The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific
and Cultural Organization

International Atomic
Energy Agency

SMR.1663- 3

SUMMER SCHOOL ON PARTICLE PHYSICS

13 - 24 June 2005

Standard Model and Higgs Physics

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The Standard Model (Electroweak Theory) and Higgs Physics

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Summer School on Particle Physics

Abdus Salam International Centre for Theoretical Physics

Trieste · 13 – 17 June 2005

A Decade of Discovery Past . . .

- ▷ Electroweak theory $\rightarrow$ law of nature
- ▷ Higgs-boson influence observed in the vacuum
- ▷ Neutrino flavor oscillations: $\nu_\mu \rightarrow \nu_\tau$, $\nu_e \rightarrow \nu_\mu/\nu_\tau$
- ▷ Understanding QCD
- ▷ Discovery of top quark
- ▷ Direct $\mathcal{CP}$ violation in $K \rightarrow \pi\pi$
- ▷ B-meson decays violate $\mathcal{CP}$
- ▷ Flat universe dominated by dark matter, energy
- ▷ Detection of ν_τ interactions
- ▷ Quarks & leptons structureless at TeV scale

A Decade of Discovery Past . . .

- ▷ Electroweak theory → law of nature
  $[Z,\, e^+e^-,\, \bar{p}p,\, \nu N,\, (g-2)_\mu,\, \ldots]$

- ▷ Higgs-boson influence observed in the vacuum
  [EW experiments]

- ▷ Neutrino flavor oscillations: $\nu_\mu \to \nu_\tau$,
  $\nu_e \to \nu_\mu/\nu_\tau$ [$\nu_\odot$, ν_{atm}, reactors]

- ▷ Understanding QCD
  [heavy flavor, Z^0, $\bar{p}p$, νN, ep, ions, lattice]

- ▷ Discovery of top quark [$\bar{p}p$]

- ▷ Direct $\mathcal{CP}$ violation in $K \to \pi\pi$ [fixed-target]

- ▷ B-meson decays violate $\mathcal{CP}$ [$e^+e^- \to B\bar{B}$]

- ▷ Flat universe dominated by dark matter, energy
  [SN Ia, CMB, LSS]

- ▷ Detection of ν_τ interactions [fixed-target]

- ▷ Quarks & leptons structureless at TeV scale
  [mainly colliders]

Goal: Understanding the Everyday

- ▷ Why are there atoms?
- ▷ Why chemistry?
- ▷ Why stable structures?
- ▷ What makes life possible?

Goal: Understanding the Everyday

- ▷ Why are there atoms?
- ▷ Why chemistry?
- ▷ Why stable structures?
- ▷ What makes life possible?

What would the world be like, without a (Higgs) mechanism to hide electroweak symmetry and give masses to the quarks and leptons?

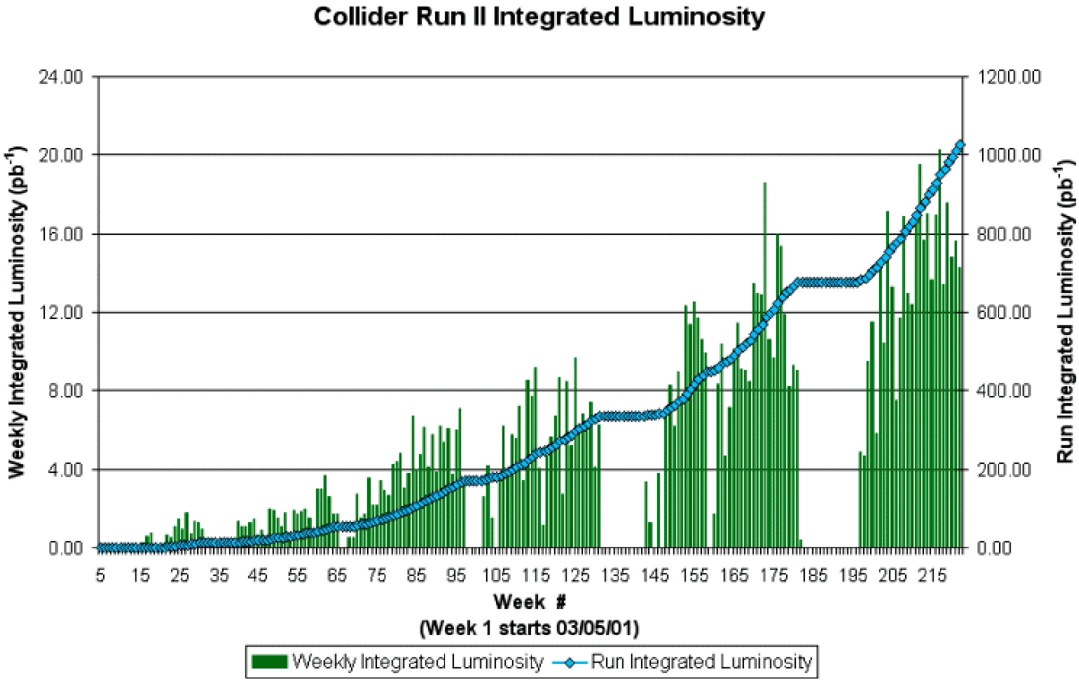
Searching for the mechanism of electroweak symmetry breaking, we seek to understand

why the world is the way it is.

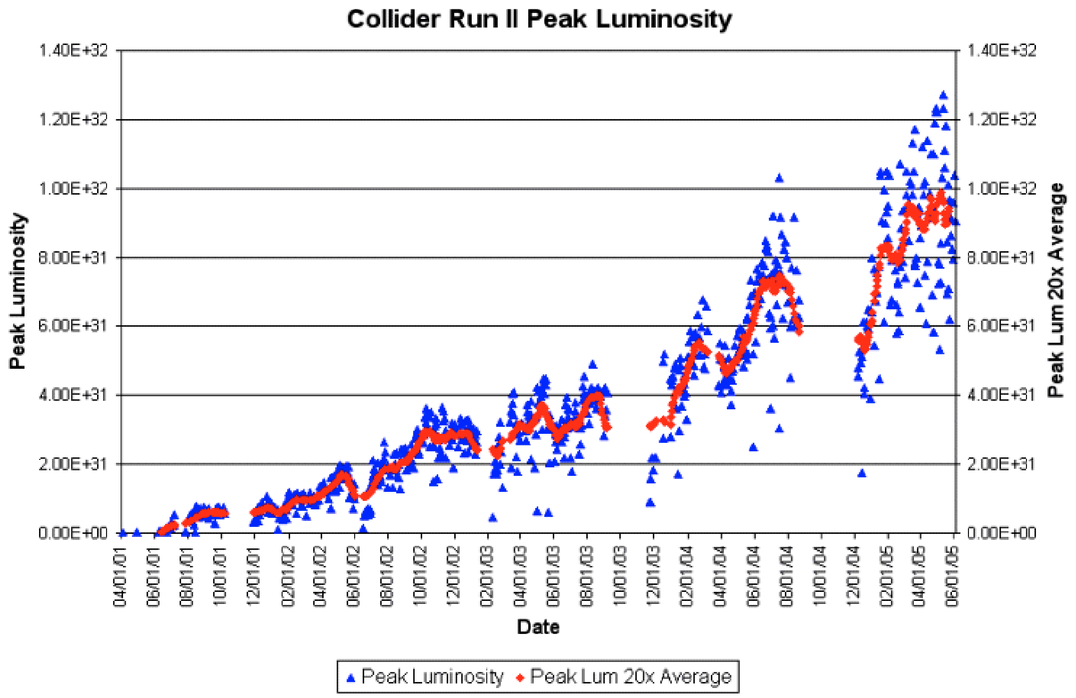
This is one of the deepest questions humans have ever pursued, and

it is coming within the reach of particle physics.

Tevatron Collider is running *now*, breaking new ground in sensitivity



Collider Run II Integrated Luminosity
Weekly Integrated Luminosity (pb-1)
Run Integrated Luminosity (pb-1)
Week #
(Week 1 starts 03/05/01)
Weekly Integrated Luminosity
Run Integrated Luminosity



Collider Run II Peak Luminosity
Peak Luminosity
Peak Lum 20x Average
Date
Peak Luminosity
Peak Lum 20x Average

Tevatron Collider in a Nutshell

980-GeV protons, antiprotons

(2π km)

frequency of revolution $\approx 45\,000$ s^{-1}

392 ns between crossings

($36 \otimes 36$ bunches)

collision rate $= \mathcal{L} \cdot \sigma_{\text{inelastic}} \approx 10^7$ s^{-1}

$c \approx 10^9$ km/h; $\quad v_p \approx c - 495$ km/h

Record $\mathcal{L}_{\text{init}} = 1.27 \times 10^{32}$ cm^{-2} s^{-1}

[ISR: pp, 1.4]

Maximum $\bar{p}$ at Low β: 1.661×10^{12}

The Large Hadron Collider will operate *soon*, breaking new ground in energy and sensitivity

LHC in a nutshell

7-TeV protons on protons (27 km);

$v_p \approx c - 10$ km/h

Novel two-in-one dipoles (≈ 9 teslas)

Startup: 43 $\otimes$ 43 bunches,

$\mathcal{L} \approx 6 \times 10^{31}$ cm^{-2} s^{-1}

Early: 936 bunches,

$\mathcal{L} \gtrsim 5 \times 10^{32}$ cm^{-2} s^{-1} [75 ns]

First year? 2808 bunches,

$\mathcal{L} \to 2 \times 10^{33}$ cm^{-2} s^{-1}

25 ns bunch spacing

Eventual $\mathcal{L} \gtrsim 10^{34}$ cm^{-2} s^{-1}:

100 fb^{-1}/year

Tentative Outline . . .

▷ Preliminaries

Current state of particle physics

A few words about QCD

Sources of mass

▷ Antecedents of the electroweak theory

What led to EW theory

What EW theory needs to explain

▷ Some consequences of the Fermi theory

μ decay

νe scattering

. . . Outline . . .

▷ $SU(2)_L \otimes U(1)_Y$ theory

Gauge theories

Spontaneous symmetry breaking

Consequences: $W^\pm$, Z^0/NC, H, m_f?

Measuring $\sin^2\theta_W$ in νe scattering

GIM / CKM

▷ Phenomena at tree level and beyond

Z^0 pole

W mass and width

Atomic parity violation

Looking for trouble

m_t, M_W, M_Z correlation

Vacuum energy problem

. . . Outline

- ▷ The Higgs boson and the 1-TeV scale
  - Why the Higgs boson must exist
  - Higgs properties, constraints
  - How well can we anticipate M_H?
  - Higgs searches
- ▷ The problems of mass
- ▷ The EW scale and beyond
  - Hierarchy problem
  - Why is the EW scale so small?
  - Why is the Planck scale so large?
- ▷ Outlook

General References

▷ C. Quigg, "Nature's Greatest Puzzles," hep-ph/0502070

▷ C. Quigg, "The Electroweak Theory," hep-ph/0204104 (TASI 2000 Lectures)

▷ C. Quigg, *Gauge Theories of the Strong, Weak, and Electromagnetic Interactions*

▷ R. N. Cahn & G. Goldhaber, *Experimental Foundations of Particle Physics*

▷ G. Altarelli & M. Grünewald, "Precision Electroweak Tests of the Standard Model," hep-ph/0404165

▷ F. Teubert, "Electroweak Physics," ICHEP04

▷ S. Eidelman et al., "Review of Particle Physics," *Phys. Lett.* **B592,** 1 (2004)

Problem sets: http://lutece.fnal.gov/TASI/default.html

Our picture of matter

Pointlike constituents ($r < 10^{-18}$ m)

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L$$

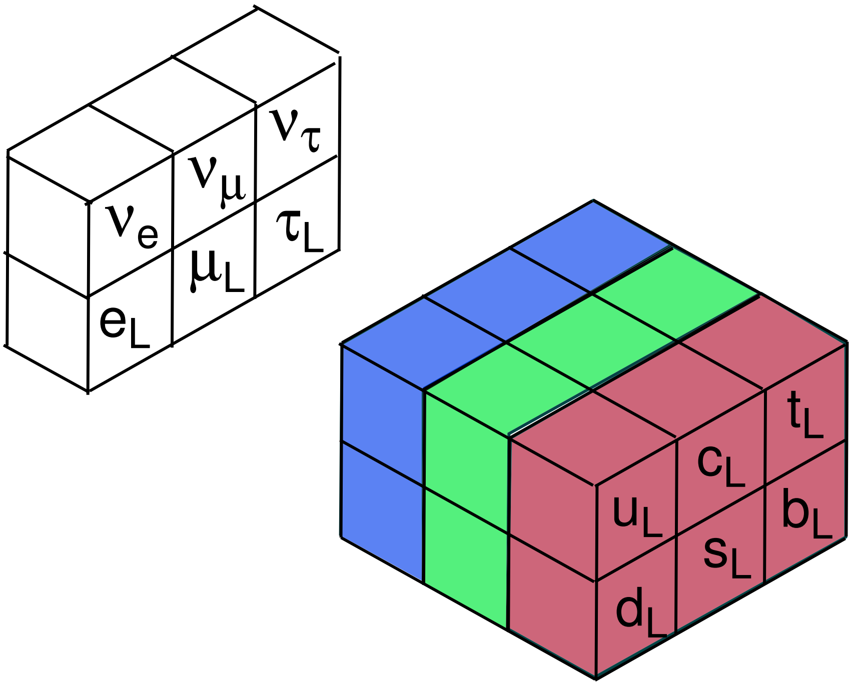
$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$$

Few fundamental forces, derived from gauge symmetries

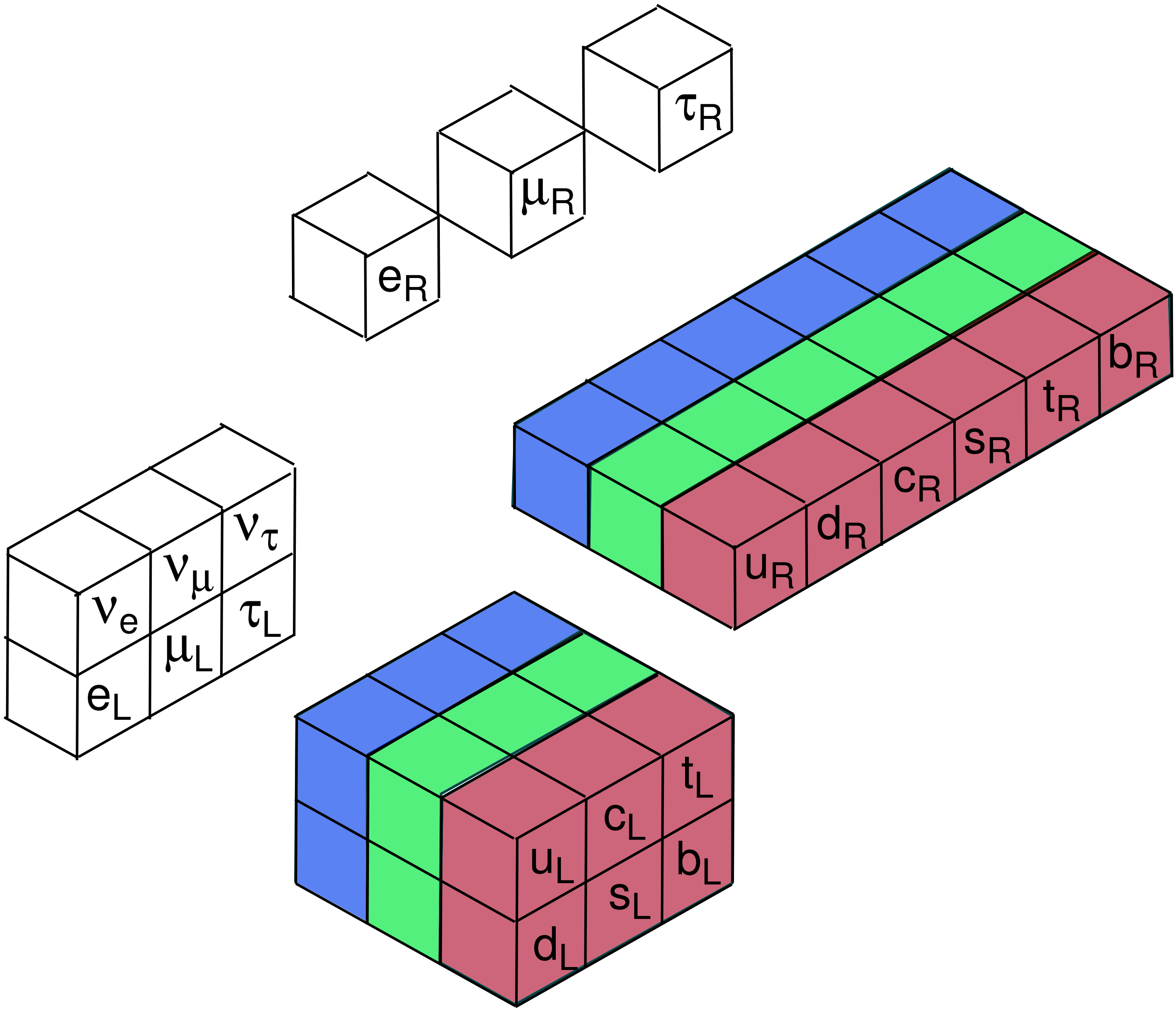
$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$$

Electroweak symmetry breaking

Higgs mechanism?



ν_e
ν_μ
ν_τ
e_L
μ_L
τ_L
t_L
c_L
u_L
b_L
s_L
d_L


τR
μR
eR
bR
tR
sR
cR
dR
uR
ντ
νμ
νe
τL
μL
eL
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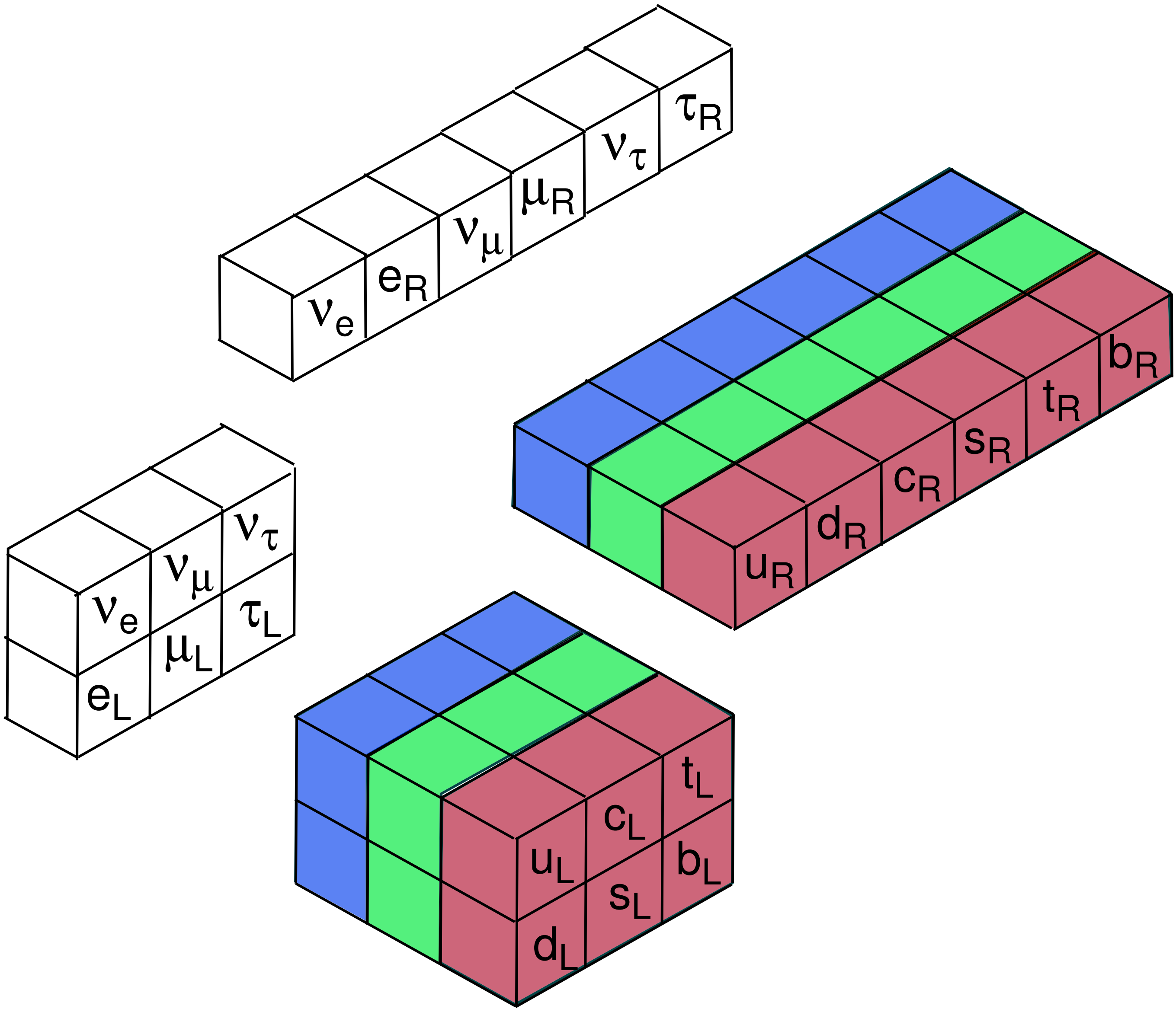
cL
uL
bL
sL
dL


ν_e
e_R
ν_μ
μ_R
ν_τ
τ_R
ν_e
ν_μ
ν_τ
e_L
μ_L
τ_L
u_R
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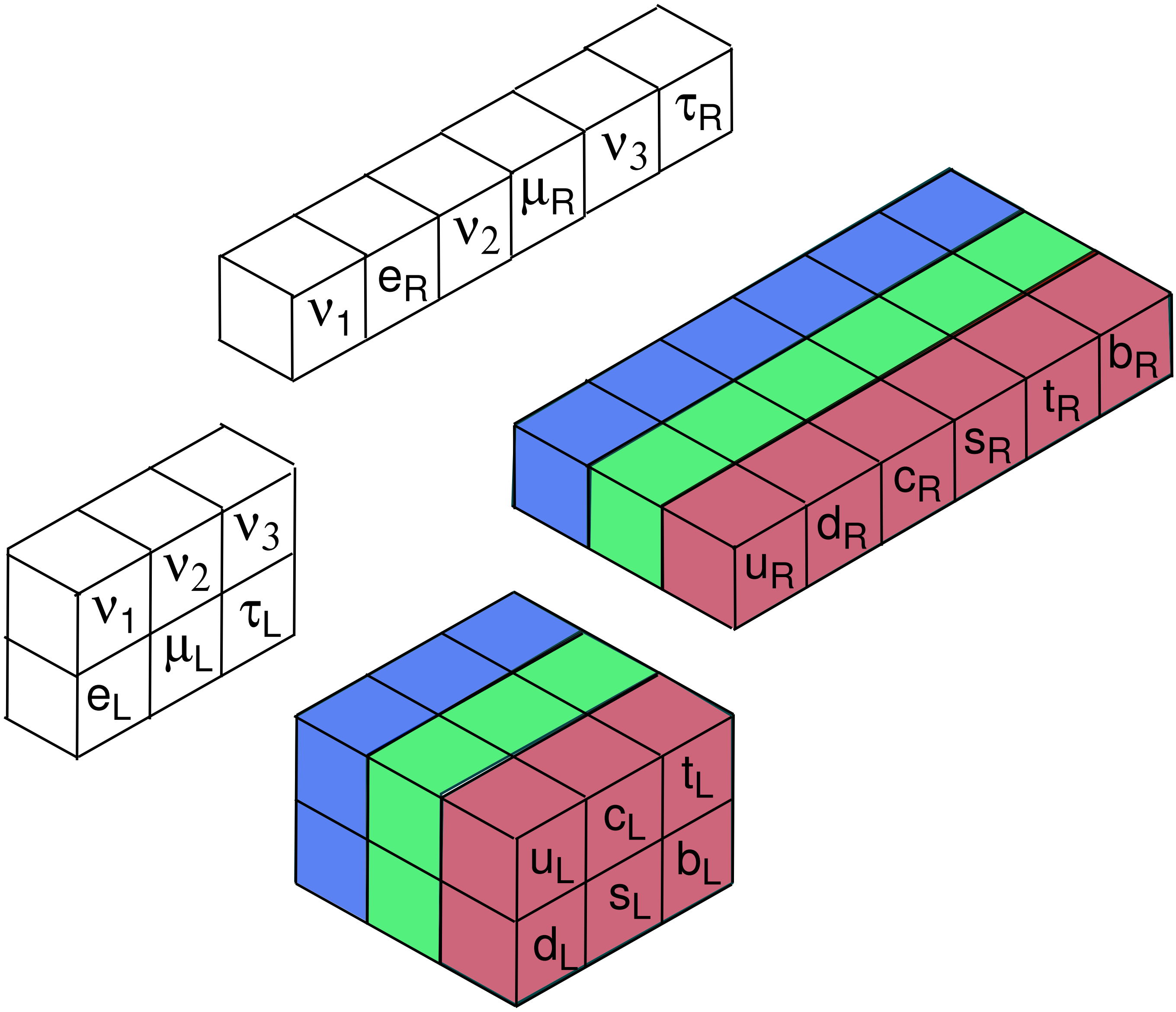
t_L
d_L
s_L
b_L


ν1
eR
ν2
μR
ν3
τR
ν1
ν2
ν3
eL
μL
τL
uR
dR
cR
sR
tR
bR
uL
cL
tL
dL
sL
bL

Elementarity

▷ Are quarks and leptons structureless?

Symmetry

▷ Electroweak symmetry breaking and the 1-TeV scale

▷ Origin of gauge symmetries

▷ Meaning of discrete symmetries

Unity

▷ Coupling constant unification

▷ Unification of quarks and leptons

(neutrality of atoms $\Rightarrow$ new forces!);

of constituents and force particles

▷ Incorporation of gravity

Identity

▷ Fermion masses and mixings; CP violation; ν oscillations

▷ What makes an electron an e and a top quark a t?

Topography

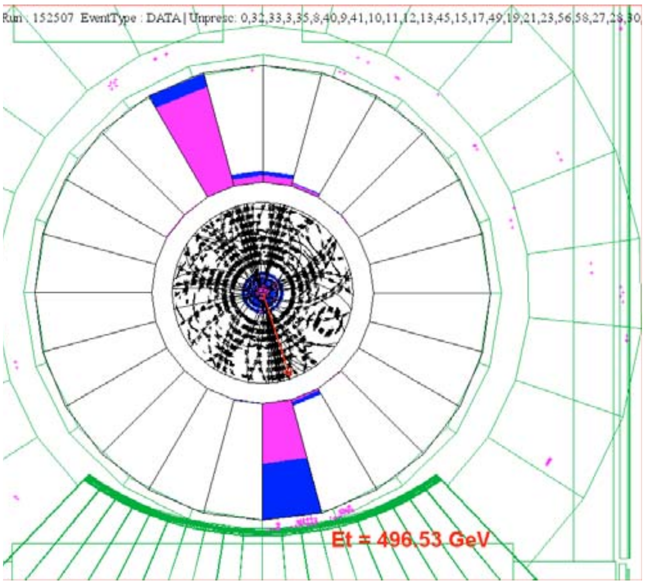
▷ What is the fabric of space and time?

