The Abdus Salam
International Centre for Theoretical Physics

United Nations
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School and Workshop on Structure and Function of Complex Networks

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Network Formation

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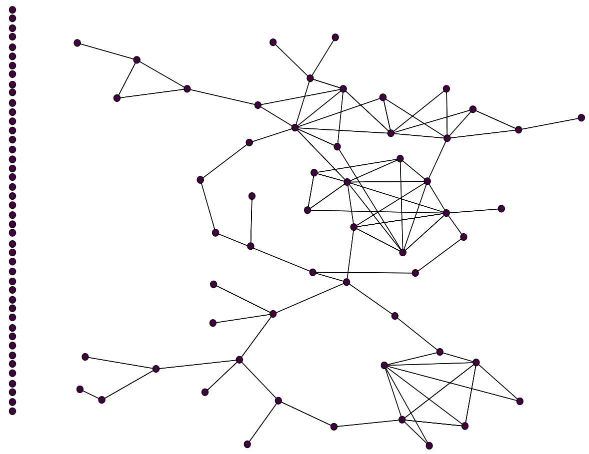
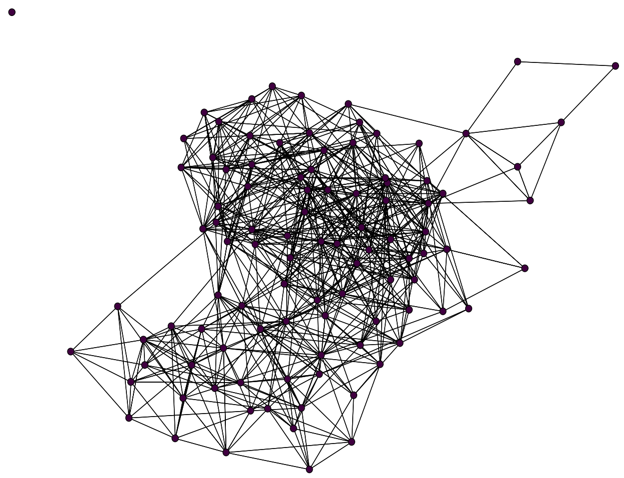


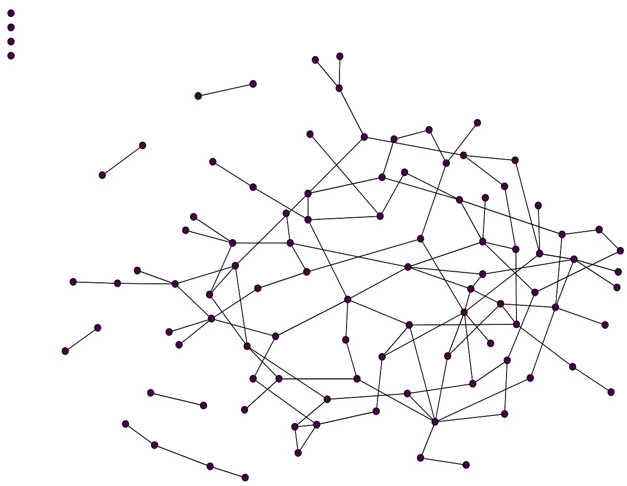
$\alpha = 0.1$, $\theta = 0.4$

Giant component	54
Average degree	1.7
Clustering coefficient	0.36
Rescaled clustering	21.1
Degree centralization	0.065



$\alpha = 0.1$, $\theta = 0.8$

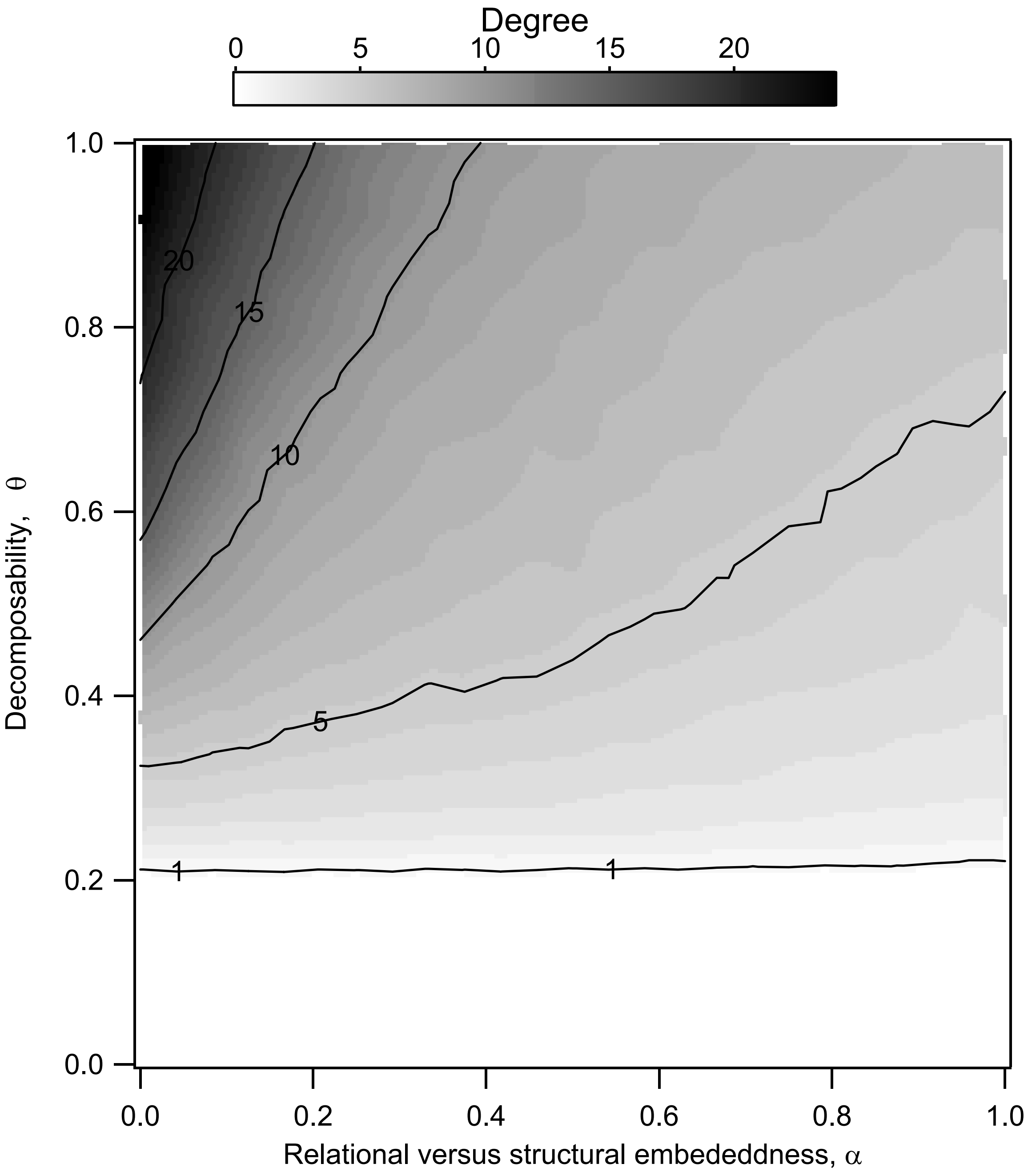
Giant component	99
Average degree	11.9
Clustering coefficient	0.34
Rescaled clustering	2.86
Degree centralization	0.105



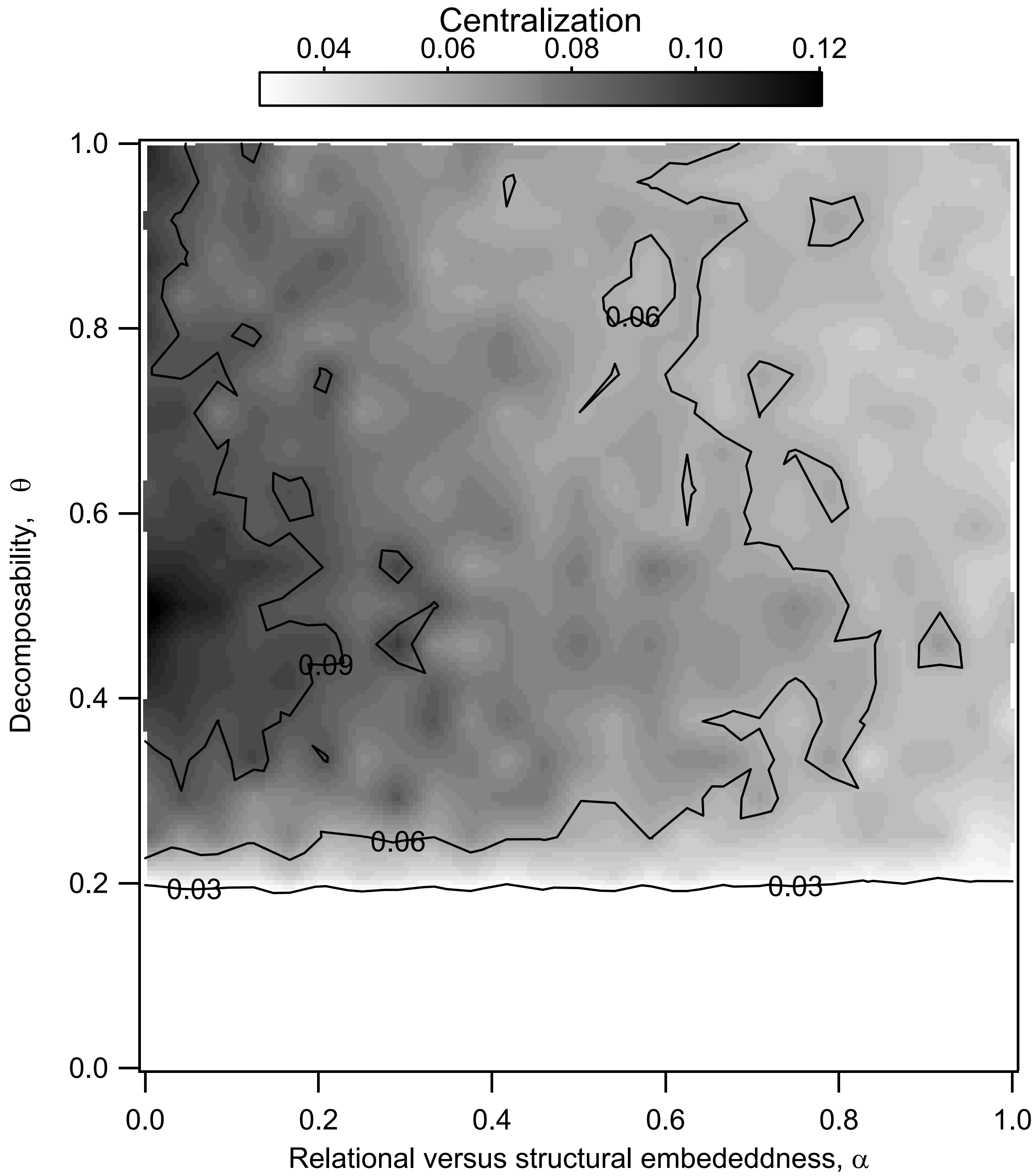
$\alpha = 0.8$, $\theta = 0.6$

Giant component	84
Average degree	2.4
Clustering coefficient	0.04
Rescaled clustering	1.67
Degree centralization	0.047

