The interacting boson model

P. Van Isacker, GANIL, France

Introduction to the IBM

Practical applications of the IBM

NSDD Workshop, Trieste, April 2005

Overview of nuclear models

- *Ab initio* methods: Description of nuclei starting from the bare nn & nnn interactions.
- Mean-field methods: Nuclear average potential with global parametrization (+ correlations).
- Nuclear shell model: Nuclear average potential + (residual) interaction between nucleons.
- Phenomenological models: Specific nuclei or properties with local parametrization, *e.g*. the interacting boson model.

A*b initio* methods

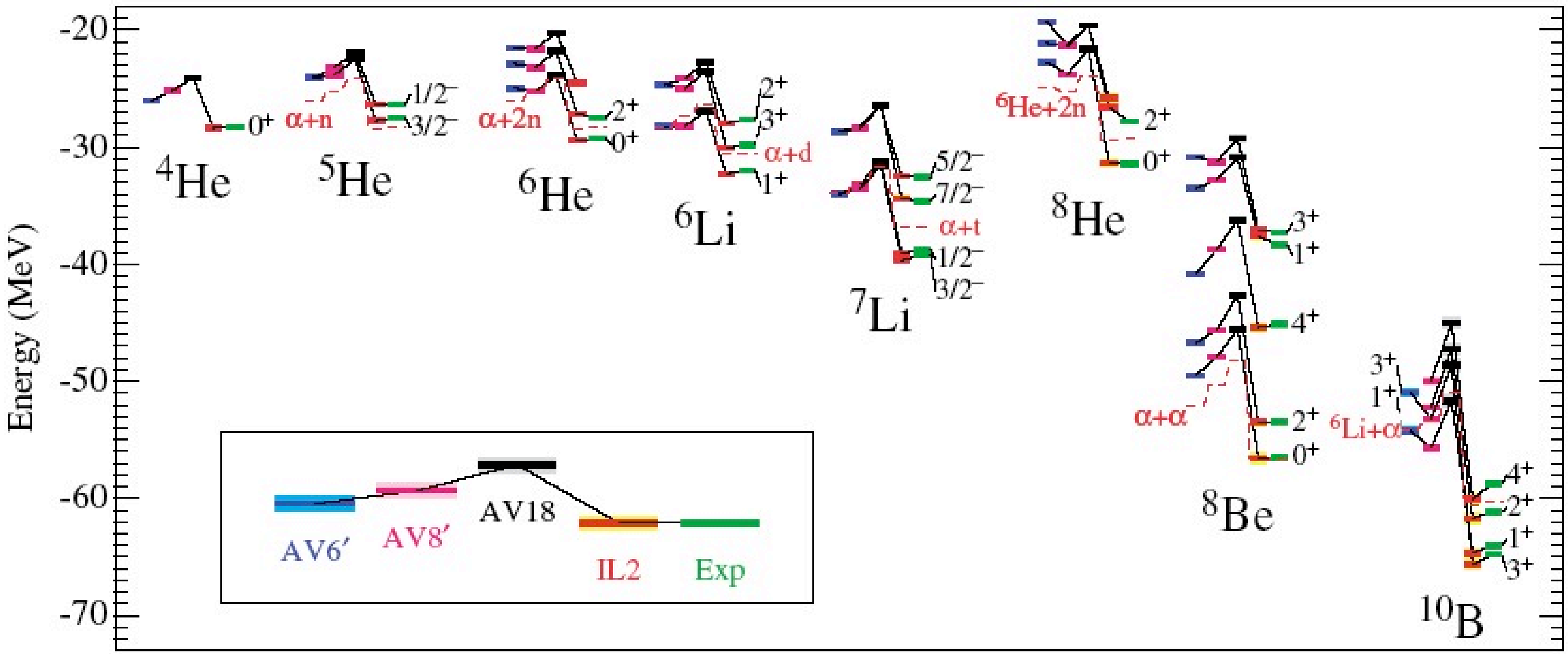
- Many *ab initio* methods exist and give consistent results.
- Example : *A=4*

Method	$\langle T \rangle$	$\langle V \rangle$	E_b	$\sqrt{\langle r^2 \rangle}$
FY	102.39(5)	−128.33(10)	−25.94(5)	1.485(3)
CRCGV	102.30	−128.20	−25.90	1.482
SVM	102.35	−128.27	−25.92	1.486
HH	102.44	−128.34	−25.90(1)	1.483
GFMC	102.3(1.0)	−128.25(1.0)	−25.93(2)	1.490(5)
NCSM	103.35	−129.45	−25.80(20)	1.485
EIHH	100.8(9)	−126.7(9)	−25.944(10)	1.486

H. Kamada *et al.*, Phys. Rev. C **64** (2001) 044001

A*b initio* calculations for light nuclei

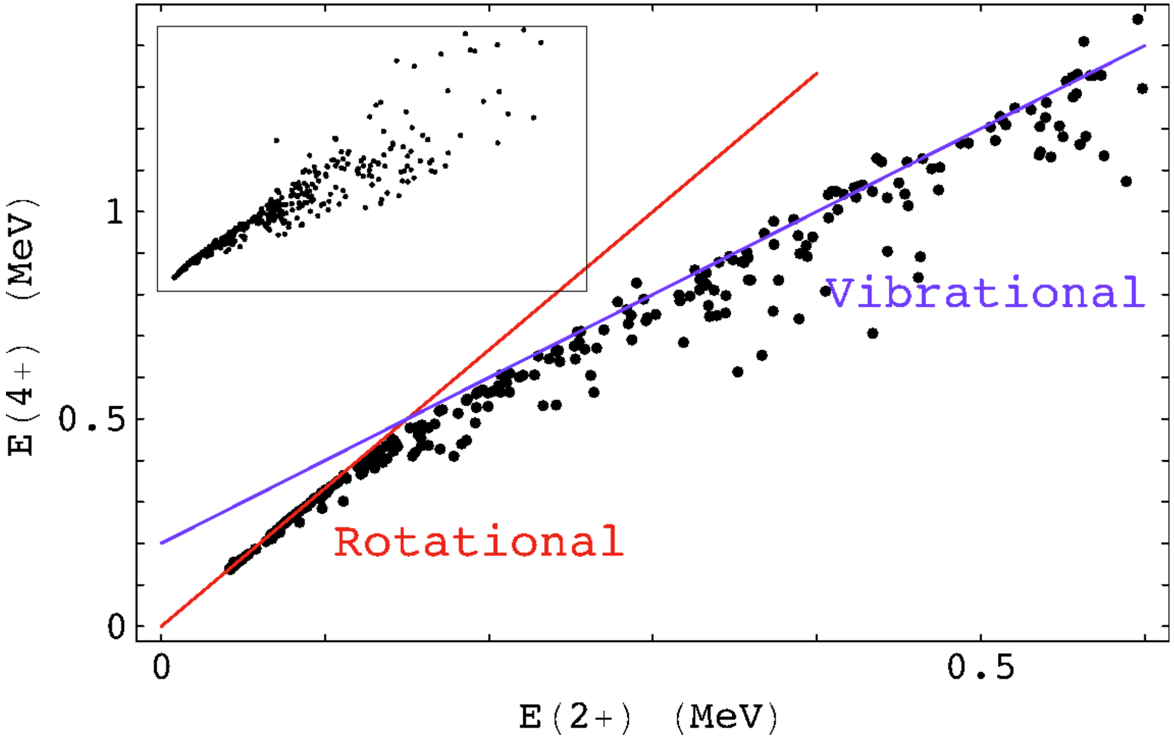
- Systematic studies of light nuclei ($A \leq 12$) ⇒ evidence for three-body nucleon interactions.



R.B. Wiringa and S.C. Pieper, Phys. Rev. Lett. **89** (2002) 182501

Tri-partite classification of nuclei

- Empirical evidence for seniority-type, vibrational- and rotational-like nuclei:



- Need for model of *vibrational* nuclei.

N.V. Zamfir *et al.*, Phys. Rev. Lett. **72** (1994) 3480

The interacting boson model

- Spectrum generating algebra for the nucleus is U(6). All physical observables (hamiltonian, transition operators,…) are expressed in terms of *s* and *d* bosons.
- Justification from
  - Shell model: *s* and *d* bosons are associated with *S* and *D* fermion (*Cooper*) pairs.
  - Geometric model: for large boson number the IBM reduces to a liquid-drop hamiltonian.