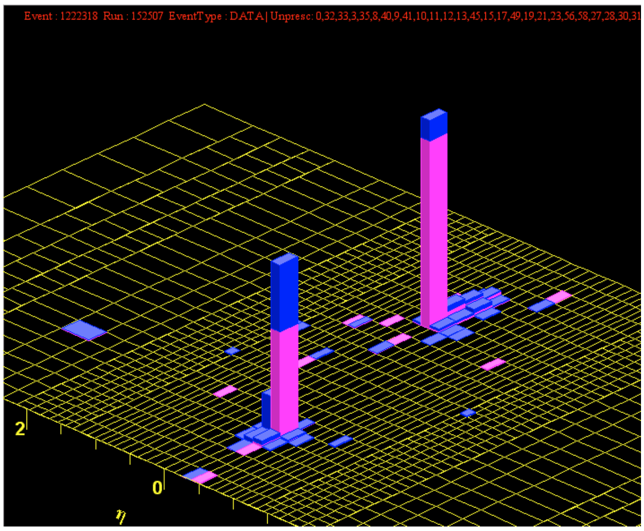
...the origin of space and time?

Elementarity

The World's Most Powerful Microscopes

nanonanophysics





CDF dijet event ($\sqrt{s} = 1.96$ TeV):

$E_T = 1.364$ TeV

$q\bar{q} \longrightarrow$ jet + jet

Elementarity

If the Lagrangian has the form $\pm \frac{g^2}{2\Lambda^2} \overline{\psi}_L \gamma_\mu \psi_L \overline{\psi}_L \gamma^\mu \psi_L$ (with $g^2/4\pi$ set equal to 1), then we define $\Lambda \equiv \Lambda^{\pm}_{LL}$. For the full definitions and for other forms, see the Note in the Listings on Searches for Quark and Lepton Compositeness in the full *Review* and the original literature.

$\Lambda^+_{LL}(eeee)$ > 8.3 TeV, CL = 95%

$\Lambda^-_{LL}(eeee)$ > 10.3 TeV, CL = 95%

$\Lambda^+_{LL}(ee\mu\mu)$ > 8.5 TeV, CL = 95%

$\Lambda^-_{LL}(ee\mu\mu)$ > 6.3 TeV, CL = 95%

$\Lambda^+_{LL}(ee\tau\tau)$ > 5.4 TeV, CL = 95%

$\Lambda^-_{LL}(ee\tau\tau)$ > 6.5 TeV, CL = 95%

$\Lambda^+_{LL}(\ell\ell\ell\ell)$ > 9.0 TeV, CL = 95%

$\Lambda^-_{LL}(\ell\ell\ell\ell)$ > 7.8 TeV, CL = 95%

$\Lambda^+_{LL}(eeuu)$ > 23.3 TeV, CL = 95%

$\Lambda^-_{LL}(eeuu)$ > 12.5 TeV, CL = 95%

$\Lambda^+_{LL}(eedd)$ > 11.1 TeV, CL = 95%

$\Lambda^-_{LL}(eedd)$ > 26.4 TeV, CL = 95%

$\Lambda^+_{LL}(eecc)$ > 1.0 TeV, CL = 95%

$\Lambda^-_{LL}(eecc)$ > 2.1 TeV, CL = 95%

$\Lambda^+_{LL}(eebb)$ > 5.6 TeV, CL = 95%

$\Lambda^-_{LL}(eebb)$ > 4.9 TeV, CL = 95%

$\Lambda^+_{LL}(\mu\mu qq)$ > 2.9 TeV, CL = 95%

$\Lambda^-_{LL}(\mu\mu qq)$ > 4.2 TeV, CL = 95%

$\Lambda(\ell\nu\ell\nu)$ > 3.10 TeV, CL = 90%

$\Lambda(e\nu qq)$ > 2.81 TeV, CL = 95%

$\Lambda^+_{LL}(qqqq)$ > 2.7 TeV, CL = 95%

$\Lambda^-_{LL}(qqqq)$ > 2.4 TeV, CL = 95%

$\Lambda^+_{LL}(\nu\nu qq)$ > 5.0 TeV, CL = 95%

$\Lambda^-_{LL}(\nu\nu qq)$ > 5.4 TeV, CL = 95%

Two views of Symmetry

1. *Indistinguishability*

One object transformed into another

Familiar (and useful!) from

Global Symmetries: isospin, $SU(3)_f$, ...

Spacetime Symmetries

Gauge Symmetries

"EQUIVALENCE"

Idealize more perfect worlds, the better to understand our diverse, changing world

Unbroken unified theory: perfect world of equivalent forces, interchangeable massless particles ... *Perfectly boring?*

Symmetry $\Leftrightarrow$ *Disorder*

A Perfect World

Two views of Symmetry

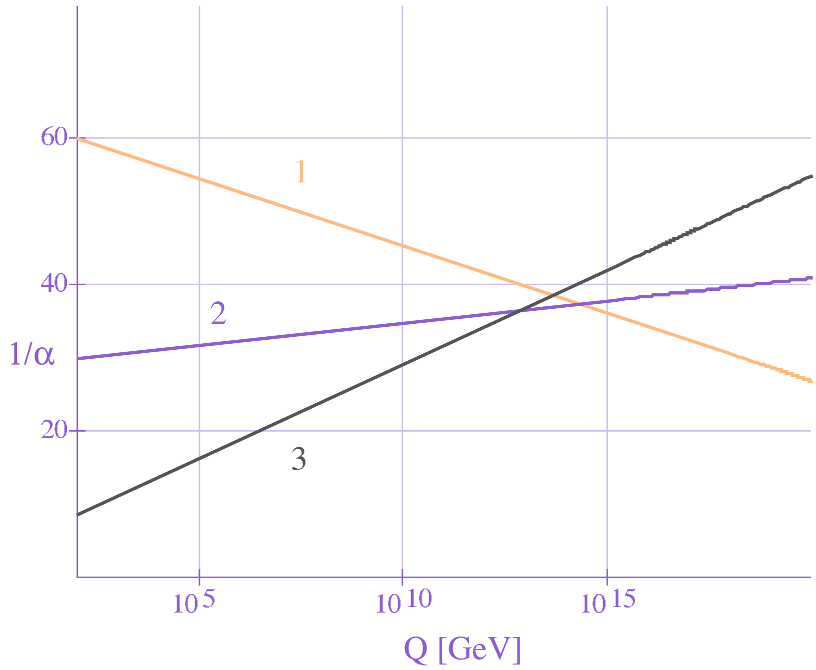
2. *Unobservable*

Goodness of a symmetry means
something cannot be measured

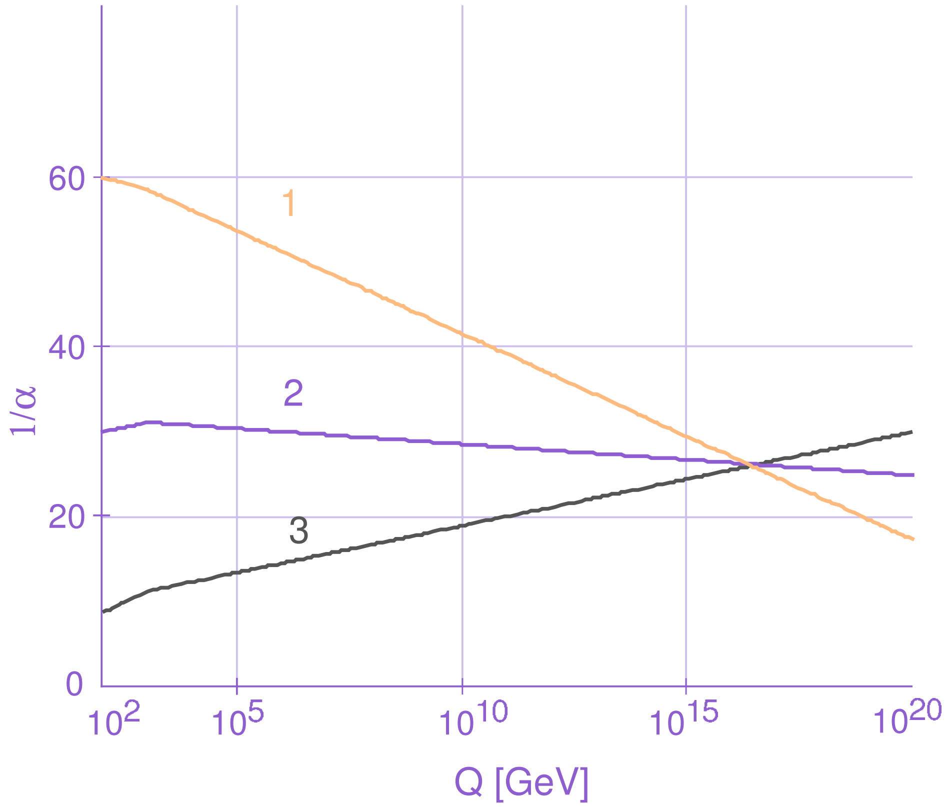
e.g., vanishing asymmetry

Unobservable	Transformation	Conserved
Absolute position	$\vec{r} \to \vec{r} + \vec{\Delta}$	$\vec{p}$
Absolute time	$t \to t + \delta$	E
Absolute orientation	$\hat{r} \to \hat{r}'$	$\vec{L}$
Absolute velocity	$\vec{v} \to \vec{v} + \vec{w}$	
Absolute right	$\vec{r} \to -\vec{r}$	P
Absolute future	$t \to -t$	T
Absolute charge	$Q \to -Q$	C
Absolute phase		
⋮		

Unity



60
40
20
1/α
1
2
3
10^5
10^10
10^15
Q [GeV]



0
20
40
60
1/α
1
2
3
10^2
10^5
10^10
10^15
10^20
Q [GeV]

QCD is part of the standard model

. . . a remarkably simple, successful, and rich theory

Wilczek, hep-ph/9907340

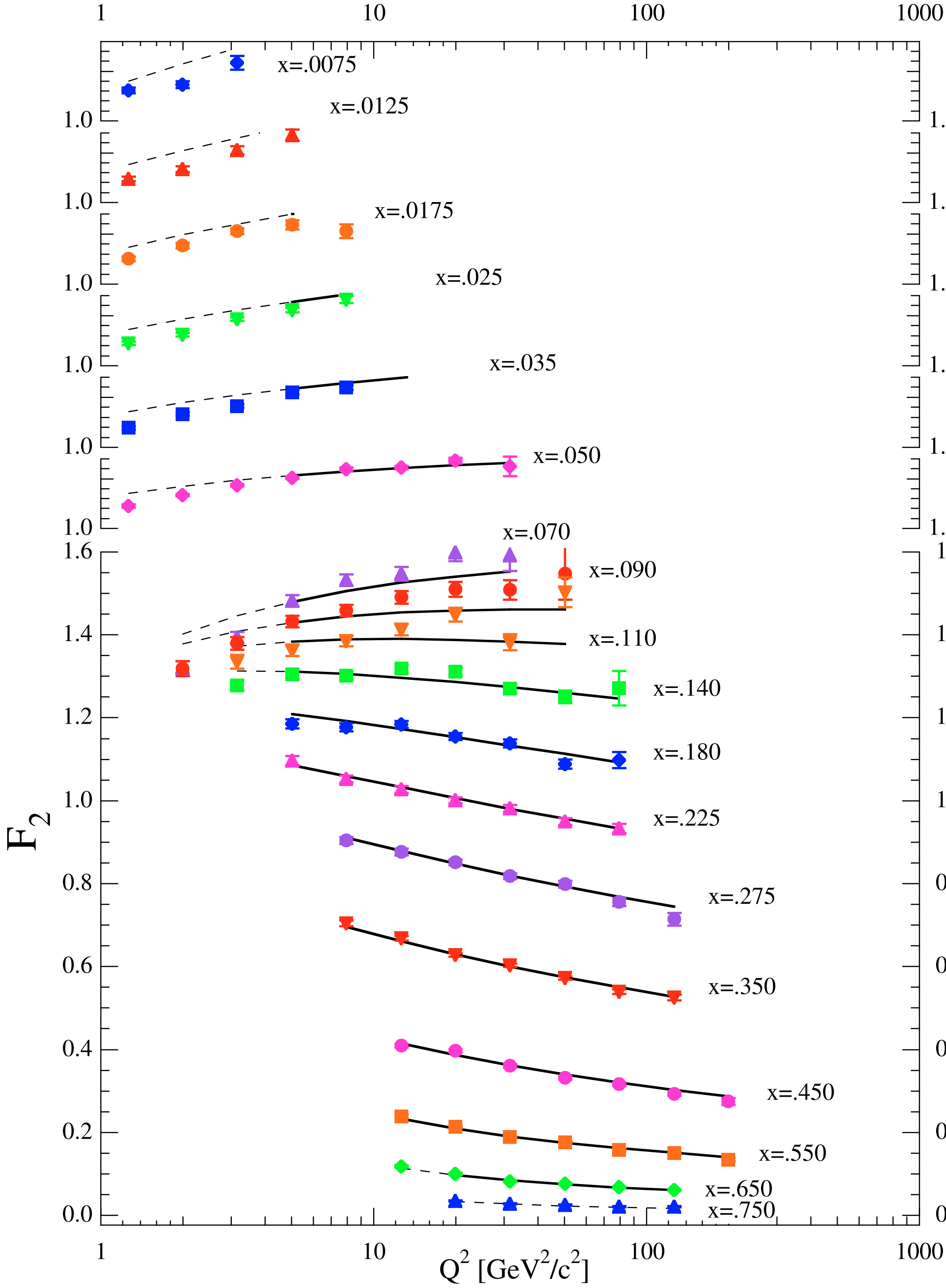
▷ Perturbative QCD

- Exists, thanks to asymptotic freedom
- Describes many phenomena in quantitative detail:
  - ▷ Q^2-evolution of structure functions
  - ▷ Jet production in $\bar{p}p$ collisions
  - ▷ Many decays, event shapes, . . .
- We can measure the running of α_s *(engineering value for unification)*

▷ Nonperturbative (lattice) QCD

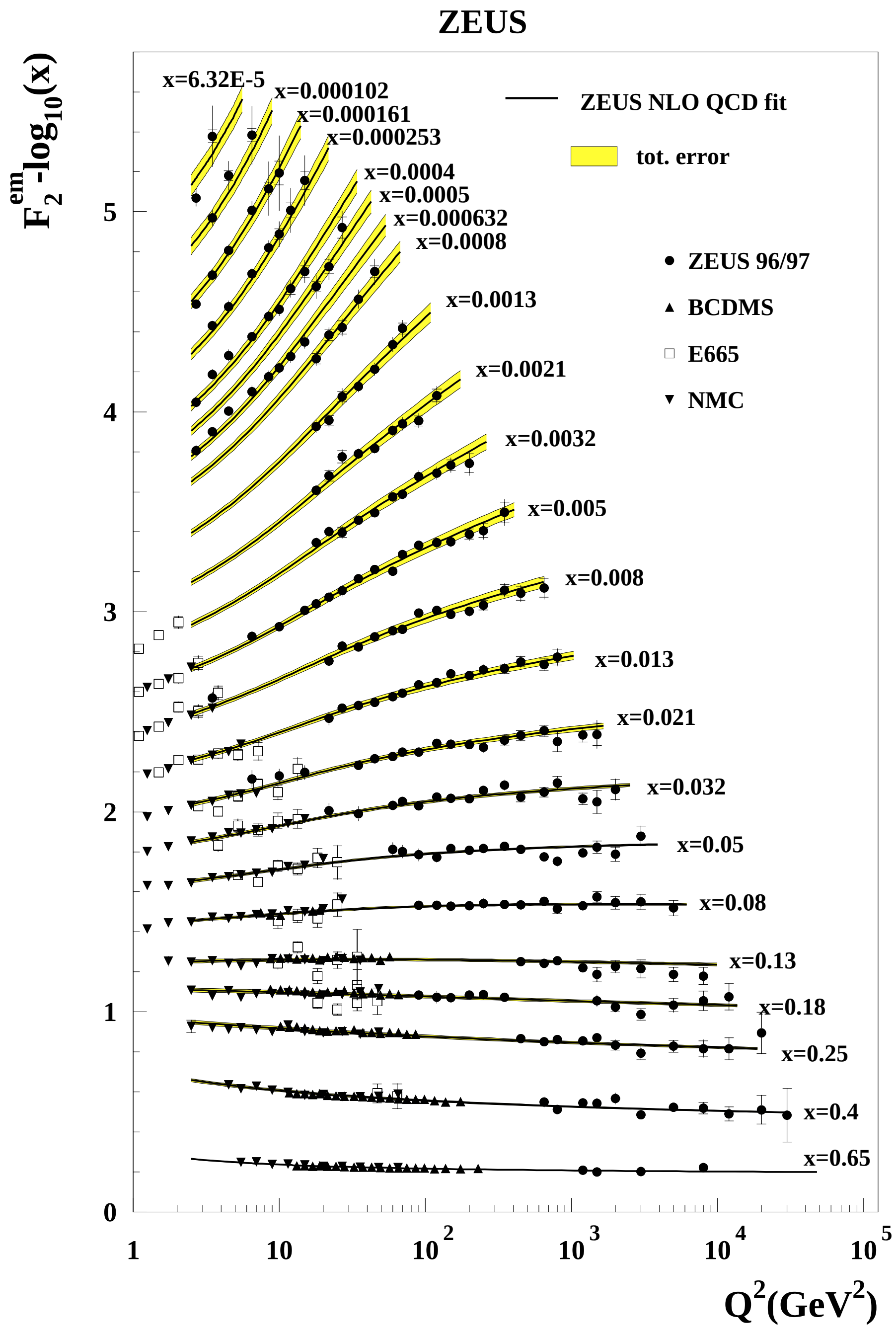
- Predicts the hadron spectrum
- Gives our best information on quark masses, etc. El-Khadra & Luke, hep-ph/0208114

$F_2(x, Q^2)$ in νN interactions (CCFR)



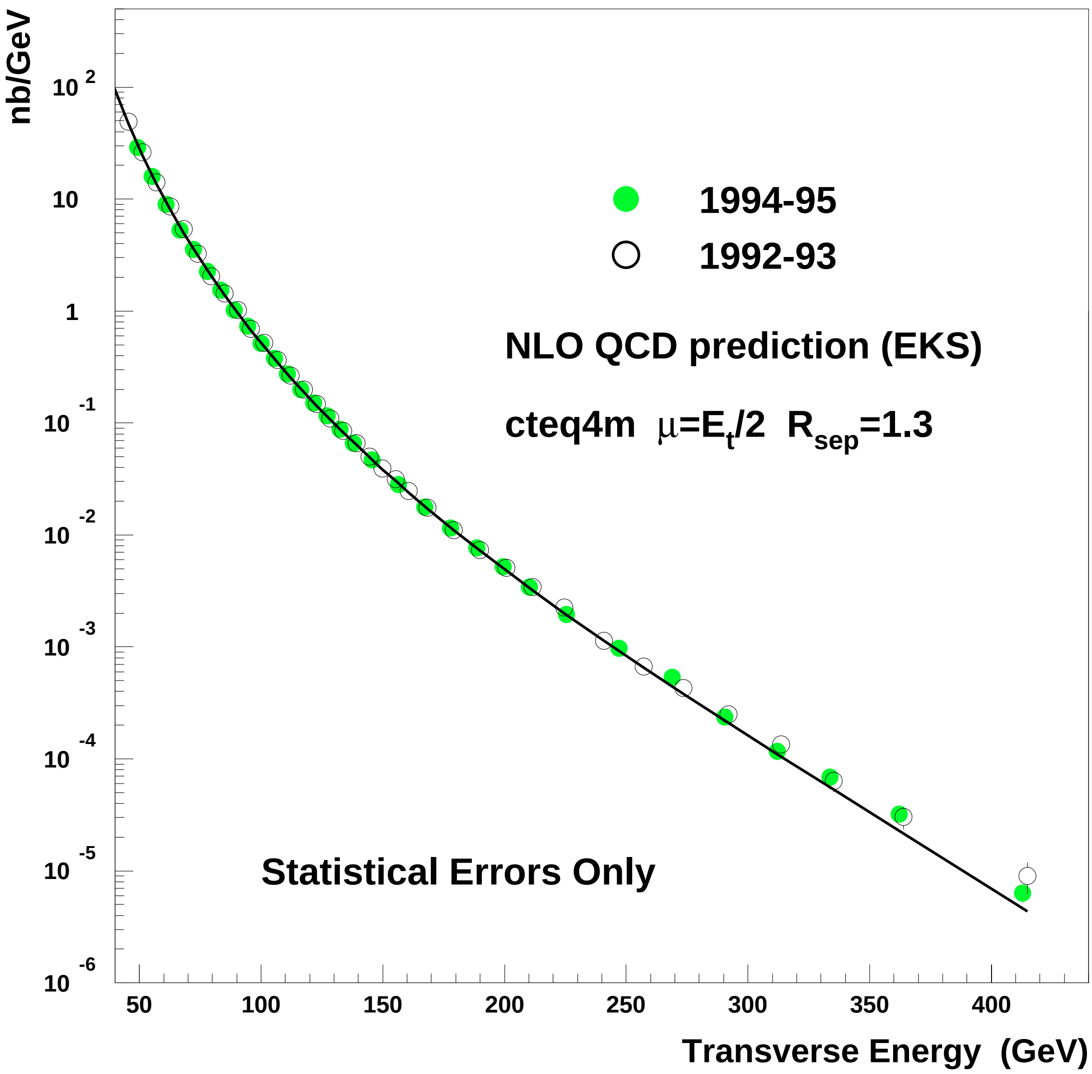
Conrad, Shaevitz, Bolton, *Rev. Mod. Phys.* **70**, 1341 (1998).

$F_2(x, Q^2)$ in ℓN interactions (ZEUS)



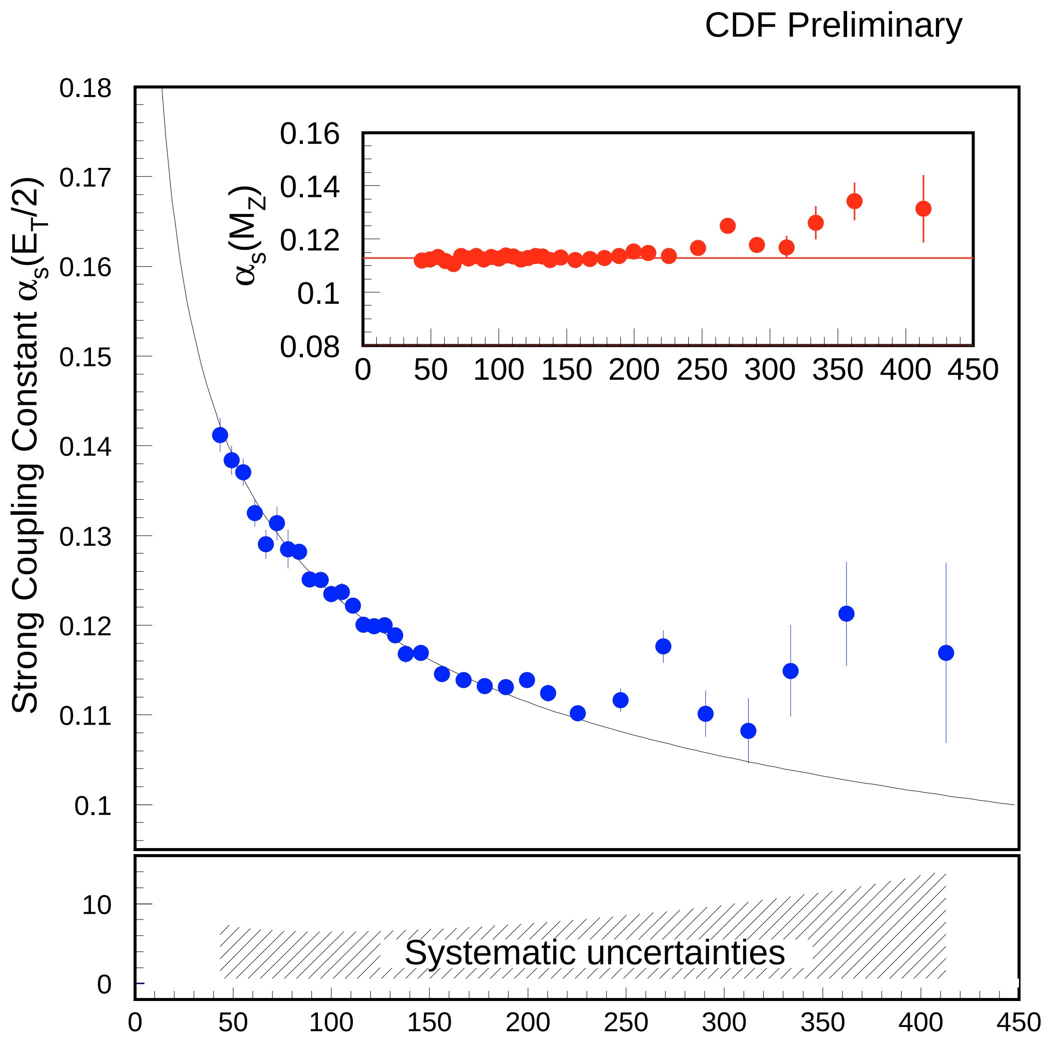
ZEUS, hep-ex/0208023.