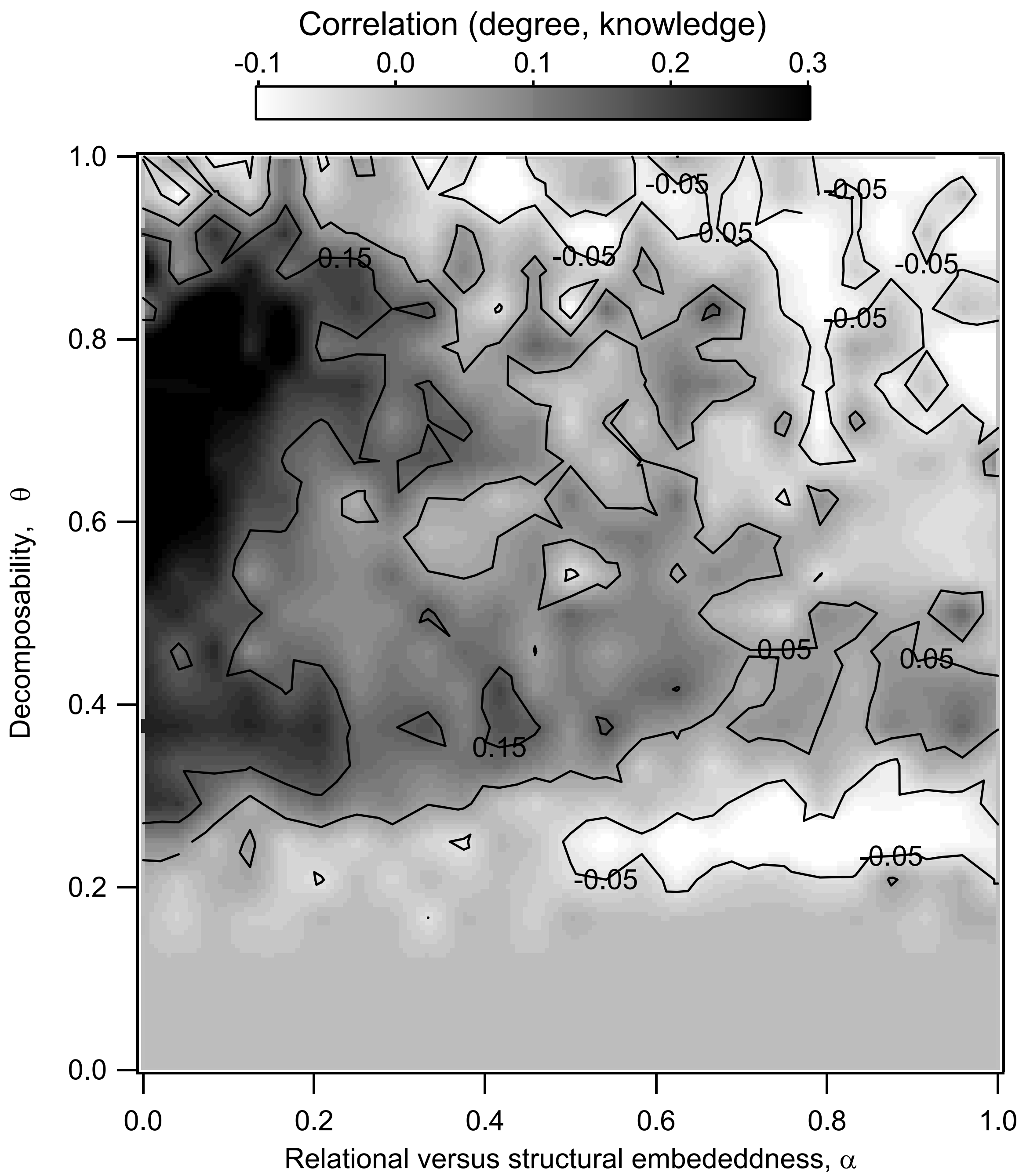
Table 1: Three characteristic networks from different parts of the parameter space.



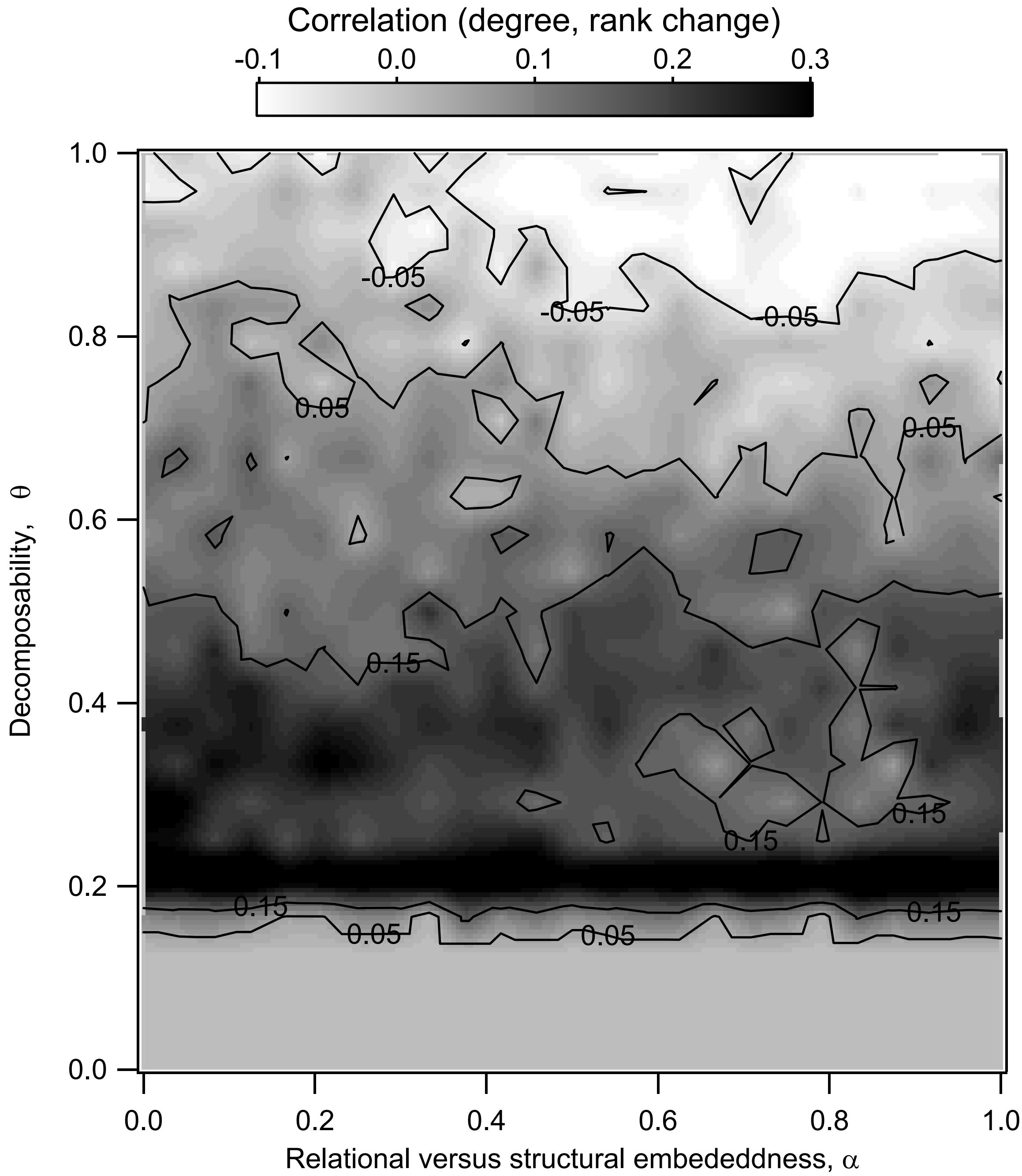
Degree
0
5
10
15
20
1.0
0.8
0.6
0.4
0.2
0.0
Decomposability, θ
20
15
10
5
1
1
0.0
0.2
0.4
0.6
0.8
1.0
Relational versus structural embededdness, α



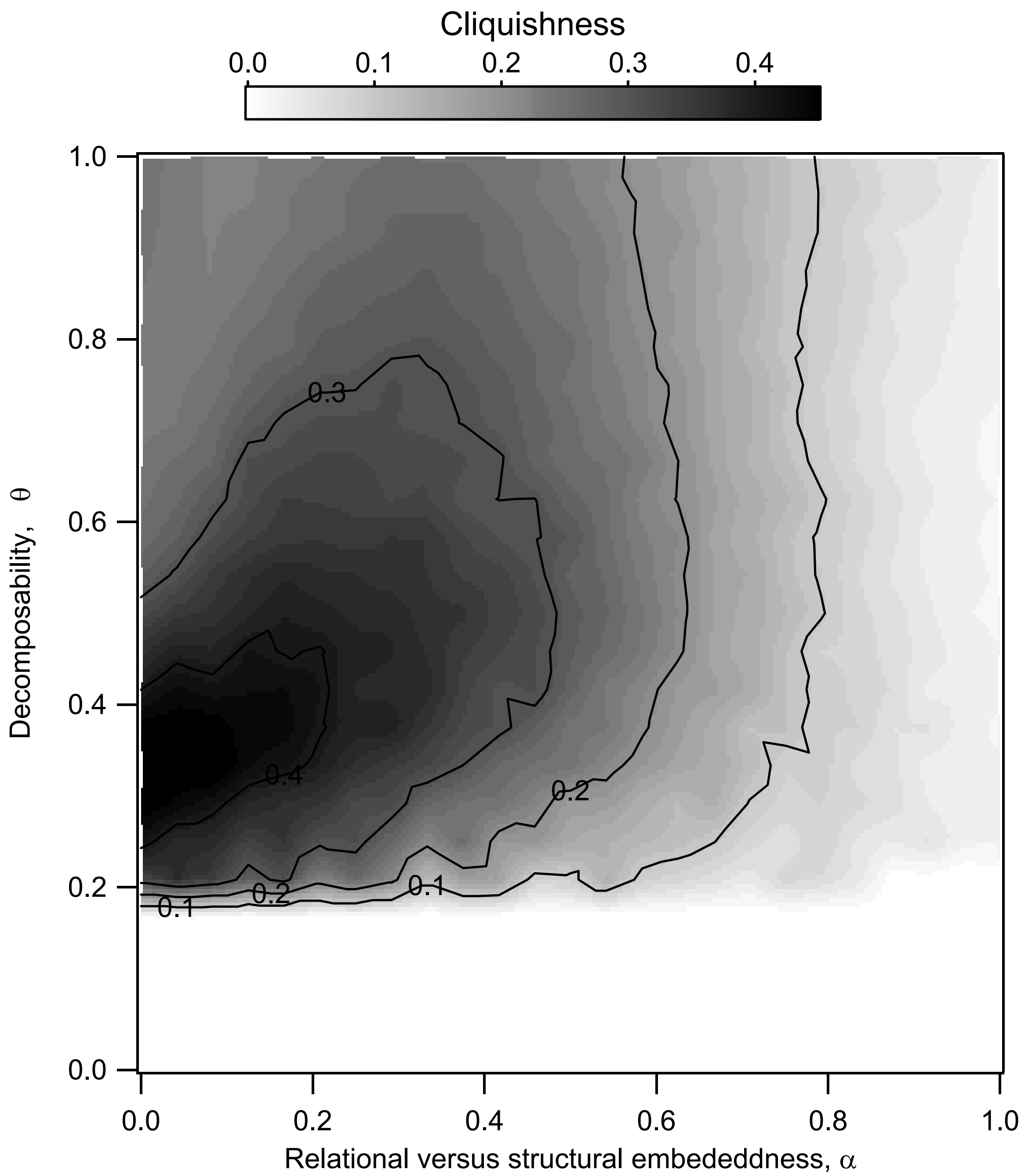
Centralization
0.04
0.06
0.08
0.10
0.12
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
Decomposability, θ
Relational versus structural embededdness, α
0.06
0.09
0.06
0.03
0.03