A. Arima & F. Iachello, Ann. Phys. (NY) **99** (1976) 253; **111** (1978) 201; **123** (1979) 468

The IBM hamiltonian

- Rotational invariant hamiltonian with up to N-body interactions (usually up to 2):

$$\hat{H}_{\mathrm{IBM}} = \varepsilon_s \hat{n}_s + \varepsilon_d \hat{n}_d + \sum_{l_1 l_2 l'_1 l'_2, L} \upsilon^L_{l_1 l_2 l'_1 l'_2} \left(b^+_{l_1} \times b^+_{l_2}\right)^{(L)} \cdot \left(\tilde{b}_{l'_1} \times \tilde{b}_{l'_2}\right)^{(L)} + \cdots$$

- For what choice of single-boson energies ε and boson-boson interactions υ is the IBM hamiltonian solvable?
- This problem is equivalent to the enumeration of all algebras G satisfying

$$\mathrm{U}(6) \supset G \supset \mathrm{SO}(3) \equiv \left\{ \hat{L}_\mu = \sqrt{10}\left(d^+ \times \tilde{d}\right)^{(1)}_\mu \right\}$$

Dynamical symmetries of the IBM

- U(6) has the following subalgebras:

$$\mathrm{U}(5)=\left\{\left(d^+\times\tilde{d}\right)^{(0)}_\mu,\left(d^+\times\tilde{d}\right)^{(1)}_\mu,\left(d^+\times\tilde{d}\right)^{(2)}_\mu,\left(d^+\times\tilde{d}\right)^{(3)}_\mu,\left(d^+\times\tilde{d}\right)^{(4)}_\mu\right\}$$

$$\mathrm{SU}(3)=\left\{\left(d^+\times\tilde{d}\right)^{(1)}_\mu,\left(s^+\times\tilde{d}+d^+\times\tilde{s}\right)^{(2)}_\mu-\sqrt{\tfrac{7}{4}}\left(d^+\times\tilde{d}\right)^{(2)}_\mu\right\}$$

$$\mathrm{SO}(6)=\left\{\left(d^+\times\tilde{d}\right)^{(1)}_\mu,\left(s^+\times\tilde{d}+d^+\times\tilde{s}\right)^{(2)}_\mu,\left(d^+\times\tilde{d}\right)^{(3)}_\mu\right\}$$

$$\mathrm{SO}(5)=\left\{\left(d^+\times\tilde{d}\right)^{(1)}_\mu,\left(d^+\times\tilde{d}\right)^{(3)}_\mu\right\}$$

- Three solvable limits are found:

$$\mathrm{U}(6)\supset\left\{\begin{array}{c}\mathrm{U}(5)\supset\mathrm{SO}(5)\\ \mathrm{SU}(3)\\ \mathrm{SO}(6)\supset\mathrm{SO}(5)\end{array}\right\}\supset\mathrm{SO}(3)$$

Dynamical symmetries of the IBM

- The general IBM hamiltonian is

$$\hat{H}_{\mathrm{IBM}} = \varepsilon_s \hat{n}_s + \varepsilon_d \hat{n}_d + \sum_{l_1 l_2 l_1' l_2', L} \upsilon^L_{l_1 l_2 l_1' l_2'} \left(b^+_{l_1} \times b^+_{l_2} \right)^{(L)} \cdot \left(\tilde{b}_{l_1'} \times \tilde{b}_{l_2'} \right)^{(L)} + \cdots$$

- An *entirely equivalent* form of H_{IBM} is

$$\begin{aligned} \hat{H}_{\mathrm{IBM}} = {} & \eta_0 \hat{C}_1[\mathrm{U}(6)] + \eta_1 \hat{C}_1[\mathrm{U}(5)] + \kappa_0' \hat{C}_1[\mathrm{U}(6)] \hat{C}_1[\mathrm{U}(5)] \\ & + \kappa_0 \hat{C}_2[\mathrm{U}(6)] + \kappa_1 \hat{C}_2[\mathrm{U}(5)] + \kappa_2 \hat{C}_2[\mathrm{SU}(3)] \\ & + \kappa_3 \hat{C}_2[\mathrm{SO}(6)] + \kappa_4 \hat{C}_2[\mathrm{SO}(5)] + \kappa_5 \hat{C}_2[\mathrm{SO}(3)] \end{aligned}$$

- The coefficients η and κ are certain combinations of the coefficients ε and υ.

The solvable IBM hamiltonians

- *Excitation* spectrum of H_{IBM} is determined by

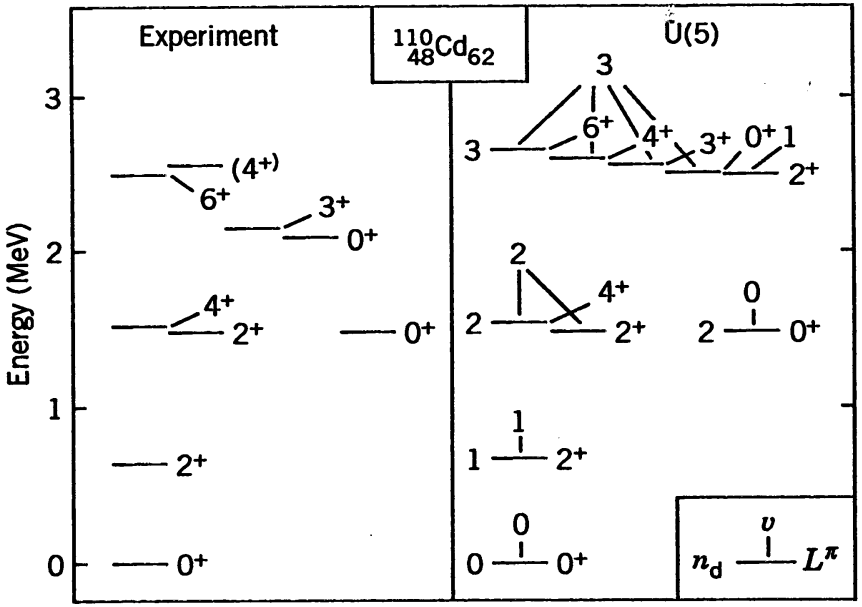
$$\hat{H}_{\mathrm{IBM}} = E_0 + \eta_1 \hat{C}_1[\mathrm{U}(5)] + \kappa_1 \hat{C}_2[\mathrm{U}(5)] + \kappa_2 \hat{C}_2[\mathrm{SU}(3)]$$
$$+ \kappa_3 \hat{C}_2[\mathrm{SO}(6)] + \kappa_4 \hat{C}_2[\mathrm{SO}(5)] + \kappa_5 \hat{C}_2[\mathrm{SO}(3)]$$

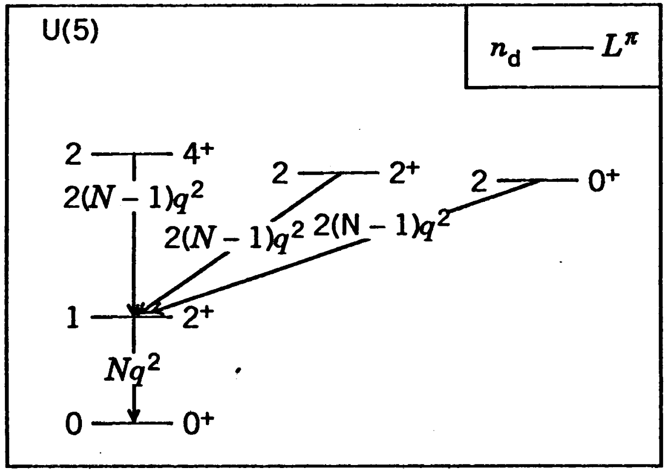
- If certain coefficients are zero, H_{IBM} can be written as a sum of commuting operators:

$$\hat{H}_{\mathrm{U}(5)} = \eta_1 \hat{C}_1[\mathrm{U}(5)] + \kappa_1 \hat{C}_2[\mathrm{U}(5)] + \kappa_4 \hat{C}_2[\mathrm{SO}(5)] + \kappa_5 \hat{C}_2[\mathrm{SO}(3)]$$
$$\hat{H}_{\mathrm{SU}(3)} = \kappa_2 \hat{C}_2[\mathrm{SU}(3)] + \kappa_5 \hat{C}_2[\mathrm{SO}(3)]$$
$$\hat{H}_{\mathrm{SO}(6)} = \kappa_3 \hat{C}_2[\mathrm{SO}(6)] + \kappa_4 \hat{C}_2[\mathrm{SO}(5)] + \kappa_5 \hat{C}_2[\mathrm{SO}(3)]$$

The U(5) vibrational limit

- Anharmonic vibration spectrum associated with the quadrupole oscillations of a spherical surface.
- Conserved quantum numbers: n_d, υ, L.

