SMR.1656 - 30

School and Workshop on Structure and Function of Complex Networks

16 - 28 May 2005

The Emergence of Paradigm Setters through Firms Interaction and Network Formation

Rainer ANDERGASSEN
Universita' degli Studi di Bologna
Dipartimento di Scienze Economiche
Piazzle Scanavilli 2
40100 Bologna
ITALY

The emergence of paradigm setters through firms‘ interaction and network formation

Rainer Andergassen, Franco Nardini,
Massimo Ricottilli
University of Bologna (ITALY)

Problem

- Innovation dynamics
- Heterogeneous firms (for example, in-house innovative capability, knowledge base, ...)
- Bounded rationality: firms cannot observe all the characteristics of all other firms
- Learning process through in-house R&D activity and through networking (spill-overs)

Purpose of the paper

- Innovative capability (innovative fitness) due to learning, searching and gathering of information.
- Emergence of paradigm setters in a context of bounded rationality.
- Aggregate performance in terms of average aggregate innovative capability.

Outline of the model and assumptions

- We consider an economy populated by J firms.
- $V_i(t)$ indicates at time t firm i 's innovative capability (innovative fitness)

$$V_i(t) \begin{cases} C_i(t) \text{ ”in-house” innovative capability} \\ \text{interaction with other firms} \end{cases}$$

- $V(t)$ is the vector $V(t) = [V_i(t)]$,i = 1,2,…,J arraying the fitness of all firms in the economy.

Autonomous part of $V_i(t)$

- $C_i(t)$ due to cumulated "in-house" R&D activity
- $C_i(t)$ undergoes stochastic variations
- $C_i(t) \in U[0,1]$
- $C(t)$ is the vector $C(t) = [C_i(t)]$, i = 1,2,…,J arraying the "in-house" innovative capability of all firms in the economy.

Interaction part of $V_i(t)$

- It is due to searching activity
- Let a_{ij} indicate the part of each firm j 's total innovative fitness that can cognitively be passed on to firm i , $a_{ii} = 0$, for each i .
- Bounded rationality: firms are not able to scan/observe the entire population of firms in the economy.
- $B(t) = [b_{ij}(t)]$ is the proximity matrix, where each $b_{ij}(t) = 1$ or $b_{ij}(t) = 0$ according to whether neighbour j has been or hasn't been identified as a useful contributor.
- A different configuration of B(t) leads to a different endogenous multiplier.

Innovative capability

- We assume a linear functional form
- Innovative capability of firm i

$$V_i(t) = \sum_{j=1}^{J} a_{ij} b_{ij}(t) V_j(t) + C_i(t)$$

- Innovative capability $V(t) = [I - M(t)]^{-1} C(t)$

  where $M(t) = [a_{ij} b_{ij}(t)]$
- Changes in $b_{ij}(t)$ lead to a change in the multiplier $[I - M(t)]^{-1}$ of the "in-house" innovative capability and to a change in the innovative capabilities.
- Changes in $b_{ij}(t)$ are triggered by firms trying individually to improve their innovative capability through networking.

Neighbourhood structure and technological paradigm setters

- Network structure as a directed graph of nodes.
- Two types of neighbourhoods:

$$\left\{\begin{array}{c} \textit{inward}\ \text{neighbourhood} \\ \textit{outward}\ \text{neighbourhood} \end{array}\right.$$

Inward neighbourhood

- Inward neighbourhood is defined by the connections each firm establishes when observing other firms.
- Let $k_{i,in}$ be the number of connections of firm i
- Bounded rationality: $k_{i,in} << J$
- *Inward* neighbourhood of firm

$$\Gamma_i(t) = \{j : j = 1,\ldots,J \wedge b_{ij} = 1\}$$

- *Inward* neighbourhood evolves over time according to firms‘ effort to improve their own innovative capability.

Outward neighbourhood

- For each firm j it is defined by firms actually observing it:

$$\Psi_j(t) = \{i : i = 1, 2\ldots, J \wedge b_{ij} = 1\}$$

- The *Outward* neighbourhood changes passively as a consequence of firms trying to increase their innovative capability.
- The *Outward* neighbourhood determines the propagation capacity of each firm.
- The cardinality of $\Psi_j(t)$ defines its impact factor on the economy.

Emergence of technological paradigm setters

- We classify the population of firms according to their impact factor

Definition *Technological paradigm setters emerge when the probability of each rank of this impact factor is positive.*

Neighbourhood and network evolution

- The evolution of the network owes two basic determinants
  - Search routines
  - Exogenous changes "in-house" innovative capabilities $C(t)$.

