The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific
and Cultural Organization

International Atomic
Energy Agency

SMR. 1649 - 13

SPRING SCHOOL ON SUPERSTRING THEORY AND RELATED TOPICS

14 - 22 March 2005

Gravitational Aspects of AdS/CFT Correspondence

0. LUNIN
Institute for Advanced Study
School of Natural Sciences
Einstein Drive
Princeton, NJ 08540
U.S.A.

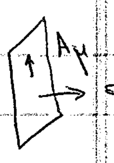
Please note: These are preliminary notes intended for internal distribution only.

I. Basics of AdS/CFT

1. Open strings and D-branes

$$S = +\frac{1}{2\pi}\int d^2\sigma \sqrt{h}\, h^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu} \rightarrow M^2 = 0: \text{ vector } \alpha^\mu_{-1}|0\rangle$$

T duality: α^m_{-1} becomes frozen $\Rightarrow$ Dp brane: $(p+1)$ vect $+ D-p-1$ scal



N D3 branes: $U(N)$: $(N^2-N)+N$ (Witten '97)

SUSY: $N=2$ in 10D $\rightarrow$ ½ BPS $\rightarrow$ $N=4$ in 4D
32 16

2. Strings in backgr. fields

Massless states: open str.: A_μ; closed str.: $h_{\mu\nu}$, $b_{\mu\nu}$, scalars + superpartn.

Vers for massless fields:
$$S = +\frac{1}{4\pi\alpha'}\int d^2\sigma\, G_{\mu\nu}\partial X^\mu \partial X^\nu + \frac{1}{4\pi\alpha'}\int d^2\sigma\, B_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu \varepsilon^{\alpha\beta} + \frac{1}{4\pi}\int d^2\sigma \sqrt{h}\, \phi(X) R^{(2)}$$

$$h_{\alpha\beta} \rightarrow e^{2\varphi}\eta_{\alpha\beta}\ ; \quad \frac{\delta S}{\delta\varphi} = 0 \Rightarrow \text{counterterms, e.g. } -\frac{1}{4\pi}\int d^2\sigma\, \varphi R_{\mu\nu}\partial X^\mu \partial X^\nu$$

Renormal. of metric: $g'_{\mu\nu} = g_{\mu\nu} - \frac{1}{2\varepsilon}R_{\mu\nu} \Rightarrow \beta$ funtion.

Eff. action:

$$\cancel{S = \int d^4x \sqrt{G}\{R - \tfrac{4}{D-2}(\nabla\phi)^2 - \tfrac{1}{12}e^{-8\phi/(D-2)}H^2\} \ldots (FT)}$$

~~Gauge fields on the brane DBI~~

$$S = \frac{1}{2\kappa_0^2}\int d^Dx \sqrt{G}\, e^{-2\Phi}\left[R + 4\nabla\Phi\nabla\Phi - \frac{1}{12}H_{\mu\nu\lambda}H^{\mu\nu\lambda} - \frac{2(D-26)}{3\alpha'} + O(\alpha')\right]$$

3. Type IIB SUGRA

$$S_{IIB} = \frac{1}{2\kappa_0^2}\int d^{10}x\sqrt{-G}\left\{e^{-2\Phi}\left[R+4(\nabla\Phi)^2-\frac{1}{12}H_{\mu\nu\lambda}H^{\mu\nu\lambda}\right]-\frac{1}{2}(dC_0)^2-\right.$$

$$\left.-\frac{1}{12}(G_3+C_0H_3)^2-\frac{1}{480}F_5^2\right\}+\frac{1}{4\kappa_0^2}\int\left(C_4+\frac{1}{2}B_2\wedge C_2\right)G_3H_3$$

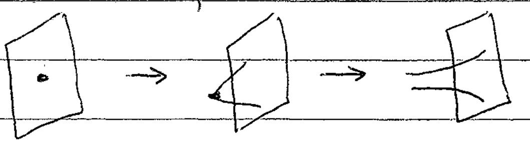
$$F_5 = F_5^* = dC_4 + H_3C_2$$

$$2\kappa^2 = 2\kappa_0^2 g_s^2 = 16\pi G_N = (2\pi)^7\alpha'^4 g_s^2\ ; \qquad G^E_{\mu\nu} = e^{(\Phi_0-\Phi)/2}G_{\mu\nu}$$

