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Structure and Function of Complex Networks

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Modeling Strategic Formation of Social Networks

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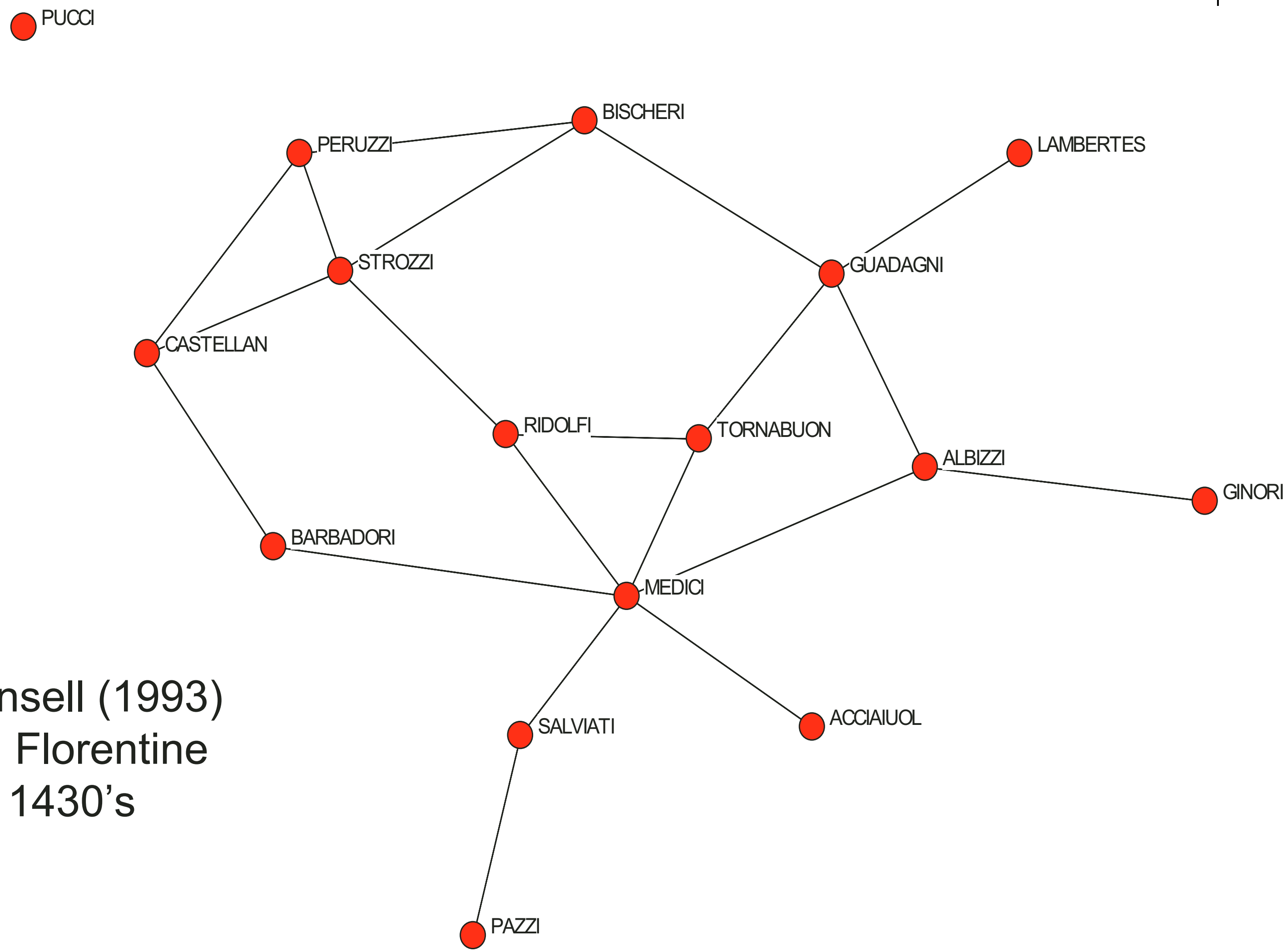
Trieste Summer School on Networks

Example of Social Network with Strategic Formation



Padgett -Ansell (1993) [from Kent] Florentine Marriages, 1430’s

What do we know?

- Networks are prevalent
  - Job contact networks, crime, trade, politics, ...
- Network position and structure matters
  - rich sociology literature
  - Padgett example – Medicis not the wealthiest nor the strongest politically, but the most central
- ``Social'' Networks have special characteristics
  - small worlds, degree distributions...

Questions:

- How does network structure affect interaction and behavior? (tomorrow, Tuesday)

- Which networks form?
  - random modeling
  - Game theoretic reasoning

- When do efficient networks form?
  - Intervention - design incentives?

