Abstracts of talks

Geometry and Topology of 3-Manifolds

ICTP, Trieste, June 20-24, 2005

Ian Agol (Monday, 9:15-10:00)
"Volumes of hyperbolic Haken 3-manifolds"
We prove a volume inequality for 3-manifolds having C^0 metrics "bent" along a hypersurface, and satisfying certain curvature pinching conditions. The result makes use of Perelman's work on Ricci flow and geometrization of closed 3-manifolds. We also use methods of Bray and Miao used for the Riemannian Penrose conjecture. Corollaries include a new proof of a conjecture of Bonahon about volumes of convex cores of Kleinian groups and estimates of volumes of certain hyperbolic Haken 3-manifolds. (Joint work with Peter Storm and William Thurston).

Mark Baker (Thursday, 14:30-15:15)
"A combination theorem for convex hyperbolic manifolds with applications to surfaces in 3-manifolds"
We prove the following convex combination theorem: there is a constant κ, such that if M is a hyperbolic n-manifold that is the union of two convex hyperbolic n-submanifolds, and if M has a κ-thickening with the same topology, then M has a convex thickening.

One application is that every slope on a torus boundary component of a compact 3-manifold with hyperbolic interior is an immersed boundary slope. (Joint work with Daryl Cooper).

Stephane Baseilhac (Monday, 16:45-17:15)
"On dilogarithms of triangles in the complex plane"
We discuss the construction of the classical Rogers dilogarithm function and the "quantum" matrix dilogarithms from the geometry of the space of tri-

angles on the plane and of quadrilaterals on the Riemann sphere, together with applications to invariants of 3-manifolds.

Steven Boyer (Tuesday, 10:15-11:00)
"Minimal hyperbolic 3-manifolds and character varieties"
The existence of a proper map of non-zero degree between two compact, connected, orientable manifolds of the same dimension is a difficult question to handle in topology. M. Gromov pointed out the interest in using the existence of such maps to define a partial ordering on the set of homeomorphism classes of compact, connected, orientable manifolds of a given dimension: we say that M *dominates* N, written $M \geq N$, if there is a continuous, proper map from M to N of non-zero degree. Moreover if N is not homeomorphic to M we say that M *strictly dominates* N. An intriguing problem is to determine 3-manifolds which are minimal with respect to strict domination. Since a closed, orientable 3-manifold dominates any closed 3-manifold with finite π_1, it is natural to consider minimality amongst 3-manifolds with infinite fundamental group. In this talk we describe several infinite families of minimal hyperbolic 3-manifolds with torus boundary and use $PSL_2(\mathbb{C})$ character variety methods and the deformation theory of Kleinian groups to show that for a manifold in one of these families, all but finitely many of its Dehn fillings are minimal amongst the class of geometrisable 3-manifolds with infinite fundamental group.

Jeffrey Brock (Friday, 11:45-12:30)
"Heegaard splittings, pants decompositions of surfaces, and volumes of hyperbolic 3-manifolds"
The combinatorics of simple closed curves on surfaces play integrally into the study of Heegaard splittings of 3-manifolds, where they serve to indicate a kind of complexity; Hempel explored the relation of Heegaard splittings to the complex of curves, eliciting necessary conditions for the existence a hyperbolic structure.

When M is assumed hyperbolic, one can explore relations between combinatorics of curves and the geometry of the 3-manifold. We establish that a related complexity arising from pants decompositions of surfaces serves to estimate the volume of M, echoing earlier results for 3-manifolds fibering over the circle.

Kenneth Bromberg (Monday, 10:15-11:00)
"The density conjecture for Kleinian groups with bounded geometry"
The density conjecture states that every finitely generated Kleinian group is approximated by geometrically finite groups. We will discuss an approach to this conjecture that does not depend on the Ending Lamination Conjecture. (Joint work with Juan Souto).

Young Choi (Friday, 16:45-17:15)
"Lines of minima in Teichmüller space"
Lines of minima are paths in Teichmüller space that resemble Teichmuller geodesics. I will explain joint work with Kasra Rafi and Caroline Series that explores the similarities and differences.