4. D3 branes in SUGRA

$$ds^2 = H^{-1/2}dx^2_{1,3} + H^{1/2}(dr^2+r^2d\Omega_5^2)\ , \qquad F_5 = dH^{-1}\wedge d^4x + \text{dual}$$

$$H = 1+\frac{4\pi gN\alpha'^2}{r^4}$$



topology change

$$\alpha'\to 0 \qquad ds^2 = \alpha'\left[\frac{U^2}{\sqrt{4\pi gN}}dx^2_{1,3}+\sqrt{4\pi gN}\,\frac{dU^2}{U^2}+\sqrt{4\pi gN}\,d\Omega_5^2\right]\ , \quad U\equiv\frac{r}{\alpha'}$$

$$\frac{R^2}{\alpha'} = \sqrt{4\pi gN}\ , \qquad \frac{1}{g_{YM}^2}+\frac{i\theta}{8\pi^2} = \frac{1}{2\pi}\left(\frac{1}{g}+\frac{i\chi}{2\pi}\right) \Rightarrow \underset{g_{YM}}{\text{strong}} - \text{weak curv.}$$

$N=4$ SYM = string theory on $AdS_5\times S^5$

$$ds^2_{AdS} = R^2\left(\frac{dU^2}{U^2}+U^2(-dt^2+d\vec{x}^2)\right) = \frac{R^2}{z^2}\left(dz^2+d\vec{x}^2-dt^2\right)$$

Boundary: $z=0 \Rightarrow R^{d-1,1}$, one point at $z=\infty$

$$X^0 = \frac{1}{2u}\left(1+u^2(R^2+\vec{x}^2-t^2)\right),\quad X^{p+2} = Rut,\quad X^i = Rux^i,\quad X^{p+1} = \frac{1}{2u}\left(1-u^2(R^2-\vec{x}^2+t^2)\right)$$

$$X_0^2+X_{p+2}^2-\sum_i^{p+1}X_i^2 = R^2,\quad ds^2 = -dX_0^2-dX_{p+2}^2+\sum dX_i^2$$ Poincare patch.

$$X_0 - X_1 = uR^2$$

5. Global AdS and SYM on S^3

Global coordinates: $X_0 = R\,\mathrm{ch}\rho\cos\tau$, $X_{p+2} = R\,\mathrm{ch}\rho\sin\tau$

$X_i = R\,\mathrm{sh}\rho\,\Omega_i$

$ds^2 = R^2\left(-\mathrm{ch}^2\rho\,d\tau^2 + d\rho^2 + \mathrm{sh}^2\rho\,d\Omega_3^2\right)$, boundary at $\rho=\infty$: $R\times S^3$.

6. Prescription for the correspondence:

1. For any field on the bound. → field in the bulk: given bound. condit. solut. in the bulk is unique

2. ϕ_0 coupled with $\mathcal{O}$ $\Rightarrow$ $\langle \exp\int_{S^d}\phi_0\mathcal{O}\rangle_{CFT} = Z_S(\phi_0) \equiv e^{-I_S(\phi)}$

$I_S(\phi)$ is a classical SUGRA action with ϕ in the bulk.

Example: $I(\varphi) = \frac{1}{2}\int_B d^{d+1}y\,\sqrt{g}\,\partial\varphi\,\partial\varphi$

$ds^2 = \frac{1}{x_0^2}\sum_0^d (dx_i)^2$; $D_\mu D^\mu\phi = 0 \Rightarrow \phi(x_0,x_i) = c\int d\vec{x}'\,\frac{x_0^d}{[x_0^2+|\vec{x}-\vec{x}'|^2]^d}\,\phi_0(\vec{x}')$

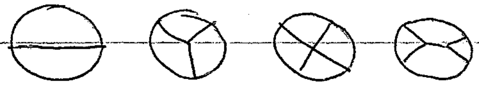
$I(\varphi) = \frac{cd}{2}\int d\vec{x}\,d\vec{x}'\,\frac{\phi_0(\vec{x})\phi_0(\vec{x}')}{|\vec{x}-\vec{x}'|^{2d}} \Rightarrow \langle\mathcal{O}\mathcal{O}\rangle \sim |\vec{x}-\vec{x}'|^{-2d}$

Generic KK modes on S^5

$(\Box + m_\Delta^2)\phi_\Delta^2 = 0$, $m_\Delta = \Delta(\Delta-4) \Rightarrow \phi_\Delta =$ z_0^Δ – normal. – vev of operat

$z_0^{4-\Delta}$ – n/norm – couplings

N/norm mode $\phi_\Delta \to \Phi_\Delta(\vec{z}) = \lim_{z_0\to 0}\phi_\Delta(z_0,\vec{z})\,z_0^{4-\Delta}$, $\Phi \equiv \phi_0$.