Inclusive jet cross section at $\sqrt{s} = 1.8$ TeV (CDF)



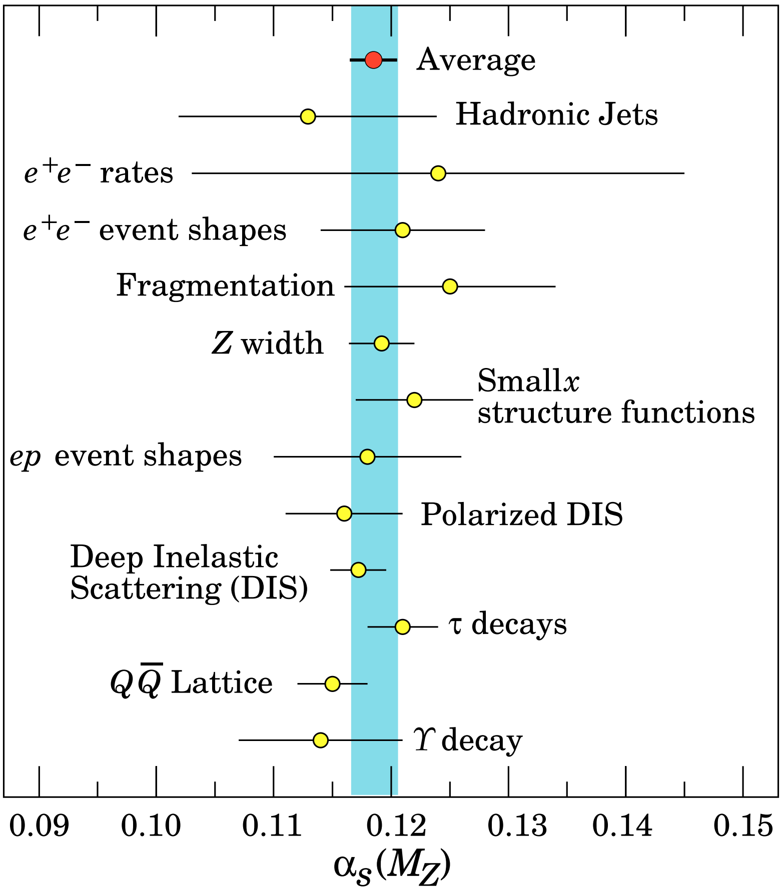
T. Affolder et al. (CDF), *Phys. Rev. D* **64**, 032001 (2001)

$\alpha_s(E_\mathrm{T}/2)$ **from** $\bar{p}p \rightarrow$ **jets**

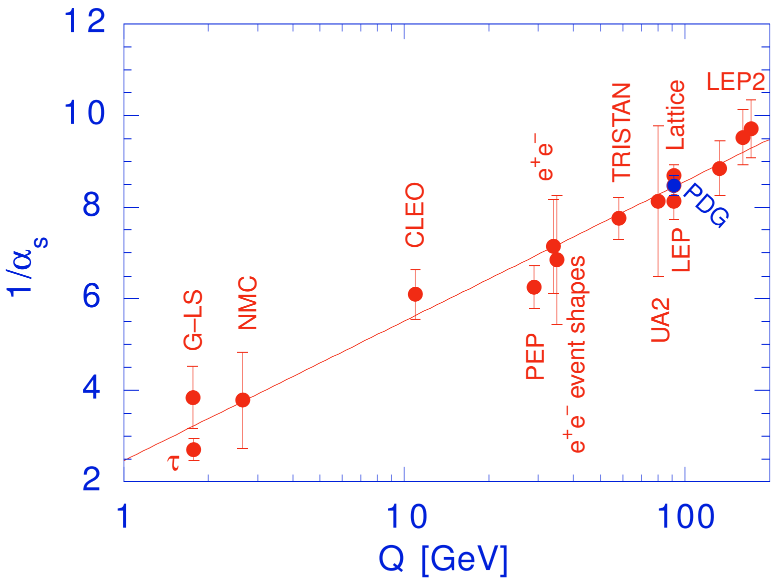


T. Affolder et al. (CDF), *Phys. Rev. Lett.* **88**, 042001 (2002)

Comprehensive survey: W. de Boer, hep-ph/0407021


Average
Hadronic Jets
e^+e^- rates
e^+e^- event shapes
Fragmentation
Z width
Small$x$
structure functions
ep event shapes
Polarized DIS
Deep Inelastic
Scattering (DIS)
τ decays
$Q\overline{Q}$ Lattice
Υ decay
0.09
0.10
0.11
0.12
0.13
0.14
0.15
$\alpha_s(M_Z)$

PDG


12
10
8
6
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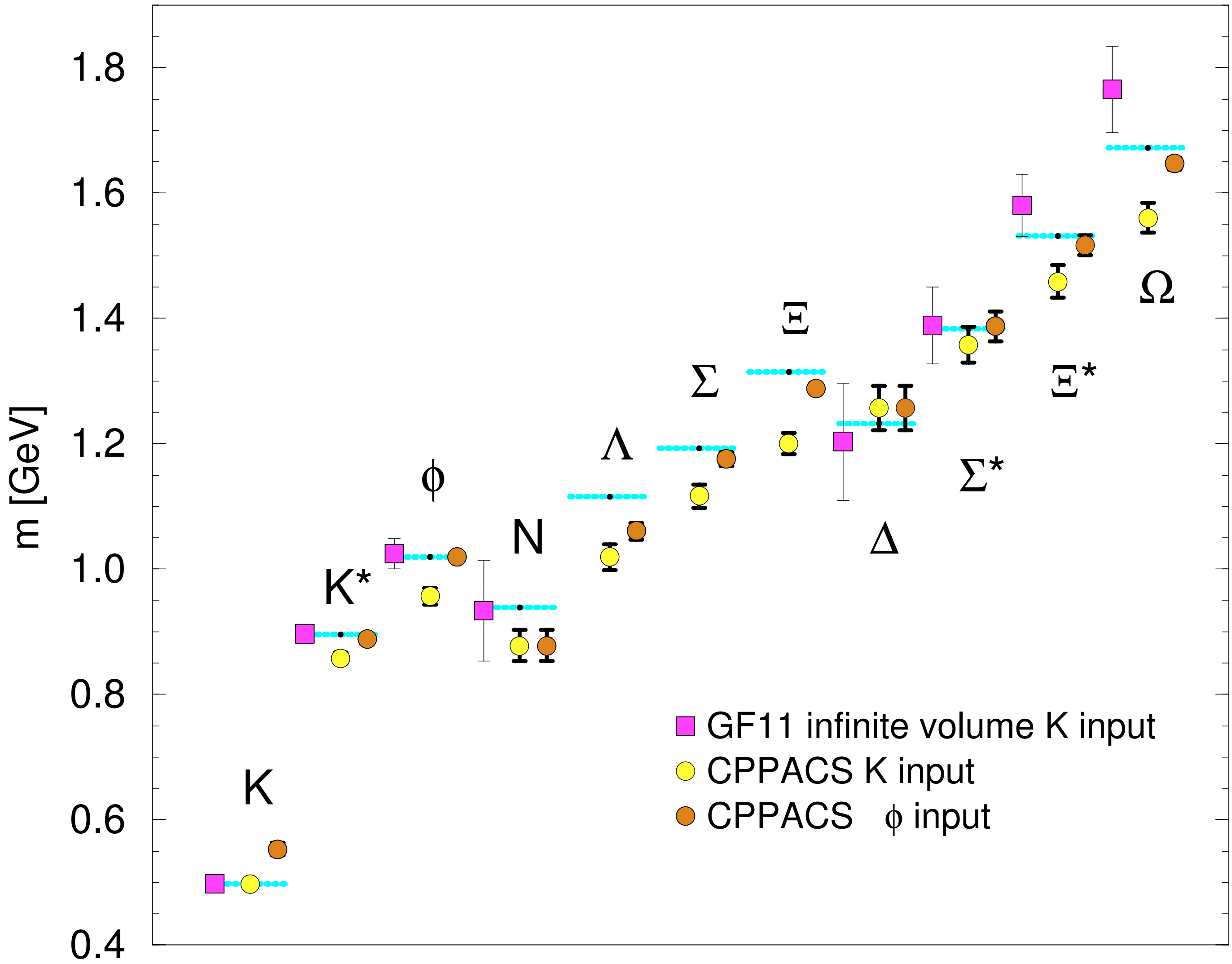
2
$1/\alpha_s$
1
10
100
Q [GeV]
τ
G–LS
NMC
CLEO
PEP
e^+e^- event shapes
e^+e^-
TRISTAN
UA2
LEP
Lattice
PDG
LEP2

Quenched hadron spectrum



S. Aoki, *et al.* (CP–PACS), *Phys. Rev. Lett.* **84**, 238 (2000)

(No dynamical fermions)

The Origins of Mass

(masses of nuclei "understood")

$p, [\pi], \rho$ understood: QCD

confinement energy is the source

"Mass without mass"

We understand the visible mass of the Universe
. . . without the Higgs mechanism

W, Z electroweak symmetry breaking

$$M_W^2 = \tfrac{1}{2} g^2 v^2 = \pi\alpha / G_F \sqrt{2} \sin^2\theta_W$$

$$M_Z^2 = M_W^2 / \cos^2\theta_W$$

$q, \ell^\mp$ EWSB + Yukawa couplings

ν_ℓ EWSB + Yukawa couplings; new physics?

All fermion masses $\Leftrightarrow$ physics beyond standard model

H ?? fifth force ??

Antecedents of EW Theory

Commins & Bucksbaum, *Weak Int^{ns} of Leptons & Quarks*

1896: Becquerel radioactivity

Several varieties, including β decay

$$^{A}\mathrm{Z} \to \; ^{A}(\mathrm{Z}+1) + \beta^{-}$$

Examples:

$$^{3}\mathrm{H}_{1} \to \; ^{3}\mathrm{He}_{2} + \beta^{-} \; ,$$

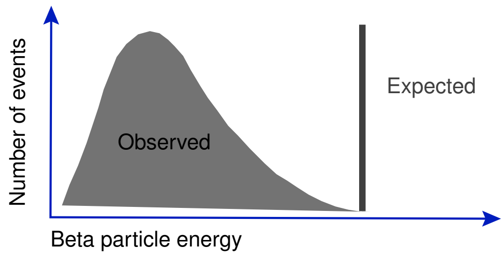
$$n \to p + \beta^{-} \; ,$$

$$^{214}\mathrm{Pb}_{82} \to \; ^{214}\mathrm{Bi}_{83} + \beta^{-} \; .$$

β^{+}-emitters, $^{A}\mathrm{Z} \to \; ^{A}(\mathrm{Z}-1) + \beta^{+}$, are rare among naturally occurring isotopes. Radio-phosphorus produced 1934 by the Joliot-Curie, *after* positron discovery in cosmic rays.

$^{19}\mathrm{Ne} \to \; ^{19}\mathrm{F} + \beta^{+}$ studied for right-handed charged currents and time reversal invariance; *positron-emission tomography*

1914: Chadwick β spectrum



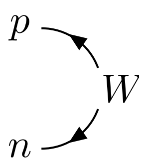
Energy conservation in question

1930: Pauli $\approx$ massless, neutral, penetrating particle

nuclear spin & statistics

$\hookrightarrow$ neutrino ν

β decay first hint for flavor



charged-current, flavor-changing interactions

1932: Chadwick neutron

$\hookrightarrow$ isospin symmetry

Neutron & flavor symmetry

$$M(n) = 939.565\,63 \pm 0.000\,28 \text{ MeV}/c^2$$
$$M(p) = 938.272\,31 \pm 0.000\,28 \text{ MeV}/c^2$$
$$\Delta M = 1.293318 \pm 0.000\,009 \text{ MeV}/c^2$$

$$\boxed{\Delta M/M \approx 1.4 \times 10^{-3}}$$

Charge-independent nuclear forces?

$$^3\text{H}(pnn) = 8.481\,855 \pm 0.000\,013 \text{ MeV}$$
$$^3\text{He}(ppn) = 7.718\,109 \pm 0.000\,013 \text{ MeV}$$
$$\Delta(\text{B.E.}) = 0.763\,46 \text{ MeV}$$

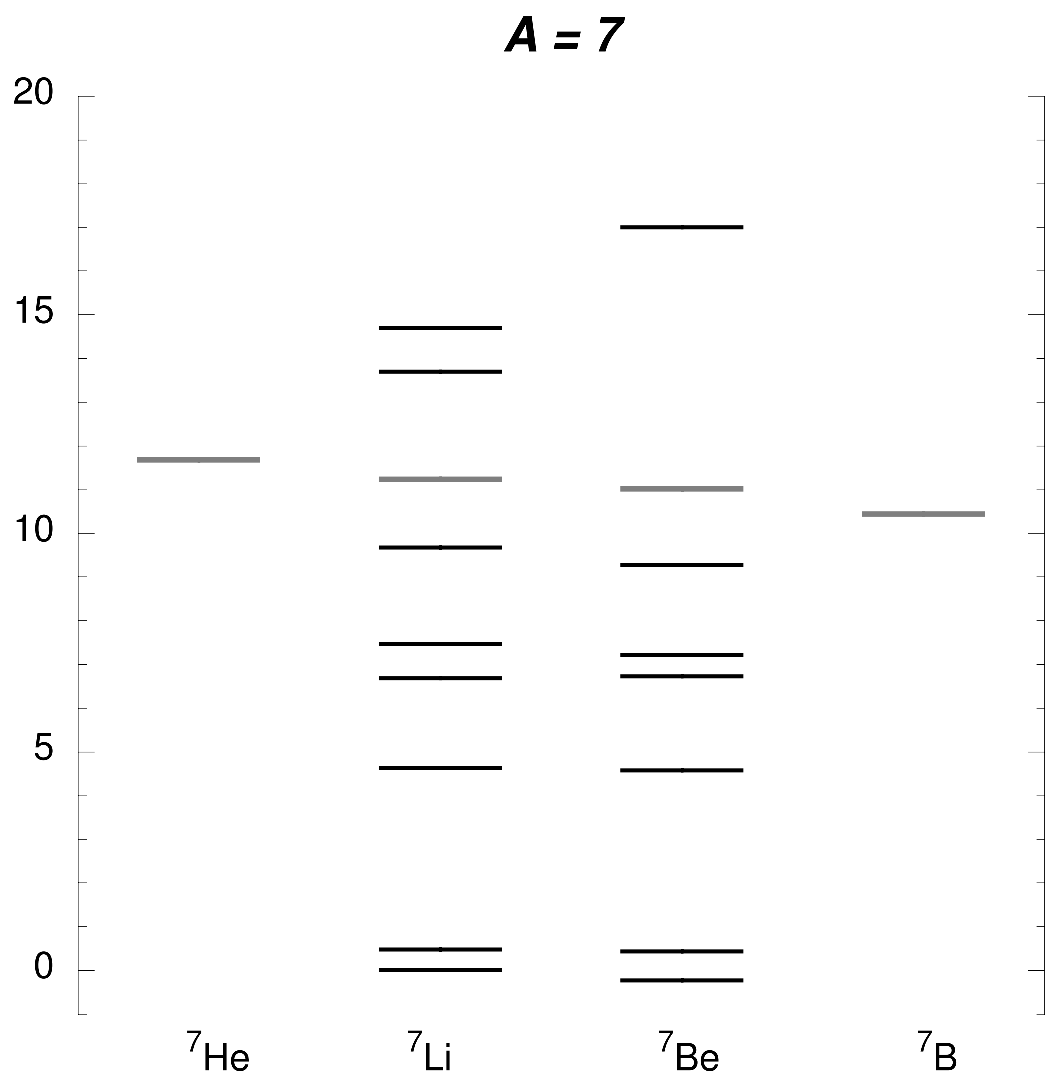
^{3}He charge radius $r = 1.97 \pm 0.015$ fm

$$\boxed{\text{Coulomb energy: } \alpha/r \approx 0.731 \text{ MeV}}$$

Level structures in mirror nuclei. 1

$I_3 = -\frac{1}{2}$: $^7\text{Li}(3p + 4n)$ $\quad$ $^7\text{Be}(4p + 3n)$: $I_3 = \frac{1}{2}$

$I_3 = -\frac{3}{2}$: $^7\text{He}(2p + 3n)$ $\quad$ $^7\text{B}(5p + 2n)$: $I_3 = \frac{3}{2}$



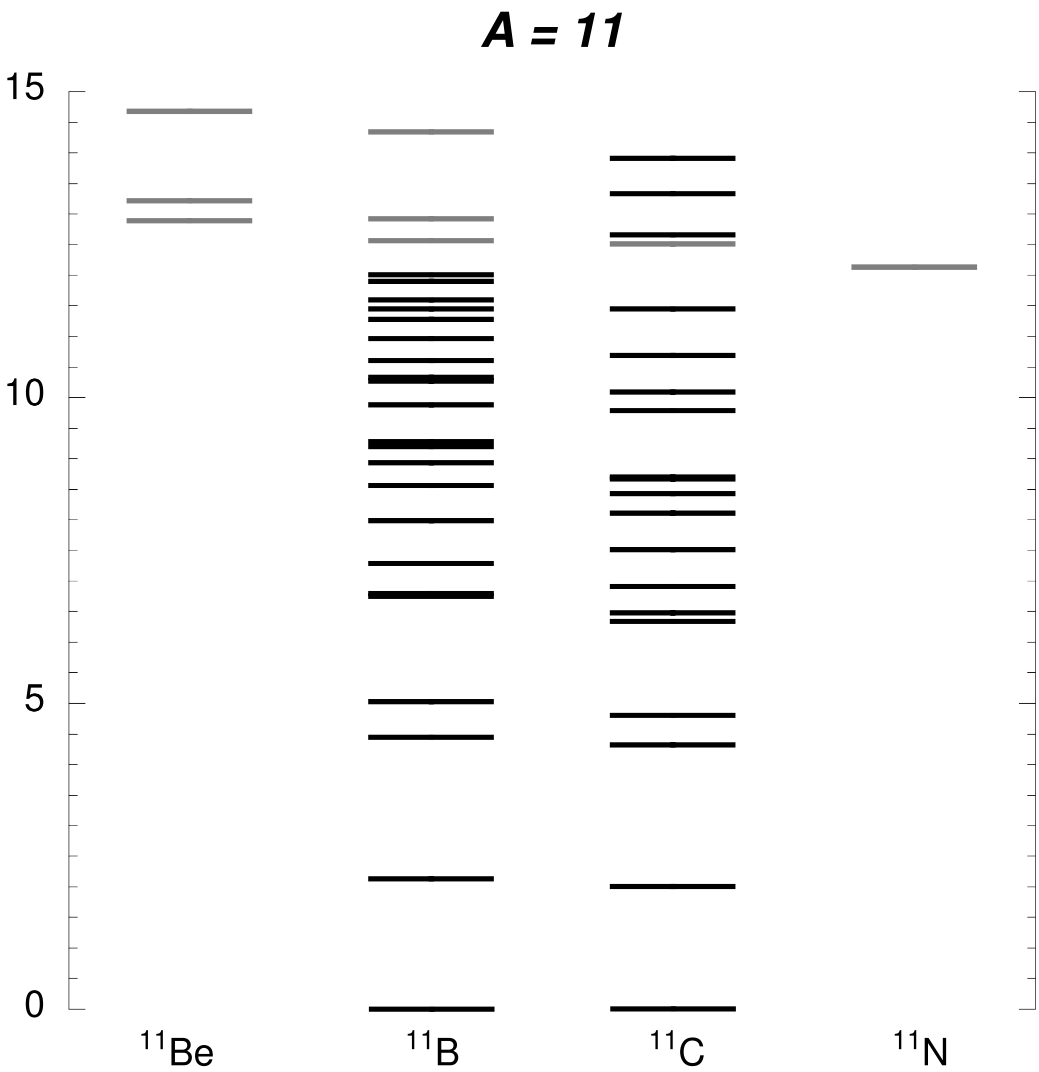
n - p mass difference, Coulomb energy removed

(isobaric analogue states)

Level structures in mirror nuclei. 2

$I_3 = -\frac{1}{2}$: ${}^{11}\mathrm{B}(5p + 6n)$ ${}^{11}\mathrm{C}(6p + 5n)$: $I_3 = \frac{1}{2}$

$I_3 = -\frac{3}{2}$: ${}^{11}\mathrm{Be}(4p + 7n)$ ${}^{11}\mathrm{N}(7p + 4n)$: $I_3 = \frac{3}{2}$

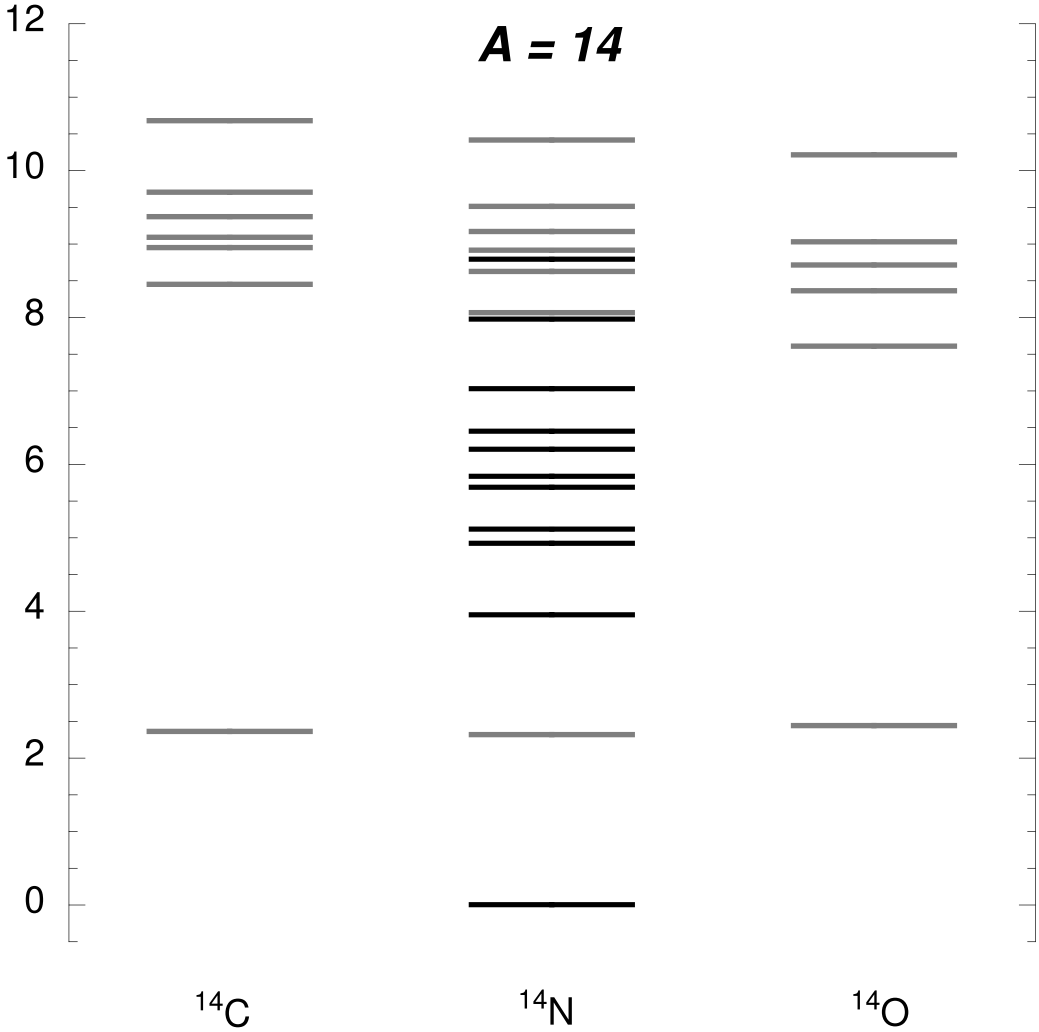


${}^{11}\mathrm{Li}(3p + 8n)$ ground state $(34.4$ MeV$)$ $I = \frac{5}{2}$ isobaric analogue

Level structures in mirror nuclei. 3

$A = 14$: NN outside closed core

$$
\begin{array}{lll}
{}^{14}\mathrm{O}: & {}^{12}\mathrm{C} + (pp) & I_3 = +1 \\
{}^{14}\mathrm{N}: & {}^{12}\mathrm{C} + (pn) & I_3 = 0 \\
{}^{14}\mathrm{C}: & {}^{12}\mathrm{C} + (nn) & I_3 = -1
\end{array}
$$



The first flavor symmetry

isospin invariance $\begin{pmatrix} p \\ n \end{pmatrix}$ isospin rotations

In the absence of EM, *convention* determines which (combination) is up

Aside: *Without EM,* how would we know there are two species of nucleons?

Parity violation in weak decays

1956 Wu *et al.*: correlation between
spin vector $\vec{J}$ of polarized $^{60}\mathrm{Co}$ and
direction $\hat{p}_e$ of outgoing β particle

Parity leaves spin (axial vector) unchanged $\boxed{\mathcal{P} : \vec{J} \rightarrow \vec{J}}$

Parity reverses electron direction $\boxed{\mathcal{P} : \hat{p}_e \rightarrow -\hat{p}_e}$

$\boxed{\text{Correlation } \vec{J} \cdot \hat{p}_e \text{ is } \textit{parity violating}}$

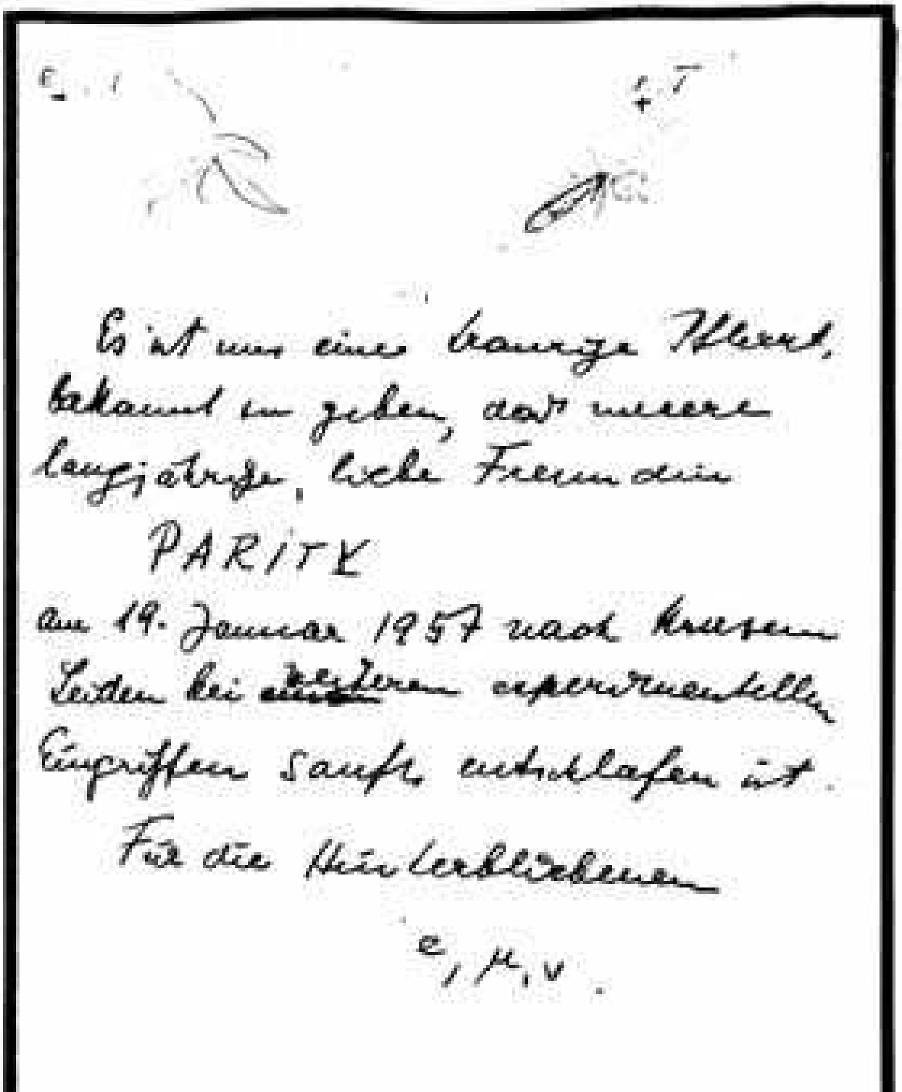
Experiments in late 1950s established that (charged-current) weak interactions are left-handed

Parity links left-handed, right-handed neutrinos,

$$\nu_L \xrightarrow{\Leftarrow} \mathcal{P} \xleftarrow{\Leftarrow} \not{\nu}_R$$

$\Rightarrow$ build a manifestly parity-violating theory with only ν_L.

Pauli's Reaction to the Downfall of Parity



Es ist uns eine traurige Pflicht, bekannt zu geben, daß unsere langjährige, liebe Freundin

PARITY

am 19. Januar 1957 nach kurzem Leiden bei weiteren experimentellen Eingriffen sanft entschlafen ist.

Für die Hinterbliebenen

e, μ, ν.

Pauli's Reaction to the Downfall of Parity

Es ist uns eine traurige Pflicht, bekannt zu geben, daß unsere langjährige ewige Freundin

PARITY

den 19. Januar 1957 nach kurzen Leiden bei weiteren experimentellen Eingriffen sanfte entschlafen ist.

Für die hinterbliebenen

$e \quad \mu \quad \nu$

It is our sad duty to announce that our loyal friend of many years

PARITY

went peacefully to her eternal rest on the nineteenth of January 1957, after a short period of suffering in the face of further experimental interventions.

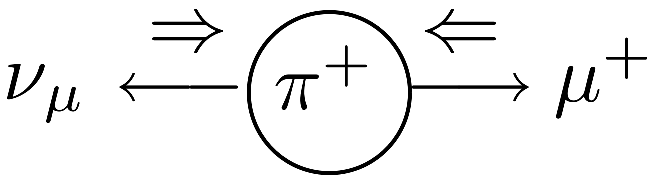
For those who survive her,

$e \quad \mu \quad \nu$

Pauli's assertiveness training . . .

How do we know ν is LH?

