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Structure and Function of Complex Networks**

16 - 28 May 2005

Human Dynamics
The Nature of Time in Networks

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These are preliminary lecture notes, intended only for distribution to participants

Human dynamics: The nature of time in networks

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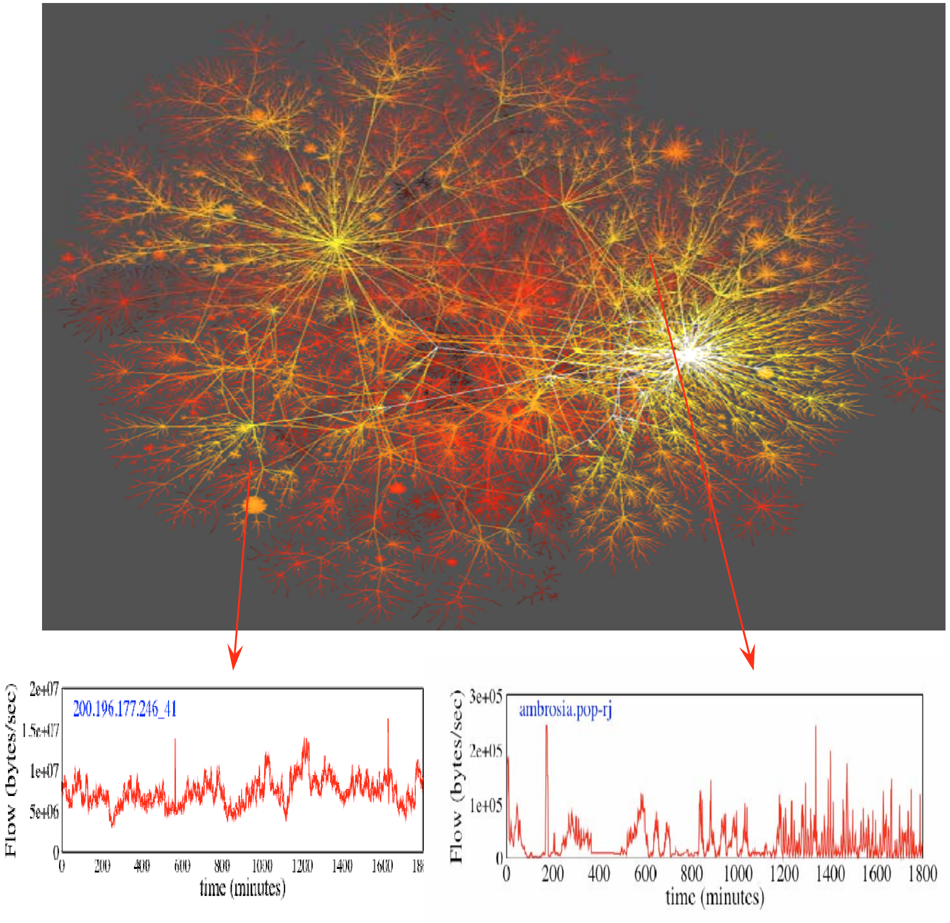
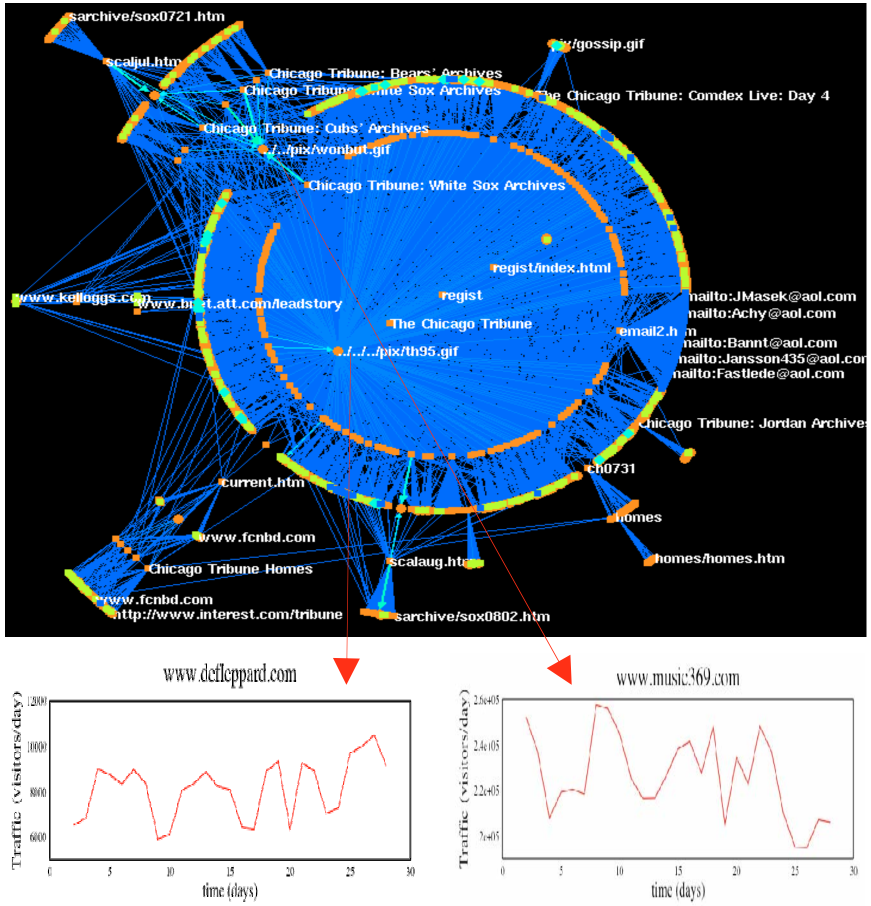
University of Notre Dame

A.L. Barabasi, Nature 435, 207-211 (2005) (**cond-mat/0505371**)
Z. Dezso et al, **physics/0505087**
Vazquez and ALB, to be published.

Internet

World Wide Web





When do things happen in networks?

When do we do things?

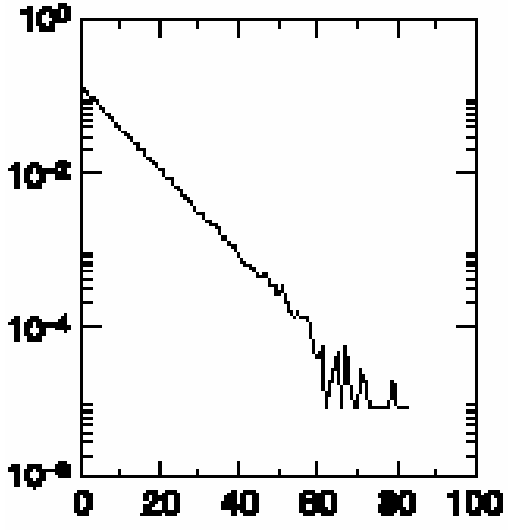
Poisson Processes

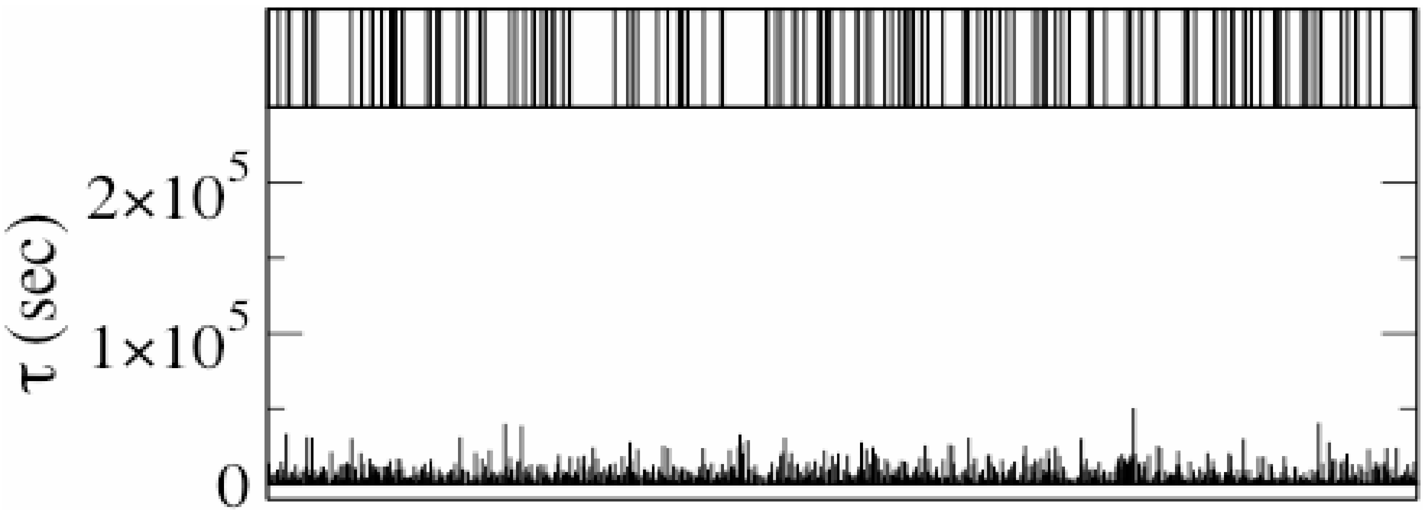
Basic assumption: events take place randomly, at a constant rate λ

Consequence: The timing of events follows a Poisson process.

