Physical grounds for improved parameterization of stable boundary layers in atmospheric models

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Abstract
In operational numerical weather prediction (NWP) models the surface layer (that is the lower 1/10 of the PBL) is always parameterized in the spirit of the Monin-Obukhov similarity theory, whereas the upper part of the PBLs is resolved (in fact only assumed to be resolved). This approach could cause considerable errors when applied to the long-loved stable PBLs (in particular those typical of Arctic Regions), whose heights are of the same order as the height of the lower computational level (~30 m). In such cases the very concept of the "constant-flux surface layer" becomes inapplicable. Instead, a new approach is proposed based on advanced PBL bulk resistance and heat/mass transfer laws and PBL depth formulations, accounting for non-local effects of the static stability and baroclinic shears in the free atmosphere (above the PBL). At the present stage, the theoretical background for this approach is developed, and the major theoretical results are verified through LES. Further efforts are needed to comprehensively validate the new theory against observational data, to develop on this basis a user-friendly PBL algorithm, to implement it in operational NWP model(s), and to perform case studies and statistical analyses of the r.m.s. error and bias of the weather forecasts using the "standard" and the "equipped" versions of the NWP model.
Basic idea

Standard approach in NWP models (e.g. HIRLAM):
- The upper part of the PBLs resolved (in fact only assumed to be resolved)
- The "constant-flux" surface layer (SL = 10% of PBL) parameterised (M-O similarity theory)

It fails in Arctic stable PBLs with heights $h$ of the same order as the height of the lower computational level $z_1 \sim 30m$ (the concept of SL inapplicable)

An alternative (based on advanced PBL theory) is to parameterise the entire PBL + capping inversion:
- Prediction equation for PBL depth $h$ accounting for the free-flow stability and baroclinicity
- Analytical capping-inversion model (for $\Delta \theta_{CI}$)
- PBL bulk resistance and heat/mass transfer law for the turbulent fluxes at the surface $\bar{\tau}, F_\theta, F_q$
- Multi-limit scaling & analytical mean profiles to determine turbulent fluxes throughout the PBL
Key Points

- Principally different types of stable PBLs: nocturnal (N) and long-lived (LL)

- LL PBL height could be of order 30 m or less – no use of the “constant-flux” surface layer (MO similarity theory inapplicable)

- Capping inversions above LL PBL (T-jump up to 20 K) – overlooked of all NWP models

- Resistance laws presented in textbooks on BLM but not practically used: poor accuracy

- Why so poor?

- Overlooked mechanisms

- Advanced theory and its validation through LES and field data
Stable and neutral planetary boundary layers (PBLs)

**Traditional local theory**

Neutral PBLs \( F_{\phi} = 0 \)

Nocturnal Stable \( F_{\phi} < 0 \)

Neither \( N \) nor \( \Gamma \) are taken into account

**Non-local theory**

Truly Neutral \( F_{\phi} = 0, \, N=0, \, \Gamma=0 \)

Conventionally Neutral \( F_{\phi} = 0, \, N>0, \, \Gamma \neq 0 \)

Nocturnal Stable \( F_{\phi} < 0, \, N=0, \, \Gamma=0 \)

Long-lived Stable \( F_{\phi} < 0, \, N>0, \, \Gamma \neq 0 \)
Traditional background

The MO-theory suggests log-linear profiles of the mean wind, $u$, potential temperature, $\theta$, and specific humidity, $q$:

$$u(z) = \frac{u_*}{k} \left( \ln \frac{z}{z_{0u}} + C_u \frac{z}{L} \right)$$

$$\theta(z) = \theta_s + \frac{\theta_*}{k_T} \left( \ln \frac{z}{z_{0T}} + C_\theta \frac{z}{L} \right)$$

$$q(z) = q_s + \frac{q_*}{k_q} \left( \ln \frac{z}{z_{0q}} + C_q \frac{z}{L} \right)$$

