



The Abdus Salam  
International Centre for Theoretical Physics



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## School and Workshop on Structure and Function of Complex Networks

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### Social Networks: Applications to Labor Markets

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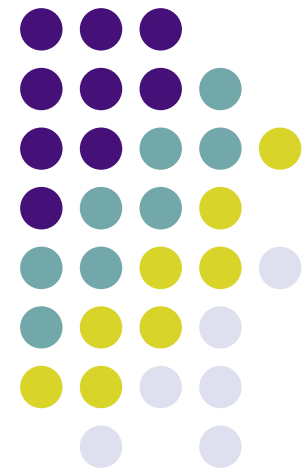
These are preliminary lecture notes, intended only for distribution to participants

# Social Networks: Applications to Labor Markets

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Matthew O. Jackson

Trieste Summer School on  
Networks





# Markets and Networks

- Traditional Models of Markets:
  - Centralized
  - Search Models
- Networks of Contacts are Important in Many Markets
  - Currency Markets
  - B to B, Supply chains,
  - Labor Markets
  - ...

# Broader Research Agenda:



- How do networks impact interactions?
  - E.g., how are networked interactions different from others?
- What are the incentives to form networks?
  - Do Pareto efficient networks form on their own?
  - How to help if they do not?

# Outline of Lecture



- Background on Networks in Labor Markets
- Stylized Facts of Labor Markets
- Network Models, Examples and Results
- Concluding Remarks



# Networks in Labor Markets

- Myers and Shultz (1951)- textile workers:
  - 62% first job from contact
  - 23% by direct application
  - 15% by agency, ads, etc.
- Rees and Shultz (1970) – Chicago market:
  - Typist 37.3%
  - Accountant 23.5%
  - Material handler 73.8%
  - Janitor 65.5%
  - Electrician 57.4%...



# More on Contact Networks

- Granovetter (1974) study across job type
  - 44% of technical jobs by contact
  - 56% of professional jobs by contact
  - 65% of managerial jobs by contact
- Corcoran et al. (1980) PSID – jobs by contact
  - White Males 52%
  - White Females 47.1%
  - Black Males 58.5%
  - Black Females 43%

# Use of Social Contacts Across Countries:



- Pellizzari (2004):
  - no systematic differences in use of contacts across European countries and US
  - more prevalent in private sector
  - more prevalent with lower education and skill
  - more prevalent in smaller, medium sized firms
  - apparent differences in wage premium/penalty to use of social contact across countries, but with much variation within EU (US at 0)



# Features of Labor Markets We Can Help Explain with Network Models



- Correlation in employment across geography, acquaintances, profession (after correction for economy)
- Serial correlation patterns in individual employment – Duration Dependence
- Gap in wages between white and black males (after correction for skills, age, education...)
- Higher dropout rate for black versus white males

Let's look at these in some detail:

# Inequality in Wages and Drop-Out Rates



- Card and Krueger (1992), Chandra (2000)...
  - Black-White wage gap is about 25 to 40%; only partly explained by differences in skill levels, quality of education
- Heckman, Lyons, and Todd (2000), Card and Krueger (1992), Chandra (2000)...
  - Dropout rates 2.5 to 3 time higher for blacks versus whites



# Inequality in wages

- Smith and Welch (1989) U.S. Census Data  
Black to white wage ratios by education level

Education:	1940	1950	1960	1970	1980
0-7 years	54.5%	63.6%	67.0%	73.9%	82.5%
8-11 years	59.7%	70.9%	70.1%	78.8%	74.8%
12+ years	56.5%	66.5%	66.2%	72.2%	78.8%



# Drop-Out Rates

- Chandra (2000) Census – males 25 to 55

	1940	1950	1960	1970	1980	1990
whites	3.3	4.2	3.0	3.5	4.8	4.9
blacks	4.2	7.5	6.9	8.9	12.7	12.7