τ: time interval between two consecutive events
(*waiting* or *inter-event* time)

$$P(\tau) = \lambda \exp(- \lambda \tau)$$





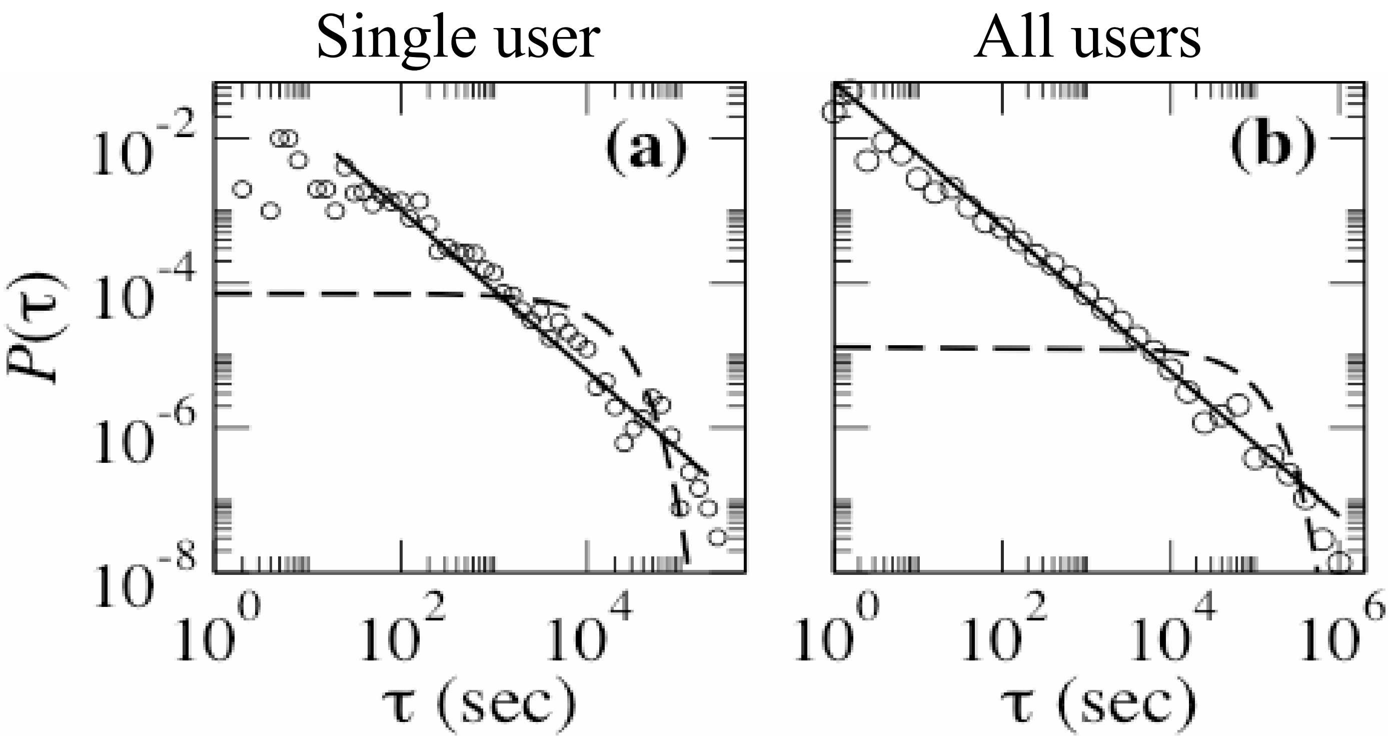
Moivre (1718)

Poisson (1837)

Erlang (1909)

Is the timing of human initiated events random?

Email communications

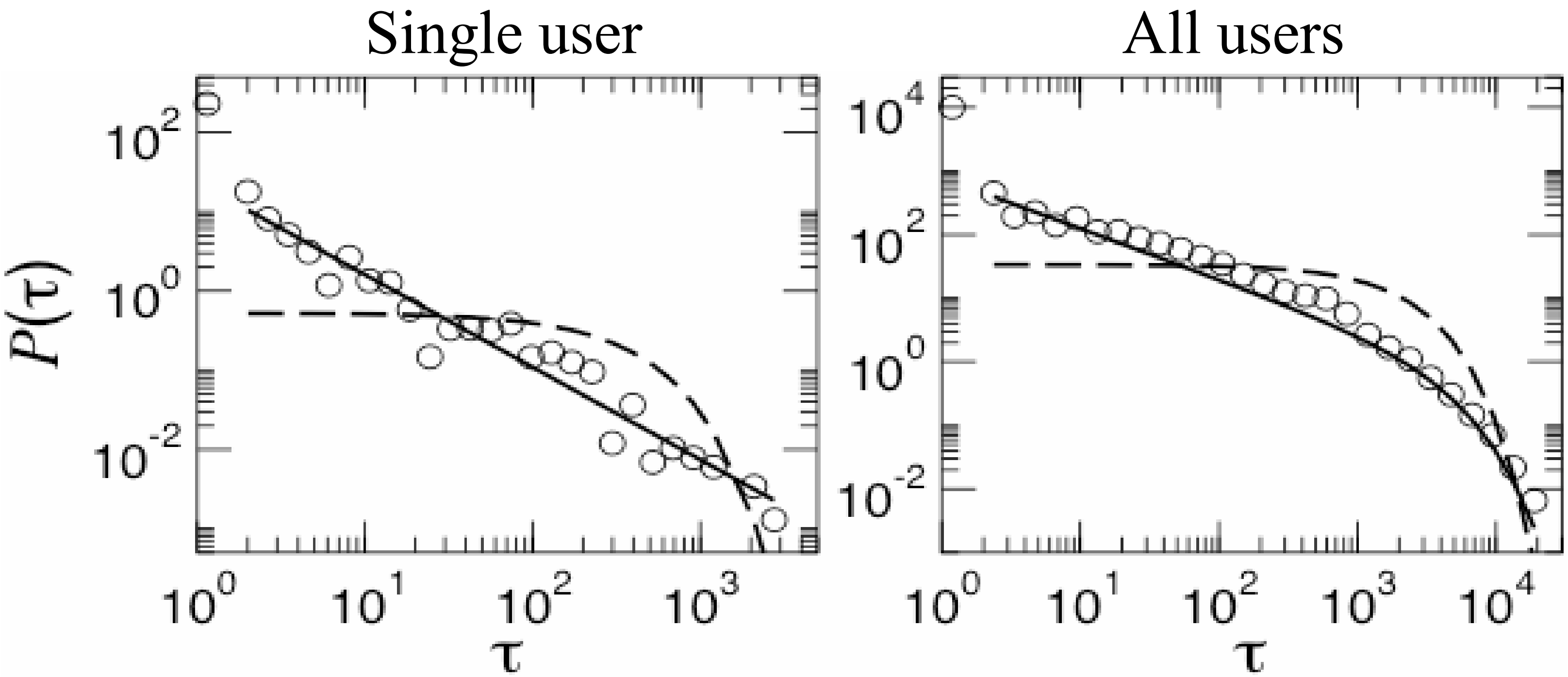


τ: time between two consecutive emails sent by a user

- Number of users: 3,180
- Number of events: 129,135
- Time period: 3 months
- Time resolution: 1 second

Eckmann et al., PNAS (2004)
Ebel et al., Phys. Rev. E (2002)

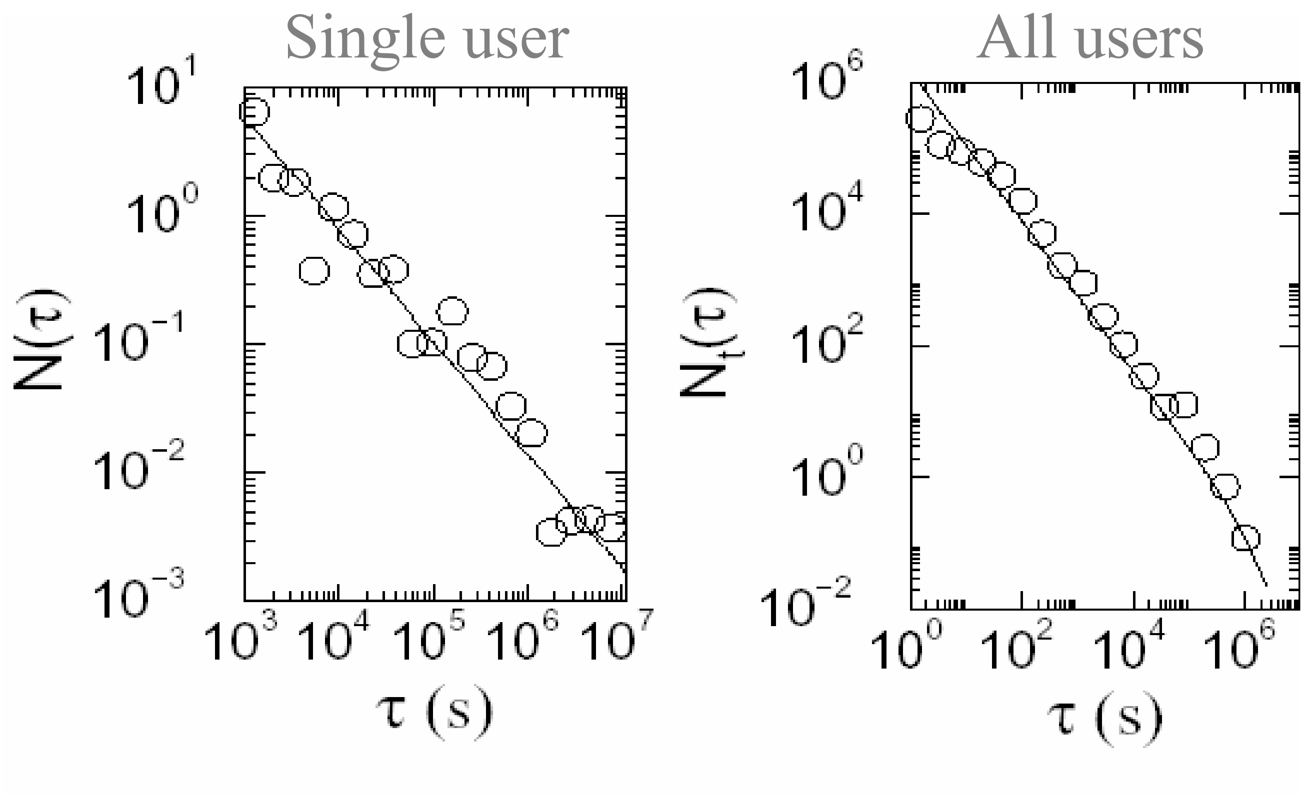
Library data



τ: time between two consecutive library visits (checkout records)

