Spring Colloquium on
'Regional Weather Predictability and Modeling'
April 11 - 22, 2005

1) Workshop on Design and Use of Regional Weather
   Prediction Models, April 11 - 19

2) Conference on Current Efforts Toward Advancing the Skill of Regional Weather
   Prediction. Challenges and Outlook, April 20 - 22

301/1652-12

Numerical Developments for Nonhydrostatic
Mesoscale Models

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Numerical Developments for Nonhydrostatic Mesoscale Models

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Trieste 2005
Plan of Lecture

• The Nonhydrostatic Model LM
• Runge Kutta and Semi-Lagrangian methods (provide sufficient accuracy for practical purposes)
• Z-coordinates with step and shaved element boundaries
• Direct implicit solvers
• Global nonhydrostatic modelling on isocahedral or similar grids
Features of the LM - Design Aspects

- Modelling Scales from 50 m to 50 km
- Nonhydrostatic Compressible Dynamics
- Efficient Numerics
- Comprehensive Physics
- Nudging Data Assimilation
- Code Portability
- Mesh Refinement
- Use for Both Research and Operations
Features of the LM - Overview

**Dynamics and Numerics**

- Advection Form
- Prognostic Variables: $u,v,w,T,q_v,q_c$, $(q_i,q_{tke},q_r,q_s)$
- Coordinates: Rotated Lat-Lon, Generalized Terrain Following
- Arakawa C / Lorenz Grid Staggering
- 2nd Order Centred Differencing
- Leapfrog / Split explicit Time Integration (HE-VI)
- Numerical Smoothing: 4th Order Horizontal Diffusion, Rayleigh Damping, 3-D Divergence Damping
- Optional Time Integration Schemes: 2-TL RK with 3rd Order Advection, 3-TL SI Scheme
Grid Structure and Time Integration

\[ \frac{\partial \phi}{\partial t} = F + S \]
Physical Parameterization

- Vertical Diffusion by
  - Diagnostic K-Closure
  - Prognostic TKE
- Grid Scale Clouds by Saturation Adjustment and
  - Warm Rain- / EM-DM- / Cloud Ice - Scheme
- Moist Convection
  - Tiedtke Mass Flux Scheme
  - Kain Fritsch Scheme with CAPE-Closure (untested)
- Radiation: Two-Stream Scheme (Ritter, Geyleyn, 1992)
- Soil Processes
  - Two Layer Extended Force Restore Soil Model
  - New Multi Layer Soil Model Including Melting and Freezing
Features of the LM - Overview

**Boundary Conditions**

- Periodic, Wall or Relaxation Boundary Conditions for Idealized Cases
- One-Way Nesting by Davies Relaxation for Real Cases Interpolated from GME (GME2LM), IFS (IFS2LM), LM (LM2LM, Current Work)
- Two Way Interactive Self Nesting (Current Work)

**Initial Conditions**

- Artificial Data for Idealized Cases (User Defined!)
- Interpolated from GME or IFS; DFI Initialization
- Continuous Data Assimilation Nudging \(\left(u,v,p,T,q_v\right)\); External Analysis of SST; Variational Soil Moisture Analysis (00UTC); Latent Heat Nudging for radar reflectivities
Monthly Precipitation Sum for September 2001
Precipitation accumulated for the period 10 Aug 06 UTC - 13 Aug 06 UTC

GME forecast

LM forecast

 Observation

The Elbe Flood Disaster, August 2002

The map shows the precipitation accumulation over the period from 10 to 13 August 2006 UTC. The areas are color-coded to indicate the amount of precipitation, ranging from white (0 mm) to red (>200 mm).
COSMO - Integration Domains
### LM - Configurations at COSMO Centres