# Duration Dependence

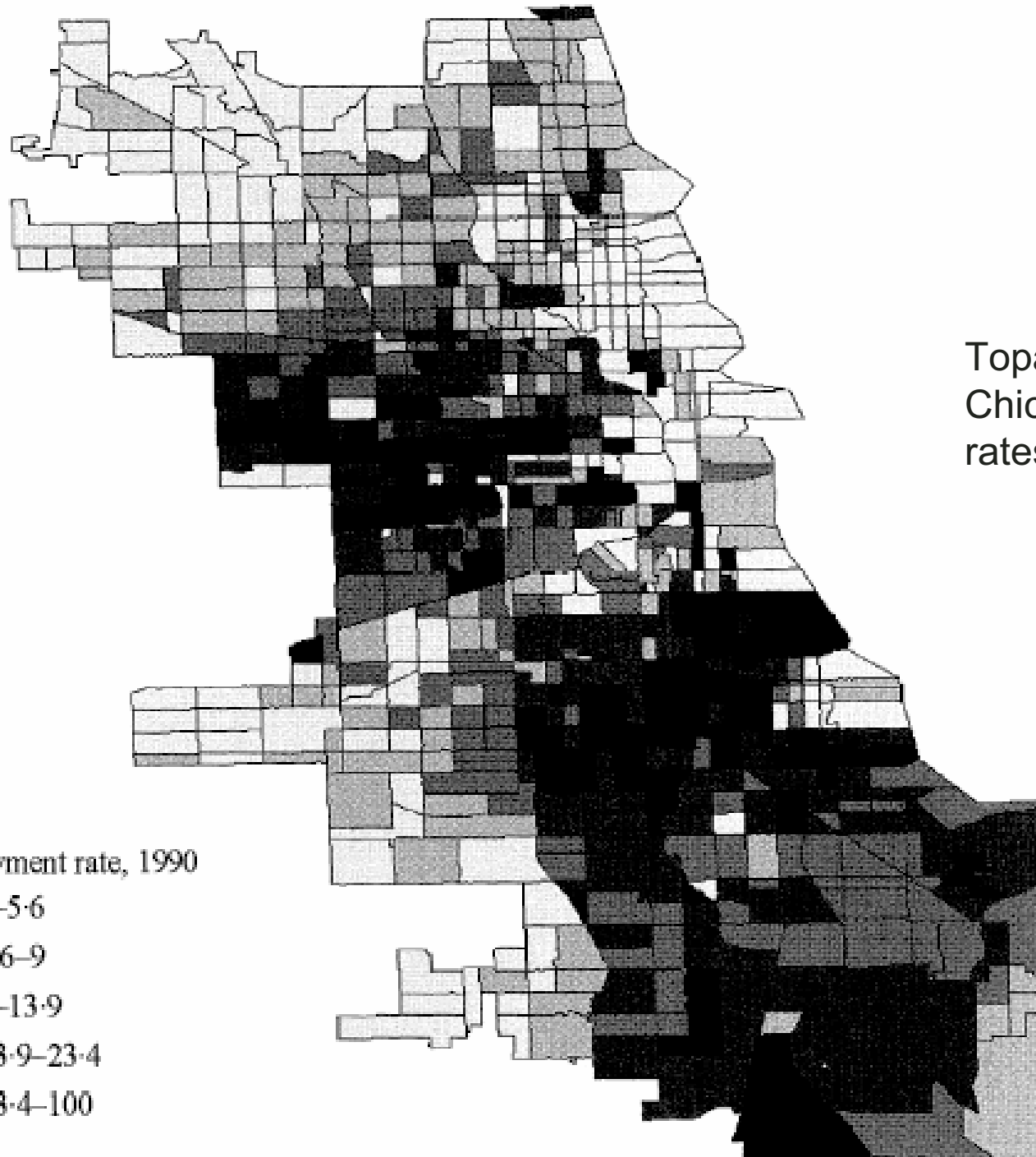
- Lynch (1989) Probability finding a job in the next week, given number of weeks unemployed:

Weeks unemployed:	1	8	52
Probability Finding Job:	.30	.08	.02



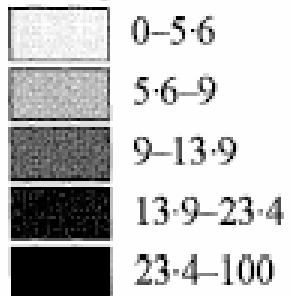
# Correlation Patterns:

- Topa (2001)
  - Geographic Correlation in Unemployment (correcting for economic conditions) - proximity to other employed agents helps explain employment
  - Strong Social Interaction within areas
- Conley and Topa (2001)
  - Correlation in proximity of travel time, occupation, ethnicity – with ethnicity and race dominant factors



Topa (2001)  
Chicago Unemployment  
rates 1990

Unemployment rate, 1990



# Models and Hypotheses regarding Networks in Labor Markets



- Granovetter (1973, 1995), Boorman (1975), Montgomery (1991, 1992, 1994), ...
  - Weak versus Strong ties – who to connect to
- Arrow and Borzekowski (2001)
  - Difference in number of offers affects wages
- Calvo-Armengol and Jackson (2004ab)
  - job offers through network, model employment, wages and dropout decisions



# Arrow-Borzekowski: Second Price Auction



- Number of opportunities is proportional to degree
- value of match for each opportunity is iid Uniform on  $[0, v]$
- expected wage if degree is  $k$  is  $(k-1)v/(k+1)$ 
  - increasing in degree
  - concave in degree
- value of match increasing and concave in degree:  
 $kv/(k+1)$

# Relating Network Structure to Outcomes



(Jackson-Rogers 2005) Degree Distributions of  
Regular networks SOSD E-R networks SOSD scale-  
free networks of same average degree.