- Number of users: 2,247
- Number of events: 48,408
- Time period: 3 years
- Time resolution: 1 minute

Web Browsing



- Number of users: 250,000
- Number of events: 22,000,000
- Time period: 1 month
- Time resolution: 1 second

Dezső et al., physics/0505087

Other Human Driven Events

→instant messages during online chats [Dewes, C. *et al.*, in Proc. 2003 ACM SIGCOMM Conf. on Internet Measurement (IMC-03) (ACM Press, New York, 2003)]

→job submissions on a supercomputer [Kleban,S.D. & Clearwater,S.H. Proc. SC2003 http://www.sc-conference.org/sc2003/paperpdfs/pap222.pdf (2003).]

→directory listings [Paxson, V. & Floyd, S. IEEE/ACM Trans. Netw. **3**, 226 (1996)]

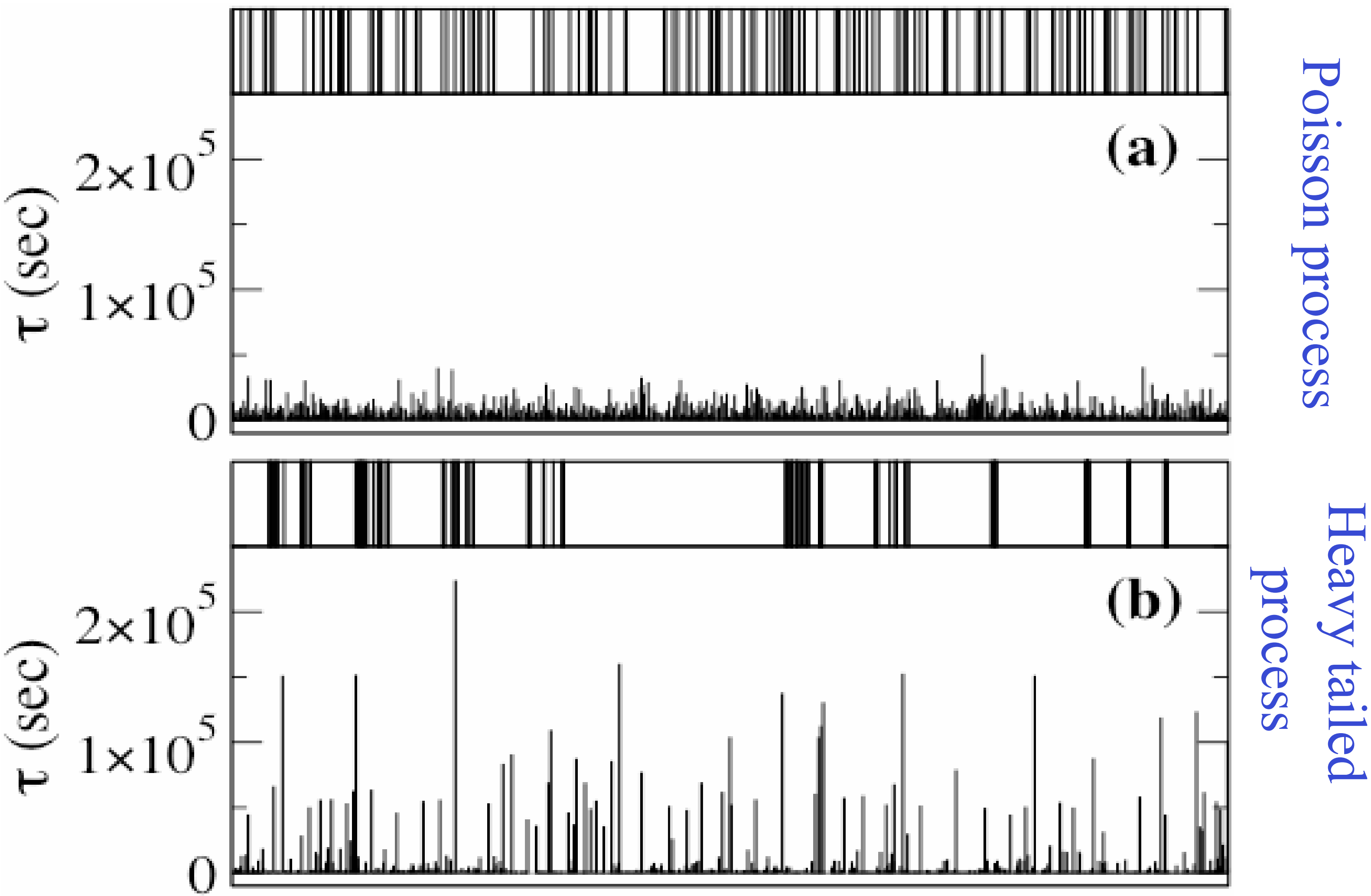
→file transfers (FTP requests) [Paxson, V. & Floyd, S. IEEE/ACM Trans. Netw. **3**, 226 (1996)]

→printing jobs [Harder, U. & Paczuski, M. http://xxx.lanl.gov/abs/cs.PF/0412027 (2004)]

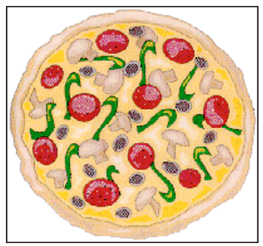
→individual trades in currency futures [Masoliver, J., Montero, M. & Weiss, G. H. *Phys. Rev. E* **67**, 021112 (2003)]

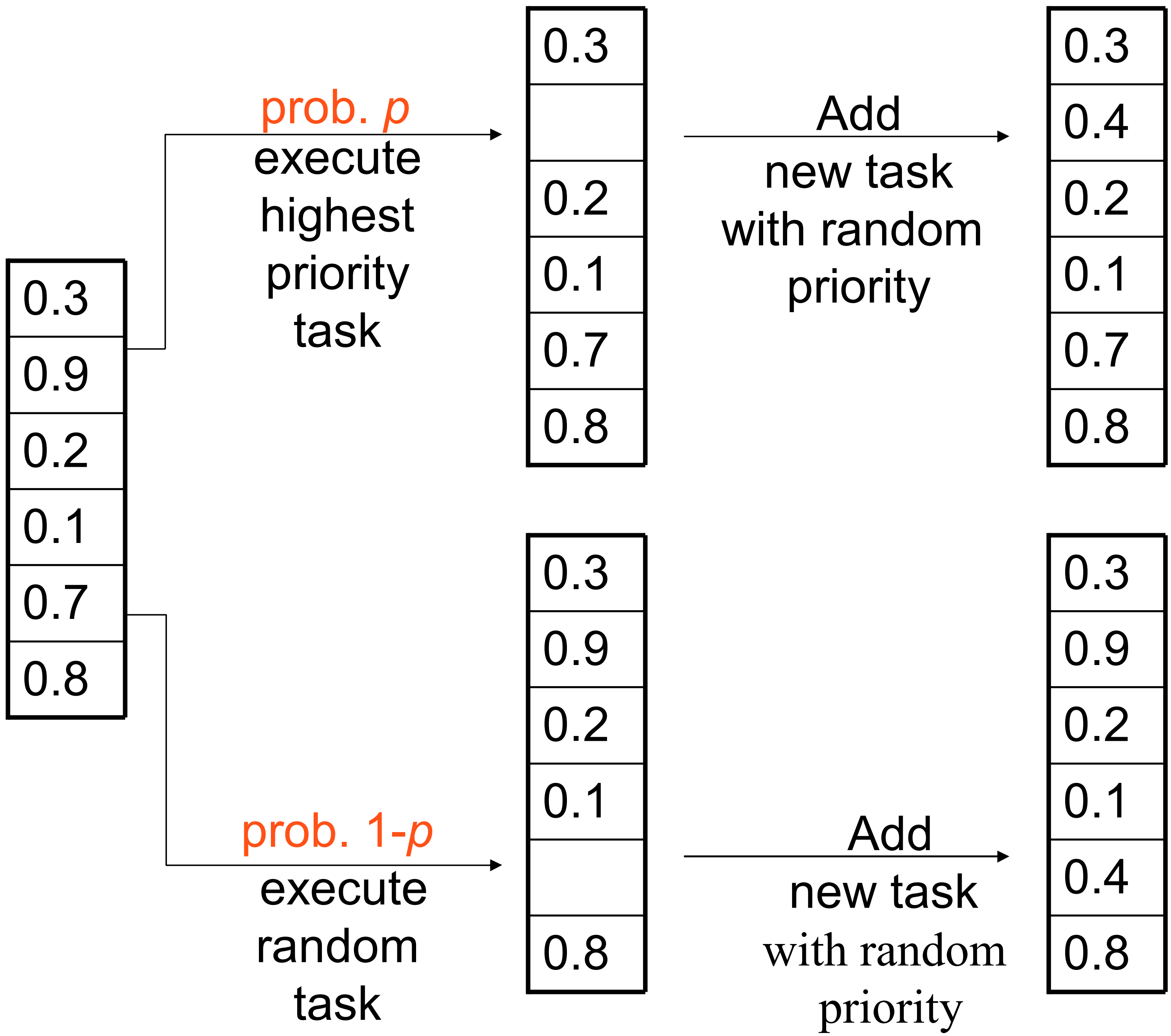
→online games played by a user [Henderson, T. & Nhatti, S. *Proc. 9th ACM Int. Conf. on* Multimetia 212–220 (ACM Press, New York, 2001)]