Olivier Collin (Thursday, 11:45-12:30)
"Floer homology and the A-polynomial"
We shall explain how to use instanton Floer homology of various Dehn surgeries along knots in homology spheres to obtain information about the A-polynomial of knots. In particular, we show that the a-polynomial of K is non-trivial whenever some $(1/k)$-surgery along K has non-vanishing Floer homology. We also show that for knots in homology spheres, the non-vanishing of the Floer homology of the 0-surgery is not a necessary condition for the non-triviality of the A-polynomial.

Daryl Cooper (Thursday, 9:15-10:00)
"Pseudo-conformal Geometry and 3-Manifolds"
We introduce Pseudo-Conformal geometry and determine which Thurston geometries are pseudo-conformal. An application to 3-manifolds is given.

Nathan Dunfield (Wednesday, 11:15-12:00)
"Does a random 3-manifold fiber over the circle?"
I will discuss the question of the title for 3-manifolds with tunnel number one — the motivation here is to try to get some handle on the Virtual Fibration Conjecture for hyperbolic 3-manifolds. I'll begin by with the question of

when a specific tunnel number one 3-manifolds fibers over the circle. In particular, I will explain a criterion of Brown which answers this question from a presentation of the fundamental group. I will describe how techniques of Agol, Hass, and W. Thurston can be adapted to compute this very efficiently by using that the relator of the group comes from an embedded curve on the boundary of a genus 2 handlebody. I will then outline a proof that the probability that a random tunnel number one 3-manifold fibers over the circle is 0, where here "random" is with respect to a natural Dehn-Thurston parameterization of the set of tunnel number one manifolds. I will conclude with some experimental evidence where "random" is instead taken with respect to a mapping class group point of view. (Joint work with Dylan Thurston).

Roberto Frigerio (Thursday, 16:45-17:15)
"Uncountably many Kleinian groups and mapping class groups"
In this talk, a Kleinian group is a discrete torsion-free subgroup of $\mathrm{PSL}(2, \mathbb{C})$, and a hyperbolic 3-manifold is the quotient of hyperbolic 3-space by the action of a Kleinian group. We show that for any countable group G there exists a hyperbolic manifold $M(G)$ such that the positive isometry group and the group of homotpy classes of positive self-homeomorphisms of $M(G)$ are both isomorphic to G. As a corollary, we deduce that there exist uncountably many isomorphism classes of (non-finitely generated) Kleinian groups. (Joint work with Bruno Martelli).

Michael Heusener (Wednesday, 12:15-13:00)
"Deforming representations of 3-manifolds into $\mathrm{PSL}_2(\mathbb{C})$"
Let M be a 3-manifold with torus boundary which is a rational homology circle. We study deformations of reducible representations of $\pi_1(M)$ into $\mathrm{PSL}_2(\mathbb{C})$ associated to a simple zero of the twisted Alexander polynomial. We also describe the local structure of the representation and character varieties. (Joint work with Joan Porti).

Craig Hodgson (Friday, 9:15-10:00)
"Computation of geometric structures on
3-dimensional orbifolds and manifolds"
We describe how the ideas behind Weeks' program SnapPea and Casson's

program Geo can be extended to give new methods for computing hyperbolic structures on a large class of 3-orbifolds and 3-manifolds. The theory is implemented in a computer program Orb developed by Damian Heard, which can start with a projection of a graph embedded in S^3 and produce a hyperbolic structure with prescribed cone angles along the edges. This makes a wealth of geometric and topological information on these objects readily accessible for the first time.

Applications include classification of knotted graphs and their symmetry groups using hyperbolic invariants, and the study of low volume hyperbolic 3-orbifolds.

Michael Kapovich (Thursday, 10:15-11:00)
"Cohomological dimension and critical exponent of Kleinian groups"
Suppose that G is a Kleinian subgroup of isometries of the hyperbolic n-space which contains no rank 2 parabolic subgroups. Let cd(G) denote the cohomological dimension of G over the real numbers. Then cd(G)-1 is less than or equal to the critical exponent of G. (In particular, if G is a finitely generated Kleinian group whose critical exponent is less than 1 then G is virtually free.)