Higher point funct.: Witten diagr. 

9. ½ BPS states

$Z = X_1 + iX_2$; primary: $\mathrm{Tr}(Z^{n_1}) \ldots \mathrm{Tr}(Z^{n_k})|0\rangle$

- Symmetry: $S^3 \times SO(4)$
- D3 branes only : metric & F_5
- SUSY $\Rightarrow$ equ for Killing spinor $\nabla_M \eta + \frac{i}{480} \not{F} \Gamma_M \eta = 0$.
  $F_5 = * F_5$

Reduction on $S^3 \times S^3$: 4d theory

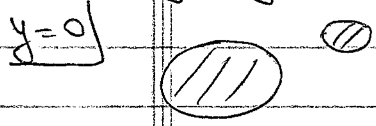
$K_\mu = -\bar{\varepsilon}\gamma_\mu \varepsilon$ – Killing ; $L_\mu = \bar{\varepsilon}\gamma^5 \gamma_\mu \varepsilon$ – exact ; K is not an R-charge of field theory

$$ds^2 = -h^{-2}(dt + V_i dx^i)^2 + h^2(dy^2 + dx^i dx^i) + y e^{G} d\Omega_3^2 + y e^{-G} d\tilde{\Omega}_3^2$$

$$Z = \frac{1}{2}\tanh G \quad ; \quad \Delta_2 Z + y\,\partial_y\left(\frac{\partial_y Z}{y}\right) = 0.$$

Global cond.: no singularities $\Rightarrow$ loci at $y=0$

$$h^2 dy^2 + y e^{-G} d\tilde{\Omega}_3^2 \approx \frac{1}{c}(dy^2 + y^2 d\tilde{\Omega}_3^2) \ , \quad Z = \frac{1}{2}$$



Ex.: $AdS_5 \times S^5$, gravit, giant, pp wave

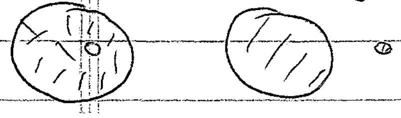
Giant gravitons

$$S = -T \int d^{p+1}\xi \, e^{-\Phi} \sqrt{\det(G_{ab} + B_{ab} + 2\pi\alpha' F_{ab})} + T \int A^{(p+1)}$$

$$G_{ab} = G_{\mu\nu} \frac{\partial X^\mu}{\partial \xi^a} \frac{\partial X^\nu}{\partial \xi^b}$$

McGreevy, Susskind, Toumbas '00

$AdS_5 \times S^5$: D3 wrapping S^3 or $\tilde{S}^3$, $r^2 = \frac{J}{N} R^2$



7. Map between modes

$\frac{1}{2}$ BPS sugra, spin ≤ 2 $\qquad$ $\mathcal{O}_2 = \mathrm{tr}\, X^{\{i} X^{j\}} + \text{desc.}$

$\frac{1}{2}$ BPS KK, spin ≤ 2 $\qquad$ $\mathcal{O}_\Delta = \mathrm{tr}\, X^{\{i_1} \ldots X^{i_\Delta\}} + \text{desc.}$

Non-chiral string modes $\qquad$ $\mathcal{O}$ with dim $\sim \lambda^{1/4}$, e.g. Konishi $\mathrm{tr}\, X^i X^i$

Multipart. $\rightarrow$ $\mathcal{O}_{\Delta_1}(x_1)\, \mathcal{O}_{\Delta_2}(x_2)$ - multi trace

$\mathrm{tr}\, X^k \rightarrow h^\alpha_\alpha,\ a_{\alpha\beta\gamma\delta}$; $\mathrm{tr}\, \lambda\bar{\lambda} X^k \rightarrow h_{\mu\alpha},\ a_{\mu\alpha\beta\gamma}$; $\mathrm{tr}\, F_+ X^k \rightarrow A_{\mu\nu}$

$\mathrm{tr}\, F_+ F_- X^k \rightarrow h'_{\mu\nu}$; $\mathrm{tr}\, \lambda\lambda\bar{\lambda}\bar{\lambda} X^k \rightarrow h_{(\alpha\beta)}$; ...