Poisson and heavy tailed activity patterns



To do list	Random order	First In First Out	With priorities
•Clean the house	1	1	0.3
•Work on a paper	6	2	0.9
•Call the dentist	3	3	0.2
•Get a date	4	4	0.1
•Check emails	5	5	0.7
•Read a book	2	6	0.8




0.3
0.9
0.2
0.1
0.7
0.8
prob. p
execute
highest
priority
task
0.3
0.2
0.1
0.7
0.8
Add
new task
with random
priority
0.3
0.4
0.2
0.1
0.7
0.8
prob. 1-p
execute
random
task
0.3
0.9
0.2
0.1
0.8
Add
new task
with random
priority
0.3
0.9
0.2
0.1
0.4
0.8

Numerical Results

p=0.99999
highest priority task

p=0.00001
random task

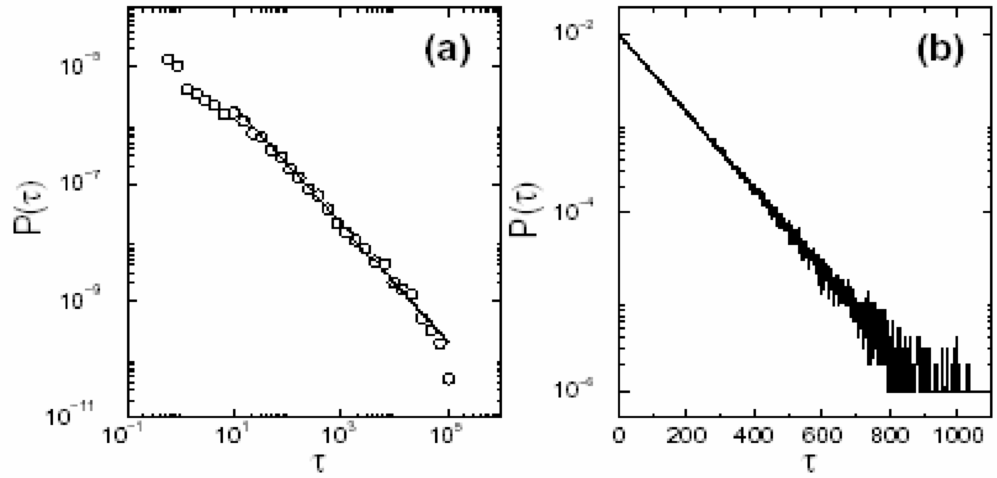


FIG. 3: The waiting time distribution predicted by the investigated queuing model. The priorities were chosen from a uniform distribution $x_i \in [0, 1]$, and we monitored a priority list of length $L = 100$ over $T = 10^6$ time steps. (a) The probability $P(\tau)$ that a task spends τ time on the list obtained for $p = 0.99999$, corresponding to the deterministic limit of the model. The continuous line on the log-log plot correspond to the scaling predicted by (2), having slope -1, indicating an excellent agreement between the numerical results and the analytical predictions. The data was log-binned, to reduce the uneven statistical fluctuations common in heavy tailed distributions, a procedure that does not alter the slope of the tail. (b) The $P(\tau)$ distribution for $p = 0.00001$, corresponding to the random choice limit of the model. The fact that the curve follows a straight line on a linear-log plot indicates that $P(\tau)$ decays exponentially (see Supplementary Material).

Stochastic Model

$$\Pi(x) = \frac{x^{\gamma}}{\sum_{i=1}^{L} x_i^{\gamma}}$$

$\gamma = 0$ **random** ***p*****=0**

$\gamma = \infty$ **priority** ***p*****=1**

◙ Probability that a task with priority *x* waits time *t*:

$$f(x,t) = (1 - \Pi(x))^{t-1}\Pi(x)$$

◙ Average waiting time τ for a task with priority *x*:

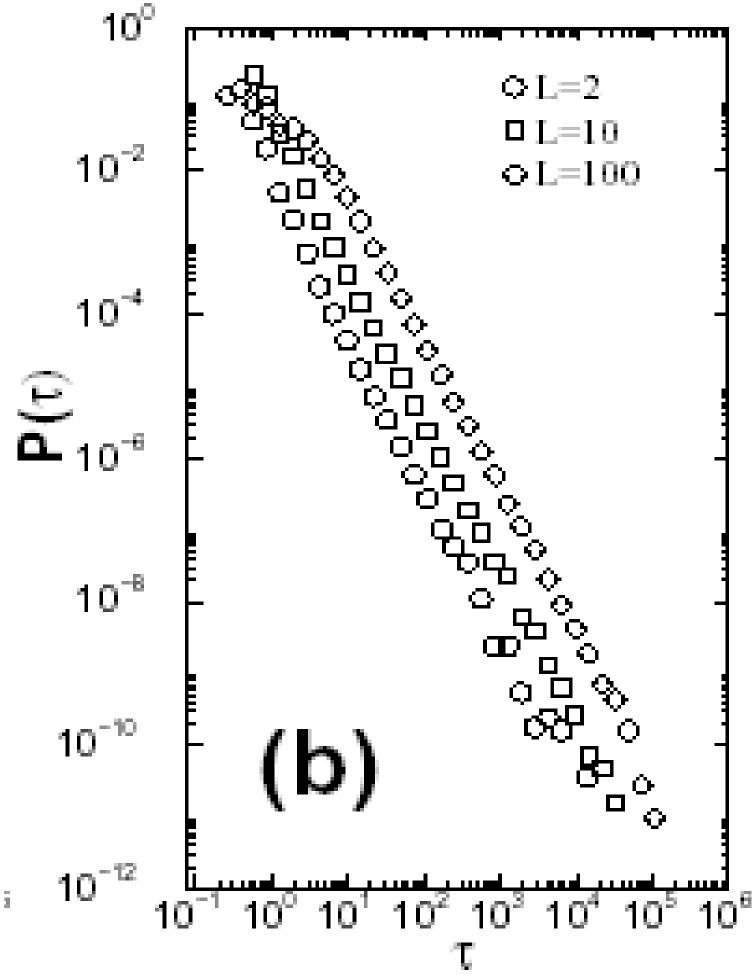
$$\tau(x) = \Pi(x)\sum_{t=1}^{\infty} t(1-\Pi(x))^{t-1} = \frac{1}{\Pi(x)} \propto \frac{1}{x^{\gamma}}$$

◙ *P(τ):* $\rho(x)dx = P(\tau)d\tau$

$$P(\tau) = \frac{1}{\gamma}\frac{\rho(\tau^{-1/\gamma})}{\tau^{1+1/\gamma}}$$

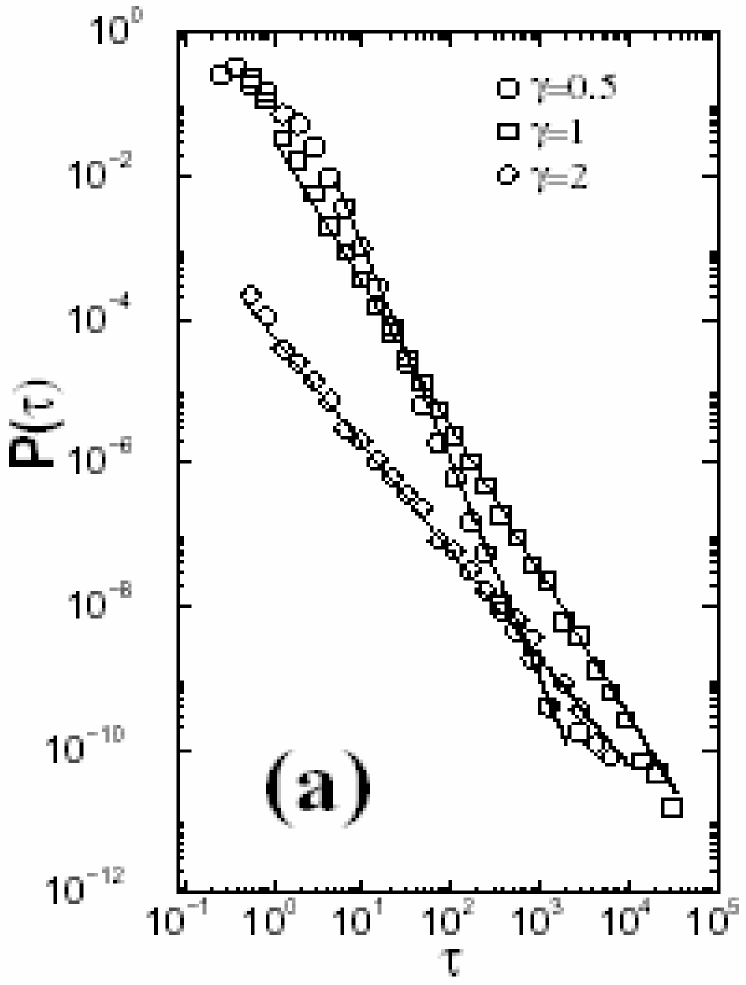
Does the queue length (L) matter?