Therefore: if wage and match value are concave in  
degree (as in the Arrow and Borzekowski model),  
then average wages and social value are ranked as  
follows (holding average degree fixed):

Regular > E-R > Scale-Free

# Aside: second order stochastic domination



$P$  and  $P'$  are distribution functions

**Theorem** (Rothschild and Stiglitz (1970)):

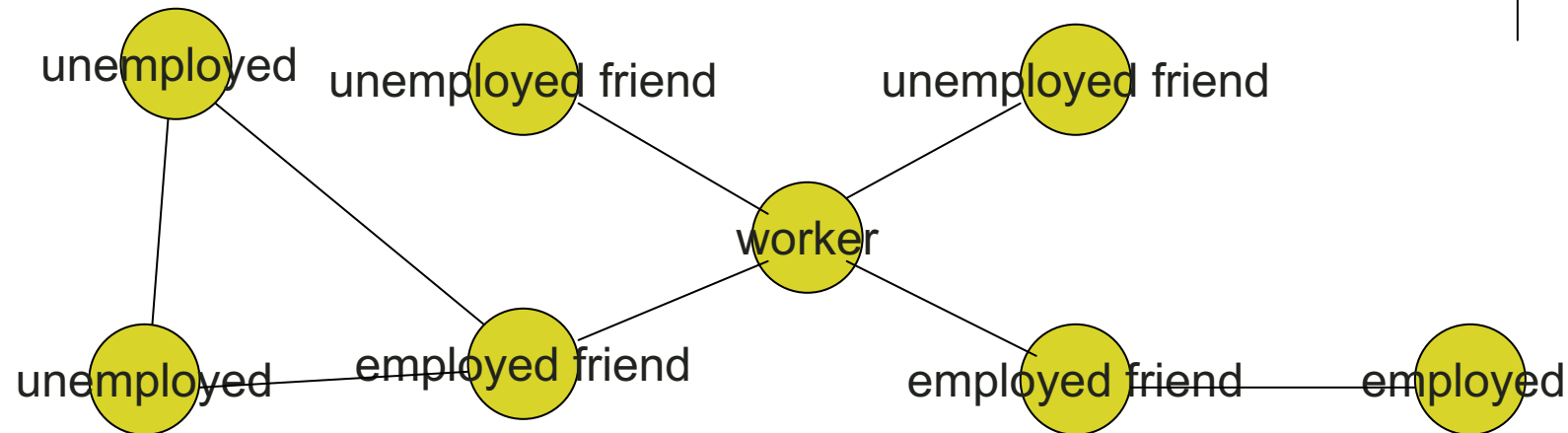
The following are equivalent

- $P$  SOSD  $P'$
- $\int f dP > \int f dP'$  for all increasing, concave  $f$
- $\int_0^x [P'(k) - P(k)] dk > 0$  for all  $x$

If  $P$  and  $P'$  have the same mean then the above are also equivalent to

- $P'$  is a mean-preserving spread of  $P$
- $\int f dP > \int f dP'$  for all concave  $f$

# Model of Labor Networks (Calvo-Jackson (2004ab))



- network describes who knows whom, intensities
- workers hear about jobs (random arrival process)
- decide whether to keep information or pass it on to a friend/relative/acquaintance - might pass repeatedly and/or to many agents
- workers lose jobs (random breakup process)
- friends of a friend help and hurt



# Model Details

- $N = \{1, \dots, n\}$  agents
- $t$  in  $\{1, 2, \dots\}$  periods
  - Two parts:
    - Hiring phase: hear about jobs and pass information
    - Breakup phase: randomly lose jobs
- Variables to keep track of
  - $S_{it}$  the end of period employment of worker  $i$
  - $W_{it}$  the end of period wage of worker  $i$

# A Simple Example – Homogeneous Jobs



- Network ( $n$  by  $n$   $\{0, 1\}$  matrix  $g$ ) describes who knows whom
- All jobs are identical and pay same wage
- Jobs arrive to an agent with probability  $a$
- Lose jobs with probability  $b$
- If employed, then randomly pick an unemployed friend to pass the job to



# Get a Markov Process

- State is list of employment/wage status of each agent
- More likely to hear about jobs when friends are (well) employed
- More likely to pass on information when have a (good) job



# Transition Probabilities

- $p_{ki}(w)$  - probability that  $k$  passes a job to  $i$
- $p_i(w)$  – expected number of jobs  $i$  hears about given current state  $w$ 
  - $p_i(w)$  is nondecreasing in  $w_{-i}$  and nonincreasing in  $w_i$
  - $p_i(w)$  is positive if  $i$  is not at highest wage level
- Wage is nondecreasing in number of offers and starting wage (e.g., from Arrow-B)
- Lose job with probability  $b_i$



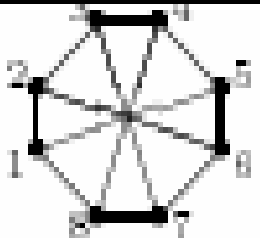
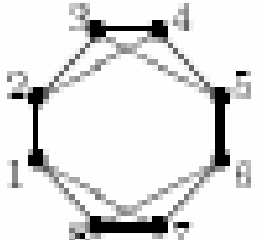
# Homogeneous Example with $a=.100$ and $b=.015$