Economic/Game Theoretic Modeling - distinguishing ingredients

- Welfare analysis – agents get utility from networks
- Decision making agents form links and/or choose actions

Notation

- {1,..,i,...,j,..,n} – nodes or players
- g - network, n by n matrix with entries 0 or 1
- ij in g iff $g_{ij}=1$
- g+ij – add link/edge ij to g; g-ij – delete link ij from g
- $u_i(g)$ utility to i from network g

Efficiency and Stability

(Jackson and Wolinsky (1996))

- Efficiency:
  - argmax $\sum u_i(g)$
- Pairwise Stable networks:
  - $u_i(g) \geq u_i(g\text{-}ij)$ for each i and ij in g
  - $u_i(g+ij) > u_i(g)$ implies $u_j(g+ij) < u_j(g)$ for each ij not in g

``Connections Model''

- $0 \leq \delta_{ij} \leq 1$ a benefit parameter for i from path connection between i and j

- $0 \leq c_{ij}$ cost to i of link to j

- d(i,j) shortest path length between i,j

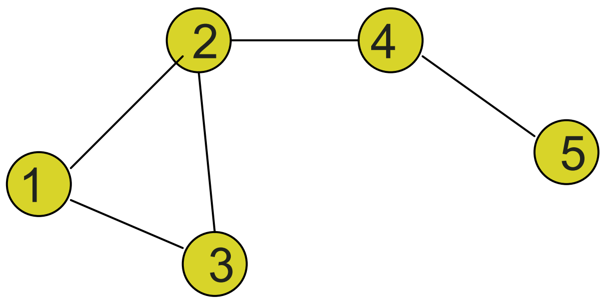
$$u_i(g) = \sum_j \delta_{ij}^{d(i,j)} - \sum_{j \text{ in } N_i(g)} c_{ij}$$

Example: Symmetric Connections Model

- benefit from a friend is $\delta<1$
- benefit from a friend of a friend is δ^2,...
- cost of a link is $c>0$

$u_2 = 3\delta + \delta^2 - 3c$

$u_1 = 2\delta + \delta^2 + \delta^3 - 2c$



$u_5 = \delta + \delta^2 + 2\delta^3 - c$

Efficient Networks

- low cost: $c < \delta - \delta^2$
  - complete network is efficient

- medium cost: $\delta - \delta^2 < c < \delta + (n-2)\delta^2/2$
  - star network is efficient

- high cost: $\delta + (n-2)\delta^2/2 < c$
  - empty network is efficient

``Proof''

- $c < \delta - \delta^2$ - obvious that complete is efficient
- $\delta - \delta^2 < c$
  - players who are connected should be connected in a star:
    - minimal number of links to connect
    - connection at length 2 is more valuable than at 1 ($\delta - c < \delta^2$)
  - value of a star is
    - $2(n-1)\delta + (n-1)(n-2)\delta^2 - 2(n-1)c = 2(n-1)[\delta + (n-2)\delta^2/2 - c]$
  - Star of size m+n more valuable than star(m)+star(n)
  - Star has positive value only when $c < \delta + (n-2)\delta^2/2$

Pairwise Stable Networks:

- low cost: $c < \delta - \delta^2$
  - complete network is pairwise stable

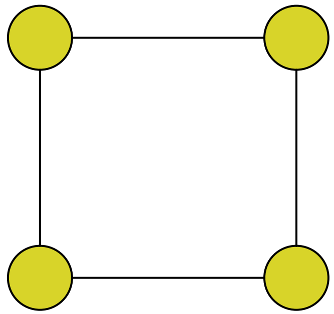
- medium/low cost: $\delta - \delta^2 < c < \delta$
  - star network is pairwise stable
  - others are also pairwise stable

- medium/high cost: $\delta < c < \delta + (n-2)\delta^2/2$
  - star network is not pairwise stable (no loose ends)
  - nonempty pairwise stable networks are over-connected and may include too few agents

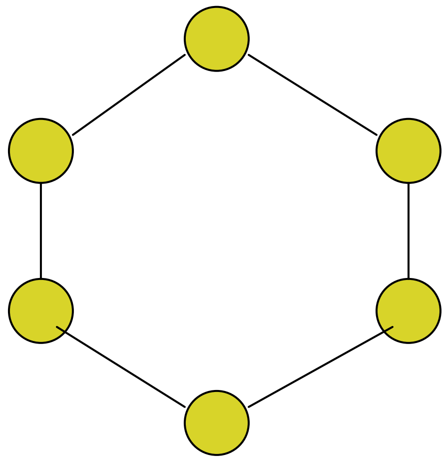
- high cost: $\delta + (n-2)\delta^2/2 < c$
  - empty network is pairwise stable

Example – Pairwise Stable but not efficient

- $\delta - \delta^2 < c < \delta - \delta^3$, n=4



- $\delta - \delta^3 < c < (\delta+\delta^2+\delta^3)(1- \delta^2)$



(unique nonempty pairwise stable structure if $\delta < c < (\delta+\delta^2+\delta^3)(1- \delta^2)$, n=6)

Directed Connections

[Bala and Goyal (2000)]

