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Physics of Extra Dimensions

V. RUBAKOV
Institute for Nuclear Research
Russian Academy of Sciences
60th October Anniversary Av.
Prospect 7A
117312 Moscow
RUSSIA

PHYSICS OF EXTRA DIMENSIONS

V. Rubakov
INSTITUTE FOR NUCLEAR RESEARCH
OF THE RUSSIAN ACADEMY OF SCIENCES,
MOSCOW

- Motivation from string/M-THEORY:
  - SUPERSTRINGS MOST EASILY CONSTRUCTED IN (9+1) dimensions
  - M-THEORY : (10+1) DIMENSIONS

EVEN WITHOUT STRINGS, INTERESTING TO UNDERSTAND PROPERTIES OF THEORIES WITH EXTRA DIMENSIONS.

- WHY EXTRA DIMENSIONS HAVE NOT BEEN OBSERVED?

A1: EXTRA DIMENSIONS COMPACT
KALUZA-KLEIN PICTURE

A2: WE LIVE ON 3-BRANE

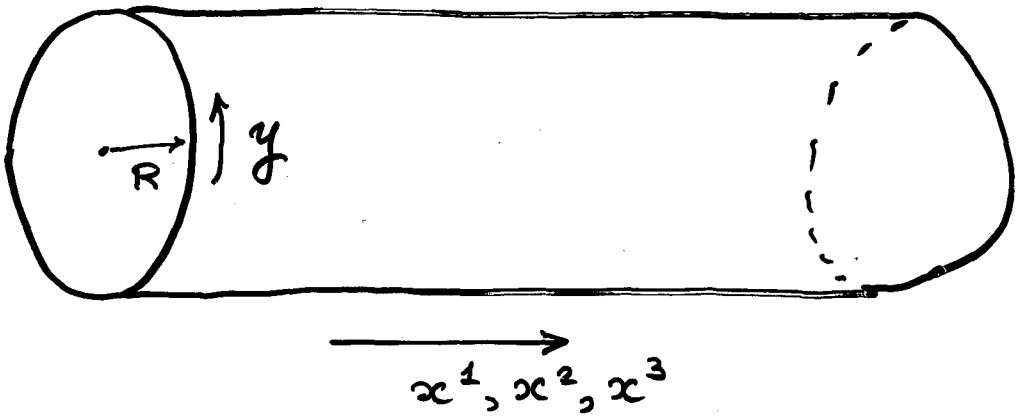
A3: BOTH.

NB: IN THESE LECTURES, EXTRA DIMENSIONS ALWAYS SPACE-LIKE.

ONE TIME.

KALUZA - KLEIN PICTURE: COMPACT EXTRA DIMENSIONS

$\eta_{\mu\nu} = +1, -1, -1, \dots, -1$; $\Box = \partial_t^2 - \vec{\partial}_x^{\,2}$



(4+1) - Dim. THEORY WITH COMPACT 5TH COORDINATE y

MASSLESS SCALAR FIELD:

$$\Box^{(5)}\phi \equiv (\partial_\mu \partial^\mu - \partial_y^2)\phi = 0 \qquad \mu = 0,1,2,3$$

ϕ periodic in y WITH PERIOD $2\pi R$

$\Downarrow$

SOLUTIONS $\phi(x,y) = e^{ip_\mu x^\mu} e^{in\frac{y}{R}\frac{2\pi}{2\pi}}$

$n = 0, \pm 1, \pm 2, \dots$ = angular momentum

$$p_\mu p^\mu = \frac{n^2}{R^2}$$

$n = 0$: HOMOGENEOUS MODE, PROPAGATING along x^1, x^2, x^3 WITH SPEED OF LIGHT $\Rightarrow$ (PROTOTYPE OF) KNOWN PARTICLES

$n \neq 0$: 4D MASSIVE PARTICLES, K-K MODES.

KK TOWER OF PARTNERS
(EXCITED ELECTRONS, PHOTONS, ETC.)

$$m_n = \frac{|n|}{R}$$

$$\downarrow$$

ACCEPTABLE $R \lesssim (\text{TeV})^{-1} \sim 10^{-17}$ cm.

CHANCE TO DISCOVER EXTRA DIMENSIONS
IFF $R \sim (\text{TeV})^{-1}$

NO HOPE FOR NATURAL (?) $R \sim M_{Pl}^{-1}$

NB: MORE THAN ONE EXTRA DIMENSIONS $\Rightarrow$
MORE K-K MODES.
E.G. $y_1, \dots, y_N$ ALL COMPACTIFIED
TO CIRCLE OF RADIUS $R_1, \dots, R_N$

$$\Downarrow$$

$$\phi = e^{ip_\mu x^\mu} e^{i n_1 \frac{y_1}{R_1}} \dots e^{i n_N \frac{y_N}{R_N}}$$

$$m^2_{\{n\}} = \sum \frac{n_i^2}{R_i^2}$$

One HOMOGENEOUS $\Leftrightarrow$ MASSLESS MODE.
MANY KK MODES. BELOW CERTAIN MASS.

EFFECTIVE 4D DESCRIPTION:

Quadratic (free) 5 dim ACTION

$$S^{(2)} = \int d^4x \int_0^{2\pi R} dy \; \frac{1}{2} \partial_A \phi \cdot \partial^A \phi$$

$$= \int d^4x \int_0^{2\pi R} dy \; \frac{1}{2} \left(\partial_\mu \phi \cdot \partial^\mu \phi - \partial_y \phi \, \partial_y \phi \right)$$

NB: DIMENSION $[\phi] = M^{3/2}$

PLUG IN KK DECOMPOSITION

$$\phi(x,y) = \sum_n \varphi_n(x) \, e^{i n \frac{y}{R}}$$

$$\Downarrow$$

$$S^{(2)} = \int d^4x \sum_n (2\pi R) \left[\frac{1}{2} |\partial_\mu \varphi_n|^2 - \frac{1}{2} \frac{n^2}{R^2} |\varphi_n|^2 \right]$$

↑ VOLUME OF EXTRA DIMENSIONS

Free 4d action of infinitely many 4d fields.

Canonically normalized 4d fields:

$$\hat{\varphi}_n(x) = \frac{1}{\sqrt{2\pi R}} \varphi_n(x) \qquad [\hat{\varphi}_n] = M \text{ as usual}$$

$$\Downarrow$$

$$S^{(2)} = \sum_n \int d^4x \left[\frac{1}{2} |\partial_\mu \hat{\varphi}_n|^2 - \frac{1}{2} \frac{n^2}{R^2} |\hat{\varphi}_n|^2 \right]$$

Sum of STANDARD 4d actions

INTERACTIONS:

5 Dim INTERACTION

$$S_{int} = \int d^4x \int_0^{2\pi R} dy \; \lambda^{(5)} \phi^4$$

NB: $[\lambda^{(5)}] = \frac{1}{M}$

5 Dim THEORY STRONGLY COUPLED AT HIGH ENERGIES

$$E_{STRONG} \sim \frac{1}{\lambda^{(5)}} \equiv M_{(5)}$$

PLUG IN FOURIER DECOMPOSITION IN y, WITH CANONICALLY NORMALIZED FIELDS

⇓

ZERO MODE ONLY, $n=0$ ("OUR" FIELD)

$$S_{int} = \int d^4x \; \underbrace{(2\pi R)}_{\int dy} \frac{\lambda^{(5)}}{(2\pi R)^2} \hat{\varphi}_0^4(x)$$

$(2\pi R)^2$ ← normalization

$$\lambda^{(4)} = \frac{\lambda^{(5)}}{(2\pi R)} \leftarrow \text{FUNDAMENTAL}$$

↑ EFFECTIVE 4D COUPLING

LESSON #1:

4D COUPINGS ARE EFFECTIVE, RELATED TO FUNDAMENTAL COUPLINGS OF ORIGINAL, HIGHER-DIM. THEORY THROUGH VOLUME OF EXTRA DIMENSIONS.

$$\lambda^{(4)} = \frac{\lambda^{(5)}}{(2\pi R)}$$

WEAKLY COUPLED 4D THEORY:

$$\lambda^{(4)} \ll 1$$

$$\Downarrow$$

$$R \gg \lambda^{(5)} \equiv \frac{1}{E_{STRONG}} \equiv \frac{1}{M_{(5)}}$$

SIZE OF EXTRA DIMENSIONS LARGE COMPARED TO (ENERGY SCALE)$^{-1}$ OF HIGH-DIM. THEORY

LESSON #2.

RECALL $m_n = \frac{|n|}{R} \Rightarrow m_n \ll M_{(5)}$ FOR $n \sim 1$

$$\Downarrow$$

TRUST KK MODES WITH RELATIVELY LOW n.

LESSON #3

Interaction of "our" field $\hat{\varphi}_0$ with KK modes $\hat{\varphi}_n$

Also comes from

$$S_{int} = \int d^4x \int_0^{2\pi R} dy\, \lambda^{(5)} \phi^4$$

Decompose:

$$\phi^4 = \left\{ \left(\hat{\varphi}_0 + \sum_{n\neq 0} \hat{\varphi}_n e^{in\frac{y}{R}} \right) \frac{1}{(2\pi R)^2} \right\}^4 \quad \leftarrow \text{zero in } \int dy$$

$$= \frac{1}{(2\pi R)^2} \left[\hat{\varphi}_0^4 + 4\hat{\varphi}_0^3 \sum_{n\neq 0} \hat{\varphi}_n e^{in\frac{y}{R}} + 6\hat{\varphi}_0^2 \sum_{n\neq 0}\sum_{n'\neq 0} \hat{\varphi}_n \hat{\varphi}_{n'} e^{i(n+n')\frac{y}{R}} \right]$$

$$\uparrow \int dy \to \delta_{n+n'}$$

$$S_{int} \Rightarrow \int d^4x \cdot 12\lambda^{(4)} \sum_{n\neq 0} \hat{\varphi}_0^2 \hat{\varphi}_n \hat{\varphi}_{-n}$$

- PAIR PRODUCTION OF KK MODES
  ⇕
  CONSERVATION OF ANGULAR MOMENTUM ALONG y

NOT TRUE IN BRANE WORLD MODELS

- UNIQUE EFFECTIVE 4d coupling $\lambda^{(4)}$

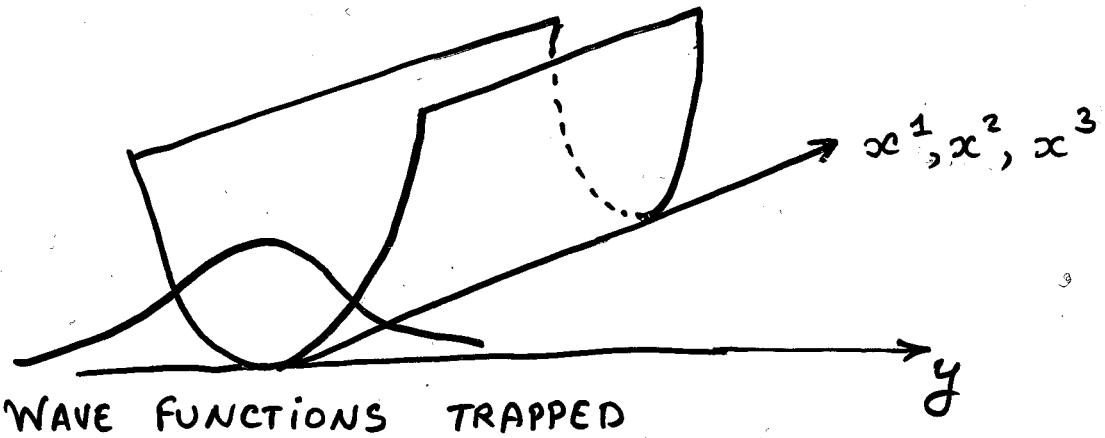
Strong coupling ≡ LARGE CROSS SECTION
AT $E \sim E_{strong}$: MANY CHANNELS OPEN
(RATHER THAN ENHANCEMENT OF INDIVIDUAL CHANNEL)

BRANE WORLD

OUR MATTER LOCALIZED ON 3 dim. HYPERSURFACE (BRANE) EMBEDDED IN HIGHER DIMENSIONAL SPACE.

MOTIVATION FROM STRING THEORY : D-BRANES, ...

Overall picture: POTENTIAL WELL IN EXTRA DIMENSIONS



WAVE FUNCTIONS TRAPPED
EXTRA DIMENSIONS MAY BE OF INFINITE SIZE.

NEED PROPERTIES OF SPECTRUM:

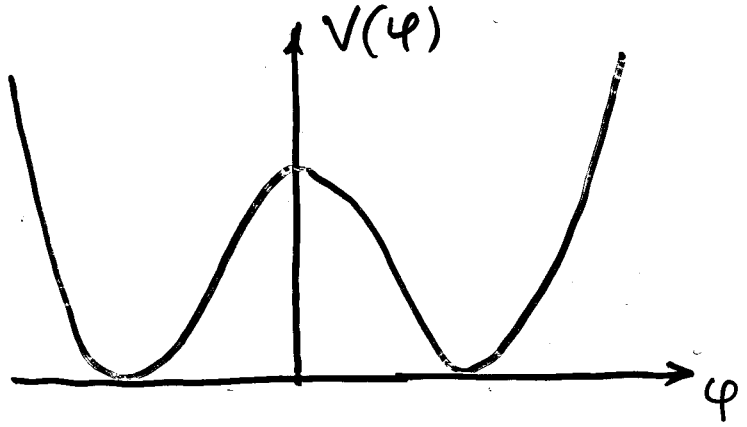
- LOWEST 4 dim. mass $\simeq 0$
- EXCITED STATES HAVE LARGE 4 dim. masses. $\Leftrightarrow$ NARROW WELL

ARE THESE POSSIBLE/NATURAL?

