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Networks on Euclidean Space

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Networks on Euclidean Space

"Growing spatial scale-free graphs by selecting local edges",
cond-mat/0503679

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Connection to the Internet

If a computer is to be connected to the Internet what one should do?

- **If it is a home computer one uses a modem and dial-up telephone line to connect to the nearest router of the telephone company.**
- **If it is the office computer, it gets a connection to the office router, which is eventually connected to the ISP.**
- **If the office is a part of the university or organization, several routers are used in the autonomous system level and are connected to the nearest ISP again.**

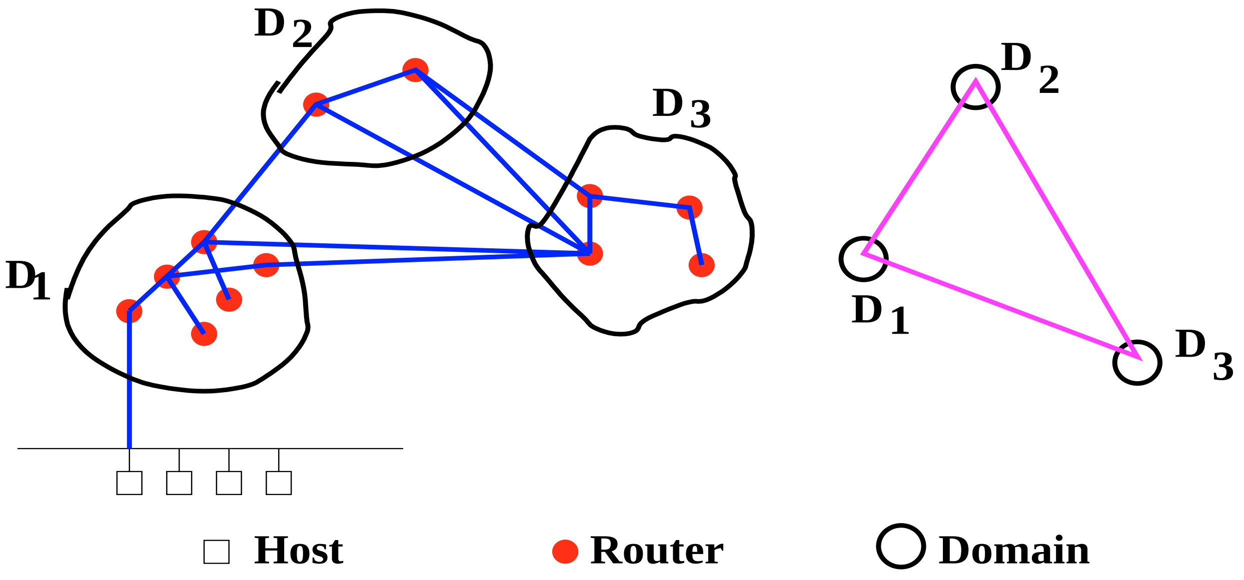
Therefore connections of the new nodes of the Internet, be it in the level of individual computers, routers or in the autonomous system level is always guided by local decisions and not global.

In the world wide Internet if we look into the time sequence in which the new nodes are popping up in time and are becoming part of the Internet then quite naturally these events should not have any spatial correlations.

Structure of the Internet

On Power-Law Relationships of the Internet Topology

M. Faloutsos, P. Faloutsos and C. Faloutsos
Proc. ACM SIGCOMM, Comput. Commun. Rev. 29, 251 (1999).
http://moat.nlanr.net/Routing/rawdata/