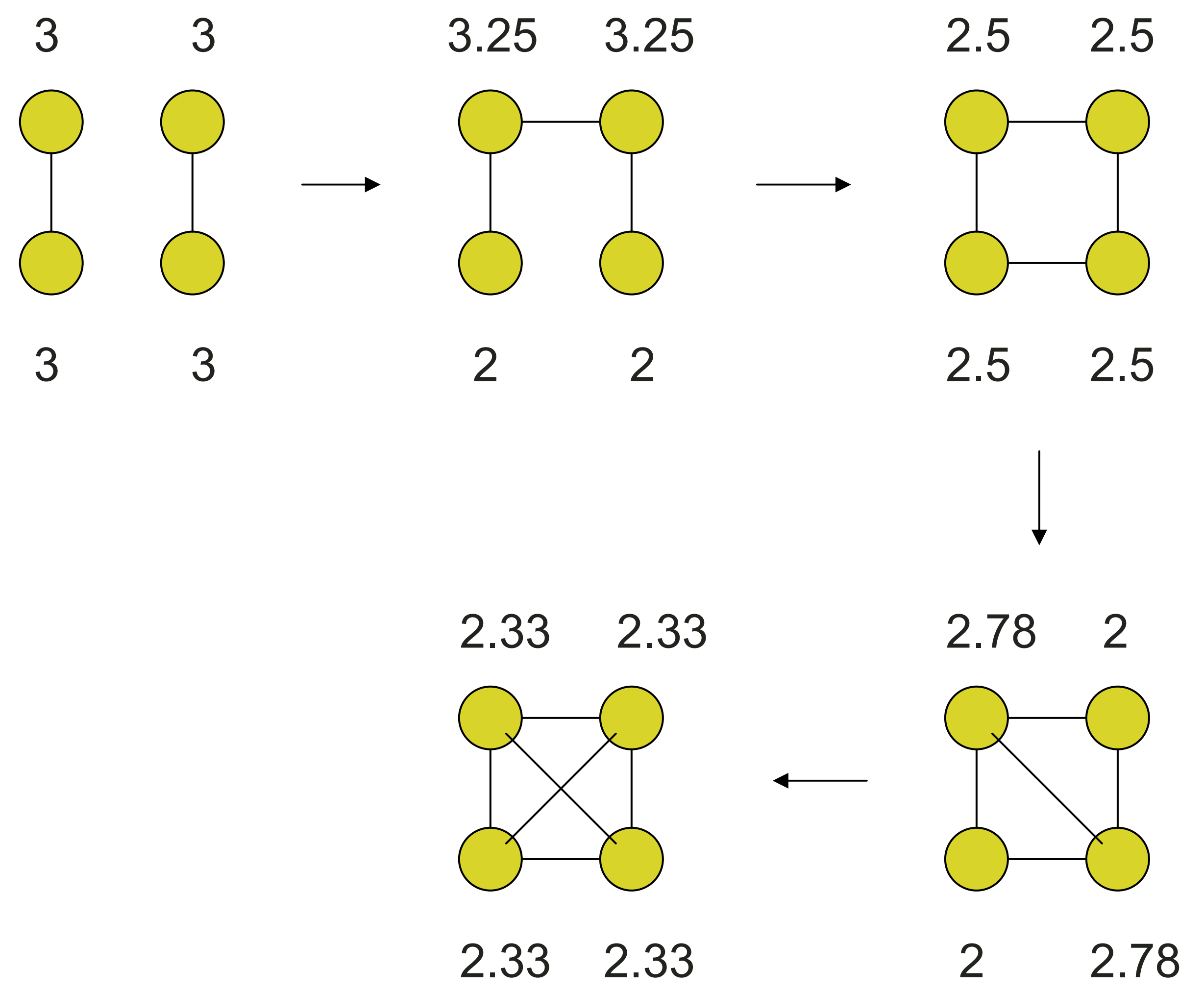
- same payoffs as before except
  - directed network and
    - one way flow – link is only useful to whom incurs cost
    - two way flow – one player pays, but link is useful to both
- Now links are formed unilaterally
  - use Nash equilibrium to model stability

Two way flow

- Efficiency as before, except c/2 and link in either direction (but not both)
- Nash Stable:
  - low cost: $c < \delta-\delta^2$
    - two-way ``complete'' networks are pairwise stable
  - medium/low cost: $\delta-\delta^2 < c < \delta$
    - all star networks are pairwise stable, plus others
  - medium/high cost: $\delta < c < \delta+(n-2)\delta^2/2$
    - peripherally sponsored star networks are stable (no other stars, but sometimes other networks)
  - efficient and stable can be empty:
    - $\delta-\delta^2 < c < 2(\delta- \delta^2)$ complete is efficient, not equilibrium