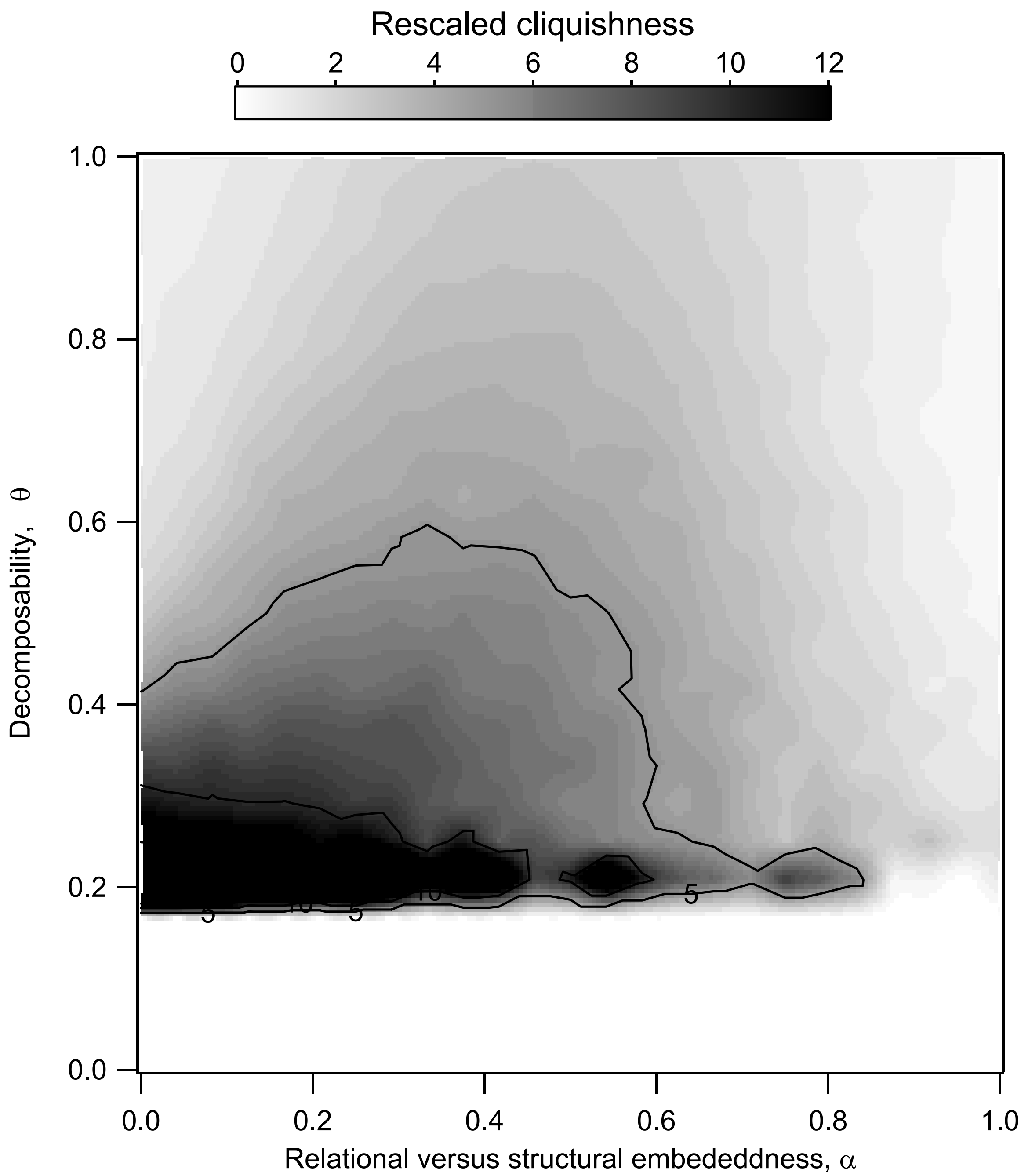
Correlation (degree, knowledge)
-0.1
0.0
0.1
0.2
0.3
1.0
0.8
0.6
0.4
0.2
0.0
Decomposability, θ
-0.05
-0.05
-0.05
-0.05
-0.05
-0.05
0.15
0.05
0.05
0.15
-0.05
-0.05
0.0
0.2
0.4
0.6
0.8
1.0
Relational versus structural embededdness, α



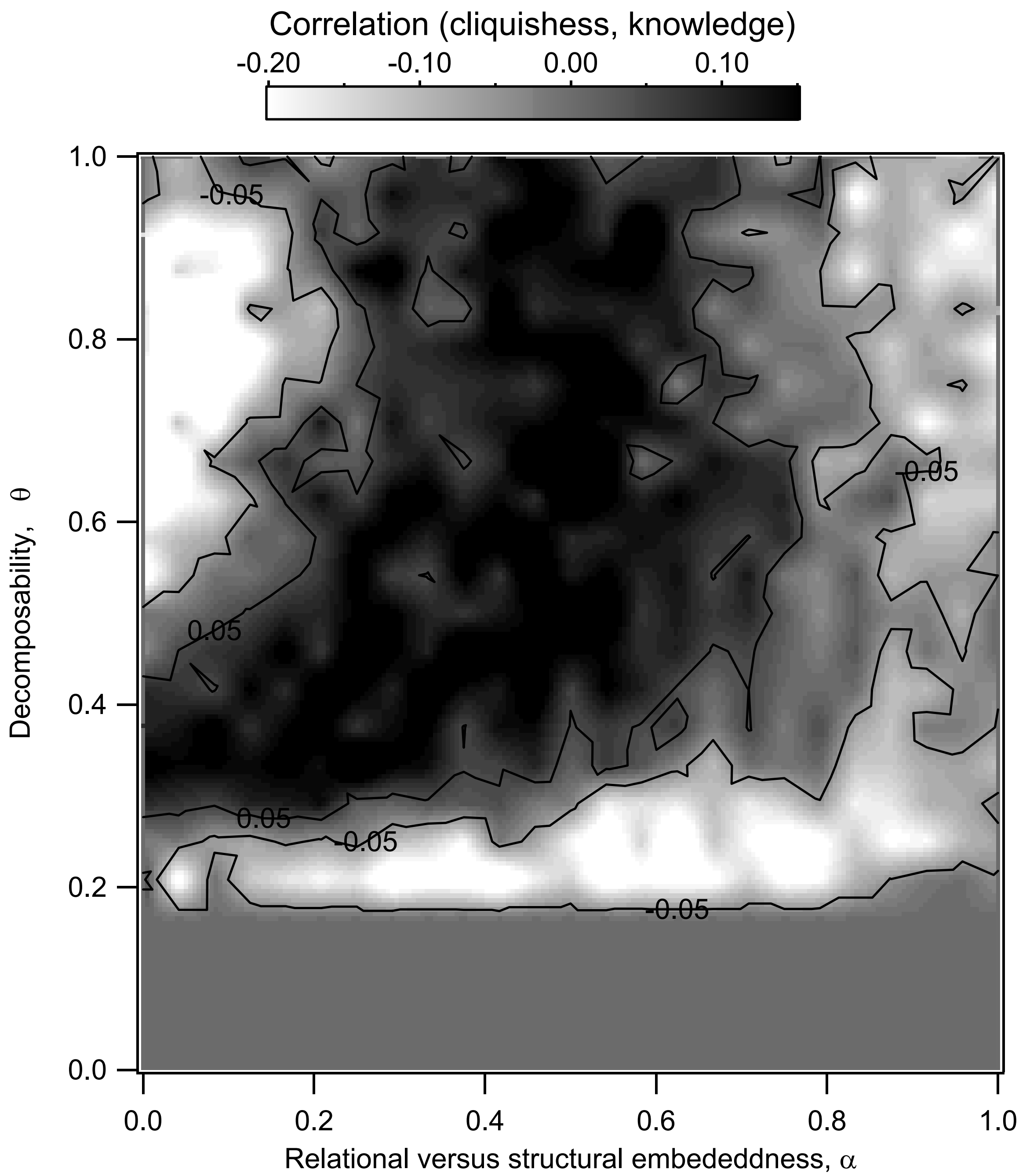
Correlation (degree, rank change)
-0.1
0.0
0.1
0.2
0.3
-0.05
-0.05
-0.05
0.05
0.05
0.15
0.15
0.15
0.15
0.15
0.05
0.05
1.0
0.8
0.6
0.4
0.2
0.0
Decomposability, θ
0.0
0.2
0.4
0.6
0.8
1.0
Relational versus structural embededdness, α