FIELD THEORY (≡ TOY MODEL) EXAMPLES: DEFECTS.

E.G. 5 DIM THEORY OF REAL SCALAR FIELD

$$S = \int d^4x\, dy \left[\frac{1}{2} \partial_A \varphi \cdot \partial^A \varphi - V(\varphi) \right]$$



VACUA: $\varphi = v$

$\varphi = -v$

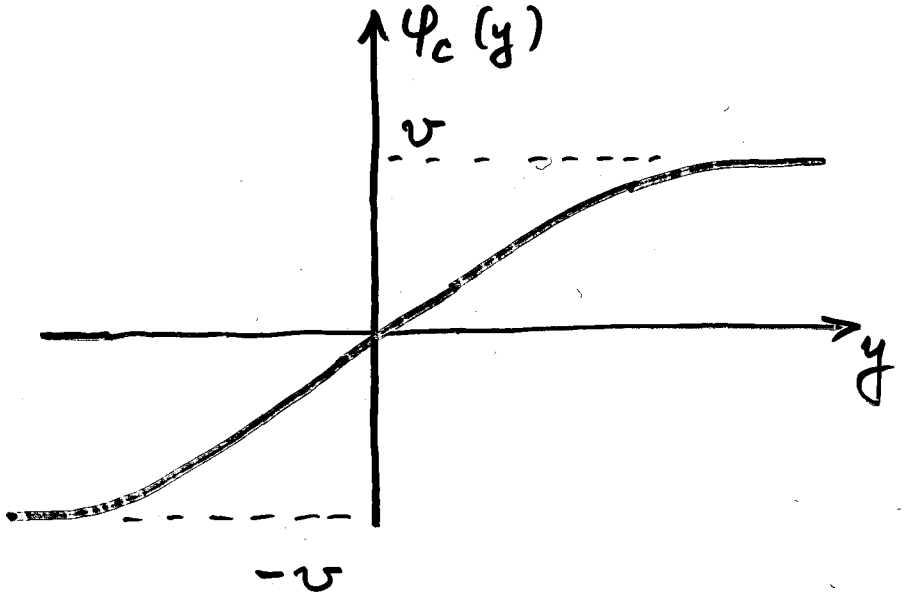
$$V(\varphi) = \lambda^{(5)} (\varphi^2 - v^2)^2$$

FIELD EQUATION: $\partial_A \partial^A \varphi + \frac{\partial V}{\partial \varphi} = 0$

DOMAIN WALL SOLUTION: depends only on y

$$\varphi_c = \varphi_c(y) = v \tanh\left(\sqrt{2\lambda^{(5)}}\, v y\right)$$

kink



BREAKS TRANSLATION ALONG y; DOES NOT BREAK 4D LORENTZ-INV.

SCALAR MODES:

$$\varphi(x,y) = \varphi_c(y) + \delta\varphi(x,y)$$

LINEARIZED FIELD EQUATION

$$\partial_A \partial^A(\delta\varphi) + \frac{\partial^2 V}{\partial\varphi^2}[\varphi=\varphi_c]\cdot\delta\varphi = 0$$

$\Downarrow$ $\qquad\qquad$ $\Uparrow$

$\partial_\mu\partial^\mu - \partial_y^2 \qquad\qquad U(y)$, POTENTIAL WELL

$$\delta\varphi = e^{ip_\mu x^\mu}\,\delta\varphi_m(y)$$

$p_\mu p^\mu \equiv m^2$: 4 dim. mass

$$m^2\,\delta\varphi_m = [-\partial_y^2 + U(y)]\,\delta\varphi_m$$

- Zero mode: $m^2 = 0$

$$\delta\varphi_0(y) = \varphi_c'(y) \propto \frac{1}{\cosh(\sqrt{2\lambda^{(5)}}\,v y)}$$

Our particles localized on a brane, traveling along the brane with speed of light.

NB: translational zero mode (modulus)

$\Updownarrow$

VIBRATIONS OF THE BRANE.

Higher modes

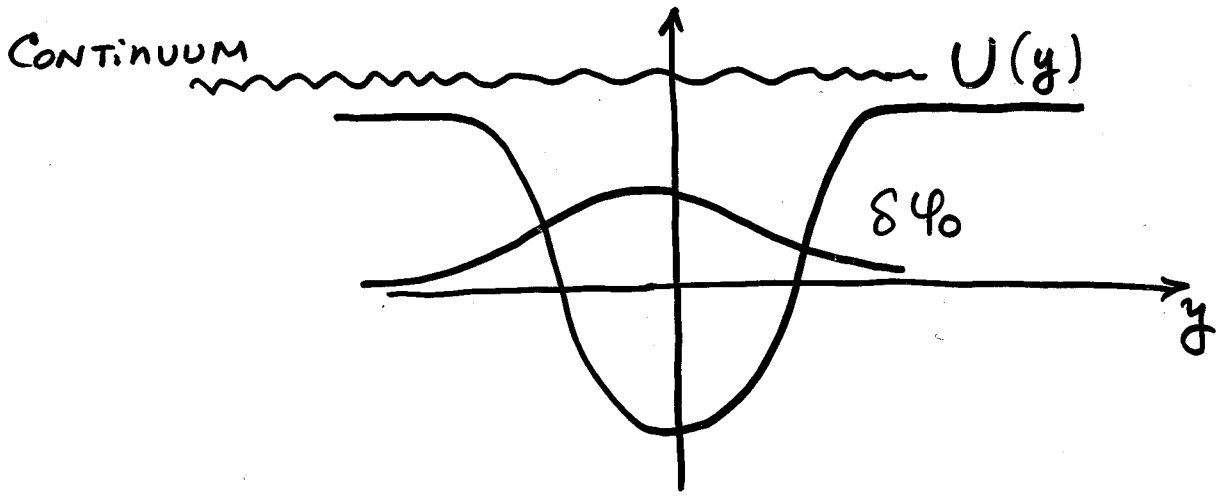
- DISCRETE, $m \sim \sqrt{\lambda^{(5)}} \cdot v$

  "excited electrons", still traveling along the brane

- CONTINUUM: STARTS AT

$$m_{cont} = \sqrt{8\lambda^{(5)}}\, v$$

Particles moving out to $y \to \pm\infty$



Particles escaping THE BRANE $\Leftrightarrow$ missing energy,

$e^+e^- \to$ NOTHING — THRESHOLD EFFECT

$e^+e^- \to \gamma$ + NOTHING (↑ SINGLE PARTICLE) — APPARENT ENERGY NONCONSERVATION POSSIBLE TO EMIT INTO BULK

ARRANGE PARAMETERS: $\sqrt{\lambda^{(5)}}\, v > \text{TeV}$

NO FIRM PREDICTION FOR THRESHOLD ENERGY...

BUT KEEP AN EYE

LOCALIZED FERMIONS

SAME KINK BACKGROUND, BUT INCLUDE 5-DIM. FERMIONS

5-dim. gamma matrices Γ^A,

$$\{\Gamma^A, \Gamma^B\} = \eta^{AB}$$

Minimal choice: 4×4 matrices,

$$\Gamma^\mu = \gamma^\mu \quad \text{(4-dim.)}$$
$$\Gamma^5 = -i\gamma^5 \quad \text{(4-dim.)}$$

Fermion = 4-column, like in 4-dim.

Choose fermion action

$$S_\Psi = \int d^4x\, dy \left(i\bar{\Psi}\Gamma^A \partial_A \Psi - h\varphi\bar{\Psi}\Psi \right)$$

DIRAC EQUATION IN KINK BACKGROUND

$$i\Gamma^A \partial_A \Psi - h\varphi_c(y)\Psi = 0$$

AGAIN, SPECTRUM 4DIM. LORENTZ-INVARIANT

4-dim. MASSLESS MODE:

$$i\gamma^\mu \partial_\mu \Psi_0 = 0$$

$$\Downarrow$$

$$i\Gamma^5 \partial_5 \Psi_0 \equiv \gamma^5 \partial_y \Psi_0 = h\varphi_c(y)\Psi_0$$

UNIQUE SOLUTION

$$\gamma^5 \Psi_0 = \pm \Psi_0$$

$$\Downarrow$$

$$\Psi_0 \propto e^{\pm \int_0^y h \varphi_c(y')\, dy'}$$

$$\hookrightarrow e^{\pm h v |y|} \quad \text{as } |y| \to \infty$$

NORMALIZABLE ZERO MODE: LEFT-HANDED

$$\gamma^5 \Psi_0 = - \Psi_0$$

$$\Psi_0(x,y) = e^{- \int_0^y h \varphi_c(y')\, dy'} \Psi_L(x)$$

↑ MASSLESS 4-DIM. FERMION

CHIRAL (BONUS)

TRAVELS ALONG BRANE.

NO ACCIDENT:

INDEX THEOREMS $\Longrightarrow$ CHIRAL FERMION ZERO MODES IN BACKGROUNDS OF TOPOLOGICAL DEFECTS.

SCALARS AND FERMIONS ARE EASY TO LOCALIZE

MORE THAN ONE EXTRA DIMENSIONS

- ABRIKOSOV-NIELSEN-OLESEN STRING (COSMIC STRING) IN (5+1) dim.
- 't Hooft-Polyakov monopole IN (6+1)-dim
- NONCOMMUTATIVE SOLITONS, D-BRANES

LOCALIZING GAUGE FIELDS: NOT AS STRAIGHTFORWARD.

Suppose THERE WERE ZERO MODE of gauge field, $A^{(0)}(y)$

4-dim. EFFECTIVE INTERACTION WOULD BE PROPORTIONAL TO OVERLAP

$$\int dy\ \Psi_0^+(y)\, A_0(y)\, \Psi_0(y)$$

Generally, depends on shape of $\Psi_0(y)$

⇓

CHARGE UNIVERSALITY NOT GUARANTEED

<u>But</u> NON-ABELIAN GAUGE THEORIES MUST OBEY CHARGE UNIVERSALITY!

WAYS OUT:

A. ZERO MODES ALL HAVE SAME SHAPES

- NONCOMMUTATIVE SOLITONS

⇕

D-BRANES WITHIN D-BRANES

B. $A^{(0)}$ INDEPENDENT OF y

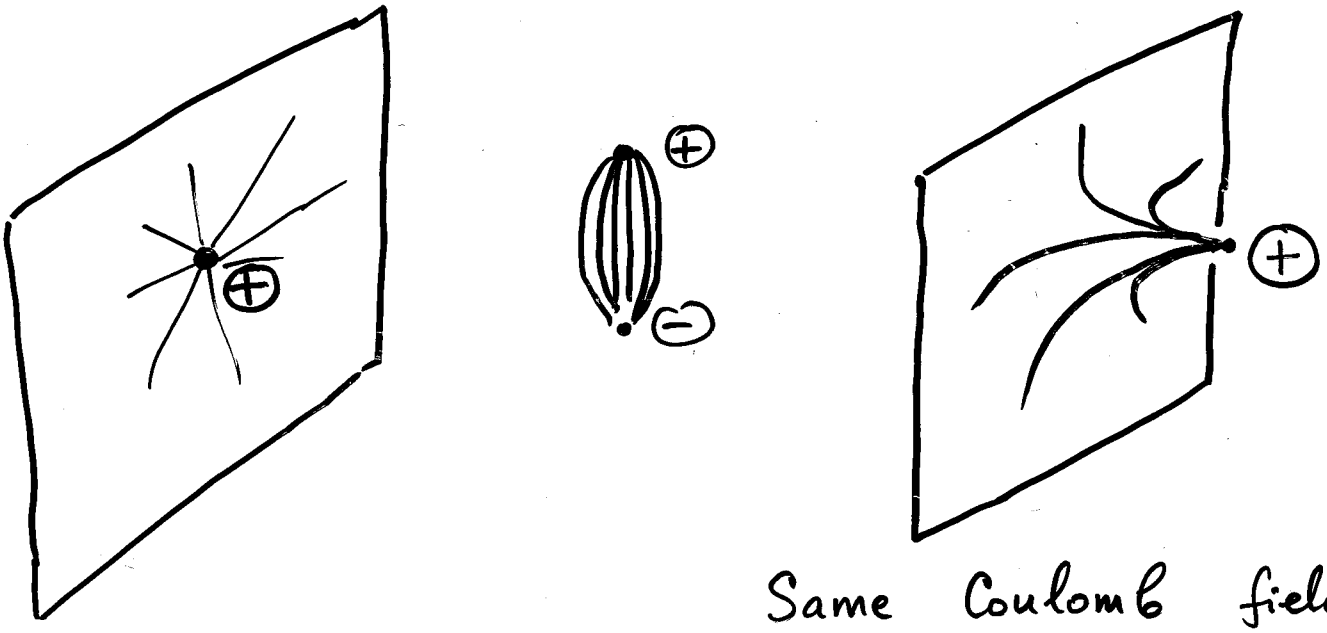
⇓

$$\text{OVERLAP} = \int dy\ \Psi_0^+ \Psi_0 = 1$$

- WARPED BULK, $A^{(0)} = \text{const}$ NORMALIZABLE

ℂ NO GAUGE FIELDS IN BULK AT ALL

- Confinement in bulk, no confinement on brane



Same Coulomb field on brane, irrespectively of position of charge in bulk.