$$P(\tau)=\frac{1}{\gamma}\frac{\rho(\tau^{-1/\gamma})}{\tau^{1+1/\gamma}}$$



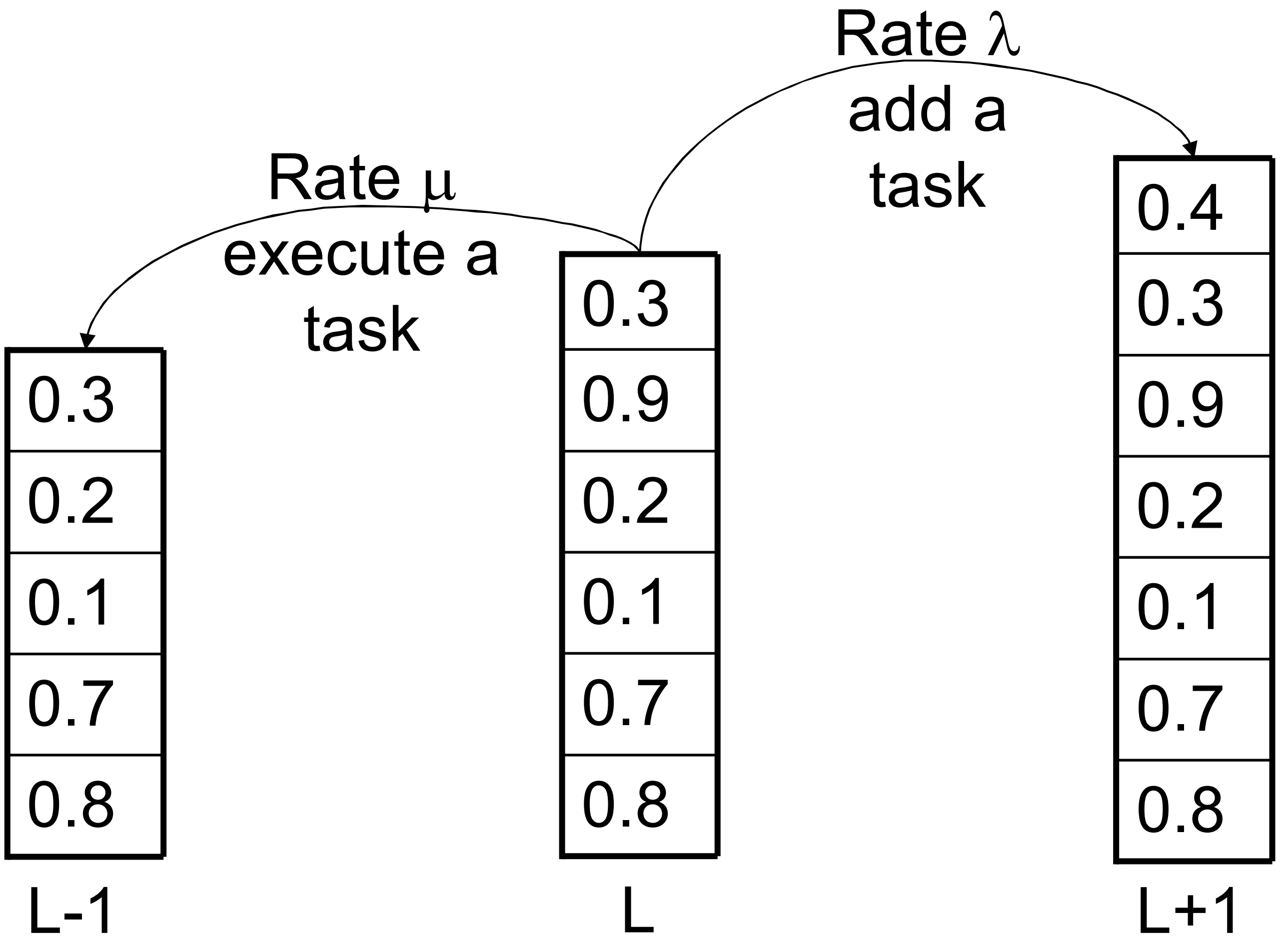
Universality Classes: Can we have arbitrary values for the exponent?

$$P(\tau) = \frac{1}{\gamma} \frac{\rho(\tau^{-1/\gamma})}{\tau^{1+1/\gamma}}$$



<u>But:</u>
only γ=∞
is realistic
P(τ)~ τ^{-1}

Cobham Model (1954)



Cobham Model

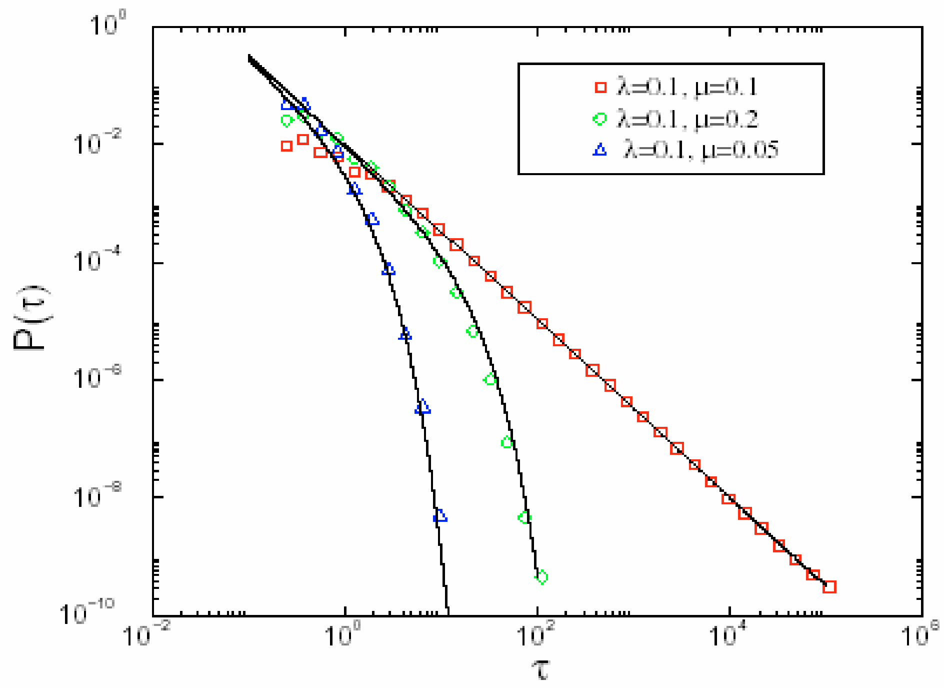


FIG. 1: The waiting time distribution generated by a priority model for which the priorities are chosen from a uniform distribution $\rho(x)$ with $x \in [0,1]$. Tasks arrive at a constant rate λ and are executed at a rate μ, whose value for each curve being shown in the figure legend. The continuous lines corresponds to $P(\tau) \sim \tau^{-3/2}$ for $\lambda = \mu = 0.1$, and to $P(\tau) \sim \tau^{-3/2} e^{\tau/\tau_0}$ for the other two curves. Note that in general a power law emerges only when $\lambda = \mu$.

$$P(\tau) \sim \tau^{-3/2} e^{-\frac{\tau}{\tau_0}}$$

$$\lambda = \mu$$

$$P(\tau) \sim \tau^{-\frac{3}{2}}$$

Two classes of processes:

L fixed:

$$P(\tau) \sim \tau^{-1}$$

L fluctuates:

$$P(\tau) \sim \tau^{-3/2} e^{-\frac{\tau}{\tau_0}}$$

Empirically:

Web visitation
Email activity
Library
Sexual activity

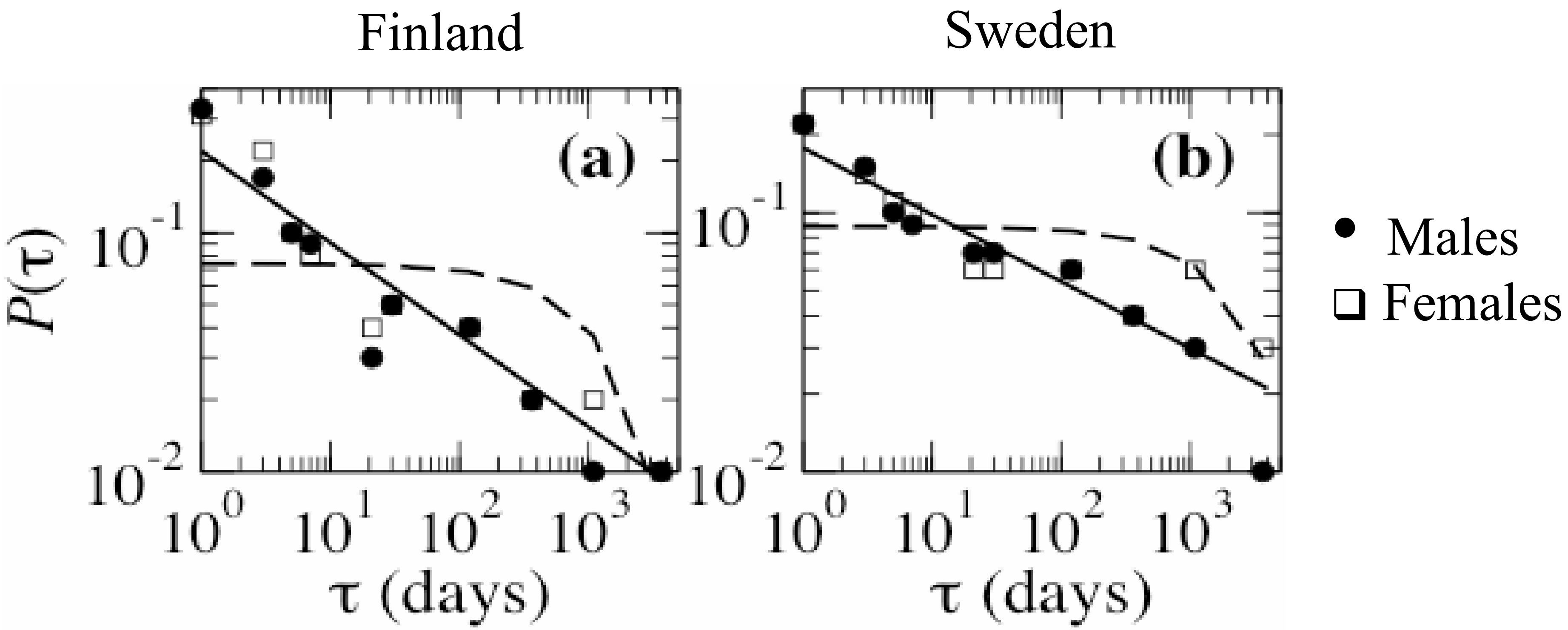
Why?

Miller 1956: The Magical Number Seven

So what?

epidemics

Sexual activity



τ: time since the last sexual intercourse

•STD

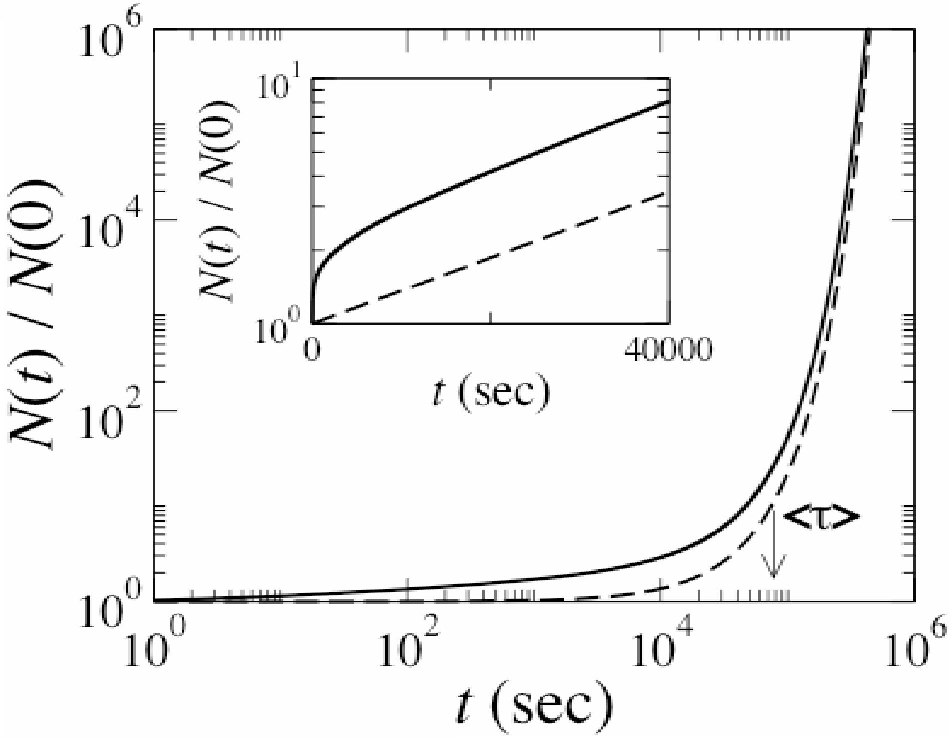
Consequences on viral dynamics

SI model:

$$n(t) = \int_0^t \beta(t')n(t-t')$$

$$\beta(t) = < k > C(t)$$

$C(t)$=probability that a patient establishes a contact at time t after it passes the first virus to other agents.



$$P(\tau) = \frac{A}{\tau^\alpha}$$

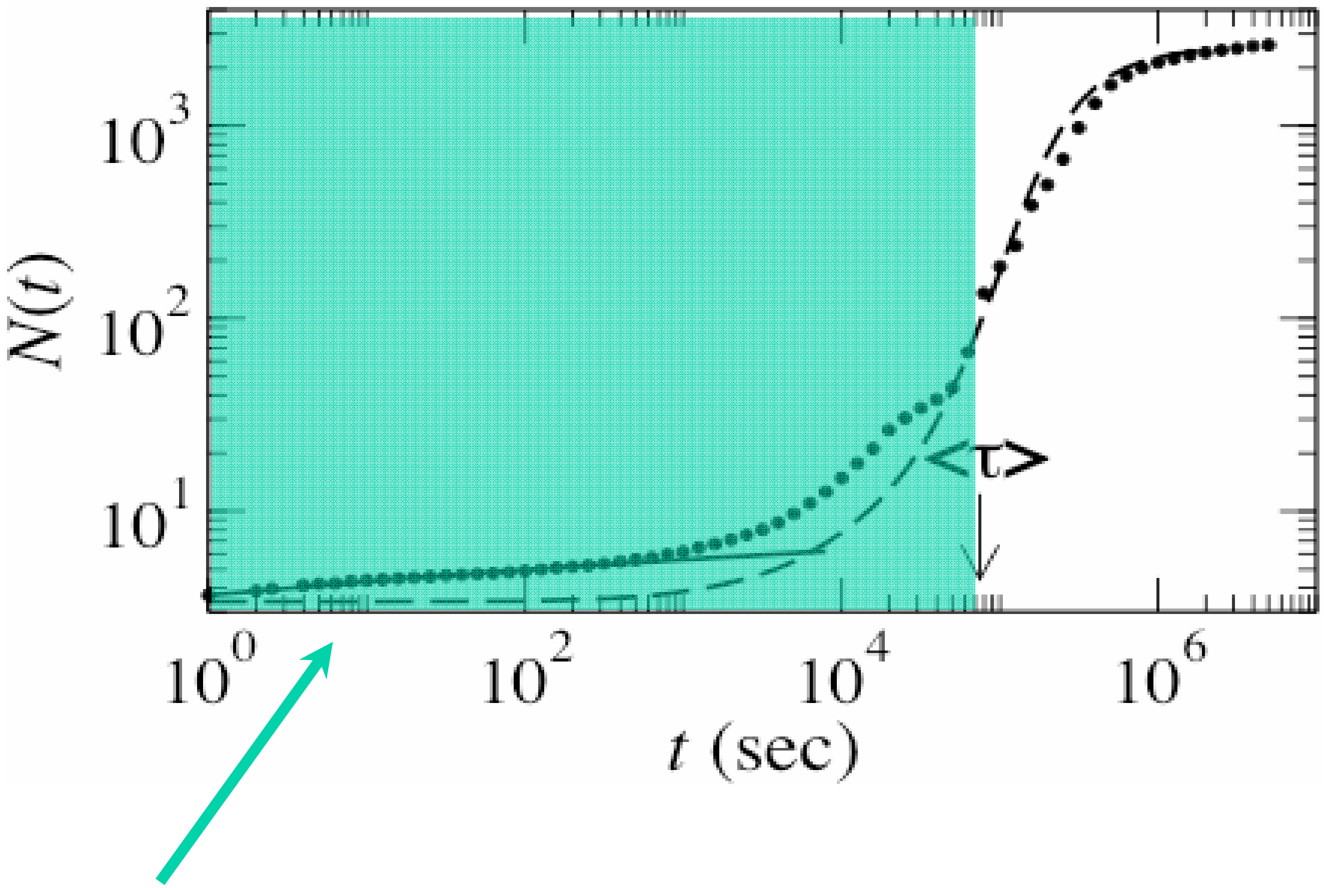
$$N(t) \approx \begin{cases} 1 + A \ln \frac{t}{\tau_0} & \alpha = 1 \\ 1 + A\left[1 - \left(\frac{t}{\tau_0}\right)^{1-\alpha}\right] & 1 < \alpha < 2 \end{cases}$$

Test: email viruses

3,188 users
3 month time interval

● pass a virus to a user, and follow its spread

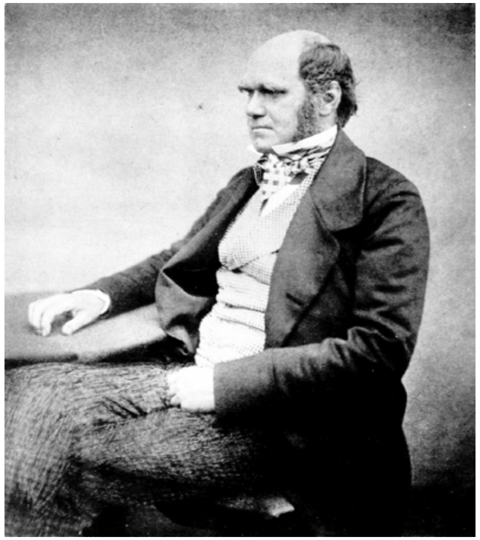
- - - Poisson timing, Same contacts as in The real data