8. Coherent states & backreaction

$\langle \mathcal{O}\mathcal{O} \rangle = \underbrace{\langle 0 | \mathcal{O}^\dagger}\, \underbrace{| \mathcal{O} | 0 \rangle} \Rightarrow$ operator-state correspondence
large $\Delta \rightarrow$ non-linearity in the bulk.

$\mathcal{N}=4$ SYM

$$\mathcal{L} = -\frac{1}{2g^2} F_{\mu\nu} F^{\mu\nu} - D_\mu X^i D^\mu X^i + \frac{g^2}{2} \sum_{i,j} [X^i, X^j]^2 + \text{ferm.}$$

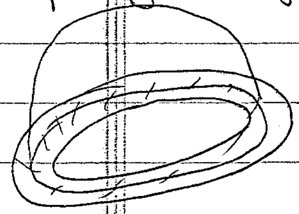
$$\delta \lambda_{b\beta} = F^+_{\mu\nu}\, \varepsilon_{\alpha b} (\sigma^{\mu\nu})^\alpha{}_\beta + [X^i, X^j]\, \varepsilon_{\beta a} (C_{ij})^a{}_b$$

$$\delta \bar{\lambda}^b_{\dot{\beta}} = \varepsilon^\alpha_a\, C^{ab}_i\, \bar{\sigma}^\mu_{\alpha\dot{\beta}}\, D_\mu X^i$$

Bosonic BPS states: $[X^i, X^j] = 0$; $D_\mu X^i = 0$

Breaking of SUSY: modific. of theory
different "vacuum" state

Topology and fluxes



- Non-contractible S^5
- Quantization of flux: $N = \int F_5 = \frac{(\text{Area})_{z=-\frac{1}{2}}}{4\pi^2 l_p^4} = \frac{(\text{Area})_{z=\frac{1}{2}}}{2\pi\hbar}$

Energy: $\Delta = J = \int \frac{d^2x}{2\pi\hbar} \frac{\frac{1}{2}(x_1^2+x_2^2)}{\hbar} - \frac{1}{2}\left(\int \frac{d^2x}{2\pi\hbar}\right)^2$

harmonic oscillator

Potential in SYM:

$$S = \int dt\, \mathrm{tr}\left(\frac{1}{2}|DZ|^2 - \frac{1}{2R^2}|Z|^2\right)$$

Map: eigenvalues $\to$ geometry.

10. BPS states in M theory

$SO(6)\times SO(3) \Rightarrow$ Toda $\Delta_2 D + \partial_y^2 e^D = 0$

$\partial_y D = 0 \quad R_2 \to 0$

$D \sim \log y, \quad R_5 \to 0$

DAC: $d\Omega_5^2 \to -ds^2_{AdS_5} \Rightarrow SO(4,2)\times SU(2)\times U(1)$

$\mathcal{N}=2$ su-conf. theory w/ 16 su-charges.

$$ds_{11}^2 = -4e^{2\lambda}(1+y^2e^{-6\lambda})(dt+V_i dx^i)^2 + e^{-4\lambda}(1+y^2e^{-6\lambda})(dy^2+e^D d\vec{x}\, d\vec{x}) + 4e^{2\lambda}d\Omega_5^2 + y^2e^{-4\lambda}d\Omega_2^2$$

Maldacena-Nunez: $e^D = \frac{1}{x_2^2}\left(\frac{1}{4}-y^2\right)$ - bound. cond. at $y=\frac{1}{2}$

Conifold and $\mathcal{N}=1$ theory

$AdS_5\times S^5 \to AdS_5\times X_5$: X_5 is Einstein $R_{\mu\nu}=\Lambda g_{\mu\nu}$

Conifold: $\sum_{a=1}^{4} z_a^2=0$, base: $\left(\sum_{a=4}^{4} z_a^2=0\right)\cup\left(\sum |z_a|^2=1\right)$