$u_* \equiv \sqrt{\tau_s}$ friction velocity

$\theta_* \equiv -F_{\theta s}/u_*$, $q_* \equiv -F_{qs}/u_*$ $\theta$ and $q$ scales

$z_{0u}, z_{0T}, z_{0q}$ roughness lengths

$k \approx 0.4$, $k_T \approx k_q \approx 0.42$ von Karman constants

$C_u \approx 2.1$ and $C_\theta \approx C_q \approx 3.2$ other constants

$$L = -\frac{u_*^3}{\beta F_{\theta s} + 0.61 g F_{qs}} = \frac{u_*^2}{\beta \theta_* + 0.61 g q_*}$$ Monin-Obukhov length
Surface fluxes in current GCMs

Let $z_1$ is the lower calculation level. A GCM predicts

$$u = u(z_1), \quad \Delta \theta = \theta(z_1) - \theta_s, \quad \Delta q = q(z_1) - q_s$$

Given $z_{0u}, z_{0T}, z_{0q}$, the MO theory equations can be solved for $u_*, \theta_*, q_*$ and $L$, and for the fluxes

$$\tau_s = u_*^2, \quad F_{\theta s} = -u_\theta \theta_*, \quad F_{qs} = -u_q q_*$$

Inconveniences:
(i) transcendental system of equations
(ii) non-zero turbulent fluxes only when $Ri < R_i_c$

Here, $Ri$ is surface-layer bulk Richardson number

$$Ri \equiv \frac{(\beta \Delta \theta + 0.61g\Delta q)z_1}{u^2}$$

and $R_i_c$ is its critical value (supposed to be ~0.3).

In GCMs decoupling at $Ri > R_i_c$ is unacceptable:
(i) technically: numerical instability
(ii) physically: principal drawbacks of the local theory
contribution from sub-grid scales
Drag & heat/mass transfer

\[ C_D \equiv \frac{\tau_s}{u^2}, \quad C_H \equiv -\frac{F_{qs}}{u\Delta\theta}, \quad C_M \equiv -\frac{F_{qs}}{u\Delta q} \]

In neutral stratification

\[ C_{Dn} = \frac{k^2}{[\ln(z_1 / z_{0u})]^2}, \]
\[ C_{Hn} = \frac{kk_T}{\ln(z_1 / z_{0u})\ln(z / z_{0T})}, \]
\[ C_{Mn} = \frac{kk_q}{\ln(z_1 / z_{0u})\ln(z / z_{0q})} \]

The effect of stratification is taken into account through correction functions dependent on only Ri

\[ f_D = \frac{C_D}{C_{Dn}}, \quad f_H = \frac{C_H}{C_{Hn}}, \quad f_M = \frac{C_M}{C_{Mn}} \]

(Louis, 1979; Källen, 1996; Beljaars and Viterbo, 1998). Generally, \( f_D \), \( f_H \) and \( f_M \) depend on \( z_{0u}, z_{0T}, z_{0q} \) and some other parameters (Z, Perov and King, 2002).

BUT ANY FLUX-CALCULATION SCHEME BASED ON THE CONCEPT OF THE CONSTANT-FLUX SURFACE LAYER IS INAPPLICABLE TO SHALLOW PBLs
The correction functions (a) to the drag coefficient, $f_D$, (b) to the heat and mass transfer coefficients, $f_H = f_M$, versus bulk Richardson number $Ri$, after measurements at Halley, Antarctica (Zilitinkevich, Perov & King, 2002). Solid lines, after op cit; dashed lines (LTG), after Louis et al. (1982); dash-and-dot lines (BV), after Beljaars and Viterbo (1998).
The correction functions (a) to the drag coefficient, $f_D$, and (b) to the heat and mass transfer coefficients, $f_H = f_M$, vs. bulk Richardson number, $Ri$, after measurements at Sodankyla, Arctic Finland (Zilitinkevich, Perov & King, 2002). Solid lines, after *op cit*; dashed lines (LTG) after Louis et al., 1982); dash-and-dot lines (BV) after Beljaars & Viterbo, 1998).
PBL Depth

