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School and Workshop on Structure and Function of Complex Networks

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Communities and Dynamics

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Communities and dynamics

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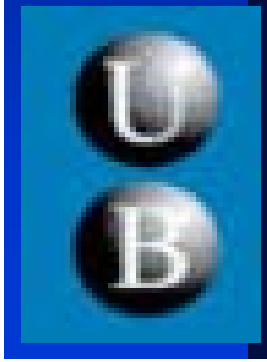
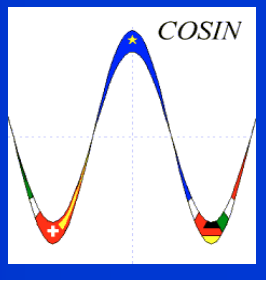
http://complex.ffn.ub.es/





Outline

- Communities in networks
- Search and congestion
- Searching within communities
- More dynamics
- Conclusions

Communities in networks

- Analysis of modular structures in networks
- Definition: subsets of nodes that are more densely linked, when compared with the rest of the network
- Community detection:
  - From computer scientists
  - To statistical physicists (Girvan-Newman, PNAS 99, 7821, 2002)

Evaluating community identification

- Modularity:

$$Q = \sum_i \left(e_{ii} - a_i^2 \right)$$

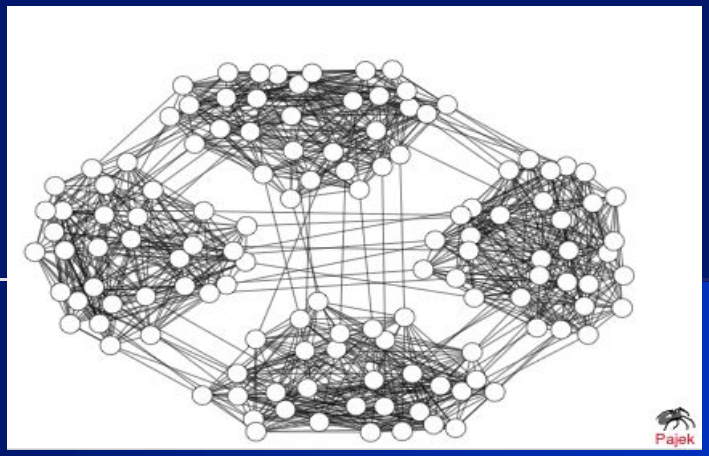
- e_{ij}: fraction of total links starting at a node in partition i and ending at a node in partition j
- a_i: fraction of links connected to i
- a_i^2: number of intracommunity links

Methods of community identification

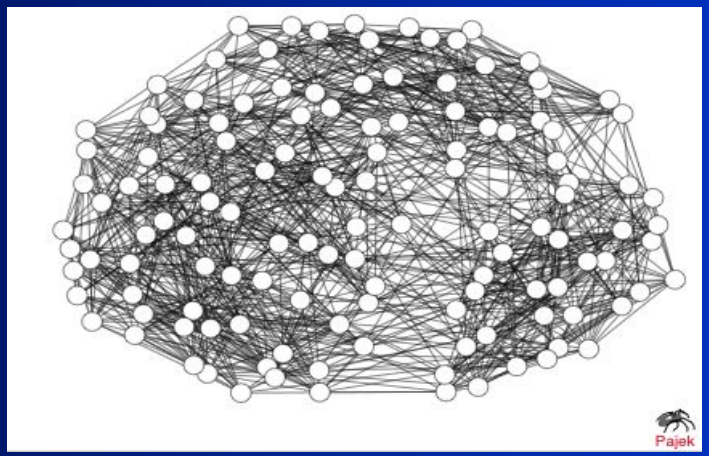
- cond-mat/0505245 to appear in COSIN book
  - Link removal methods
  - Agglomerative methods
  - Maximizing modularity
  - Spectral analysis methods
  - ……

Comparing algorithms

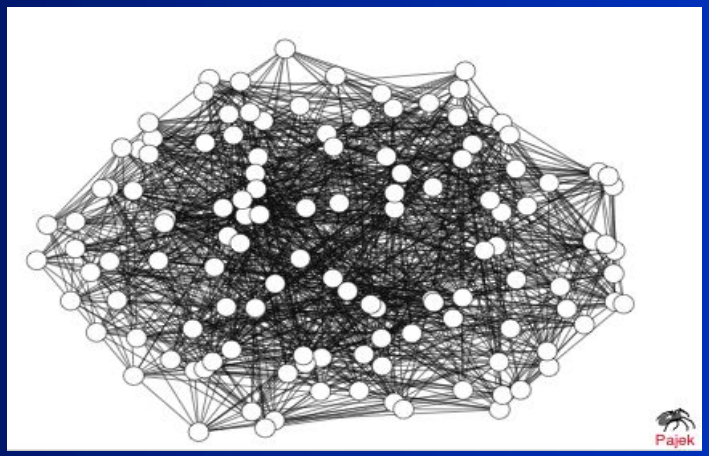
- *ad-hoc* networks (Newman-Girvan, PRE 69, 026113, 2004)
  - 128 nodes
  - 4 communities of 32 nodes each
  - Each node has 16 links:
    - z_{in} internal nodes within the community
    - z_{out} nodes out of its community