A similar result holds if G has parabolic elements. This generalizes a theorem of Besson, Courtois and Gallot.

Marc Lackenby (Tuesday, 11:45-12:30)
"Covering spaces of 3-orbifolds"
We propose the following conjecture: the fundamental group of any closed hyperbolic 3-orbifold with non-empty singular locus is large. We will give several results that provide quite strong supporting evidence. The first states that if M is a compact orientable 3-manifold and K is a knot in M such that M - K is hyperbolic, then the orbifold with an order n singularity along K has large fundamental group, for infinitely many values of n. The second results states that any closed orientable hyperbolic 3-orbifold with non-empty singular locus has at least exponential subgroup growth. The third result establishes that it has a tower of finite-sheeted covers where the rank of the mod p homology grows linearly in the covering degree, for some prime p; these covers therefore have positive Heegaard gradient. The final result claims that the conjecture would follow from a group-theoretic conjecture of Lubotzky and Zelmanov.

Sylvain Maillot (Tuesday, 16:00-16:30)
"A spherical decomposition for open Riemannian 3-manifolds"
It is known that the Kneser Prime Decomposition Theorem does not hold for noncompact manifolds: an open 3-manifold need not have a locally finite decomposition along 2-spheres into prime manifolds.

I will give a characterization of those open 3-manifolds which do have such a decomposition by the existence of a Riemannian metric with respect to which the second homotopy group is generated by small elements.

Bruno Martelli (Monday, 15:30-16:00)
"Dehn surgery on links in 3-manifolds"
A link in a closed 3-manifold, coloured with a rational number on each component, describes another 3-manifold M via Dehn surgery. In general, the same 3-manifold M can be obtained via infinitely many distinct colourings on the same link. However, we can prove that the colourings become finitely many after twisting along discs and annuli embedded in some "partial surgery".

This result has the following applications:

- finitely many links in a closed 3-manifold share the same complement, up to twists along discs and annuli;
- by adding 2-handles on the same link we get only finitely many smooth cobordisms between two given closed 3-manifolds.

The second application in turn implies that finitely many smooth closed 4-manifolds have a Kirby diagram with bounded number of crossings.

Sergei Matveev (Tuesday, 14:30-15:15)
"Roots of 3-manifolds, cobordisms, knotted graphs, and orbifolds"
We describe a new proof of a weaker version of the Milnor prime decomposition theorem for 3-manifolds. The main trick of the proof works for many other geometric objects like cobordisms and knotted graphs. (Joint work with Cynthia Hog-Angeloni).

Mattia Mecchia (Friday, 16:00-16:30)
"Finite groups acting on homology spheres"

We consider the problem to determine which finite groups admit an action on integer and $\mathbb{Z}_2$-homology 3-spheres (arbitrary i.e. not necessarily free action); we report on some recent results obtained in joint work with B.Zimmermann. In particular we give a list of nonsolvable groups which are the candidates for action on $\mathbb{Z}_2$-homology 3-spheres. The interest in $\mathbb{Z}_2$-homology 3-spheres is in part motivated by the fact that they appear often in low dimensional topology; for example this class of 3-manifolds includes the 2-fold branched coverings of knots. We extend the methods we applied in dimension three also to the case of homology 4-spheres.

Alexander Mednykh (Monday, 11:45-12:30)
"Hyperbolic and spherical volume for knots, links and polyhedra"
A new geometrical method is developed to calculate hyperbolic and spherical volume for knots and links cone-manifolds and polyhedra. It is based on trigonomerical identities relating lengths of singular geodesics of cone-manifolds and its cone-angles. Then the Schlaefli formula is used to obtain an integral representation for the volume. We apply this approach to find volume of an arbitrary tetrahedron and the Lambert cube in the hyperbolic and spherical spaces. The latter application are the volume formulas for the Figure eight knot, the Whitehead link, the Borromean Rings, and the Twist link cone-manifolds.