2002

2003
Motivation

• Contradicting PBL depth formulations: diagnostic, prognostic, bulk $R_i$ concept... no consensus

• Multiple mechanisms and scales:

**Universally accepted:**
- Earth's rotation
- negative buoyancy flux at the surface

**Overlooked:**
- free-flow stability
- baroclinic shear
- large-scale vertical velocity
- non-steady developments
CNPBL depth ($N > 0$)

![Chart showing dimensionless CNPBL depth versus imposed-stability parameter, $\mu_N$]

**Dimensionless CNPBL depth** $|f| h_{LES}/u_*$

vsersus imposed-stability parameter, $\mu_N$

after new LES (Esau, 2003), earlier LES (Mason & Thomson, 1987; Lin et al., 1997) and DNS (Coleman, 1999). The theoretical line is

$$h_E = 0.7 \frac{u_*}{|f| (1 + 0.28N/|f|)^{1/2}}$$

Over decades, the neutral PBL depth was calculated as $h_E = C_R u_*/|f|$, which resulted in wide spread of empirical estimates of the empirical coefficient: $0.1 < C_R < 0.7$. 

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Baroclinic fluid

Free-atmosphere parameters

Shear
\[ \Gamma = \left| \frac{du_g}{dz} \right| = \frac{g}{f} \frac{1}{T} \left[ \left( \frac{\partial T}{\partial y} \right)^2 + \left( \frac{\partial T}{\partial x} \right)^2 \right]^{1/2} \]

Brunt-Väisälä frequency
\[ N \equiv \left( \frac{g \left( \frac{\partial \theta_v}{\partial z} \right)}{T \left( \frac{\partial T}{\partial z} \right)} \right)^{1/2} \]

Parameter of baroclinicity
\[ \mu_T = \frac{\Gamma}{N} = \text{Ri}^{-1/2} \]

Richardson number
\[ 1 < \text{Ri} = \left( \frac{N}{\Gamma} \right)^2 < 10 \]

Critical \( \text{Ri}_c = 0.25 \Rightarrow \) When \( N \) diminishes, wind shear generates turbulence throughout

Baroclinic turbulent velocity scale
\[ u_T^2 = u_\ast^2 (1 + C_0 \mu_T) \]
\[ C_0 = \text{constant} \]

Derived from
\[ u_T^2 = u_\ast^2 + C' K_M^* \Gamma, \quad l_T \sim u_T / N, \quad K_M^* = u_T l_T \]
Baroclinic, conventionally neutral

\[ h_E = C_R \frac{u_*}{|f|} \left[ \frac{C_R^2 C_{uN} \mu_N}{C_S^2} \left( 1 + \frac{C_R^2 C_{uN} \mu_N}{C_S^2} \right)^{-1/2} \right] \]

\[ \equiv [h_E \text{ (barotropic Eq.6)}] \left( 1 + C_0 \mu_T \right)^{1/2} \]

The ratio of the baroclinic to barotropic PBL depth \( R = \frac{h_{LES}}{h_E \text{ (Eq.6)}} \) versus \( \mu_T = \Gamma / N \)

The upper scale shows Richardson numbers, \( \text{Ri} \). Theoretical curve, \( R = \left( 1 + C_0 \mu_T \right)^{1/2} \) with \( C_0 = 0.7 \), closely matches data from new LES (squares) over the whole range of \( \text{Ri} \) or \( \mu_T \).
Conclusions (PBL depth)

- **Equilibrium barotropic PBL**: $h_E$ depends on
  - earth’s rotation (Coriolis parameter $f$)
  - surface-layer stability (buoyancy flux $F_{bs}$; internal-stability parameter $\mu = u_*/|f|L$)
  - free-flow stability (Brunt-Väisälä frequency $N$; imposed stability parameter $\mu_N = N/|f|$)