Search routines

- Bounded rationality and *satisficing* procedures.
- Firm i tries to substitute the least contributing neighbour j.
- We allow two types of searching routines leading to neighbourhood $\Gamma'_i(t) \neq \Gamma_i(t)$: randomly drawing from
  - the pool of the neighbour's neighbours.
  - the remaining $J - k_{i,in} - 1$ members of the entire economy.
- The substitution occurs if the firms innovative capability increases $V_i(t) > V_i(t-1)$
- In each period of time one firm has the opportunity of changing its least contributing neighbour.

Exogenous changes in "in house" innovative capabilities $C(t)$

- We fix the mean-waiting time of exogenous changes of in-house innovative capabilities $C(t) : \mu$
- The firm is chosen at random and its C_i is randomly drawn from a uniform distribution, i.e. $C_i(t) \in U[0,1]$

Two different time scales

$$\text{Time scales}\begin{cases}\text{Network evolution through changes in edges}\\ \text{Exogenous changes in } C(t)\end{cases}$$

- Network evolution through changes in edges:
  - In each period of time a randomly seleceted firm has the opportunity of changing one of its neighbours
- Exogenous changes in "in-house" innovative capabilities $C(t)$:
  - Occurs with mean waiting time μ

Performance and emergence of paradigms setters

- Firms try to improve their innovative capability through networking.
- Overall, the innovative capability of each firm depends, ceteris paribus, on the network configuration, and on the ability of the economy to converge rapidly to the most efficient network.
- Networking decisions are taken at the individual firm level and are rationally bounded.
- Two search routines are considered:
  - randomly selecting from the pool of all firms
  - randomly selecting from the pool of neighbours' of neighbours
- The larger μ , the more time the economy has to find the most efficient network.

Parameters - Average innovative capability

- $k_{i,inn} = k_{inn}\ ,\ \forall i$
- $a_{ij} = a \leq \frac{1}{k_{inn}}\ ,\ \forall i \neq j\ ,\ a_{ii} = 0\ ,\ \forall i$.
- At time t, the average innovative capability is given by

$$v(t) = \frac{1}{J} \sum_{j=1}^{J} V_j(t)$$

Benchmarking: most efficient network I

It is obtained by ranking the element of the vector $C(t)$ in a decreasing way, obtaining a vector $\overline{C}(t)$. Thus

- $\overline{B} = \left\{ \overline{b}_{ij} = 1, \forall i \neq j \leq k_{inn} + 1, \overline{b}_{ij} = 1, \right.$
  $\left. \forall i > k_{inn} + 1, j \leq k_{inn} + 1 \text{ and } \overline{b}_{ij} = 0 \text{ for all other} \right\}$
- $\overline{M} = a\overline{B}$
- $\overline{V}(t) = [I - \overline{M}]^{-1}\overline{C}(t)$
- $\overline{v}(t) = \frac{1}{J}\sum_{j=1}^{J} \overline{V}_j(t)$

Benchmarking: most efficient network II

Example

$$\overline{B} = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Matrix $\overline{B}$ if $J = 7$ and $k_{inn} = 3$.

Benchmarking: Least efficient network I

It is obtained if each firm choses its neighbours at random from the pool of firms. Thus

- $\underline{\underline{B}} = \left\{ \underline{b}_{ij} = \frac{k_{inn}}{J-1}, \forall i \neq j, \underline{b}_{ij} = 0 \forall i \right\}$
- $\underline{\underline{M}} = a\underline{\underline{B}}$
- $\underline{V}(t) = [I - \underline{\underline{M}}]^{-1} C(t)$
- $\underline{v}(t) = \frac{1}{J} \sum_{j=1}^{J} \underline{V}_j(t)$

Benchmarking: Least efficient network II

Example

$$\underline{B} = \begin{bmatrix} 0 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0 & 0.5 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0 & 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 & 0 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0 & 0.5 \\ 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0 \end{bmatrix}$$

Matrix $\underline{B}$ if $J = 7$ and $k_{inn} = 3$.

