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SPRING SCHOOL ON SUPERSTRING THEORY
AND RELATED TOPICS

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Exact Results in SUSY Gauge Theories

PART II

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Lecture 2 | Non-perturbative Results

Recall: In a SUSY gauge thy, we can usefully think of Λ, the strong coupling scale, as complex:

$$-\frac{8\pi}{g^2} + i\theta = b\ln\left(\frac{\Lambda}{\mu}\right) \Rightarrow e^{-S_{inst} + i\theta} = \left(\frac{\Lambda}{\mu}\right)^b.$$

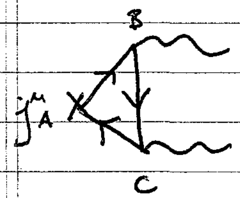
We can see that instanton effects are weighted by Λ^b.

Review: Relation to anomalies.

Say we have a current j^μ that's anomalous,

$$\partial_\mu j^\mu = \frac{A}{32\pi^2}\mathrm{Tr}F\tilde{F} \quad \text{at one loop}$$

$\Rightarrow$ Take j^μ to be a global current, and compute w/ 2 gauge bosons too



$$A = \mathrm{Tr}(T^A_{global})\,\mathrm{Tr}(T^B_{gauge}T^C_{gauge})$$

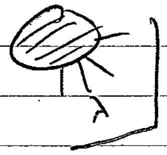
So non-abelian global currents are not anomalous.

For a $U(1)$ global current, $A = \sum_i q_i T(r_i)$

q_i: $U(1)$ charge; $T(r_i)$: Casimir of rep r_i of Fermion

Ex SYM ($\mathcal{N}=1$): Only ferms are gauginos λ_α, $R(\lambda_\alpha) = 1$

$\Rightarrow A = T(G) \equiv 2h$



2h gauginos soaked up

Integrate both sides of $\partial_\mu j^\mu = \frac{A}{32\pi^2} \mathrm{Tr} F\tilde{F}$

$$\Rightarrow \quad \Delta Q = \underbrace{2h}_{\text{Anomaly coeff.}} \cdot \underbrace{k}_{\text{Instanton \#}}$$

ΔQ: Charge violation

$\therefore$ Must insert $2h$ gauginos to get nonzero amplitude

$$\Rightarrow \quad \langle (\lambda_\alpha \lambda^\alpha)^h \rangle = c\Lambda^b$$

This is one way of seeing that the R-symmetry is anomalous.

But, as before, we can assign Λ an R-charge & think of this as spontaneous breaking.

$$\left.\begin{array}{lll} & U(1)_R & \\ \lambda_\alpha & 1 & \lambda \to e^{i\phi}\lambda \\ \Lambda^b & 2h & \Lambda^b \to e^{i\phi(2h)}\Lambda^b \end{array}\right] \Rightarrow \text{This shifts } \Theta : \Lambda = |\Lambda| e^{i\Theta/b}$$

<u>Another way</u>: $\frac{\delta S}{\delta \phi} = \partial_\mu j^\mu = \frac{A F\tilde{F}}{32\pi^2}$ ← anomaly.

So when $\lambda \to e^{i\phi}\lambda$, $S \to S - \phi \frac{A}{32\pi^2}(F\tilde{F})$

i.e. changing Λ^b, shifting Θ, undoes this. $\Theta \to \Theta + \phi A$.

$\boxed{\text{Notice}}$ We have broken $U(1)_R \to \mathbb{Z}_{2h}$ since Θ is periodic.

[Why are we doing this? In order to find nonpert effects — these will get Λ contributions]

But for expectation values of elts. of the chiral ring, positions don't matter:

$$\frac{\partial}{\partial x_j}\left\langle \prod_i \mathcal{O}_i(x_i)\right\rangle = 0$$

$\Rightarrow$ Can separate $\langle (\lambda_\alpha \lambda^\alpha)^h\rangle = \langle \lambda_\alpha\lambda^\alpha\rangle^h = c\Lambda^b$

So for SYM, $b = 3N_c$, $h = N_c$, $\langle \lambda_\alpha \lambda^\alpha\rangle = c\Lambda^3$

$\Rightarrow$ gaugino condensation

$$\boxed{U(1)_R \to \mathbb{Z}_{2h} \to \mathbb{Z}_2}$$ (Can recover via other methods).