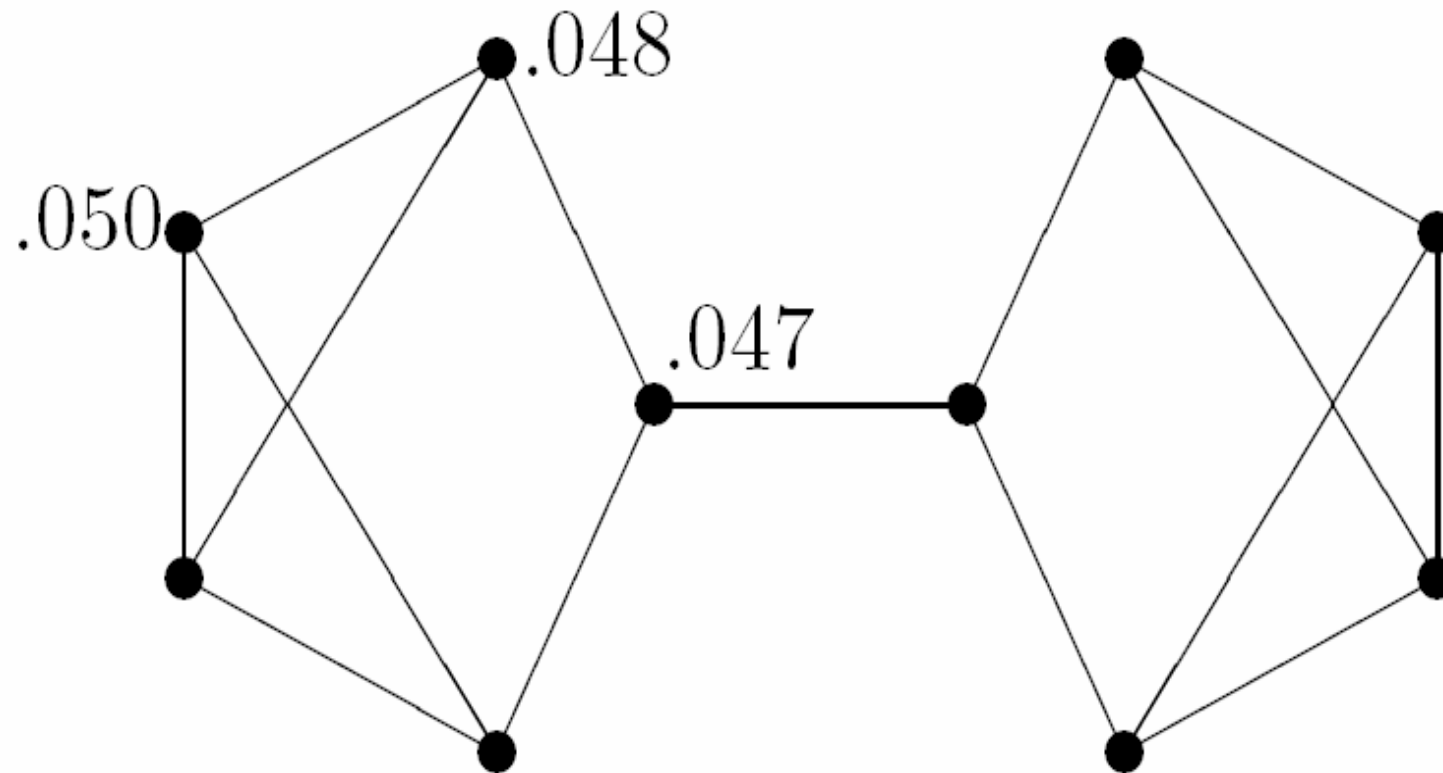
	Prob Unemployed	Corr 1 and 2	Corr 1 and 3	Corr 1 and 4
	.132	—	—	—
	.083	.041	—	—
	.063	.025	.019	—
	.050	.025	.025	—
	.060	.023	.003	.001
	.030	.014	.014	.014

# Structure Matters



$g$	average path length	average unemployment
	1.571	.048
	1.786	.049

# Position Matters





## Result on wage correlation:

Consider an economy  $(N,p,b)$ . The wages and employment of any path-connected agents are positively correlated (in fact, strongly-associated) under the unique steady state distribution. The same is true across any times starting from the steady state distribution.

- Proof challenge: Do friends of a friend help or hurt?
  - Compete with me for information about jobs
  - But help keep my friends well-employed

# Duration Dependence: Prob of Getting Job Conditional on Length of Unemployment



Network	1 period	2 periods	10 periods	limit
	.099	.099	.099	.099
	.176	.175	.170	.099
	.305	.300	.278	.099

# Result on duration dependence



Consider an economy  $(N, p, b)$  such that  $p$  is a function of employment. Agents' employment exhibits duration dependence; that is,

$$\begin{aligned} & \text{Prob}(S_{i,t}=1 \mid S_{i_0}=0, \dots, S_{i_{t-1}}=0) \\ & < \text{Prob}(S_{i,t}=1 \mid S_{i_{t'}}=0, \dots, S_{i_{t-1}}=0) \end{aligned}$$

# Remarks on the Patterns and Dynamics



- Correlation gives the observed clustering and correlation patterns
- Duration dependence comes out due to the inferred state of the network
- There will be natural cycles in this setting:
  - Cyclic unemployment even if arrival/breakup are stationary
  - More employed leads to higher probability stick there, and similarly for less employed

# Relate Employment to Network Structure:



(Jackson and Lopez-Pintado):

In the homogeneous job setting, if  $P(k) \text{ SOSD } P'(k)$  then average employment is higher under  $P$  than under  $P'$ , as is the employment rate for nodes of any fixed degree.