Performance index

$$\phi(T) = \frac{1}{T}\sum_{t=1}^{T}\frac{v(t) - \underline{v}(t)}{\overline{v}(t) - \underline{v}(t)}$$

$$\phi(T) \in (-\infty, 1]$$

- if $\phi(T) = 0$, then, on average, performance is equivalent to random selection;
- if $\phi(T) = 1$, performance is equivalent to max performance

Simulation results

- Number of firms J = 64,   32 quantiles
- Number of *inward* neighbours $k_{inn} = 3$; $a_{ij} = a = 0.25$
- On average, each τ times a firm selects randomly from the pool of all firms
  - For $\tau = 1$ each time the firm selects randomly from the pool of all firms
  - For $\tau = \infty$ each time the firm selects randomly from the pool of the neighbours' neighbour's.
- We investigate the features of the model in the parameter space (μ, τ) :
  - Emergence of paradigm setters
  - Average aggregate performance of economic system

Emergence of Paradigm setters

- $\mu = 8$

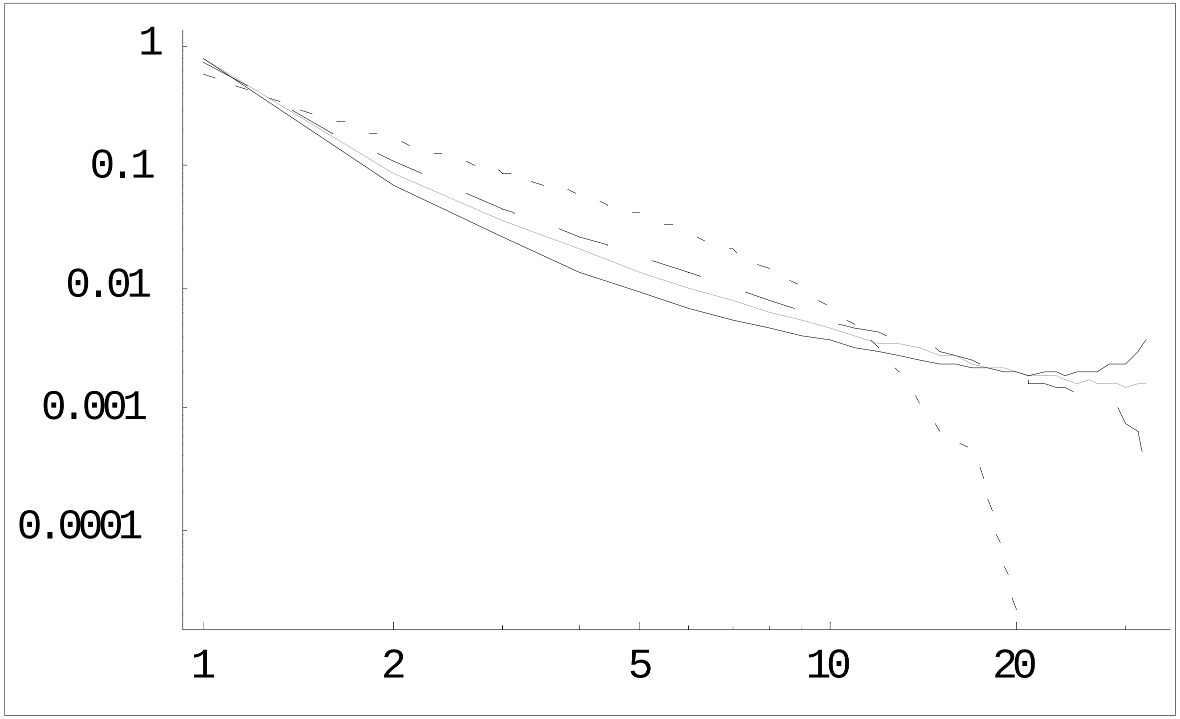

Figure 1: Average frequency distribution of quantiles per time unit, in log – log space; $\mu = 8$, $\tau = 1$ (dots), $\tau = 2$ (dashed line), $\tau = 3$ (gray line) and $\tau = 5$ (black line).