$z_{in}=15$



$z_{in}=12$



$z_{in}=8$

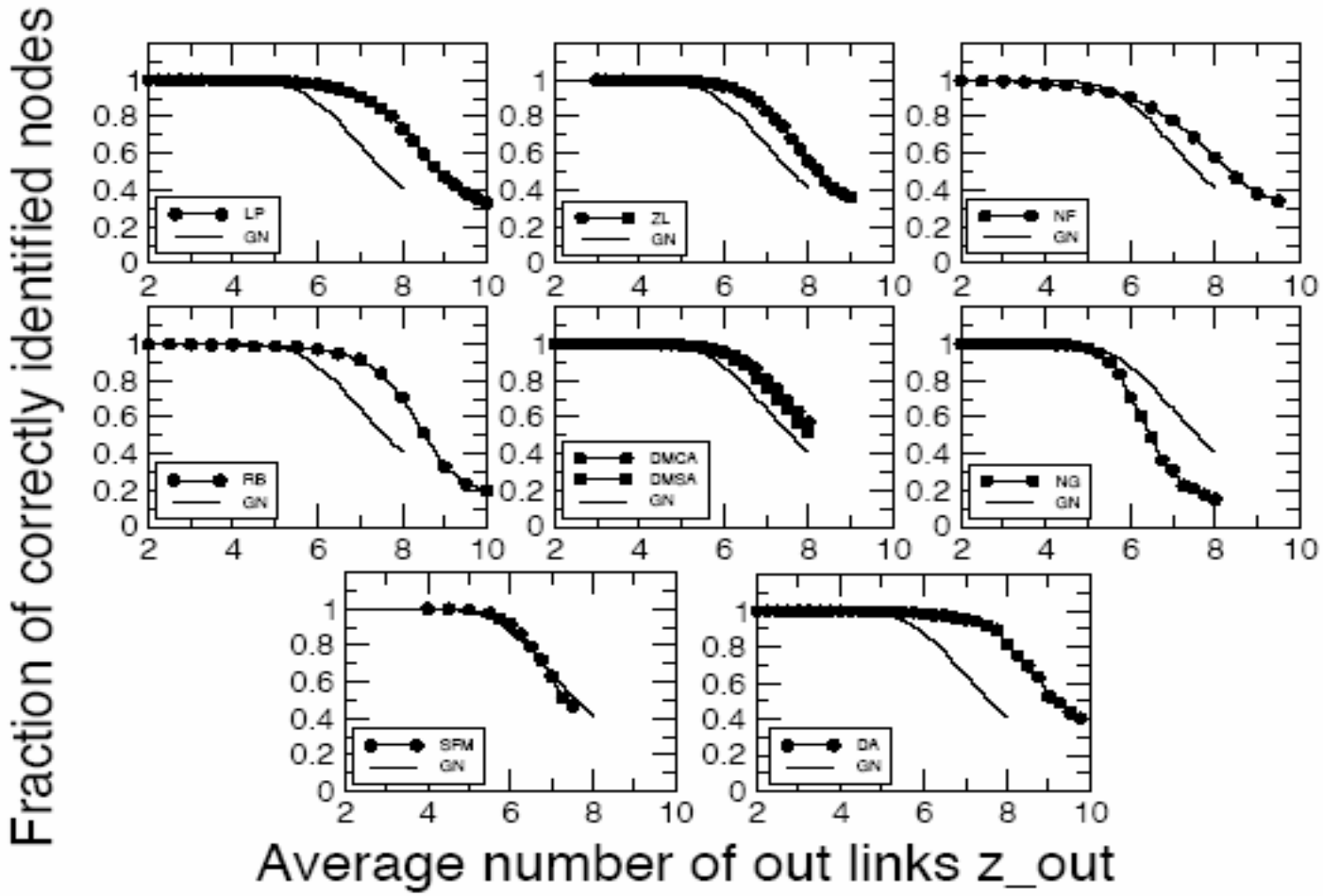


Figure 7: Comparing algorithm sensitivity using *ad hoc* networks with predetermined community structure with $n = 128$, the network divided into four communities with 32 nodes each and total average degree of 16. The x-axis is the average number of connections to outside communities z_{out} and the y-axis is the fraction of nodes correctly identified by the method.