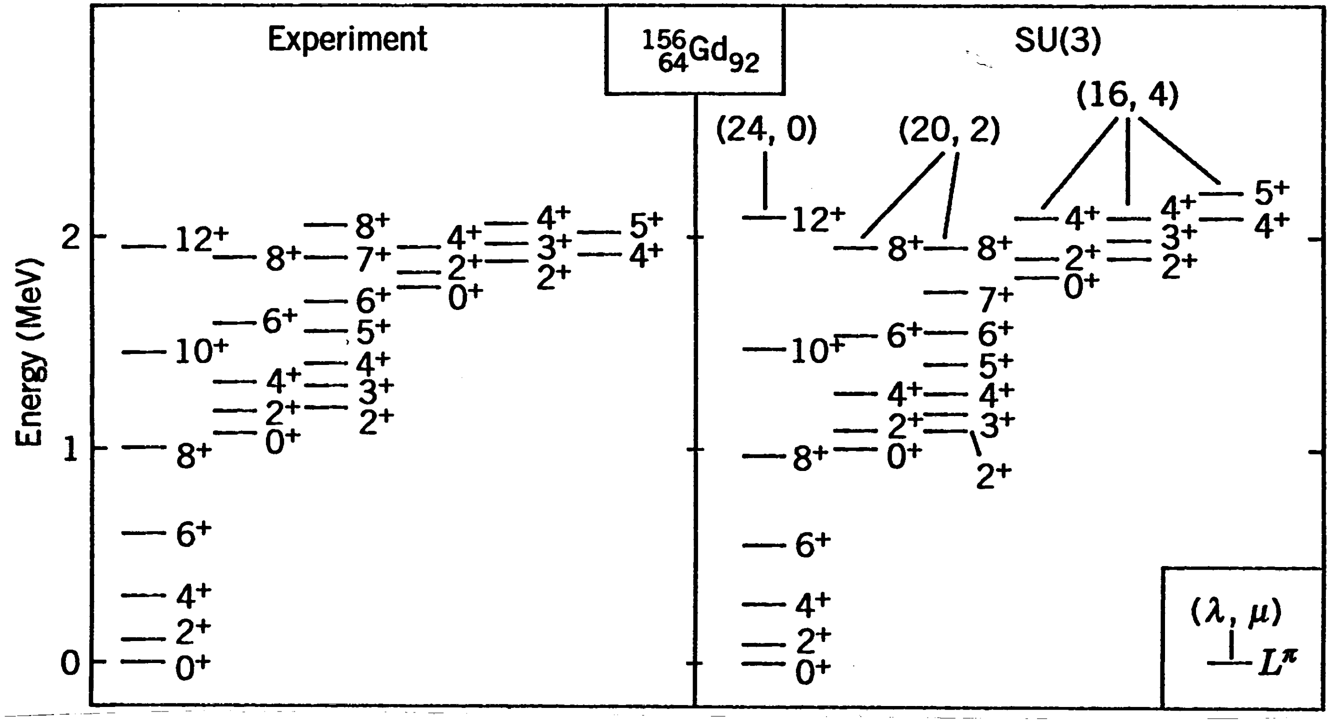




A. Arima & F. Iachello, Ann. Phys. (NY) **99** (1976) 253
D. Brink *et al.*, Phys. Lett. **19** (1965) 413

The SU(3) rotational limit

- Rotation-vibration spectrum of quadrupole oscillations of a spheroidal surface.
- Conserved quantum numbers: (λ,μ), L.

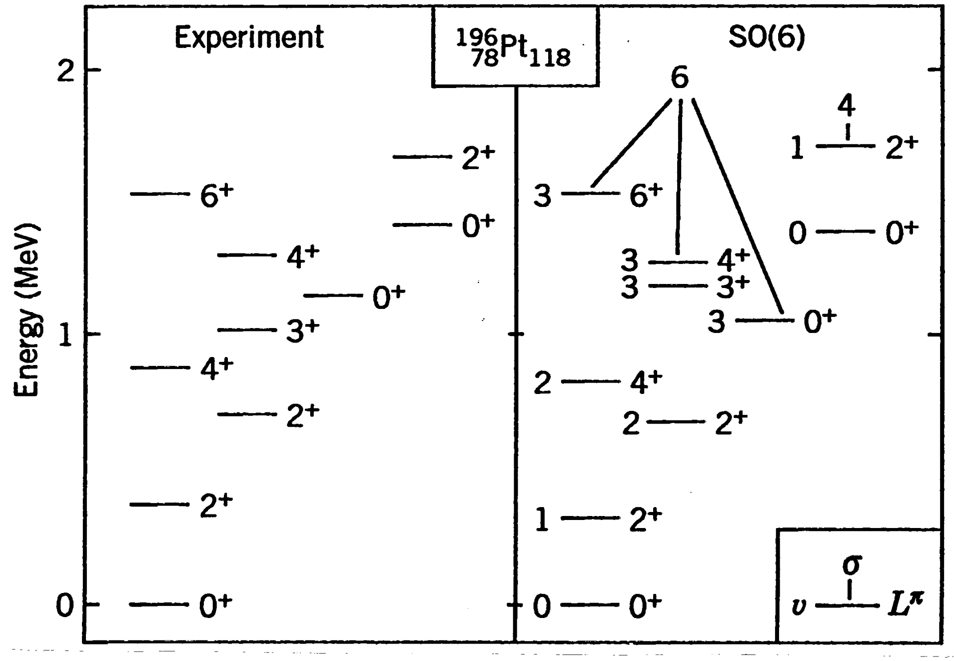


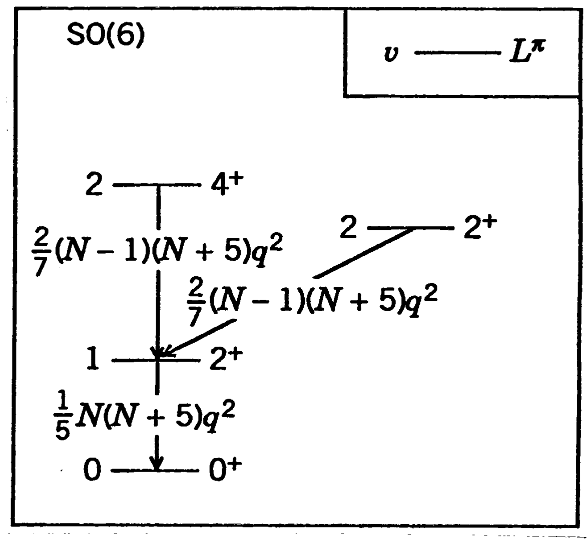
A. Arima & F. Iachello, Ann. Phys. (NY) **111** (1978) 201

A. Bohr & B.R. Mottelson, Dan. Vid. Selsk. Mat.-Fys. Medd. **27** (1953) No 16

The SO(6) γ-unstable limit

- Rotation-vibration spectrum of quadrupole oscillations of a γ-unstable spheroidal surface.
- Conserved quantum numbers: σ, υ, L.





A. Arima & F. Iachello, Ann. Phys. (NY) **123** (1979) 468
L. Wilets & M. Jean, Phys. Rev. **102** (1956) 788

Modes of nuclear vibration

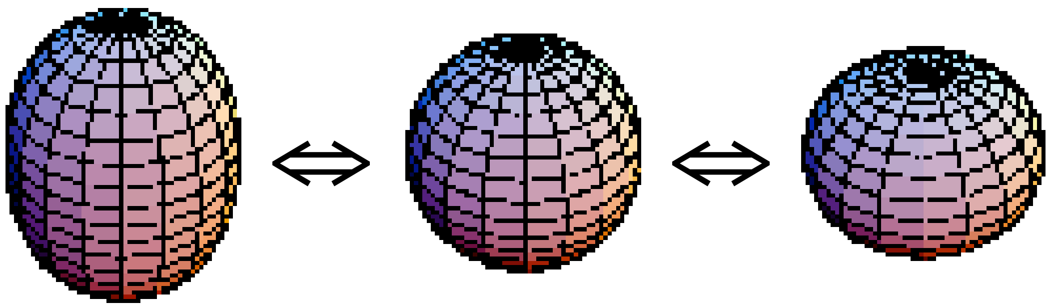
- Nucleus is considered as a droplet of nuclear matter with an equilibrium shape. Vibrations are modes of excitation around that shape.
- Character of vibrations depends on symmetry of equilibrium shape. Two important cases in nuclei:
  - Spherical equilibrium shape
  - Spheroidal equilibrium shape

Vibrations about a spherical shape

- Vibrations are characterized by a multipole quantum number λ in surface parametrization:

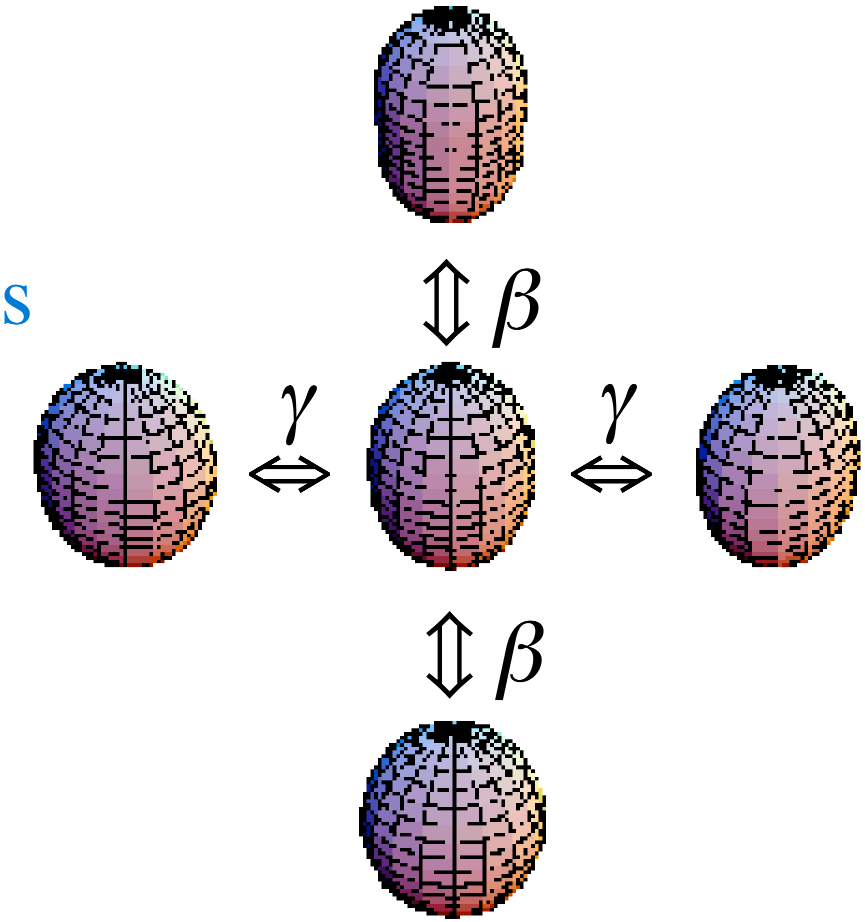
$$R(\theta,\varphi) = R_0\left(1 + \sum_{\lambda}\sum_{\mu=-\lambda}^{+\lambda}\alpha_{\lambda\mu}Y^{*}_{\lambda\mu}(\theta,\varphi)\right)$$

  - $\lambda=0$: compression (high energy)
  - $\lambda=1$: translation (not an intrinsic excitation)
  - $\lambda=2$: quadrupole vibration



Vibrations about a spheroidal shape

- The vibration of a shape with axial symmetry is characterized by $a_{\lambda\nu}$.
- Quadrupolar oscillations
  - $\nu=0$: along the axis of symmetry (β)
  - $\nu=\pm 1$: spurious rotation
  - $\nu=\pm 2$: perpendicular to axis of symmetry (γ)



Synopsis of IBM symmetries

- Three standard solutions: U(5), SU(3), SO(6).
- Analytic solution for U(5) → SO(6) via SU(1,1) Richardson-Gaudin integrability.
- Hidden symmetries because of parameter transformations: $SU_{\pm}(3)$ and $SO_{\pm}(6)$.
- Partial dynamical symmetries.
- Critical-point symmetries?

Classical limit of IBM

- For large boson number N, a *coherent* (or *intrinsic*) state is an approximate eigenstate,

$$\hat{H}_{\mathrm{IBM}}\left|N;\alpha_\mu\right\rangle \approx E\left|N;\alpha_\mu\right\rangle, \qquad \left|N;\alpha_\mu\right\rangle \propto \left(s^+ + \sum_\mu \alpha_\mu d_\mu^+\right)^N \left|\mathrm{o}\right\rangle$$

- The real parameters α_μ are related to the three Euler angles and shape variables β and γ.
- Any IBM hamiltonian yields energy surface:

$$\left\langle N;\alpha_\mu\right|\hat{H}_{\mathrm{IBM}}\left|N;\alpha_\mu\right\rangle = \left\langle N;\beta\gamma\right|\hat{H}_{\mathrm{IBM}}\left|N;\beta\gamma\right\rangle \equiv V(\beta,\gamma)$$

J.N. Ginocchio & M.W. Kirson, Phys. Rev. Lett. **44** (1980) 1744.
A.E.L. Dieperink *et al.*, Phys. Rev. Lett. **44** (1980) 1747.
A. Bohr & B.R. Mottelson, Phys. Scripta **22** (1980) 468.

Geometry of IBM

- A simplified, much used IBM hamiltonian:

$$\hat{H}_{\mathrm{CQF}} = \varepsilon_d \hat{n}_d - \kappa \hat{Q}^\chi \cdot \hat{Q}^\chi, \qquad \hat{Q}^\chi_\mu = s^+ \tilde{d}_\mu + d^+_\mu s + \chi\left(d^+ \times \tilde{d}\right)^{(2)}_\mu$$

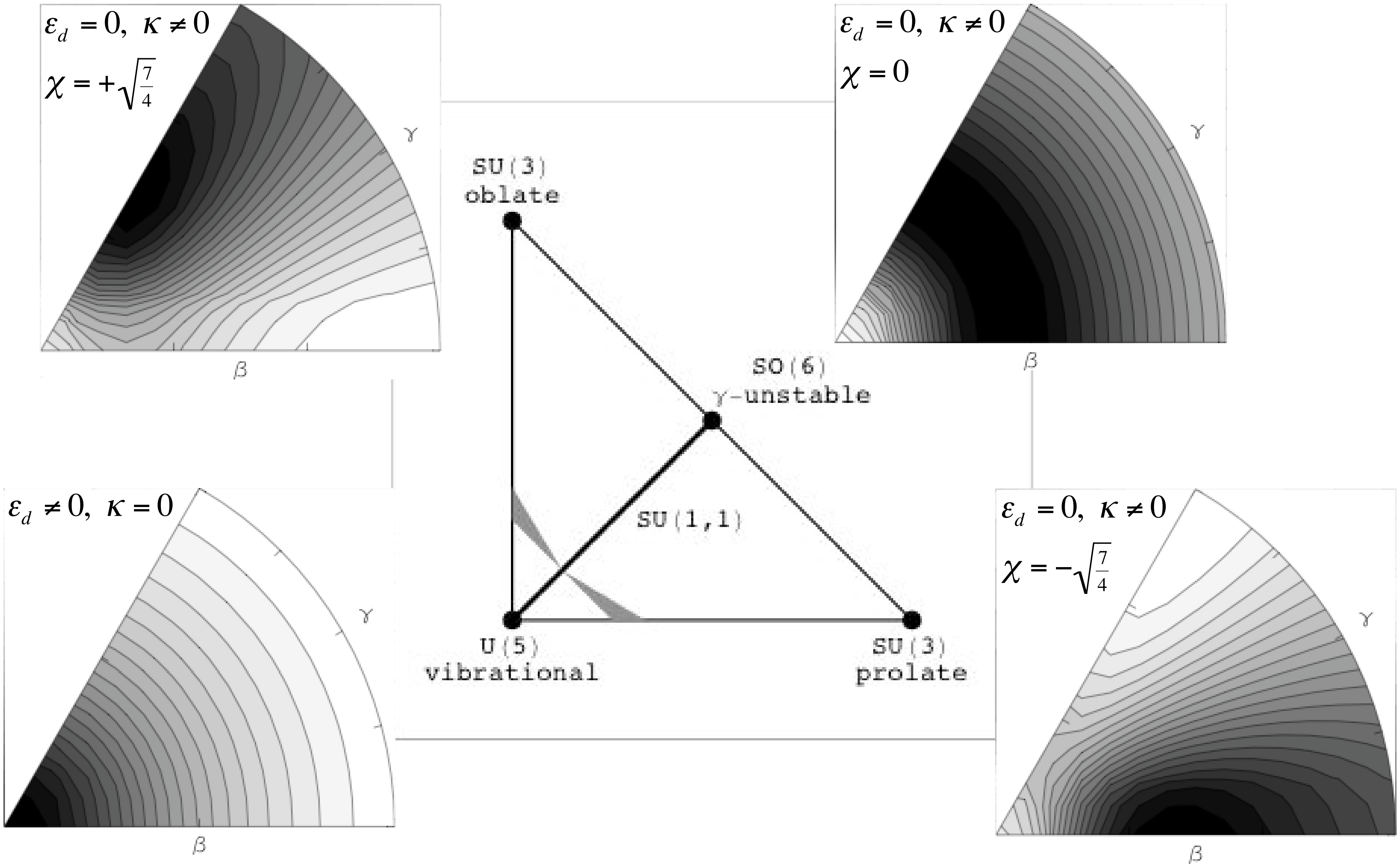
- H_{CQF} can acquire the three IBM symmetries.
- H_{CQF} has the following classical limit:

$$V_{\mathrm{CQF}}(\beta,\gamma) \equiv \langle N;\beta\gamma|\hat{H}_{\mathrm{CQF}}|N;\beta\gamma\rangle$$

$$= \varepsilon_d N \frac{\beta^2}{1+\beta^2} - \kappa N \frac{5+\left(1+\chi^2\right)\beta^2}{1+\beta^2}$$

$$-\kappa \frac{N(N-1)}{1+\beta^2}\left(\frac{2}{7}\chi^2\beta^4 - 4\sqrt{\frac{2}{7}}\chi\beta^3\cos 3\gamma + 4\beta^2\right)$$

Phase diagram of IBM



Microscopy of IBM

- In a boson mapping, fermion pairs are represented as bosons:

$$s^{+} \Leftrightarrow S^{+} \equiv \sum_{j} \alpha_{j} \left(a_{j}^{+} \times a_{j}^{+} \right)_{0}^{(0)}, \quad d_{\mu}^{+} \Leftrightarrow D_{\mu}^{+} \equiv \sum_{jj'} \beta_{jj'} \left(a_{j}^{+} \times a_{j'}^{+} \right)_{\mu}^{(2)}$$

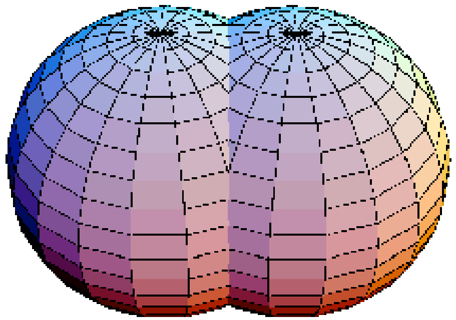
- Mapping of operators (such as hamiltonian) should take account of Pauli effects.
- Two different methods by
  - – requiring same commutation relations;
  - – associating state vectors.