A toy co-author model

- $u_i(g)=\sum_{j:\ ij\ in\ g} [1/n_i + 1/n_j + 1/(n_i\ n_j)]$
- n is even:
  - efficient networks: pairs
  - pairwise stable networks: completely connected components, where for any two components, one has more than the square of the number of nodes in the other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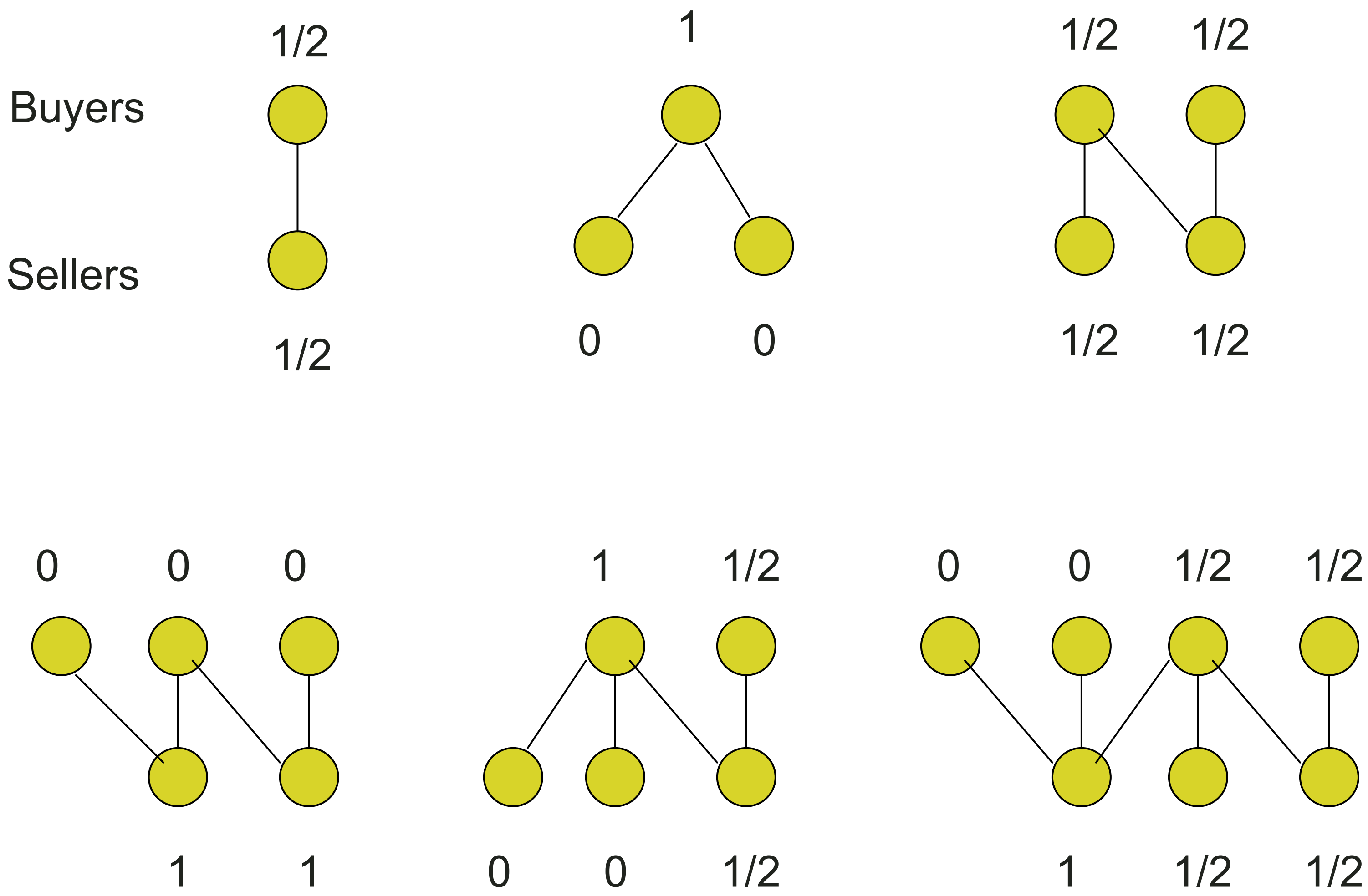
Some Settings stable=efficient

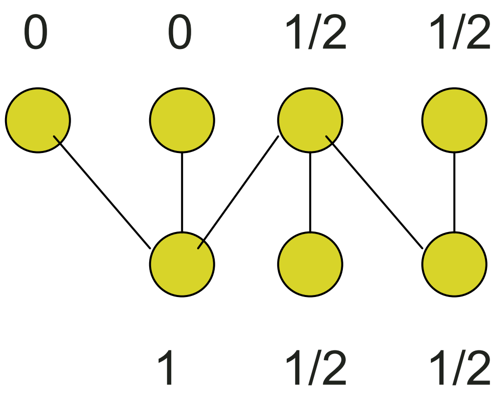
Buyer-Seller Networks: Corominas Bosch (2002):

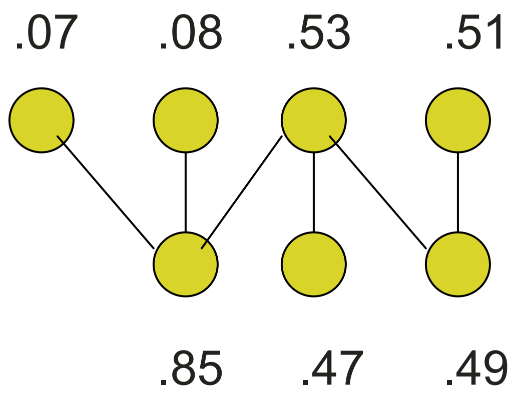
- Sellers each with one identical object value 0
- Buyers each desire one object at value 1
- Alternating offers bargaining

Payoffs as function of network



Experiments
Charness, C-B, and Frechette





(normalized from their 2500 base and 200 reservation value)

general algorithm for solution

- is there group of at least two sellers connected to just one buyer?
  - if so, remove, buyer gets 1, sellers 0
- is there group of at least k+1 sellers connected to just k buyers?
  - if so, remove, buyers get 1, sellers 0
- iterate
- repeat with buyers/sellers reversed
- remaining players get 1/2

Pairwise stable = efficient

[I] buyer gets 1 implies some linked seller gets 0 (and vice versa)

[II] component – all buyers get ½ if and only if for all k and subsets of k buyers they are linked to at least k distinct sellers and vice versa

- pairwise stable and cost to link implies no 0's. so by [I] must be that all players get 1/2
- if not pairs find subnetwork that links them into pairs – extra links can be deleted and still satisfy [II], same payoff at lower cost

- If bargaining is such that split is not 0,1 in ``unbalanced'' networks, then could get inefficiency

Stable and Efficient only coincide in special cases

- Can transfers help in other cases?
- What can we say about when conflict exists
- What can we say about transfers helping?
- What about other formation processes?