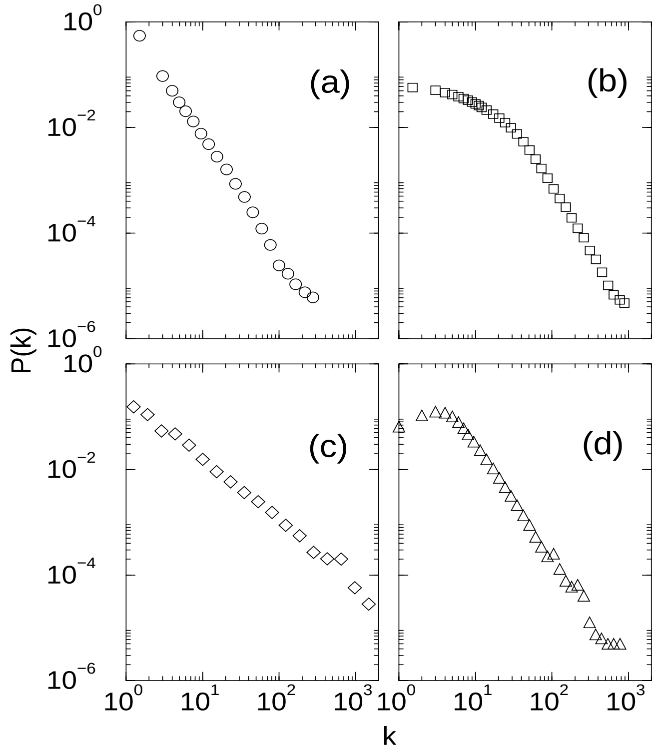
	Nov. 1997 N=3015,L=5156	Apr. 1998 N=3530,L=6432	Dec. 1998 N=4389,L=8256
Degree Dist. $P(k) \sim k^{-\gamma}$	γ=2.15	γ=2.16	γ=2.20
Rank Dist. $k \sim r^{-R}$	R=0.81	R=0.82	R=0.74
Hop-plot $N(l) \sim l^{H}$	H=4.6	H=4.7	H=4.9

Scale-free Networks

A network whose degree distribution has a power law tail

$$P(k) \propto k^{-\gamma}$$

(a) Internet (2.1 – 2.5)
(b) Movie actor network (2.3)
(c) Co-authorship network of high energy physicists (1.2)
(d) Co-authorship network of neuroscientists (2.1)



R. Albert and A.-L.Barabasi, Rev. Mod. Phys. **74**, 47, 2002.

Barabasi-Albert (BA) model of Scale-free Networks

A.–L. Barabási and R. Albert, Science, **286**, 509 (1999)

Why a few nodes have very large degrees?
Barabási-Albert: *Rich get richer*

a. ***Incremental Growth:***

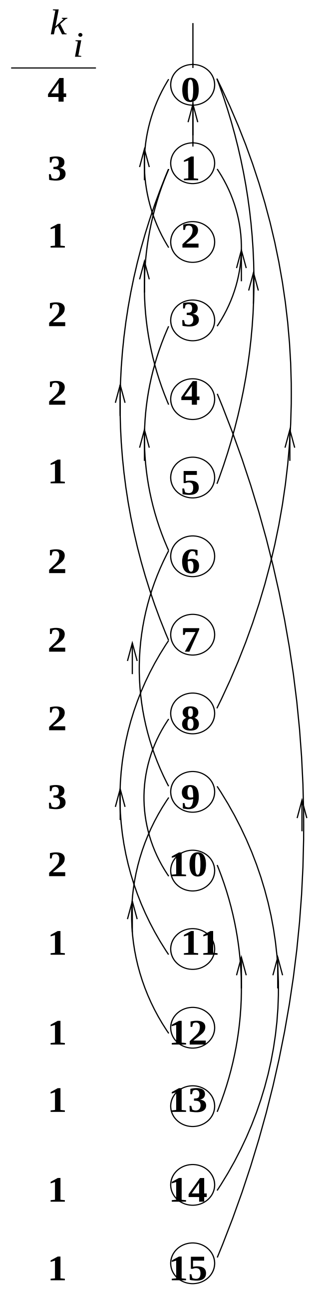
Starting from a small connected network, at every time step a new node is added and linked to *m* distinct nodes.

b. ***Preferential Attachment:***

A new node is linked to an older node *i* with degree k_i with an attachment probability:

$$\pi_i(t) \propto k_i(t)$$

For BA SFN $\gamma = 3$



Growth of a BA SFN with N=16,m=1

CPU Requirement in BA network

- Let there be a BA SFN of N nodes and m=1
- Let the nodal degrees be $k_1, k_2, k_3, \ldots, k_N$
- We try to introduce the (N+1)-th node
- Calculate the cumulative degree sums, like:

$s_1 = k_1$
$s_2 = k_1+k_2$
$s_3 = k_1+k_2+k_3$
$s_4 = k_1+k_2+k_3+k_4$
….
$s_N = k_1+k_2+k_3+\ldots+k_N$

- Call a random no. r between 0 and s_N
- Check if $s_{i-1} < r \le s_i$, then select node i
- This requires CPU ~ N
- Therefore for N nodes network, CPU ~ N^2

Local Connections

Though the BA algorithm is very much successful to generate SFNs, the requirement of linear attachment probability in case of Internet implies that a new node is linked by looking at the whole world wide Internet network and then selecting one of the nodes seems difficult to be plausible.