<u>Moduli Spaces</u> When finding SUSY Vacua, need to solve F & D equations: $F: \left|\frac{\partial W}{\partial \phi}\right|^2 = 0$ (really, $g^{i\bar{j}}\frac{\partial W}{\partial \phi_i}\frac{\partial \bar{W}}{\partial \bar{\phi}_{\bar{j}}} = 0$)

$$D^A: \sum_i \phi_i^\dagger T^A \phi_i = 0$$

<u>Ex</u> SUSY QCD: $SU(N_c)$ gauge group, ($W_{tree} = 0$)

	$SU(N_c)$
Q	$\square$
$\tilde{Q}$	$\bar{\square}$

Then since $T^A_{\bar{\square}} = -(T^A_{\square})^T$,

redundant $\rightarrow$ $\mathrm{Tr}\left[(T^A)_a{}^b\left(Q_b Q^{\dagger a} - \tilde{Q}^\dagger_b \tilde{Q}^a\right)\right] = 0$

But $Q_b Q^{\dagger a} - \tilde{Q}^\dagger_b\tilde{Q}^a \equiv M_b{}^a$ may be expanded as $M_b{}^a = c_0\delta_b{}^a + \sum_A c_A (T^A)_b{}^a$

Taking the trace gives, since $\mathrm{Tr}\,T^A = 0$, $\mathrm{Tr}\,T^A T^B = \frac{1}{2}T(r)\delta^{AB}$

$$c_A = 0$$

$$\Rightarrow \boxed{Q_b Q^{\dagger a} - \tilde{Q}^\dagger_b \tilde{Q}^a = c_0 \delta_b{}^a}$$

Let's take the case of 1 flavor: Can use $SU(N_c)$ rotations to say

$$Q = \begin{pmatrix} q_1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \Big\updownarrow N_c \ , \quad \tilde{Q}^{+} = \begin{pmatrix} \tilde{q}_1 \\ q_2 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \Big\updownarrow N_c$$

$$S_a \ QQ^{+} - \tilde{Q}^{+}\tilde{Q} = \begin{pmatrix} |q_1|^2 - |\tilde{q}_1|^2 & -\tilde{q}_1 \tilde{q}_2 & 0 \cdots 0 \\ -\tilde{q}_1 \tilde{q}_2 & -|\tilde{q}_2| & 0 \\ 0 & 0 & 0 \end{pmatrix} = c_0 \delta_a^{\ b}$$

$$\Rightarrow \boxed{c_0 = 0, \ \tilde{q}_2 = 0, \ |q_1|^2 = |\tilde{q}_1|^2}$$

Can describe moduli space of vacua by $\dfrac{\text{gauge invts}}{\text{classical rel'ns}}$.

$$\Rightarrow \langle M \rangle = \langle Q\tilde{Q} \rangle = q^2$$

There's a 1 complex-dimensional space of vacua.

(Classically, this Higgses $SU(N_c) \to SU(N_c - 1)$).

<u>But now</u> What's the effective superpotential?

Assign charges:

	$SU(N_c)$	$U(1)_B$	$U(1)_A$	$U(1)_R$
	Q	1	1	r
	$\tilde{Q}$	-1	1	r
	M	0	2	$2r$
Assign charges to kill anom. $\longrightarrow$	Λ^b	0	2	$2N_c + 2(r-1)$
	λ_α	0	0	1

Can assign $r = 1 - N_c$, then it's anomaly-free.

So the only thing possible is

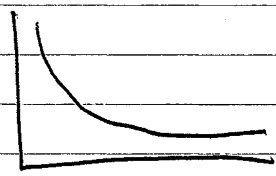
$$W_{eff} = c\left(\frac{\Lambda^{3N_c-1}}{M}\right)^{\#}$$

$$R(W) = 2 = -(\#)(2)(1-N_c) \Rightarrow \# = \frac{1}{N_c-1}$$

$$\boxed{W_{eff} = c\left(\frac{\Lambda^{3N_c-1}}{M}\right)^{1/N_c-1}}$$

Notice: $W \sim (M)^{-1/N_c-1}$

So $|\partial W|^2 \sim |M|^{-2N_c/N_c-1}$

No vaccuum state!! → 

Quantum effects completely ~~[illegible]~~ remove the ground state.

More generally, for N_f flavors, $(N_f < N_c)$.

$$W_{dyn} = c\left(\frac{\Lambda^{3N_c-N_f}}{\det M}\right)^{\frac{1}{N_c-N_f}}.$$

for $N_f \geq N_c$, this does not arise.