⇓

charge universality.

PHENOMENOLOGY DIFFERENT:

A, ℂ: CHARGE CONSERVATION ON BRANE

B: (APPARENT) CHARGE NON-CONSERVATION FOR BRANE-BASED OBSERVER.

$e^- \longrightarrow$ NOTHING.

𝔻 GAUGE FIELDS LIVE IN BULK.
BULK IS COMPACT. BACK TO KALUZA-KLEIN.

To summarize:

It is not excluded that our matter fields are bound to 3-brane in extra dimensions.

- Apparent energy non-conservation (until now: above some threshold)
  Missing energy (into bulk).
  Possibly no suppression compared to conventional processes
- Apparent non-conservation of electric charge

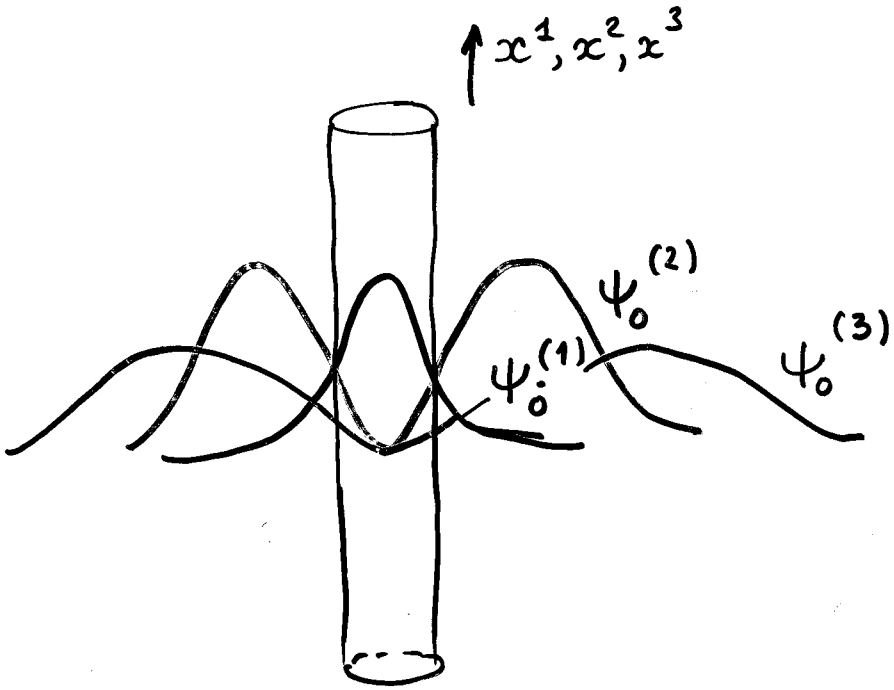
Lots of novel possibilities for model building.

Example: three 4-dim. fermion generations out of a single fundamental (higher-dimensional) generation

Three types of zero fermion modes in background of a single defect.

E.G.: FLUX-3 VORTEX (COSMIC STRING) IN 6-DIM'S.

THREE ZERO MODES (BY INDEX THEOREM) FOR FERMIONS



DIFFERENT SHAPES OF ZERO MODES.

DIFFERENT ANGULAR DEPENDENCE IN TRANSVERSE SPACE

4 DIM'S

$$\psi_0^{(1)} = f^{(1)}(\rho) \qquad \text{1ST GENERATION}$$

$$\psi_0^{(2)} = e^{i\theta} f^{(2)}(\rho) \qquad \text{2ND}$$

$$\psi_0^{(3)} = e^{2i\theta} f^{(3)}(\rho) \qquad \text{3D}$$

SMALL MIXING BETWEEN GENERATIONS

$\Updownarrow$ IN TRANSVERSE SPACE

(APPROXIMATE) SYMMETRY UNDER ROTATIONS, CONSERVATION OF ANGULAR MOMENTUM

- DIFFERENT MASSES OF FERMIONS

$\Updownarrow$

DIFFERENT RADIAL SHAPES OF ZERO MODES
$\Rightarrow$ DIFFERENT OVERLAPS WITH HIGGS MODE LOCALIZED ON VORTEX

- K-K COPIES OF W, Z:

$$\tilde{W}, \tilde{Z} \propto e^{-i\theta} W(\rho)$$

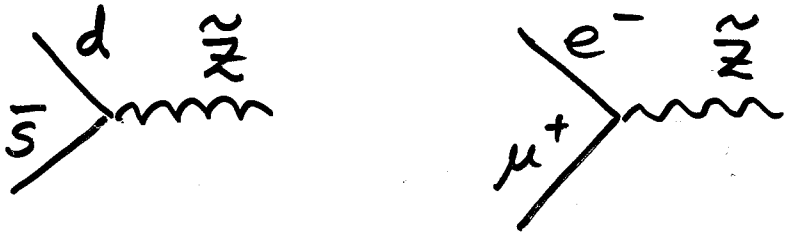
$\Downarrow$

Dominant interactions: angular momentum conserved

$$\bar{\psi}^{(1)} \tilde{W}, \tilde{Z} \psi^{(2)} \leftarrow e^{i\theta} f^{(2)}(\rho) \quad , \quad \bar{\psi}^{(2)} \tilde{W}, \tilde{Z} \psi^{(3)}$$

$$\uparrow f^{(1)}(\rho) \qquad \uparrow e^{-i\theta} W(\rho)$$

FCNC, LEPTON # VIOLATION



ANGULAR MOMENTUM (GENERATION #) CONSERVED (TO LEADING ORDER)

$\Downarrow$

$K_L \to \mu e$ LARGEST

RARE DECAYS SENSITIVE TO EXTRA DIMENSIONS

MANY MORE possibilities

FOR MODEL BUILDING, FANTASY...

A NOTE ON SUPERSYMMETRY:

IN SUSY THEORIES WITH EXTRA DIMENSIONS, BRANE MAY RESPECT 4-dim. SUSY (BPS-BRANE) OR NOT RESPECT (non-BPS BRANE).

UNTIL NOW: NO GRAVITY

GRAVITY PHENOMENOLOGICALLY INTERESTING FOR HIGH ENERGY PHYSICS, IFF FUNDAMENTAL GRAVITY SCALE IS LOW.

RECALL: 4-dim. couplings are *effective*. THEY ARE RELATED TO FUNDAMENTAL couplings IN A PECULIAR WAY THAT INVOLVES SIZE OF EXTRA DIMENSIONS.

4-dim's: GAUGE HIERARCHY PROBLEM; WHY $M_{WEAK} \ll M_{Planck}$?

EXCELLENT SOLUTION: GRAND UNIFICATION + SUSY.

EXTRA DIM'S: POSSIBLE TWIST

UNIFICATION OF couplings, GUTs PROBLEMATIC $\longrightarrow$ $M_{GRAVITY} \sim M_{WEAK}$

WHILE M_{Pl} = EFFECTIVE 4dim. parameter

IF SO, EXPECT QUANTUM GRAVITY EFFECTS AT $E \sim$ TeV

IS THIS SCENARIO POSSIBLE AT ALL?

WHAT IS THE PRICE TO PAY?

WHAT ARE POSSIBLE SIGNATURES?

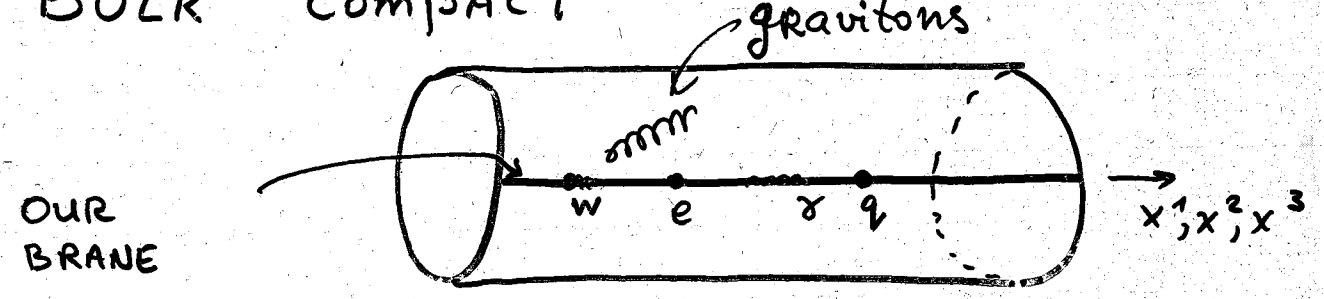
NEED CONCRETE EXAMPLES.

EXAMPLE #1: LARGE EXTRA DIMENSIONS, ADD.

Matter on brane, gravity in Bulk

BRANE WITHOUT TENSION (MASS) not so great!

BULK COMPACT



KALUZA - KLEIN PICTURE FOR GRAVITONS.
THEY HAVE 4-dim. masses

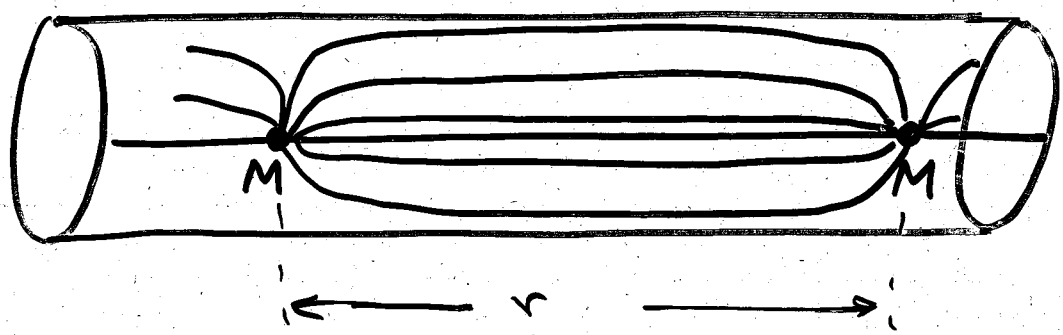
$$m^2_{\{n\}} = \frac{\sum_{i=1}^{N} n_i^2}{(2\pi R)^{\blacksquare 2}}$$

N EXTRA DIM'S; ALL OF SAME SIZE R for simplicity.

Long-distance 4-dim. gravity due to graviton zero mode

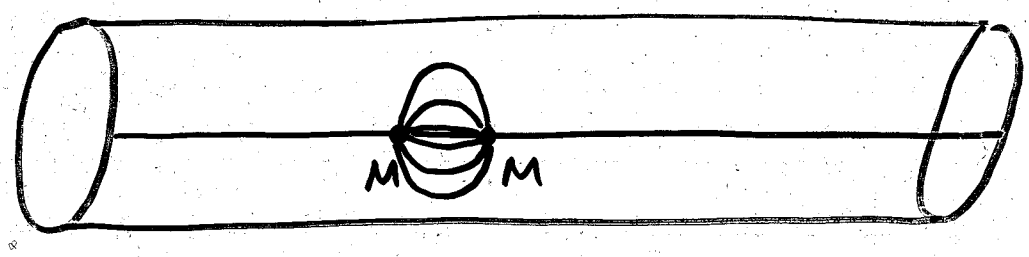
$$n_1 = n_2 = \ldots = n_N = 0 \iff m^2 = 0$$

GRAVITY DEVIATES FROM 4-dim. Newton's law AT $r \lesssim R$:



4-dim. PICTURE OF LINES OF FORCE FOR $r \gg R$

$V(r) \propto \frac{1}{r}$



$D \equiv (4+N)$ -dim. PICTURE FOR $r \ll R$; $V(r) \propto \frac{1}{r^{N+1}}$

(more detail later)

EFFECTIVE 4-dim. Planck mass:

FUNDAMENTAL ACTION FOR GRAVITY

$$S_G = \frac{1}{16\pi G_D} \int R^{(D)} \sqrt{g^{(D)}}\, d^4x\, d^N y$$

LARGE DISTANCES: ONLY HOMOGENEOUS MODE RELEVANT

$$\Rightarrow \quad g_{\mu\nu} = g_{\mu\nu}(x)$$

Plug THIS into S_G $\Rightarrow$ OBTAIN effective 4-dim. action relevant at $r \gg R$

$$S_{eff} = \frac{V_N}{16\pi G_D} \int \sqrt{g^{(4)}}\, R^{(4)}\, d^4x$$

$$\boxed{G_4 = \frac{G_D}{V_N}} \quad ; \quad V_N = (2\pi R)^N \quad \text{if all sizes are equal.}$$

$$G_4 = \frac{1}{M_{Pl}^2} \qquad G_D = \frac{1}{M^{2+N}}$$

↑ FUNDAMENTAL GRAVITY SCALE

N.B: $\frac{G_4}{r} \sim \frac{G_D}{r^{N+1}}$ @ $r \sim R$

$$\boxed{M_{Pl} = M \cdot (MR)^{N/2}}$$

NEGLECTING 2's AND π's

CHOOSE $M \sim$ TeV $\Rightarrow$ 4-dim. gravity weak

BECAUSE VOLUME OF EXTRA DIMENSIONS LARGE.