Symmetry: $SO(4)$; R-symm. $U(1) \Rightarrow T^{1,1}=\frac{SU(2)\times SU(2)}{U(1)}$

$$ds_5^2 = dr^2 + r^2\left[\frac{1}{9}\left(d\psi+\sum_1^2\cos\theta_i d\phi_i\right)^2+\frac{1}{6}\sum\left(d\theta_i^2+\sin^2\theta_i d\phi_i^2\right)\right]$$

D3 branes at the appex: $ds^2=\left(1+\frac{L^4}{r^4}\right)^{-1/2}\eta_{\mu\nu}dx^\mu dx^\nu+\left(1+\frac{L^4}{r^4}\right)^{1/2}(ds_6^2)$

Near-horizon: $AdS_5\times T^{1,1}$; # of susy 1/4 of $AdS_5\times S^5$ due to CY.

Field theory: $z_{ij}=\frac{1}{\sqrt{2}}\sum\sigma^n_{ij}z_n \Rightarrow \det z_{ij}=0 \Rightarrow z_{ij}=A_iB_j$

D3 brane: $\not\exists$ $U(1)\times U(1)$ charges $\begin{pmatrix}A_1\\A_2\end{pmatrix}\sim(1,-1)$; $\begin{pmatrix}B_1\\B_2\end{pmatrix}\sim(-1,1)$

Unique marginal su-pot: $W=\varepsilon^{ij}\varepsilon^{kl}\,\mathrm{tr}\,A_iB_kA_jB_l \Rightarrow SU(N)\times SU(N)$
$SU(2)\times SU(2)\times U(1)_R$ (KW'98)

Chiral operat: $\mathrm{Tr}\,A_{k_1}B_{l_1}A_{k_2}\dots B_{l_n}$; from su-pot: $B_1A_kB_2=B_2A_kB_1$
coeff. - compl. symmetric. $A_1B_kA_2=A_2B_kA_1$

Relation to $AdS_5\times S^5/\mathbb{Z}_2$: $\sum_1^6 x_i^2=1 \to \mathbb{Z}_2$: $(x_1\to -x_1;\dots,x_4\to -x_4, x_5\to x_5, x_6\to x_6$

$U(N)\times U(N)$; $W=g\,\mathrm{Tr}\,\Phi(A_1B_1+A_2B_2)+g\,\mathrm{Tr}(\tilde\Phi(B_1A_1+B_2A_2))$

Relev. pert.: $\delta W=\frac{m}{2}(\mathrm{Tr}\,\Phi^2-\mathrm{Tr}\,\tilde\Phi^2) \Rightarrow \frac{g^2}{2m}\left[\mathrm{Tr}(A_1B_1A_2B_2)-\mathrm{Tr}(B_1A_1B_2A_2)\right]$

Fractional branes on conifold

$$ds^2 = \frac{1}{9}\left(d\psi + \sum \cos\theta_i\, d\varphi_i\right)^2 + \frac{1}{6}\underbrace{\sum\left(d\theta_i^2 + \sin^2\theta_i\, d\varphi_i^2\right)}_{S^2\times S^2}$$

N D3 branes w/volume $x_0 .. x_3 \Rightarrow SU(N)\times SU(N)$

M D5 branes w/volume S^2, $x_0 .. x_3$ – "frac. D3" $\Rightarrow SU(M+N)\times SU(N)$

$$\frac{1}{4\pi^2\alpha'}\int_{S^3} F_3 = M\,;\quad \frac{1}{(4\pi^2\alpha')^2}\int_{T^{1,1}} F_5 = N$$

Linearized in $\frac{M}{N}$ solut. (Kleb-Nekrasov), exact (Klebanov-Tseytlin):

$$ds^2 = \frac{r^2}{L^2\sqrt{\ln(r/r_s)}}\, dx_\mu dx^\mu + \frac{L^2\sqrt{\ln(r/r_s)}}{r^2}\, dr^2 + L^2\sqrt{\ln(r/r_s)}\, ds^2_{T^{1,1}} \qquad (*)$$

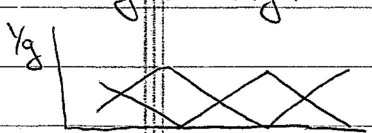
$$F_3 = \dots,\quad H_3 = \dots,\quad F_5 = K(r)\,\mathrm{vol}\, T^{1,1} + *\left(K(r)\,\mathrm{vol}\, T^{1,1}\right)$$

$$K(r) = N + a g_s M^2 \ln\frac{r}{r_0}\ ;\quad B_2 = 3 g_s M \omega_2 \ln\frac{r}{r_0}$$