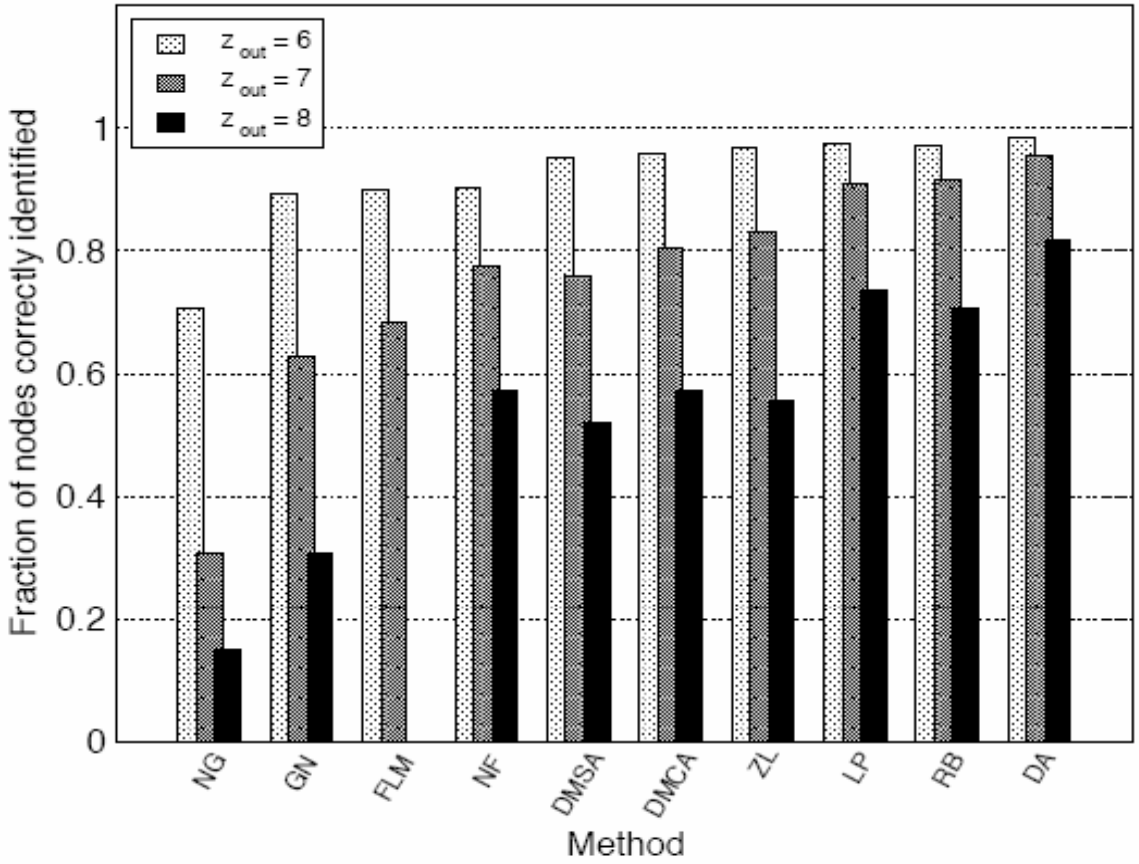


Figure 8: The fraction of correctly identified nodes at three specific values of z_{out}, 6, 7 and 8 for all available methods. Here we can see that most of the methods are very good at finding the "correct" community structure for values of z_{out} up to 6. At $z_{out} = 7$ some methods begin to falter but most still identify more than half of the nodes correctly. At $z_{out} = 8$, only three methods are still able to identify the correct structure.

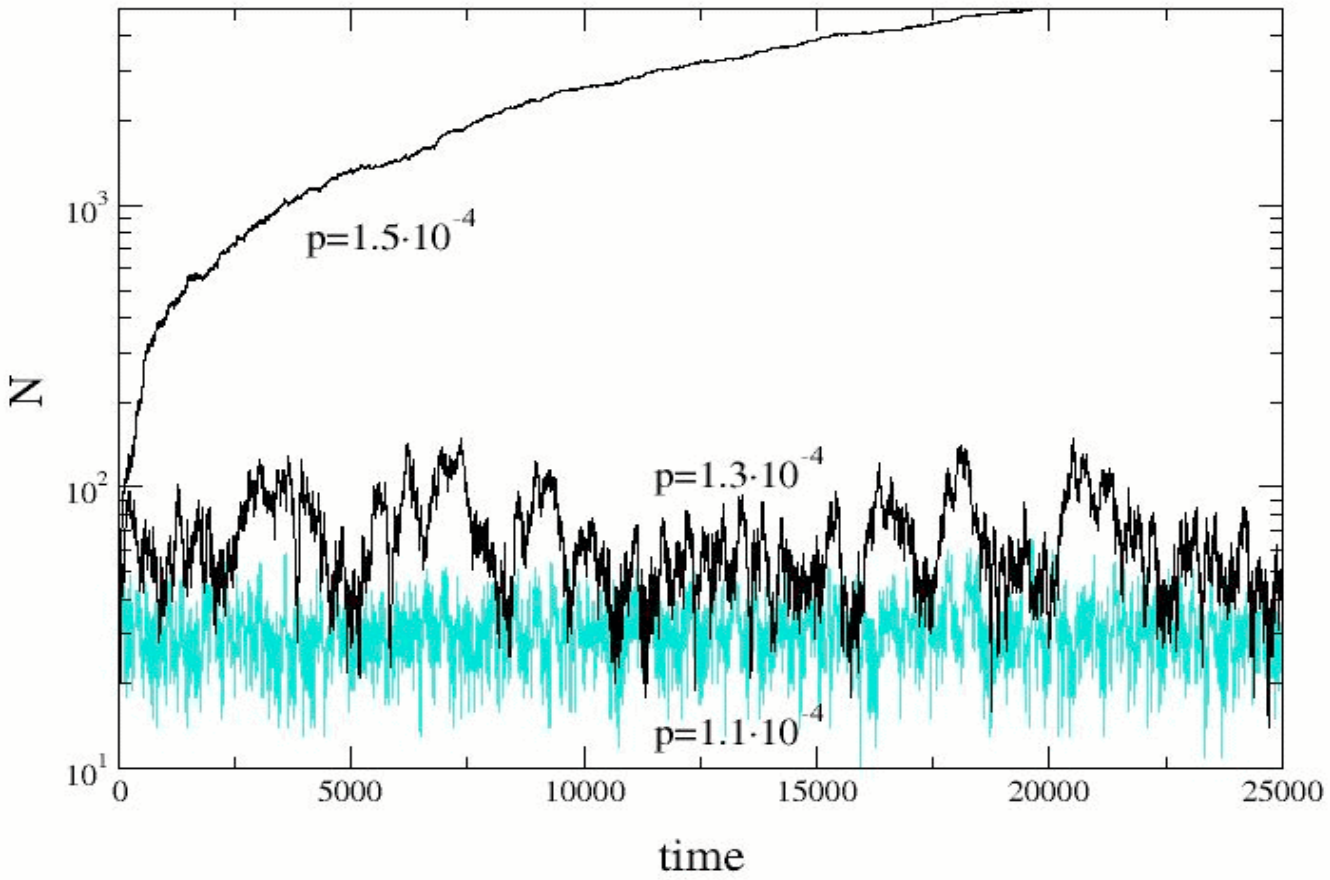
Search and congestion (I)