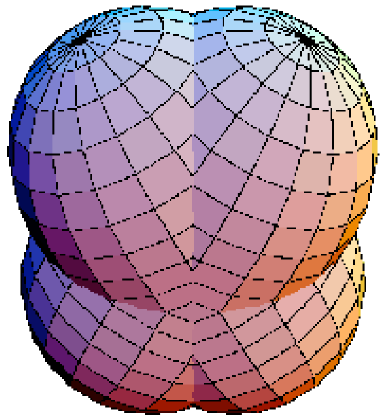
Extensions of the IBM

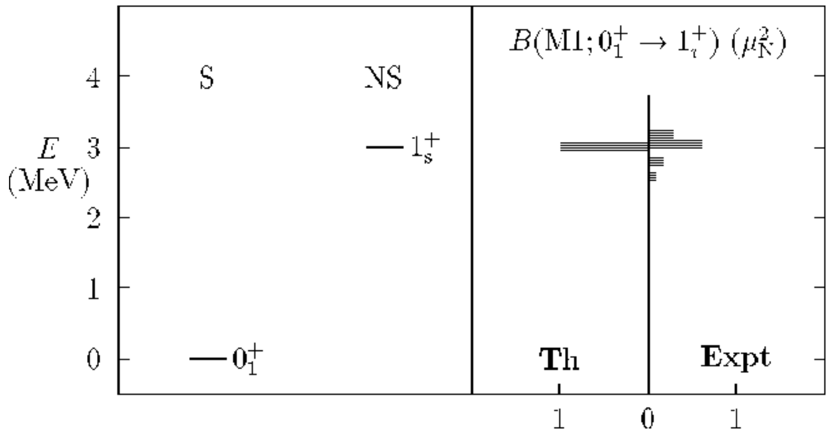
- Neutron and proton degrees freedom (IBM-2):
  - *F*-spin multiplets ($N_\nu+N_\pi$=constant).
  - Scissors excitations.
- Fermion degrees of freedom (IBFM):
  - Odd-mass nuclei.
  - Supersymmetry (doublets & quartets).
- Other boson degrees of freedom:
  - Isospin $T=0$ & $T=1$ pairs (IBM-3 & IBM-4).
  - Higher multipole (g,…) pairs.

Scissors excitations

- Collective displacement modes between neutrons and protons:
  - *Linear* displacement (giant dipole resonance): $\boldsymbol{R}_\nu$-$\boldsymbol{R}_\pi \Rightarrow E1$ excitation.
  - *Angular* displacement (scissors resonance): $\boldsymbol{L}_\nu$-$\boldsymbol{L}_\pi \Rightarrow M1$ excitation.



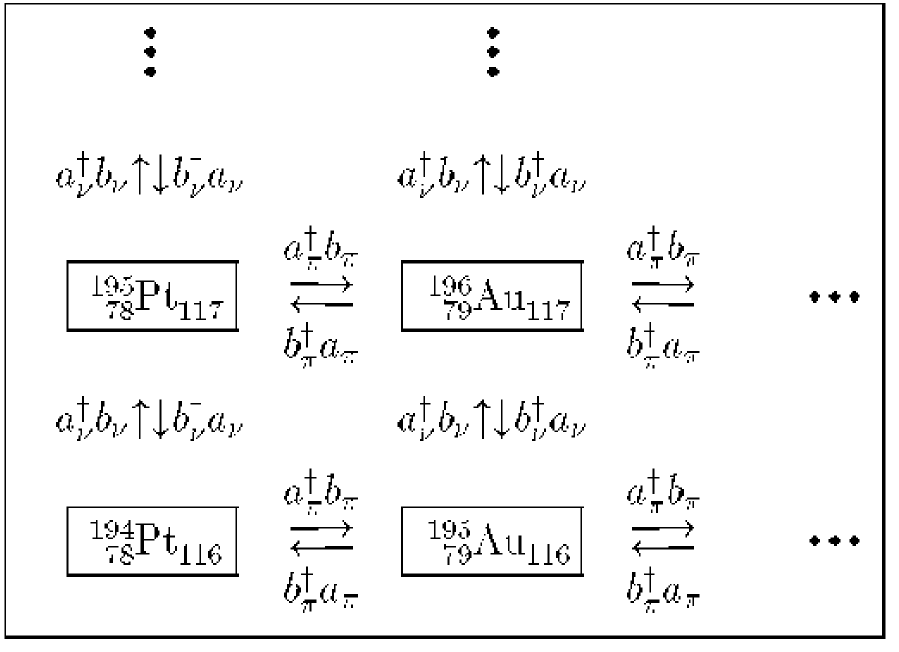




N. Lo Iudice & F. Palumbo, Phys. Rev. Lett. **41** (1978) 1532
F. Iachello, Phys. Rev. Lett. **53** (1984) 1427
D. Bohle *et al.*, Phys. Lett. B **137** (1984) 27