Emergence of Paradigm setters

- $\mu = 16$

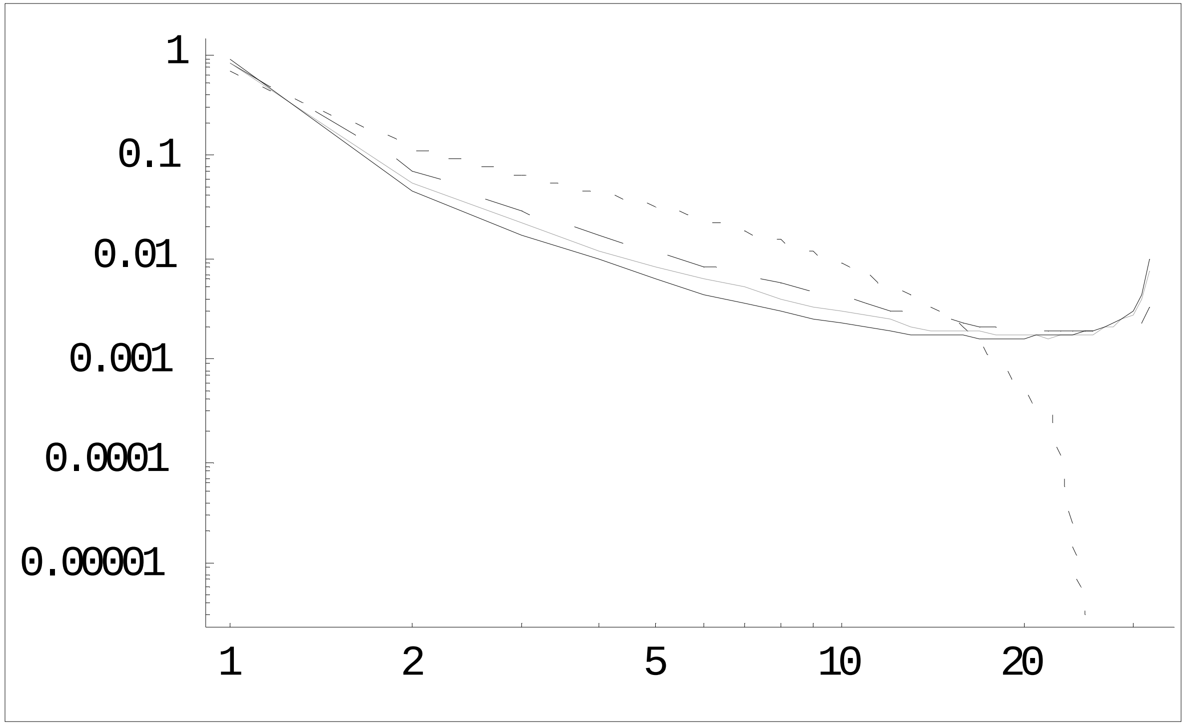

Figure 2: Average frequency distribution of quantiles per time unit, in the log – log space; $\mu = 16$, $\tau = 1$ (dots), $\tau = 2$ (dashed line), $\tau = 3$ (gray line) and $\tau = 5$ (black line).