$\int_{S^2} B_2$ changes by a period $\Rightarrow K \to K - M \Rightarrow$ cascade:

$$SU(N+M)\times SU(N) \to SU(N)\times SU(N-M)$$

Seiberg duality: $SU(N+M)$ w/ $2N$ flavors $\Rightarrow SU(2N-(M+N)) = SU(N-M)$



Solution $(*)$: $r \gg r_s$

~~Now~~ Small r: $U(1)_R \to \mathbb{Z}_2$; deformed conif: $\sum z_i^2 = \varepsilon^2$

Exact solut: Kleb-Strassler '00.

Marginal deformations of N=4 SYM

Marginal deform.: Leigh-Strassler (95): $W = A\,\mathrm{tr}\left[XYZ - q\,XZY + h(X^3+Y^3+Z^3)\right]$

$h=0, q=1 \Rightarrow \mathcal{N}=4$ SYM, generically: $\mathcal{N}=1$ SU-conf theory.

q-pure phase: $\mathrm{tr}\left[e^{i\pi\beta}XYZ - e^{-i\pi\beta}XZY\right]$

geometry: $AdS_5\times X$ - linear. solut. Aharony-Kol-Yankielowicz '02.

	X	Y	Z
Symmetry: $U(1)\times U(1)$:	0	1	-1
	-1	1	0

noncomm. product: $f * g = \exp\left[i\pi\beta(Q_f^1 Q_g^2 - Q_f^2 Q_g^1)\right] fg$

gravity side: $\tau = B + i\sqrt{g} \to \tau_\beta = \dfrac{\tau}{1+\beta\tau}$ on T^2.

spinor: $\varepsilon = e^{(i\phi_1+i\phi_2+i\phi_3)/2}$; $ds_5^2 = d\alpha^2 + s_\alpha^2 d\theta^2 + c_\alpha^2 d\phi_1^2 + s_\alpha^2(c_\theta^2 d\phi_2^2 + s_\theta^2 d\phi_3^2)$

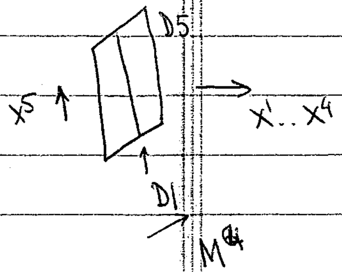
$\phi_1 = \psi - \varphi_2$; $\phi_2 = \psi + \varphi_1 + \varphi_2$; $\phi_3 = \psi - \varphi_1$

T → rot → T: $ds^2 = R^2\left[ds^2_{AdS} + \sum(d\mu_i^2 + G\mu_i^2 d\phi_i^2) + \gamma^2 G\mu_1^2\mu_2^2\mu_3^2\left(\sum d\phi_i\right)^2\right]$

$G^{-1} = 1 + \gamma^2(\mu_1^2\mu_2^2 + \mu_2^2\mu_3^2 + \mu_1^2\mu_3^2)$

Other fields: $e^{2\phi}$, B_2, C_2, C_4. More general: $SL(3,R)$

II. AdS_3/CFT_2 and black holes.



$$ds^2 = \frac{1}{\sqrt{f_1 f_5}} dx_{1,1}^2 + \sqrt{f_1 f_5}\,(dr^2 + r^2 d\Omega_3^2) + \sqrt{\frac{f_1}{f_5}}\, ds_4^2$$

T^4 or $K3$

$$f_1 = 1 + \frac{g\alpha' Q_1}{V r^2}\,; \quad f_5 = 1 + \frac{g\alpha' Q_5}{r^2}$$

$$ds^2 = \alpha'\left[\frac{u^2}{g_6\sqrt{Q_1 Q_5}} dx_{1,1}^2 + g_6\sqrt{Q_1 Q_5}\frac{du^2}{u^2} + g_6\sqrt{Q_1 Q_5}\, d\Omega_3^2\right] + .. ds_4^2$$

Field theory: σ model on moduli space of instantons: $\mathcal{M} = (T^4)^{N_1 N_5}/S_{N_1 N_5}$

(N_1 instant. w/ N_5 orient).