- **Baroclinicity increases PBL depth**
  - $h_E$ depends on geostrophic shear $\Gamma$, which involves the free-flow $\text{Ri} = (N/\Gamma)^2$, or $\mu_\Gamma = \Gamma/N$

- **Widely used critical-Ri approach is poorly grounded** (overlooks the roles of $N, \Gamma, z_0, f$)

- **Recommended prognostic $h$-equation:**
  \[
  \frac{\partial h}{\partial t} + u \cdot \nabla h - w_h = -\frac{h - h_E}{t_E} + K_h \nabla^2 h
  \]

    Needed:
    
    $t_E, K_h$, empirical / LES validation, testing in GCMs
Capping inversion

2004
A page from a document discussing "Capping Inversions" and "Vertical Profiles of Potential Temperature". The text includes graphs and annotations related to temperature profiles and potential temperature gradients, with references to LES (LES, 2004-2005) and other related studies.

Graphical content includes:
- Vertical profiles of potential temperature.
- Vertical profiles of the potential temperature gradient.
- Annotations indicating depth and height indicators.

Handwritten notes include:
- "θ(2)"
- "θ/θ_
- "h in the PBL depth"
VERTICAL PROFILES OF THE GRADIENT RICHTERSON NUMBER

\[
N = \left( \frac{\partial \theta}{\partial z} \right)^2 > h
\]

\[
R_i = \frac{\partial \theta}{\partial z} \left[ \frac{1}{1 + \left( \frac{z}{h} \right)^2} \right]
\]

DIFFERENT \( \langle \theta' \rangle \) \( z=0 \)

CAPPING INVERSION IN \( \text{LES} \) (ESAU, 2005)
VERTICAL PROFILES OF THE GRADIENT RICHARDSON NUMBER

\[ Ri = \frac{\frac{\partial u}{\partial z}}{\left(\frac{\partial \theta}{\partial z}\right)^2} \]

\( \langle w_0 \rangle_{z=0} \) = surface flux of potential temperature

different values of the free-flow Brunt-Väisälä frequency \( N \)

CAPPING INVERSIONS IN LES (ESAU, 2005)
SIMPLE MODEL OF CAPPING INVERSION

LES data on temporal changes in the aerodynamic surface potential temperature $\theta_0$ and the increment $\Delta \theta_{CI} = \theta(h + \frac{1}{2} \delta) - \theta(h - \frac{1}{2} \delta)$ across the capping inversion of depth $\delta = 0.5h$

(a) Temporal changes of the normalised surface temperature drop $(\theta_0 |_{t=0} - \theta_0) h / F_{\theta}$

(b) The increment $\Delta \theta_{CI}$ versus the drop $(\theta_0 |_{t=0} - \theta_0)$

The lines are $(\theta_0 |_{t=0} - \theta_0) = - F_{\theta} h^{-1} t$

and $\Delta \theta_{CI} = (\theta_0 |_{t=0} - \theta_0)$

CONCLUSION: FURTHER WORK NEEDED
Resistance and heat/mass transfer

PBL mean profiles $u(z)$, $T(z)$, $q(z)$

1967

1968

1989

1998

2002

2004
History (local models)

Rossby-Montgomery (1935) **neutral** resistance law for $C_g = u_*/G$ and cross-isobaric angle $\alpha$:

$$\tilde{A} = \ln(C_g \text{Ro}) - \frac{k}{C_g} \cos \alpha \quad \tilde{B} = \mp \frac{k}{C_g} \sin \alpha$$

$\tilde{A}, \tilde{B}$ dimensionless coefficients

$\text{Ro} \equiv G / |f| z_0$ surface Rossby number

Z (1967); Z & Chalikov (1968) **nocturnal** $\tilde{A}, \tilde{B}$ depend on the internal stability parameter

$$\mu = \frac{u_*}{|f| L} \left( L = \frac{u_*^3}{-\beta F \theta} \right) \quad (-10^3 < \mu < 10^3)$$