▷ Measure μ^+ helicity in (spin-zero) $\pi^+ \to \mu^+\nu_\mu$



$$h(\nu_\mu) = h(\mu^+)$$

Bardon, *Phys. Rev. Lett.* **7**, 23 (1961)

Possoz, *Phys. Lett.* **70B**, 265 (1977)

μ^+ forced to have "wrong" helicity

. . . inhibits decay, and inhibits $\pi^+ \to e^+\nu_e$ more

$$\Gamma(\pi^+ \to e^+\nu_e)/\Gamma(\pi^+ \to \mu^+\nu_\mu) = 1.23 \times 10^{-4}$$

▷ Measure longitudinal polarization of recoil nucleus in $\mu^{-12}\mathrm{C}(J = 0) \to {}^{12}\mathrm{B}(J = 1)\nu_\mu$

Infer $h(\nu_\mu)$ by angular momentum conservation

Roesch, *Am. J. Phys.* **50**, 931 (1981)

▷ Measure longitundinal polarization of recoil nucleus in

$$e^- \; {}^{152}\mathrm{Eu}^m(J=0) \;\; \to \;\; {}^{152}\mathrm{Sm}^*(J=1)\;\nu_e$$
$$\hookrightarrow \gamma \; {}^{152}\mathrm{Sm}$$

Infer $h(\nu_e)$ from γ polarization

Goldhaber, *Phys. Rev.* **109**, 1015 (1958)

Charge conjugation is also violated . . .

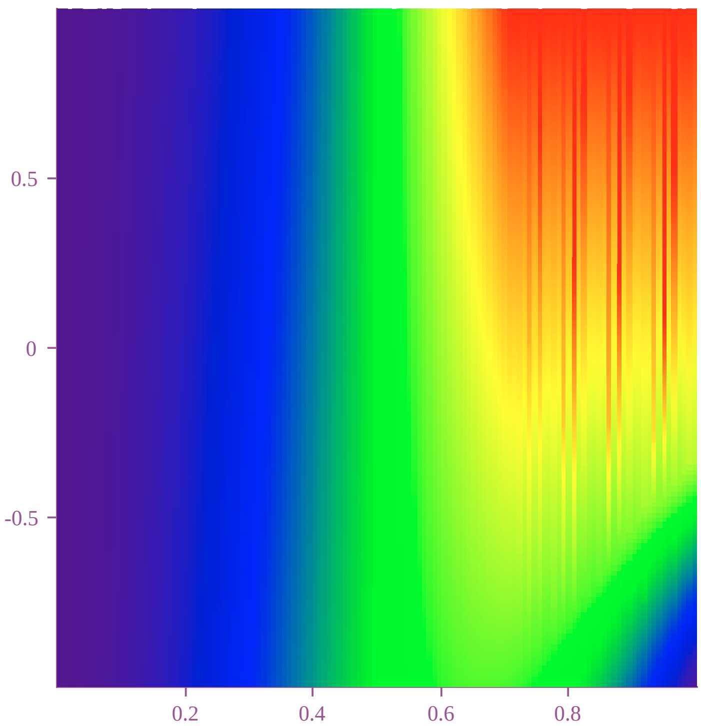
$$\nu_L \overset{\Leftarrow}{\overset{\cdot}{\longrightarrow}} \;\mathcal{C}\; \overset{\Leftarrow}{\longrightarrow} \not{\bar{\nu}}_L$$

$\mu^{\pm}$ decay: angular distributions of $e^{\pm}$ reversed

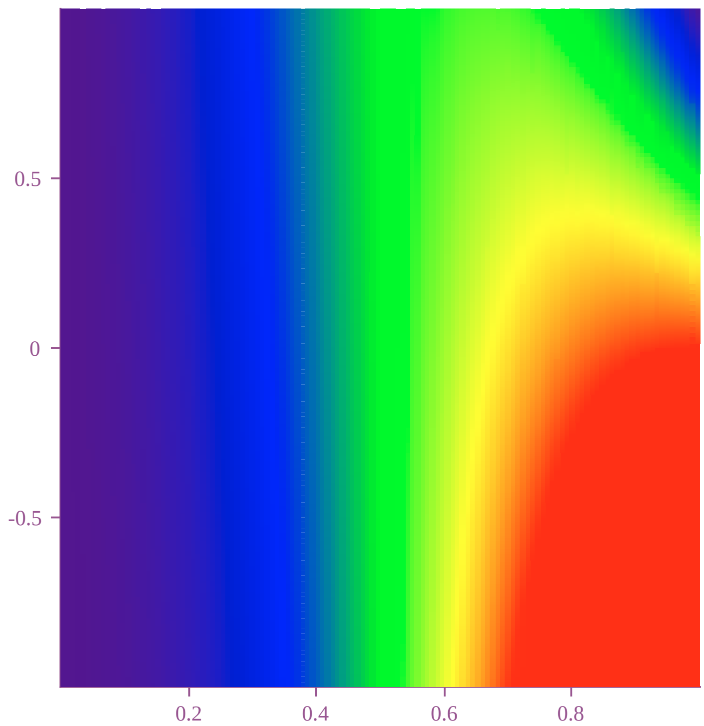
$$\frac{dN(\mu^{\pm} \to e^{\pm} + \ldots)}{dxdz} = x^2(3-2x)\left[1 \pm z\frac{(2x-1)}{(3-2x)}\right]$$

$$x \equiv p_e/p_e^{\max},\; z \equiv \hat{s}_\mu \cdot \hat{p}_e$$

e^+ follows μ^+ spin



e^- avoids μ^- spin



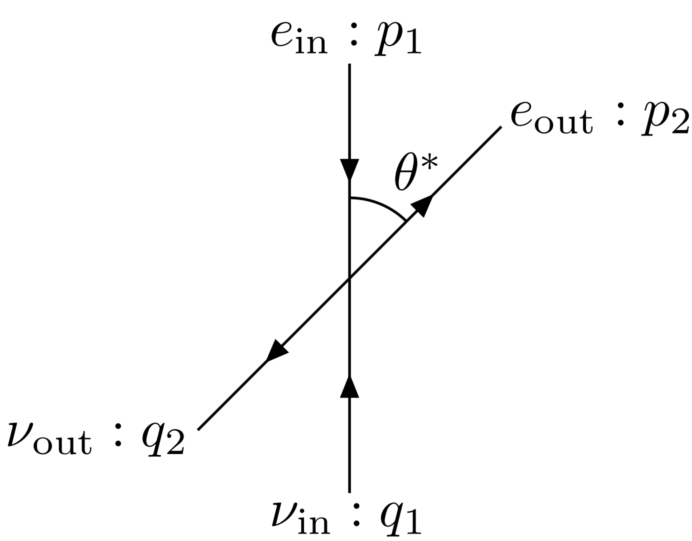
Effective Lagrangian . . .

Late 1950s: current-current interaction

$$\mathcal{L}_{V-A} = \frac{-G_F}{\sqrt{2}} \bar{\nu}\gamma_\mu(1-\gamma_5)e \; \bar{e}\gamma^\mu(1-\gamma_5)\nu + \text{h.c.}$$

$$G_F = 1.16632 \times 10^{-5} \text{ GeV}^{-2}$$

Compute $\bar{\nu}e$ scattering amplitude:



$$\begin{aligned} \mathcal{M} \quad = \quad & -\frac{iG_F}{\sqrt{2}} \bar{v}(\nu, q_1)\gamma_\mu(1-\gamma_5)u(e, p_1) \\ & \cdot \bar{u}(e, p_2)\gamma^\mu(1-\gamma_5)v(\nu, q_2) \end{aligned}$$

$\bar{\nu} e \longrightarrow \quad \bar{\nu} e$

$$\frac{d\sigma_{V-A}(\bar{\nu}e \to \bar{\nu}e)}{d\Omega_{\rm cm}} = \frac{\overline{|\mathcal{M}|^2}}{64\pi^2 s} = \frac{G_F^2 \cdot 2mE_\nu (1-z)^2}{16\pi^2}$$

$z = \cos\theta^*$

$$\sigma_{V-A}(\bar{\nu}e \to \bar{\nu}e) = \frac{G_F^2 \cdot 2mE_\nu}{3\pi}$$

$$\approx 0.574 \times 10^{-41} \text{ cm}^2 \left(\frac{E_\nu}{1 \text{ GeV}}\right)$$

Small! $\approx 10^{-14}\, \sigma(pp)$ at 100 GeV

$\nu e \longrightarrow \quad \nu e$

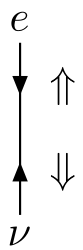
$$\frac{d\sigma_{V-A}(\nu e \to \nu e)}{d\Omega_{\rm cm}} = \frac{G_F^2 \cdot 2mE_\nu}{4\pi^2}$$

$$\sigma_{V-A}(\nu e \to \nu e) = \frac{G_F^2 \cdot 2mE_\nu}{\pi}$$

$$\approx 1.72 \times 10^{-41} \text{ cm}^2 \left(\frac{E_\nu}{1 \text{ GeV}}\right)$$

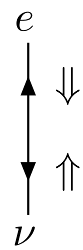
Why $3\times$ difference?

incoming



$J_z = 0$

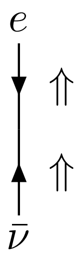
outgoing, $z = +1$



$J_z = 0$

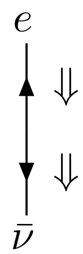
allowed at all angles

incoming



$J_z = +1$

outgoing, $z = +1$



$J_z = -1$

forbidden (angular momentum) at $z = +1$

1962: Lederman, Schwartz, Steinberger $\nu_\mu \neq \nu_e$

▷ Make HE $\pi \rightarrow \mu\nu$ beam

▷ Observe $\nu N \rightarrow \mu +$ anything

▷ Don't observe $\nu N \rightarrow e +$ anything

Danby, *et al.*, *Phys. Rev. Lett.* **9**, 36 (1962)

Suggests family structure

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L$$

$\approx$ no interactions known to cross boundaries

Generalize effective (current-current) Lagrangian:

$$\mathcal{L}_{V-A}^{(e\mu)} = \frac{-G_F}{\sqrt{2}} \bar{\nu}_\mu \gamma_\mu (1 - \gamma_5) \mu \; \bar{e} \gamma^\mu (1 - \gamma_5) \nu_e + \text{h.c.}\ ,$$

Compute muon decay rate

$$\Gamma(\mu \rightarrow e \bar{\nu}_e \nu_\mu) = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

accounts for the 2.2-μs muon lifetime

Citation: S. Eidelman *et al.* (Particle Data Group), Phys. Lett. B **592**, 1 (2004) (URL: http://pdg.lbl.gov)

TESTS OF NUMBER CONSERVATION LAWS

LEPTON FAMILY NUMBER

Lepton family number conservation means separate conservation of each of L_e, L_μ, L_τ.

$\Gamma(Z \to e^\pm \mu^\mp)/\Gamma_{\text{total}}$	[i]	$<1.7 \times 10^{-6}$, CL = 95%
$\Gamma(Z \to e^\pm \tau^\mp)/\Gamma_{\text{total}}$	[i]	$<9.8 \times 10^{-6}$, CL = 95%
$\Gamma(Z \to \mu^\pm \tau^\mp)/\Gamma_{\text{total}}$	[i]	$<1.2 \times 10^{-5}$, CL = 95%
limit on $\mu^- \to e^-$ conversion		
$\sigma(\mu^- \, {}^{32}\text{S} \to e^- \, {}^{32}\text{S})$ / $\sigma(\mu^- \, {}^{32}\text{S} \to \nu_\mu \, {}^{32}\text{P}^*)$		$<7 \times 10^{-11}$, CL = 90%
$\sigma(\mu^- \text{Ti} \to e^- \text{Ti})$ / $\sigma(\mu^- \text{Ti} \to \text{capture})$		$<4.3 \times 10^{-12}$, CL = 90%
$\sigma(\mu^- \text{Pb} \to e^- \text{Pb})$ / $\sigma(\mu^- \text{Pb} \to \text{capture})$		$<4.6 \times 10^{-11}$, CL = 90%
limit on muonium $\to$ antimuonium conversion $R_g = G_C \, / \, G_F$		<0.0030, CL = 90%
$\Gamma(\mu^- \to e^- \nu_e \overline{\nu}_\mu)/\Gamma_{\text{total}}$	[j]	$<1.2 \times 10^{-2}$, CL = 90%
$\Gamma(\mu^- \to e^- \gamma)/\Gamma_{\text{total}}$		$<1.2 \times 10^{-11}$, CL = 90%
$\Gamma(\mu^- \to e^- e^+ e^-)/\Gamma_{\text{total}}$		$<1.0 \times 10^{-12}$, CL = 90%
$\Gamma(\mu^- \to e^- 2\gamma)/\Gamma_{\text{total}}$		$<7.2 \times 10^{-11}$, CL = 90%
$\Gamma(\tau^- \to e^- \gamma)/\Gamma_{\text{total}}$		$<2.7 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \mu^- \gamma)/\Gamma_{\text{total}}$		$<1.1 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to e^- \pi^0)/\Gamma_{\text{total}}$		$<3.7 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \mu^- \pi^0)/\Gamma_{\text{total}}$		$<4.0 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to e^- K^0_S)/\Gamma_{\text{total}}$		$<9.1 \times 10^{-7}$, CL = 90%
$\Gamma(\tau^- \to \mu^- K^0_S)/\Gamma_{\text{total}}$		$<9.5 \times 10^{-7}$, CL = 90%
$\Gamma(\tau^- \to e^- \eta)/\Gamma_{\text{total}}$		$<8.2 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \mu^- \eta)/\Gamma_{\text{total}}$		$<9.6 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to e^- \rho^0)/\Gamma_{\text{total}}$		$<2.0 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \mu^- \rho^0)/\Gamma_{\text{total}}$		$<6.3 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to e^- K^*(892)^0)/\Gamma_{\text{total}}$		$<5.1 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \mu^- K^*(892)^0)/\Gamma_{\text{total}}$		$<7.5 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to e^- \overline{K}^*(892)^0)/\Gamma_{\text{total}}$		$<7.4 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \mu^- \overline{K}^*(892)^0)/\Gamma_{\text{total}}$		$<7.5 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to e^- \phi)/\Gamma_{\text{total}}$		$<6.9 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \mu^- \phi)/\Gamma_{\text{total}}$		$<7.0 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to e^- e^+ e^-)/\Gamma_{\text{total}}$		$<2.9 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to e^- \mu^+ \mu^-)/\Gamma_{\text{total}}$		$<1.8 \times 10^{-6}$, CL = 90%

TOTAL LEPTON NUMBER

Violation of total lepton number conservation also implies violation of lepton family number conservation.

$\Gamma(Z \to p e)/\Gamma_{\text{total}}$		$<1.8 \times 10^{-6}$, CL = 95%
$\Gamma(Z \to p \mu)/\Gamma_{\text{total}}$		$<1.8 \times 10^{-6}$, CL = 95%
limit on $\mu^- \to e^+$ conversion		
$\sigma(\mu^- \, ^{32}\text{S} \to e^+ \, ^{32}\text{Si}^*)$ / $\sigma(\mu^- \, ^{32}\text{S} \to \nu_\mu \, ^{32}\text{P}^*)$		$<9 \times 10^{-10}$, CL = 90%
$\sigma(\mu^- \, ^{127}\text{I} \to e^+ \, ^{127}\text{Sb}^*)$ / $\sigma(\mu^- \, ^{127}\text{I} \to \text{anything})$		$<3 \times 10^{-10}$, CL = 90%
$\sigma(\mu^- \text{Ti} \to e^+ \text{Ca})$ / $\sigma(\mu^- \text{Ti} \to \text{capture})$		$<3.6 \times 10^{-11}$, CL = 90%
$\Gamma(\tau^- \to e^+ \pi^- \pi^-)/\Gamma_{\text{total}}$		$<1.9 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \mu^+ \pi^- \pi^-)/\Gamma_{\text{total}}$		$<3.4 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to e^+ \pi^- K^-)/\Gamma_{\text{total}}$		$<2.1 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to e^+ K^- K^-)/\Gamma_{\text{total}}$		$<3.8 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \mu^+ \pi^- K^-)/\Gamma_{\text{total}}$		$<7.0 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \mu^+ K^- K^-)/\Gamma_{\text{total}}$		$<6.0 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \overline{p} \gamma)/\Gamma_{\text{total}}$		$<3.5 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \overline{p} \pi^0)/\Gamma_{\text{total}}$		$<1.5 \times 10^{-5}$, CL = 90%
$\Gamma(\tau^- \to \overline{p} 2\pi^0)/\Gamma_{\text{total}}$		$<3.3 \times 10^{-5}$, CL = 90%
$\Gamma(\tau^- \to \overline{p} \eta)/\Gamma_{\text{total}}$		$<8.9 \times 10^{-6}$, CL = 90%
$\Gamma(\tau^- \to \overline{p} \pi^0 \eta)/\Gamma_{\text{total}}$		$<2.7 \times 10^{-5}$, CL = 90%
$t_{1/2}(\,^{76}\text{Ge} \to \,^{76}\text{Se} + 2\,e^-)$		$>1.9 \times 10^{25}$ yr, CL = 90%
$\Gamma(\pi^+ \to \mu^+ \overline{\nu}_e)/\Gamma_{\text{total}}$	[k]	$<1.5 \times 10^{-3}$, CL = 90%
$\Gamma(K^+ \to \pi^- \mu^+ e^+)/\Gamma_{\text{total}}$		$<5.0 \times 10^{-10}$, CL = 90%
$\Gamma(K^+ \to \pi^- e^+ e^+)/\Gamma_{\text{total}}$		$<6.4 \times 10^{-10}$, CL = 90%
$\Gamma(K^+ \to \pi^- \mu^+ \mu^+)/\Gamma_{\text{total}}$	[k]	$<3.0 \times 10^{-9}$, CL = 90%
$\Gamma(K^+ \to \mu^+ \overline{\nu}_e)/\Gamma_{\text{total}}$	[k]	$<3.3 \times 10^{-3}$, CL = 90%
$\Gamma(K^+ \to \pi^0 e^+ \overline{\nu}_e)/\Gamma_{\text{total}}$		$<3 \times 10^{-3}$, CL = 90%
$\Gamma(D^+ \to \pi^- e^+ e^+)/\Gamma_{\text{total}}$		$<9.6 \times 10^{-5}$, CL = 90%
$\Gamma(D^+ \to \pi^- \mu^+ \mu^+)/\Gamma_{\text{total}}$		$<4.8 \times 10^{-6}$, CL = 90%
$\Gamma(D^+ \to \pi^- e^+ \mu^+)/\Gamma_{\text{total}}$		$<5.0 \times 10^{-5}$, CL = 90%
$\Gamma(D^+ \to \rho^- \mu^+ \mu^+)/\Gamma_{\text{total}}$		$<5.6 \times 10^{-4}$, CL = 90%
$\Gamma(D^+ \to K^- e^+ e^+)/\Gamma_{\text{total}}$		$<1.2 \times 10^{-4}$, CL = 90%
$\Gamma(D^+ \to K^- \mu^+ \mu^+)/\Gamma_{\text{total}}$		$<1.3 \times 10^{-5}$, CL = 90%
$\Gamma(D^+ \to K^- e^+ \mu^+)/\Gamma_{\text{total}}$		$<1.3 \times 10^{-4}$, CL = 90%
$\Gamma(D^+ \to K^*(892)^- \mu^+ \mu^+)/\Gamma_{\text{total}}$		$<8.5 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to \pi^- \pi^- e^+ e^+ + \text{c.c.})/\Gamma_{\text{total}}$		$<1.12 \times 10^{-4}$, CL = 90%
$\Gamma(D^0 \to \pi^- \pi^- \mu^+ \mu^+ + \text{c.c.})/\Gamma_{\text{total}}$		$<2.9 \times 10^{-5}$, CL = 90%
$\Gamma(D^0 \to K^- \pi^- e^+ e^+ + \text{c.c.})/\Gamma_{\text{total}}$		$<2.06 \times 10^{-4}$, CL = 90%

Cross section for inverse muon decay

$$\sigma(\nu_\mu e \to \mu\nu_e) = \sigma_{V-A}(\nu_e e \to \nu_e e)\left[1 - \frac{(m_\mu^2 - m_e^2)}{2m_e E_\nu}\right]^2$$

agrees with CHARM II, CCFR data ($E_\nu \lesssim 600$ GeV)

PW unitarity: $|\mathcal{M}_J| < 1$

$V - A$ theory:

$$\mathcal{M}_0 = \frac{G_F \cdot 2m_e E_\nu}{\pi\sqrt{2}}\left[1 - \frac{(m_\mu^2 - m_e^2)}{2m_e E_\nu}\right]$$

satisfies pw unitarity for

$$E_\nu < \pi/G_F m_e\sqrt{2} \approx 3.7 \times 10^8 \text{ GeV}$$

$\Rightarrow V - A$ theory cannot be complete

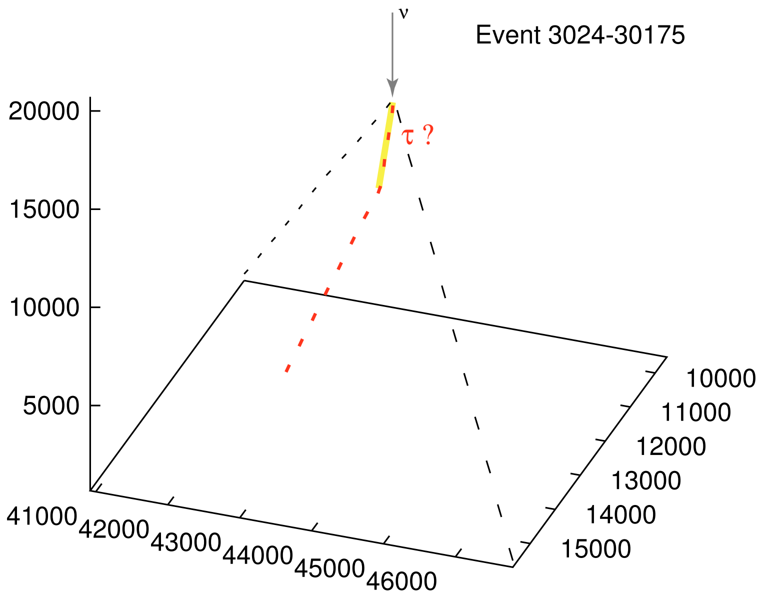
physics must change before $\sqrt{s} \approx 600$ GeV

2000: DONuT Three-Neutrino Experiment

▷ Prompt (beam-dump) ν_τ beam produced in

$$D_s^+ \rightarrow \tau^+\nu_\tau$$
$$\hookrightarrow X^+\bar{\nu}_\tau$$

▷ Try to observe $\nu_\tau N \rightarrow \tau +$ anything in emulsion; τ lifetime is 0.3 ps



Candidate event in ECC1. The three tracks with full emulsion data are shown. The red track shows a 100 mrad kink 4.5mm from the interaction vertex. The scale units are microns.

Kodama, *et al.*, *Phys. Lett.* **B504**, 218 (2001)

Leptons are seen as free particles

Table 1: Some properties of the leptons.

Lepton	Mass	Lifetime
e^-	$0.510\,998\,92(4)$ MeV/c^2	$> 4.6 \times 10^{26}$ y $(90\%\ \mathrm{CL})$
ν_e	< 3 eV/c^2	$\tau/m > 7 \times 10^9$ s/eV
μ^-	$105.658\,369(9)$ MeV/c^2	$2.197\,03(4) \times 10^{-6}$ s
ν_μ	< 0.19 MeV/c^2 $(90\%\ \mathrm{CL})$	$\tau/m > 15.4$ s/eV
τ^-	$1776.99^{+0.29}_{-0.26}$ MeV/c^2	$290.6 \pm 1.1 \times 10^{-15}$ s
ν_τ	< 18.2 MeV/c^2 $(95\%\ \mathrm{CL})$	

All spin-$\frac{1}{2}$, pointlike ($\lesssim$ few $\times\ 10^{-17}$ cm)

kinematically determined ν masses consistent with 0

(ν oscillations $\Rightarrow$ nonzero, unequal masses)

Universal weak couplings

Rough and ready test

Fermi constant from muon decay

$$G_\mu = \left[\frac{192\pi^3\hbar}{\tau_\mu m_\mu^5}\right]^{\frac{1}{2}} = 1.1638 \times 10^{-5}\ \text{GeV}^{-2}$$

Meticulous analysis yields $G_\mu = 1.16637(1) \times 10^{-5}$ GeV^{-2}

Fermi constant from tau decay

$$G_\tau = \left[\frac{\Gamma(\tau \to e\bar{\nu}_e\nu_\tau)}{\Gamma(\tau \to \text{all})}\frac{192\pi^3\hbar}{\tau_\tau m_\tau^5}\right]^{\frac{1}{2}} = 1.1642{\times}10^{-5}\ \text{GeV}^{-2}$$

Excellent agreement with $G_\beta = 1.16639(2) \times 10^{-5}$ GeV^{-2}

Charged currents acting in leptonic and semileptonic interactions are of universal strength; $\Rightarrow$ *universality of current-current form, or whatever lies behind it*

Nonleptonic enhancement

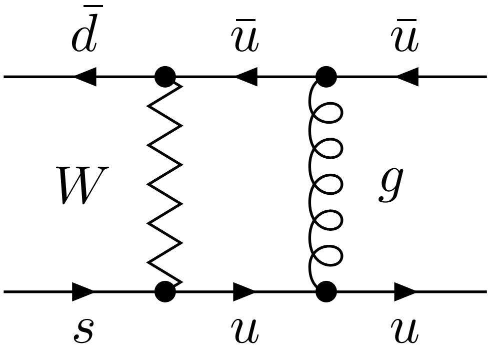
Certain NL transitions are more rapid than universality suggests

$$\underbrace{\Gamma(K_S \to \pi^+\pi^-)}_{I=0,2} \approx 450 \times \underbrace{\Gamma(K^+ \to \pi^+\pi^0)}_{I=2}$$

$$\boxed{A_0 \approx 22 \times A_2}$$

$|\Delta I| = \frac{1}{2}$ rule; "octet dominance" (over **27**)

Origin of this phenomenological rule is only partly understood. Short-distance *(perturbative)* QCD corrections arise from



... explain $\approx \sqrt{\text{enhancement}}$

SYMMETRIES $\Longrightarrow$ INTERACTIONS

Phase Invariance (Symmetry) in Quantum Mechanics

QM STATE: COMPLEX SCHRÖDINGER WAVE FUNCTION $\psi(x)$

OBSERVABLES

$$\langle O \rangle = \int d^n x \psi^* O \psi$$

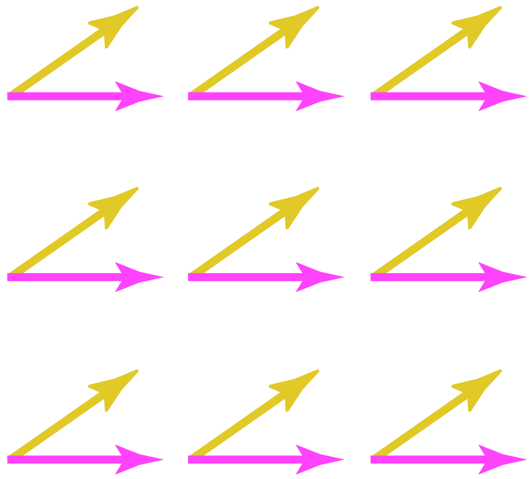
ARE UNCHANGED

UNDER A GLOBAL PHASE ROTATION

$$\psi(x) \to e^{i\theta}\psi(x)$$
$$\psi^*(x) \to e^{-i\theta}\psi^*(x)$$

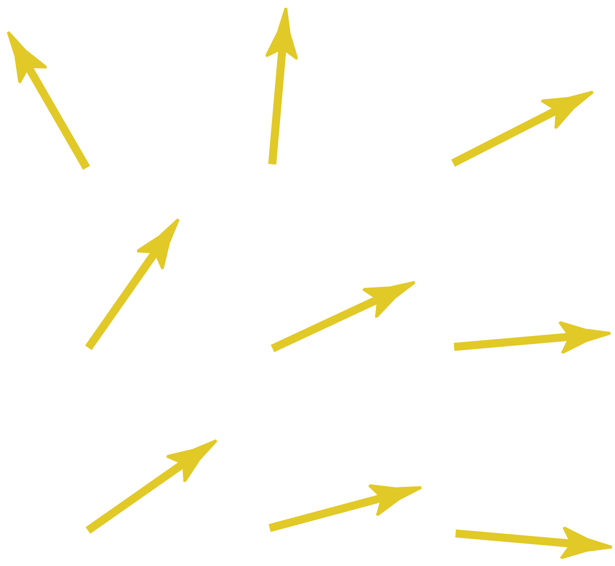
- Absolute phase of the wave function cannot be measured (is a matter of convention).
- Relative phases (interference experiments) are unaffected by a global phase rotation.