<table>
<thead>
<tr>
<th>Configurations</th>
<th>ARPA–SMR</th>
<th>DWD</th>
<th>HNMS</th>
<th>MeteoSwiss</th>
<th>IMGW</th>
</tr>
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<tbody>
<tr>
<td><strong>Domain Size</strong></td>
<td>234 x 272</td>
<td>325 x 325</td>
<td>95 x 113</td>
<td>385 x 325</td>
<td>385x321, 193x161</td>
</tr>
<tr>
<td><strong>Grid Spacing</strong></td>
<td>0.0625°</td>
<td>0.0625°</td>
<td>0.1250°</td>
<td>0.0625°</td>
<td>0.0625°, 0.125°</td>
</tr>
<tr>
<td><strong>Number of Layers</strong></td>
<td>35</td>
<td>35</td>
<td>35</td>
<td>45</td>
<td>35</td>
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<tr>
<td><strong>Time Step</strong></td>
<td>40 sec</td>
<td>40 sec</td>
<td>80 sec</td>
<td>40 sec</td>
<td>40 sec, 80 sec</td>
</tr>
<tr>
<td><strong>Forecast Range</strong></td>
<td>48 hrs</td>
<td>48 hrs</td>
<td>48 hrs</td>
<td>48 hrs</td>
<td>36 hrs, 72 hrs</td>
</tr>
<tr>
<td><strong>Model Runs</strong></td>
<td>00, 12 UTC</td>
<td>00, 12, 18 UTC</td>
<td>00 UTC</td>
<td>00, 12 UTC</td>
<td>00, 12 UTC</td>
</tr>
<tr>
<td><strong>Boundaries</strong></td>
<td>GME</td>
<td>GME</td>
<td>GME</td>
<td>GME</td>
<td>GME</td>
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<tr>
<td><strong>LBC Updates</strong></td>
<td>1 hr</td>
<td>1 hr</td>
<td>1 hr</td>
<td>1 hr</td>
<td>1 hr</td>
</tr>
<tr>
<td><strong>Initial State</strong></td>
<td>GME</td>
<td>Nudging</td>
<td>GME</td>
<td>Nudging</td>
<td>GME</td>
</tr>
<tr>
<td><strong>Initialization</strong></td>
<td>Digital Filtering</td>
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<td>Digital Filtering</td>
<td>None</td>
<td>Digital Filtering</td>
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<td><strong>External Analyses</strong></td>
<td>None</td>
<td>SST, S–Depth, Soil Moisture</td>
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<td>SST, S–Depth</td>
<td>None</td>
</tr>
<tr>
<td><strong>Hardware</strong></td>
<td>IBM SP (Pwr3)</td>
<td>IBM SP (Pwr3)</td>
<td>CONVEX</td>
<td>NEC SX–5</td>
<td>SGI 3800</td>
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<tr>
<td><strong>No. of Processors</strong></td>
<td>32 / 64</td>
<td>160 / 1280</td>
<td>14 / 16</td>
<td>12 / 16</td>
<td>96 / 100</td>
</tr>
</tbody>
</table>
COSMO Limited Area Ensemble Prediction System

(Regionalization of the ECMWF Ensemble using LM)

Probability for 2m-Temperature exceeding 20°C

Probability for 24-h precipitation exceeding 20 mm
Recent and Current Numerical Developments

- Mesoscale ensemble
- LM_K: operational cloud prediction model at a scale of dx=2.8 km (includes supporting developments in analysis, physics model interpretation)
- LM-Z
- Runge Kutta
- Implicit and SL
- Next generation model and global mesoscale model: ICON project
The Runge Kutta scheme (NCAR)

- RK is a two time level 3rd order in time scheme, involving substepping for fast waves
- Spacial order is 3 or 5 (upstream differencing)
- Approximation conditions concern vert. coordinate and phys. interface
- Semi-lagrange: 2nd order in time, 3rd order in space, could be easier to achieve efficiency with large dt
Expected advantages of the Z-coordinate

- The atmosphere at rest can be represented in Z-coordinates, but not in terrain following coordinates
- Stratiform clouds and low stratus are predicted better in LM-Z
- Mountain and valley winds are better with LM-z
- Precipitation amplitudes should be better with LM-Z, in particular maxima and minima near mountains
The mountain related bias of convectional clouds and precipitation is supposed to disappear with the Z-coordinate.
The step-orography

• The shaved elements are mathematically more correct than step boundaries

• By shaved elements the z-coordinate is improved such that the criticism of Gallus and Klemp (2000), Mon. Wea. Rev. 128, 1153-1164 no longer applies
Cross Section for Flow Over the Alps, Forecasted by Eta Model with Step Orography (Right) and with Terrain Following Orography (Left)
The finite volume method for the treatment of the fast waves in LM-Z

