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*SPRING SCHOOL ON SUPERSTRING THEORY
AND RELATED TOPICS*

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Exact Results in SUSY Gauge Theories

PART I

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Please note: These are preliminary notes intended for internal distribution only.

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Exact Results in SUSY Gauge Theories B. Wecht

Rough breakdown: Lecture 1: Perturbative Non-renormalization
Lecture 2: Non-perturbative Effects
Lecture 3: SCFTs & a-maximization.

Lecture 1

Motivation: SUSY is awesome! Maybe it's physical, maybe it's not. Either way it is a good testing ground for ideas in QFT.

Additional Motivation: Available for a fee.

I. Brief Introduction/Review: $\mathcal{N}=1$, $d=4$.

I'll use superfields. There are two kinds here, chiral & vector

Chiral Superfield: $\underset{}{\Phi} = \underset{\substack{\uparrow \\ \text{cplx boson} \\ \text{(scalar)}}}{\phi} + \sqrt{2}\theta^\alpha \underset{\substack{\uparrow \\ \text{Weyl fermion}}}{\psi_\alpha} + \theta^2 \underset{\substack{\uparrow \\ \text{Auxiliary field}}}{F} + \cdots$

Vector Superfield: (in WZ gauge) $V = \theta\sigma^\mu\bar\theta \underset{\substack{\uparrow \\ \text{vector} \\ \text{(e.g. gauge boson)}}}{A_\mu} + i\theta^2\bar\theta\bar\lambda - i\bar\theta^2\theta\lambda + \frac{1}{2}\theta^2\bar\theta^2 \underset{\substack{\uparrow \\ \text{auxiliary field}}}{D}$

gaugino

The auxiliary fields are not dynamical d.o.f. & may be integrated out

NB: The top component of these superfields is invariant under SUSY, up to a total derivative, e.g. $\delta F \sim \partial\psi$.

$\exists$ a $U(1)$ that does not commute w/ SUSY, the R-charge.

if $R(\phi)=r$, $R(\psi)=r-1$, using normalization $R(\theta)=1$.

SUSY action (not gauge thy yet):

$$\mathcal{L} = \int d^4\theta\, K(\Phi^\dagger, \Phi) + \left[\int d^2\theta\, W(\Phi) + h.c.\right]$$

for 1 Φ, easy to generalize for more

Kähler potential e.g. $\Phi^\dagger\Phi$ — Superpotential.

Note: $R(W) = 2$ in our conventions.

The superpotential is manifestly only a function of Φ, not $\Phi^\dagger$.

$\Rightarrow$ It is a $\boxed{\text{holomorphic}}$ function of Φ.

II. Exact Result #1:

Theorem: The superpotential is not renormalized to any order in perturbation theory!!

Classic Example: $W = \frac{1}{2} m \Phi^2 + \frac{1}{2} g \Phi^3$, the Wess-Zumino model.

Pf of theorem: There are two $U(1)$ symmetries, $U(1)_R$ & $U(1)_\Phi$.

R — Phase to Φ.

Assign charges:

	$U(1)_R$	$U(1)_\Phi$
ϕ	0	1
m	2	-2
g	2	-3

Think of m, g as vevs. Assign charges as if fields, thinking of spont. breaking.

But W must be a hol. function of ϕ, m, g s.t. $R(W) = 2$, $U(1)_\Phi$ charge of $W = 0$.

$$\Rightarrow W = \frac{1}{2} m \Phi^2 f\left(\frac{g\Phi}{m}\right)$$ where $f(t)$ is some function of t (not t^*!)

1^{st} take $g \to 0$. Then $f(t) \to f(0)$ must recover $\frac{1}{2} m \Phi^2$.

$\Rightarrow f(t) = 1 + \sum_{n=1}^{\infty} c_n t^n$.

Now (at same time) take $m \to 0$ s.t. $\frac{m}{g} \to 0$.

Can't have divergences, so no powers ≥ 1 are permitted

$\Rightarrow f(t) = 1 + \frac{2}{3} t. \Rightarrow \boxed{W_{eff} = \frac{1}{2} m \Phi^2 + \frac{1}{3} g \Phi^3}$

match classically

And in fact here, $W_{exact} = W_{tree} = \frac{1}{2} m \Phi^2 + \frac{1}{3} g \Phi^3$.