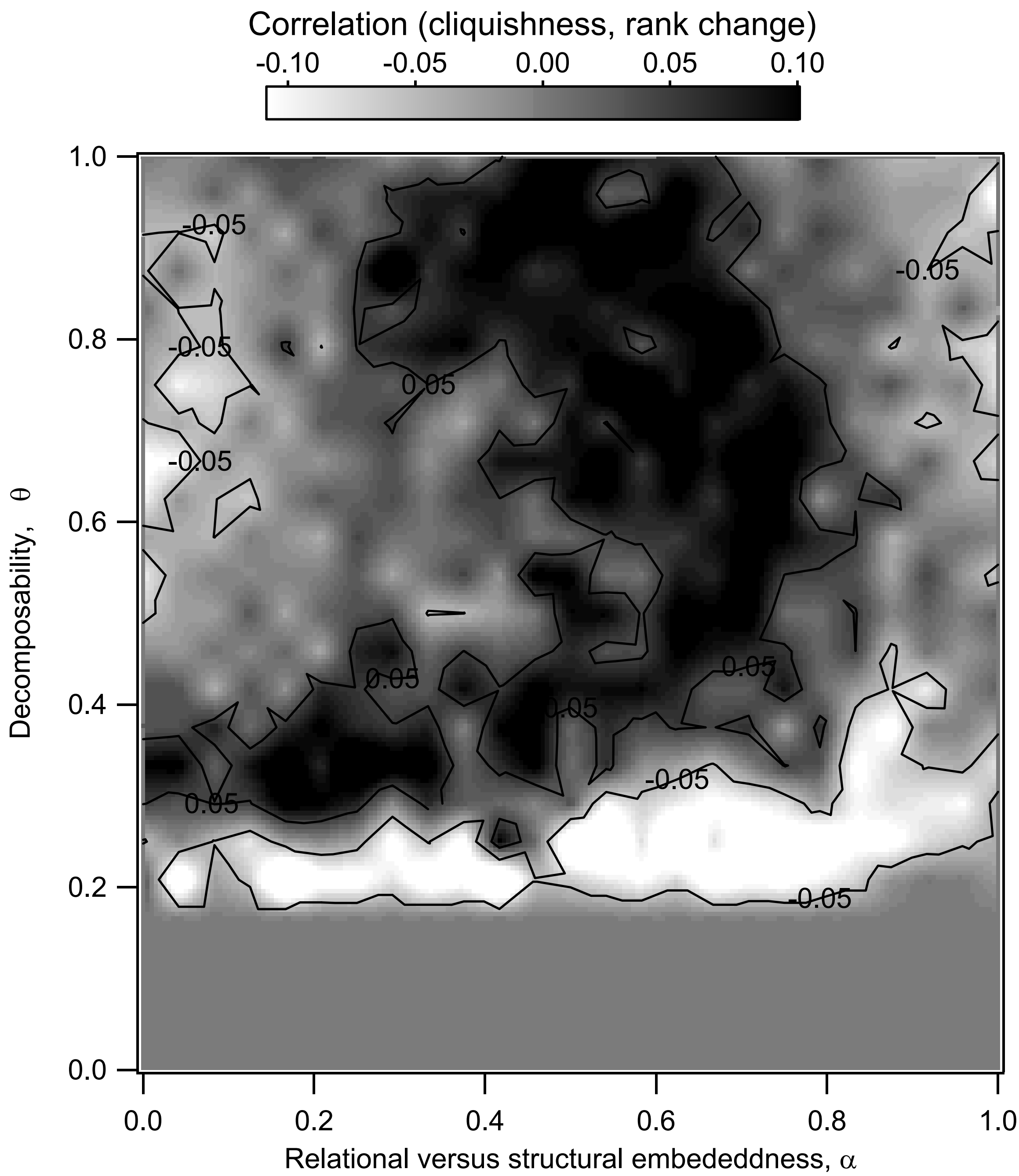


Cliquishness
0.0
0.1
0.2
0.3
0.4
1.0
0.8
0.6
0.4
0.2
0.0
Decomposability, θ
0.3
0.4
0.2
0.1
0.2
0.1
0.0
0.2
0.4
0.6
0.8
1.0
Relational versus structural embededdness, α

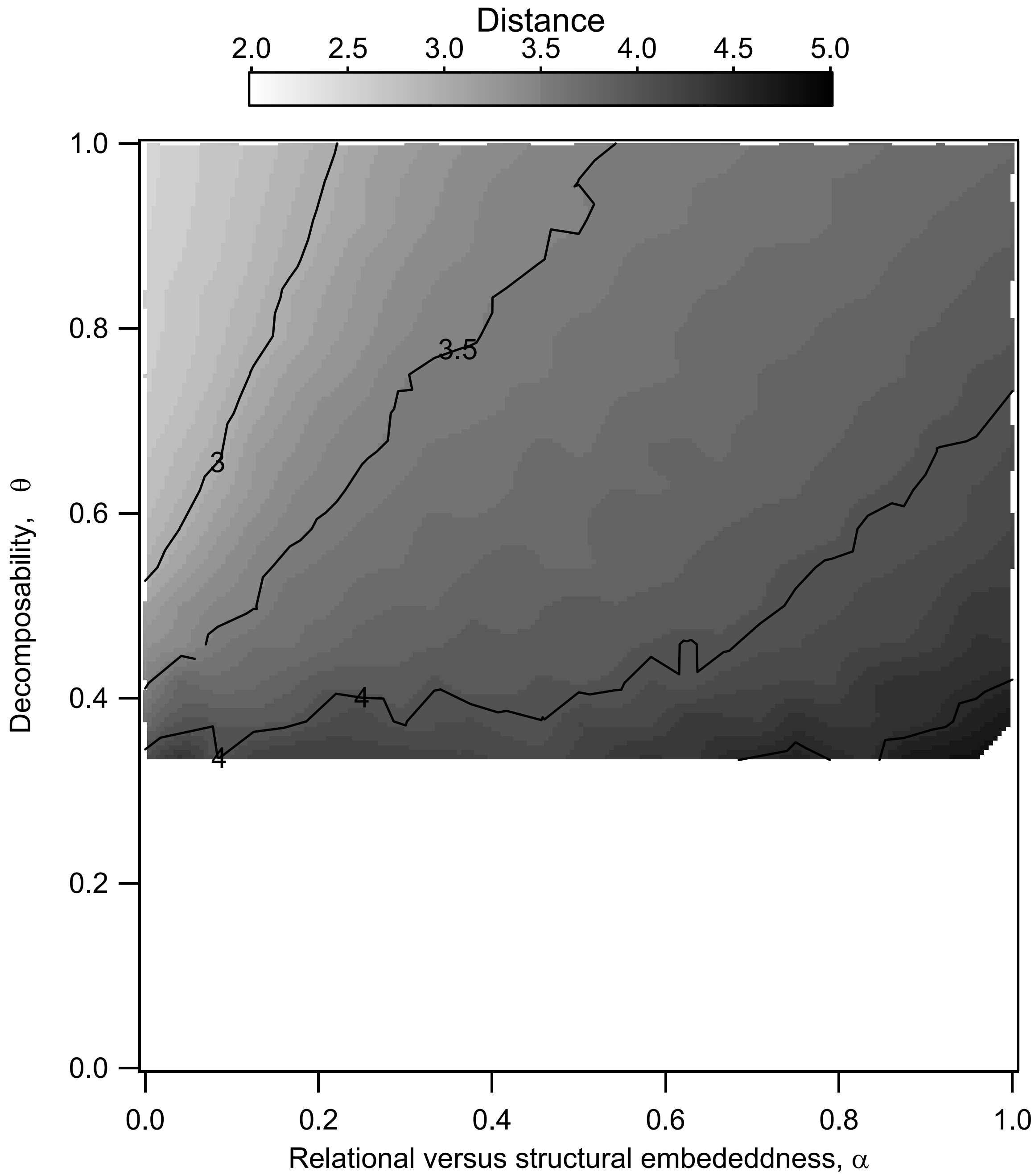


Rescaled cliquishness
0
2
4
6
8
10
12
Decomposability, θ
1.0
0.8
0.6
0.4
0.2
0.0
5
10
5
10
5
0.0
0.2
0.4
0.6
0.8
1.0
Relational versus structural embededdness, α

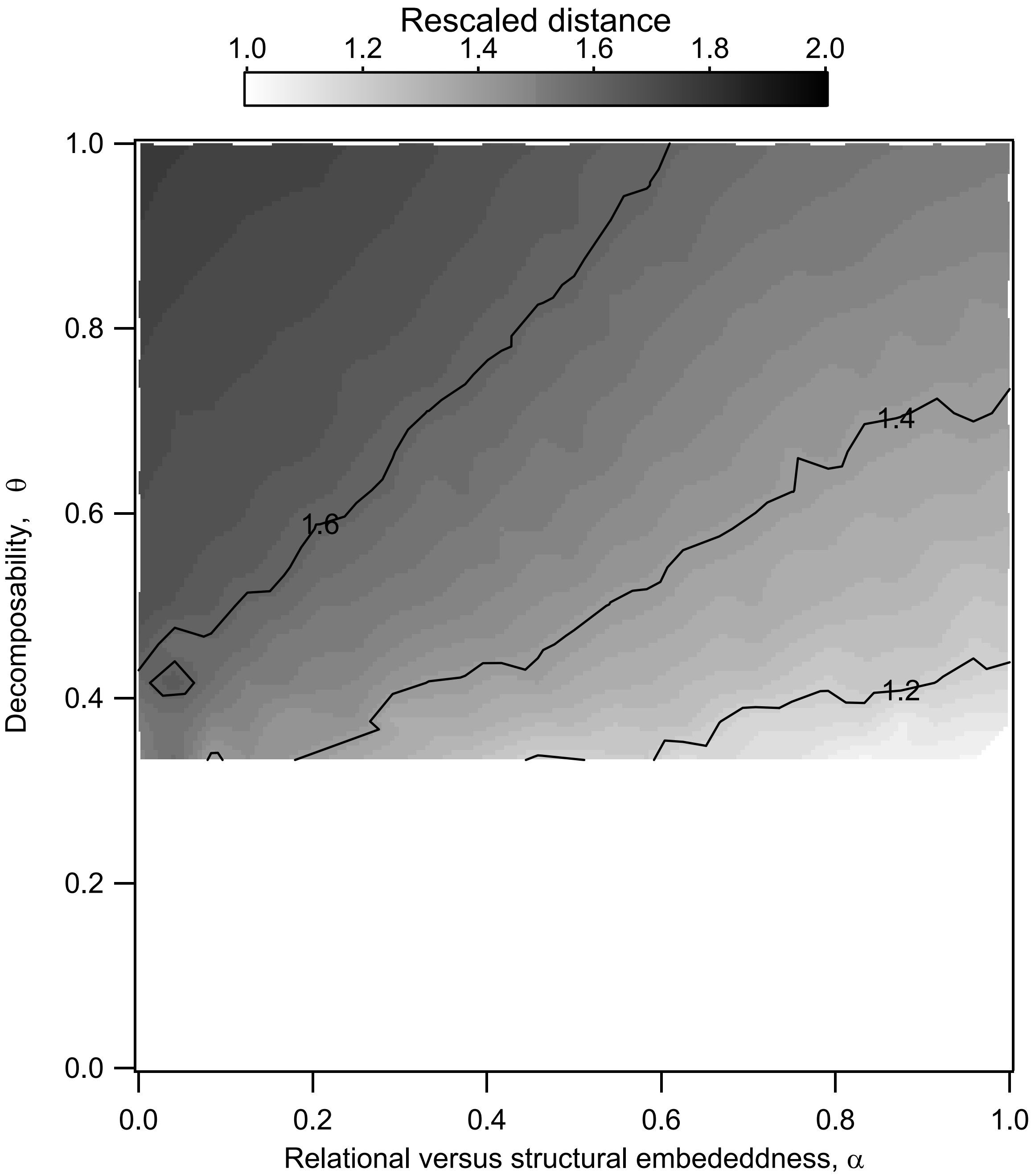

Correlation (cliquishess, knowledge)
-0.20
-0.10
0.00
0.10
1.0
0.8
0.6
0.4
0.2
0.0
Decomposability, θ
-0.05
0.05
0.05
-0.05
-0.05
-0.05
0.0
0.2
0.4
0.6
0.8
1.0
Relational versus structural embededdness, α