- Dealing simultaneously with search and congestion (PRL, 89, 248701, 2002)
- Flow of information packets (computer networks, organization, …)
- Nodes act as queues

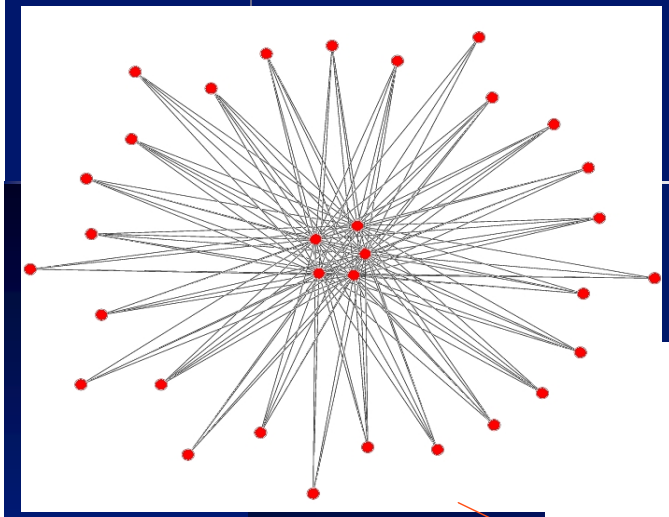
Search and congestion (II)

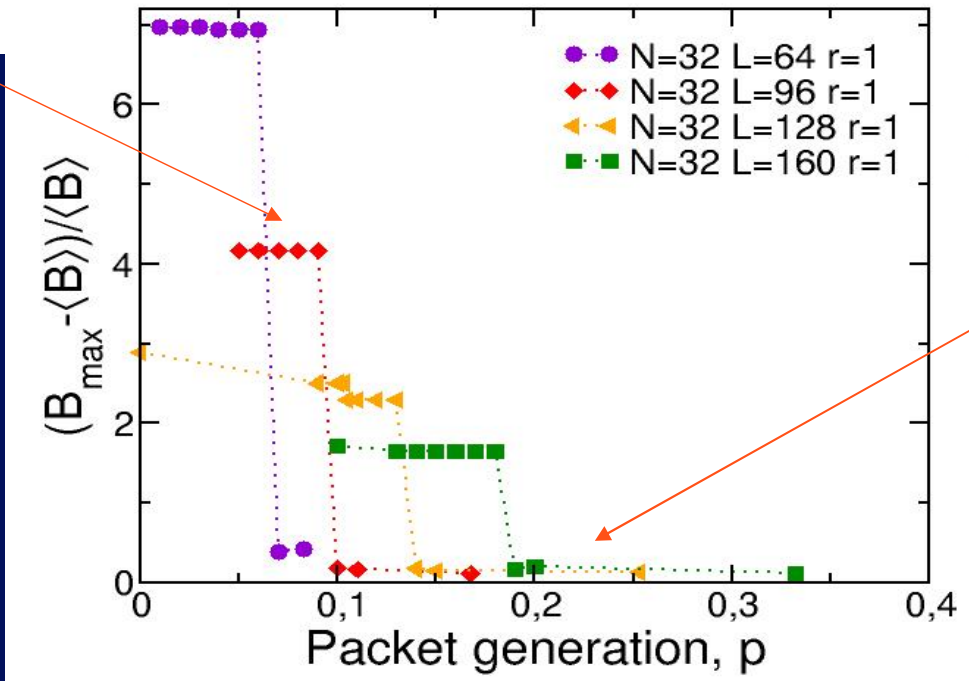
- Packets are created at random with probability ρ
- Each packet has an assigned random destination
- Packets flow from node to node
- Nodes process packets independently, depending on its queue length