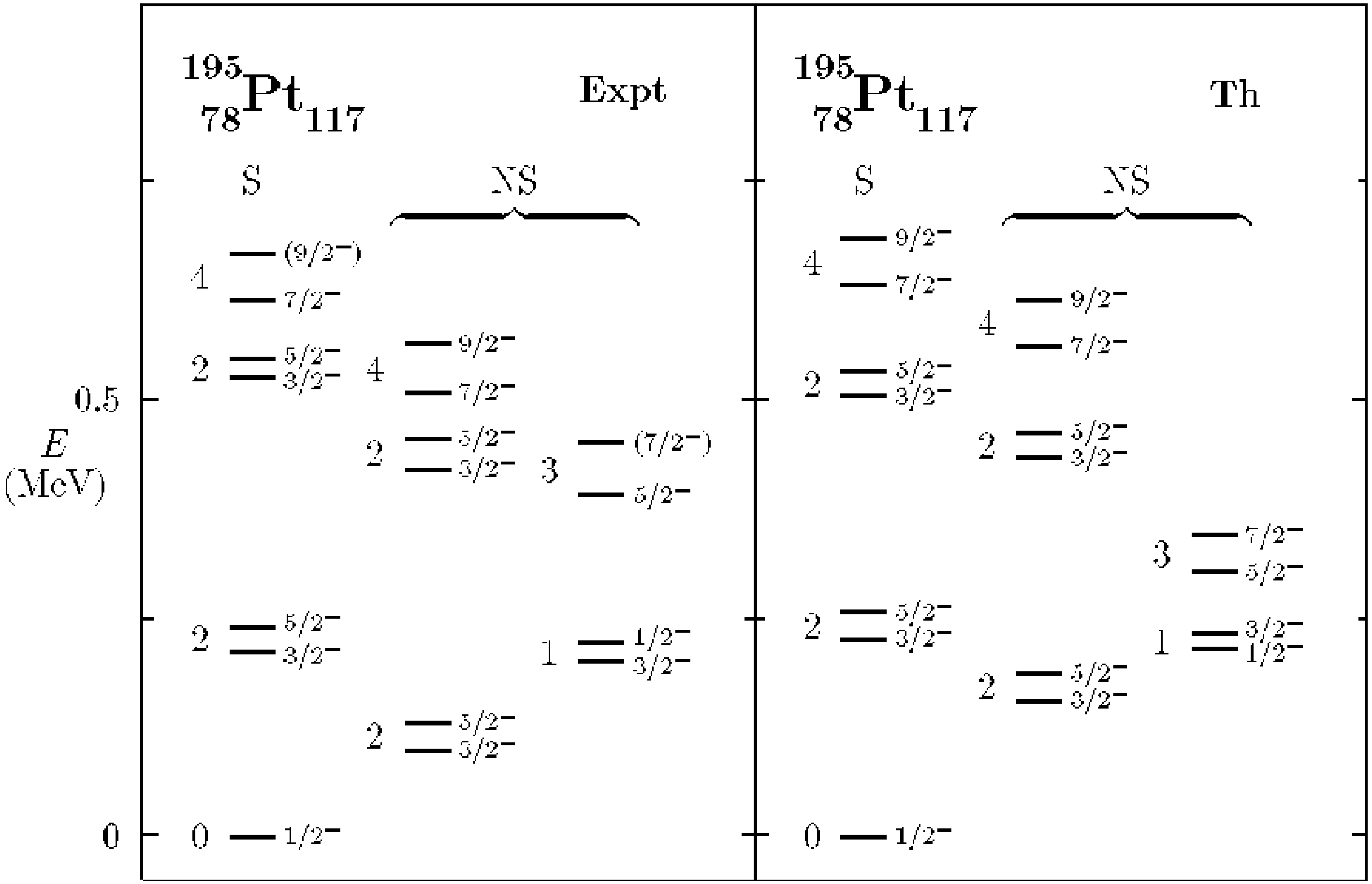
Supersymmetry

- A simultaneous description of even- and odd-mass nuclei (*doublets*) or of even-even, even-odd, odd-even and odd-odd nuclei (*quartets*).
- Example of ^{194}Pt, ^{195}Pt, ^{195}Au & ^{196}Au:

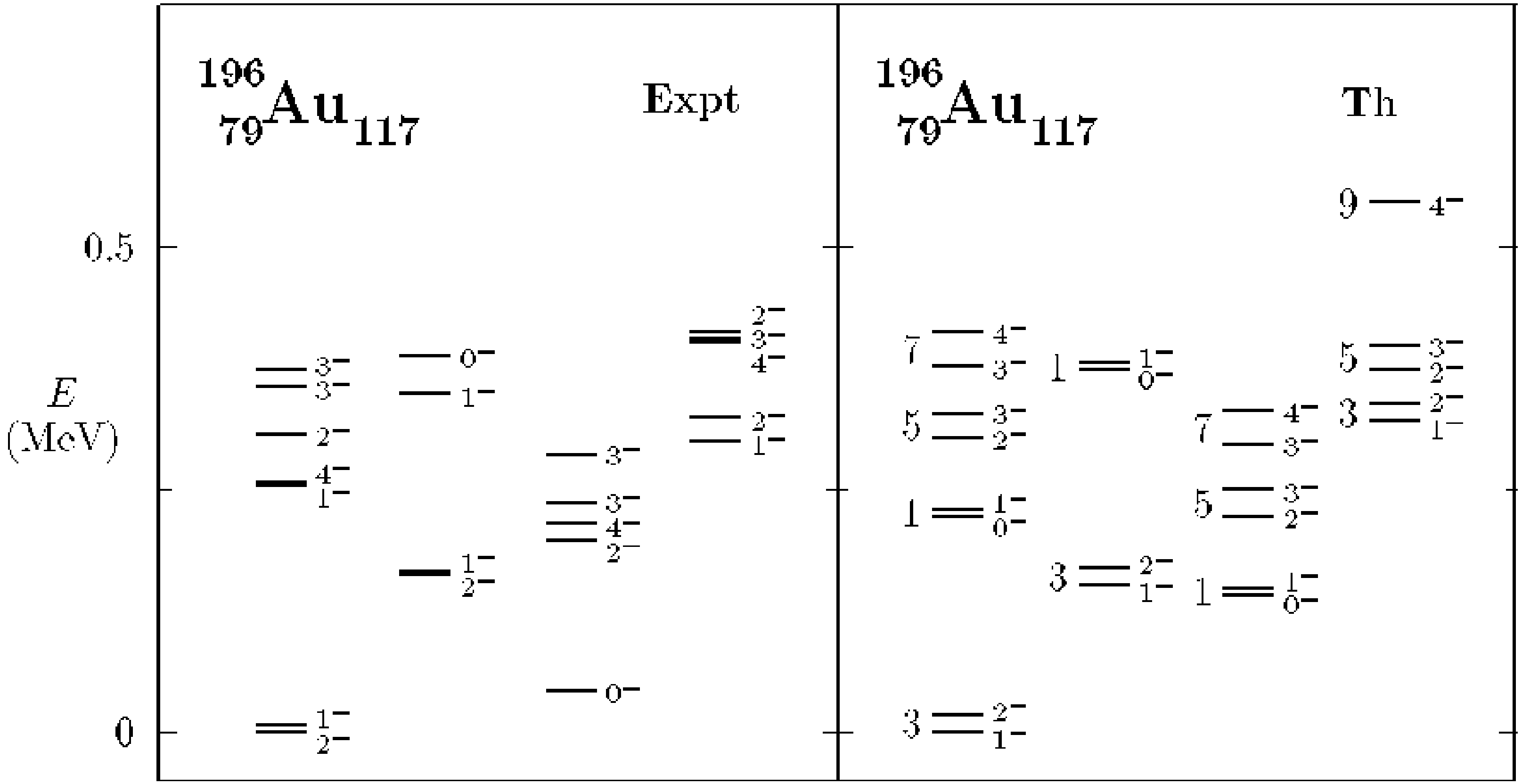


F. Iachello, Phys. Rev. Lett. **44** (1980) 772
P. Van Isacker *et al.*, Phys. Rev. Lett**. 54** (1985) 653
A. Metz *et al.*, Phys. Rev. Lett. **83** (1999) 1542

Example of ^{195}Pt



Example of ^{196}Au



Isospin invariant boson models

- Several versions of IBM depending on the fermion pairs that correspond to the bosons:
  - IBM-1: single type of pair.
  - IBM-2: $T=1$ nn ($M_T=-1$) and pp ($M_T=+1$) pairs.
  - IBM-3: full isospin $T=1$ triplet of nn ($M_T=-1$), np ($M_T=0$) and pp ($M_T=+1$) pairs.
  - IBM-4: full isospin $T=1$ triplet and $T=0$ np pair (with $S=1$).
- Schematic IBM-k has only S ($L=0$) pairs, full IBM-k has S ($L=0$) and D ($L=2$) pairs.

Algebraic many-body models

- The integrability of quantum many-body (bosons and/or fermions) systems can be analyzed with algebraic methods.
- Two nuclear examples:
  - Pairing *vs*. quadrupole interaction in the nuclear shell model.
  - Spherical, deformed and γ-unstable nuclei with *s*,*d*-boson IBM.

$$\mathrm{U}(6) \supset \left\{ \begin{array}{c} \mathrm{U}(5) \supset \mathrm{SO}(5) \\ \mathrm{SU}(3) \\ \mathrm{SO}(6) \supset \mathrm{SO}(5) \end{array} \right\} \supset \mathrm{SO}(3)$$

Other fields of physics

- Molecular physics:
  - U(4) vibron model with *s*,*p*-bosons.

$$\mathrm{U}(4) \supset \begin{Bmatrix} \mathrm{U}(3) \\ \mathrm{SO}(4) \end{Bmatrix} \supset \mathrm{SO}(3)$$

  - Coupling of many SU(2) algebras for polyatomic molecules.
- Similar applications in hadronic, atomic, solid-state, polymer physics, quantum dots…

F. Iachello, 1975 to now

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P. Van Isacker, GANIL, France

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The IBM hamiltonian

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- Explicit forms of the hamiltonian: multipole expansion and "standard representation".

The IBM hamiltonian

- Standard representation:

$$
\begin{aligned}
\hat{H} \quad = \quad & \mathtt{C(1)}\hat{N} + \mathtt{C(2)}\hat{n}_d + \mathtt{C(3)}\tfrac{1}{2}[[d^\dagger \times d^\dagger]^{(0)} \times [\tilde{d} \times \tilde{d}]^{(0)}]^{(0)} \\
& + \mathtt{C(4)}\sqrt{5}\tfrac{1}{2}[[d^\dagger \times d^\dagger]^{(2)} \times [\tilde{d} \times \tilde{d}]^{(2)}]^{(0)} \\
& + \mathtt{C(5)}\tfrac{3}{2}[[d^\dagger \times d^\dagger]^{(4)} \times [\tilde{d} \times \tilde{d}]^{(4)}]^{(0)} \\
& + \mathtt{C(6)}[[s^\dagger \times d^\dagger]^{(2)} \times [\tilde{d} \times \tilde{d}]^{(2)} + [d^\dagger \times d^\dagger]^{(2)} \times [\tilde{s} \times \tilde{d}]^{(2)}]^{(0)} \\
& + \mathtt{C(7)}[[s^\dagger \times s^\dagger]^{(0)} \times [\tilde{d} \times \tilde{d}]^{(0)} + [[d^\dagger \times d^\dagger]^{(0)} \times [\tilde{s} \times \tilde{s}]^{(0)}]^{(0)} \\
& + \mathtt{C(8)}\sqrt{5}[[s^\dagger \times d^\dagger]^{(2)} \times [\tilde{s} \times \tilde{d}]^{(2)}]^{(0)} \\
& + \mathtt{C(9)}[[s^\dagger \times s^\dagger]^{(0)} \times [\tilde{s} \times \tilde{s}]^{(0)}]^{(0)}.
\end{aligned}
$$