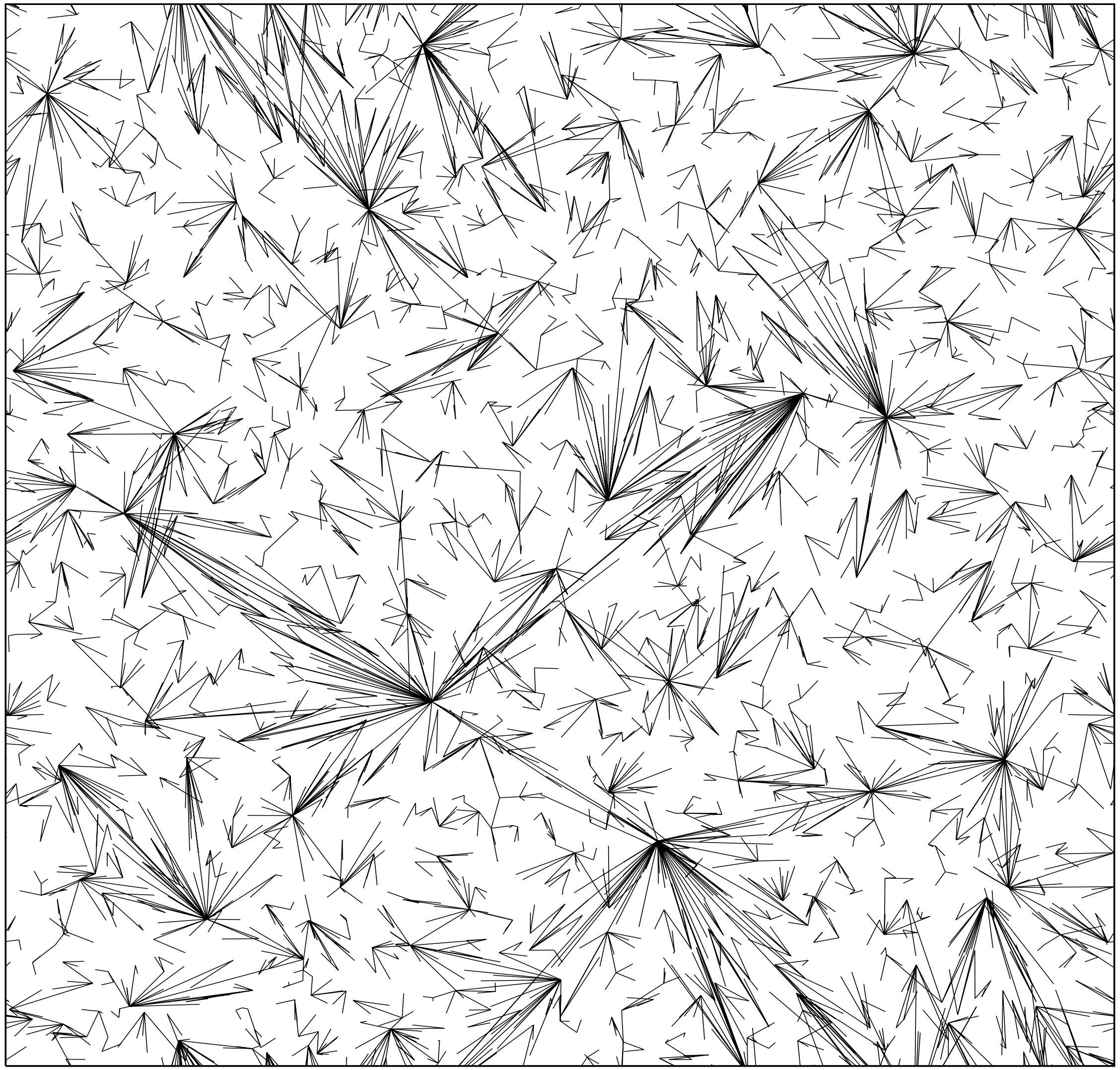
We argue that for Internet, the availability of local facilities determines the decision and therefore the new nodes always get connected to existing local nodes.