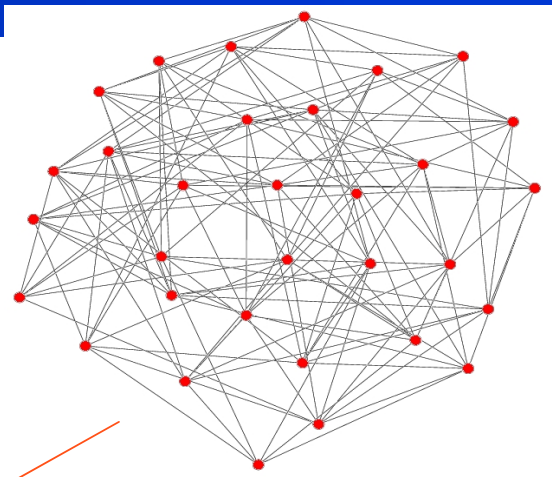
Congestion


10^3
10^2
10^1
N
p=1.5·10^{-4}
p=1.3·10^{-4}
p=1.1·10^{-4}
0
5000
10000
15000
20000
25000
time

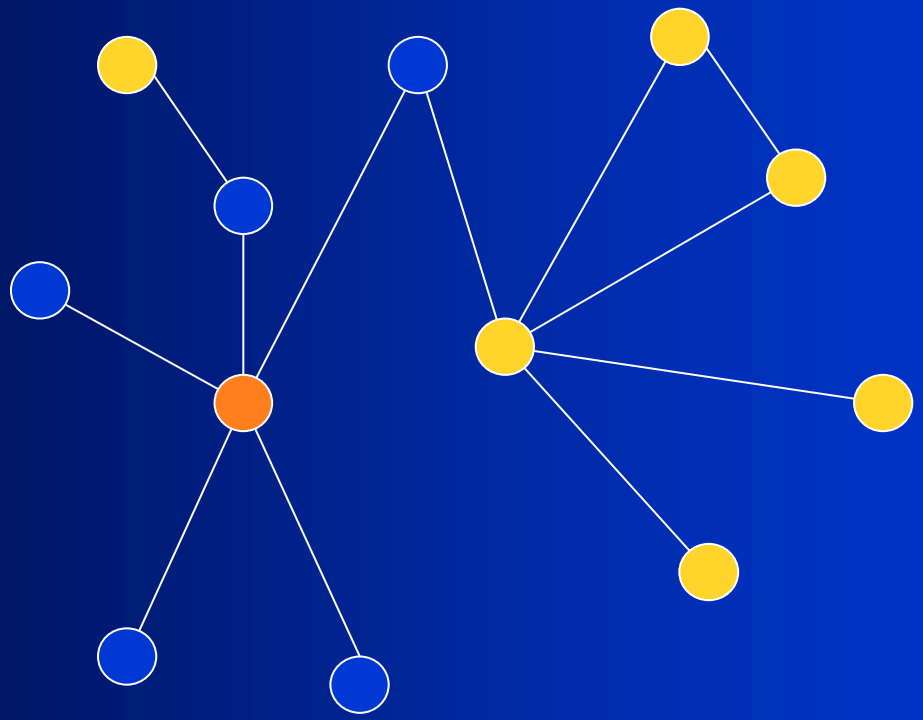
Optimization





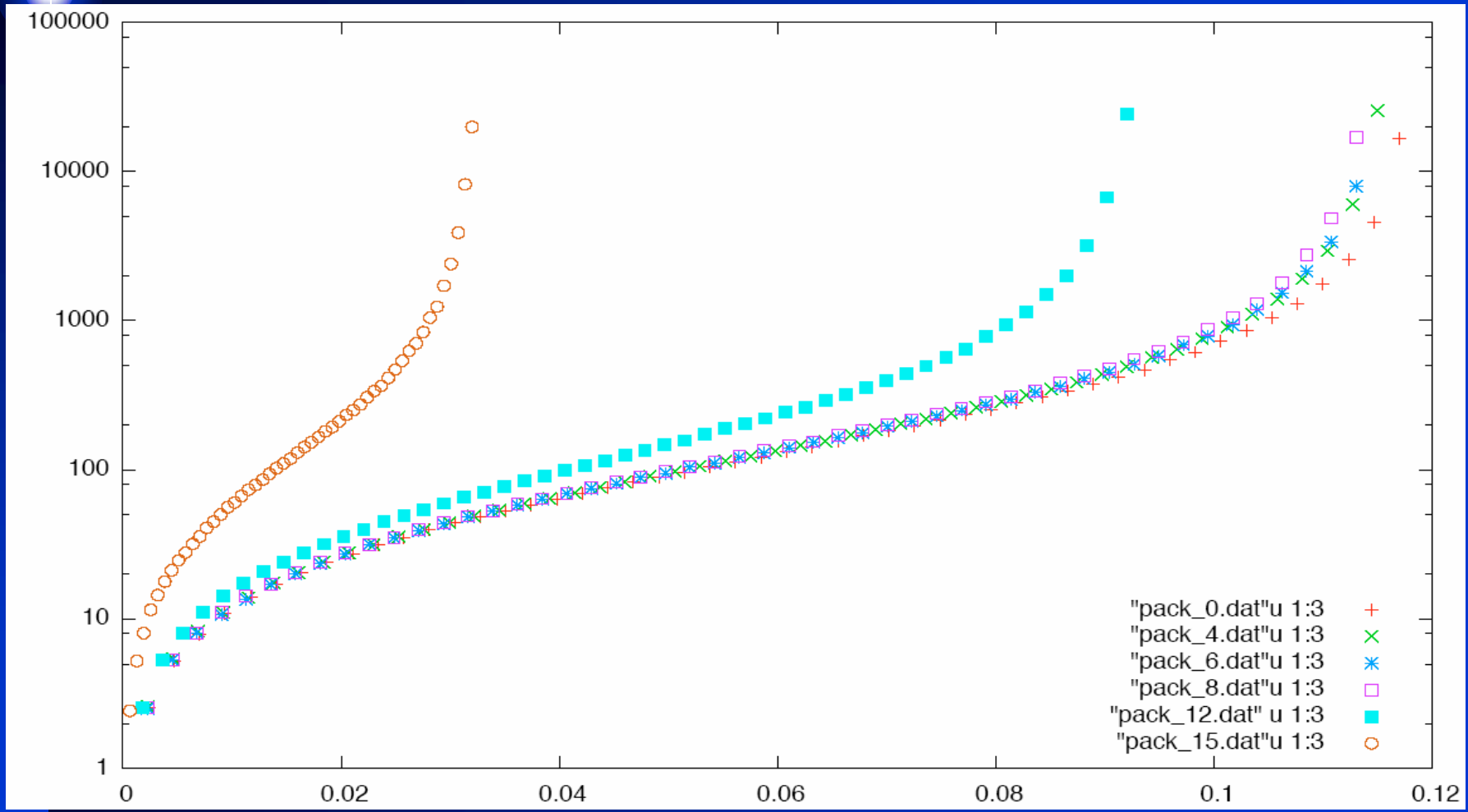