GLOBAL ROTATION — SAME EVERYWHERE



MIGHT WE CHOOSE ONE PHASE CONVENTION
IN MIRAMARE AND ANOTHER IN BATAVIA?

A DIFFERENT CONVENTION AT EACH POINT?



$$\boxed{\psi(x) \to e^{iq\alpha(x)}\psi(x)}$$

THERE IS A PRICE.

Some variables (e.g., momentum) and the Schrödinger equation itself contain derivatives. Under the transformation

$$\psi(x) \to e^{iq\alpha(x)}\psi(x)$$

the gradient of the wave function transforms as

$$\nabla\psi(x) \to e^{iq\alpha(x)}[\nabla\psi(x) + iq(\nabla\alpha(x))\psi(x)]$$

The $\nabla\alpha(x)$ term spoils local phase invariance.

TO RESTORE LOCAL PHASE INVARIANCE ...

Modify the equations of motion and observables.

Replace ∇ by $\nabla + iq\vec{A}$

"Gauge-covariant derivative"

If the vector potential $\vec{A}$ transforms under local phase rotations as

$$\vec{A}(x) \to \vec{A}'(x) \equiv \vec{A}(x) - \nabla\alpha(x),$$

then $(\nabla + iq\vec{A})\psi \to e^{iq\alpha(x)}(\nabla + iq\vec{A})\psi$ and $\psi^*(\nabla + iq\vec{A})\psi$ is invariant under local rotations.

NOTE . . .

- $\vec{A}(x) \to \vec{A}'(x) \equiv \vec{A}(x) - \nabla\alpha(x)$ has the form of a gauge transformation in electrodynamics.
- The replacement $\nabla \to (\nabla + iq\vec{A})$ corresponds to $\vec{p} \to \vec{p} - q\vec{A}$

FORM OF INTERACTION IS DEDUCED
FROM LOCAL PHASE INVARIANCE

$\Longrightarrow$ MAXWELL'S EQUATIONS

DERIVED

FROM A SYMMETRY PRINCIPLE

QED is the gauge theory based on
$U(1)$ phase symmetry

GENERAL PROCEDURE

- Recognize a symmetry of Nature.
- Build it into the laws of physics.
  (Connection with conservation laws)
- Impose symmetry in stricter (local) form.

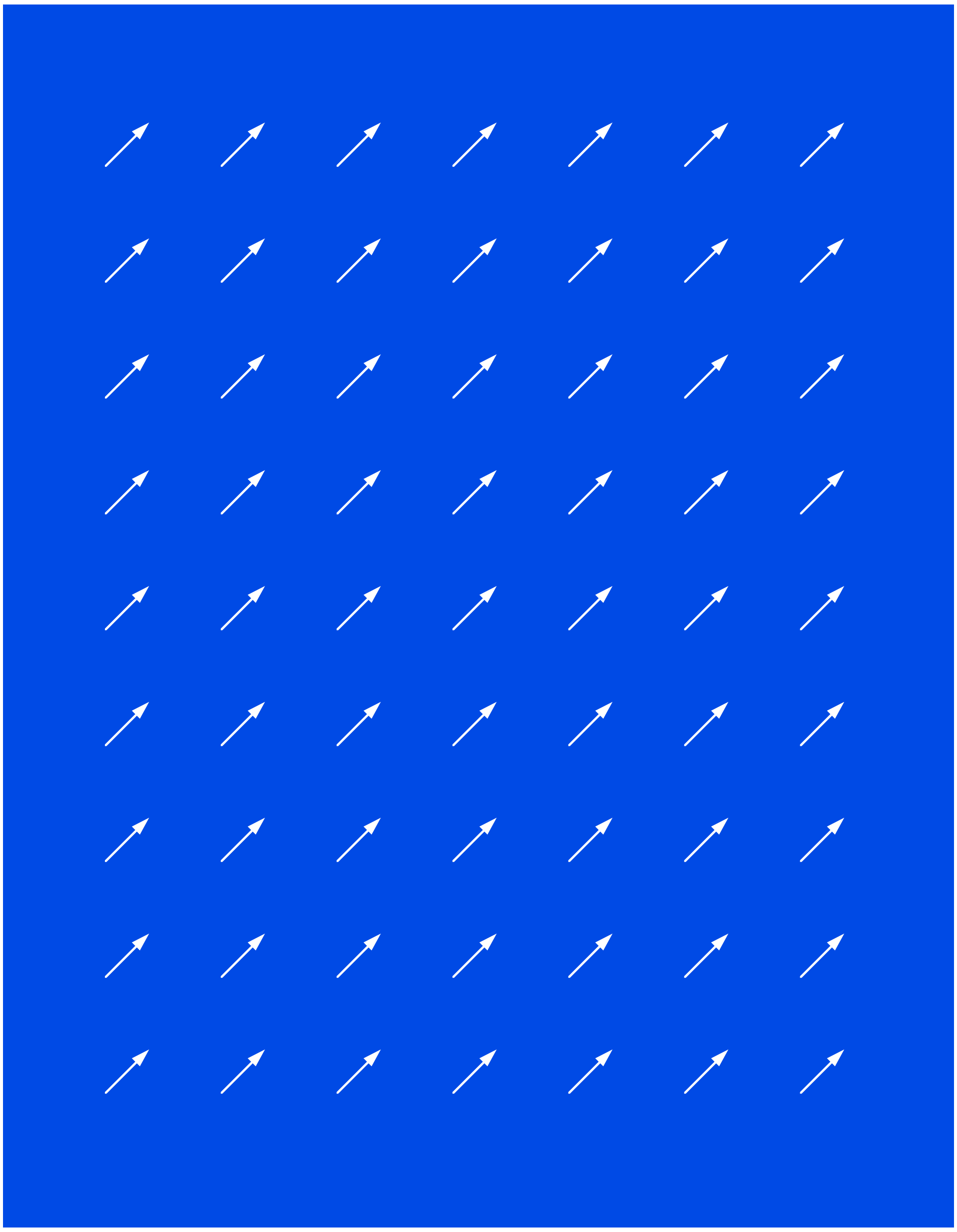
$\Longrightarrow$ INTERACTIONS

- Massless vector fields (gauge fields)
- Minimal coupling to the conserved current
- Interactions among the gauge fields, if symmetry is non-Abelian

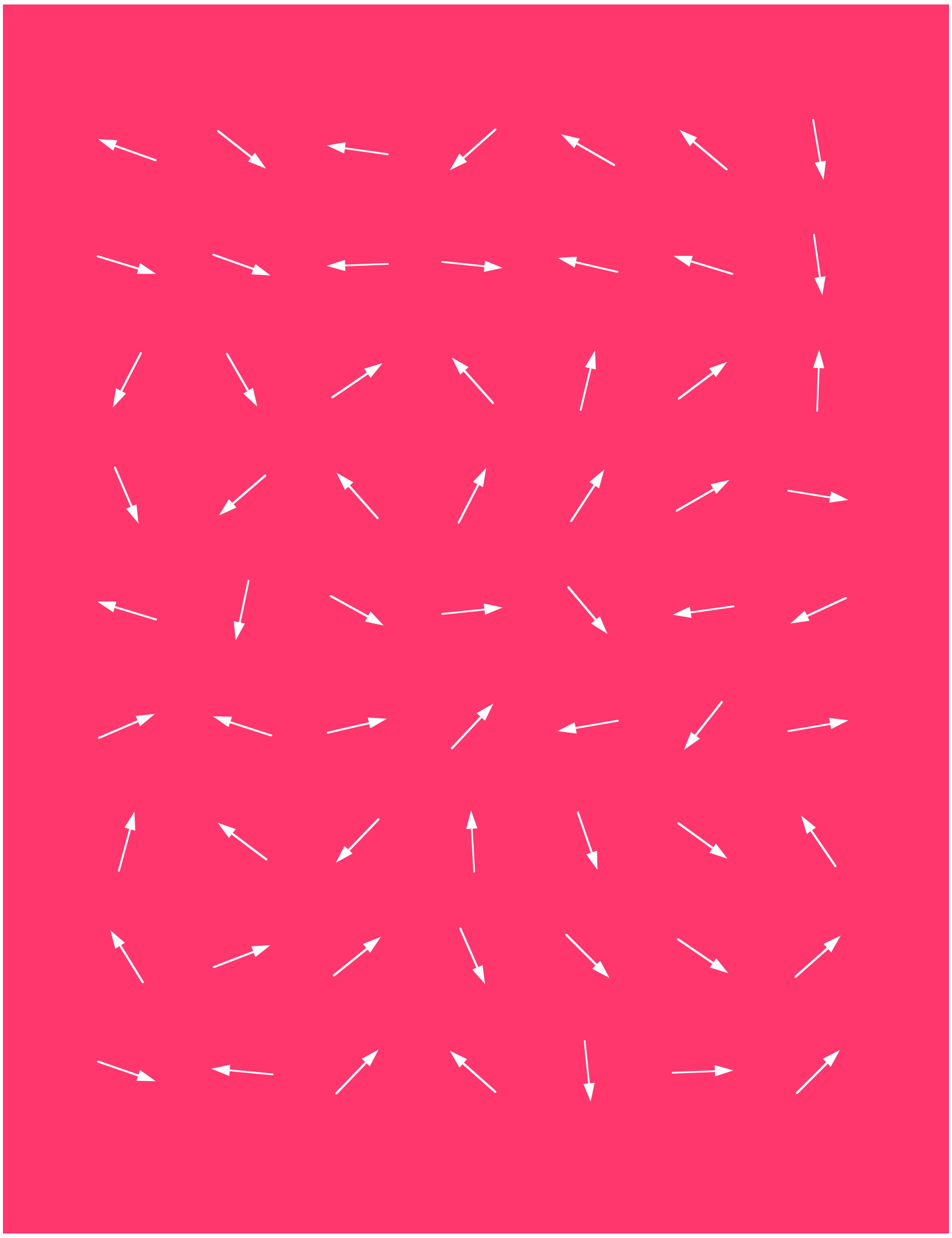
Posed as a problem in mathematics, construction of a gauge theory is always possible (at the level of a classical $\mathcal{L}$; consistent quantum theory may require additional vigilance).

Formalism is no guarantee that the gauge symmetry was chosen wisely.

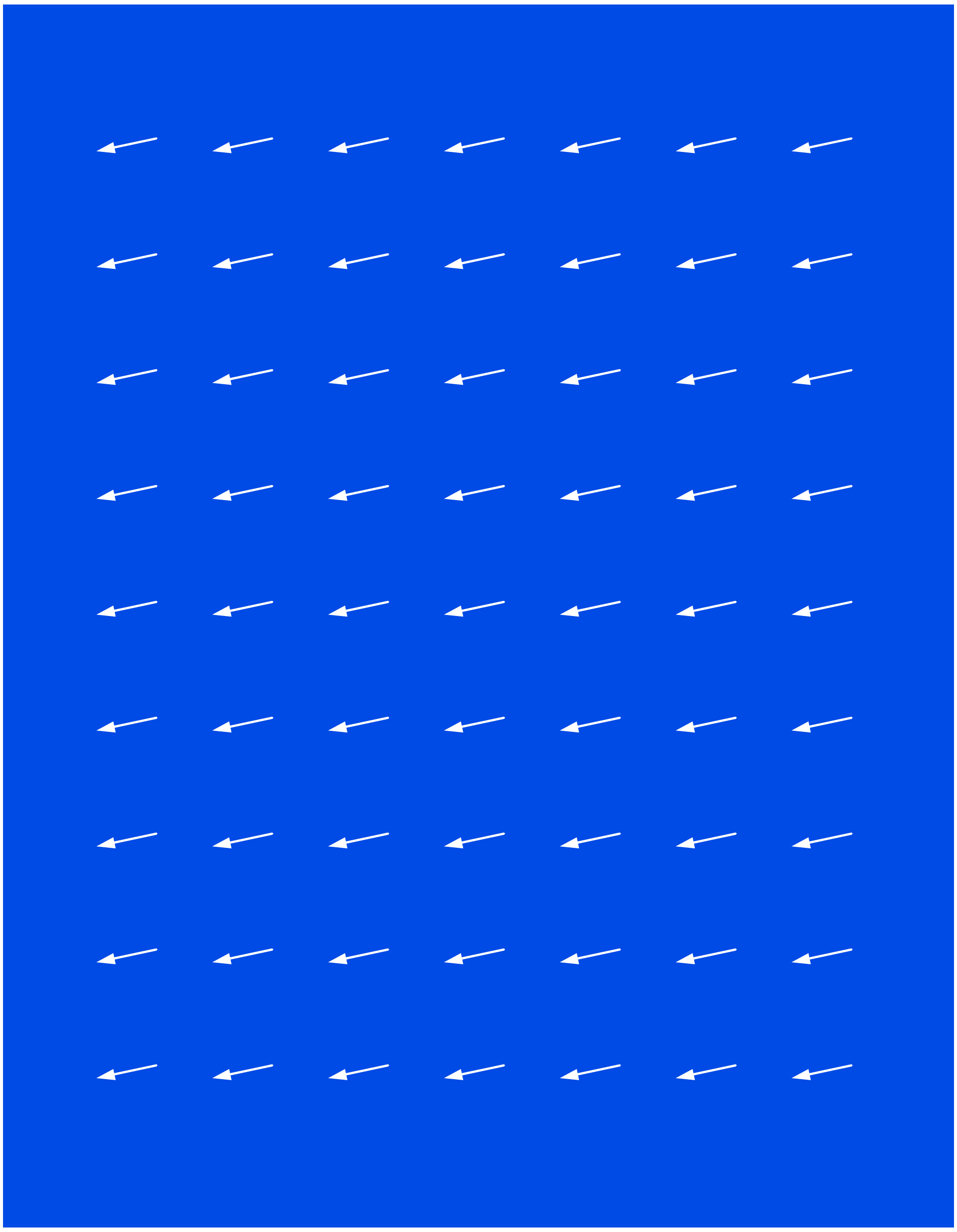
The Crystal World



The Crystal World



The Crystal World



The Perfect World

Massive Photon? *Hiding Symmetry*

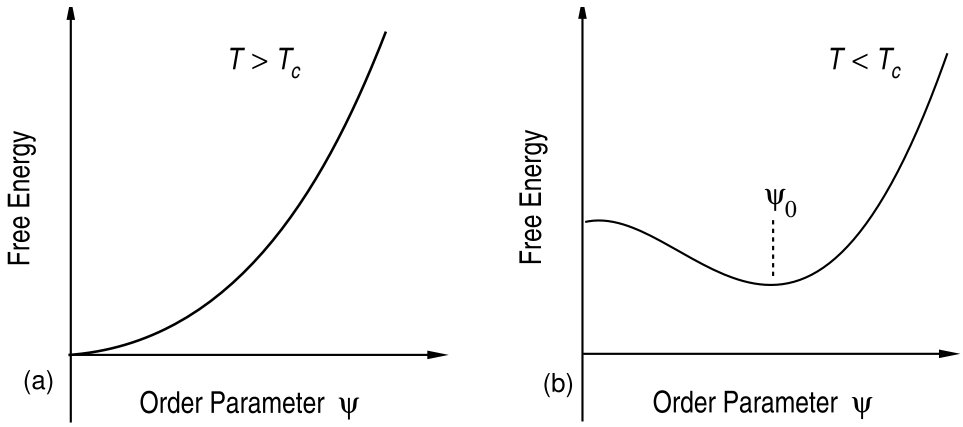
Recall 2 miracles of superconductivity:

- ▷ No resistance
- ▷ Meissner effect (exclusion of $\mathbf{B}$)

Ginzburg–Landau Phenomenology
(not a theory from first principles)

normal, resistive charge carriers . . .

. . . + superconducting charge carriers



$\mathbf{B} = 0$:

$$G_{\text{super}}(0) = G_{\text{normal}}(0) + \alpha\,|\psi|^2 + \beta\,|\psi|^4$$

$$T > T_c: \quad \alpha > 0 \quad \langle|\psi|^2\rangle_0 = 0$$

$$T < T_c: \quad \alpha < 0 \quad \langle|\psi|^2\rangle_0 \neq 0$$

NONZERO MAGNETIC FIELD

$$G_{\text{super}}(\mathbf{B}) = G_{\text{super}}(0) + \frac{\mathbf{B}^2}{8\pi} + \frac{1}{2m^*}\left|-i\hbar\nabla\psi - \frac{e^*}{c}\mathbf{A}\psi\right|^2$$

$$\left.\begin{array}{c} e^* = -2 \\ m^* \end{array}\right\} \text{of superconducting carriers}$$

Weak, slowly varying field

$$\psi \approx \psi_0 \neq 0,\ \nabla\psi \approx 0$$

Variational analysis $\Longrightarrow$

$$\boxed{\nabla^2\mathbf{A} - \frac{4\pi e^*}{m^* c^2}\left|\psi_0\right|^2\mathbf{A} = 0}$$

wave equation of a *massive photon*

Photon— *gauge boson* — acquires mass
within superconductor

origin of Meissner effect

Formulate electroweak theory

three crucial clues from experiment:

▷ Left-handed weak-isospin doublets,

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \qquad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \qquad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

and

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \qquad \begin{pmatrix} c \\ s' \end{pmatrix}_L \qquad \begin{pmatrix} t \\ b' \end{pmatrix}_L ;$$

▷ Universal strength of the (charged-current) weak interactions;

▷ Idealization that neutrinos are massless.

First two clues suggest $SU(2)_L$ gauge symmetry

A theory of leptons

$$\mathsf{L} = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \qquad \mathsf{R} \equiv e_R$$

weak hypercharges $Y_L = -1$, $Y_R = -2$
Gell-Mann–Nishijima connection, $Q = I_3 + \frac{1}{2}Y$

$SU(2)_L \otimes U(1)_Y$ gauge group $\Rightarrow$ gauge fields:

⋆ weak isovector $\vec{b}_\mu$, coupling g

⋆ weak isoscalar $\mathcal{A}_\mu$, coupling $g'/2$

Field-strength tensors

$$F^\ell_{\mu\nu} = \partial_\nu b^\ell_\mu - \partial_\mu b^\ell_\nu + g\varepsilon_{jk\ell} b^j_\mu b^k_\nu \; , SU(2)_L$$

and

$$f_{\mu\nu} = \partial_\nu \mathcal{A}_\mu - \partial_\mu \mathcal{A}_\nu \; , U(1)_Y$$

Interaction Lagrangian

$$\mathcal{L} = \mathcal{L}_{\rm gauge} + \mathcal{L}_{\rm leptons} \ ,$$

with

$$\mathcal{L}_{\rm gauge} = -\tfrac{1}{4}F^{\ell}_{\mu\nu}F^{\ell\mu\nu} - \tfrac{1}{4}f_{\mu\nu}f^{\mu\nu},$$

and

$$\begin{aligned}\mathcal{L}_{\rm leptons} &= \overline{\mathsf{R}}\, i\gamma^{\mu}\left(\partial_{\mu} + i\frac{g'}{2}\mathcal{A}_{\mu}Y\right)\mathsf{R} \\ &+ \overline{\mathsf{L}}\, i\gamma^{\mu}\left(\partial_{\mu} + i\frac{g'}{2}\mathcal{A}_{\mu}Y + i\frac{g}{2}\vec{\tau}\cdot\vec{b}_{\mu}\right)\mathsf{L}.\end{aligned}$$

Electron mass term

$$\mathcal{L}_{e} = -m_{e}(\bar{e}_{\mathsf{R}}e_{\mathsf{L}} + \bar{e}_{\mathsf{L}}e_{\mathsf{R}}) = -m_{e}\bar{e}e$$

would violate local gauge invariance Theory has four massless gauge bosons

$$\mathcal{A}_{\mu} \quad b^{1}_{\mu} \quad b^{2}_{\mu} \quad b^{3}_{\mu}$$

Nature has but one (γ)

Hiding EW Symmetry

Higgs mechanism: relativistic generalization of Ginzburg-Landau superconducting phase transition

▷ Introduce a complex doublet of scalar fields

$$\phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad Y_\phi = +1$$

▷ Add to $\mathcal{L}$ (gauge-invariant) terms for interaction and propagation of the scalars,

$$\mathcal{L}_{\text{scalar}} = (\mathcal{D}^\mu \phi)^\dagger (\mathcal{D}_\mu \phi) - V(\phi^\dagger \phi),$$

where $\mathcal{D}_\mu = \partial_\mu + i\frac{g'}{2}\mathcal{A}_\mu Y + i\frac{g}{2}\vec{\tau} \cdot \vec{b}_\mu$ and

$$\boxed{V(\phi^\dagger \phi) = \mu^2(\phi^\dagger \phi) + |\lambda| (\phi^\dagger \phi)^2}$$

▷ Add a Yukawa interaction

$$\mathcal{L}_{\text{Yukawa}} = -\zeta_e \left[\overline{\mathsf{R}}(\phi^\dagger \mathsf{L}) + (\overline{\mathsf{L}}\phi)\mathsf{R} \right]$$

▷ Arrange self-interactions so vacuum corresponds to a broken-symmetry solution: $\mu^2 < 0$

Choose minimum energy (vacuum) state for vacuum expectation value

$$\langle\phi\rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \quad v = \sqrt{-\mu^2/|\lambda|}$$

Hides (breaks) $SU(2)_L$ and $U(1)_Y$

but preserves $U(1)_{\mathrm{em}}$ invariance

Invariance under $\mathcal{G}$ means $e^{i\alpha\mathcal{G}}\langle\phi\rangle_0 = \langle\phi\rangle_0$, so $\mathcal{G}\langle\phi\rangle_0 = 0$

$$\tau_1\langle\phi\rangle_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \quad \text{broken!}$$

$$\tau_2\langle\phi\rangle_0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} -iv/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \quad \text{broken!}$$

$$\tau_3\langle\phi\rangle_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ -v/\sqrt{2} \end{pmatrix} \neq 0 \quad \text{broken!}$$

$$Y\langle\phi\rangle_0 = Y_\phi\langle\phi\rangle_0 = +1\langle\phi\rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \neq 0 \quad \text{broken!}$$

Examine electric charge operator Q on the (electrically neutral) vacuum state

$$\begin{aligned} Q\langle\phi\rangle_0 &= \tfrac{1}{2}(\tau_3 + Y)\langle\phi\rangle_0 \\ &= \tfrac{1}{2}\begin{pmatrix} Y_\phi + 1 & 0 \\ 0 & Y_\phi - 1 \end{pmatrix}\langle\phi\rangle_0 \\ &= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \\ &= \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \textit{unbroken!} \end{aligned}$$

Four original generators are broken

electric charge is not

▷ $SU(2)_L \otimes U(1)_Y \to U(1)_{\mathrm{em}}$ (will verify)

▷ Expect massless photon

▷ Expect gauge bosons corresponding to

$$\tau_1,\ \tau_2,\ \tfrac{1}{2}(\tau_3 - Y) \equiv K$$

to acquire masses

Expand about the vacuum state

Let $\phi = \begin{pmatrix} 0 \\ (v+\eta)/\sqrt{2} \end{pmatrix}$; in *unitary gauge*

$$\begin{aligned} \mathcal{L}_{\text{scalar}} &= \tfrac{1}{2}(\partial^\mu \eta)(\partial_\mu \eta) - \mu^2 \eta^2 \\ &\quad + \frac{v^2}{8}[g^2 \left|b_1 - ib_2\right|^2 + (g' \mathcal{A}_\mu - g b_\mu^3)^2] \\ &\quad + \text{interaction terms} \end{aligned}$$

Higgs boson η has acquired (mass)^2 $M_H^2 = -2\mu^2 > 0$

$$\frac{g^2 v^2}{8}(\left|W_\mu^+\right|^2 + \left|W_\mu^-\right|^2) \Longleftrightarrow M_{W^\pm} = gv/2$$

Now define othogonal combinations

$$Z_\mu = \frac{-g' \mathcal{A}_\mu + g b_\mu^3}{\sqrt{g^2 + g'^2}} \qquad A_\mu = \frac{g \mathcal{A}_\mu + g' b_\mu^3}{\sqrt{g^2 + g'^2}}$$

$$M_{Z^0} = \sqrt{g^2 + g'^2}\, v/2 = M_W \sqrt{1 + g'^2/g^2}$$

A_μ remains massless

$$
\begin{aligned}
\mathcal{L}_{\text{Yukawa}} &= -\zeta_e \frac{(v+\eta)}{\sqrt{2}}(\bar{e}_{\mathsf{R}} e_{\mathsf{L}} + \bar{e}_{\mathsf{L}} e_{\mathsf{R}}) \\
&= -\frac{\zeta_e v}{\sqrt{2}} \bar{e}e - \frac{\zeta_e \eta}{\sqrt{2}} \bar{e}e
\end{aligned}
$$

electron acquires $m_e = \zeta_e v/\sqrt{2}$

Higgs coupling to electrons: m_e/v ($\propto$ mass)

Desired particle content $\ldots +$ Higgs scalar

Values of couplings, electroweak scale v?

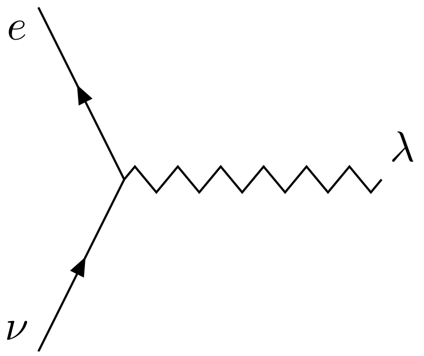
What about interactions?

Interactions . . .

$$\mathcal{L}_{W\text{-}\ell} = -\frac{g}{2\sqrt{2}}[\bar{\nu}\gamma^{\mu}(1-\gamma_5)eW_{\mu}^{+} + \bar{e}\gamma^{\mu}(1-\gamma_5)\nu W_{\mu}^{-}]$$

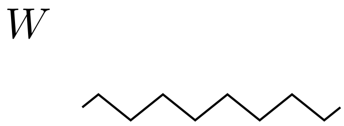
$+$ similar terms for μ and τ

Feynman rule:



$$\frac{-ig}{2\sqrt{2}}\gamma_{\lambda}(1-\gamma_5)$$

gauge-boson propagator:



$$= \frac{-i(g_{\mu\nu} - k_{\mu}k_{\nu}/M_W^2)}{k^2 - M_W^2} .$$

Compute $\nu_\mu e \rightarrow \mu\nu_e$

$$\sigma(\nu_\mu e \to \mu\nu_e) = \frac{g^4 m_e E_\nu}{16\pi M_W^4} \frac{[1-(m_\mu^2 - m_e^2)/2m_e E_\nu]^2}{(1+2m_e E_\nu/M_W^2)}$$

Reproduces 4-fermion result at low energies if

$$\frac{g^4}{16M_W^4} = 2G_F^2$$

$$\Rightarrow g^4 = 32(G_F M_W^2)^2 = 64\left(\frac{G_F M_W^2}{\sqrt{2}}\right)^2$$

$$\Rightarrow \frac{g}{2\sqrt{2}} = \left(\frac{G_F M_W^2}{\sqrt{2}}\right)^{\frac{1}{2}}$$

Using $M_W = gv/2$, determine

$$v = (G_F\sqrt{2})^{-\frac{1}{2}} \approx 246 \text{ GeV}$$

the electroweak scale

$$\Rightarrow \langle\phi^0\rangle_0 = (G_F\sqrt{8})^{-\frac{1}{2}} \approx 174 \text{ GeV}$$

W-propagator modifies HE behavior

$$\sigma(\nu_\mu e \to \mu\nu_e) = \frac{g^4 m_e E_\nu}{16\pi M_W^4} \frac{[1-(m_\mu^2 - m_e^2)/2m_e E_\nu]^2}{(1+2m_e E_\nu/M_W^2)}$$

$$\lim_{E_\nu \to \infty} \sigma(\nu_\mu e \to \mu\nu_e) = \frac{g^4}{32\pi M_W^2} = \frac{G_F^2 M_W^2}{\sqrt{2}}$$

independent of energy!