Transfers can help

- Change utilities from $u_i(g)$ to $u_i(g)+t_i(g)$
  - $\sum_{i \text{ in } C} t_i(g) = 0$ within each component C
  - respect anonymity
- E.g., peripheral players pay center of star in connections model to maintain connections

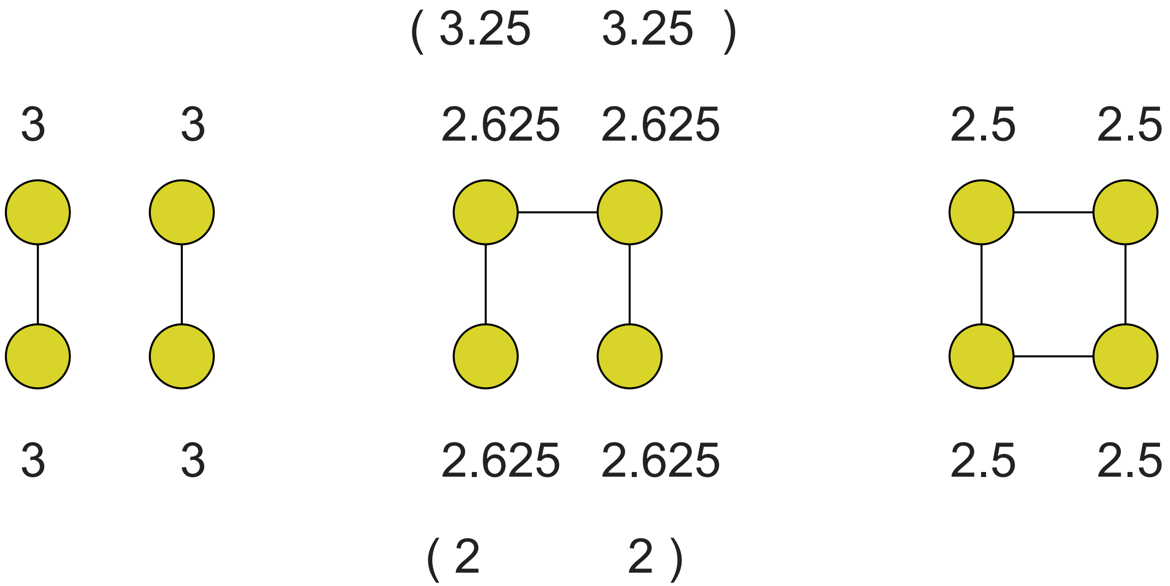
Can Prove: Transfers can lead to at least one efficient network being pairwise stable provided:

- v is anonymous, and
- in some efficient network all nodes have degree >1.

[ugly proof]

Transfers in Co-author - equalizing works

[tax on having a second link]