Information radius

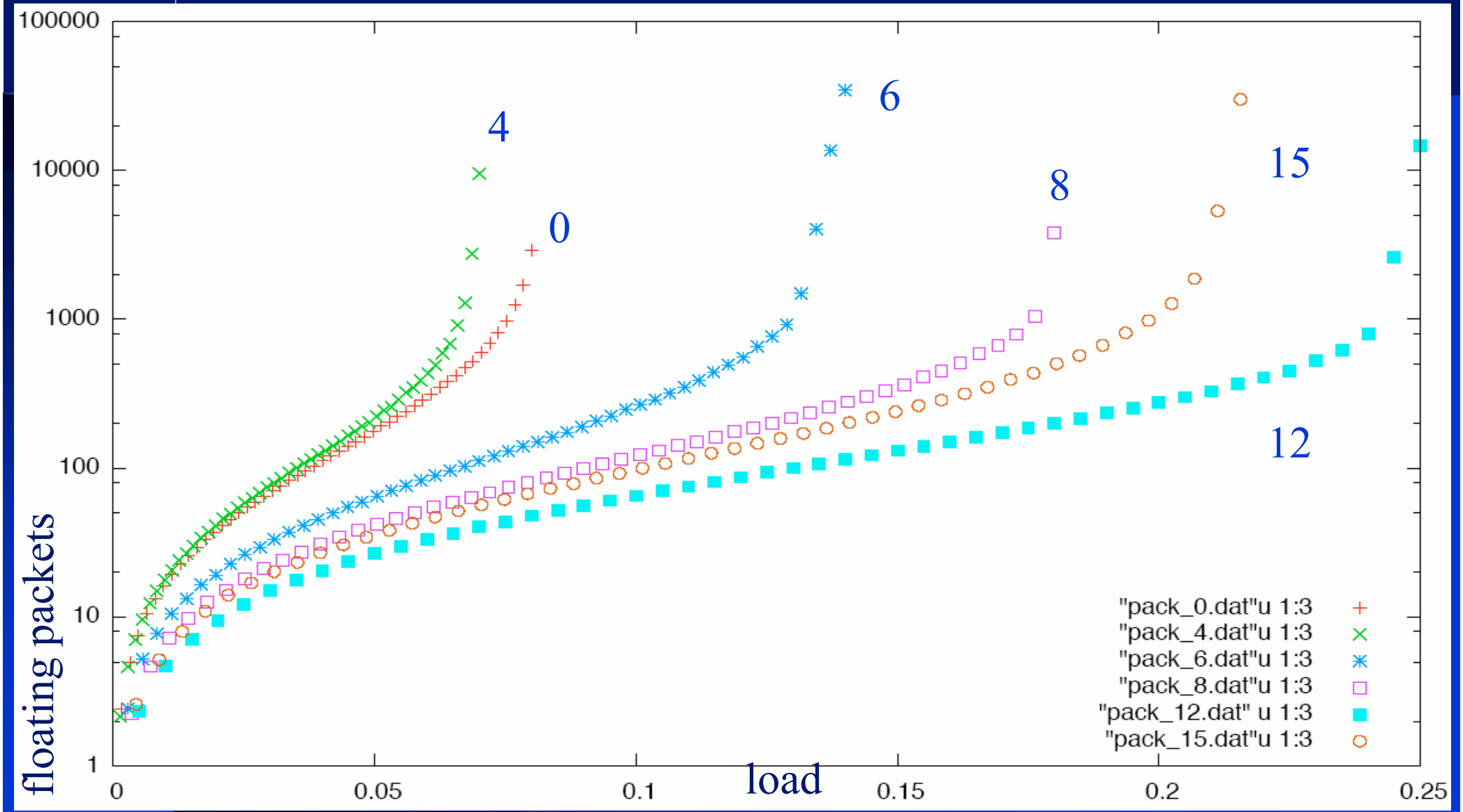


r=1

Searching within communities



Adding community information



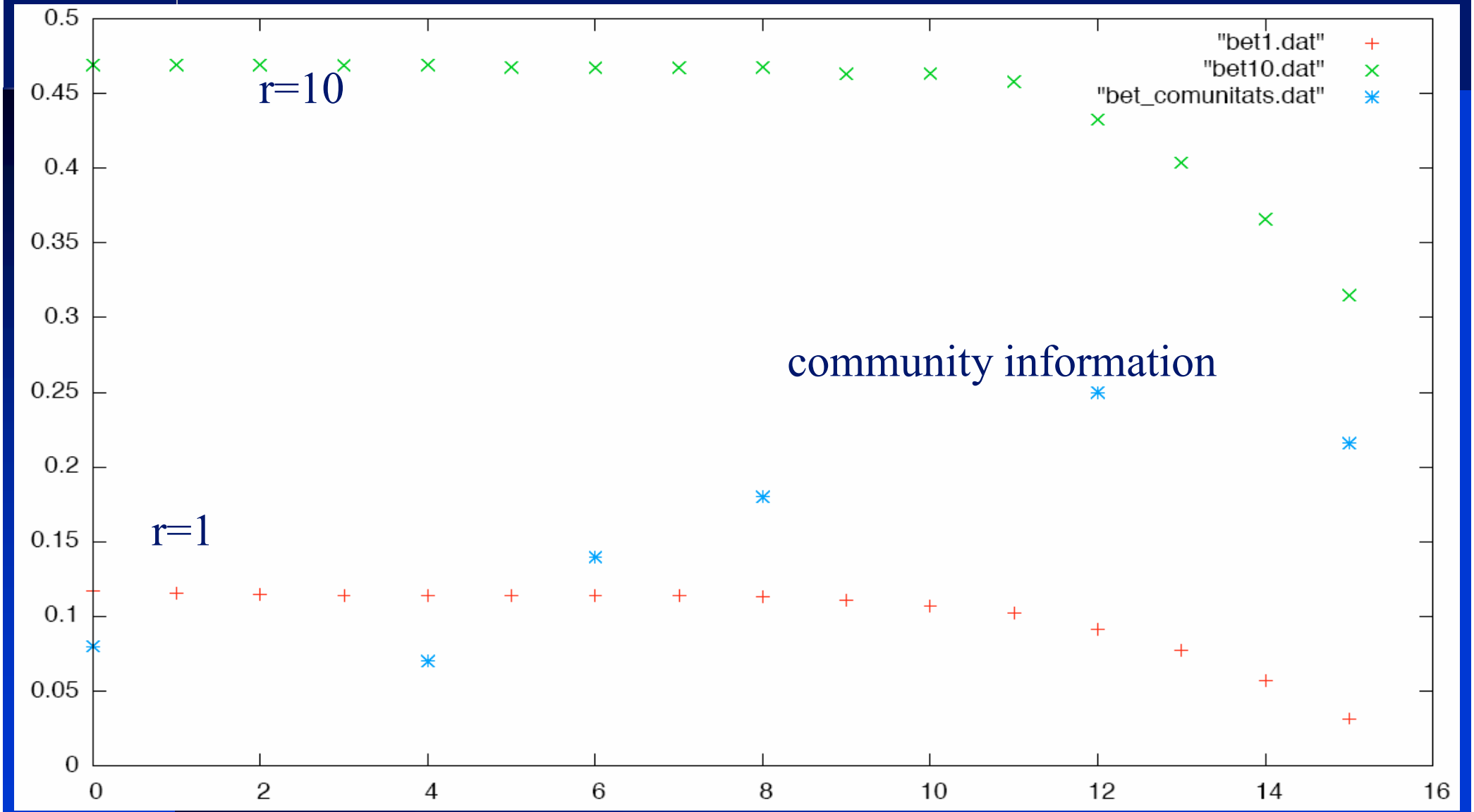
Betweenness

- Dynamics and topology

$$< N(t) >= \sum_i n_i(t) = \sum_i \frac{pB_i / (N-1)}{1 - pB_i / (N-1)} =$$

$$= \begin{cases} \dfrac{p}{(N-1)} \sum_i B_i = Np\langle d \rangle & \text{small } p \\ \approx B^* \text{ (node with maximum betweenness)} & \text{large } p \end{cases}$$