Heat transfer: $\frac{k_T}{C_{Td}} = \ln(C_g \text{Ro}) - \tilde{C}, \quad C_{Td} = \frac{\theta_*}{\Delta \theta_0}$

PBL is considered as neutral when $\mu = 0$

Z & Deardorff (1974) **non-steady nocturnal**

$$\frac{k}{C_{gd}} \cos \alpha = \ln \frac{h}{z_{0u}} - A \quad \frac{k}{C_{gd}} \sin \alpha = -\frac{fh}{u_*} B$$

$$\frac{k_T}{C_{Td}} = \ln \frac{h}{z_{0u}} - C \quad A \text{ and } B \text{ depend on } \frac{h}{L}$$
Advanced PBL scaling

Local M-O *nocturnal* PBL \[ L = -\tau^{3/2} (\beta F_\theta)^{-1} \]

Non-local *long-lived* PBL \[ L_N = u_* / N \]

Rotational *truly neutral* PBL \[ L_f = u_* / |f| \]

General
\[
\frac{1}{L_{\{M,H\}}} = \left[ \left( \frac{1}{L} \right)^2 + \left( \frac{C_{\{NM,NH\}}}{L_N} \right)^2 + \left( \frac{C_{\{FM,\theta \}}}{L_f} \right)^2 \right]^{1/2}
\]

Flux profiles \[ \frac{\tau}{u_*^2} = f_\tau \left( \frac{z}{h} \right), \quad \frac{F_\theta}{F_\theta^*} = f_{F\theta} \left( \frac{z}{h} \right) \]

**LES** × *nocturnal*; ○ *long-lived*; □ *convent.neutral*

\[ L/L_N \text{ decreases with increasing height } \zeta = z/h \]
Velocity shear ($L_M$ instead of $L$)

Scaling

Log & $z$-less

$$\frac{\partial u}{\partial z} = \frac{\tau^{1/2}}{kz} \left[ 1 + \left( \frac{C_u z}{L_M} \right) \right] \approx \frac{C_u \tau^{1/2}}{kL_M}$$

LES

$$\Phi_M = \frac{kz}{\tau^{1/2}} \frac{\partial u}{dz}$$

versus $z/L$ (a) and $z/L_M$ (b)

\[\Phi_M = 1 + 2.5 \frac{z}{L}\]

$C_{NM} = 0.1$, $C_M = 1$

$x$ nocturnal; $o$ long-lived; $\square$ conventionally neutral
Geostrophic-drag function

\[ A = \ln \frac{h}{z_{0u}} - k \frac{u_g}{u_*} \] vs. \( m_A = \left[ \left( \frac{h}{L_s} \right)^2 + \left( \frac{C_{NA} h}{L_N} \right)^2 \right]^{1/2} \]

\[ x \text{ nocturnal; } o \text{ long-lived; } \square \text{ conventionally neutral} \]

Theory \[ A = -1.4m_A + \ln(e^{0.5} + m_A) \], \( C_{NA} = 0.09 \)

New LES: \( x, o \) and \( \square \) for nocturnal, long-lived and conventionally neutral PBLs. Earlier LES: \( \diamond \) (Brown et al., 1994) and \( \star \) (Kosovic and Curry, 2000). Error bars show ±3 standard deviation intervals for each LES run (i.e. 96% statistical confidence). Semi-log coordinates demonstrates how the theory performs in near-neutral and moderate-stability regimes.

\[ \Rightarrow \text{ALGORITHM : } U_* \text{ through PBL external parameters} \]
Traditional presentation of geostrophic-drag function

\[ \tilde{A} \equiv A - \ln(\frac{|f| h_E}{u_*}) \] versus \( \mu = \frac{u_*}{|f| L} \).