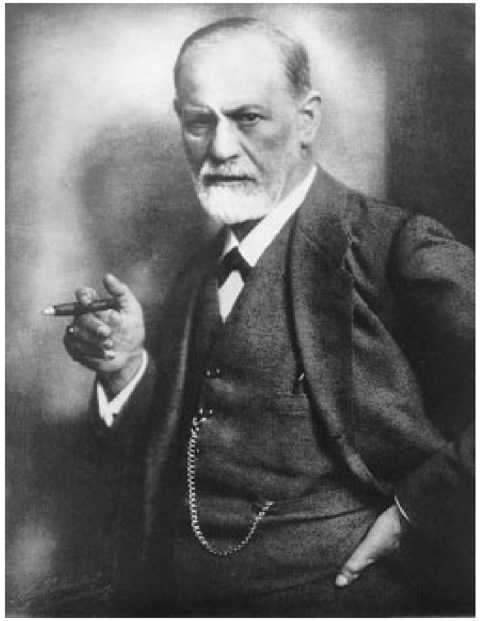
Pre-outbreak regime: $t < <\tau>$

Let’s have some fun…

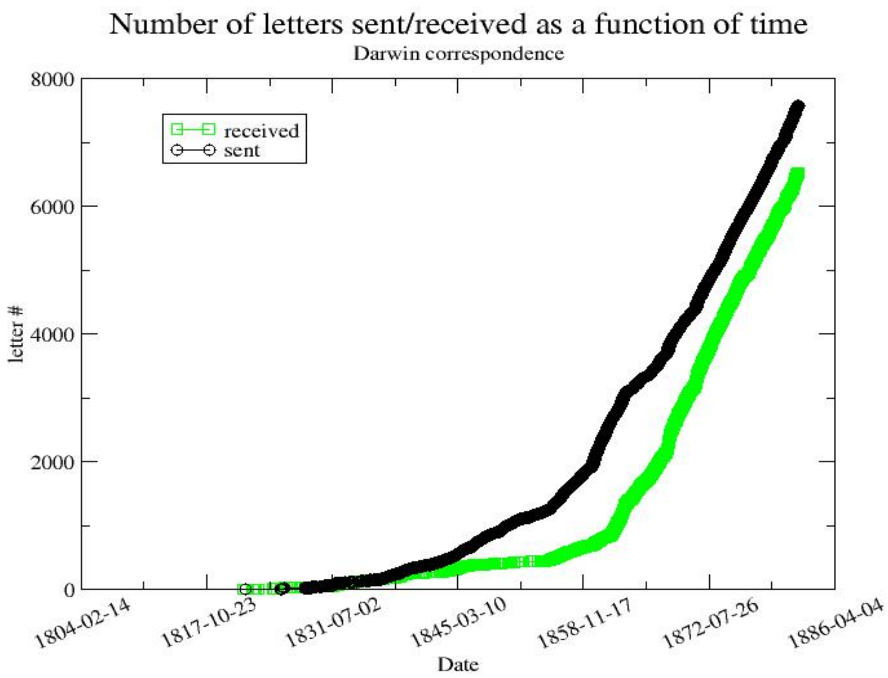
Charles Darwin (1809 – 1882)

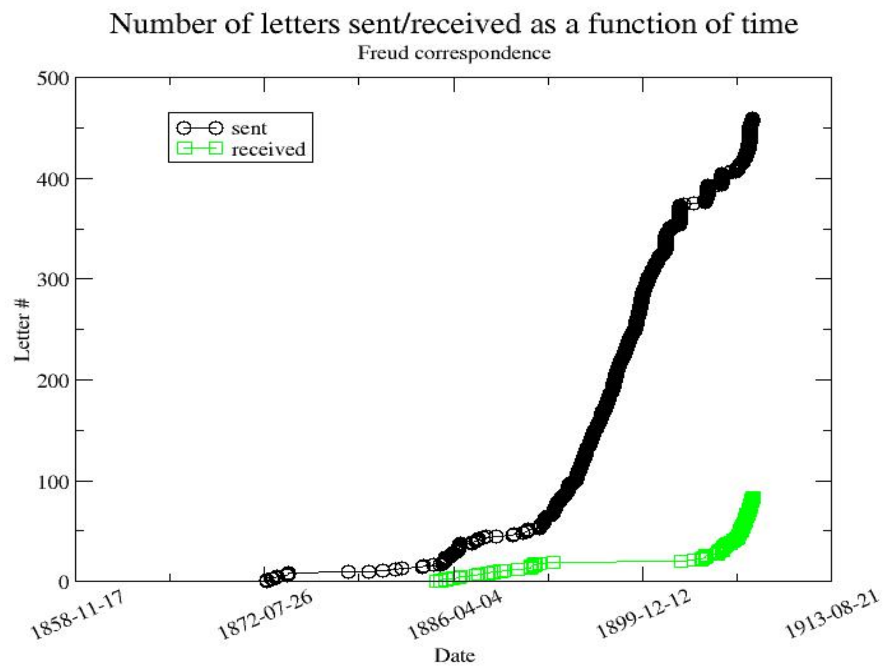


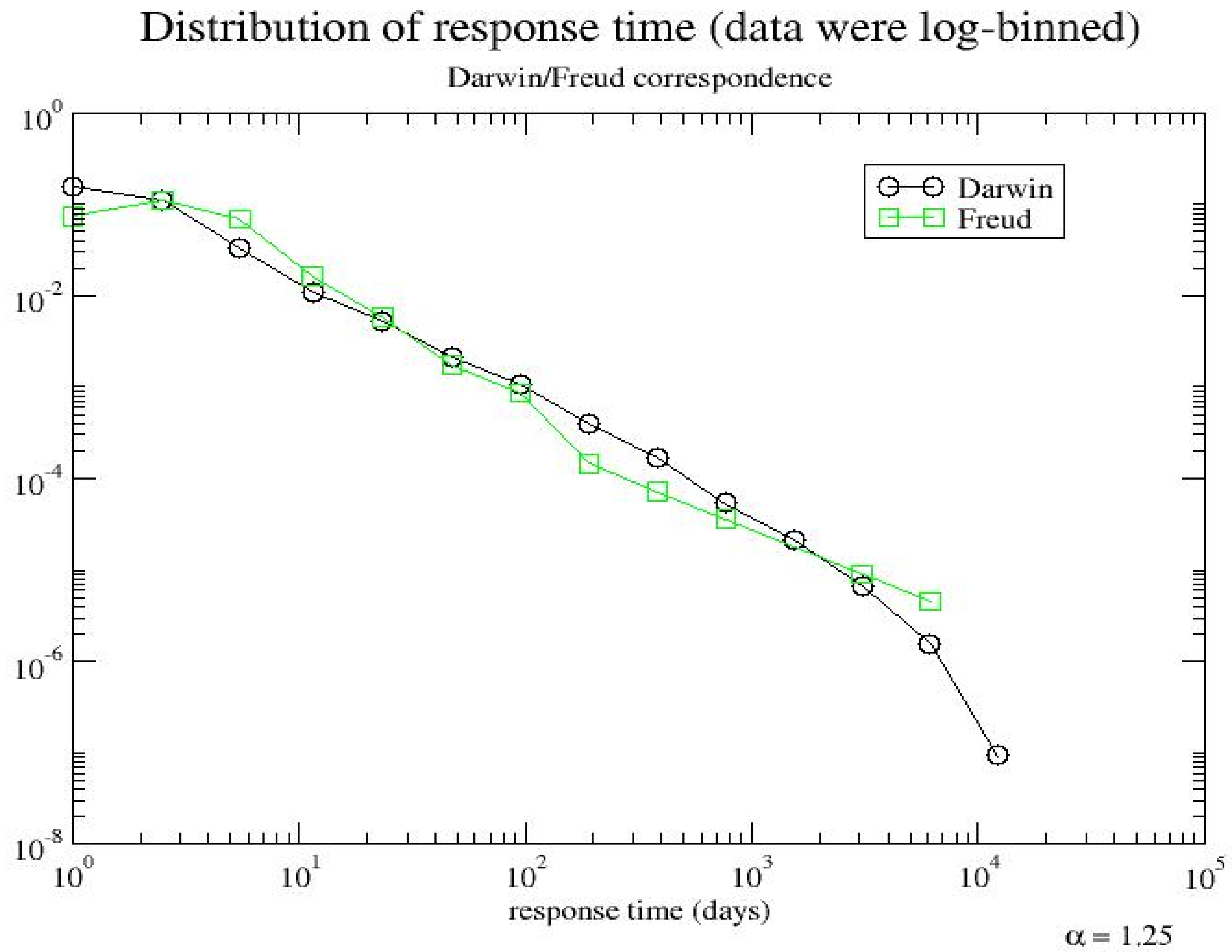
Sigmund Freud (1856 – 1939)



	Letters sent	Letters received
Darwin	7588	6529
Freud	460	87


Number of letters sent/received as a function of time
Darwin correspondence
received
sent
letter #
8000
6000
4000
2000
0
1804-02-14
1817-10-23
1831-07-02
1845-03-10
1858-11-17
1872-07-26
1886-04-04
Date