- Recall, keeping average degree fixed:  
Regular network SOSD Exp SOSD Scale-free.

[no critical  $a/b$ , unless only employed hear, then need  $a/b > 1$  regardless of degree distribution]





# Dropping Out

- Cost to getting education, maintaining skills, looking for employment, etc.
- Dropout if outlook for employment is poor
- Outlook is poor if bad starting conditions
- Outlook is poor if network connections are not diverse
- Contagion: Dropout if many acquaintances dropout



# Model as a game

- Agents decide whether or not to drop out
- Compare discounted value of expected future wages to cost of staying in.
- This game is supermodular – if more agents stay in, then  $i$ 's payoff from staying in is higher
- There exists a “maximal” equilibrium – we focus on that, (but could equally well compare minimal equilibria)

# Player's decision:



Number of Contacts In

	0	1	2	... n
Stay In	$E[w 0,s]-c_i$	$E[w 1,s]-c_i$	$E[w 2,s]-c_i$	$E[w n,s]-c_i$
Drop Out	0	0	0	0

# Dropout Example

$c_i \sim U[.8, 1]$ , discount .9,  
 $w=1$ ,  $a=.1$ ,  $b=.015$



All Start Employed	$n = 1$	$n = 2$	$n = 4$	$n = 16$	$n \rightarrow \infty$
Dropout Percentage	58.3	44.5	26.2	9.7	6.8
Percentage Due to Contagion	0	8.8	5.0	0.4	0

All Start Unemployed	$n = 1$	$n = 2$	$n = 4$	$n = 16$	$n \rightarrow \infty$
Dropout Percentage	100	97.8	92.9	68.0	56.8
Percentage Due to Contagion	0	12.1	21.7	8.7	0



## Result on dropouts:

Consider an economy  $(N, p, b, c, d)$ . Consider two different starting states  $w$  and  $w'$  with  $w' \geq w$ .

- (i) The maximal equilibrium dropouts following  $w'$  are a subset of those under  $w$ , with strict inclusion for some costs and discount rates.
- (ii) For agents who stay in, expected wages starting from  $w'$  strictly first order stochastically dominate those starting from  $w$ , at all dates.

# Application to Social Mobility



- Correlation in parent-child  $\log(\text{earnings})$   
[Solon (1992), Zimmerman:
  - in  $[\cdot 4, \cdot 6]$  US and UK
  - in  $[\cdot 2, \cdot 4]$  in Germany, Sweden
- Higher numbers if look at income
- Numbers for father-son, higher for daughters

# Application to Social Mobility



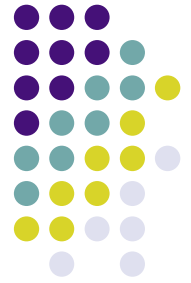
- Parent child log earnings correlations:
  - [.4,.6] in the US [Solon (1992), Zimmerman (1992),Mulligan (1997)...]
  - [.1,.4] Canada, Finland, Germany, Sweden [Corak and Heisz (1999) ; Osterbacka (2001); Couch and Dunn (1997), Wiegrand (1997); Bjorklund and Jantti (1997), Gustafsson (1994), Osterberg (2000)...]
  - [.4,.6] U.K. [Atkinson, Maynard, and Trinder (1983), Dearden, Machin, Reed (1997)]
  - .45 South Africa [Hertz (2001)]
  - ....

# Income and Class Mobility



- Income correlations are higher: .7 and above
  - Higher when include daughters [Mulligan (1997)]
  - Non-linear in income levels - higher in the tails [Cooper, Durlauf, Johnson (1994)]
- Odds Ratios [ $\text{prob}(\text{same class})/\text{prob}(\text{change class})$ ] are very high: ranging from 1 to 15 depending on class, etc.
  - Hout (1983), Erikson and Goldthorpe (1992), Breen (1994), Ishida (1995)...