Emergence of Paradigm setters

- $\mu = 32$

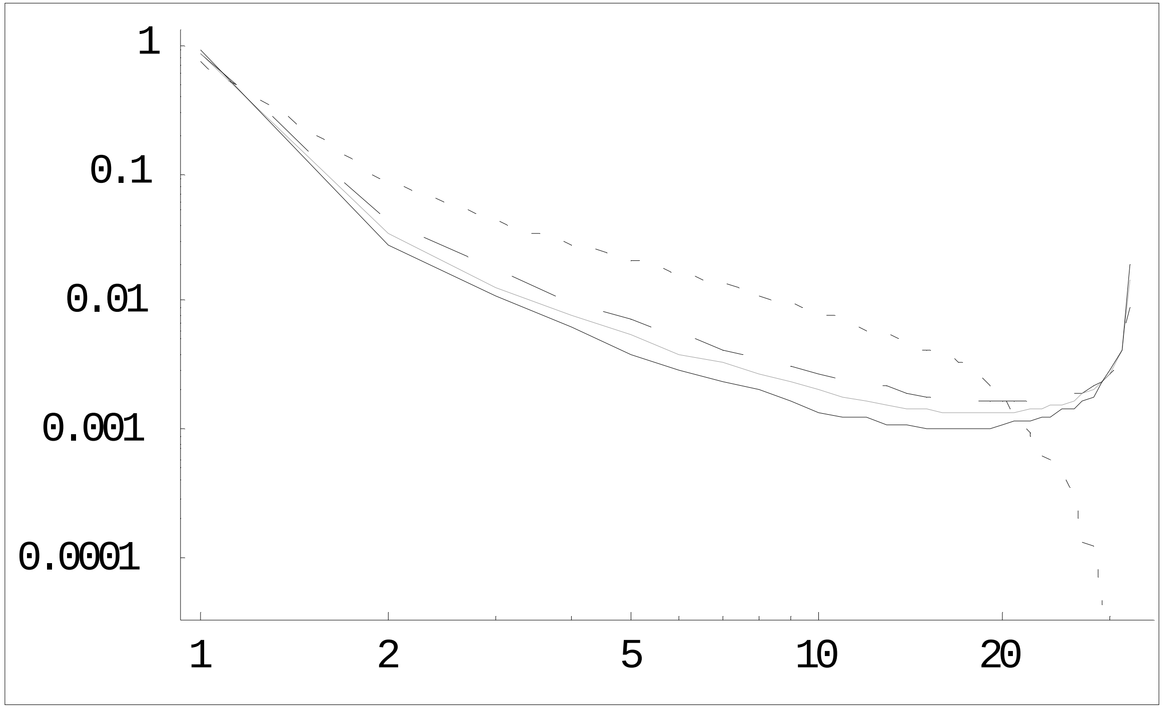

Figure 4: Average frequency distribution of quantiles per time unit, the $\log - \log$ space; $\mu = 32$, $\tau = 1$ (dots), $\tau = 2$ (dashed line), $\tau = 3$ (gray line) and $\tau = 5$ (black line).

Performance index $\phi(T)$ as a function of τ

- For $\tau \rightarrow \infty$, $\phi(T) \rightarrow 0$, for each value of μ

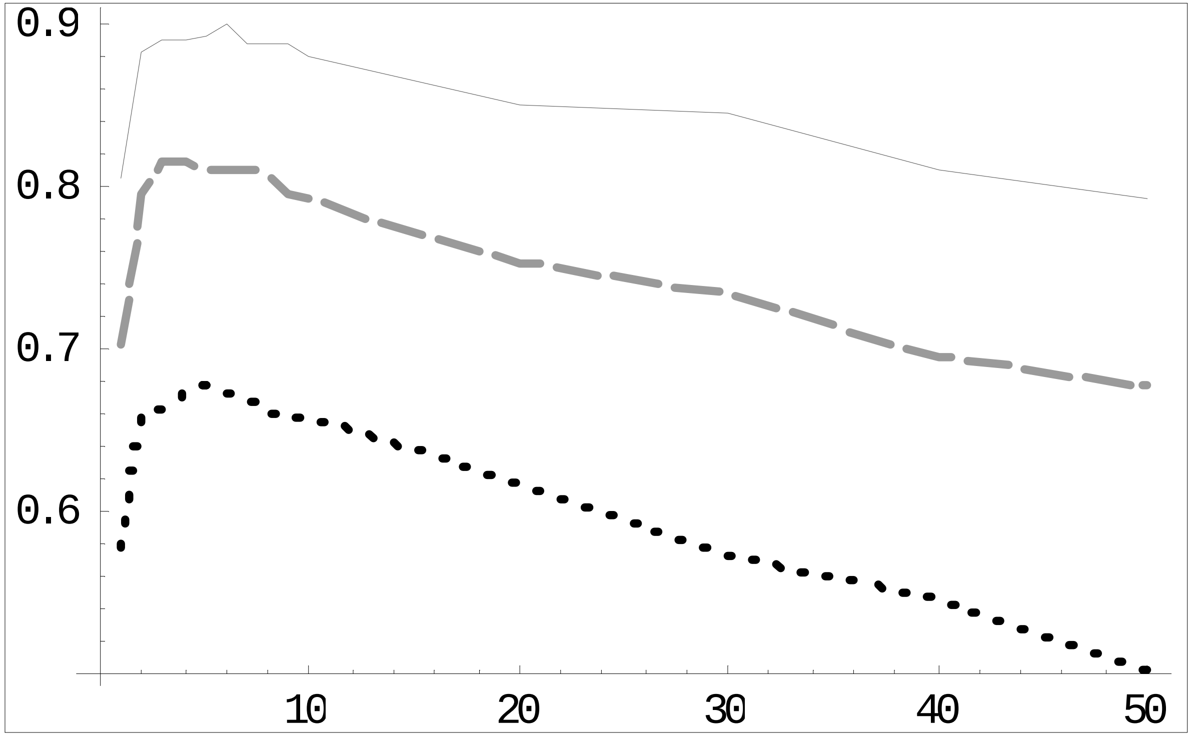

Figure 9: $\phi(T)$ Efficiency index $\phi(T)$ as a function of τ for mean-waiting time $\mu = 8$ (dots), $\mu = 16$ (dashed line) and $\mu = 32$ (continuous line).

Conclusions

A larger μ increases

- the probability that technological paradigm setters emerge
- the efficiency of the system

A larger τ

- increases the probability that technological paradigm setters emerge
- increases the persistence of technological paradigm setters
- has an ambiguous effect on the efficiency of the system
  - the higher concentration (due to the increased persistence of paradigm setters) reduces the cost of search
  - higher concentration leads to a possible lock-in into an inefficient neighbourhood