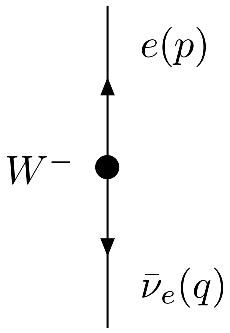
partial-wave unitarity respected for

$$s < M_W^2 [\exp{(\pi\sqrt{2}/G_F M_W^2)} - 1]$$

W-boson properties

No prediction yet for M_W (haven't determined g)

Leptonic decay $W^- \to e^-\nu_e$



$$p \approx \left(\frac{M_W}{2}; \frac{M_W \sin\theta}{2}, 0, \frac{M_W \cos\theta}{2}\right)$$

$$q \approx \left(\frac{M_W}{2}; -\frac{M_W \sin\theta}{2}, 0, -\frac{M_W \cos\theta}{2}\right)$$

$$\mathcal{M} = -i\left(\frac{G_F M_W^2}{\sqrt{2}}\right)^{\frac{1}{2}} \bar{u}(e,p)\gamma_\mu(1-\gamma_5)v(\nu,q)\,\varepsilon^\mu$$

$\varepsilon^\mu = (0; \hat{\varepsilon})$: W polarization vector in its rest frame

$$|\mathcal{M}|^2 = \frac{G_F M_W^2}{\sqrt{2}} \mathrm{tr}\left[\not{\varepsilon}(1-\gamma_5)\not{q}(1+\gamma_5)\not{\varepsilon}^*\not{p}\right]\ ;$$

$$\mathrm{tr}[\cdots] = \left[\varepsilon\cdot q\,\varepsilon^*\cdot p - \varepsilon\cdot\varepsilon^*\, q\cdot p + \varepsilon\cdot p\,\varepsilon^*\cdot q + i\epsilon_{\mu\nu\rho\sigma}\varepsilon^\mu q^\nu \varepsilon^{*\rho} p^\sigma\right]$$

decay rate is independent of W polarization; look first at longitudinal pol. $\varepsilon^\mu = (0;0,0,1) = \varepsilon^{*\mu}$, eliminate $\epsilon_{\mu\nu\rho\sigma}$

$$|\mathcal{M}|^2 = \frac{4G_F M_W^4}{\sqrt{2}}\sin^2\theta$$

$$\frac{d\Gamma_0}{d\Omega} = \frac{|\mathcal{M}|^2}{64\pi^2}\,\frac{\mathcal{S}_{12}}{M_W^3}$$

$$\mathcal{S}_{12} = \sqrt{[M_W^2 - (m_e + m_\nu)^2][M_W^2 - (m_e - m_\nu)^2]} = M_W^2$$

$$\frac{d\Gamma_0}{d\Omega} = \frac{G_F M_W^3}{16\pi^2\sqrt{2}}\sin^2\theta$$

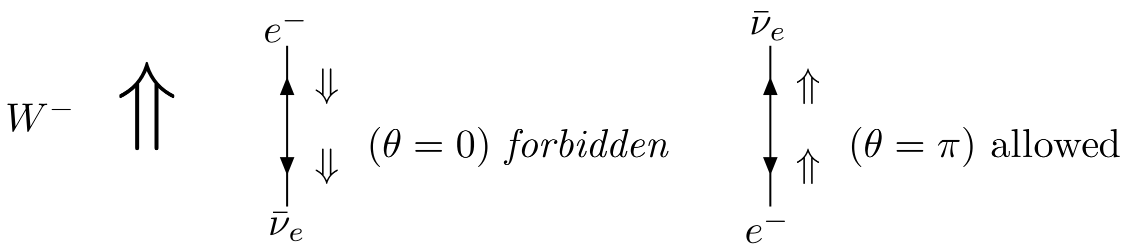
and

$$\boxed{\Gamma(W \to e\nu) = \frac{G_F M_W^3}{6\pi\sqrt{2}}}$$

Other helicities: $\varepsilon^\mu_{\pm 1} = (0; -1, \mp i, 0)/\sqrt{2}$

$$\frac{d\Gamma_{\pm 1}}{d\Omega} = \frac{G_F M_W^3}{32\pi^2\sqrt{2}}(1 \mp \cos\theta)^2$$

Extinctions at $\cos\theta = \pm 1$ are consequences of angular momentum conservation:



(situation reversed for $W^+ \to e^+\nu_e$)

e^+ follows polarization direction of W^+

e^- avoids polarization direction of W^-

important for discovery of $W^\pm$ in $\bar{p}p$ $(\bar{q}q)$ C violation

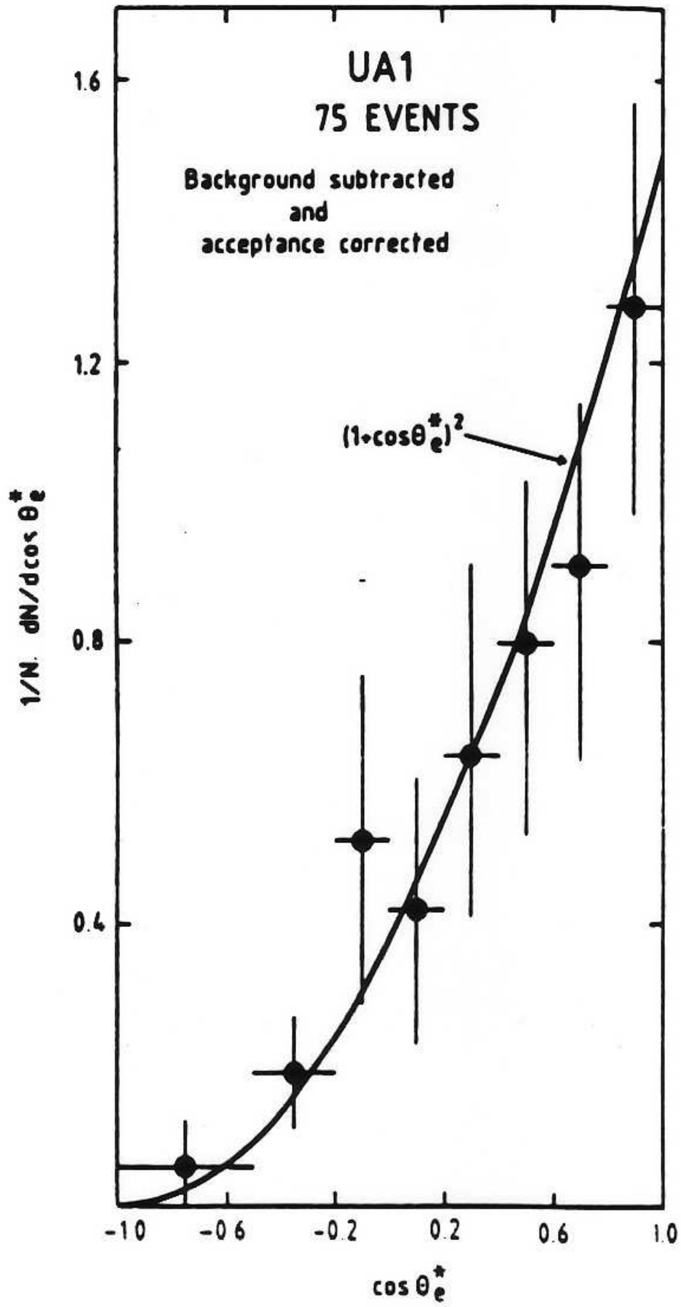


Fig. 2. The W decay angular distribution of the emission angle θ^* of the electron (positron) with respect to the proton (antiproton) direction in the rest frame of the W. Only those events for which the lepton charge and the decay kinematics are well determined have been used. The curve shows the $(V - A)$ expectation of $(1 + \cos\theta^*)^2$.

Interactions . . .

$$\mathcal{L}_{A\text{-}\ell} = \frac{gg'}{\sqrt{g^2 + g'^2}} \bar{e}\gamma^\mu e A_\mu$$

. . . vector interaction; $\Rightarrow A_\mu$ as γ, provided

$$\boxed{gg'/\sqrt{g^2 + g'^2} \equiv e}$$

Define $g' = g \tan\theta_W$ θ_W: weak mixing angle

$$\begin{aligned} g &= e/\sin\theta_W \geq e \\ g' &= e/\cos\theta_W \geq e \end{aligned}$$

$Z_\mu = b_\mu^3 \cos\theta_W - \mathcal{A}_\mu \sin\theta_W$ $A_\mu = \mathcal{A}_\mu \cos\theta_W + b_\mu^3 \sin\theta_W$

$$\mathcal{L}_{Z-\nu} = \frac{-g}{4\cos\theta_W} \bar{\nu}\gamma^\mu(1-\gamma_5)\nu\, Z_\mu$$

$$\mathcal{L}_{Z-e} = \frac{-g}{4\cos\theta_W} \bar{e}\left[L_e\gamma^\mu(1-\gamma_5) + R_e\gamma^\mu(1+\gamma_5)\right] e Z_\mu$$

$$\begin{aligned} L_e &= 2\sin^2\theta_W - 1 = 2x_W + \tau_3 \\ R_e &= 2\sin^2\theta_W = 2x_W \end{aligned}$$

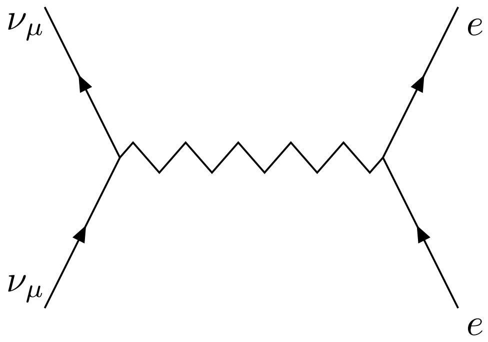
Z-boson properties

Decay calculation analogous to $W^\pm$

$$\Gamma(Z \to \nu\bar{\nu}) = \frac{G_F M_Z^3}{12\pi\sqrt{2}}$$
$$\Gamma(Z \to e^+e^-) = \Gamma(Z \to \nu\bar{\nu}) \left[L_e^2 + R_e^2\right]$$

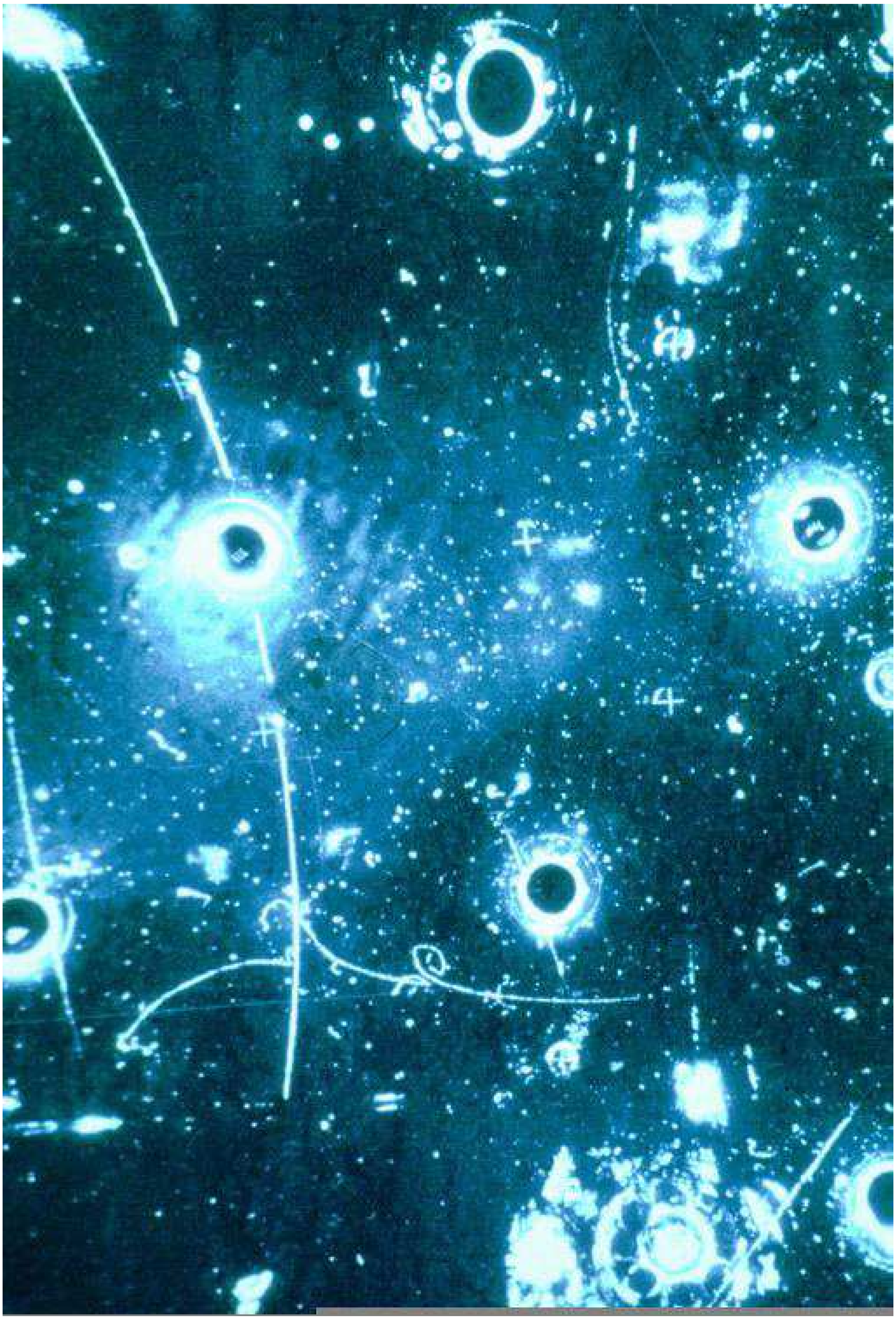
Neutral-current interactions

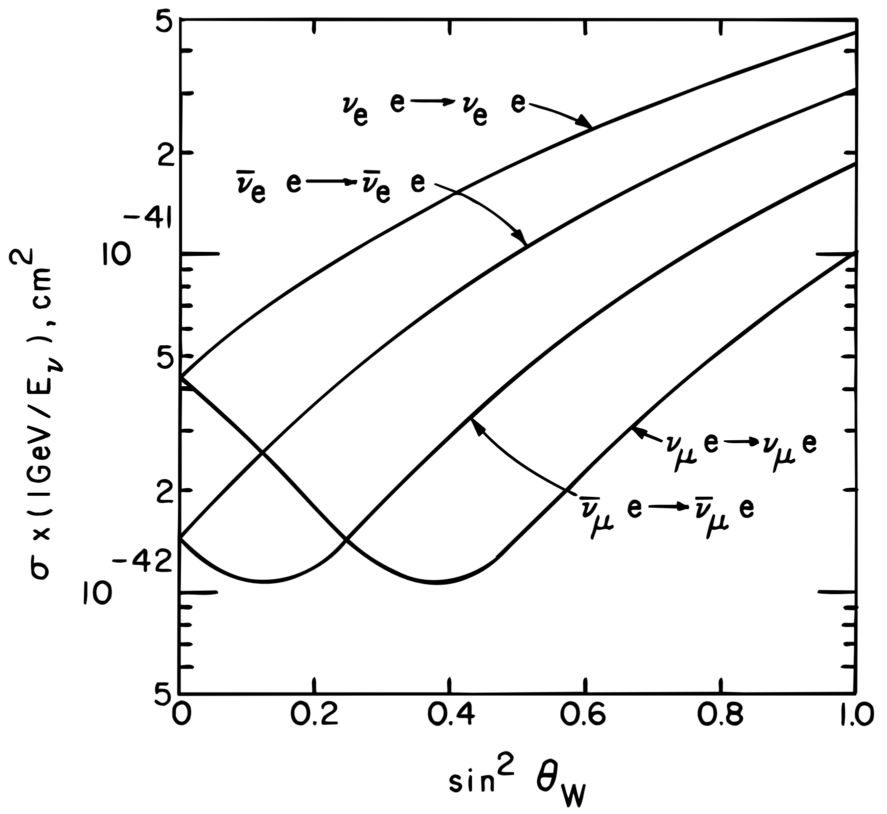
New νe reaction, not present in $V - A$



$$\sigma(\nu_\mu e \to \nu_\mu e) = \frac{G_F^2 m_e E_\nu}{2\pi} \left[L_e^2 + R_e^2/3\right]$$
$$\sigma(\bar{\nu}_\mu e \to \bar{\nu}_\mu e) = \frac{G_F^2 m_e E_\nu}{2\pi} \left[L_e^2/3 + R_e^2\right]$$
$$\sigma(\nu_e e \to \nu_e e) = \frac{G_F^2 m_e E_\nu}{2\pi} \left[(L_e + 2)^2 + R_e^2/3\right]$$
$$\sigma(\bar{\nu}_e e \to \bar{\nu}_e e) = \frac{G_F^2 m_e E_\nu}{2\pi} \left[(L_e + 2)^2/3 + R_e^2\right]$$

Gargamelle $\nu_\mu e$ Event





"Model-independent" analysis

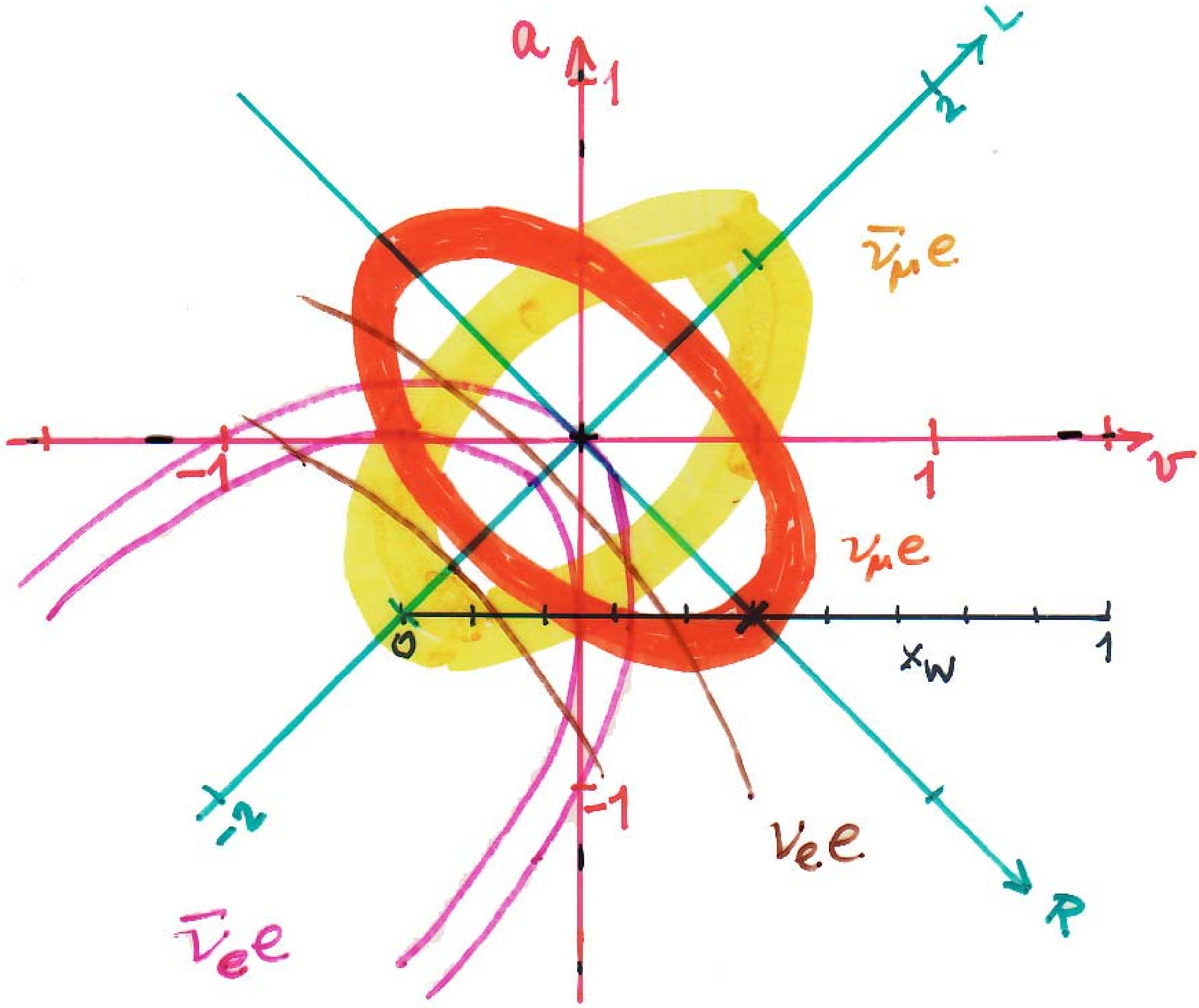
Measure all cross sections to determine chiral couplings L_e and R_e or traditional vector and axial couplings v and a

$$a = \tfrac{1}{2}(L_e - R_e) \qquad v = \tfrac{1}{2}(L_e - R_e)$$

$$L_e = v + a \qquad R_e = v - a$$

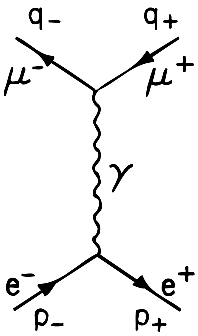
model-independent in V, A framework

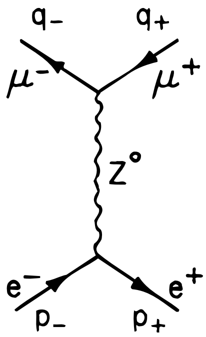
Neutrino–electron scattering



Twofold ambiguity remains even after measuring all four cross sections: same cross sections result if we interchange $R_e \leftrightarrow -R_e$ $(v \leftrightarrow a)$

Consider $e^+e^- \rightarrow \mu^+\mu^-$





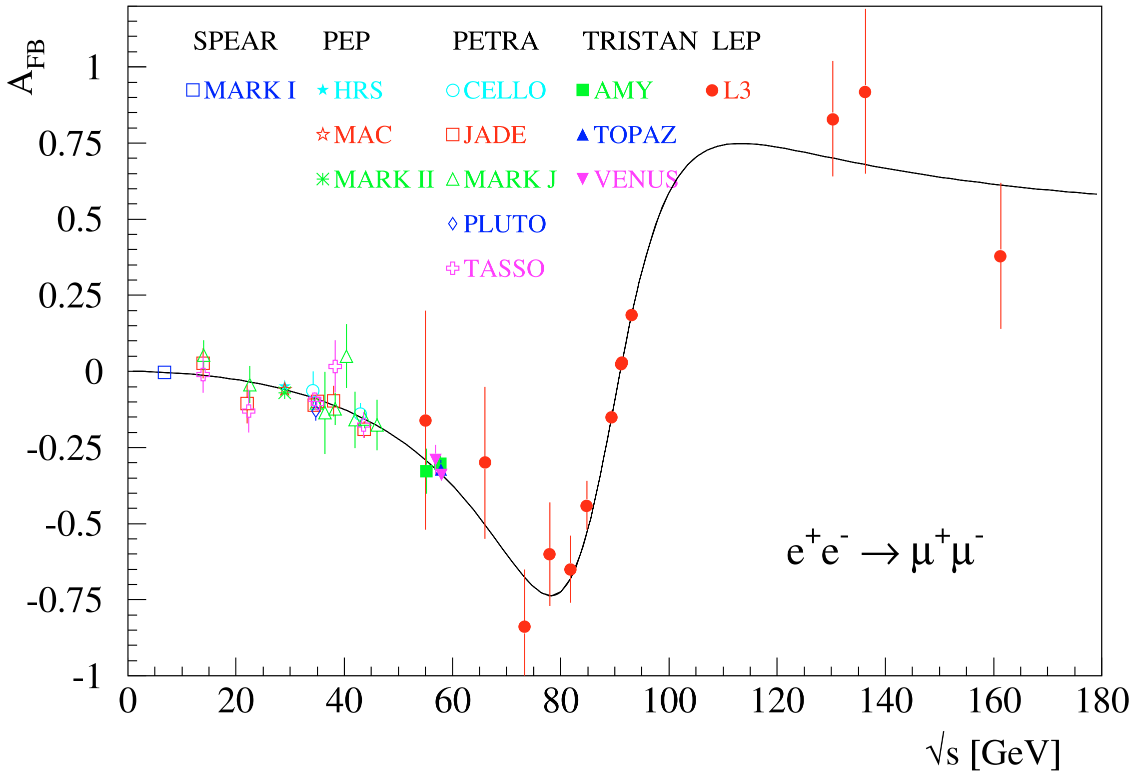
$$\begin{aligned}\mathcal{M} &= -ie^2\bar{u}(\mu,q_-)\gamma_\lambda Q_\mu v(\mu,q_+)\frac{g^{\lambda\nu}}{s}\bar{v}(e,p_+)\gamma_\nu u(e.p_-)\\ &+\frac{i}{2}\left(\frac{G_F M_Z^2}{\sqrt{2}}\right)\bar{u}(\mu,q_-)\gamma_\lambda[R_\mu(1+\gamma_5)+L_\mu(1-\gamma_5)]v(\mu,q_+)\\ &\times\frac{g^{\lambda\nu}}{s-M_Z^2}\bar{v}(e,p_+)\gamma_\nu[R_e(1+\gamma_5)+L_e(1-\gamma_5)]u(e,p_-)\end{aligned}$$

muon charge $Q_\mu = -1$

$$\begin{aligned}\frac{d\sigma}{dz} &= \frac{\pi\alpha^2 Q_\mu^2}{2s}(1+z^2)\\ &-\frac{\alpha Q_\mu G_F M_Z^2(s-M_Z^2)}{8\sqrt{2}[(s-M_Z^2)^2+M_Z^2\Gamma^2]}\\ &\times[(R_e+L_e)(R_\mu+L_\mu)(1+z^2)+2(R_e-L_e)(R_\mu-L_\mu)z]\\ &+\frac{G_F^2 M_Z^4 s}{64\pi[(s-M_Z^2)^2+M_Z^2\Gamma^2]}\\ &\times[(R_e^2+L_e^2)(R_\mu^2+L_\mu^2)(1+z^2)+2(R_e^2-L_e^2)(R_\mu^2-L_\mu^2)z]\end{aligned}$$

$$\text{F-B asymmetry } A \equiv \frac{\int_0^1 dz\, d\sigma/dz - \int_{-1}^0 dz\, d\sigma/dz}{\int_{-1}^1 dz\, d\sigma/dz}$$

$$\begin{aligned} \lim_{s/M_Z^2 \ll 1} A &= \frac{3G_F s}{16\pi\alpha Q_\mu \sqrt{2}} (R_e - L_e)(R_\mu - L_\mu) \\ &\approx -6.7 \times 10^{-5} \left(\frac{s}{1 \text{ GeV}^2} \right) (R_e - L_e)(R_\mu - L_\mu) \\ &= -3G_F s a^2 / 4\pi\alpha\sqrt{2} \end{aligned}$$