- The computation of the fast waves is based on the evaluation of fluxes into a cell
- The evaluation of the fluxes requires weights associated with the cell surfaces, which depend on their open part
- Advection terms (slow waves) are computed by finite difference methods
SKANIA Test: Korrect (left) faulty (right)
SKANIA Test for Z_LM
Without Physics

Dry Physics
NO physics

„dry“ Physics
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The Atmosphere at Rest Computed with the Z_LM

Day

Night
Test

Day

Night
Cloud water  
Precipitation
No physics / Physics

date: 09.01.04 00UTC
Low stratus
Z-coordinate

NO HGB 04.01.09 12:00 12 h accumulation. Threshold: 0.00 Corrected: I
28039700 6 to 6

- LM-Z
- OBS
- LM-tf
• LM_Z

28039700+12

• LM_tf
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28039700+12

- LM_Z
- LM_tf
Current state of LM-Z

- Idealised tests with bell shaped mountain show the expected feature of improved wind, temperatures and precipitation.
- SKANIA Test with LM-Z reveals no problems.
- First realistic runs are encouraging. LM-Z compared to LM-tf showed large improvements concerning the prediction of clouds and precipitation.
Some Desirable features of Next Generation NH Models

- Sufficient Accuracy/high (Spatial) Approximation order (Achieved with RK and SL)
- A problem is the combination of high order and conservation
- Approximation condition orography: **Z-Coordinate**
- Both RK and SL have currently the computational expense of second order centred differences / Leapfrog
- One of the keys to numerical efficiency is **implicit time integration**
- Global mesoscale models (achieved on Isocaheronlike grids) may be feasible within 15 years
Implicit Approaches

Example: \( \frac{\partial h}{\partial t} = \frac{\partial (uh)}{\partial x} \) \quad u \text{: constant field; } h \text{: dynamic field}

Nonlinear: \( h^{n+1} - h^n = dt([uh]_x^n + [uh]_x^{n+1}) / 2 \)

Tangent linear: \( h^{n+1} - h^n = dt(u_x h_x^{n+1} + u_x h^n + u^{n+1} h_x + u^n h_x) / 2 \)

Locally homogenised: \( h^{n+1} - h^n = dt(u h_x^{n+1} + u h_x^n) / 2 \)
Direct Methods for Locally Homogenized SI

- The Equations of Motion are homogeneously linearised at each Grid Point
- At Each Grid-Point a Problem of Constant Coefficients is Defined
- For Each Grid-Point The Associated Linear Problem Can be Solved Using an FT and a Linear Problem Specific to Each Grid-Point
- The GFT (Generalised FT) Computes the Results of the Different FTs Using One Transform
- The numerical cost of GFT is Similar to that of an FT
- A Fast GFT exists similar to Fast FT
Boundary- and Exterior Points

- Redundant points can be included in the FT
- The result of the time-step does not depend on the continuation of the field to redundant points
Organisation of the Implicit Time-Step

- The Fourier Coefficients are the same for the grid Points of a Subregion
- The Linearised Eqs. are different for each Gridpoint
- In Case of only One Subregion the Support Points of the derivative $\phi_{0x}$ are the Boundary Values
- $\phi_{0x}$ does not create time-Step Limitations
- GFT Returns the Grid-Point-Values after doing a Different Eigenvalue Calculation at Each Point
The SI Timestep

Subtract Large Scale Part of each function

Use Result Only for Chosen Grid Point

Do the Above for all Other Grid Points

Compute Fourier Coefficients

Transform Back

Set Boundary Values as in Finite Difference Methods

Choose Gridpoint

Compute next time level in Fourier Space
Example: 1-d Schallow Water Eq

\[
\frac{\partial u}{\partial t} = -u \frac{\partial u}{\partial x} - \frac{\partial h}{\partial x} \\
\frac{\partial h}{\partial t} = -\frac{\partial (hu)}{\partial x}
\]