HOW LARGE IS R?

$$R \sim \frac{1}{M}\left(\frac{M_{Pl}}{M}\right)^{\frac{2}{N}} \sim 10^{-17}\,\text{cm} \cdot 10^{\frac{32}{N}}$$

$M_{Pl} = 10^{19}$ GeV

$M = 10^3$ GeV

- $N=1 \Rightarrow R \sim 10^{15}$ cm : UNACCEPTABLE
- $N=2 \Rightarrow R \sim$ mm Interesting:

NEWTON'S LAW CHECKED DOWN TO 0.2 mm

MOTIVATION for measuring gravity at shorter distances.

GRAIN OF SOLT: ASTROPHYSICS at $N=2 \Rightarrow M > 30$ TeV,

$R \lesssim 10^{-4}$ cm $\simeq \mu$m Very difficult, BUT NOT impossible

- $N=3 \Rightarrow R \sim 10^{-6}$ cm

  Hardly possible to reach in gravity measurements

IN ANY CASE, R QUITE LARGE

E.g. $N=6$ $(D=10) \Rightarrow R \sim 10^{-12}$ cm $\sim (10\ \text{MeV})^{-1}$

NB: IF EXTRA DIMENSIONS HAVE UNEQUAL SIZES $R_1, \dots, R_N$, THEN

$$V_N = \frac{1}{M^N} \frac{M_{Pl}^2}{M^2}$$

$$\parallel$$

$$R_1 \cdot R_2 \cdot \dots \cdot R_N$$

R_1 MAY BE LARGE, $R_2, \dots, R_N$ SMALL

$\Downarrow$

DEVIATION FROM 4Dim NEWTON'S LAW AT $r \sim R_1$, POSSIBLY $r \sim$ mm

ASTROPHYSICS/COSMOLOGY DOES NOT FORBID.

TWIST IN HIERARCHY PROBLEM:
WHY R IS SO LARGE?

NO COMPELLING ANSWER.

NEW PHENOMENOLOGICALLY INTERESTING PARTICES: KALUZA-KLEIN GRAVITONS.

MASS SPLITTING $\sim \frac{1}{R}$

VERY SMALL: $N=2 \Rightarrow \frac{1}{R} \sim \frac{1}{mm} \sim 10^{-4}\,eV \sim {}^{\circ}K$

$\vdots$

$N=6 \Rightarrow \frac{1}{R} \sim 10\,MeV$

$\mathcal{N}(E)$: NUMBER OF GRAVITONS WITH $m < E$

$$m^2 = \frac{\sum_{i=1}^{N} n_i^2}{R^2} < E^2$$

$\mathcal{N}(E)$ = # OF POINTS WITH INTEGER COORDINATES $n_1, \ldots, n_N$ (IN N DIM'S) INSIDE A SPHERE OF RADIUS (RE)

$= (ER)^N \times$ (VOLUME OF UNIT N-dim. BALL).

$$\boxed{\mathcal{N}(E) = (E \cdot R)^N}$$

NEGLECTING 2'S AND π'S

LARGE # OF K-K GRAVITON SPIECES.

E.G. $E = GeV$, $M = TeV$, $N=4 \Rightarrow \mathcal{N}(E) \sim 10^{20}$

NB: $R^N = \frac{1}{M^N} \cdot \frac{M_{Pe}^2}{M^2} \Rightarrow \mathcal{N}(E) = \left(\frac{E}{M}\right)^N \cdot \frac{M_{Pe}^2}{M^2}$

THE LARGER N, THE SMALLER $\mathcal{N}(E)$ FOR $E \ll M$

INTERACTION WITH MATTER

FUNDAMENTAL ACTION

Neglect brane thickness ↓

$$S = \frac{1}{16\pi G_D}\int R^{(D)}\sqrt{g^{(D)}}\,d^4x\,d^Ny + \int \mathcal{L}_{MATTER}\left(\begin{matrix}\text{matter}\\ \text{fields}\end{matrix}, g_{\mu\nu}\right) d^4x\,\delta^N(y)\,d^Ny$$

$g_{\mu\nu}$: metric on BRANE

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}(x,y)$$

$$h(x,y) = \sum_{\{\vec{n}\}} e^{i\vec{n}\vec{y}/R}\, h_n(x)$$

OMITTING INDICES, IGNORING TENSOR STRUCTURE

$$\Downarrow$$

$$S_{eff} = \frac{V_N}{16\pi G_D}\sum_{\{\vec{n}\}}\int d^4x\left[\partial_\mu h_n(x)\right]^2 + \text{higher order terms}$$

$$+ \sum_{\{n\}}\int d^4x\, T^{\mu\nu} h_{n,\mu\nu}$$

$$\frac{1}{16\pi G_4} = M_{Pl}^2$$

Canonically normalize: $h_n = \frac{1}{M_{Pl}}\hat{h}_n$

$$\Downarrow$$

$$S_{eff} = \sum_{\{n\}}\int d^4x\left(\partial_\mu \hat{h}_n\right)^2 + \frac{1}{M_{Pl}}\int d^4x\, T_{\mu\nu}\hat{h}_n^{\mu\nu}$$

EACH K-K graviton interacts with OUR matter at 4-dim. gravitational strength (i.e. suppressed by $\frac{1}{M_{Pl}}$). SINGLE K-K graviton PRODUCTION POSSIBLE

Q: WHY GRAVITY IS NOT M_{Pl} SUPPRESSED AT $r \ll R$?

POTENTIAL BETWEEN TWO UNIT MASSES:

$$V(r) = -G_4 \sum_{\{n\}} \frac{e^{-m_{\{n\}} \cdot r}}{r} \qquad m_{\{n\}} = \frac{1}{R}\sqrt{\vec{n}^2}$$

$r \ll R$: MANY KK GRAVITONS CONTRIBUTE

$$\Downarrow$$

$$V(r) = -G_4 \int d^N n \, \frac{e^{-\frac{r}{R}\sqrt{\vec{n}^2}}}{r}$$

$$= -S_{N-1}\, G_4 \int n^{N-1} dn \, \frac{e^{-\frac{r}{R} n}}{r}$$

area of $(N-1)$-dim. sphere $\Rightarrow$ 2's and π's

$$= -G_4 \cdot \frac{1}{r} \cdot \left(\frac{R}{r}\right)^N$$

CRUCIAL FACTOR: NUMBER OF K-K graviton SPIECES WITH $m \lesssim \frac{1}{r}$

$$G_D = G_4 \cdot R^N$$

$$V(r) = -G_D \frac{1}{r^{N+1}}, \qquad D = (N+4)\text{-dim.}$$

NEWTON'S LAW

A: BECAUSE THERE ARE PLENTY OF GRAVITON SPIECES CONTRIBUTING.

SUBTLETY OF K-K DECOMPOSITION

ORIGINAL FIELDS:

h_{AB} , $A,B = 0,1,\ldots D-1$ TAKE $D=(N+1)$ VALUES

OF THESE:

$h_{\mu\nu}$ are 4-dim. tensors $\mu = 0,1,2,3$
(K-K gravitons proper)

$h_{\mu i}$ are 4-vectors $i = 4,\ldots, N+3$
Do not interact with our matter

h_{ij} are SCALARS
One at each KK LEVEL
INTERACTS WITH T^{μ}_{μ} at gravitational strength.

Dangerous: $\vec{n}=0 \Rightarrow$ MASSLESS SCALAR $\equiv$ BRANS-DICKE FIELD.
INTERACTS WITH T^{μ}_{μ}, SPOILS BENDING OF LIGHT.

THIS IS MODULUS CORRESPONDING TO VOLUME OF EXTRA DIMENSIONS.

$\Downarrow$

NEED MECHANISM TO FIX (STABILIZE) SIZE OF EXTRA DIMENSIONS

THERE ARE MANY MECHANISMS. SCALARS OFTEN LIGHT, $m_{min} \sim \frac{1}{R}$; K-K spectrum UNCHANGED AT $n \gg 1$.
We'll DISCUSS POSSIBLE MANIFESTATIONS LATER.

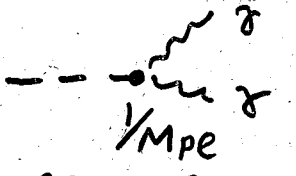
BACK TO K-K gravitons

- DECAY TO OUR MATTER,
  E.G. $G \to \gamma\gamma$, $G \to e^+e^-$, etc.

  SUPPRESSED BY M_{Pl}^{-1}:

$$\Gamma \sim \frac{1}{M_{Pl}^2} M_G^3$$

↑ ON DIMENSIONAL GROUNDS



Even for $M_G \sim$ TeV LIFETIME AGAINST DECAY INTO OUR MATTER LARGE

$$\tau \sim \frac{1}{M_G}\left(\frac{M_{Pl}}{M_G}\right)^2 \sim 10^{-27}\,s \cdot 10^{32} \sim 10^5\,s$$

MUCH LONGER FOR LIGHTER GRAVITONS.

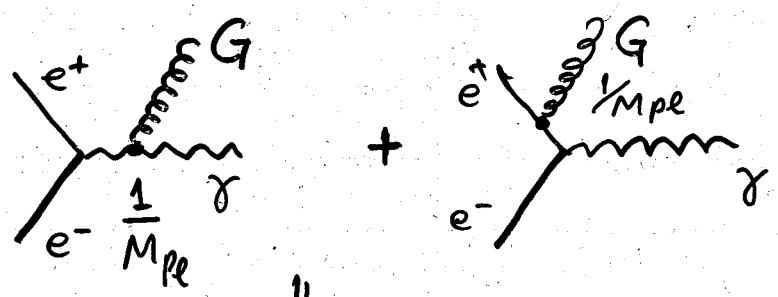
- DECAY OF K-K gravitons into BULK/OTHER BRANES MATTER (if ANY) AND K-K gravitons THEMSELVES IS FASTER, MODEL DEPENDENT. THIS IS INVISIBLE MODE ANYWAY.
- INTERACTION OF K-K GRAVITONS WITH OUR MATTER IS VERY WEAK, SUPPRESSED BY M_{Pl}^{-1}.

⇓

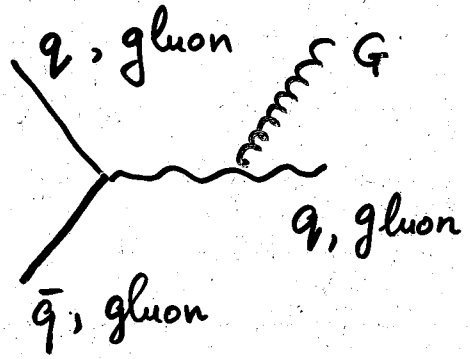
K-K GRAVITONS ARE INVISIBLE, DO NOT LEAVE TRACKS in DETECTORS

COLLIDERS:

K-K GRAVITON = MISSING ENERGY



$$e^+e^- \to \gamma + \not{E}$$



: $pp \to \text{jet} + \not{E}_T$

- EFFECTIVELY CONTINUOUS SPECTRUM of KK gravitons (mass splitting less than MeV)

⇓

BOTH ENERGY AND ANGLE OF PHOTON (jet) ARBITRARY

- MORE HEAVY GRAVITONS THAN LIGHT

⇓

LOWER ENERGY PHOTONS (jETS) ENHANCED.

CROSS SECTIONS RAPIDLY GROW WITH ENERGY.

TWO WAYS TO CALCULATE CROSS SECTION

1: 4-dim. viewpoint

EACH TYPE OF K-K gravitons created at 4-dim. gravitational strength,

$$\sigma \sim \frac{\alpha}{M_{Pl}^2} \leftarrow \text{gauge coupling}$$

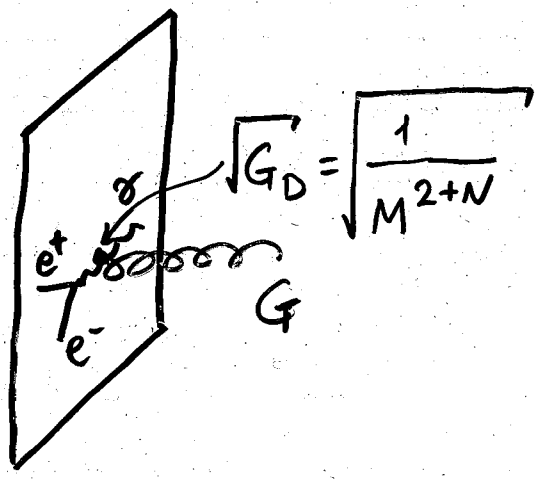
BUT ALL GRAVITONS WITH $m < E$ CAN BE EMITTED:

$$\sigma_{tot} \sim \frac{\alpha}{M_{Pl}^2} N(E) \sim \frac{\alpha}{M_{Pl}^2} (RE)^N = \boxed{\frac{\alpha}{M^{2+N}} E^N}$$

↑ TeV scale!

- Large (saturates unitarity) at $E \sim M$ (cf: D-dim. gravity strongly coupled at $E \sim M$). TRUST RESULT AT $E < M$
- D-DIM. BEHAVIOR

2: D-Dim. viewpoint. EXTRA Dim's EFFECTIVELY INFINITELY LARGE



$$\sigma_{tot} \propto \frac{\alpha}{M^{2+N}} E^N$$

LHC: $pp \to \text{jet} + \not{E}_T$

BACKGROUND $pp \to \text{jet} + Z$, $Z \to \nu\bar{\nu}$

Fig.