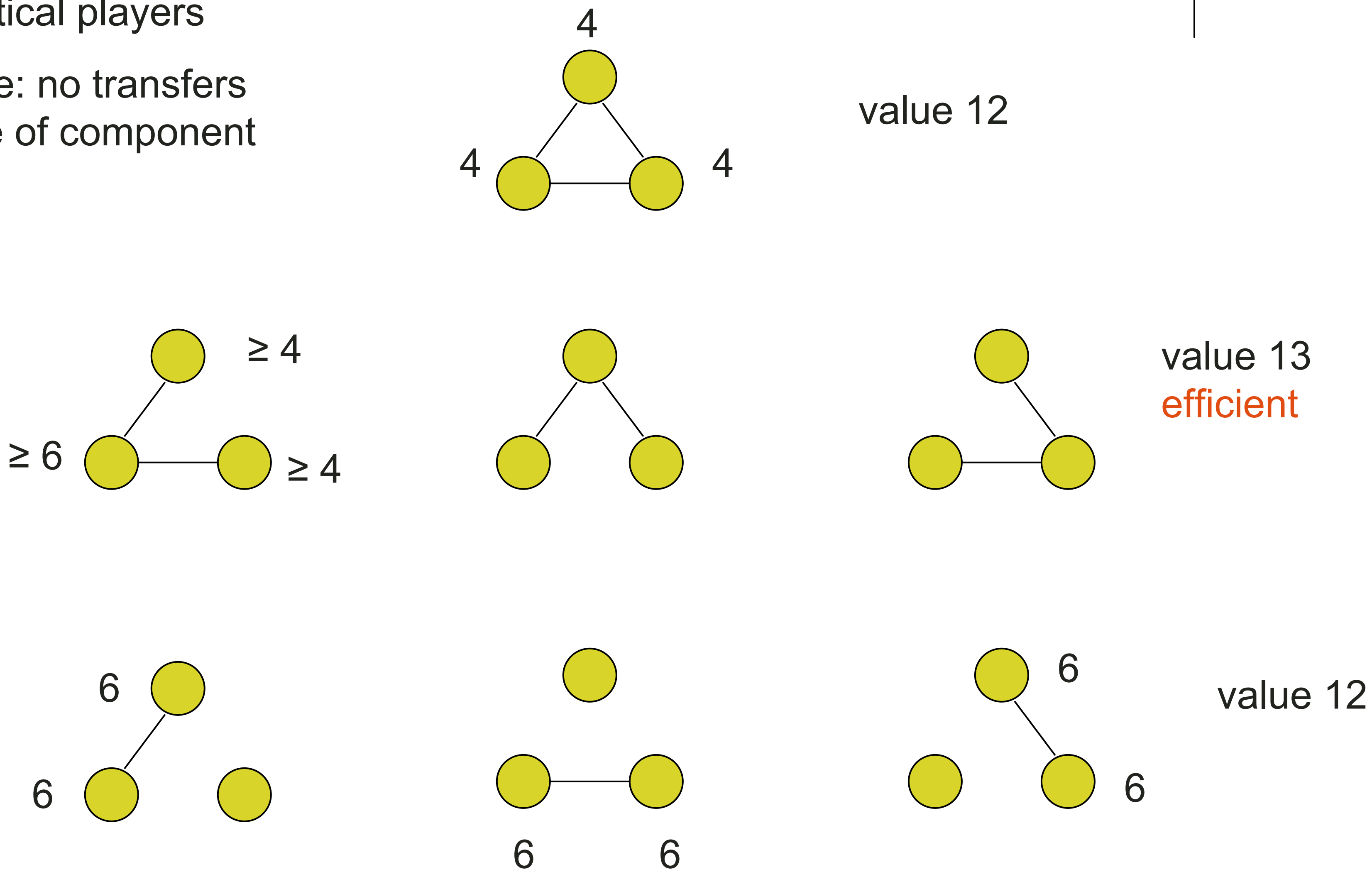


But note: need to charge players who form links - not just subsidize (here penalizing for forming links)

Transfers cannot always help

anonymity: same transfers to identical players

balance: no transfers outside of component



What is needed to avoid this?

- Break Anonymity
- Make transfers part of formation process or allow transfers to be contingent on network

Bargaining when forming links Can Help

- Currarini and Morelli (2000)
  - Order players (breaks anonymity)
  - Player demands payoff and suggests links
  - Link is formed if both players announce it, and component is feasible in terms of demands

Bargaining in the example

- 1 announces {2,3} and demands 6.5
  - 1 cannot ask for more than x>6.5 or 2 can respond with {3} and slightly below x and will get x and 1 will get nothing
- 2 announces {1} and demands 6.5
  - 2 cannot say {3} (or {1} or {1,3}) and ask for more than 6.5
- 3 announces {1} and demands 0
  - 3 cannot do better

If v(g+ij)> v(g) whenever g+ij has more components than g, then all (subgame perfect) equilibria of the above game lead to efficient networks.

- Game is contrived – introduces asymmetries, requires endpoint
- But ability to endogenize transfers as part of the formation process is important

Can economic models match observables?

- Small worlds derived from costs/benefits
  - low costs to local links – high clustering
  - high value to distant connections – low diameter
  - high cost of distant connections – few distant links

Geographic Connections (Johnson-Gilles (2000), Carayol-Roux (2003), Galeotti-Goyal-Kamphorst (2004), ...)

Islands connections model (Jackson-Rogers (2004))

- players live on islands
- cost c of link to player on the island
- cost C>c of link to player on another island