Not a gauge theory yet – that's later.

Convince yourself diagramatically that e.g. Φ^4 corrections are not holomorphic.

II. Gauge Theories

$$S = \int d^4x \, d^2\theta \, \tau \left[\frac{\mathrm{Tr} W_\alpha W^\alpha}{32\pi^2} \right]$$

field strength chiral s.f.

$\tau = -\frac{8\pi^2}{g^2} + i\theta$ is the holomorphic gauge coupling;

this gives $-\frac{1}{4g^2} F_{\mu\nu} F^{\mu\nu} + \frac{\theta}{32\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}$ + gauginos ...

There's a 1-loop β function (can see diagramatically):

$$\mu \frac{\partial g}{\partial \mu} = -\frac{b g^3}{16\pi^2}, \quad b = \frac{11}{6} T(adj) - \frac{1}{3} \sum_a T(r_a) - \frac{1}{6} \sum_n T(r_n)$$

Weyl ferm   cplx scalar

$\left(T(r): \ \mathrm{Tr}\, T_r^a T_r^b = \frac{1}{2} T(r) \delta^{ab}. \quad T(\text{adj of } SU(N)) = 2N \right.$
$\left. T(\square) = 1. \right)$

SUSY

$$\left(\frac{11}{6}-\overset{\text{gauginos}}{\frac{1}{3}}\right)T(\text{adj}) = \frac{3}{2}T(\text{adj})$$

$$\underbrace{-\frac{1}{3}T(r_i) - \frac{1}{6}T(r_i)}_{\text{supermatter}} = -\frac{1}{2}T(r_i)$$

$$\boxed{b = \frac{3}{2}T(\text{adj}) - \frac{1}{2}\sum_i T(r_i)}$$

But τ is a cplx parameter, so can treat as χ.s.f. $\Rightarrow$ everything must be holomorphic in τ!

$$\beta(\tau) = \frac{\partial\tau}{\partial\ln\mu} = f(\tau) \Rightarrow \frac{\partial}{\partial\ln\mu}\left[\frac{-8\pi^2}{g^2}+i\Theta\right] = f(\tau).$$

But Θ doesn't run, b/c we have a massless fermion - can shift into phase of gauginos.

$\Rightarrow f(\tau)$ indep. of imaginary part

$\Rightarrow f(\tau) = $ real constant $= -b$.

$$\Rightarrow \boxed{\frac{\partial}{\partial\ln\mu}\left[\frac{-8\pi^2}{g^2}+i\Theta\right] = -b}$$

The holomorphic 1-loop β function is exact!

Integrate: $-\frac{8\pi^2}{g^2}+i\Theta = b\ln\left(\frac{\Lambda}{\mu}\right)$, So $\Lambda = |\Lambda|e^{i\Theta/b}$.

Can usefully think of the strong coupling scale Λ as complex, with phase Θ/b.

IV. Does *anything* run?

Yes, the *physical* couplings. $\boxed{\text{Kähler potential not under control!!}}$

$$K_{cl} \to K_{quantum} = Z \Phi^\dagger \Phi .$$

Canonically normalizing means $\Phi_{can} = Z^{1/2} \Phi$

not holomorphic.

$$\boxed{Ex} \quad g\Phi^3 \to \overbrace{g Z^{-3/2}}^{g_{phys}} \Phi_{can}^3$$

So
$$\frac{\partial}{\partial \ln \mu} g_{phys} = g \cdot \left(-\frac{3}{2} Z^{-5/2} \frac{\partial Z}{\partial \ln \mu} \right)$$
$$= \frac{3}{2} g Z^{-3/2} \left(-\frac{\partial \ln Z}{\partial \ln \mu} \right) = \boxed{\frac{3}{2} g_{phys} \gamma_\Phi(g)}$$

The physical couplings do run!

A similar canonical renorm of gauge fields gives

$$\boxed{\beta_{NSVZ}(g) = \frac{-g^3}{16\pi^2} \frac{3T(G) - \sum_i T(r_i)(1-\gamma_i(g))}{1 - g^2 \frac{T(adj)}{8\pi^2}}}$$

This is the running of the physical gauge coupling!

STOP