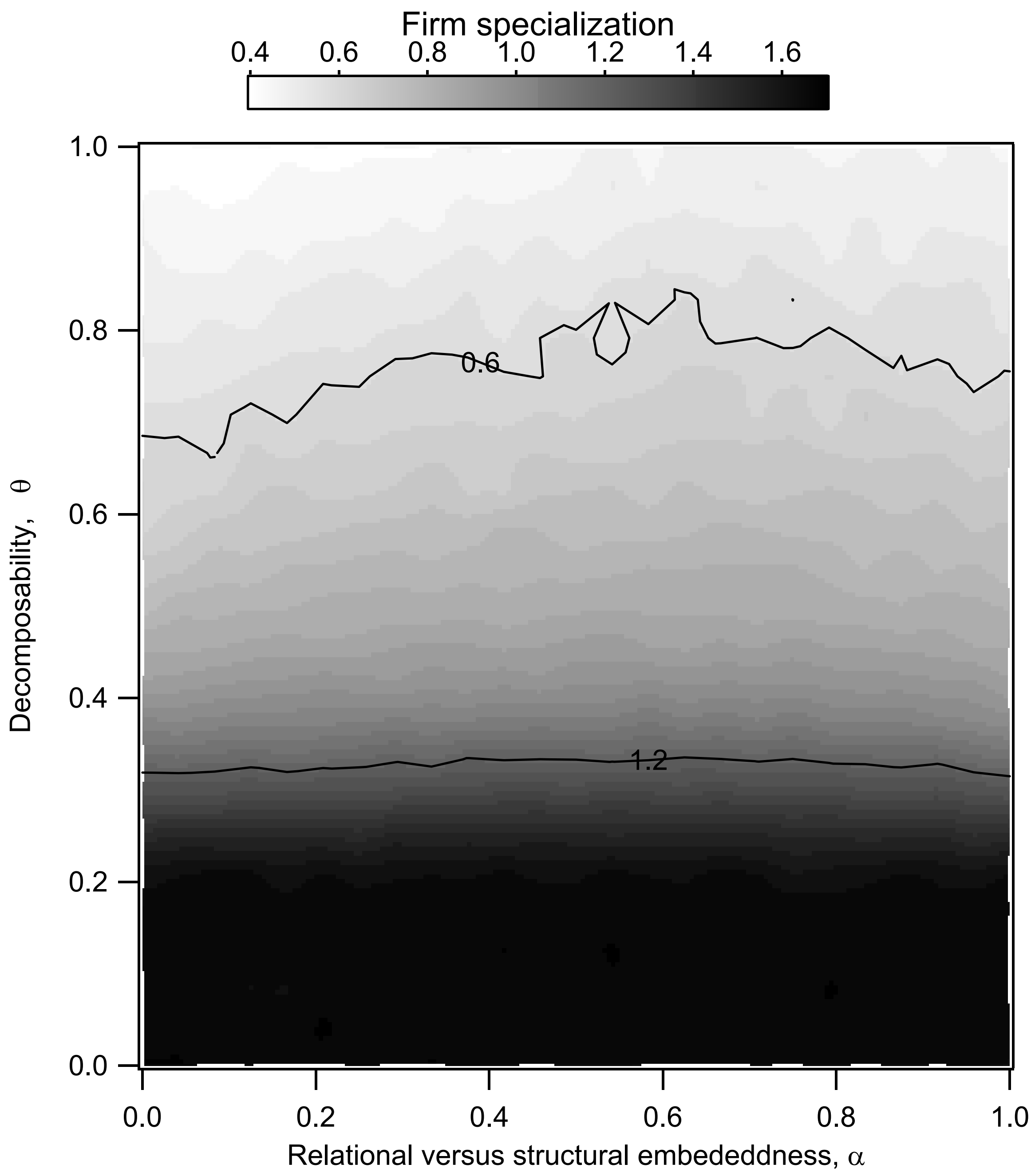
Correlation (cliquishness, rank change)
-0.10
-0.05
0.00
0.05
0.10
-0.05
-0.05
-0.05
-0.05
0.05
0.05
0.05
0.05
0.05
-0.05
-0.05
1.0
0.8
0.6
0.4
0.2
0.0
Decomposability, θ
0.0
0.2
0.4
0.6
0.8
1.0
Relational versus structural embededdness, α



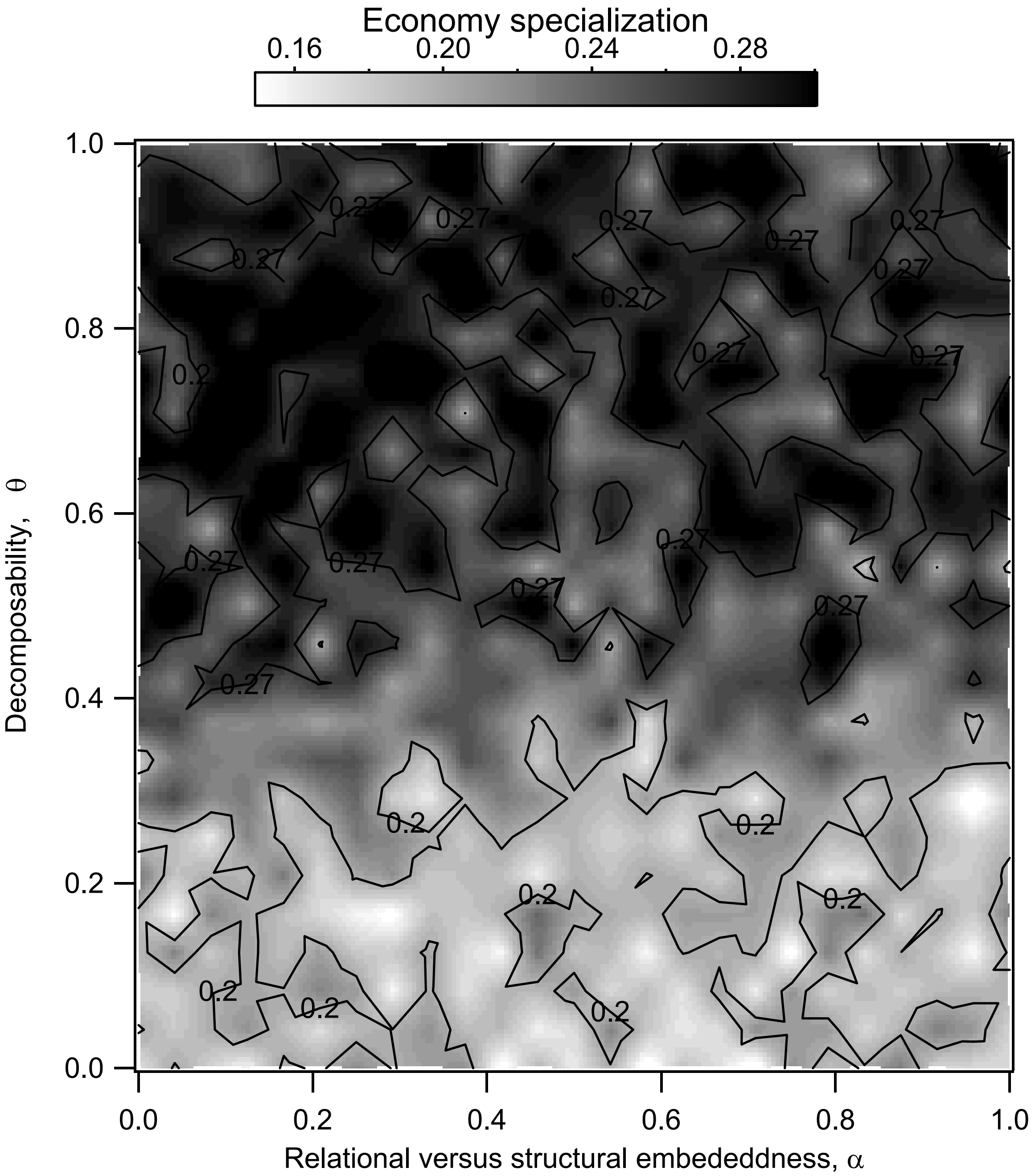
Distance
2.0
2.5
3.0
3.5
4.0
4.5
5.0
1.0
0.8
0.6
0.4
0.2
0.0
3
3.5
4
4
Decomposability, θ
0.0
0.2
0.4
0.6
0.8
1.0
Relational versus structural embededdness, α



Rescaled distance
1.0
1.2
1.4
1.6
1.8
2.0
1.6
1.4
1.2
Decomposability, θ
0.0
0.2
0.4
0.6
0.8
1.0
Relational versus structural embededdness, α



Firm specialization
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.0
0.8
0.6
0.4
0.2
0.0
0.6
1.2
Decomposability, θ
0.0
0.2
0.4
0.6
0.8
1.0
Relational versus structural embededdness, α



Economy specialization
0.16
0.20
0.24
0.28
1.0
0.8
0.6
0.4
0.2
0.0
Decomposability, θ
0.0
0.2
0.4
0.6
0.8
1.0
Relational versus structural embededdness, α
0.27
0.2

Background

Boundaries of the firm

Localized nature of knowledge transmission

Networks and collaborative agreements in R&D or production

Structural vs. relational embededness

Networks and Small Worlds

Network Formation

Game-theoretic

Nash equilibria

stability and efficiency

prevalence of empty, complete and star networks

Evolutionary

network formation/evolution by melioration

Hebb learning rule

probabilistic link formation

The Model

N agents indexed by $i \in S = \{1..N\}$

Each agent has knowledge endowment:

$v_i(t)$ where $v_i(t)$ is a vector of knowledge types

Each period:

- $N/2$ pairs (i, j) form; (Matching)
- i and j jointly innovate, producing new knowledge; (Innovation)
- this new knowledge is added to their respective knowledge stocks; (Knowledge dynamics)
- the $N/2$ partnerships are dissolved.

Properties of Innovation

If i and j jointly innovate:

- the knowledge amounts held by i and j will increase;
- the knowledge types of i and j will change;
- the distance between the knowledge types of i and j will fall;
- most often, innovation produces knowledge of a type which is "between" the types of the two contributors; however, innovation sometimes produces very different knowledge.

When agents (i, j) come together they pool their knowledge:

$$v_c = (1 - \theta) \min(v_{i,c}, v_{j,c}) + \theta \max(v_{i,c}, v_{j,c}).$$

Pooled knowledge used to create new knowledge:

$$r(i, j) = \left(\sum_c v_c^\beta\right)^{1/\beta}$$

β: elasticity of substitution across knowledge types:

$\beta \to -\infty$: Leontieff production function,

$\beta = 1$: perfect substitutability

$\beta \to 0$: Cobb-Douglas production function.

Innovation Success

Innovation may fail; success probability:

Direct credit (relational embededness)

$$r(i,j) = \rho^{\tau(i,j)}\chi(i,j)$$

Indirect credit (structural embededness)

$$s(i,j) = \sum_{k \neq i \neq j} r(i,k)\, r(k,j)$$

$$
\begin{aligned}
c\,(i,j) \;&=\; \alpha\gamma(i,j) \\
&\quad + \frac{(1-\alpha)\,s\,(i,j)}{\#\,\{k : r\,(i,k) > 0, \tau\,(k,j) < \infty\}}
\end{aligned}
$$

α: weight of relational embededness.

$$
\pi\,(i,j) = \pi_L + (\pi_H - \pi_L)c\,(i,j)
$$

$$
F(i,j) = \pi\,(i,j) \cdot r\,(i,j)
$$

Knowledge Dynamics

If i and j innovate, producing knowledge $v(i, j)$ (a scalar) then:

$v\,(i, j)$ is allocated to the mth knowledge category with probability:

$$\frac{v_m}{\Sigma_c\, v_c}, \forall m$$

Matching

A matching is a bijection of S, forming $N/2$ disjoint pairs: $\mu : S \to S$ such that $\mu(\mu(i)) = i$

Every $i \in S$ has a preference ordering $\succ_i$ over all the individuals in $S - \{i\}$.