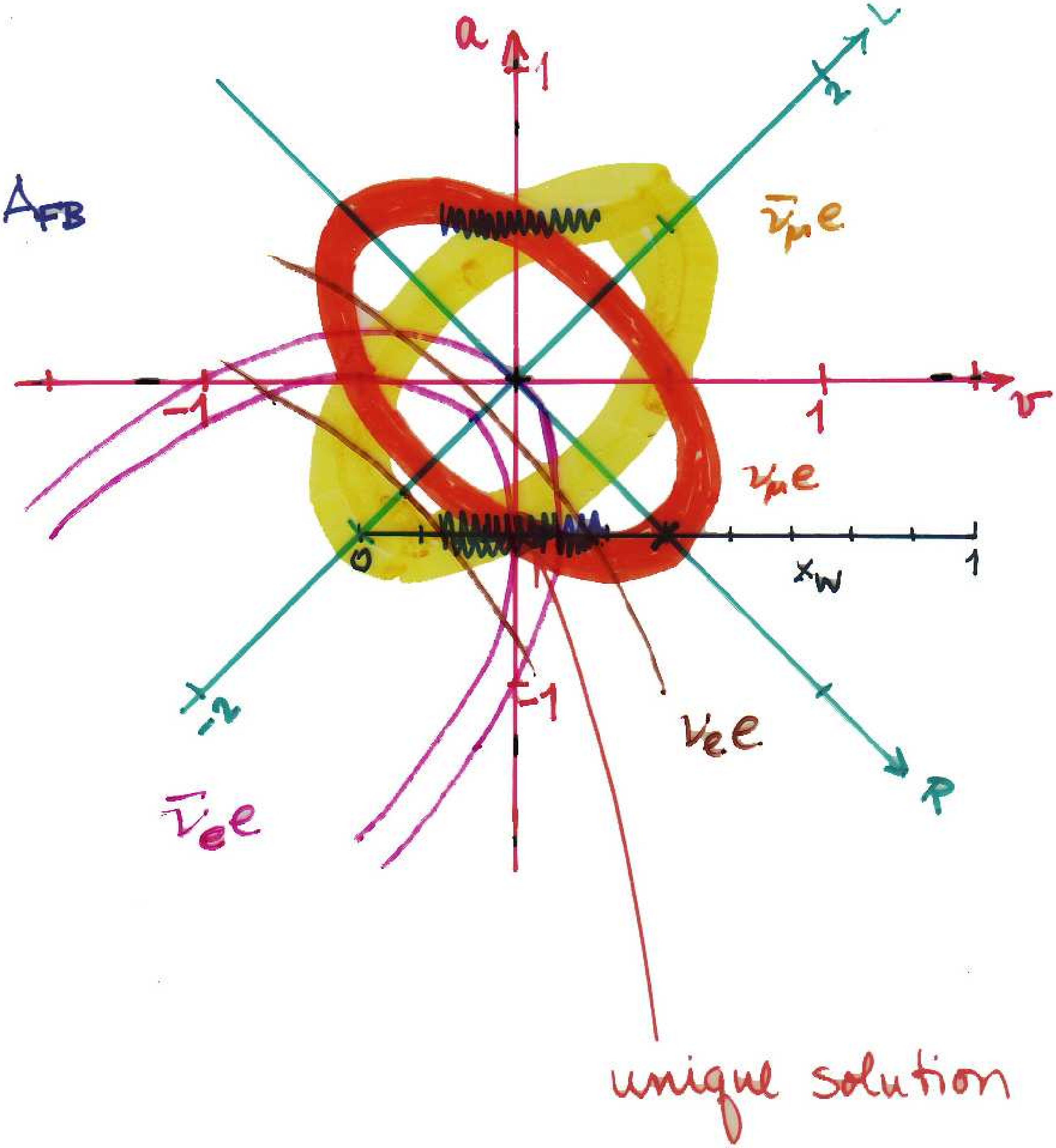


J. Mnich *Phys. Rep.* **271**, 181-266 (1996)

Measuring A resolves ambiguity

Validate EW theory, measure $\sin^2\theta_W$

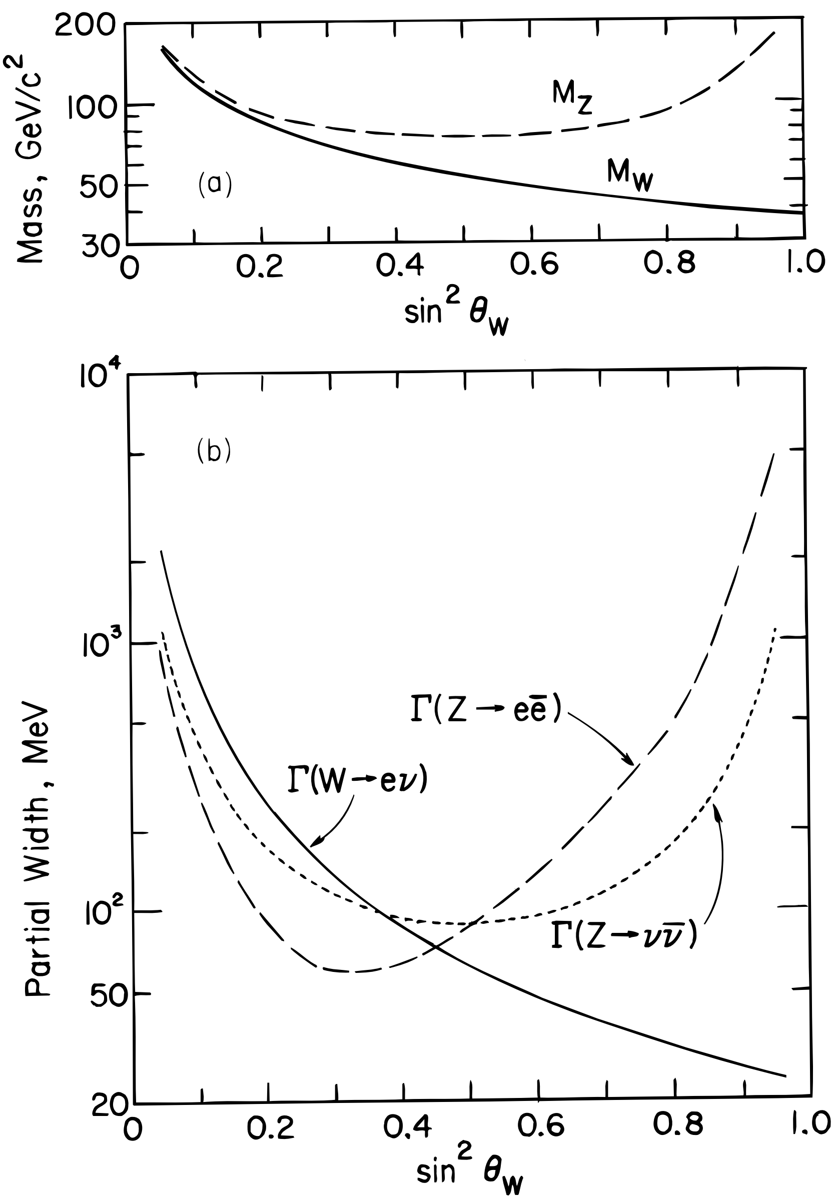
Neutrino–electron scattering



With a measurement of $\sin^2\theta_W$, predict

$$M_W^2 = g^2v^2/4 = e^2/4G_F\sqrt{2}\sin^2\theta_W \approx (37.3\ \text{GeV}/c^2)^2/\sin^2\theta_W$$

$$M_Z^2 = M_W^2/\cos^2\theta_W$$



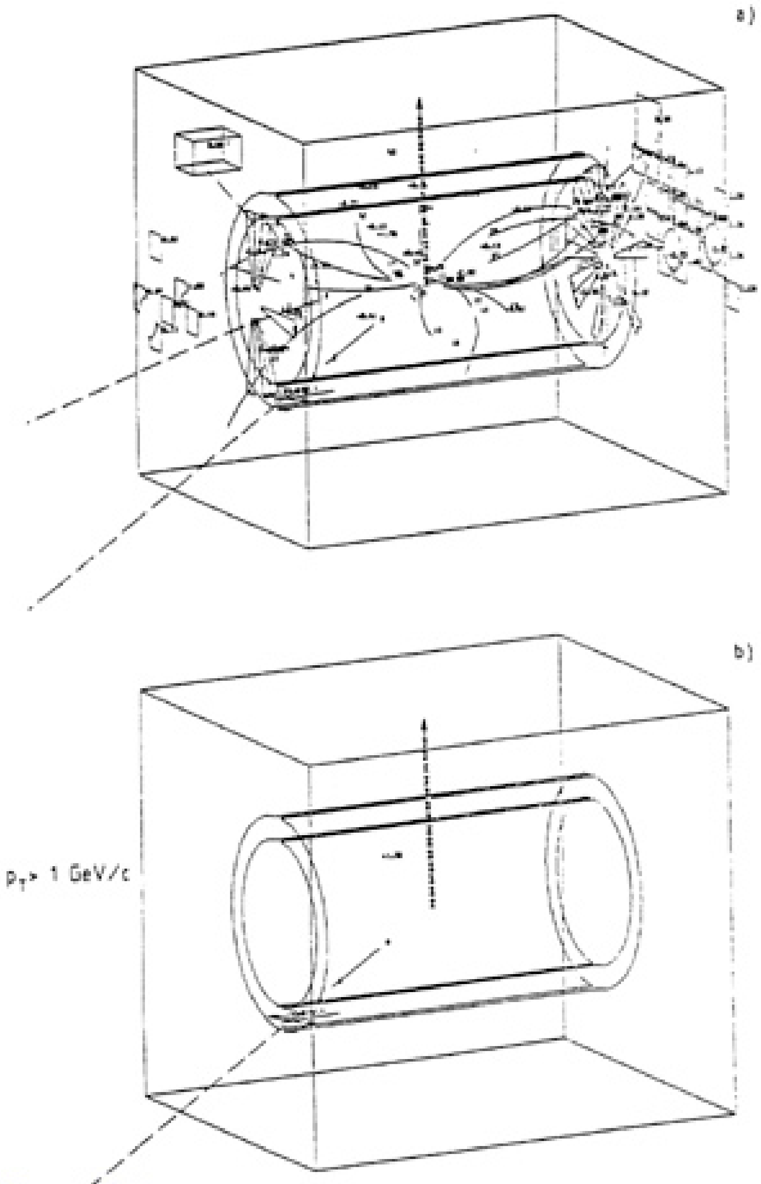
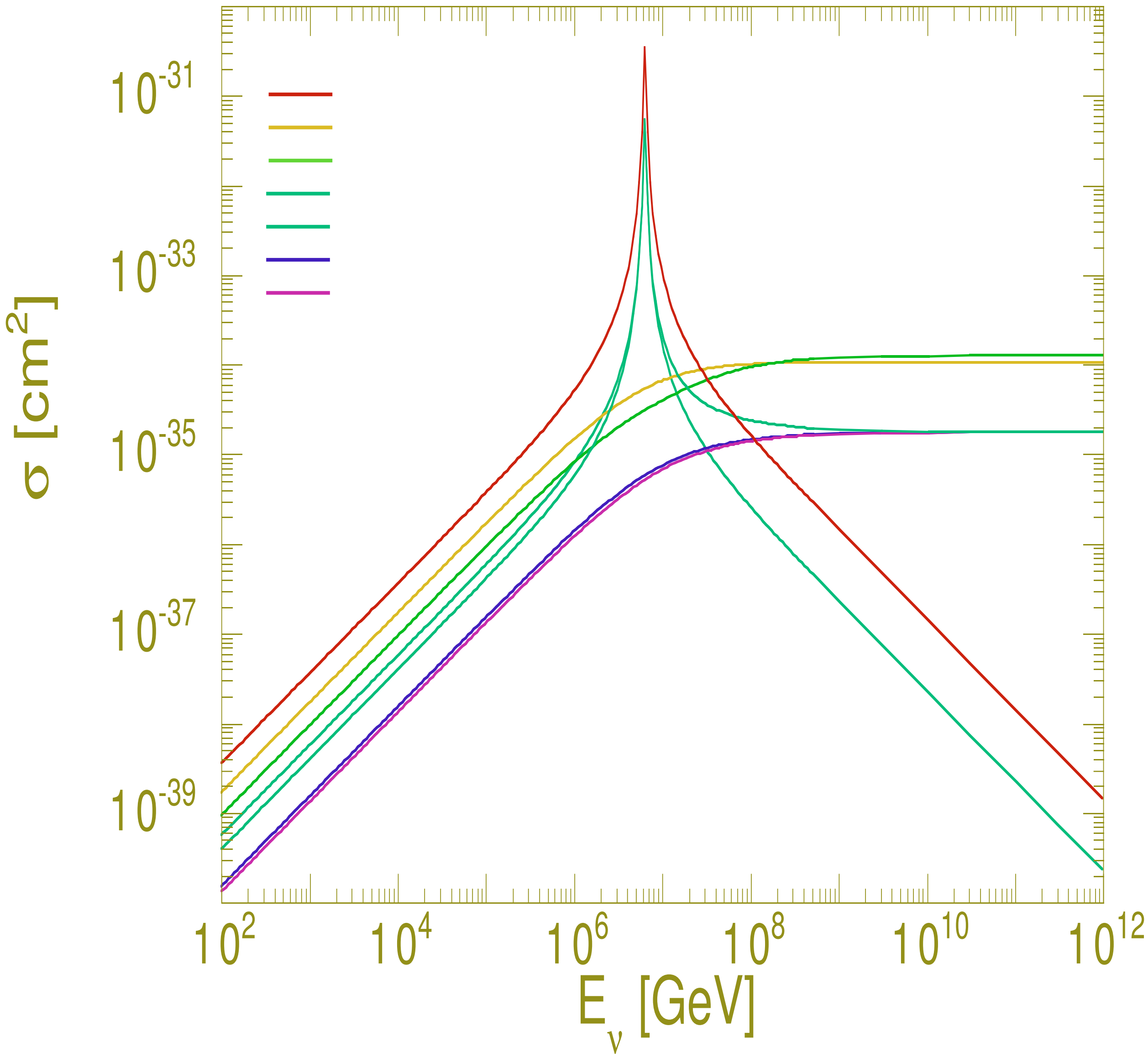


Fig. 16a,b

UA1



At low energies: $\sigma(\bar{\nu}_e e \to$ hadrons$) > \sigma(\nu_\mu e \to \mu\nu_e) >$ $\sigma(\nu_e e \to \nu_e e) > \sigma(\bar{(}\nu_e e \to \bar{\nu}_\mu \mu) > \sigma(\bar{\nu}_e e \to \bar{\nu}_e e) >$ $\sigma(\nu_\mu e \to \nu_\mu e) > \sigma(\bar{\nu}_\mu e \to \bar{\nu}_\mu e)$

EW interactions of quarks

▷ Left-handed doublet

	I_3	Q	$Y = 2(Q - I_3)$
$\mathsf{L}_q = \begin{pmatrix} u \\ d \end{pmatrix}_L$	$\frac{1}{2}$	$+\frac{2}{3}$	$\frac{1}{3}$
	$-\frac{1}{2}$	$-\frac{1}{3}$	

▷ two right-handed singlets

	I_3	Q	$Y = 2(Q - I_3)$
$\mathsf{R}_u = u_R$	0	$+\frac{2}{3}$	$+\frac{4}{3}$
$\mathsf{R}_d = d_R$	0	$-\frac{1}{3}$	$-\frac{2}{3}$

▷ CC interaction

$$\mathcal{L}_{W\text{-}q} = \frac{-g}{2\sqrt{2}} \left[\bar{u}_e \gamma^\mu (1 - \gamma_5) d\, W_\mu^+ + \bar{d} \gamma^\mu (1 - \gamma_5) u\, W_\mu^- \right]$$

identical in form to $\mathcal{L}_{W\text{-}\ell}$: universality ⇔ weak isospin

▷ NC interaction

$$\mathcal{L}_{Z\text{-}q} = \frac{-g}{4 \cos\theta_W} \sum_{i=u,d} \bar{q}_i \gamma^\mu \left[L_i (1 - \gamma_5) + R_i (1 + \gamma_5) \right] q_i Z_\mu$$

$$L_i = \tau_3 - 2Q_i \sin^2\theta_W \quad R_i = -2Q_i \sin^2\theta_W$$

equivalent in form (not numbers) to $\mathcal{L}_{Z\text{-}\ell}$

Trouble in Paradise

Universal $u \leftrightarrow d$, $\nu_e \leftrightarrow e$ *not quite right*

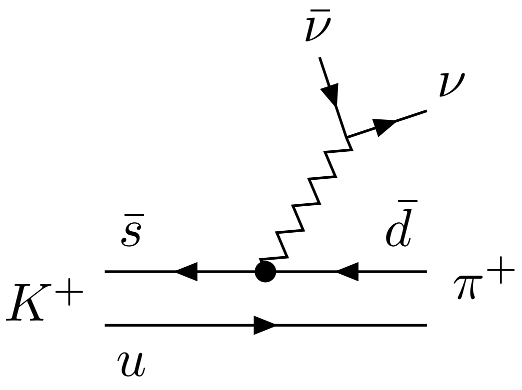
$$\text{Good: } \begin{pmatrix} u \\ d \end{pmatrix}_L \longrightarrow \text{Better: } \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L$$

$$d_\theta \equiv d\,\cos\theta_C + s\,\sin\theta_C \qquad \cos\theta_C = 0.9736 \pm 0.0010$$

"Cabibbo-rotated" doublet perfects CC interaction (up to small third-generation effects) but $\Rightarrow$ serious trouble for NC

$$\begin{aligned}\mathcal{L}_{Z\text{-}q} \quad = \quad & \frac{-g}{4\cos\theta_W} Z_\mu \left\{ \bar{u}\gamma^\mu \left[L_u(1-\gamma_5) + R_u(1+\gamma_5) \right] u \right. \\ & + \bar{d}\gamma^\mu \left[L_d(1-\gamma_5) + R_d(1+\gamma_5) \right] d\,\cos^2\theta_C \\ & + \bar{s}\gamma^\mu \left[L_d(1-\gamma_5) + R_d(1+\gamma_5) \right] s\,\sin^2\theta_C \\ & + \bar{d}\gamma^\mu \left[L_d(1-\gamma_5) + R_d(1+\gamma_5) \right] s\,\sin\theta_C\cos\theta_C \\ & \left. +\ \bar{s}\gamma^\mu \left[L_d(1-\gamma_5) + R_d(1+\gamma_5) \right] d\,\sin\theta_C\cos\theta_C \right\}\end{aligned}$$

Strangeness-changing NC interactions highly suppressed!



(SM: 0.8 ± 0.3)

BNL E-787 detected two $K^+ \to \pi^+\nu\bar{\nu}$ candidates, with $\mathcal{B}(K^+ \to \pi^+\nu\bar{\nu}) = 1.57^{+1.75}_{-0.82} \times 10^{-10}$

Phys. Rev. Lett. **88**, 041803 (2002)

Glashow–Iliopoulos–Maiani

two left-handed doublets

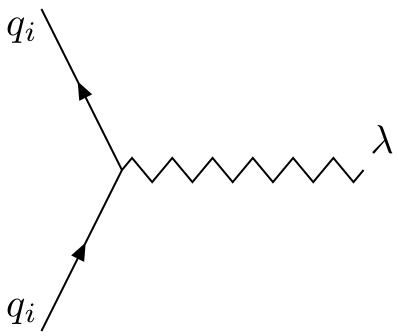
$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \qquad \begin{pmatrix} u \\ d_\theta \end{pmatrix}_L \quad \begin{pmatrix} c \\ s_\theta \end{pmatrix}_L$$

$$(s_\theta = s\,\cos\theta_C - d\,\sin\theta_C)$$

$+$ right-handed singlets, e_R, μ_R, u_R, d_R, c_R, s_R

Required new charmed quark, c

Cross terms vanish in $\mathcal{L}_{Z\text{-}q}$,



$$\frac{-ig}{4\cos\theta_W}\gamma_\lambda[(1-\gamma_5)L_i + (1+\gamma_5)R_i] \quad ,$$

$$L_i = \tau_3 - 2Q_i\sin^2\theta_W \qquad R_i = -2Q_i\sin^2\theta_W$$

flavor-diagonal interaction!

Straightforward generalization to n quark doublets

$$\mathcal{L}_{W\text{-}q} = \frac{-g}{2\sqrt{2}} \left[\bar{\Psi}\gamma^{\mu}(1-\gamma_5)\mathcal{O}\Psi\, W_{\mu}^{+} + \text{h.c.}\right]$$

composite $\Psi = \begin{pmatrix} u \\ c \\ \vdots \\ \\ d \\ s \\ \vdots \end{pmatrix}$

flavor structure $\mathcal{O} = \begin{pmatrix} 0 & U \\ 0 & 0 \end{pmatrix}$

U: unitary quark mixing matrix

Weak-isospin part: $\mathcal{L}_{Z\text{-}q}^{\text{iso}} = \dfrac{-g}{4\cos\theta_W} \bar{\Psi}\gamma^{\mu}(1-\gamma_5)\left[\mathcal{O},\mathcal{O}^{\dagger}\right]\Psi$

Since $\left[\mathcal{O},\mathcal{O}^{\dagger}\right] = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \propto \tau_3$

$\Rightarrow$ NC interaction is flavor-diagonal

General $n \times n$ quark-mixing matrix U:

$n(n-1)/2$ real $\angle$, $(n-1)(n-2)/2$ complex phases

3×3 (Cabibbo–Kobayashi-Maskawa): $3\ \angle + 1$ phase

$\Rightarrow$ CP violation

Qualitative successes of $SU(2)_L \otimes U(1)_Y$ theory:

- ▷ neutral-current interactions
- ▷ necessity of charm
- ▷ existence and properties of $W^\pm$ and Z^0

Decade of precision tests EW (one-per-mille)

M_Z	$91\,187.6 \pm 2.1$ MeV/c^2
Γ_Z	2495.2 ± 2.3 MeV
$\sigma^0_{\text{hadronic}}$	41.541 ± 0.037 nb
Γ_{hadronic}	1744.4 ± 2.0 MeV
Γ_{leptonic}	83.984 ± 0.086 MeV
$\Gamma_{\text{invisible}}$	499.0 ± 1.5 MeV

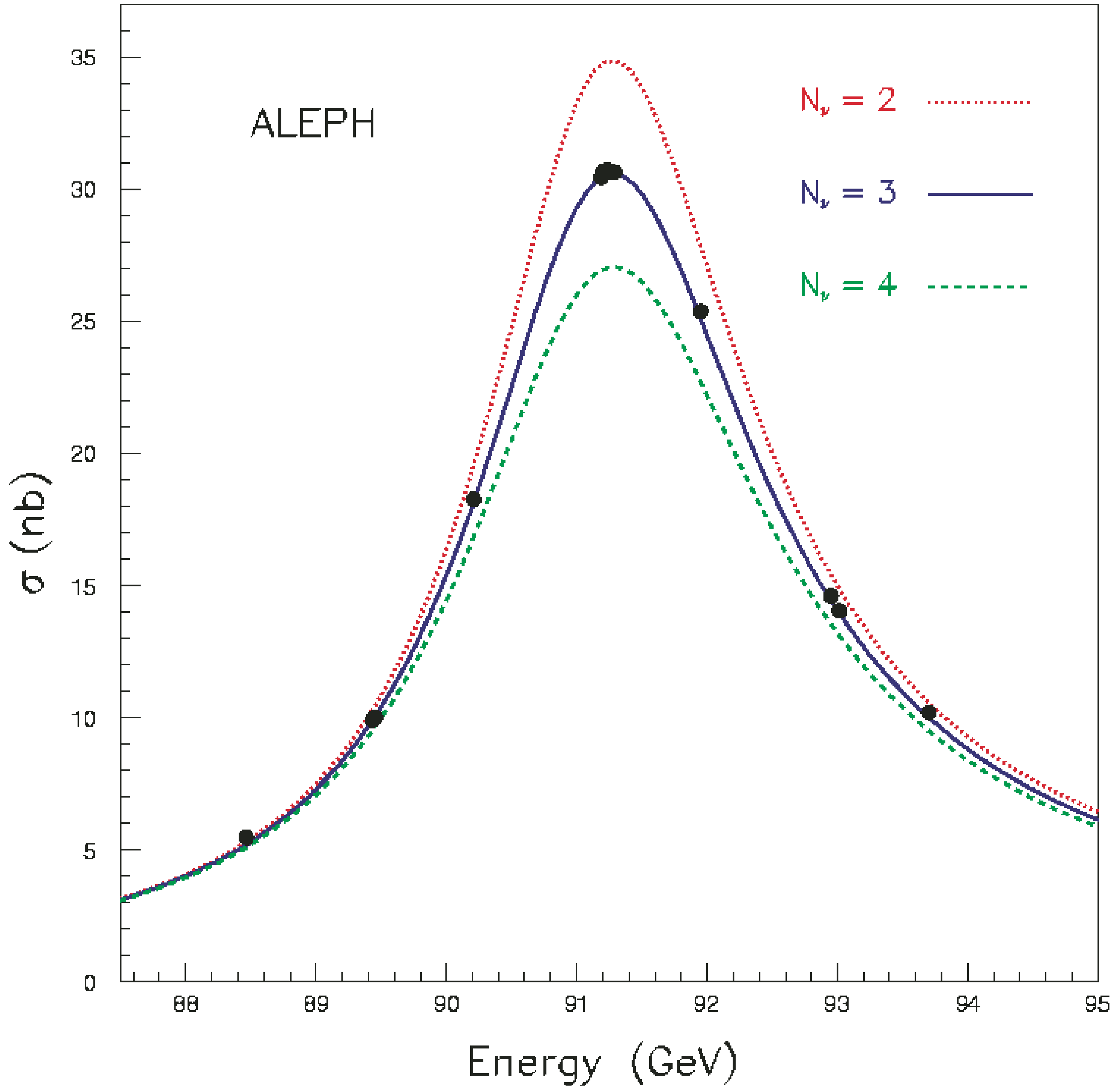
where $\Gamma_{\text{invisible}} \equiv \Gamma_Z - \Gamma_{\text{hadronic}} - 3\Gamma_{\text{leptonic}}$

light neutrinos $N_\nu = \Gamma_{\text{invisible}}/\Gamma^{\text{SM}}(Z \to \nu_i\bar{\nu}_i)$

Current value: $\boxed{N_\nu = 2.994 \pm 0.012}$

. . . excellent agreement with ν_e, ν_μ, and ν_τ

Three light neutrinos



The top quark must exist

▷ Two families

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L$$

don't account for CP violation. Need a third family ... or another answer.

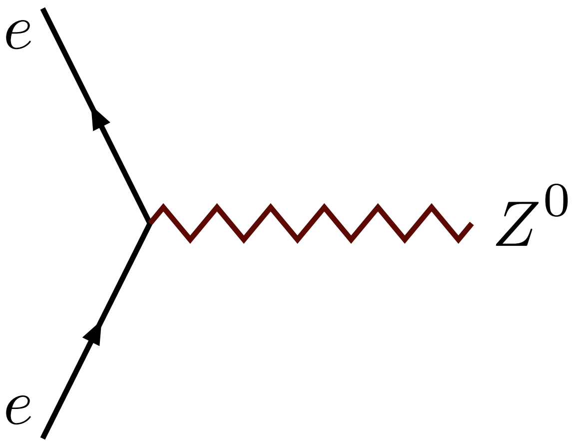
Given the existence of b, (τ)

▷ top is needed for an anomaly-free EW theory

▷ absence of FCNC in b decay ($b \not\to s\ell^+\ell^-$, etc.)