Betweenness

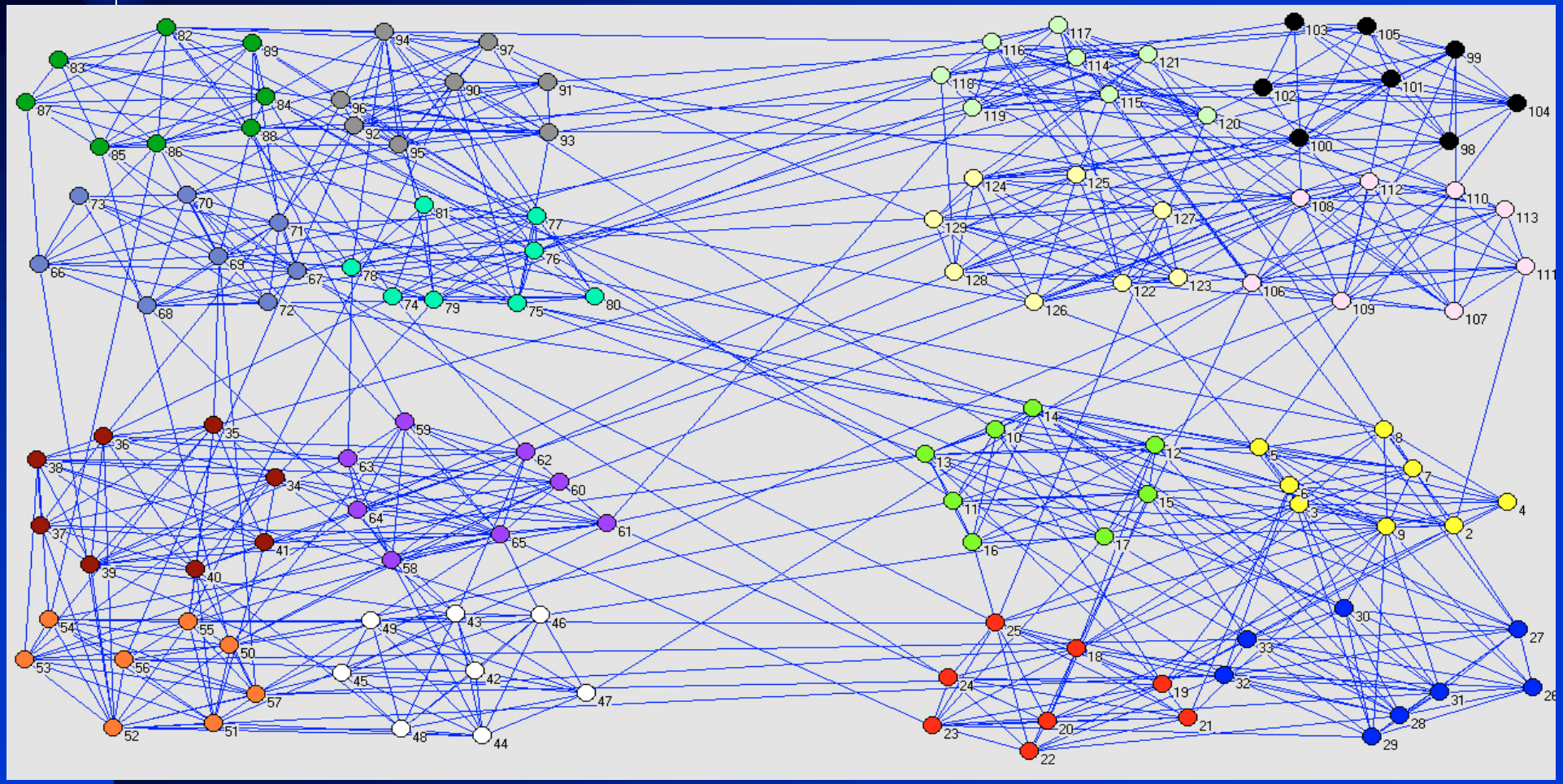


More dynamics

- Synchronization of Kuramoto oscillators

$$\frac{d\varphi_i}{dt} = \frac{K}{k_i} \sum_{j \in \Gamma(i)} \sin(\varphi_j - \varphi_i)$$

Hierarchical structure of communities



Conclusions

- Strong communities are not efficient for the dynamics
- Balance between inter- and intra-community links
- Dynamics reflects community structure