Central charge: $c = n_{bose} + \frac{1}{2} n_{fermi} = 6 N_1 N_5$

Operators: $X_i^{(m)}, \psi_\alpha^{(m)}, \sigma_n^{+-}$: $h = j = \frac{n+1}{2}$; $\bar{h} = \bar{j} = \frac{n-1}{2}$

S^3: $SO(4) \sim SU(2)_L \times SU(2)_R$

# of SUSY: 8 for D1-D5 $\to$ 16 in the near-horizon.

Adding 3-rd charge : $ds^2 = \frac{1}{\sqrt{f_1 f_5}}\left[-dt^2 + dy^2 + K(dt+dy)^2\right] + \dots$

$R_y = \frac{K}{\sqrt{f_1 f_5}} \to$ const ; $K = \frac{Q_p}{r^2} \Rightarrow$ reduction to 4D:

$$ds_E^2 = -(f_1 f_5 (1+K))^{-2/3} dt^2 + (f_1 f_5 (1+K))^{1/3} (dr^2 + r^2 d\Omega_3^2)$$

$$S_{BH} = \frac{A}{4G_5} = \frac{1}{4G_5}\pi^2 r^3 \left[f_1 f_5 (1+K)\right]^{3/5}\Big|_{r=0} = 2\pi\sqrt{N_1 N_5 N_m}$$

Strom-Vafa count:

$$E = \frac{N_m}{R}, \quad Z = \left[\prod_1^\infty \frac{1+w^{N_m}}{1-w^{N_m}}\right]^{4N_1N_5} = \sum \Omega(N_m) w^{N_m}$$

Cardy formula: $\Omega(N_m) \sim \exp\left[\frac{\pi c E(2\pi R)}{3}\right] = \exp\left(2\pi\sqrt{\frac{c}{6}N_m}\right)$

$S_{micro} = \log \Omega(N_m) = 2\pi\sqrt{N_1 N_5 N_m}$

<u>Question</u>: How to see microstates on AdS side?

<u>Two-charge system</u>: $h=j,\ \bar{h}=\bar{j}$.

$e^S \sim e^{2\pi\sqrt{N_1N_5}}$, zero area of the horizon.

$$\begin{pmatrix} D1 \\ D5\end{pmatrix} \xrightarrow{S} \begin{pmatrix} F1 \\ NS5\end{pmatrix} \xrightarrow{T56} \begin{pmatrix} P(5) \\ NS5\end{pmatrix} \xrightarrow{S} \begin{pmatrix} P \\ D5\end{pmatrix} \xrightarrow{T^4} \begin{pmatrix} P \\ D1\end{pmatrix} \xrightarrow{S} \begin{pmatrix} P \\ F1\end{pmatrix} \qquad \int \vec{F}(v)$$

<u>Metric for D1-D5</u>: $ds^2 = \frac{1}{\sqrt{f_1 f_5}}\left[-(dt+A)^2 + (dy+B)^2\right] + \sqrt{f_1 f_5}\,d\vec{x}_4 + \sqrt{\frac{f_1}{f_5}}\,d\vec{z}d\vec{z}$

$$f_1 = 1 + \frac{Q}{l}\int\frac{dv}{(\vec{x}-\vec{F})^2}\ ;\ f_5 = 1 + \frac{Q}{l}\int\frac{|\dot{F}|^2 dv}{(\vec{x}-\vec{F})^2}\ ;\ A_i = -\frac{Q}{l}\int\frac{\dot{F}_i\,dv}{(\vec{x}-\vec{F})^2}$$

$$dB = -*dA. \qquad \text{(OL, Mathur, 01)}$$

<u>Metric is regular</u>:

$$ds^2 = \frac{R}{2x_1}\left[dx_1^2 + x_1^2(d\theta^2 + \sin^2\theta\, d\varphi^2)\right] + 2Rx_1\left(\frac{dy}{R} + \frac{1}{2}(1-\cos\theta)d\varphi\right)^2$$

$$+ C\left[(2dx_2 - dt)^2 - dt^2\right] \ -\ \text{KK monopole.}$$

<u>Example</u>: $\left[\sigma_1^{--}\right]^N \Rightarrow F_1 = a\cos\omega v\ ;\ F_2 = a\sin\omega v,\ F_3, F_4 = 0.$