▷ b has weak isospin $I_{3L} = -\frac{1}{2}$; needs partner

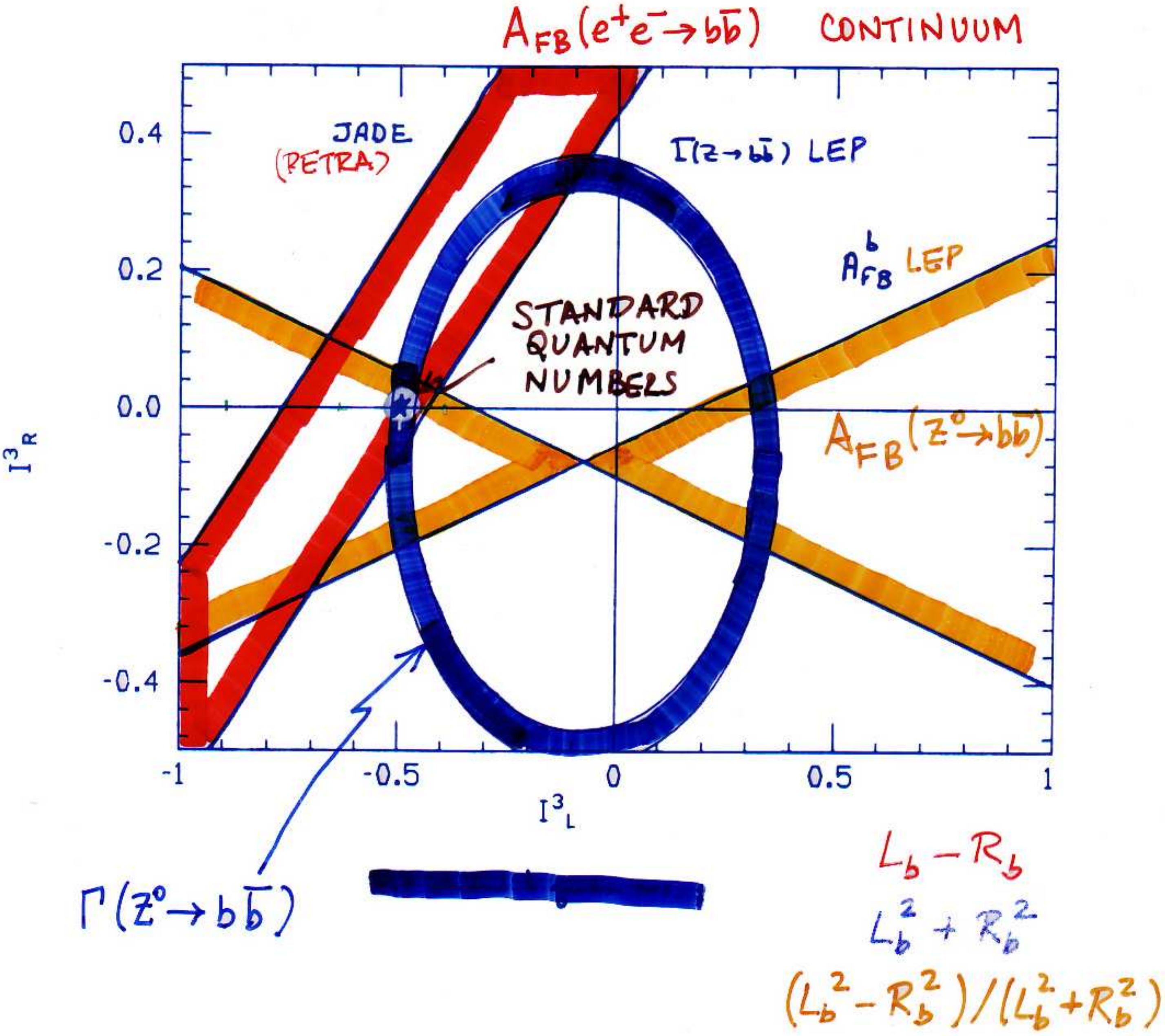
$$\begin{pmatrix} t \\ b \end{pmatrix}_L$$



$$L_b = I_{3L} - Q_b \sin^2\theta_W$$

$$R_b = I_{3R} - Q_b \sin^2\theta_W$$

$$\text{Measure } I_{3L}^{(b)} = -0.490^{+0.015}_{-0.012} \; I_{3R}^{(b)} = -0.028 \pm 0.056$$



Needed: top with $I_{3L} = +\frac{1}{2}$

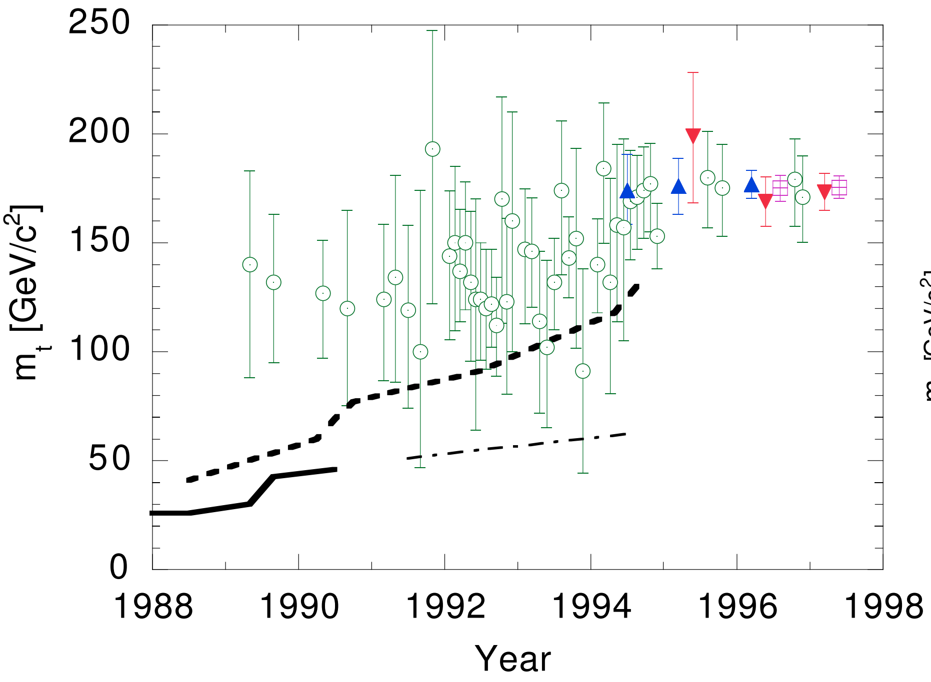
D. Schaile & P. Zerwas, *Phys. Rev.* D**45**, 3262 (1992)

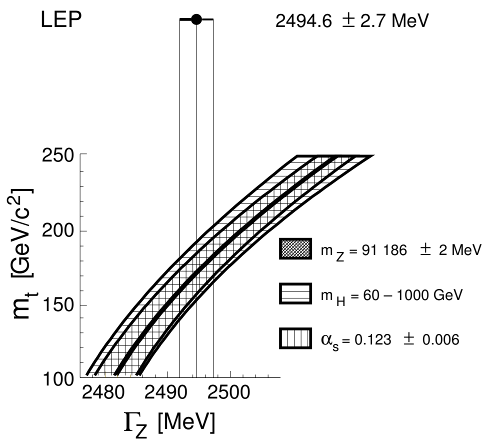
Global fits . . .

to precision EW measurements:

▷ precision improves with time

▷ calculations improve with time

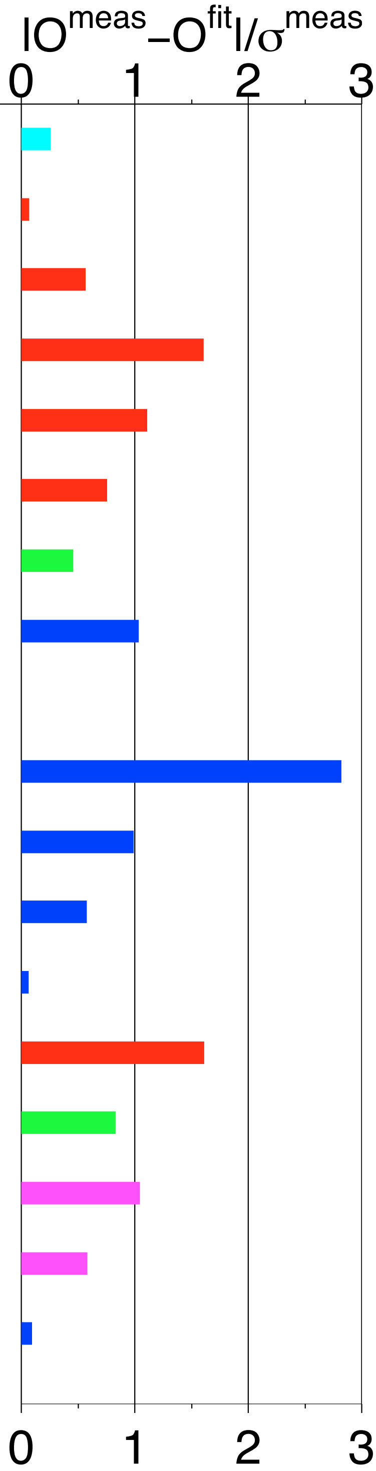




11.94, LEPEWWG: $m_t = 178 \pm 11^{+18}_{-19}$ GeV/c^2

Direct measurements: $m_t = 174.3 \pm 5.1$ GeV/c^2

	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02761 ± 0.00036	0.02770
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4965
σ^0_{had} [nb]	41.540 ± 0.037	41.481
R_l	20.767 ± 0.025	20.739
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01642
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1480
R_b	0.21630 ± 0.00066	0.21562
R_c	0.1723 ± 0.0031	0.1723
$A^{0,b}_{fb}$	0.0992 ± 0.0016	0.1037
$A^{0,c}_{fb}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1480
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.425 ± 0.034	80.390
Γ_W [GeV]	2.133 ± 0.069	2.093
m_t [GeV]	178.0 ± 4.3	178.4



LEP Electroweak Working Group, Winter 2005

Parity violation in atoms

Nucleon appears elementary at very low Q^2; effective Lagrangian for nucleon β-decay

$$\mathcal{L}_\beta = -\frac{G_F}{\sqrt{2}}\bar{e}\gamma_\lambda(1-\gamma_5)\nu\,\bar{p}\gamma^\lambda(1-g_A\gamma_5)n$$

$g_A \approx 1.26$: axial charge

NC interactions ($x_W \equiv \sin^2\theta_W$):

$$\begin{aligned}\mathcal{L}ep &= \frac{G_F}{2\sqrt{2}}\bar{e}\gamma_\lambda(1-4x_W-\gamma_5)e\,\bar{p}\gamma^\lambda(1-4x_W-\gamma_5)p\,,\\ \mathcal{L}en &= \frac{G_F}{2\sqrt{2}}\bar{e}\gamma_\lambda(1-4x_W-\gamma_5)e\,\bar{n}\gamma^\lambda(1-\gamma_5)n\end{aligned}$$

▷ Regard nucleus as a noninteracting collection of Z protons and N neutrons ▷ Perform NR reduction; nucleons contribute coherently to A_eV_N coupling, so dominant P-violating contribution to eN amplitude is

$$\mathcal{M}_{\text{pv}} = \frac{-iG_F}{2\sqrt{2}}Q^W\bar{e}\rho_N(\mathbf{r})\gamma_5 e$$

$\rho_N(\mathbf{r})$: nucleon density at e^- coordinate $\mathbf{r}$

$Q^W \equiv Z(1-4x_W)-N$: weak charge

Bennett & Wieman (Boulder) determined weak charge of Cesium by measuring 6S-7S transition polarizability

$$Q_W(\text{Cs}) = -72.06 \pm 0.28\ (\text{expt}) \pm 0.34\ (\text{theory})$$

about 2.5σ above SM prediction

The vacuum energy problem

Higgs potential $V(\varphi^\dagger\varphi) = \mu^2(\varphi^\dagger\varphi) + |\lambda|\,(\varphi^\dagger\varphi)^2$

At the minimum,

$$V(\langle\varphi^\dagger\varphi\rangle_0) = \frac{\mu^2 v^2}{4} = -\frac{|\lambda|\,v^4}{4} < 0.$$

Identify $M_H^2 = -2\mu^2$

contributes field-independent vacuum energy density

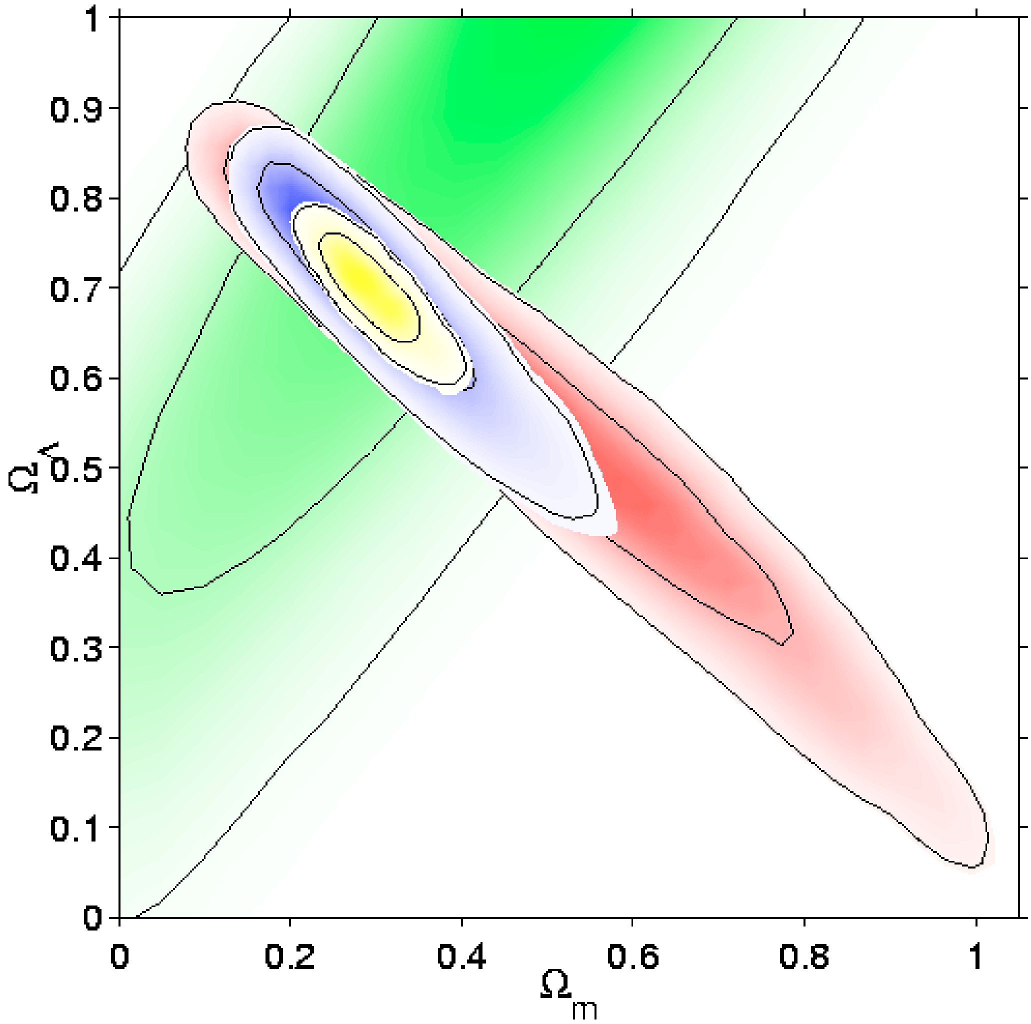
$$\varrho_H \equiv \frac{M_H^2 v^2}{8}$$

Adding vacuum energy density $\varrho_{\text{vac}} \Leftrightarrow$ adding cosmological constant Λ to Einstein's equation

$$R_{\mu\nu} - \tfrac{1}{2}Rg_{\mu\nu} = \frac{8\pi G_N}{c^4}T_{\mu\nu} + \Lambda g_{\mu\nu}$$

$$\Lambda = \frac{8\pi G_N}{c^4}\varrho_{\text{vac}}$$

observed vacuum energy density $\varrho_{\text{vac}} \lesssim 10^{-46}$ GeV^4



Lewis & Bridle, astro-ph/0205436

But $M_H \gtrsim 114$ GeV/c^2 $\Rightarrow$

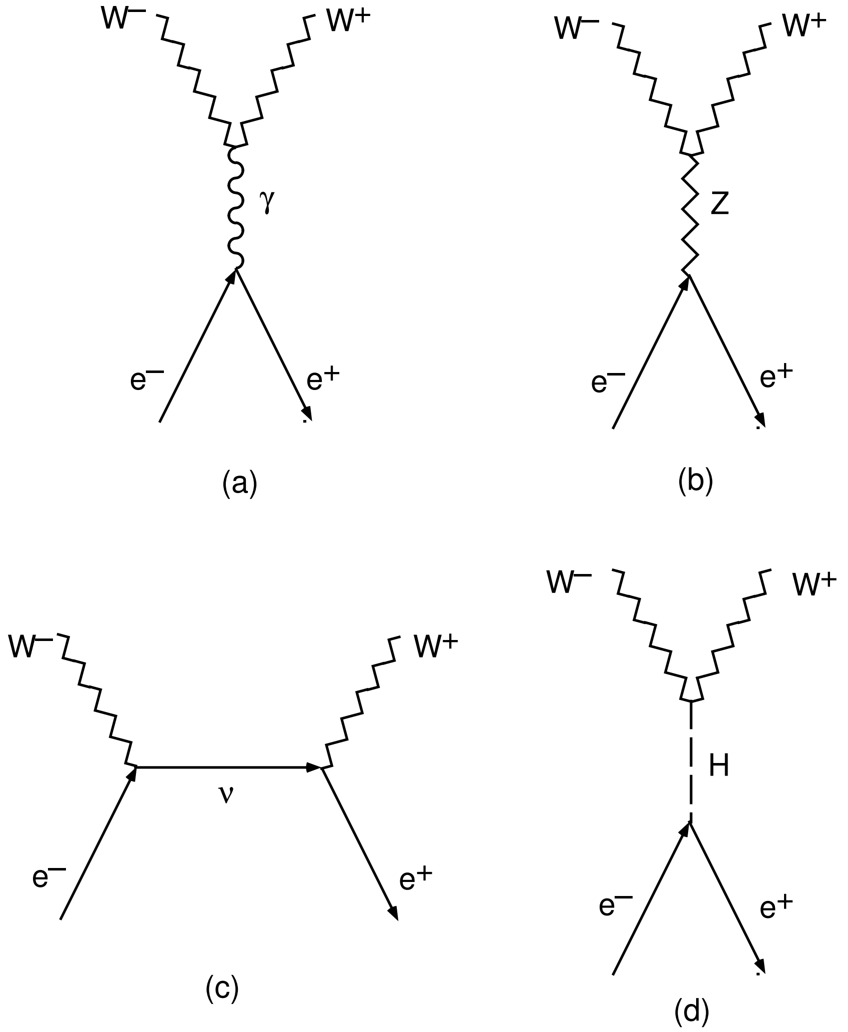
$$\varrho_H \gtrsim 10^8 \text{ GeV}^4$$

MISMATCH BY 54 ORDERS OR MAGNITUDE

Why a Higgs Boson Must Exist

▷ Role in canceling high-energy divergences

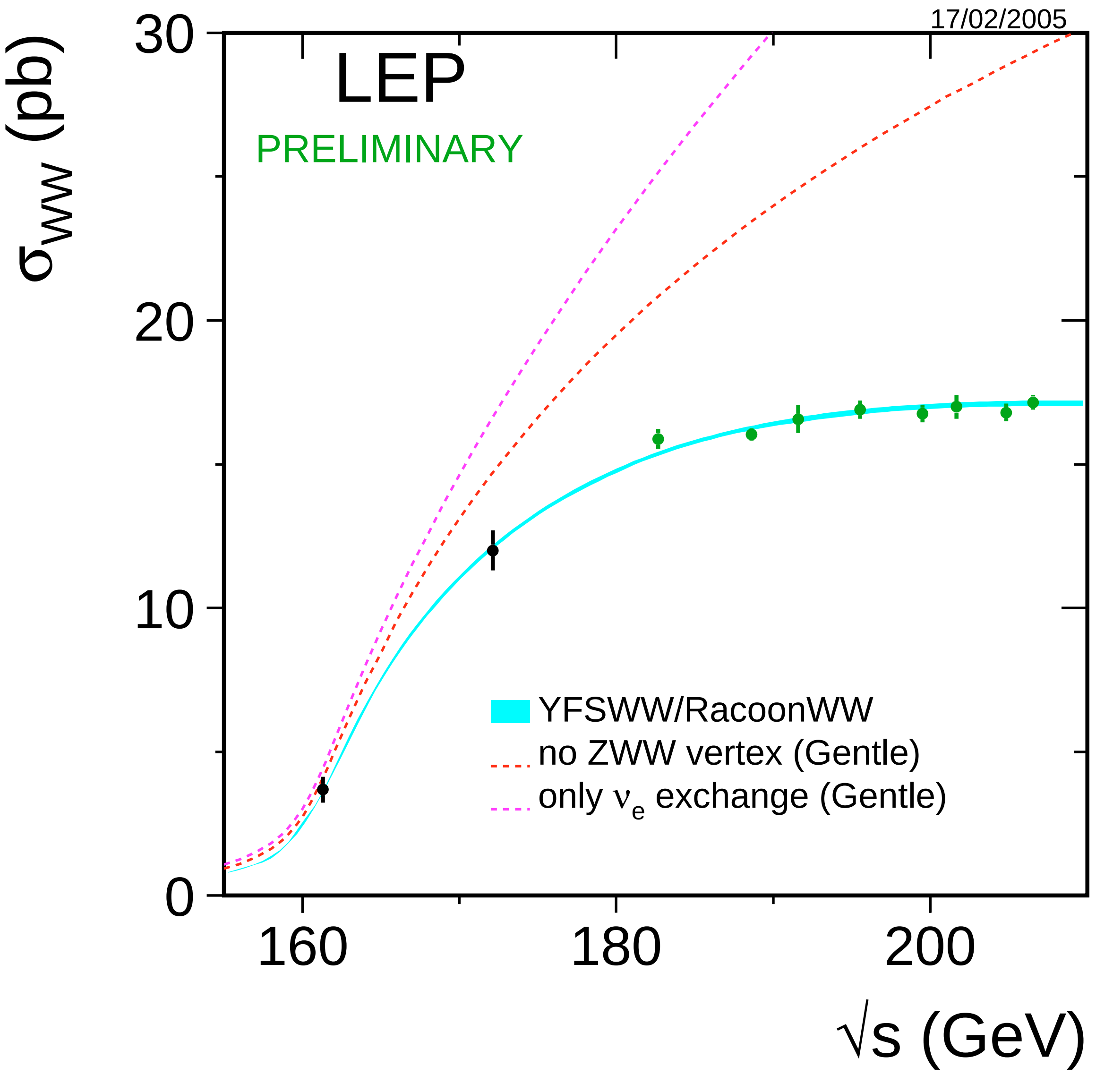
S-matrix analysis of $e^+e^- \to W^+W^-$



$J = 1$ partial-wave amplitudes $\mathcal{M}_\gamma^{(1)}$, $\mathcal{M}_Z^{(1)}$, $\mathcal{M}_\nu^{(1)}$ have—individually—unacceptable high-energy behavior $(\propto s)$

. . . But sum is well-behaved

"Gauge cancellation" observed at LEP2, Tevatron



$J = 0$ amplitude exists because electrons have mass, and can be found in "wrong" helicity state

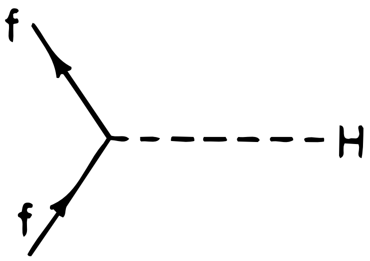
$$\mathcal{M}_{\nu}^{(0)} \propto s^{\frac{1}{2}} \quad \text{: unacceptable HE behavior}$$

(no contributions from γ and Z)

This divergence is canceled by the Higgs-boson contribution

$\Rightarrow He\bar{e}$ coupling must be $\propto m_e$,

because "wrong-helicity" amplitudes $\propto m_e$



$$\frac{-im_f}{v} = -im_f (G_F \sqrt{2})^{1/2}$$

If the Higgs boson did not exist, *something else* would have to cure divergent behavior

IF gauge symmetry were unbroken . . .

▷ no Higgs boson

▷ no longitudinal gauge bosons

▷ no extreme divergences

▷ no wrong-helicity amplitudes

. . . and no viable low-energy phenomenology

In spontaneously broken theory . . .

▷ gauge structure of couplings eliminates the most severe divergences

▷ lesser—but potentially fatal—divergence arises because the electron has mass

. . . due to the Higgs mechanism

▷ SSB provides its own cure—the Higgs boson

A similar interplay and compensation *must exist* in any acceptable theory

Bounds on M_H

EW theory does not predict Higgs-boson mass

Self-consistency $\Rightarrow$ plausible lower and upper bounds

▷ Conditional *upper bound* from Unitarity

Compute amplitudes $\mathcal{M}$ for gauge boson scattering at high energies, make a partial-wave decomposition

$$\mathcal{M}(s,t) = 16\pi \sum_J (2J+1) a_J(s) P_J(\cos\theta)$$

Most channels decouple—pw amplitudes are small at all energies (except very near the particle poles, or at exponentially large energies)—for any M_H.

Four interesting channels:

$$W_L^+ W_L^- \quad Z_L^0 Z_L^0/\sqrt{2} \quad HH/\sqrt{2} \quad HZ_L^0$$

L: longitudinal, $1/\sqrt{2}$ for identical particles

In HE limit,[a] s-wave amplitudes $\propto G_F M_H^2$

$$\lim_{s \gg M_H^2} (a_0) \to \frac{-G_F M_H^2}{4\pi\sqrt{2}} \cdot \begin{bmatrix} 1 & 1/\sqrt{8} & 1/\sqrt{8} & 0 \\ 1/\sqrt{8} & 3/4 & 1/4 & 0 \\ 1/\sqrt{8} & 1/4 & 3/4 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$$

Require that largest eigenvalue respect the partial-wave unitarity condition $|a_0| \leq 1$

$$\Longrightarrow \quad M_H \leq \left(\frac{8\pi\sqrt{2}}{3G_F}\right)^{1/2} = 1 \text{ TeV}/c^2$$

condition for perturbative unitarity

[a] Convenient to calculate using *Goldstone-boson equivalence theorem*, which reduces dynamics of longitudinally polarized gauge bosons to scalar field theory with interaction Lagrangian given by $\mathcal{L}_{\text{int}} = -\lambda v h(2w^+w^- + z^2 + h^2) - (\lambda/4)(2w^+w^- + z^2 + h^2)^2$, with $1/v^2 = G_F\sqrt{2}$ and $\lambda = G_F M_H^2/\sqrt{2}$.

▷ If the bound is respected

- ⋆ weak interactions remain weak at all energies
- ⋆ perturbation theory is everywhere reliable

▷ If the bound is violated

- ⋆ perturbation theory breaks down
- ⋆ weak interactions among $W^{\pm}$, Z, and H become strong on the 1-TeV scale

⇒ features of *strong* interactions at GeV energies will characterize *electroweak* gauge boson interactions at TeV energies

Threshold behavior of the pw amplitudes a_{IJ} follows from chiral symmetry

$a_{00} \approx$	$G_F s/8\pi\sqrt{2}$	attractive
$a_{11} \approx$	$G_F s/48\pi\sqrt{2}$	attractive
$a_{20} \approx$	$-G_F s/16\pi\sqrt{2}$	repulsive

New phenomena are to be found in the EW interactions at energies not much larger than 1 TeV