SI Scheme:

\[
u^{n+1} - u^n = -dt(\left( u^n u^{n+1} + u^n \right) / 2 + (h^{n+1}_x + h^n_x) / 2) \\
h^{n+1} - h^n = -dt(h^n (u^{n+1}_x + u^n_x) / 2 + u^n (h^{n+1}_x + h^n_x) / 2)
\]

Definitions:

\[
\Delta u = u^{n+1} - u^n ; \Delta h = h^{n+1} - h^n ; \\
rsu = -u^n u^n_x - h^n_x ; rsh = u^n h^n_x - h^n u^n_x
\]

SI Scheme:

\[
\Delta u = rsu - dt u^n \Delta u_x - dt \Delta h_x = rsu - dt(u^n \Delta u^0_x - \Delta h^0_x) - dt u^n \Delta u'_x - dt \Delta h'_x \\
\Delta h = rsh - dt u^n \Delta h_x - dt h^n \Delta u_x = rsh - dt(u^n \Delta h^0_x - h^n \Delta u^0_x) - dt u^n \Delta h'_x - dt h^n \Delta u'_x
\]

Periodicity Terms (=0 in the following)
Operation in Fourier Space

\[ \Delta u = rsu - dtu^n \Delta u_x - dt \Delta h_x = rsu - dtu^n \Delta u'_x - dt \Delta h'_x \]

**SI Scheme:**
\[ \Delta h = rsh - dtu^n \Delta h_x - dth^n \Delta u_x = rsh - dtu^n \Delta h'_x - dth^n \Delta u'_x \]

In the following \( rsu, rsh, u^n, h^n \) will be taken at some chosen gridpoint.

**Definition:**
\[ \sim \phi = F \phi \]

**Linear Equations at the chosen gridpoint:**

\[ \sim \Delta u - dtu^n \sim \Delta u_x - dt \sim \Delta h_x = \sim rsu \]
\[ - dtu^n \sim \Delta h_x + \sim \Delta h - dth^n \sim \Delta u_x = \sim rsh \]

\( \sim \Delta u, \sim \Delta h \) are obtained from the solution of the linear equation.

The back transformed fields \( \sim \Delta u, \sim \Delta h = F^{-1} \sim \Delta u, F^{-1} \sim \Delta h \) will be taken at the chosen gridpoint only.
Computational Example: 1-d Shock Wave

The time step could be increased up to the CFL of advection (10 m/sec)
1-d Shallow Water Equus. Periodic boundaries with bell shaped initial disturbance
2-d Shallow Water Equations with Barrier
Implicit Conclusions

• A direct si- method was proposed
• The method is based on a generalised Fourier Transform
• The generalised FT is potentially as efficient as the FT (fast FT)
• 1-d and 2-d tests have been performed
A family Grids on the sphere

• Current approach: lat lon and Kurihara will not be discussed
• The Baumgardner principle
• Great circle grids for triangles and rhomboids/ Isocahedron, cube and 8 surface body grid stencils
• Computation of directional derivatives of a given order
• Overview of numerical concepts
• Interpolation
• Rooftile grid
• Dual grid and conservation
• Choice of options and modular workplan
• Conclusions
Points of concern in this lecture

• Is o3 important? ("next generation Dynamics"?) Can issues addressed in WRF and LMK be incorporated? Can current nh-solutions/solutions under research be incorporated: i.e. FT Preconditioners?)

• Model development with limited resources? (Are current developments incorporated easily? Fallback positions? Baumgardner doctrine?)
The Baumgardner principle

(Rules of good behaviour on triangels)
Proven for o2, not yet for o3

Supported by the success of Skamarock nesting)

• No global coordinate
• Keep approximation order at grid interfaces
• The faithful are rewarded by having no problems carrying plane discretisations to the sphere
Desirable features of discretisations on the sphere

- **Nonhydrostatic**
- **Accuracy: Order 3 or higher in space [and time]**
  - (Observation of approximation conditions: smoothness \( \frac{\partial^4}{\partial x^4} \) (for third order schemes), Smooth physics interface, smooth orography (dh<dz) or z-coordinate)
- **Conservation: mass, energy**
- **Efficiency (computer time and development time)**
  - (Positivity of advection: flux correction)
  - (Nesting option: Skamarock method)
- **Ability to incorporate developments for nh models**
Quasi regular grids