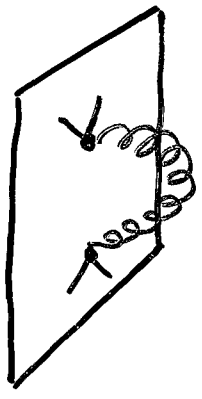
Sensitivity of LHC: $M = 8 \longrightarrow 4.5$ TeV

FOR $N = 2 \longrightarrow 6$

—— o ——

CONTACT INTERACTIONS $\Longleftrightarrow$ VIRTUAL GRAVITON EXCHANGE.

D-Dim. CALCULATION



$$\mathcal{L}_{eff} = \frac{1}{M^{N+2}} T_{\mu\nu}(p)\, T^{\mu\nu}(-p) \times \int_0^\infty \frac{d^N q_y}{p^2 + q_y^2}$$

ULTRAVIOLET DIVERGENT!

Need UV completion (UV behavior of FUNDAMENTAL THEORY) TO CALCULATE.

CAN ONLY GUESS...

Generally:

$$\left(T_{\mu\nu} T^{\mu\nu} - \frac{1}{N+2} T_\lambda^\lambda T_\mu^\mu \right) \cdot \frac{1}{\Lambda^4}$$

$\Lambda \sim M$? OR SOMEWHAT LOWER? OR SOMEWHAT LARGER?

– HARDLY

INTERESTING AT LHC: Standard story ...

$pp \to \gamma\gamma$, LARGE $E_{T,\gamma}$, $M_{\gamma\gamma}$

SENSITIVITY SIMILAR TO DIRECT PRODUCTION

(32a

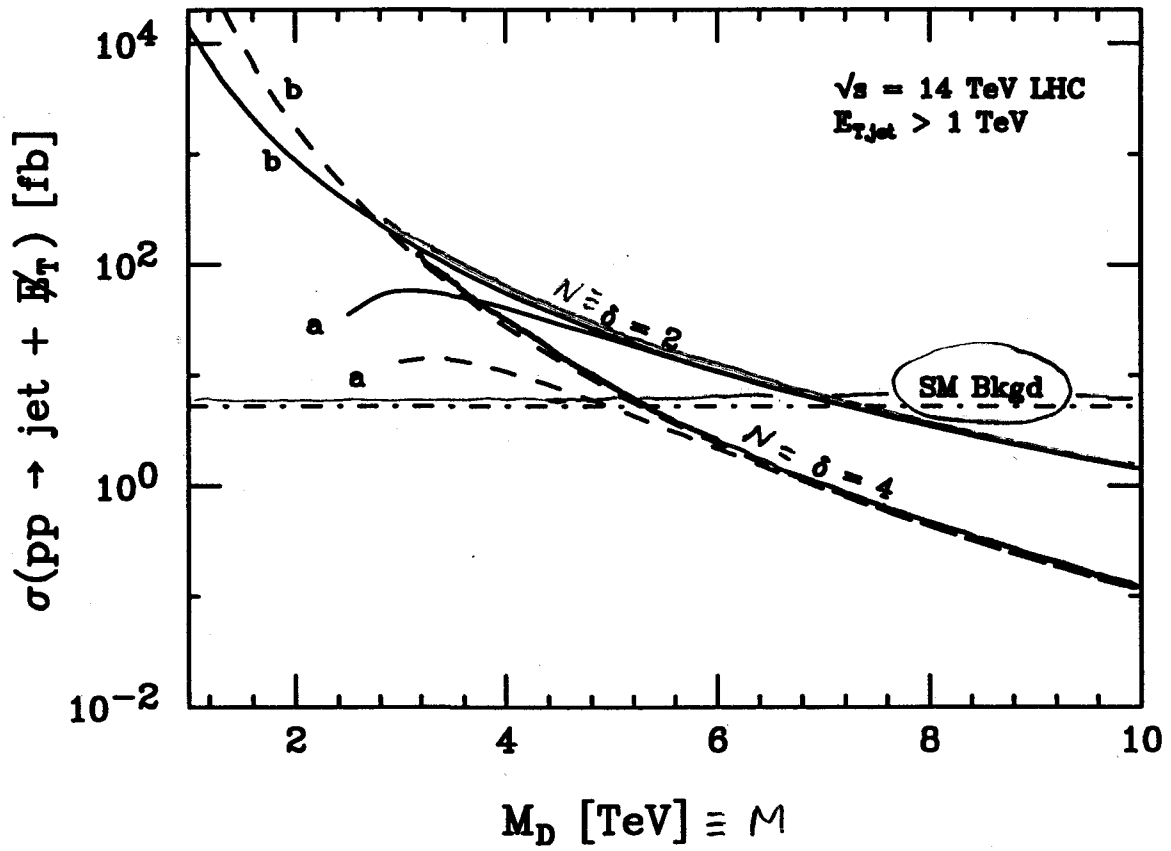


Figure 4: The total jet + nothing cross-section versus M_D at the LHC integrated for all $E_{T,\text{jet}} > 1$ TeV with the requirement that $|\eta_{\text{jet}}| < 3.0$. The Standard Model background is the dash-dotted line, and the signal is plotted as solid and dashed lines for $\delta = 2$ and 4 extra dimensions. The **a** (**b**) lines are constructed by integrating the cross-section over $\hat{s} < M_D^2$ (all $\hat{s}$).

GIUDICE, RATTAZZI, WELLS

GRAVISCALARS:

Higgs - graviscalar mixing

EXAMPLE OF DECAY INTO BULK.

4 dim's: SCALAR PART OF METRIC PERTURBATION h^{λ}_{λ} NOT DYNAMICAL

NO LONGER TRUE IN EXTRA dim's.

OUR HIGGS + GRAVITY: ALLOWED TERM

$$\int d^4x \sqrt{g^{(4)}} \cdot \xi R^{(4)} \cdot \varphi^+\varphi$$

ξ: free dimensionless parameter; φ: our Higgs field

Lowest order:

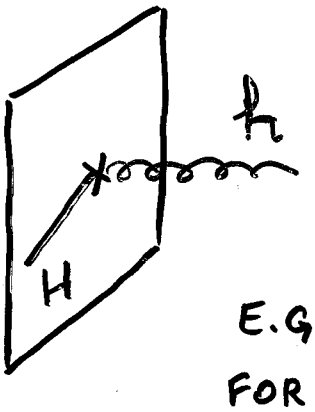
$$R^{(4)} = \Box^{(4)} h^{\lambda}_{\lambda} + \dots$$

$$\varphi = \begin{pmatrix} 0 \\ v+H \end{pmatrix}$$

$$\Downarrow$$

MIXING $\quad \xi v \partial_\mu h \cdot \partial_\mu H$

Higgs converts into scalar graviton and escapes FROM OUR BRANE $\Rightarrow$ INVISIBLE Higgs DECAY



$$\Gamma \sim \xi^2 \frac{v^2}{M^{2+N}} \cdot m_H^{1+N}$$

NOT SO SMALL for $M \sim$ TeV

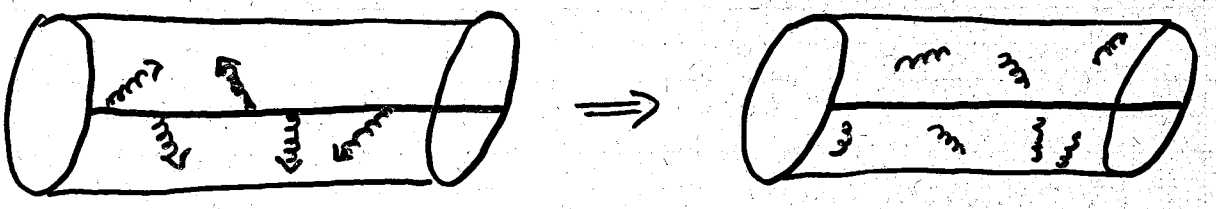
E.G. $N=4$, $Br(H \to h) \gtrsim 10\%$

FOR $m_H < 160$ GeV (BELOW WW THRESHOLD).

ADD: COSMOLOGY AND ASTROPHYSICS

OFTEN MAKES THINGS PROBLEMATIC (if not kills).

- COSMOLOGY: K-K gravitons (≡ BULK GRAVITONS) PRODUCED IN EARLY UNIVERSE. SHOULD NOT DOMINATE NUCLEOSYNTHESIS EPOCH (expansion AT)



PRODUCTION RATE/UNIT VOLUME AT TEMPERATURE T:

$$\frac{dn_{grav}}{dt} \sim \frac{1}{M^{N+2}} T^{N+6}$$

$$\Downarrow$$

PRODUCTION OF ENERGY DENSITY IN BULK GRAVITONS

$$\frac{d\rho_{grav}}{dt} \sim \frac{1}{M^{N+2}} T^{N+7}$$

HUBBLE EXPANSION RATE (RADIATION DOMINATED) $H(T) = \frac{T^2}{M_{Pl}^*}$ $M_{Pl}^* \sim 10^{18}$ GeV

ENERGY TRANSFERRED INTO BULK GRAVITONS IN HUBBLE TIME:

$$H^{-1}\frac{d\rho_{grav}}{dt} \sim \frac{M_{Pl}^{*}}{M^{N+2}} T^{N+5}$$

Let T_{max} = maximum temperature of Universe (e.g. reheat temperature after inflation)

Maximum production at $T \sim T_{max}$.

$$\rho_{grav}(T_{max}) \sim \frac{M_{Pl}^{*}}{M^{N+2}} T_{max}^{N+5}$$

M_{Pl} enhanced: slow expansion

$$H \sim \frac{1}{t_{universe}} \sim \frac{T^2}{M_{Pl}^{*}}$$

Then ρ_{grav} decreases due to expansion of Universe

At BEST AS T^4 (if all gravitons decay into something massless. If not, $\rho \propto T^3 \Rightarrow$ stronger limits)

$\Downarrow$

AT NUCLEOSYNTHESIS ($T_{NS} \sim 1$ MeV)

$$\rho_{GRAV}(T_{NS}) \sim \frac{M_{Pl}^{*}}{M} \cdot \left(\frac{T_{max}}{M}\right)^{N+1} \cdot T_{NS}^{4}$$

MUST BE SMALLER THAN T_{NS}^4 (ENERGY DENSITY OF STANDARD PARTICLE

$\Downarrow$

$$\frac{M_{Pl}^{*}}{M} \cdot \left(\frac{T_{max}}{M}\right)^{N+1} < 1$$

BOUND ON T_{max}:

$$T_{max} < M \left(\frac{M}{M_{Pl}^*} \right)^{\frac{1}{N+1}}$$

$M \sim 10^3$ GeV, $M_{Pl}^* \sim 10^{-15}$

$N = 2 \Rightarrow T_{max} \lesssim 10$ MeV

$\vdots$

$N = 6 \Rightarrow T_{max} \lesssim 10$ GeV

NO DIRECT CONTRADICTION: DATA ON EARLY UNIVERSE FOR $T \sim 1$ MeV ONLY (YET!)

STILL NOT VERY PLAUSIBLE:

- BARYOGENESIS DIFFICULT
- DARK MATTER GENERATION DIFFICULT

NB: IF WE KNEW WHAT ARE DARK MATTER PARTICLES, WE WOULD RULE OUT LOW T_{max}
(STANDARD SCENARIO: DARK MATTER GENERATED AT $T \gtrsim 100$ GeV)

• ASTROPHYSICS

SN 1987a : NEUTRINOS EMITTED AT COLLAPSE

Central core heated to $T = (\text{a few}) \cdot 10$ MeV for time $\Delta t = (\text{a few}) \cdot 10$ s. ENERGY IN ν's

Bulk gravitons SHOULD NOT TAKE AWAY ALL ENERGY (LEAVE GOOD PART TO ν's)

$$\frac{d\rho_{grav}}{dt} \sim \frac{1}{M^{N+2}} T^{N+7}$$

$$\rho_{grav} \sim \left(\frac{T}{M}\right)^{N+2} \cdot (T\Delta t) \cdot T^4 \quad < T^4$$

(T^4: energy density in ν's)

$$\Downarrow$$

$$M > T \cdot (T \cdot \Delta t)^{\frac{1}{N+2}}$$

($T \sim 10$ MeV, $T \cdot \Delta t \sim 10^{23}$)

$N=2 \Rightarrow M > 10$ TeV (MORE ACCURATE $M > 30$ TeV)

NB: $N=3 \Rightarrow M > 1$ TeV, not RESTRICTIVE.

$N=2$: $M \gtrsim 30$ TeV, quite a BIT HIGHER THAN M_{EW}

To summarize:

ADD:
- Interesting collider signatures
- Deviation from Newton's law at relatively large distances
- $N=2$: gravity scale $M \gtrsim 30$ TeV
  Difficult for all experiments
- Cosmology problematic

Does not need SUSY, but not inconsistent with SUSY.

EXAMPLE #2:
WARPED EXTRA DIMENSION, RS

Until now: BRANE WITHOUT ENERGY DENSITY ASSOCIATED TO IT

NOT VERY REALISTIC.

WHAT HAPPENS WHEN BRANE CARRIES ENERGY DENSITY (TENSION)?

