R}^3$.

In particular the general solution has

$$
\begin{aligned}
f^{-1} &= H^{-1}K^2 + L \\
\omega &= (H^{-2}K^3 + \frac{3}{2}H^{-1}KL + M)(d\psi + \cos\theta d\phi) \\
&\qquad + \hat{\omega}_i dx^i
\end{aligned}
$$

with ω obtained by solving

$$
\nabla \times \hat{\boldsymbol{\omega}} = H\nabla M - M\nabla H + \frac{3}{2}(K\nabla L - L\nabla K)
$$

Nice.

Black Holes and Black Rings

Gibbons-Hawking base is $\mathbb{R}^4$: $H = 1/r$

$$ds^2(\mathbb{R}^4) = \frac{1}{r}[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)] \\ + r(d\psi + \cos\theta d\phi)^2$$

Set $\rho = 2\sqrt{r}$ to get usual coordinates for $\mathbb{R}^4$ with θ, ϕ, ψ Euler angles on S^3.

Later:

$$\begin{aligned} r_1 &= \rho\cos\frac{\theta}{2}, \qquad \phi_1 = \frac{1}{2}(\psi + \phi) \\ r_2 &= \rho\sin\frac{\theta}{2}, \qquad \phi_2 = \frac{1}{2}(\psi - \phi) \end{aligned}$$

then

$$ds^2(\mathbb{R}^4) = dr_1^2 + r_1^2 d\phi_1^2 + dr_2^2 + r_2^2 d\phi_2^2$$

Black Hole [Breckenridge, Myers, Peet, Vafa]

Lies in the Gibbons-Hawking class with

$$
\begin{aligned}
H &= \frac{1}{r} \\
K &= -\frac{q}{2}\frac{1}{r} \\
L &= 1 + \frac{Q - q^2}{4}\frac{1}{r} \\
M &= \frac{3q}{4}
\end{aligned}
$$

⋆ Asymptotically flat

⋆ Horizon topology S^3 $(r = 0)$

⋆ 2 parameter solution: Q, q

⋆ 2 conserved charges:

Q -electric charge. (Mass$\sim Q$ by susy)

$J_1 = J_2 \sim q(3Q - q^2) \equiv j.$

⋆ Entropy$\sim (Q^3 - j^2)^{1/2}$

⋆ No CTC's: $j^2 \leq Q^3$

Single Black Ring [Elvang, Emparan, Mateos, Reall]

Lies in the Gibbons-Hawking class with

$$
\begin{aligned}
H &= \frac{1}{r} \\
K &= -\frac{q}{2}h_1 \\
L &= 1 + \frac{Q - q^2}{4}h_1 \\
M &= \frac{3q}{4} - \frac{3qR^2}{16}h_1
\end{aligned}
$$

where

$$h_1 = \frac{1}{|\mathbf{x} - \mathbf{x}_1|}$$

and

$$\mathbf{x}_1 \equiv (0, 0, -R^2/4)$$

⋆ Asymptotically flat

⋆ Horizon topology $S^1 \times S^2 (\mathbf{x} = \mathbf{x}_1)$

⋆ Three parameter solution: Q, q, R

⋆ Three independent conserved charges:

Q -electric charge. (Mass$\sim Q$)

$$J_1 = \frac{\pi}{8G} q(3Q - q^2), \qquad J_2 = J_1 + \frac{3\pi}{4G} qR^2$$

⋆ Single ring in the minimal theory does not violate uniqueness ($J_1 \neq J_2$).

⋆ Radius of $S^2$is $q/2$. Radius of S^1 is

$$L = \sqrt{3[\frac{(Q - q^2)^2}{4q^2} - R^2]}$$

⋆ Entropy $\sim q^2 L$

⋆ No CTCs $\Leftrightarrow \frac{(Q-q^2)^2}{4q^2} > R^2$

⋆ q is a dipole charge (which in general are not conserved).

⋆ Setting $R = 0$ one obtains the black hole solution. However, it is not a smooth limit.

Concentric Black Rings [Gauntlett, Gutowski]

The natural generalization is to stay in the Gibbons-Hawking class with

$$\begin{aligned} H &= \frac{1}{r} \\ K &= -\frac{1}{2}\sum_{i=1}^{N} q_i h_i \\ L &= 1 + \frac{1}{4}\sum_{i=1}^{N} (Q_i - {q_i}^2) h_i \\ M &= \frac{3}{4}\sum_{i=1}^{N} q_i - \frac{3}{4}\sum_{i=1}^{N} q_i |\mathbf{x}_i| h_i \end{aligned}$$

with $h_i = 1/|\mathbf{x} - \mathbf{x}_i|$

⋆ Asymptotically flat

⋆ If $|\mathbf{x}_i| \neq 0$ then we have N Killing horizons with topology $S^1 \times S^2$

Interpretation

The S^1 direction of each horizon lies on an orbit of the Killing-vector field ∂_ψ.

In $\mathbb{R}^4$ (asymptotic infinity) such orbits lie in a two plane and are specified by a point in $\mathbb{R}^3$ labelled by (r_0, θ_0, ϕ_0): the two-plane is specified by (θ_0, ϕ_0) and the radius of the S^1 orbit is given by r_0.

All these S^1 orbits are concentric, with common centre $r = 0$. E.g. $\theta = \pi$ and $r = R^2/4$, corresponding to the single ring solution, defines an S^1 lying in the (r_2, ϕ_2) plane, whereas $\theta = 0$ and $r = R^2/4$ defines an S^1 lying in the orthogonal (r_1, ϕ_1) two-plane, both centred at the origin.

$\star$ Set one of the $\mathbf{x}_i = \mathbf{0}$ then we get a single black hole sitting at the centre of the rings.