A matching μ is *stable* in $(S, \succ)$ if there is no $(i, j) \notin \mu$ such that

$$j \succ_i \mu(i) \text{ and } i \succ_j \mu(j).$$

Preferences:

$$j \succ_i k \Leftrightarrow r(i, j) > r(i, k),$$

$\forall i, j, k \in S, i \neq j \neq k,$

Proposition: A stable matching always exists and is unique if preference orderings are strict.

Dynamics (again)

Each period:

- At most $N/2$ pairs (i, j) form (isolation authorized);
- i and j jointly innovate, producing new knowledge;
- this new knowledge is added to their respective knowledge stocks;
- the $N/2$ partnerships are dissolved.

Monte Carlo Experiment

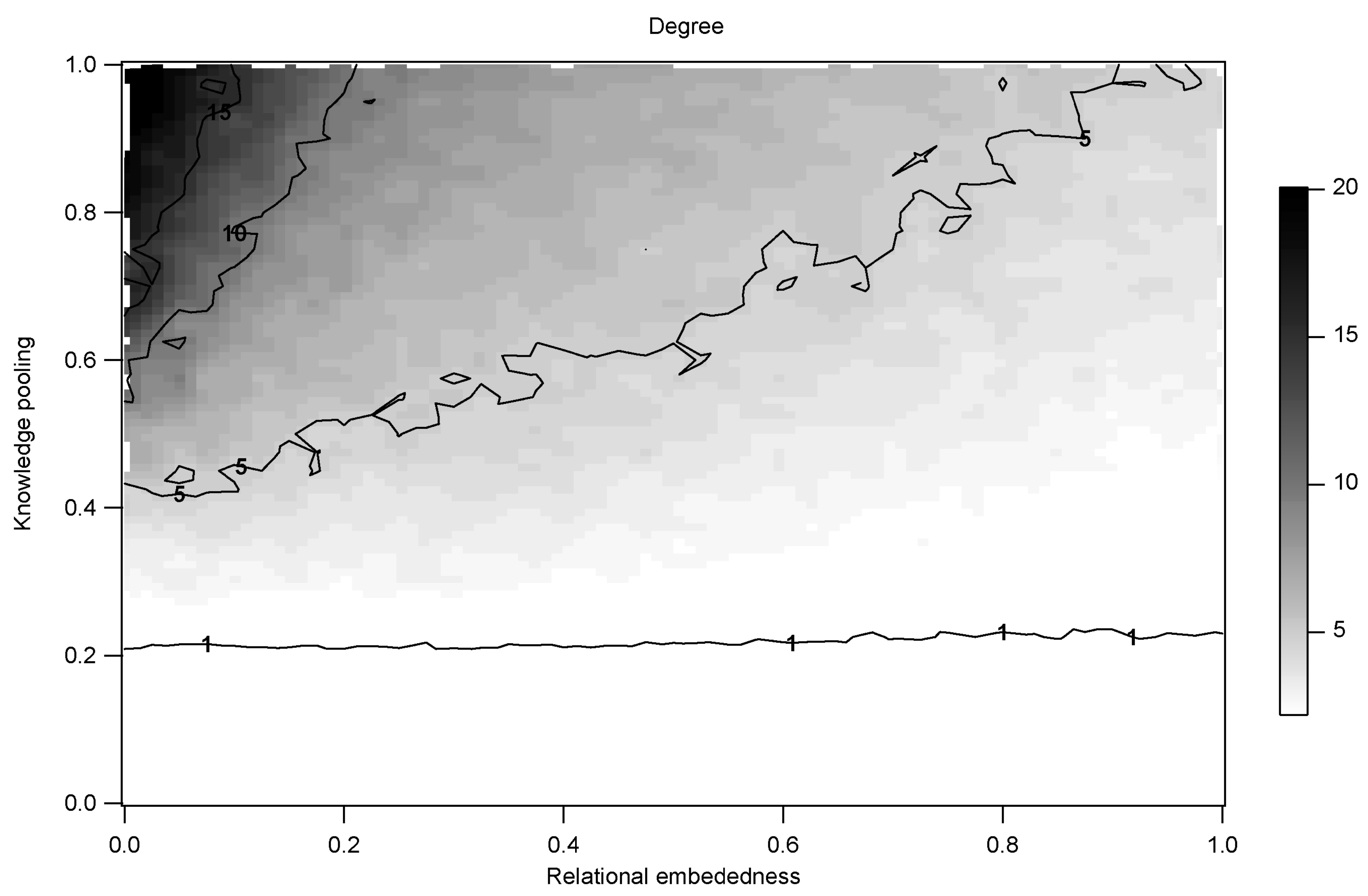
$N = 100$ agents

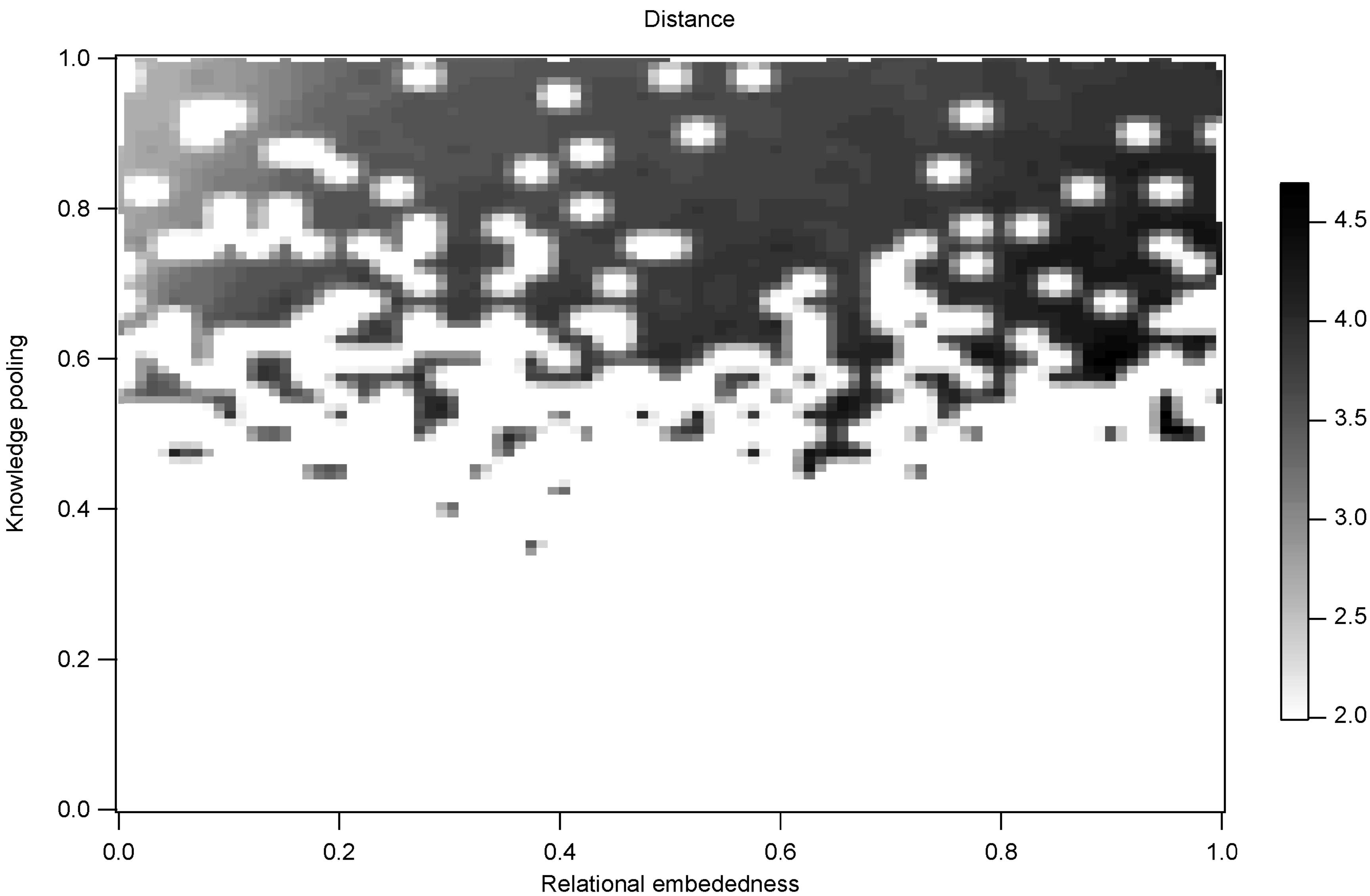
100 random $\alpha \in [0, 1]$

100 random $\theta \in [0, 1]$

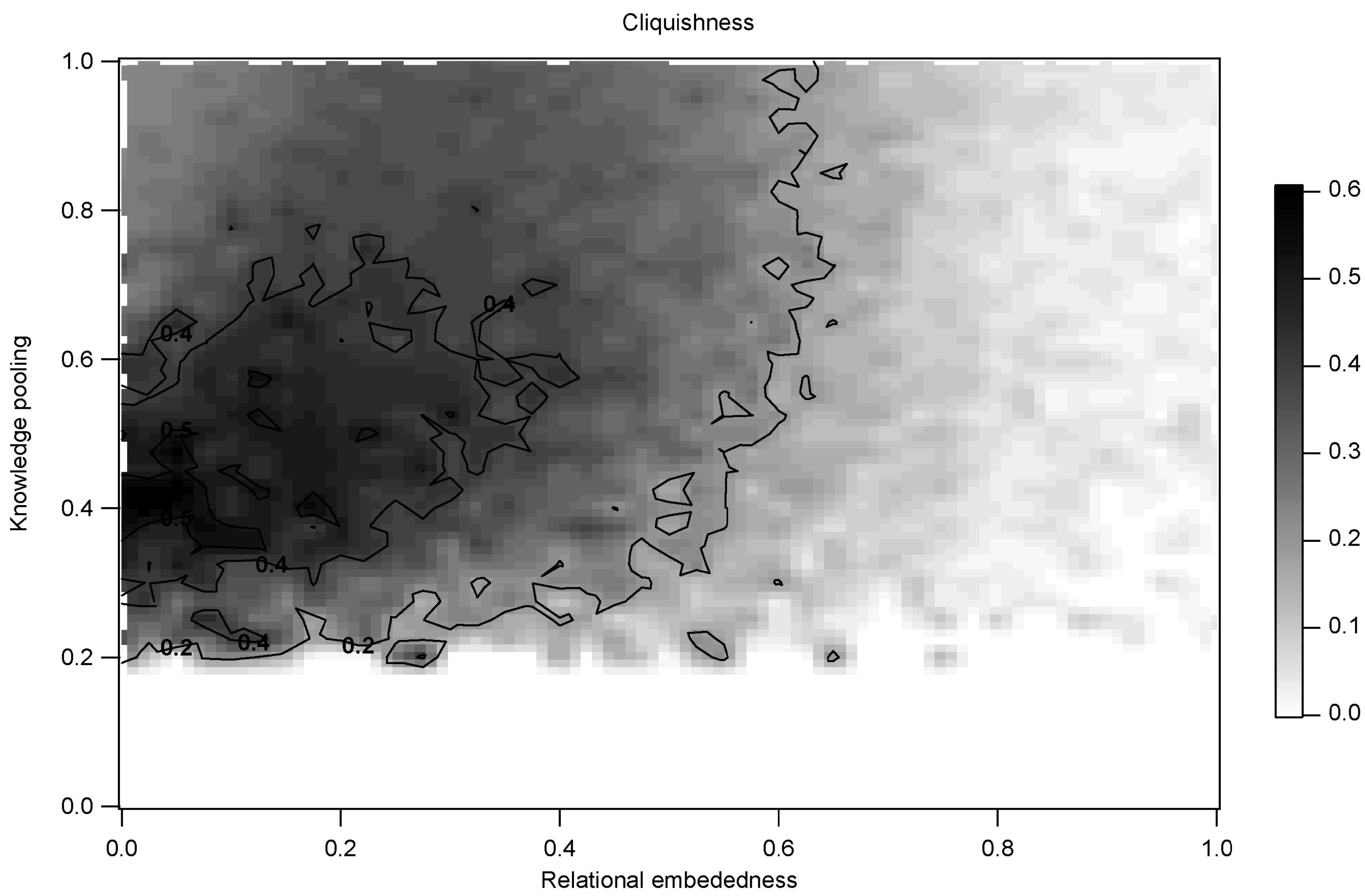
For each α, θ pair, network evolves for $T = 1000$ periods