Introducing brane tension:

4 DIM'S: VACUUM IN FLAT SPACE-TIME IS LORENTZ-INVARIANT.

ENERGY-MOMENTUM OF VACUUM $\Leftrightarrow$ LORENTZ-INVARIANCE

$$T_{\mu\nu} = \sigma \cdot \eta_{\mu\nu}$$

σ: constant ≡ vacuum energy ≡ cosmological constant

$\eta_{\mu\nu}$: Minkowski tensor

NB: FLUID $\rightarrow T_{\mu\nu} = \begin{pmatrix} \rho & & \\ & p & \\ & & p \\ & & & p \end{pmatrix} \Rightarrow$ VACUUM $p = -\rho$

ρ: energy density; p: pressure

In CURVED SPACE-TIME

$$T_{\mu\nu} = \sigma \cdot g_{\mu\nu}$$ FOR VACUUM.

BRANE IN EXTRA DIMENSIONS

If 4-dim. LORENTZ SYMMETRY IS NOT VIOLATED

$$T_{\mu\nu}(x,y) = \sigma \cdot g_{\mu\nu}(x) \cdot \delta^N(y)$$

- σ: constant: brane tension, energy per unit area
- $g_{\mu\nu}(x)$: METRIC ON BRANE, INDUCED FROM BULK
- $\delta^N(y)$: if brane THICKNESS is negligibly small

BRANE OF FINITE TENSION PRODUCES gravitational field in bulk $\Leftrightarrow$ self-gravitates.

IS THERE INDEED A SOLUTION WITH FLAT (LORENTZ-INVARIANT) BRANE?

5 DIM'S (ONE EXTRA)

- NEED COSMOLOGICAL CONSTANT Λ IN BULK
- Fine-tuning of Λ and σ

• EINSTEIN EQS. IN 5 DIM'S:

$$R^{(5)}_{AB} - \frac{1}{2} g^{(5)}_{AB} R^{(5)} = 8\pi G_5 \left(\Lambda \cdot g^{(5)}_{AB} + T^{\text{brane}}_{AB} \right)$$

NON-zero components

$$T_{\mu\nu} = \sigma \cdot g^{(4)}_{\mu\nu} \delta(y)$$

(in COORDINATES WITH brane SITTING AT $y=0$)

4-dim. LORENTZ-INV. SOLUTION EXISTS FOR

$$\Lambda = -\frac{4\pi}{3} G_5 \cdot \sigma^2$$

Fine-tuning, similar to fine-tuning of cosmological constant in 4-dim. THEORIES.

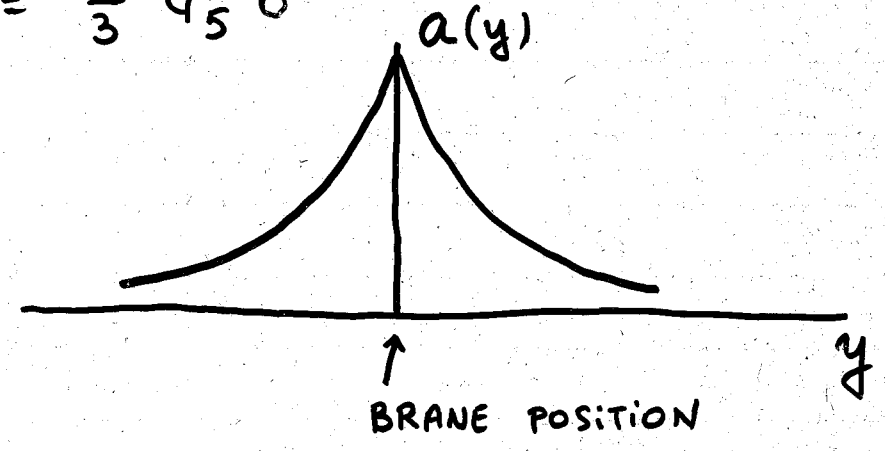
SOLUTION

$$ds^2 = a^2(y) \cdot \eta_{\mu\nu} dx^\mu dx^\nu - dy^2$$

↑ WARP FACTOR

$$a(y) = e^{-k|y|}$$

$$k = \frac{4\pi}{3} G_5 \sigma$$



NB: $\partial_y^2 a(y) \propto \delta(y) \Leftrightarrow T_{\mu\nu}^{\text{brane}} \propto \delta(y)$

NB: BULK METRIC IS PART OF adS_5

k = INVERSE "RADIUS" of adS_5

NB: Δx^μ = PHYSICAL DISTANCE, TIME INTERVAL AT THE BRANE ONLY

AWAY FROM BRANE PHYSICAL DISTANCE

$\Delta \ell^\mu = a(y) \cdot \Delta x^\mu$, smaller than Δx^μ

↑ $e^{-k|y|}$

SOLUTION TO FIELD EQS. IN RS BACKGROUND WILL CONTAIN FACTOR

$$e^{ip_\mu x^\mu}$$

p^μ = physical 4-momentum at brane only. AWAY FROM IT

$$p_\mu^{\text{phys}} = \frac{1}{a(y)} \cdot p_\mu$$ HARDER THAN p_μ.

GRAVITON PERTURBATIONS ABOUT RS BACKGROUND:

WRITE PERTURBED METRIC

Gauge choice ↓

$$ds^2 = [\, a^2(y)\cdot\eta_{\mu\nu} + h_{\mu\nu}(x,y)\,]dx^\mu dx^\nu - dy^2$$

$a^2(y) = e^{-k|y|}$ ↑ gravitational perturbations

PLUG INTO 5D EINSTEIN EQS., LINEARIZE

⇓ CHOOSING GAUGE, OMITTING INDICES IN $h_{\mu\nu}$

$$-a^2(y)\,\partial_y^2 h + \eta^{\mu\nu}\partial_\mu\partial_\nu h + 4k^2a^2 h - 2k\delta(y)\cdot h = 0$$

FROM Einstein tensor

FROM $\Lambda(a^2\eta_{\mu\nu}+h_{\mu\nu})$

FROM $T^{brane}_{\mu\nu} = \sigma\cdot\delta(y)[a^2\eta_{\mu\nu}+h_{\mu\nu}]$

SOLUTIONS: $h = e^{ip_\mu x^\mu} h_m(y)$; $p_\mu p^\mu = m^2$

⇓ Eigenvalue equation

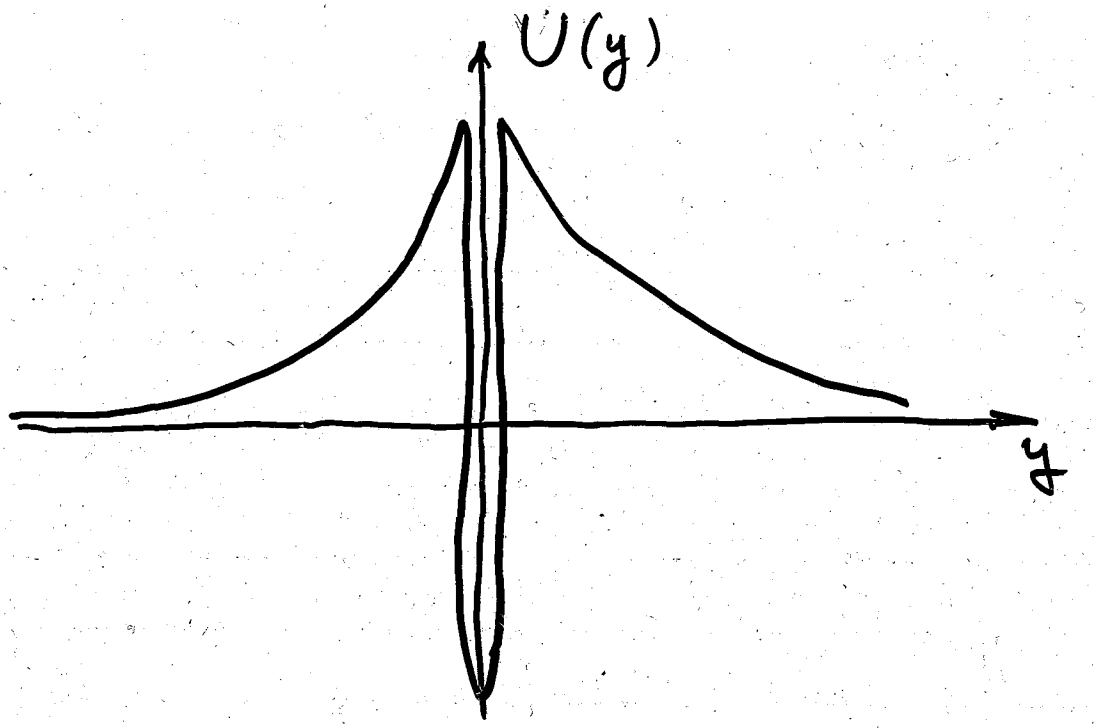
$$-a^2(y)\cdot\partial_y^2 h_m + [4k^2a^2(y) - 2k\delta(y)]h_m = m^2 h_m$$

- NORMALIZATION $\int \frac{1}{a^2}|h_m|^2\,dy$
- UPTO SOMEWHAT UNUSUAL DERIVATIVE TERM

SCHRÖDINGER EQUATION IN POTENTIAL

$$U(y) = 4k^2a^2(y) - 2k\delta(y)$$

$$= 4k^2 e^{-k|y|} - 2k\delta(y)$$



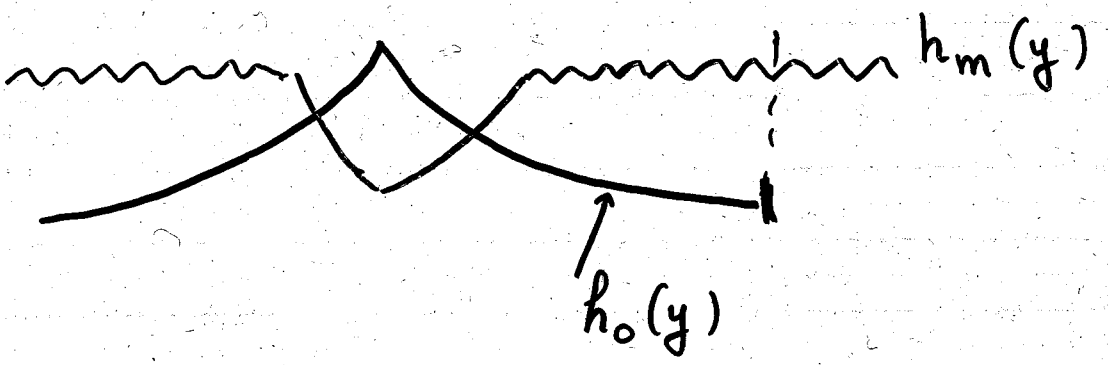
- ZERO MODE: $m = 0$
  MASSLESS 4-DIM. GRAVITON, DESCRIBES OUR GRAVITY.

$$h_0 = a^2(y) = e^{-2k|y|}$$

- OTHER MODES: DAMPED NEAR BRANE AT $m \ll k$
  OSCILLATE AWAY FROM BRANE,

$$h_m \propto \cos\left(\frac{m}{k} e^{ky}\right) \quad , \; y \gg \frac{1}{k} \ln \frac{m}{k}$$

HOW CAN ONE USE THIS SET UP TO CONSTRUCT INTERESTING BRANE-WORLD MODELS?


$h_m(y)$
$h_o(y)$

RS1: COMPACT EXTRA DIMENSIONS, EXPONENTIALLY EASIER TWISTED HIERARCHY PROBLEM.

Put ANOTHER, NEGATIVE TENSION BRANE AT $y = y_c$.

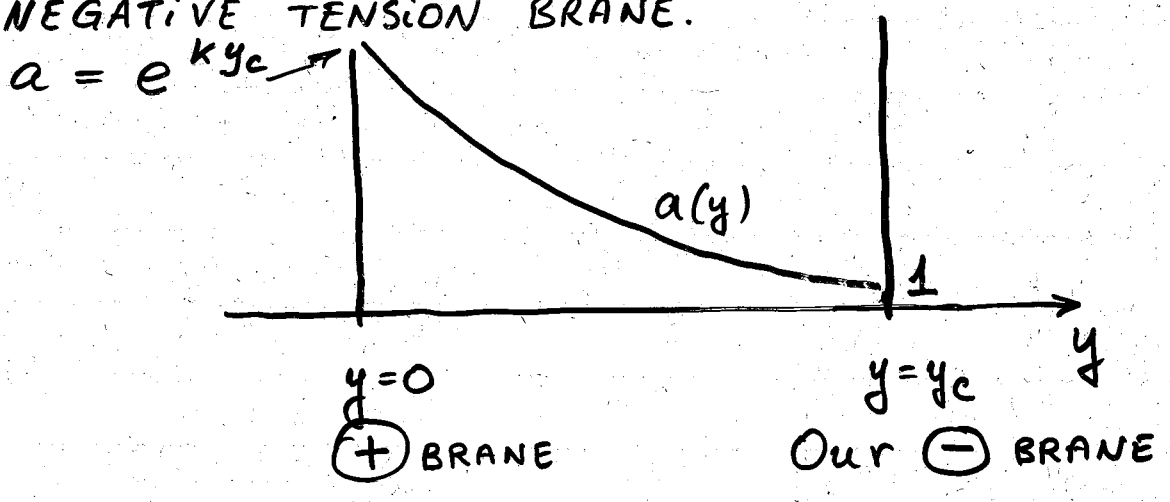
↑ weird, BUT HAS STRING THEORY interpretation

$$ds^2 = a^2(y)\,\eta_{\mu\nu}\,dx^\mu dx^\nu - dy^2$$

$$a(y) = e^{-k(|y| - y_c)}$$

COORDinates x^μ rescaled for convenience, measure physical distances on negative tension BRANE

ASSUME OUR MATTER LIVES ON NEGATIVE TENSION BRANE.