The new nodes pop up at random geographical locations with uniform probability near already grown nodes of the Internet and get connected with local nodes.

Euclidean Network

- Unit square area on the *x-y* plane.
- 2N random variables $\{x_1, x_2, \dots, x_N\}$ and $\{y_1, y_2, \dots, x_N\}$ are i.i.d. within {0,1}. N such pairs $\{(x_i,y_i), i=1,N\}$ denote the positions of N randomly distributed vertices.
- Initially only one edge between vertices 1 and 2.
- Vertices 3 to N are added one by one.

- Suppose (t+2)-th vertex is being added to a network of t edges and (t+1) vertices.
- t distinct distances d_n, (n=1,t) are measured from the (t+2)-th vertex to the centre of each edge.
- The edge corresponding to the minimal distance is selected.
- One end of this edge is randomly chosen with prob. ½ and is connected to the vertex (t+2).



N=2^{12}, each vertex is connected randomly to one end of the nearest edge.

CALCULATIONS

Degree Distribution:

- **A direct measure of the slopes of ln P(k) vs. ln k plots gives $\gamma = 3.00 \pm 0.05$ for $N = 2^{12}$, 2^{14} and 2^{16}.**

- **A finite size scaling varying no. of vertices N in the graph gives:**

$$P(k) \sim N^{-\alpha} G(k/N^{\beta})$$

with $\alpha \approx 1.48$, $\beta \approx 0.48$ giving $\gamma = \alpha/\beta \approx 3.0$

Strength Distribution:

Strength s_i of a vertex i is defined as the total sum of the lengths l_{ij} of all edges meeting at this vertex.