- Structured (index i,j)
- Each line of points j,i  j,i+1  j,i+2............ is on a great circle
- Obtained by projecting bilinear grids to the sphere
- Projection of any vector $\mathbf{r}$ to the sphere with image $\mathbf{r}_{\text{a}}$: $\mathbf{r}_{\text{a}} = \frac{\mathbf{r}}{|\mathbf{r}|}$
Bilinear grids

- Four points $r_1,r_2,r_3,r_4$ may have any position in space
- Divide the sides of the rhomboid equally and connect opposite points
- Bilinear grid theorem: each coordinate line intersects each line of the crossing coordinate line family. The grid is regular in each direction.
Orange cut grids

- NP=3
- NP=4
- NP=5
Grid Stencils Baumgardner

- Edges grid
- Order 3
  Redundancy 19:9

- Edges grid
- Order 2
  Redundancy 6:5 or 5:5
Grid Stencils Baumgardner

- Area grid
- Order 3
  Redundancy 13:9

Area grid
Order 1 (Finite Volume)
Redundancy 4:3
Great Circle Grid Stencils

- Edges grid
- Order 3
- Local coordinate, for example local geographic
  - Possibility 1:
    irregular, but locally nearly regular grid
  Non orthogonal grid
  - Possibility 2:
    Rooftile grid: regular and nearly orthogonal
Interpolation

- Grid redundancy is an issue for all methods relying on interpolation
- Cascade interpolation for regular grids
- Serendipidity interpolation: the part going into 2d and 3d look like linear.
- Serendipidity grids replace forecasts of some points by order consistent interpolation
Rooftile Grids

- Grid matching at most boundaries
- Nearly orthogonal

- Interpolation O3 for boundary values
- For 100 points per tile grid, irregularity is 1%
Rooftile Grids, 4-Body

- Triangles are used to match areas (implying irregular shaped cells or double grid covering). All other boundaries match
Discretisation Options, Based on Interpolation

- Finite Volumes: Bonaventura choice, best on regularised grids, conservation possible, often low order, tested by Ringler and Steppeler
- Baumgardner: suitable for somewhat irregular grids, tested for order 2
- Baumgardner Order2: Amplitudes on edges; small grid redundancy
- Baumgardner Order3: Amplitudes on triangle surfaces, very irregular grid for plane waves, yet untested, (some grid redundancy)
- Great circle grids: very similar to limited area discretisations, order 2,3 easily possible, RK, SI, SL, adaptation of all local developments easy (grid redundancy no problem)
- Tiled grids: very uniform grids (~1%), less elegant look, spectral elements possible
- Serendipidity grids (can be derived as a further development of SE)
Saving factors of Discretisations

- Finite Volumes: 1
- Baumgardner Order2: 1
- Baumgardner Order3: 1
- Great circle grids: RK, SI, SL 1 now 3 seem possible
- Tiled grids: 1.5
- Serendipidity grids 3 $^{1.3^{-1}}$
- Unstructured
- Conservation
Dual grid and conservation

- Use conservation form, compute fluxes
- 1st possibility: WRF-method
- 2nd: Flux correction
- Issue: order of representation of the conserved quantity
Serendipidity grids

- Grid interpolation: 3rd order
  - In a square grid grid redundancy is so large (64:15) that it becomes a problem
  - Some of the redundant grid points can be interpolated from the non redundant ones in an order consistent way, resulting in:
    - efficient interpolation
    - Saving of 27:7 from redundantly forecasted points
    - Easier for posing boundary values and theoretically more satisfying in this respect
  - The field is known to 4 rth order error at all points, meaning that c-grid structures for fast waves can be generated.

- Spectral elements:
  - Some of the basis functions can be shown to contribute in a negligible way
  - avoiding these contributions leads to the same grid structures and saving as above
Conclusions

• RK achieves sufficient accuracy, all schemes discussed are about equal in efficiency
• Increase of efficiency by a factor of 10 is possible by implicit methods and serendipidity grids
• Combined Order 3 and conservation is possible using the dual grid method
• Great circle grids based on the isocahedron allow to make local models global