Number of letters sent/received as a function of time
Freud correspondence
sent
received
Letter #
500
400
300
200
100
0
1858-11-17
1872-07-26
1886-04-04
1899-12-12
1913-08-21
Date


Distribution of response time (data were log-binned)
Darwin/Freud correspondence
Darwin
Freud
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
10^0
10^1
10^2
10^3
10^4
10^5
response time (days)
$\alpha = 1.25$

Conclusions

•In human driven processes the waiting time distribution has a heavy tail with exponent $\alpha=1$

→ bursty activity pattern

•Origin: tasks queuing on the priority list have very uneven waiting time

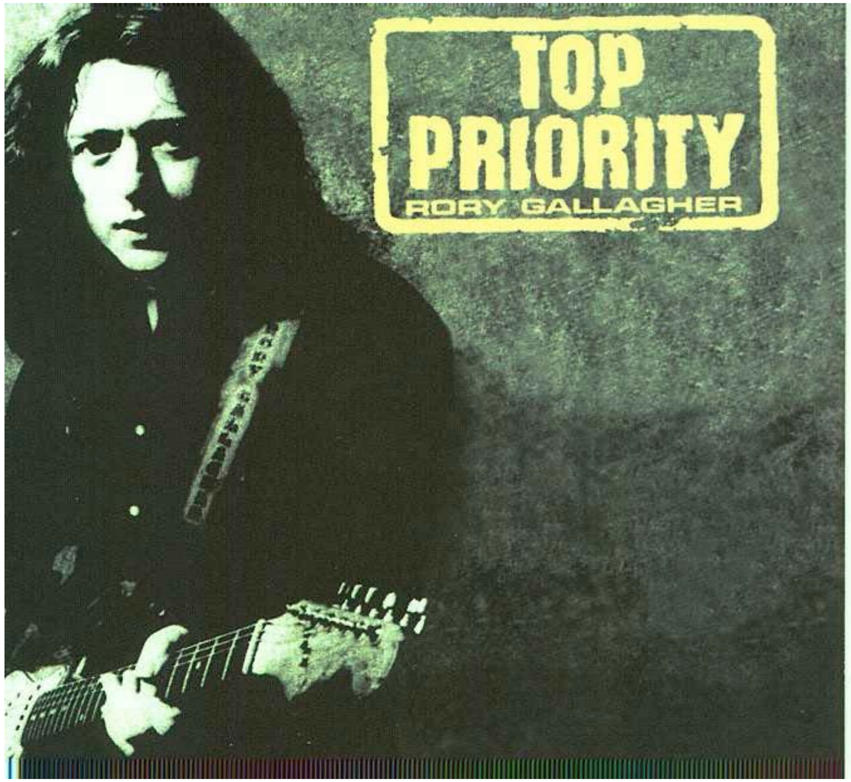
Two universality classes:

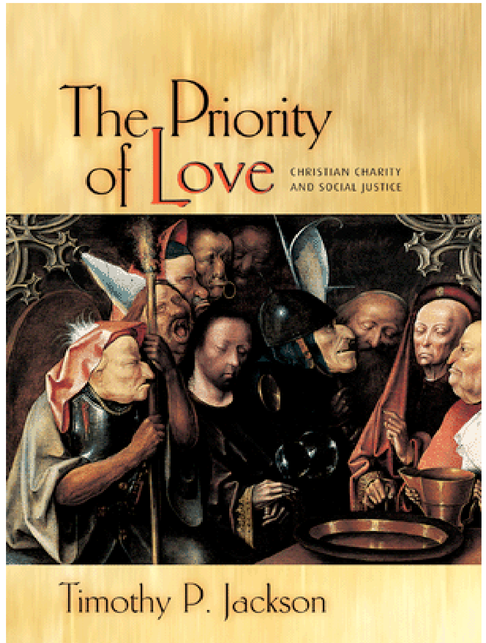
→ L fixed: $\alpha=1$

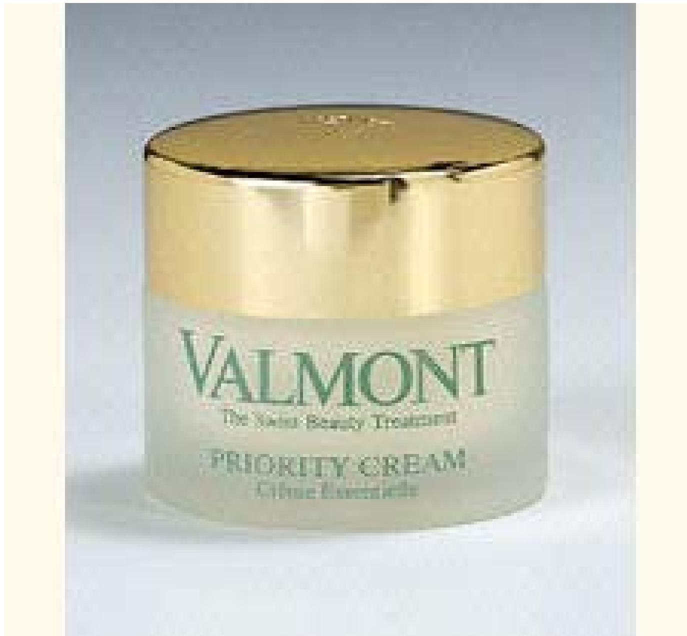
→ L fluctuates: $\alpha=3/2$

•Bursty contact timing alters the dynamics of pre-outbreak regime in epidemics

Priorities do matter…







But how do we model it…?

Execution time...

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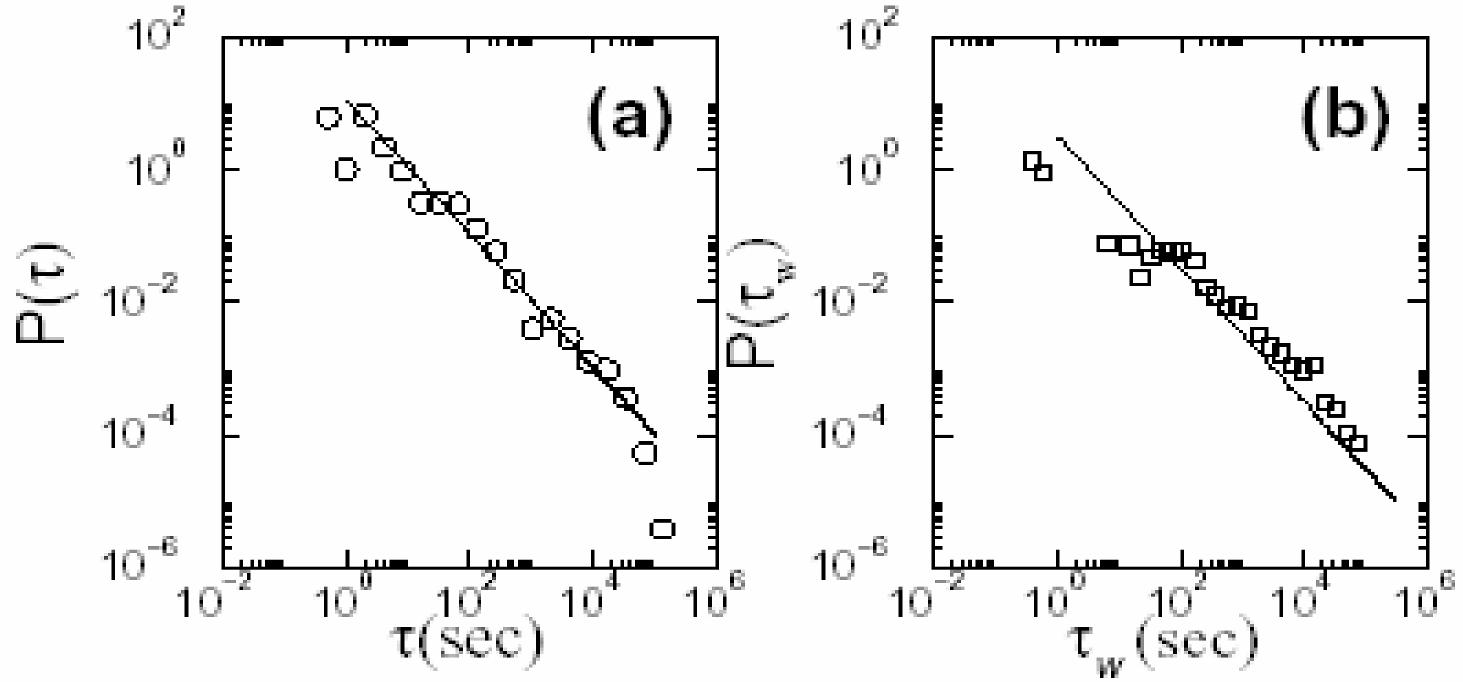
"My report took me five hours—one hour for research and typing and four hours to get my sister off the computer."

Emailing time vs. response time



Conclusions

- In many contact processes the waiting time distribution has a heavy tail.
- Bursty contact timing alters the dynamics of pre-outbreak regime
- Need for a better understanding of contact timing:
  - ➢Empirical measurements on the contact statistics for major viral processes
  - ➢Better analytical understanding of the impact of contact timing
  - ➢ Reevaluate the current epidemic models:
    a) What assumptions do they use for contact timing?
    b) Are their predictions sensitive to contact timing?