NB: WAVE FUNCTION OF ZERO MODE GRAVITON (CARRYING 4-dim. gravity)

$h_0 \propto a^2(y)$ DECAYS TOWARDS OUR BRANE.

4-dim. Newton constant:

$$S_{eff} = \frac{1}{16\pi G_5} \int_{-y_c}^{y_c} \frac{dy}{a^2(y)} \, d^4x \, (\partial_\mu h)^2$$

$h = a^2(y) \cdot h^{(4)}(x)$

recall measure in normalization of graviton modes

$$= \underbrace{\frac{1}{16\pi G_5} \left[\int_{-y_c}^{y_c} a^2(y) dy \right]}_{\frac{1}{16\pi^2 G_4}} \cdot \int d^4x \left[\partial_\mu h^{(4)}(x) \right]^2$$

$\frac{1}{k}(e^{2ky_c} - 1)$

$$G_4 = \frac{G_5 \cdot k}{e^{2ky_c} - 1} \qquad \leftarrow 1/M^3$$

$\frac{1}{M_{Pl}^2}$

FOR $y_c > \frac{1}{k}$, G_4 and $G_5 k$ exponentially different,

$ky_c > 1$

$$\boxed{M_{Pl}^2 = \frac{M^3}{k} e^{2ky_c}}$$

Exponential twist of HIERARCHY PROBLEM:

FOR M, k IN TeV RANGE

CORRECT $M_{Pl} = 10^{16}$ TeV IS OBTAINED FOR

$y_c \sim 35\, k^{-1}$

Still: WHY y_c is quite a bit larger than (TeV)$^{-1}$

KEY PROPERTY: WAVE FUNCTION OF GRAVITON (ZERO MODE) IS exponentially smaller on our brane than on (+) BRANE.

—o—

K-K GRAVITON MODES

$$h_m \propto \ldots \cos\left(\frac{k}{m} e^{k(|y|-y_c)}\right)$$

at large enough $|y|$.

- BOUNDARY CONDITIONS AT OUR BRANE, $y=y_c$ RELATE $\partial_y h_m$ and $k h_m$

  ⇒ DISCRETE SPECTRUM

  $m_n \approx n \cdot k$

  n: integer

TRUST FIRST K-K MODES FOR $k < M$

FOR $k \sim$ TeV, MASSES IN TeV RANGE.

- WAVE FUNCTIONS OF K-K GRAVITONS DO NOT DECREASE TOWARDS OUR BRANE

  ⇓

  K-K gravitons interact with SM particles at strength $\frac{1}{M}$

  (TeV scace interactions)

- K-K GRAVITONS = RESONANCES, e.g. in

$pp \to \text{GRAVITON} \to \mu^+\mu^-, e^+e^-$

$pp \to \text{GRAVITON} \to \text{jet} + \text{jet}$

Fig.

DIFFERENT FROM OTHER NEW PARTICLES, since

- K-K gravitons = TENSOR PARTICLES
  ANGULAR DISTRIBUTION of LEPTON PAIRS, JETS WRT BEAM direction

$gg \to G \to \ell^+\ell^- \;:\; 1 - \cos^4\theta$

$q\bar{q} \to G \to \ell^+\ell^- \;:\; 1 - 3\cos^2\theta + 4\cos^4\theta$

cf. $1 + \cos^2\theta$ for vector resonance
const for scalar.

- UNIVERSAL COUPLING TO ALL SM PARTICLES
- SERIES OF RESONANCES

(47a

$m_{1\text{-st KK graviton}} = 1.5$ TeV

$\ell^+\ell^-$-pairs at LHC

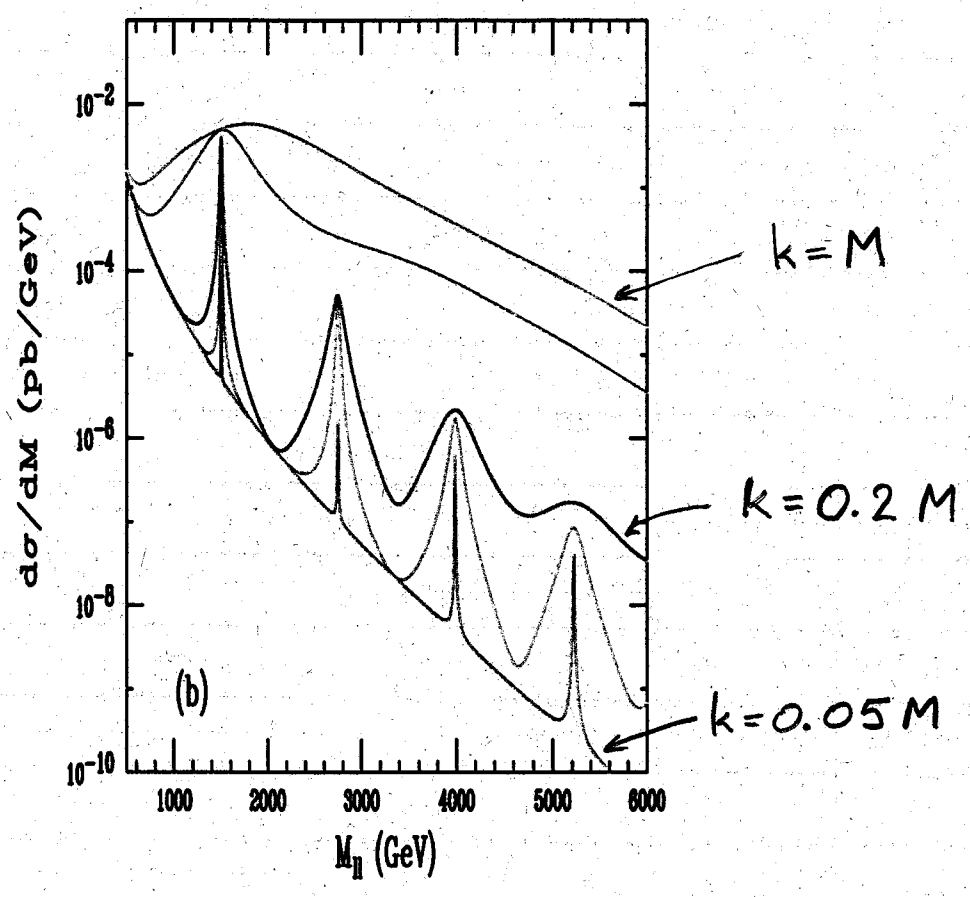


Figure 13: Drell-Yang production of the KK graviton with $M_1 = 1500$ GeV and its subsequent tower states at the LHC for $\eta = 1, 0.5, 0.1, 0.05, 0.2$, and 0.01, respectively, from top to bottom [72].

Davoudiasl, Hewett, Rizzo

- COSMOLOGY MUCH EASIER THAN IN ADD: maximum temperature OF UNIVERSE JUST BELOW k is allowed.
- NO ASTROPHYSICAL CONSTRAINTS FOR $k \sim$ TeV.

RS1 IN SOME SENSE MORE SIMILAR TO KALUZA-KLEIN WITH $(\text{TeV})^{-1}$ size of EXTRA DIMENSION(S). EXCEPT FOR HIERARCHY RELATION

$$G_4 = G_5 \cdot k \cdot e^{-2kyc}$$

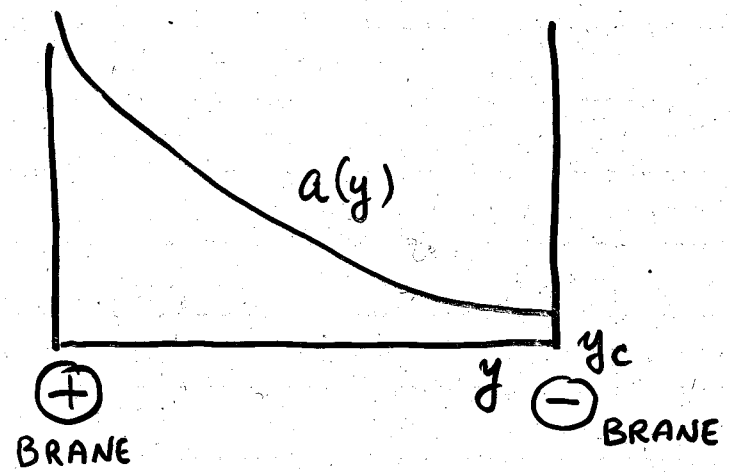
TO SUMMARIZE:

- WITH COMPACT EXTRA DIMENSIONS, fundamental gravity scale may be in TeV range
  (ALSO MODELS WITH NON-COMPACT EXTRA DIM'S)
  Quantum gravity may be within reach

- UNIFICATION OF COUPLINGS IN 4-DIM. SUSY AT GUT SCALE IS AN ACCIDENT

REMINDER:

RS1 SET UP



Gauge hierarchy solved due to

$$\frac{a(y_c)}{a(y=0)} \simeq e^{-ky_c}$$

IF $y_c \simeq 35\,k^{-1}$.

This is not the only option.

E.G. PUT SM fields in BULK

⇓

MASS SCALE ~ $M_{Pl} \cdot e^{-ky_c}$ emerges,

even IF $k \sim M_{Pl}$, and M_{Pl} is THE ONLY MASS SCALE IN ACTION (i.e. $M = M_{Pl}$).

This is BECAUSE K-K MODES BEHAVE AS $\cos\left(\frac{m}{k} e^{ky}\right)$

AWAY FROM ⊕ BRANE ⇒ BOUNDARY CONDITIONS AT ⊖ BRANE GIVE ROUGHLY

$$\frac{m}{k} e^{ky_c} = n + \mathcal{O}(1)$$

The scale

$$M_{Pl} \cdot e^{-kyc}$$

MAY WELL BE ASSOCIATED WITH WEAK SCALE.

- K-K EXCITATIONS OF SM FIELDS IN TeV ENERGY RANGE.

LIKEWISE, SUPERSYMMETRY BREAKING SCALE MAY ALSO BE

$$\sim M_{Pl}\, e^{-kyc}$$

E.G., SUSY BROKEN AT ⊖ BRANE.

PHENOMENOLOGICALLY VIABLE.

BESIDES K-K GRAVITONS OF MASSES IN TeV RANGE, CONTAINS K-K partners of our particles (and superpartners) IN SAME RANGE.

TOO GOOD ...

—o—

EVEN IF GRAVITY IS STRONG AT TeV, ANY CONCRETE MODEL IS HARDLY THE TRUTH.

BUT THE MODELS INDICATE VARIETY OF PHENOMENA NOT BE MISSED.

EXTRA DIMENSIONS OF INFINITE SIZE : RS2

Consider RS set up again, but

- FORGET ABOUT HIERARCHY PROBLEM. LET IT BE SOLVED, E.G. IN A usual manner BY SUSY/GUT.
- FORGET ABOUT NEGATIVE TENSION BRANE. LET THE BULK EXTEND TO $|y| = \infty$.
- ASSUME OUR matter lives on remaining (+) Brane.

STILL SENSIBLE THEORY: GRAVITY LOCALIZED.

Recall

- GRAVITON ZERO mode

$m^2 = 0$ $\quad h_{\mu\nu}(x,y) = \sqrt{k}\, e^{-2k|y|}\, h^{(4)}_{\mu\nu}(x)$

$\sqrt{k}$: normalization factor; $h^{(4)}_{\mu\nu}(x)$: 4dim. graviton

- "K-K" modes WITH <u>CONTINUOUS</u> SPECTRUM OF m^2.