Strength $s_i = \Sigma_j \, l_{ij}$

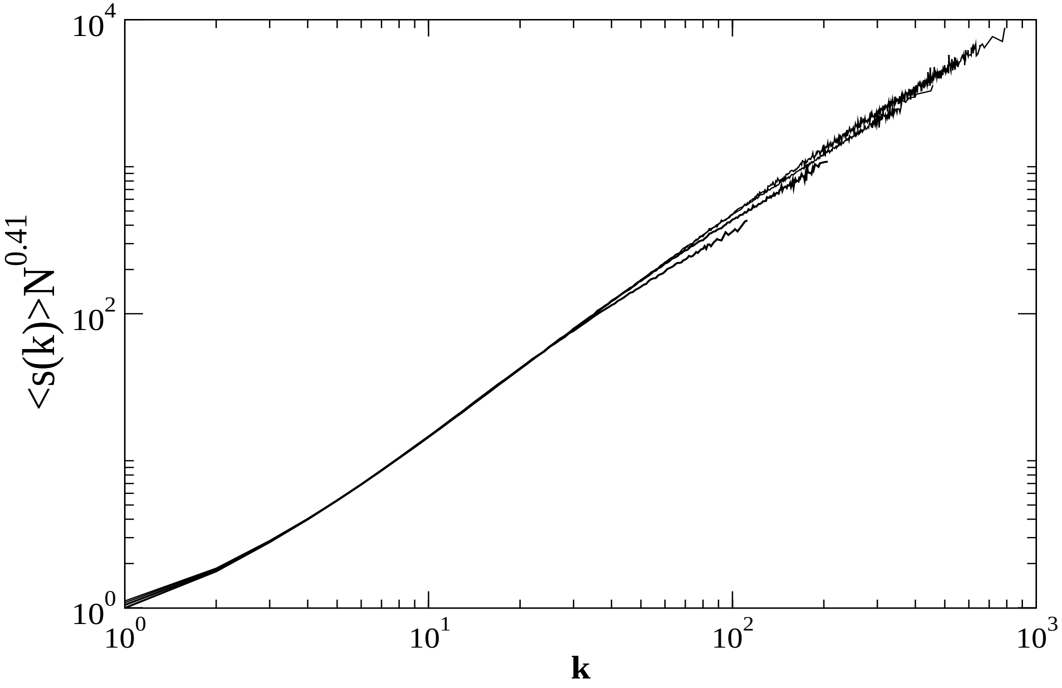
For a fixed N

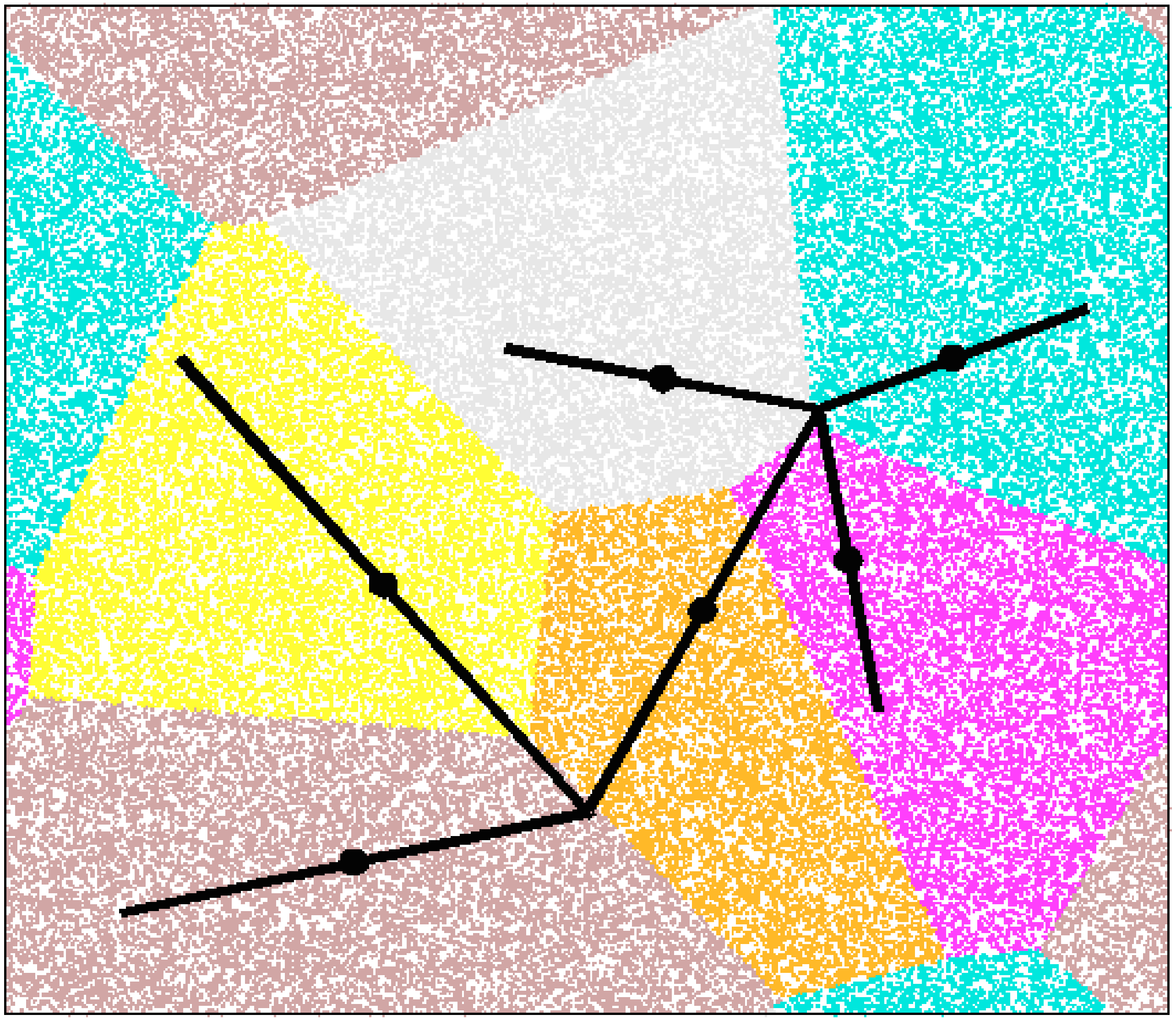
$\langle s \rangle \sim k^{\nu}$

Varying
$N = 2^{12}, 2^{14}, 2^{16}$ we get

$\langle s(k,N) \rangle \sim k^{\nu} N^{-\mu}$

with $\nu \approx 1.5$ and $\mu \approx 0.41$





Alternative route to BA model

- **Select one link j randomly with uniform prob.**
- **Add the new node m and connect to one end of j with equal prob.**
- **The resulting network is a BA network, since one end node i of the link j whose degree is k_i is selected with prob. proportional to k_i**

Efficient Algorithm

- **The CPU α N compared to N^2 in BA algorithm.**