$$\frac{ds^2}{\sqrt{Q_1Q_5}} = -\frac{r^2+a^2}{Q_1Q_5}dt^2 + \frac{dr^2}{r^2+a^2} + \frac{r^2}{Q_1Q_5}dy^2 + d\theta^2 + \cos^2\theta\left(d\psi - \frac{a}{\sqrt{Q_1Q_5}}dy\right)^2$$

$$+ \sin^2\theta\left(d\phi - \frac{a}{\sqrt{Q_1Q_5}}dt\right)^2 \ -\ AdS_3\times S^3 \text{ in } \underline{\text{global}} \text{ coord.}$$

Asympt. - flat space $\qquad a = \frac{2j}{n_1 n_5}\frac{\sqrt{Q_1Q_5}}{R} = \gamma\frac{\sqrt{Q_1Q_5}}{R}$

Asympt. flat space $\Rightarrow$ ferm. are periodic in y (R)

CFT: from R to NS sector - spectral flow

R vacuum $\rightarrow$ chiral primary

$$h^R = h^{NS} - j_3^{NS} + \frac{c}{24}\ ; \quad j_3^R = j_3^{NS} - \frac{c}{12}$$

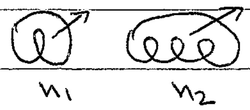
Spectral flow in SUGRA:

1. Go to the near-hor. limit: $H = 1 + \frac{Q}{\ell}\int .. \rightarrow \frac{Q}{\ell}\int ..$
2. Flat connection at ∞
3.

Example: NS vacuum : $\psi' = \psi - \frac{y}{R}$; $\phi' = \phi - \frac{t}{R}$.

$$L_0 = \frac{\ell M + J}{2} + \frac{c}{24} + \frac{6(j)^2}{c}$$ - gravity param. are invar. under SF.

Map in NS sector:

$\sigma^{+-}_{n_1}\ \sigma^{++}_{n_2} \dots \xrightarrow{SF}$  $\rightarrow$ Fourier transform of $\vec{F}(v)$

NS

R

$\downarrow$

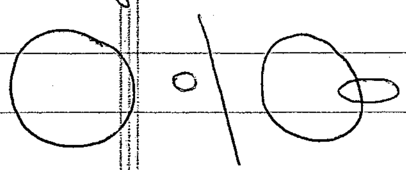
R sector solution

$\downarrow$ Gravit. SF

NS sector solution.

Tests: travel time, emission rate

Giant gravitons: D1(D5) brane anywhere in $AdS_3 \times S^3$



Exact metrics.

Addition of momentum

Minimal SUGRA in 6D: SUSY solutions have the form

$$ds^2 = -2H^{-1}(du+\beta_m dx^m)\left(dv + \omega_m dx^m + \frac{F}{2}(du+\beta_m dx^m)\right) + H h_{mn} dx^m dx^n$$

h_{mn} - hyper-Kähler; $d\beta = {}^*d\beta$

$$d^*dH + d\beta \wedge G^+ = 0 \qquad G^+ \equiv H^{-1}\left(d\omega^+ + \tfrac{1}{2}F d\beta\right)$$

$$dG^+ = 0$$

$$^*d^*dF = (G^+)_{mn}(G^+)^{mn}$$

We assumed u-independence.

"Near horizon map" (not limit!): $H = \hat{H}+1$; $(d\omega)^+ = \frac{\hat{H}+1}{\hat{H}}(d\hat{\omega})^+ + \frac{1}{2}\frac{F}{\hat{H}}d\beta$

If $F=0$, $(d\omega)^+ = 0 \Rightarrow$ usual $H = \hat{H}+1$

Procedure: $\underset{F=0,\,(d\omega)^+=0}{\text{asympt. flat}} \longrightarrow \underset{\text{no moment.}}{AdS_3\times S^3 \text{ asympt.}} \longrightarrow \underset{\text{by diffeom.}}{\text{add mom.}} \rightarrow$ asympt. flat

New solution with 3 charges, trivial in AdS region.

Open problem: $\frac{1}{4}$ BPS states: $h_L^{NS} = j_{3L}$, $h_R^{NS} > j_{3R}$

$h_L^{R} = \frac{c}{24}$, $h_R^{R} > \frac{c}{24}$