Cross-isobaric angle function

\[ B = k \frac{v_g}{\varphi h} \text{ vs. } m_{\{A,B\}} = \left[ \left( \frac{h}{L_s} \right)^2 + \left( \frac{C_{\{NA,NB\}} h}{L_N} \right)^2 \right]^{1/2} \]

\[ x \text{ nocturnal; } o \text{ long-lived; } \square \text{ conventionally neutral} \]

Theory \[ B = 1.5 + 7.5m_{NB}^2 , \text{ with } C_{NB} = 0.13 \]

\[ \tan(\theta) = \frac{v_g}{u_g} \Rightarrow \text{ALGORITHM: (1) thorough PBL external parameters} \]
Traditional presentation of cross-sobaric angle function

\[ \tilde{B}(\mu) \equiv (|f| h_E / u_*) B \text{ versus } \mu = u_*/|f|L \]
Potential temperature ($L_H$ or $L$)

Inflation point on top of PPBL $z = h - \frac{1}{2} \delta_{ci}$: $\partial \theta / \partial z$ approaches minimum and starts growing

**Scaling**

\[
\frac{\partial \theta}{\partial z} = \frac{\tau^{1/2}}{k_T z} \left[ 1 + \left( \frac{C_\theta z}{L_H} \right) \right] \approx \frac{C_\theta \tau^{1/2}}{k_T L_H}
\]

**LES**

\[
\Phi_M = \frac{k_T z \partial \theta}{\theta_* d\zeta}
\]

(a) $z/L$ (b) $z/L_H$ with $C_{NH} = 0.6$

\[\Phi_H = 1 + 2z/L \times \text{nocturnal}\]

\[\Phi_H = 1 + 2z/L_H \circ \text{long-lived}\]
Resistance law for potential temperature

\[ C = \ln \frac{h}{z_{0u}} - k_T \frac{\Delta \theta_{PBL}}{\theta_{*S}} \]

versus \( m_c = \left[ \left( \frac{h}{L_s} \right)^2 + \left( \frac{C_{NC} h}{L_N} \right)^2 \right]^{1/2} \) \( (C_{NC}=1.2) \)

\[ C = -4.1m_c + \ln(e^{12} + m_c) \]

\( x - \text{nocturnal} \) PBLs; \( o - \text{long-lived} \) PBLs

\[ \Theta_\ast = \frac{F_{os}}{u_\ast} \quad \text{through PBL external parameters} \]
Traditional presentation of the potential temperature resistance function

\[ \tilde{C} (\mu) \equiv C - \ln (|f| h_E / u_*) \text{ versus } \mu = u_* / |f| L \]
Conclusions (Resistance Laws)

• Account for free-flow stability, baroclinicity and non-steady evolution

• Revised surface-layer scaling \( L \Rightarrow L_{\{M,H\}} \)

• Limited applicability of earlier models: \( A, B, C \) as functions of \( \frac{u_*}{|f| L_s} \) or \( \frac{h}{L_s} \)

• Advanced formulation for cross-isobaric angle: role of the Coriolis parameter

• Physical basis for new surface-flux scheme applicable to shallow PBLs (for use in NWP)

• Application to oceanic PBLs: dominant role of external stability parameter \( \mu_N = \frac{N}{|f|} \)

• Analytical model (theory + LES) for wind, temperature, eddy viscosity / conductivity, applicable to different types of stable PBLs
GENERAL CONCLUSIONS

- Advanced scaling for wind, temperature and eddy viscosity/conductivity applicable to stable PBLs of different nature

- Comprehensive revision of earlier PBL depth equations: account for free-flow stability, baroclinicity and non-steady evolution

- Advanced resistance and heat-transfer laws, limited applicability of earlier models considered $A, B, C$ as functions of $\mu = u_*/fL$

- Physical background for improved surface-flux scheme applicable to shallow PBLs (demand from operational models)

- Applications to oceanic PBLs: dominant role of external stability parameter $\mu_N = N / |f|$