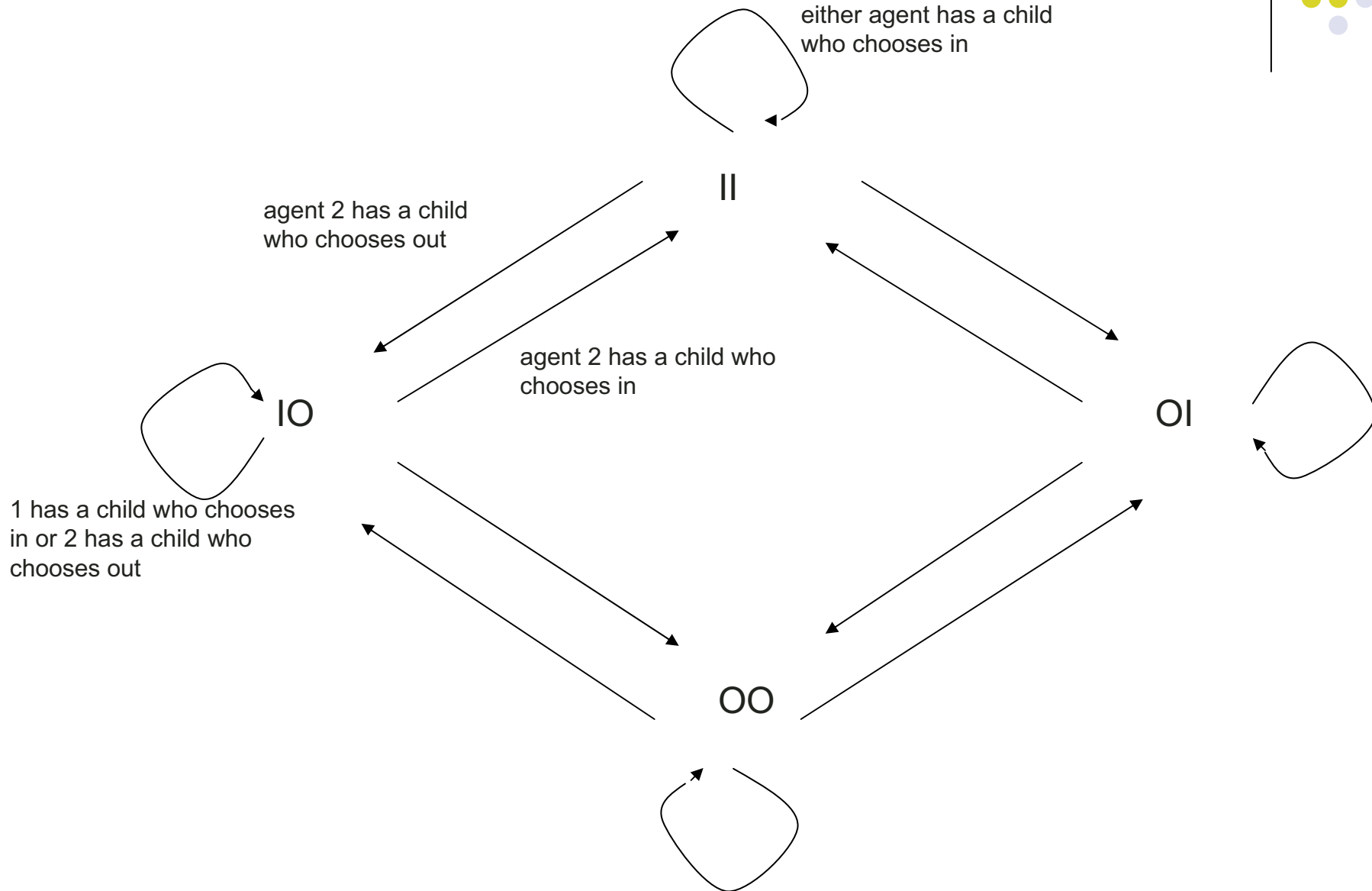


# Drop Outs over time

- Drop out decisions over time
- Parent replaced by child
- Randomly pick node to replace
- Correlation comes from similarity of network

# Correlations

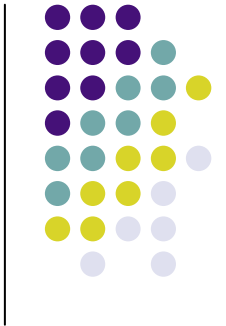
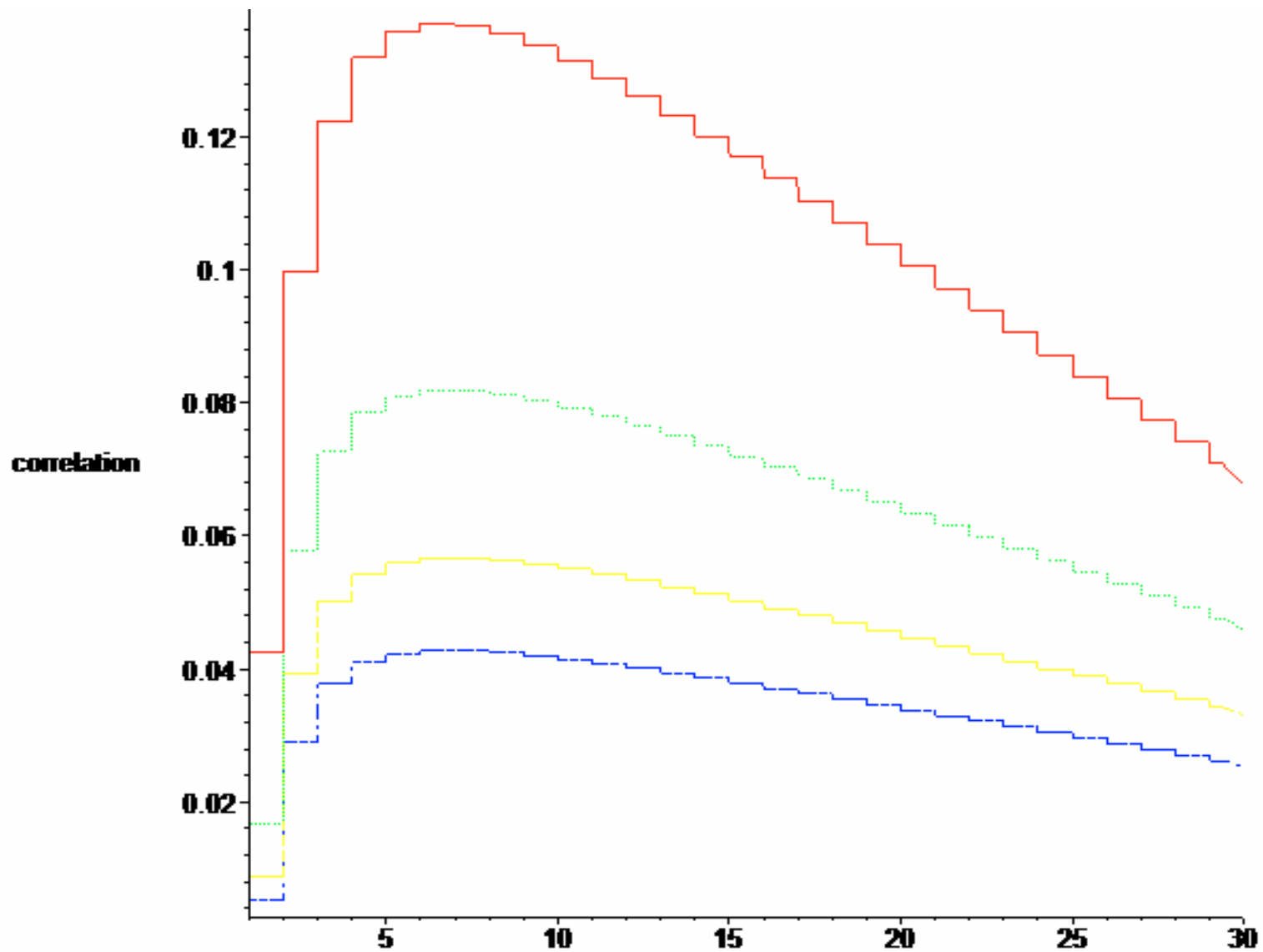
## A Dyad Example:





## Dyad probabilities:

- II occurs with prob  $p_0 p_1 / (1 + p_0 - p_1)$ 
  - increasing in both  $p_0$  and  $p_1$ ,
- IO,OI occurs with prob  $p_0 (1 - p_1) / (1 + p_0 - p_1)$ 
  - increasing in  $p_0$  and decreasing in  $p_1$ ,
- OO occurs with prob  $(1 - p_0)(1 - p_1) / (1 + p_0 - p_1)$ 
  - decreasing in both  $p_0$  and  $p_1$ ,



arrival rate a as a factor of the breakdown rate b

- n=4
- ⋯ n=8
- - n=12
- · - n=16

# Conclusions



- Network effects help understand labor market patterns:
  - clustering and correlation
  - duration dependence
  - drop-out rates
  - persistent inequality in wages and employment
  - social mobility patterns
- Network structure matters: more regular networks have higher average employment, higher average utility
- Policy: Local Increasing Returns to Subsidies - subsidize group of neighbors all at once



# Things to Think About

- Richer endogeneity of network
- Equilibrium model of wages and job arrival process
- Why are social contacts/networks used?



## Role of Ethnicity/Race:

- Ethnic group status (as measured by average of father's cohort) plays a large role in economic status of child
  - Borjas (1992): correlation in status of father-child on order of .2+, and father's cohort-child on order of .2 to .5
- Wealth, race, education are more important than IQ in explaining heritability of earnings; parental status still important when education is well-measured
  - Bowles and Gintis (2002)



# Wage Differentials:

- Holding education constant, class correlates with wages [Goux and Maurin (1997)...]
- College enrollment differential by race narrows in US [Kan (1994)], but black-white wage differential does not [Card and Krueger (1992), Chandra (1992) - on order of 25 to 40%...]