Luisa Paoluzzi (Thursday, 15:30-16:00)
"Determining prime satellite knots
by means of their cyclic branched covers"
In 1981 Y. Nakanishi and M. Sakuma developed a method to construct, for each given $n \geq 2$, pairs of prime knots having the same n-fold cyclic branched cover. This construction was then shown to be the only way to get pairs of non equivalent hyperbolic knots with the same n-fold cyclic branched cover if $n \geq 3$ by B. Zimmermann in 1998. As a consequence of Zimmermann's result one can prove that a hyperbolic knot is determined by any three cyclic branched covers of orders ≥ 3.

In this talk I shall discuss the behaviour of deck transformations for n-fold cyclic covers of the 3-sphere branched along a prime knot, when n is an odd prime. I shall start by illustrating examples of non equivalent prime knots which do not arise from Nakanishi and Sakuma's construction, before showing how one can prove that a prime knot is determined by any three cyclic branched covers of odd prime orders. (Joint work with Michel Boileau).

Bernard Perron (Wednesday, 8:45-9:30)
"A homotopic intersection theory on surfaces.
Applications to mapping class groups and braids"
Let S be a compact, connected, oriented surface with boundary. We denote by G its fundamental group. To each element of $G \times G$, by a geometric construction we associate an "intersection number" with values in $\mathbb{Z}G$, the group ring of G. This is a biderivation in the sense of Fox, which has a simple behaviour under exchange of the arguments. Composed with the natural projection onto $\mathbb{Z}$, we recover the usual algebraic intersection number. This geometric construction allows us to reprove geometrically and very simply some old results (on the planarity of surface coverings), some recent results (on the symplectic character of the Fox matrix of a homeomorphism of surface). We also obtain new results on the Magnus representation of the Torelli group of a surface. Finally, we obtain new informations on the Burau (resp. Gassner) representation on the braid (resp. pure braid) groups: we prove that it is "unitary" with respect to a matrix which is much simpler than Squier's one.

Riccardo Piergallini (Tuesday, 16:45-17:15)
"Wild branching surfaces and topological 4-manifolds"
It is known that every closed orientable smooth 4-manifold is a 4-fold (resp. 5-fold) simple cover of S^4 branched over a transversally immersed (resp. non-singular) smooth surface. The main ingredients of the proof are a Montesinos representation theorem of 4-dimensional 2-handlebodies as 3-fold simple covers of B^4 branched over ribbon surfaces and an equivalence theorem for 3-fold simple coverings of S^3 representing the same 3-manifold.

In a recent joint paper with I. Bobtcheva, we imporved both these theorems. Then, we are now able to provide a relative version of the above covering representation result for bounded 4-manifolds and a suitable extension of it to the non-compact case. As a consequence, we get that any closed orientable topological 4-manifold is a simple cover of S^4 branched over a possibly wild surface.

Joan Porti (Friday, 14:30-15:15)
"Spherical cone structures on 2-bridge knots and links"
This talk studies spherical cone structures on hyperbolic 2-bridge knots and links with cone angle $> \pi$. It is known that such structures exist for cone angle $\alpha \in (\alpha_0, \pi]$, and that they become Euclidean when α approaches α_0. For smaller cone angles, they are hyperbolic . Here we prove that these structures exist for cone angle $\alpha \in [\pi, 2\pi - \alpha_0)$, where α_0 is the Euclidean cone angle. When $\alpha \to 2\pi - \alpha_0$, the singular locus crosses transversally with itself along the tunnels; hence the tunnels collapse, giving a spherical suspension over a sphere with four cone points.

Alan Reid (Friday, 10:15-11:00)
"LERF, extensions and covers of 3-manifolds"
Marden's conjecture was recently proven by Agol and independently Calegari and Gabai. In this talk we show how LERF can be used to improve this in some cases in the following sense:

Theorem: Let $M = \mathbb{H}^3/G$ be a non-compact orientable finite volume hyperbolic 3-manifold and H a non-elementary subgroup of G of infinite index generated by two parabolic elements. If G is LERF, then there is a finite index subgroup $G_H < G$, and an epimorphism $f : G_H \to \mathbb{Z}$ for which H is contained in the kernel of f.