- Multipole expansion:

$$
\hat{H} = \mathtt{EPS}\,\hat{n}_d + \mathtt{A(0)}\hat{P}^\dagger\hat{P} + \mathtt{A(1)}\hat{L}\cdot\hat{L} + \mathtt{A(2)}\hat{Q}_\chi\cdot\hat{Q}_\chi + \mathtt{A(3)}\hat{T}_3\cdot\hat{T}_3 + \mathtt{A(4)}\hat{T}_4\cdot\hat{T}_4,
$$

The U(5) vibrational limit

- U(5) Hamiltonian:

$$\hat{H}_{\mathrm{U}(5)} = \varepsilon \hat{n}_d + \sum_{L=0,2,4} c^L \tfrac{1}{2}\left(d^+ \times d^+\right)^{(L)} \cdot \left(\tilde{d} \times \tilde{d}\right)^{(L)}$$

- Energy eigenvalues:

$$E(n_d,\upsilon,L) = \varepsilon n_d + \kappa_1 n_d (n_d + 4) + \kappa_4 \upsilon(\upsilon + 3) + \kappa_5 L(L+1)$$

with

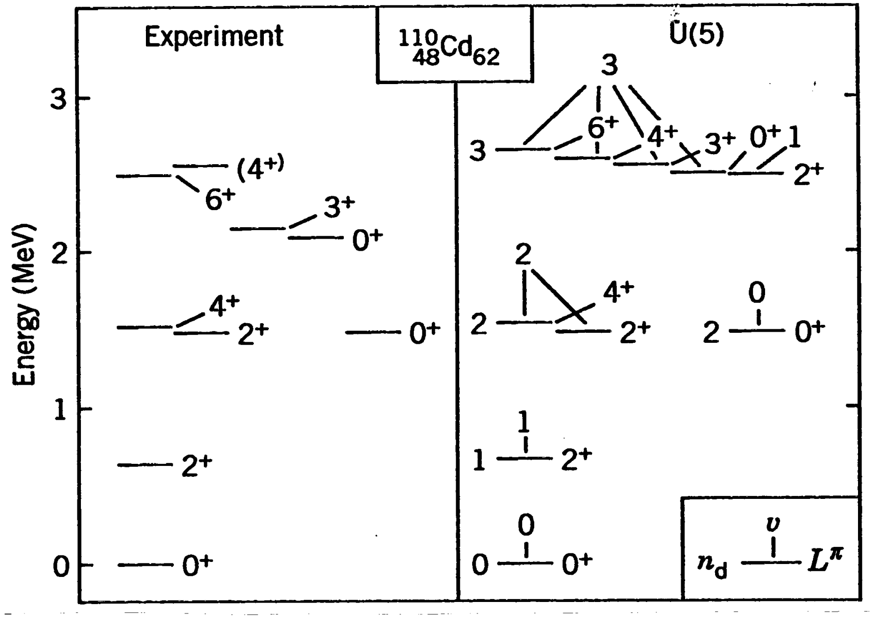
$$\kappa_1 = \tfrac{1}{12} c_0$$

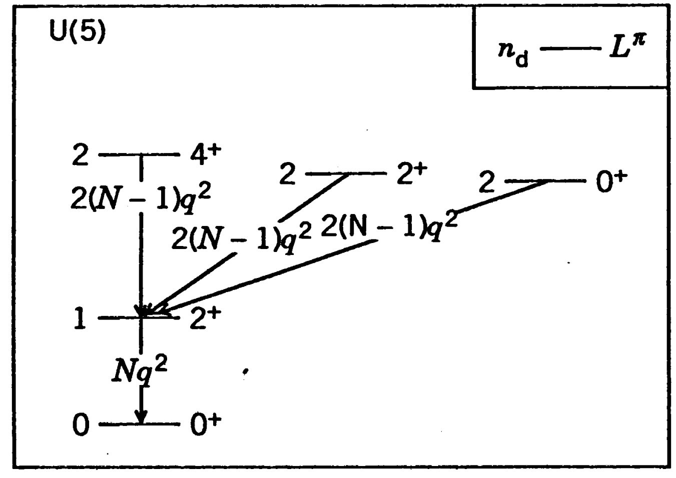
$$\kappa_4 = -\tfrac{1}{10} c_0 + \tfrac{1}{7} c_2 - \tfrac{3}{70} c_4$$

$$\kappa_5 = -\tfrac{1}{14} c_2 + \tfrac{1}{14} c_4$$

The U(5) vibrational limit

- Conserved quantum numbers: n_d, υ, L.





The SU(3) rotational limit

- SU(3) Hamiltonian:

$$\hat{H}_{\mathrm{SU}(3)} = a\hat{Q}_\chi \cdot \hat{Q}_\chi + b\hat{L} \cdot \hat{L}$$

- Energy eigenvalues:

$$E(\lambda,\mu,L) = \kappa_2\left(\lambda^2 + \mu^2 + 3\lambda + 3\mu + \lambda\mu\right) + \kappa_5 L(L+1)$$

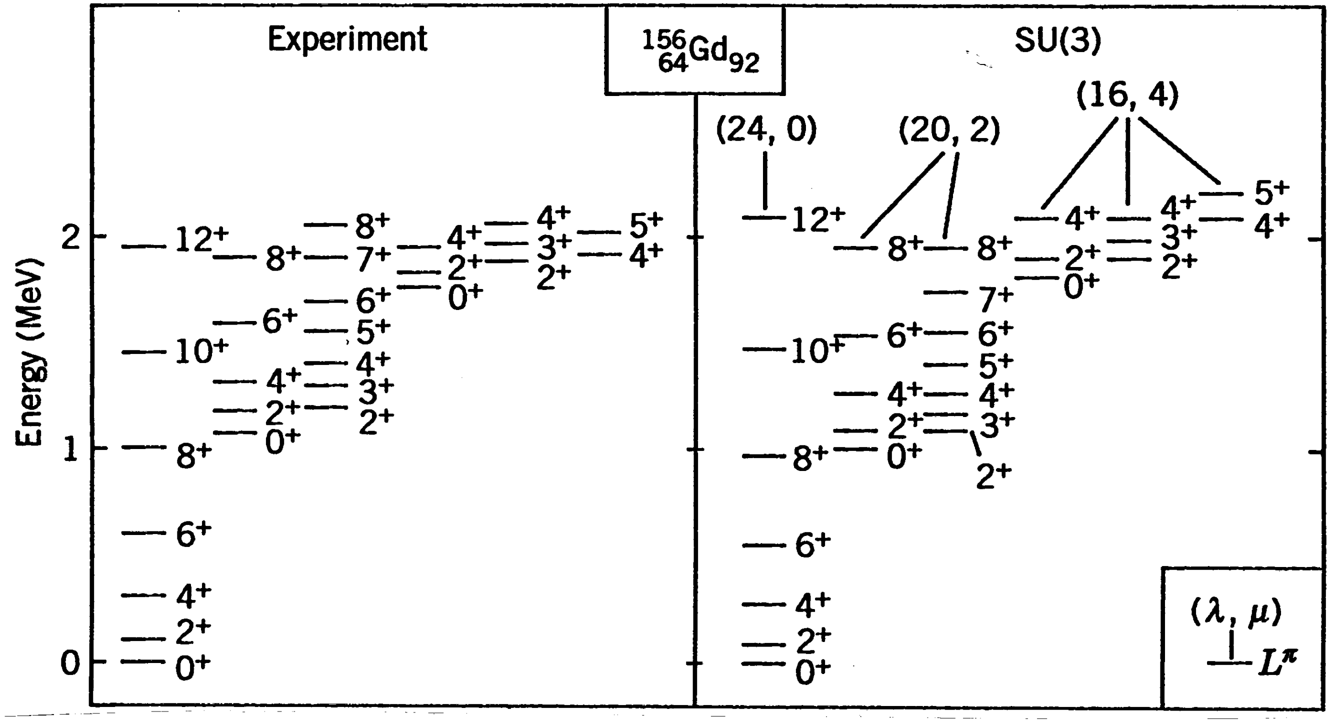
with

$$\kappa_2 = \tfrac{1}{2}a$$

$$\kappa_5 = b - \tfrac{3}{8}a$$

The SU(3) rotational limit

- Conserved quantum numbers: (λ,μ), L.