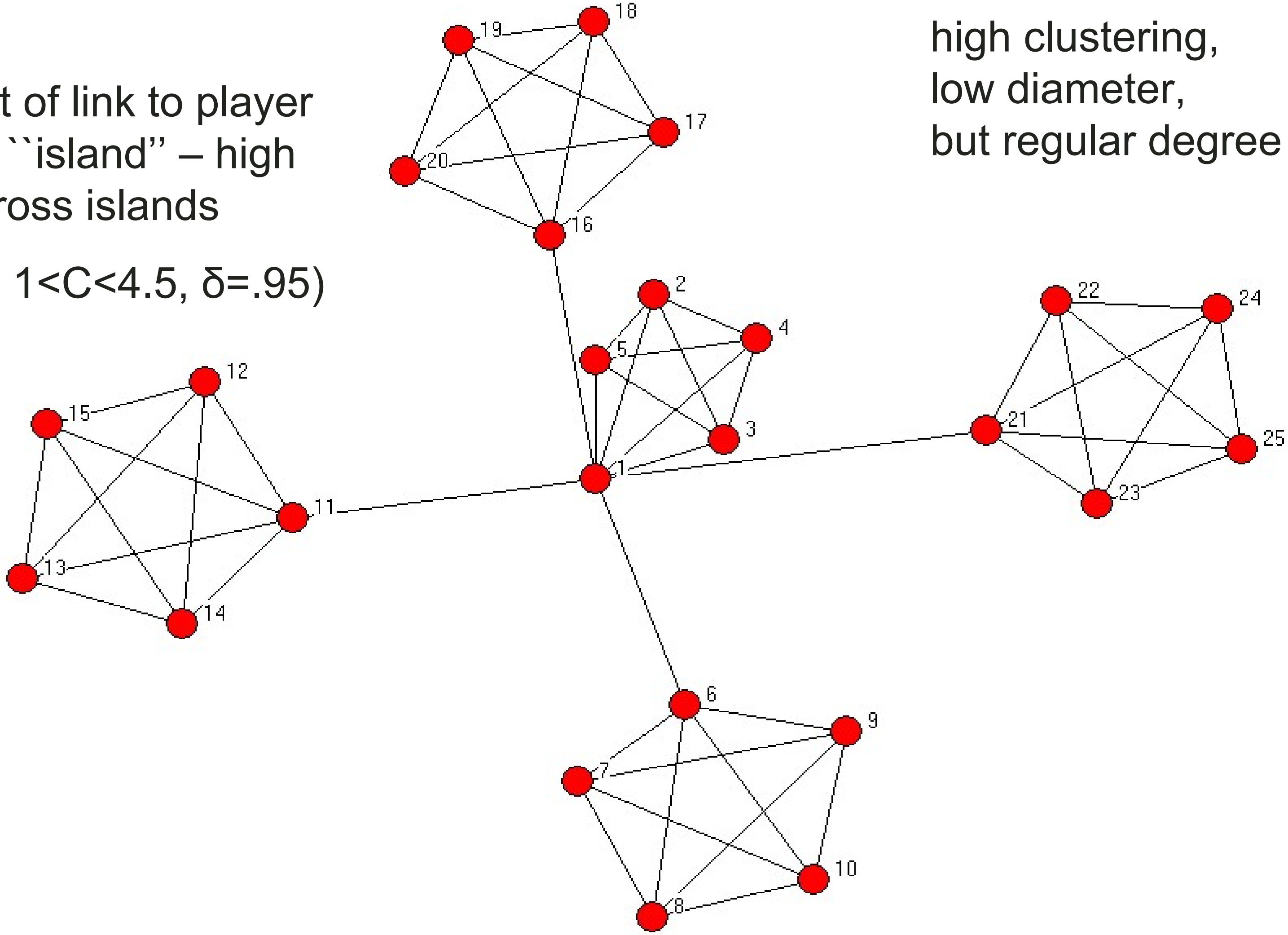
Results:

- High clustering within islands, few links across
- small distances

low cost of link to player on own ``island'' – high cost across islands

(c<.04, 1<C<4.5, δ=.95)

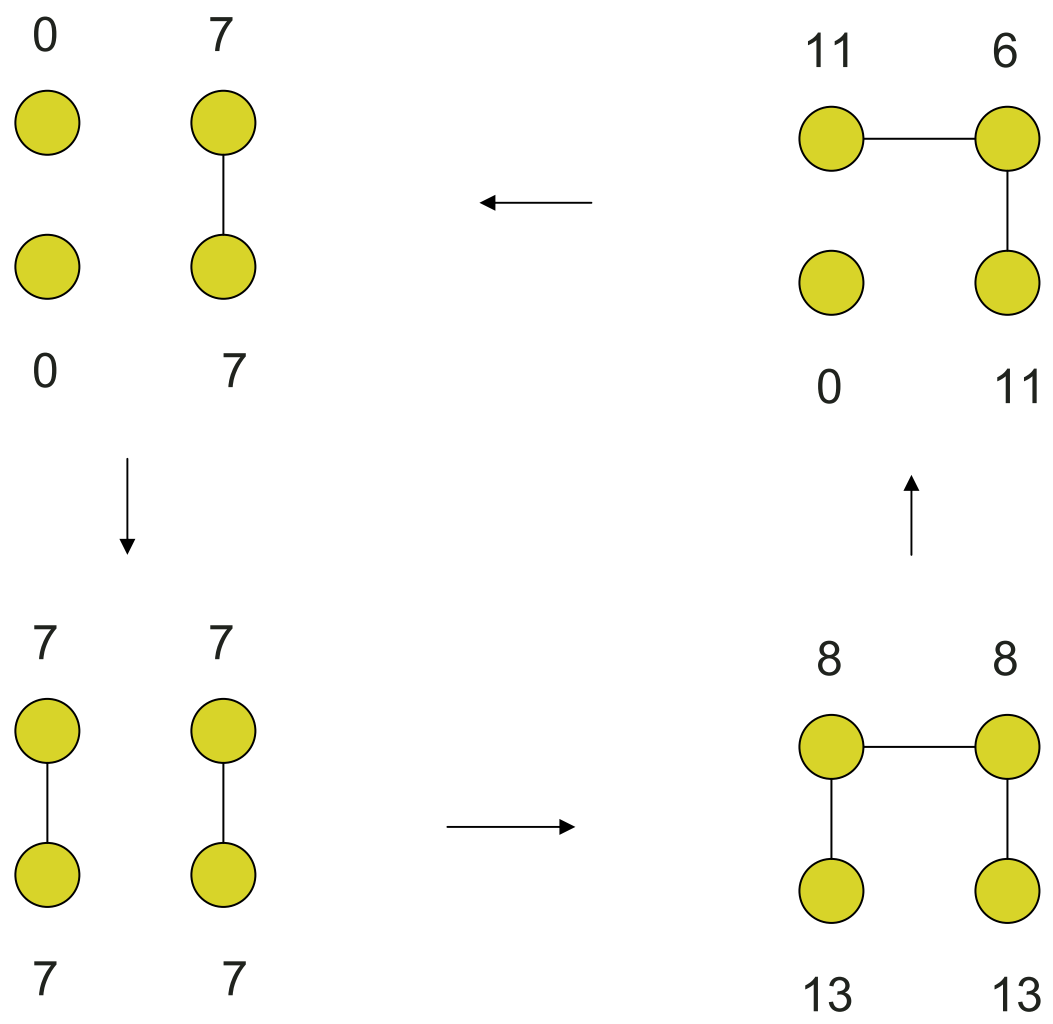
high clustering,
low diameter,
but regular degree