Non-parametric (Kernel) estimate of relationships between α, θ and several structural parameters extracted from the data.

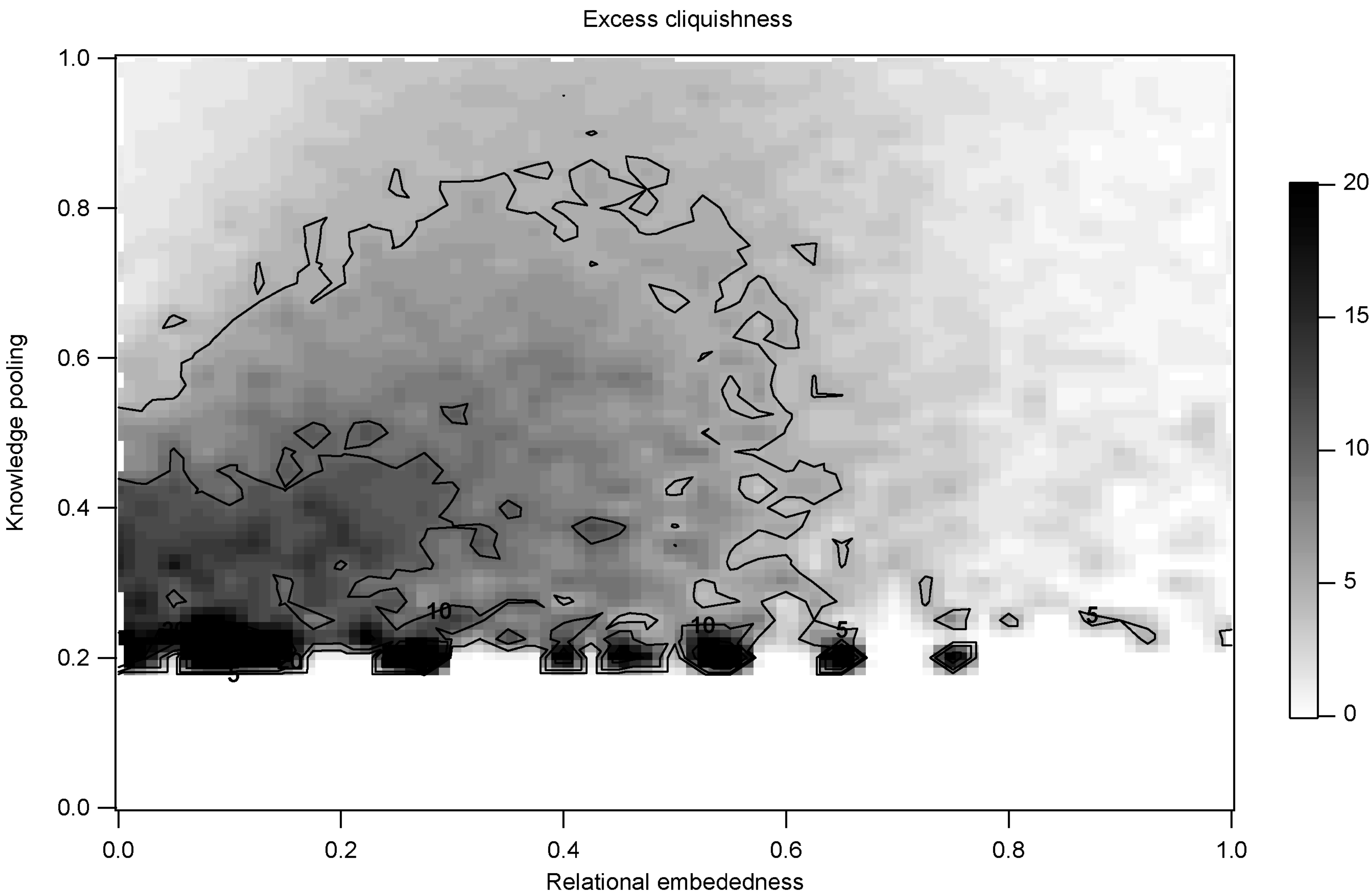

Degree
Knowledge pooling
Relational embededness
0.0
0.2
0.4
0.6
0.8
1.0
1
5
10
15
20