The SO(6) γ-unstable limit

- SO(6) Hamiltonian:

$$\hat{H}_{\mathrm{SO(6)}} = a\hat{P}^+ \cdot \hat{P} + b\hat{T}_3 \cdot \hat{T}_3 + c\hat{L} \cdot \hat{L}$$

- Energy eigenvalues:

$$E(\sigma,\upsilon,L) = \kappa_3\left[N(N+4) - \sigma(\sigma+4)\right] + \kappa_4\upsilon(\upsilon+3) + \kappa_5 L(L+1)$$

with

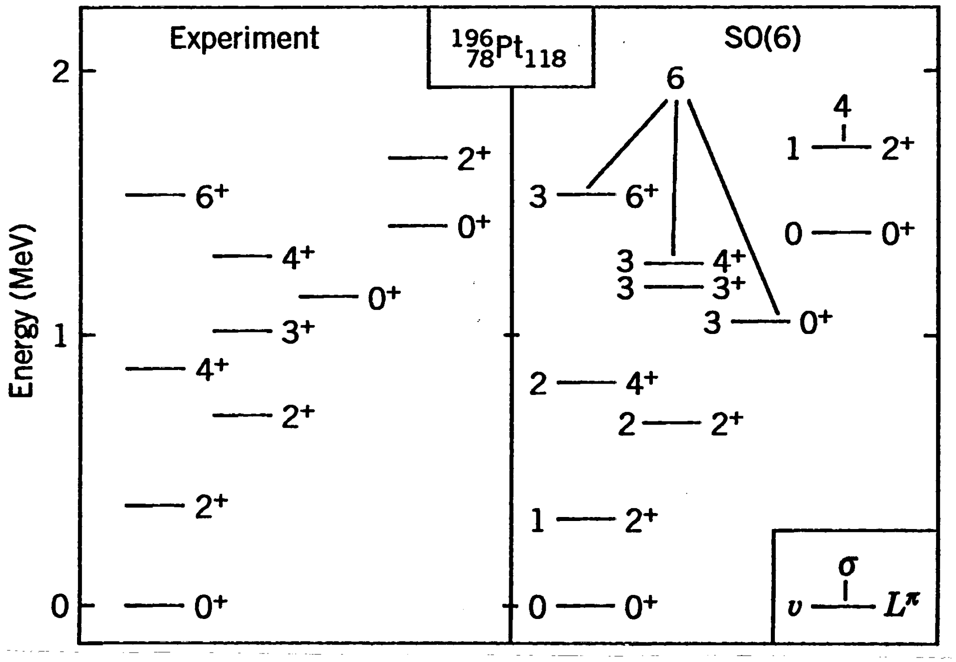
$$\kappa_3 = \tfrac{1}{4}a$$

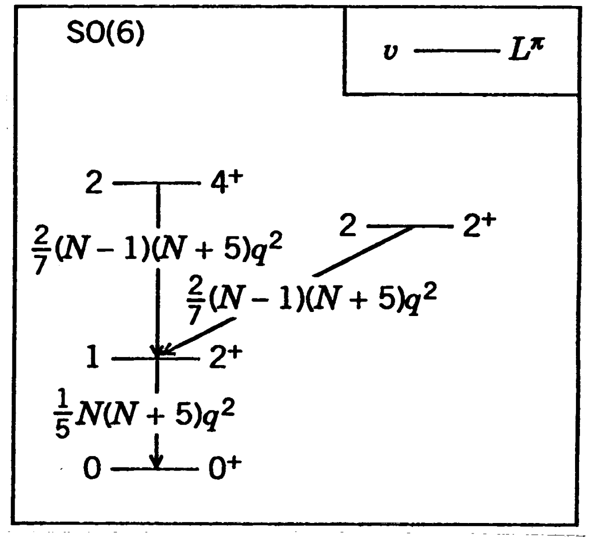
$$\kappa_4 = \tfrac{1}{2}b$$

$$\kappa_5 = -\tfrac{1}{10}b + c$$

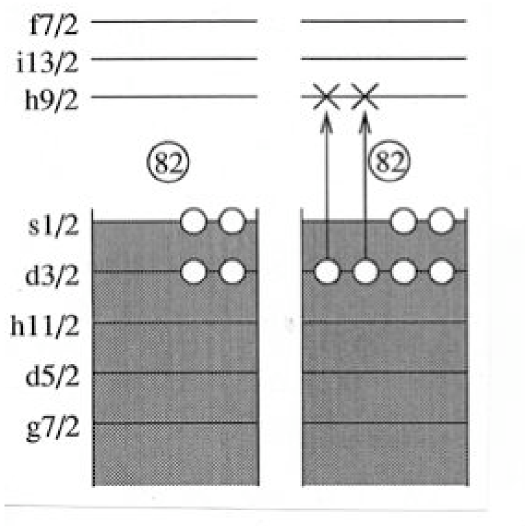
The SO(6) γ-unstable limit

- Conserved quantum numbers: σ, υ, L.





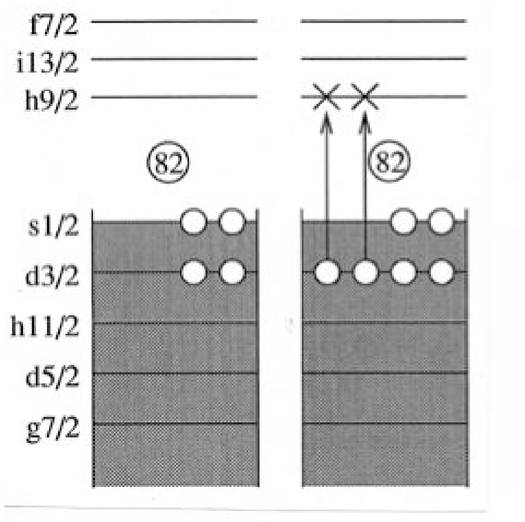
Configuration mixing in shell model



- Example of platinum isotopes (*Z=78, 82<N<126*):
  - Regular configuration: 4 proton holes in 50-82 shell.
  - Deformed configuration: 6 proton holes in 50-82 shell and 2 protons in the 82-126 shell.
  - Neutrons always in 82-126 shell.

P. Federman & S. Pittel, Phys. Lett. B **69** (1977) 385.

Configuration mixing in IBM

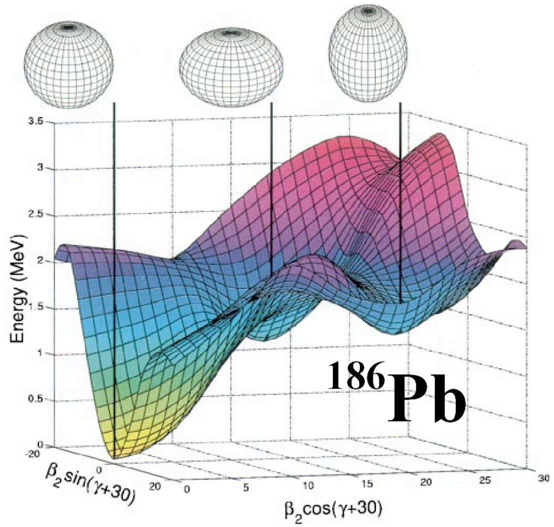


- Example of platinum isotopes ($Z=78$, $82<N<126$):
  - Regular configuration: $N_\pi=2$ proton bosons.
  - Deformed configuration: $N_\pi=4$ proton bosons.
  - Always N_ν neutron bosons.
- IBM-1: configurations with N and $N+2$ bosons.

P.D. Duval & B.R. Barrett, Nucl. Phys. A **376** (1982) 213.

Example: Coexistence in ^{186}Pb

- Observation: triplet of differently shaped 0^+ states in ^{186}Pb.
- Mean-field theory predicts three minima.
- IBM calculation for Pb isotopes yields
  - spectroscopy;
  - geometry.



F.R. May *et al.*, Phys. Lett. B **68** (1977) 113.
W. Nazarewicz, Phys. Lett. B **305** (1993) 195.
A.N. Andreyev *et al.*, Nature **405** (2000) 430.

Lead isotopes in the IBM

- Hamiltonian for *three* configurations:

$$H = H_{0\text{p}-0\text{h}} + H_{2\text{p}-2\text{h}} + H_{4\text{p}-4\text{h}} + H_{\text{mix}}^{02} + H_{\text{mix}}^{24}$$

$$H_{i\text{p}-i\text{h}} = \varepsilon_i n_d + \kappa_i Q_i \cdot Q_i, \quad Q_i = \left(s^+\tilde{d} + d^+\tilde{s}\right)^{(2)} + \chi_i\left(d^+\tilde{d}\right)^{(2)}$$

$$H_{\text{mix}}^{ii'} = \omega_0^{ii'}\left(s^+s^+ + \tilde{s}\tilde{s}\right) + \omega_2^{ii'}\left(d^+ \cdot d^+ + \tilde{d}\cdot\tilde{d}\right)$$

- Single parameter set for *all* Pb isotopes.
- Parameters for 2p-2h and 4p-4h configurations obtained from *I*-spin considerations.

R. Fossion *et al.*, Phys. Rev. C **67** (2003) 024306.

Spectroscopy of lead isotopes