$\star$ $3N$ parameters

$\star$ 3 conserved charges: electric charge$\sim$ Mass, J_1, J_2.

$$Mass = \frac{3\pi}{4G}[\sum_{i=1}^{N}(Q_i - q_i^2) + (\sum_{i=1}^{N} q_i)^2] \ .$$

Usual non-uniqueness.

$\star$ Entropy$\sim$ $\sum_{i=1}^{N} q_i^2 L_i$where

$$L_i = \sqrt{3[\frac{(Q_i - q_i^2)^2}{4q_i^2} - R_i^2]}$$

$\star$ Analysis of CTC's requires ω, which requires an integration.

Poles on the z-Axis

Define

$$\Lambda_i \equiv \frac{Q_i - q_i^2}{2q_i}$$

No CTCs $\Rightarrow \Lambda_i = \Lambda$ for all i. Then the ring radii are

$$L_i \equiv \sqrt{3\left[\Lambda^2 - R_i^2\right]} \ .$$

Rings can be placed anywhere on the z-axis provided that $R_i^2 < \Lambda^2$.

As R_i increases, the circumference of the rings get uniformly smaller!

Comparing a black hole with two rings

Black hole with parameters Q, q corresponding to charge Q and $J_1 = J_2 = \frac{q}{2}(3Q - q^2)$.

Cannot match these conserved charges with a single black ring which has $J_1 \neq J_2$.

Can match with e.g. one black ring on the negative z-axis with parameters Q_1, q_1, R_1 and another on the positive z-axis with parameters Q_2, q_2, R_2.

Can choose the parameters so that

$$S_{BH} > S_{Rings}, \quad S_{BH} = S_{Rings}, \quad S_{BH} < S_{Rings}$$

Black rings can be entropically $\underline{preferred}$ to the black hole!

Challenge for state counting intepretation.

Contrast with extreme Reissner-Nordstrom black holes in $D = 4$

e.g. Two $Mass = Q$ R-N black holes have same charges as a single $Mass = 2Q$ R-N black hole. However, The entropy of the former scales like Q^2 while the latter scales like $4Q^2$. Thus a single R-N black hole is always preferred entropically.

Multi-Charged Black Rings

$D = 5$ minimal supergravity (g, A) coupled to $n-1$ vector multiplets (A^i, ϕ^i).

Use the classification of solutions for these models: timelike and null case. When timelike case has a Gibbons-Hawking base space the solution is specified in terms of $2n+2$ harmonic fucntions H, M, L^I, K^I

Focus on 3 $U(1)$'s. Can arise from D=11 reduced on T^6 or type IIB reduced on T^5. If we set all $U(1)$'s equal $\rightarrow$ D=5 minimal theory.

Black Hole [BMPV]

⋆ 4 parameter solution $Q^1, Q^2, Q^3,\ J_1 = J_2$.

⋆ Brane interpretation

Q^1 M2:	0	1	2				
Q^2 M2:	0			3	4		
Q^3 M2:	0					5	6

or

Q^1 D1:	0					5
Q^2 D5:	0	1	2	3	4	5
Q^3 P:	0					5

Microscopic state counting interpretation of entropy [Vafa, Strominger]; [BMPV]

Three-Charge Black Ring [Gauntlett, Gutowski]; [Bena, Warner]; [Elvang, Emparan, Mateos, Reall]

⋆ 7 Parameter Solution: Q^1, Q^2, Q^3, three dipole charges q^1, q^2, q^3 and R.

⋆ 5 Conserved charges: Q^1, Q^2, Q^3, $J_1 \neq J_2$

⋆ Non-unique - they have hair. But are distinguished from black holes.

⋆ Challenge for state counting interpretation of entropy.

⋆ Multi concentric generalisation [Gauntlett, Gutowski]

Black Rings and supertubes

Recall the basic supertube in type IIA [Mateos, Townsend]

D0:	0		
F1:	0	1	
d2:	0	1	ψ

Carries $D0$ and $F1$ charge, and dipole $D2$-charge.

2 T-dualities and then uplift to $D = 11$

M2:	0	1	2			
M2:	0			3	4	
m5:	0	1	2	3	4	ψ

The general black ring is a three-charge, three-dipole charge supertube:

Q^1 M2:	0	1	2					
Q^2 M2:	0			3	4			
Q^3 M2:	0					5	6	
q^1 m5:	0			3	4	5	6	ψ
q^2 m5:	0	1	2			5	6	ψ
q^3 m5:	0	1	2	3	4			ψ

Conclusions and Future Directions

⋆ Concentric black ring solutions

⋆ Concentric black rings with multiple charges

⋆ Interesting uniqueness and entropy properties.

⋆ Microscopic state counting interpretation of entropy?

The near horizon limit of the type IIB solutions has a decoupling limit (not the same as the near ring horizon limit) that is locally $AdS_3 \times S^3 \times T^4$, so the states should be found in the CFT. [Bena, Kraus]

Also, entropy can be recovered in M-theory by viewing the ring as a loop of $(0,4)$ CFT arising on wrapped $m5$-branes. [Cyrier, Mateos, Strominger]

$\star$ More general ring solutions.

Classification $\Rightarrow$ that black holes have near horizon geometry of (i) BMPV, (ii) $AdS_3 \times S^2$ or (iii) $\mathbb{R}^{1,4}$. Only AF solutions in class (i) are BMPV [Real]; [Gutowski]

Non-circular rings with varying charge density - functions worth of non-uniqueness? [Bena, Warner] Non- Regular [Horowitz, Reall] ? CTCs?

Multi-black ring solutions with different centres?

$\star$ Supersymmetric black ring solutions in $D = 5$ ADS supergravity?

$\star$ Supersymmetric exotica in other dimensions?