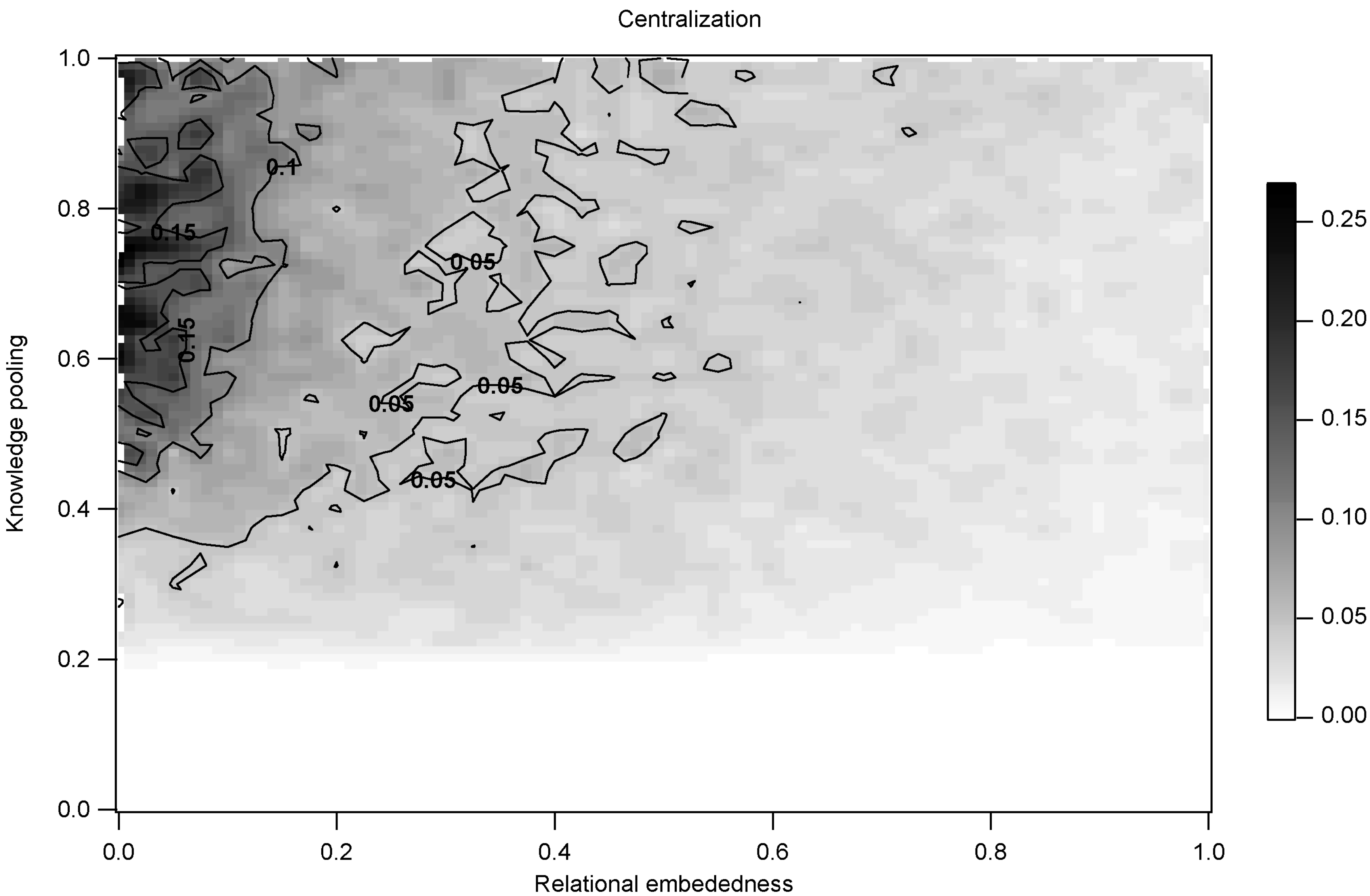


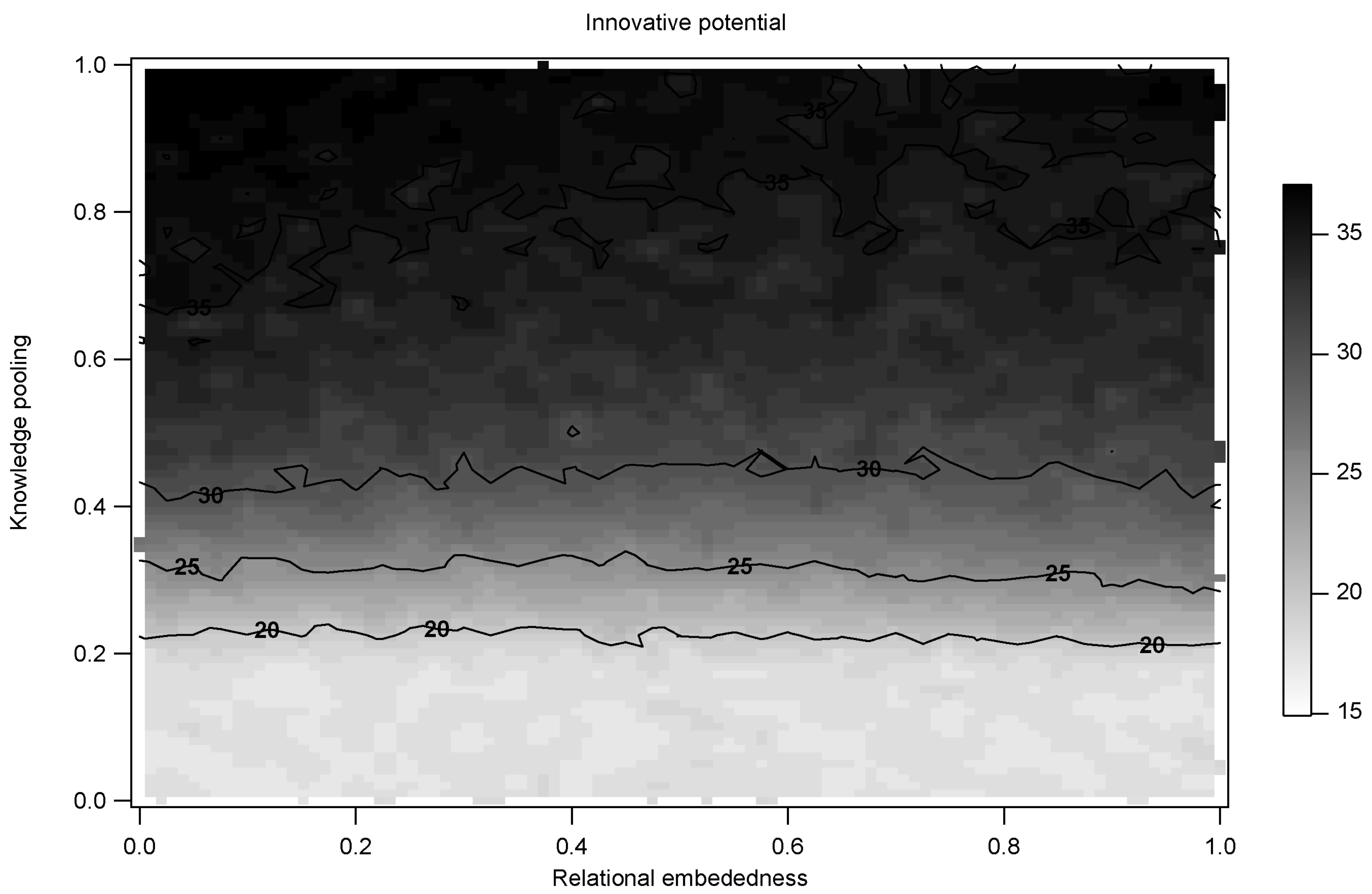
Distance
Knowledge pooling
Relational embededness
1.0
0.8
0.6
0.4
0.2
0.0
4.5
4.0
3.5
3.0
2.5
2.0



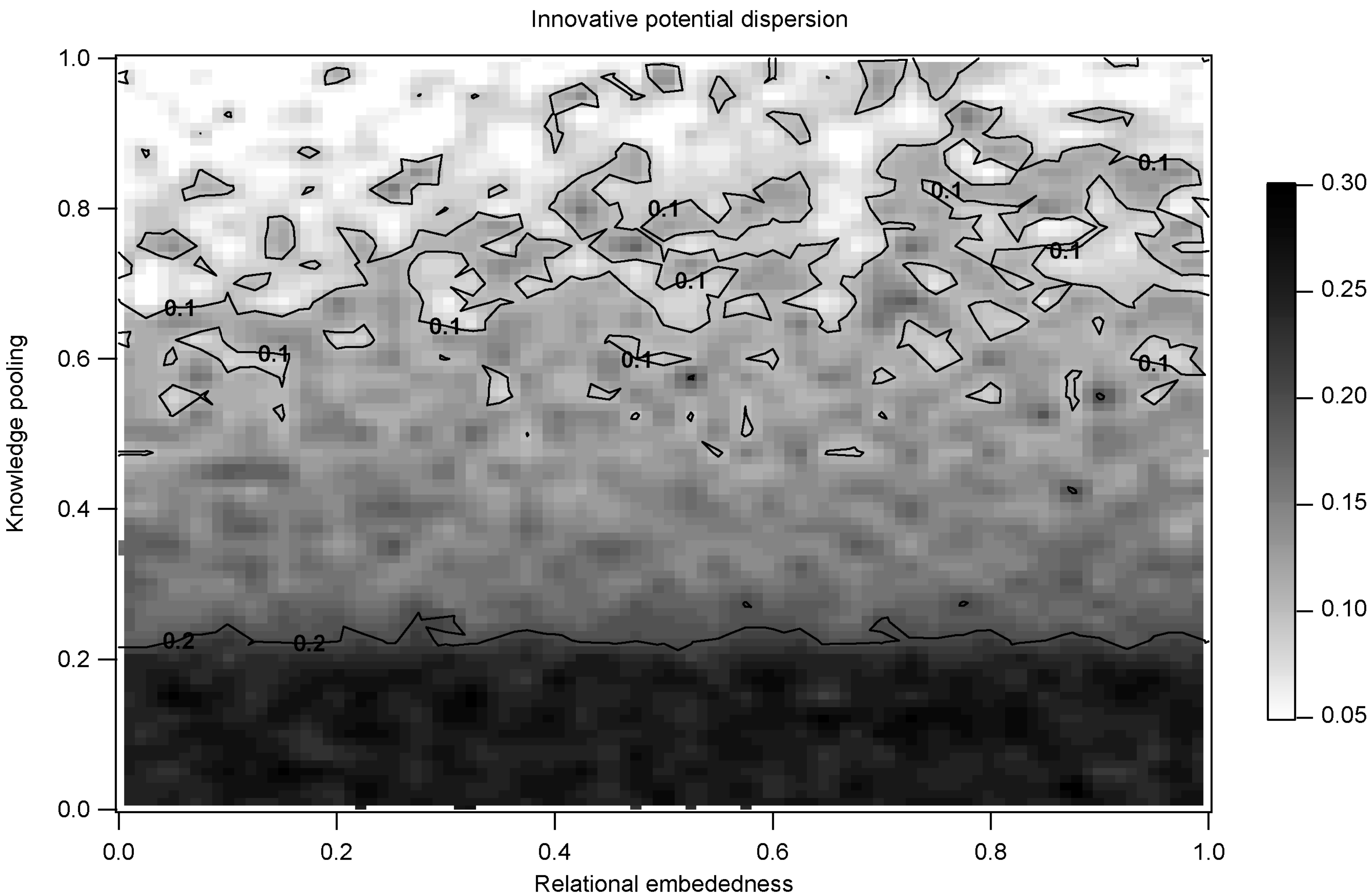
Cliquishness
Knowledge pooling
Relational embededness
1.0
0.8
0.6
0.4
0.2
0.0
0.4
0.5
0.2

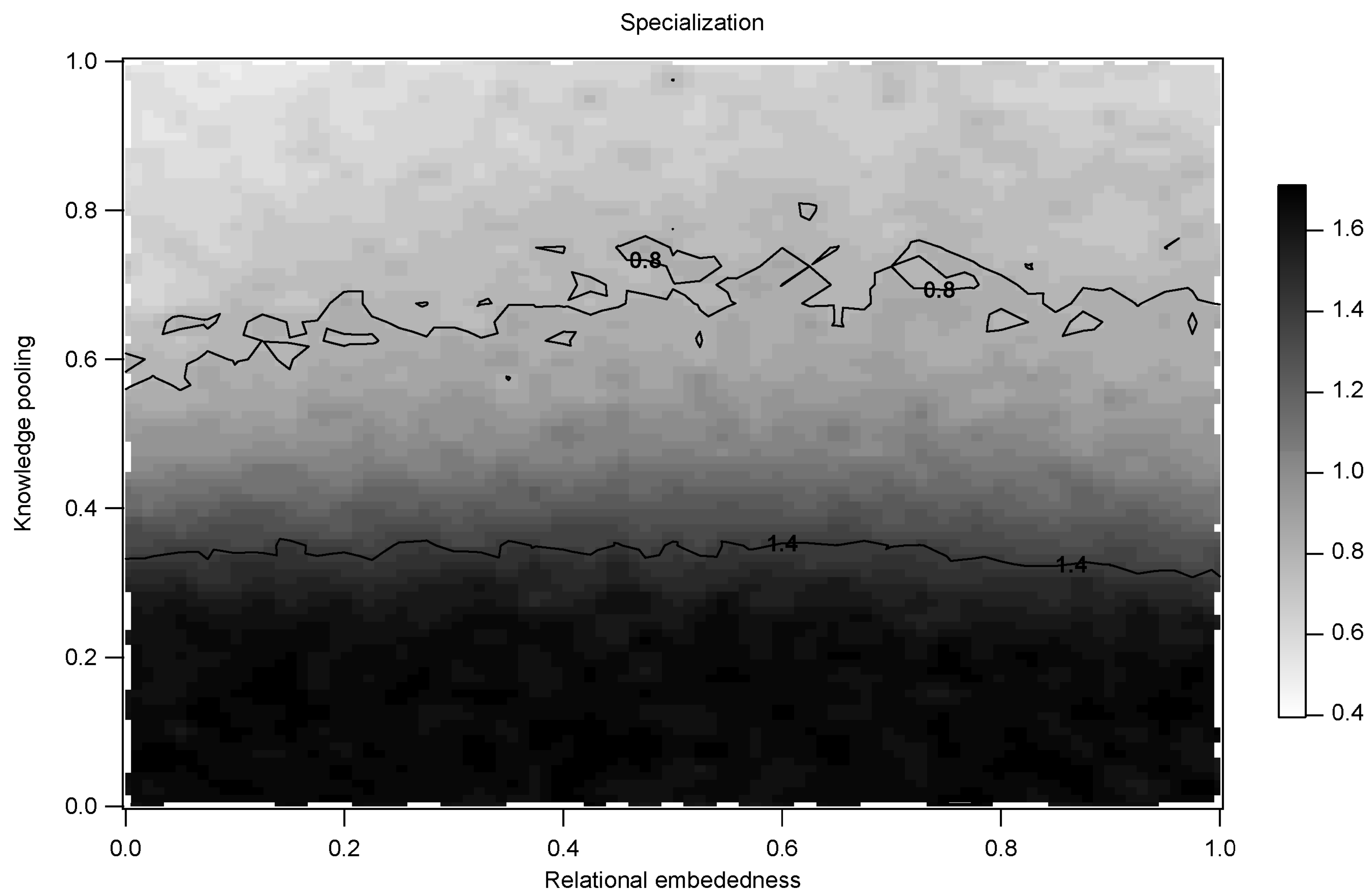

Excess cliquishness
Knowledge pooling
Relational embededness
0.0
0.2
0.4
0.6
0.8
1.0
0
5
10
15
20


Centralization
Knowledge pooling
Relational embededness
0.0
0.2
0.4
0.6
0.8
1.0
0.15
0.1
0.05
0.00
0.05
0.10
0.15
0.20
0.25



Innovative potential
Knowledge pooling
Relational embededness
0.0
0.2
0.4
0.6
0.8
1.0
35
30
25
20
15


Innovative potential dispersion
Knowledge pooling
Relational embededness
0.0
0.2
0.4
0.6
0.8
1.0
0.1
0.2
0.05
0.10
0.15
0.20
0.25
0.30



Specialization
Knowledge pooling
Relational embededness
0.0
0.2
0.4
0.6
0.8
1.0
0.8
1.4
1.6
1.4
1.2
1.0
0.8
0.6
0.4