The relevance to tameness is given by the following consequence of Canary's covering theorem: $M = \mathbb{H}^3/G$ is any finite volume hyperbolic and $H < G$ a finitely generated subgroup which is contained in the kernel of an epimorphism $f : G \to \mathbb{Z}$. Then the cover of M corresponding to H is tame.

Peter Shalen (Monday, 14:30-15:15)
"Hyperbolic volume and classical topology"
We show that if M is a closed orientable hyperbolic 3-manifold such that $H_1(M; \mathbf{Z}/2\mathbf{Z})$ has rank at least 7, then the volume of M is greater than 3.08. The main new ingredient in the proof is a purely topological theorem about resolving singularities of an immersed π_1-injective genus-2 surface. This topological theorem is proved by combining the tower arguments developed by Papakyriakopoulos and Shapiro-Whitehead in the middle of the last century with estimates for homology of covering spaces proved by Shalen and Wagreich. The volume estimate is proved by combining the topological the-

orem with Agol's tameness theorem, a co-volume estimate for 3-tame, 3-free Kleinian groups due to Anderson, Canary, Culler and Shalen, and a volume estimate for hyperbolic Haken manifolds recently proved by Agol, Storm and Thurston. (Joint work with Marc Culler).

Juan Souto (Tuesday, 9:15-10:00)
"The rank of the fundamental group of hyperbolic 3-manifolds fibering over the circle"
If M is a hyperbolic 3-manifold fibering over the circle where the monodromy is a sufficiently high power of a pseudo-Anosov map of a closed genus g surface then the rank of the fundamental group of M is $2g + 1$. Moreover, there is only a Nielsen equivalence class of minimal generating sets.

Dylan Thurston (Thursday 17:30-18:00)
"How efficiently do 3-manifolds bound 4-manifolds?"
It is known since 1954 that every 3-manifold bounds a 4-manifold. Thus, for instance, every 3-manifold has a surgery diagram. There are many proofs of this fact, including several constructive ones, but they do not bound the complexity of the 4-manifold. (By "complexity" of a manifold we mean the minimum number of simplices in a triangulation.) Given a 3-manifold M of complexity n we show how to construct a 4-manifold bounded by M of complexity $O(n^2)$. It is an open question whether this quadratic bound can be replaced by a linear bound.

The natural setting for this result is shadow surfaces, a representation of 3- and 4-manifolds that generalizes many other representations of these manifolds. One consequence of our results is some intriguing connections between the complexity of a shadow representation and the hyperbolic volume of a 3-manifold.

Our results can also be phrased in terms of singularities of smooth maps. In particular, the minimum number of crossing singularities" of a map from a hyperbolic 3-manifold to the plane is bounded below by the hyperbolic volume. (Joint work with Francesco Costantino.)

Shicheng Wang (Wednesday, 9:45-10:30)
"1-domination between 3-manifolds and between knots"

I will report recent results indicated by the title, which are jointly with Boileau-Rubinstein and with Boileau-Boyer-Rolfsen.

Hartmut Weiss (Monday, 17:30-18:00)
"Rigidity of cone-3-manifolds"
We prove global rigidity for compact hyperbolic and spherical cone-3-manifolds with cone-angles $\leq \pi$ (which are not Seifert fibered in the spherical case), furthermore for a class of hyperbolic cone-3-manifolds of finite volume with cone-angles $\leq \pi$, possibly with boundary consisting of totally geodesic hyperbolic turnovers. To that end we first generalize our local rigidity result to the setting of hyperbolic cone-3-manifolds of finite volume as above. We then use the geometric techniques developed by Boileau, Leeb and Porti to deform the cone-manifold structure to a complete non-singular or a geometric orbifold structure, where global rigidity holds due to Mostow-Prasad rigidity in the hyperbolic case, resp. a result of de Rham in the spherical case. This strategy has already been implemented successfully by Kojima in the compact hyperbolic case if the singular locus is a link using Hodgson-Kerckhoff local rigidity.