Rich literature on strategic formation

- loosen anonymity (Dutta-Mutuswami (1997))
- directed networks (Bala-Goyal (2000), Dutta-Jackson (2000),...)
- bargaining when forming links (Currarini-Morelli(2000), Slikker-van den Nouweland (2000), Mutuswami-Winter(2002), Bloch-Jackson (2004))
- dynamic models (Aumann-Myerson (1988), Watts (2001), Jackson-Watts (2002ab), Goyal-Vega-Redondo (2004), Feri (2004), Lopez-Pintado (2004),...)
- farsighted models (Page-Wooders-Kamat (2003), Dutta-Ghosal-Ray (2003), Deroian (2003),...)
- allocating value (Myerson (1977), Meessen (1988), Borm-Owen-Tijs (1992), van den Nouweland (1993), Qin (1996), Jackson-Wolinsky (1996), Slikker (2000), Jackson (2005)...)
- modeling stability (Dutta-Mutuswami (1997), Jackson-van den Nouweland (2000), Gilles-Sarangi (2003ab), Calvo-Armengol and Ikilic (2004),...)
- experiments (Callander-Plott (2001), Corbae-Duffy (2001), Pantz-Zeigelmeyer (2003), Charness-Corominas-Bosch-Frechette (2001), Falk-Kosfeld (2003), ...)

Nonexistence of Pairwise Stable



(Lending Model
Jackson-Watts (2002))

Advantages of an economic approach

- Payoffs allow for a welfare analysis
  - Identify tradeoffs – incentives versus efficiency
- Tie the nature of externalities to network formation...
- Put network structures in context - outcomes of network interaction
- Account for (and *explain*) some observables

What’s missing from Game theoretic formation models?

- Stark network structures emerge
  - need to mix with random models
- over-emphasize choice versus chance determinants for some *large* applications?

Models of Networks in Context

- crime networks (Glaeser-Sacerdote-Scheinkman (1996), Ballester, Calvo, Zenou (2003),...)
- markets (Kirman (1997), Tesfatsion (1997), Weisbach-Kirman-Herreiner (2000), Kranton-Minehart (2002), Corominas-Bosch (2005), Wang-Watts (2002), Galeotti (2005),Kakade et al (2005)...)
- labor networks (Boorman (1975), Montgomery (1991, 1994), Calvo (2000), Arrow-Borzekowski (2002), Calvo-Jackson (2004ab,2005), Cahuc-Fontaine (2004),...)
- insurance (Fafchamps-Lund (2000), DeWeerdt (2002), Bloch-Genicot-Ray (2004),...
- IO (Bloch (2001), Goyal-Moraga (2001), Goyal-Joshi (2001), Belleflamme-Bloch (2002),Billard-Bravard (2002), ...)
- international trade (Casella-Rauch (2001), Furusawa-Konishi (2003),...)
- public goods (Bramoulle-Kranton (2004), Galeotti and Vega (2005),...)
- airlines (Starr-Stinchcombe (1992), Hendricks-Piccione-Tan (1995))
- network externalities in goods (Katz-Shapiro (1985), Economides (1989, 1991) , Sharkey (1991)...)
- organization structure (Radner (), Radner-van Zandt (), Demange (2004)...)
- learning (Bala-Goyal (1998), Morris (2000), DeMarzo-Vayanos-Zweibel (2003), Gale-Kariv (2003), Choi-Gale-Kariv (2004),...)

Whither now?

- Bridging random/mechanical – economic/strategic
- Networks in Applications - predictions of behavior as dependent on network structure
  - Labor, mobility, voting, trade, collaboration, crime, public goods, ...
- Empirical/Experimental
  - many case studies lack key economic variables, tie networks to outcomes, utilities
  - enrich modeling of social interactions from a structural perspective
- Furthering game theoretic modeling, equilibrium, dynamics,...
- Foundations and Tools– centrality, power, allocation rules, community structures, ...

Setting stable=efficient depends on who bears link cost

Buyer-Seller Networks: Kranton-Minehart (2002):

- Sellers each with one identical object
- Buyers each desire one object, private valuation
- links only costly to buyers
- sellers hold simultaneous ascending auctions

Example: values iid U[0,1], 1 seller

	Each buyer's expected utility	Seller's expected utility	Total social value
n buyers	1/[n(n+1)]	(n-1)/(n+1)	n/(n+1)
n+1 buyers	1/[(n+1)(n+2)]	n/(n+2)	(n+1)/(n+2)
change	-2/[n(n+1)(n+2)]	2/[(n+1)(n+2)]	1/[(n+1)(n+2)]