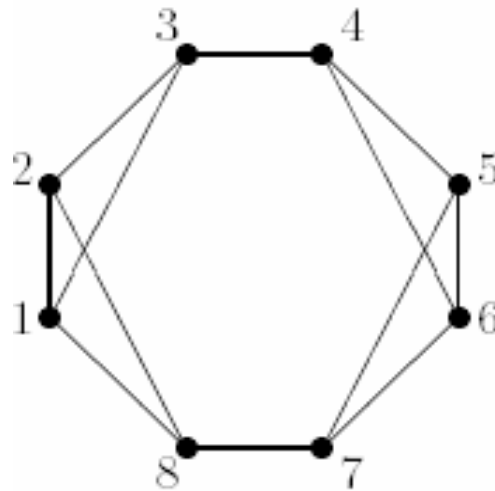


# Previous Models of Social Mobility



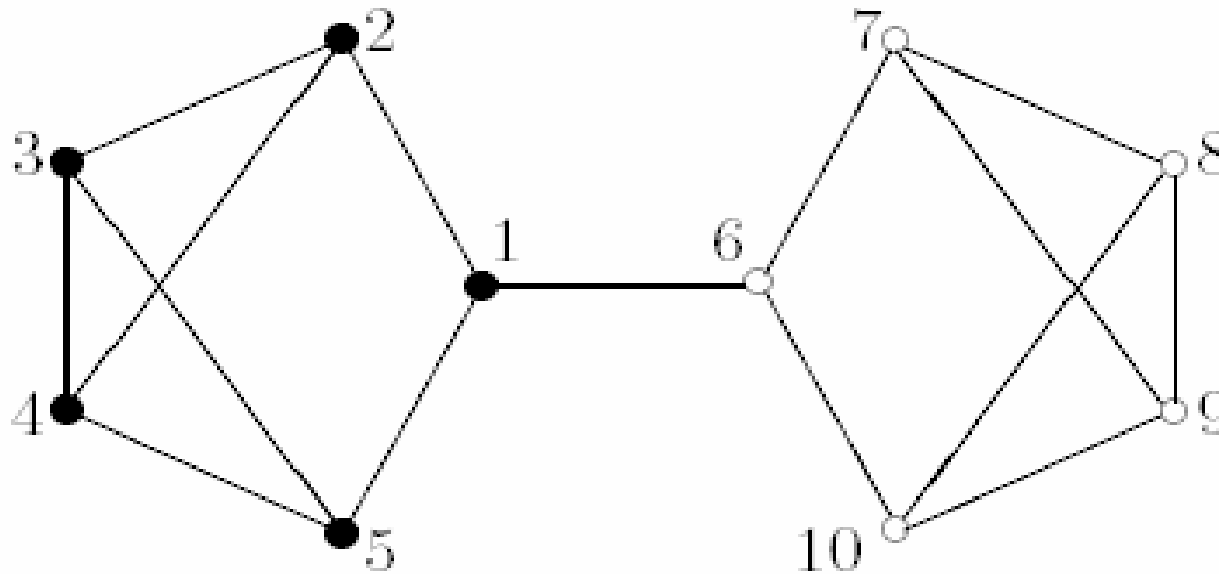
- Becker and Tomes (1979),
  - wealth transfers by altruistic parents
  - inheritance of abilities correlates human capital and earnings
- Loury (1981),
  - Imperfect credit markets - child's education decisions affected by parental wealth
- Benabou (1993, 1996), Durlauf (1996),
  - Education is a local public good – investment is lower in lower income areas – mobility tied to residential choice

# Drop-out rates as a function of who is subsidized



Agents Subsidized	Cost Range		
	.80 to 1	.84 to 1	.88 to 1
1 and 2	47.1	72.2	75.0
1 and 5	<b>42.1</b>	73.0	75.0
3 and 8	43.8	<b>70.9</b>	75.0

# Dropouts in a connected Network



Agent	1	2	3	4	5	6	7	8	9	10
Drop-out Rate	.47	.42	.42	.42	.42	.91	.93	.93	.93	.93

# Increasing Returns to Subsidies



- four individuals
- start in complete network
- cost actually decreasing if subsidize more - so this underestimates

Number subsidized	Percentage Staying In
0	7.1
1	26.6
2	52.3

$$\underbrace{\frac{1}{2} [SI(2) + SI(0)]}_{29.7} > \underbrace{SI(1)}_{26.6} .$$

# Dropout Rates - Comparative Statics



Costs [.8, 1]		b					
		.015	.045	.075	.105	.135	.165
a	.05	69:27	100:0	100:0	100:0	100:0	100:0
	.10	27:5	99:27	100:0	100:0	100:0	100:0
	.15	17:2	76:27	99:12	100:0	100:0	100:0
	.20	13:1	52:13	97:28	100:0	100:0	100:0
	.25	11:1	42:10	83:26	100:16	100:0	100:0
	.30	10:1	37:9	68:18	98:24	100:2	100:0
	.35	9:1	29:3	61:15	88:24	100:12	100:0
	.40	9:1	27:2	53:11	84:23	96:20	100:0
	.45	7:0	25:2	46:7	76:18	91:20	100:6

# Basic Elements of Calvo-Jackson Model



- Jobs obtained either directly hearing or from social contact hearing and passing information
- More and better placed social contacts lead to better information and prospects for jobs
- Friends of Friends help and hurt:
  - Help keep friends employed
  - But compete with me for job information

# Results from a Network Model:



- Positive correlation across networked individuals, decaying with distance
- Future wage prospects depend on status of network – can get persistent gaps in wages based on starting conditions
- Drop-out decisions affected by network status – have contagion among dropouts
- Duration of employment depends on network status – get duration dependence



## Other examples

- Agents are heterogeneous
  - they hear about jobs at different rates
  - they have different skills and qualifications
- Jobs are heterogeneous
  - keep good ones that improve current situation
- Pass information selectively
- Pass information to several agents
- Information is relayed through the network
- ...





## Theorem 2

Consider an economy  $(N, p, b)$ . The wages of any path connected agents are positively correlated across any times starting from the steady state distribution.

Again, a similar result holds for employment.

# Drop-outs at various turnover rates:



Scaled by factor of	1	3	5	7	9
a= b=	.05 .015	.15 .045	.25 .075	.35 .105	.45 .135
Dropout and contagion	69 27	76 27	83 26	88 24	96 20