AT $m < k$ BEHAVE like

$$h_m = \sqrt{\frac{m}{k}}, \quad |y| \to 0$$

$$h_m = \dots \cos\left(\frac{m}{k}\, e^{k|y|}\right) \quad |y| \to \infty$$

GRAVITATIONAL POTENTIAL BETWEEN UNIT MASSES ON ⊕ BRANE:

$$V(r) = -G_5 \left| h_0(y=0) \right|^2 \cdot \frac{1}{r} - G_5 \int_0^\infty dm \left| h_m(y=0) \right|^2 \frac{e^{-mr}}{r}$$

(k: $h_0(y=0)$ — zero mode contribution; $\frac{m}{k}$: contribution of "K-K" continuum, decays faster than $\frac{1}{r}$)

$$V(r) = -\frac{G_5 k}{r} - \frac{G_5}{k}\frac{1}{r^3} = -\frac{G_5 k}{r}\left(1 + \frac{\text{const}}{k^2 r^2}\right)$$

4-dim. NEWTON'S LAW for $r \ll k^{-1}$

$$G_4 = G_5 \cdot k$$

$$\Updownarrow \qquad G_5 = \frac{1}{M^3}\,, \quad G_4 = \frac{1}{M_{Pl}^2}$$

$$M^3 = M_{Pl}^2 \cdot k$$

k may be as large as mm^{-1} (though no good reason for that)

EVEN FOR $k = mm^{-1}$ FUNDAMENTAL ENERGY SCALE LARGE, $M \sim 10^8$ GeV

FORGET ABOUT TeV QUANTUM GRAVITY

NOT SO INTERESTING FOR COLLIDER EXPERIMENTS

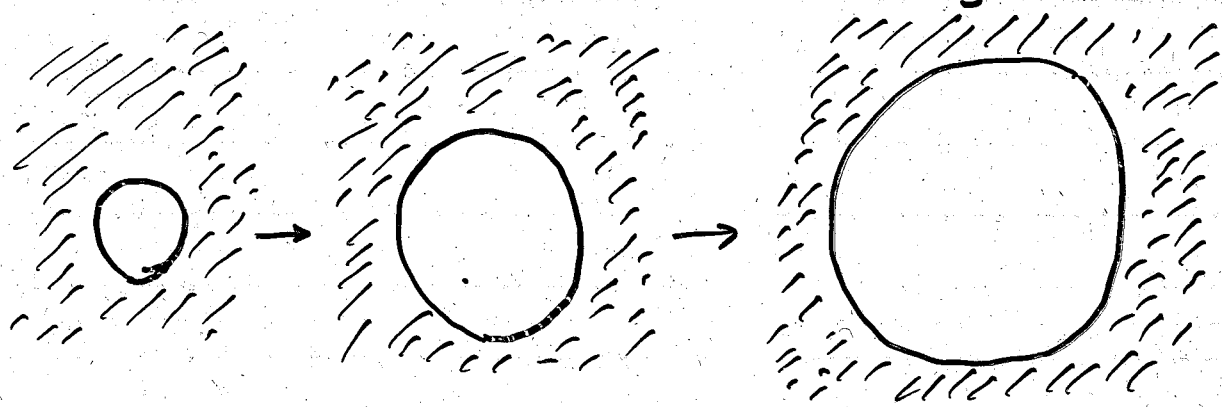
BUT MAY BE POTENTIALLY INTERESTING

- COSMOLOGY (ESPECIALLY IF k IS SMALL)
- BETTER UNDERSTANDING OF APPARENT ENERGY NON-CONSERVATION, ELECTRIC CHARGE NON-CONSERVATION
- RARE PHENOMENA AT LOW ENERGIES
- POSSIBILITY OF WEAK VIOLATION OF LORENTZ-INVARIANCE

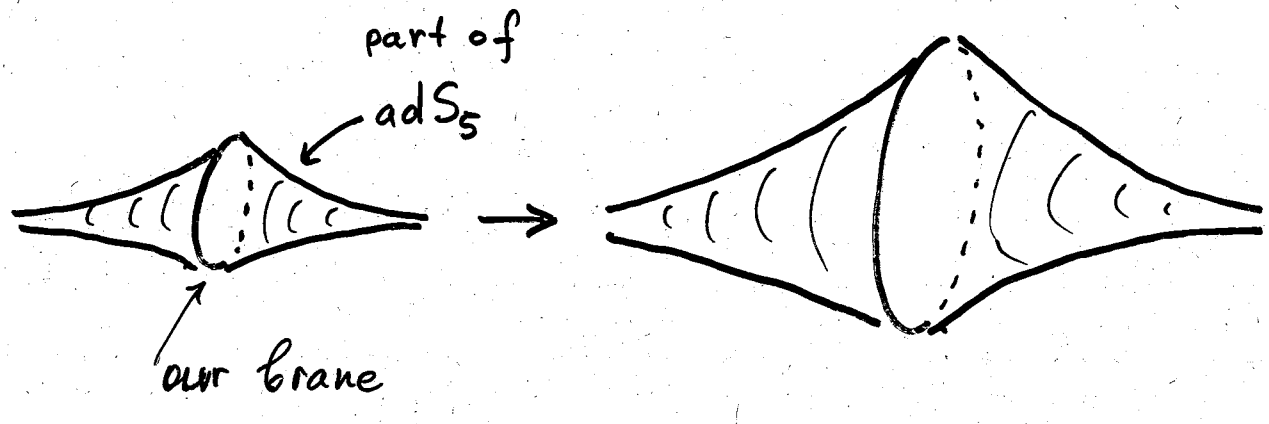
• • • • •

- COSMOLOGY:

OUR UNIVERSE LITERALLY EXPANDING BRANE IN "STATIC" BULK ADS$_5$.



BETTER PICTURE



Expansion rate:

Usual Friedmann equation

$$H^2 = \frac{8\pi}{3} G_4 \rho \quad , H \ll k$$

Neglecting spatial curvature

$$H^2 = \frac{16\pi^2}{9} G_5^2 \cdot \rho^2 \quad , H \gg k$$

ρ ← energy density of matter on our brane

i.e. (omitting 2's and π's)

$$H = \frac{\sqrt{\rho}}{M_{Pl}} \quad , H \ll k$$

$$H = \frac{\rho}{M^3} \quad , H \gg k$$

TRANSITION OCCURS (assuming radiation domination, $\rho \sim T^4$)

AT $T_* \sim \sqrt{M_{Pl}^* \cdot k}$

$\sim$ (A few) $\cdot$ 100 GeV FOR $k^{-1} \sim$ mm

Higher temperatures at larger k.

AT EARLY TIMES, i.e., AT $T > T_*$

UNIVERSE COOLS DOWN FASTER THAN IN USUAL COSMOLOGY:

- FASTER EXPANSION RATE, $H \propto T^4$ INSTEAD OF $H \propto T^2$
- EMISSION OF K-K gravitons into BULK.

For $H > k$, temperature definitely larger than k.

⇓

"K-K" gravitons from continuum have UNSUPPRESSED WAVE FUNCTIONS NEAR BRANE

⇓

ENERGY LOSS

$$\left(\frac{d\rho}{dt}\right)_{\text{Loss}} = - \frac{\text{const}}{M^3} \cdot T^8$$

ENERGY DILUTION DUE TO EXPANSION

$$\left(\frac{d\rho}{dt}\right)_{\text{expansion}} = -4H\rho = - \frac{\text{const}}{M^3} \cdot T^8$$

$H \sim \frac{\rho}{M^3}$, $\rho \sim T^4$

ROUGHLY COMPARABLE

NB: AT CONVENTIONAL STAGE, $H \sim \frac{\sqrt{\rho}}{M_{Pl}} \sim \frac{T^2}{M_{Pl}}$

$$\frac{\text{DILUTION RATE}}{\text{LOSS RATE}} \sim \frac{T^6/M_{Pl}}{T^8/M^3} = \frac{M^3}{M_{Pl} T^2} = \frac{M_{Pl} k}{T^2} = \frac{T_*^2}{T^2}$$

⇓

DILUTION $\gg$ LOSS

HOT BIG BANG GROSSLY MODIFIED AT $T > T_*$, POSSIBLY AT $T \sim$ TeV

WILL BE INTERESTING IF WE EVER HAVE DATA ON UNIVERSE AT $T \gtrsim$ TeV

(HOMEWORK: CALCULATE DARK MATTER DENSITY IN YOUR FAVORABLE EXTENSION OF SM)

—o—

INFLATION

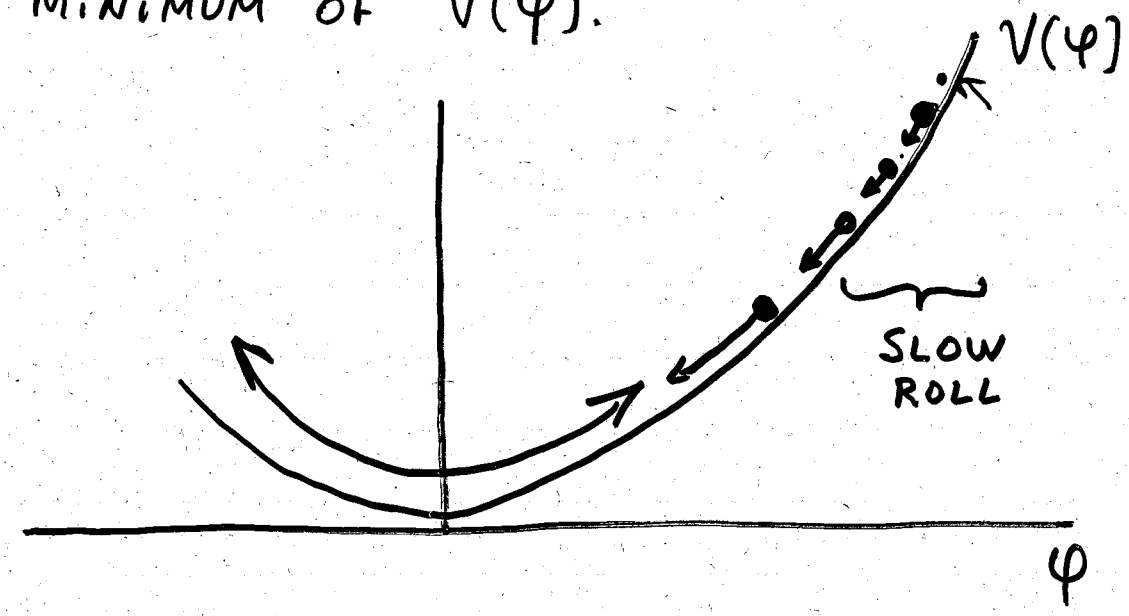
- IS IT AT ALL POSSIBLE, WITH $H > k$ AT INFLATION?

If yes:
- WHAT DENSITY PERTURBATIONS AND GRAVITATIONAL WAVES ARE GENERATED? OFF HAND: NO REASON TO BE SIMILAR TO 4-dim. THEORY.

INFLATION IN NUTSHELL

- INFLATION IS DRIVEN BY A SCALAR FIELD, INFLATON, φ WITH SUITABLE SCALAR POTENTIAL $V(\varphi)$ ALTHOUGH OTHER MECHANISMS EXIST
- DURING INFLATION, φ IS (CLASSICALLY) HOMOGENEOUS, $\varphi = \varphi(t)$

- φ SLOWLY ROLLS TOWARDS MINIMUM OF $V(\varphi)$.



- WHEN φ BECOMES SUFFICIENTLY SMALL, IT STARTS TO ROLL FAST, INFLATION ENDS

- DURING INFLATION, ENERGY DENSITY COMES FROM $V(\varphi)$, CHANGES VERY SLOWLY IN TIME.
- HUBBLE PARAMETER $H = \frac{\dot{a}}{a}$

  STAYS ALMOST CONSTANT (SLOWLY DECREASES), UNIVERSE EXPANDS EXPONENTIALLY

✓ SAME

$$H = \frac{4\pi}{3} G_5 \rho = \frac{\rho}{M^3}$$

✓

✓

✓ SCALAR FIELD EQ

$$H = \frac{V}{M^3}$$

✓

$$\dot{\varphi}^2 = \frac{V'^2}{H^2} = \frac{V'^2}{V^2} \cdot M^6 \sim V$$

$$\Downarrow$$

$$\frac{V'^2}{V^3} \sim \frac{1}{M^6}$$

$$\varphi \sim \lambda^{-1/6} M$$

OK : $V \sim \lambda^{1/3} M^4 \ll M^4$

EQUATION OF MOTION FOR HOMOGENEOUS INFLATON FIELD

$$\ddot{\varphi} + 3H\dot{\varphi} = -\frac{\partial V}{\partial \varphi} \equiv -V'$$

FRIEDMANN EQN.

$$H^2 = \frac{8\pi}{3} G_4 \rho \overset{2's,\pi's}{=} \frac{\rho}{M_{Pl}^2}$$

$$\rho = V(\varphi) + \frac{1}{2}\dot{\varphi}^2$$

SLOW ROLL CONDITIONS

$$\ddot{\varphi} \ll H\dot{\varphi} \; ; \quad \dot{\varphi}^2 \ll V(\varphi)$$

GIVE ALMOST THE SAME FOR POWER-LAW $V(\varphi)$

$$\Downarrow$$

EQ'S AT INFLATIONARY STAGE

$$3H\dot{\varphi} = -V'$$

$$\begin{cases} H^2 = \dfrac{V(\varphi)}{M_{Pl}^2} \end{cases}$$

INFLATION ENDS WHEN

$$\dot{\varphi}^2 \sim V(\varphi)$$

$$\dot{\varphi}^2 = \frac{V'^2}{H^2} = \frac{V'^2 M_{Pl}^2}{V} \sim V$$

$$\Downarrow$$

$$\frac{V'}{V} \sim \frac{1}{M_{Pl}}$$

FOR $V = \lambda\varphi^4$

$\varphi \sim M_{Pl}$, STILL $V \sim \lambda M_{Pl}^4 \ll M_{Pl}^4$

DURING INFLATION, THE INFLATON FIELD DEVELOPS QUANTUM FLUCTUATIONS, ALMOST SCALE INVARIANT, OF AMPLITUDE

$$\delta\varphi \sim H$$

(AT interesting length scales, i.e., beyond Hubble horizon of inflationary epoch).

$\Downarrow$

AT DIFFERENT PLACES OF UNIVERSE, THE INFLATON ROLLS DOWN EFFECTIVELY FROM DIFFERENT VALUES OF $\varphi = \varphi(t) \pm \delta\varphi$

$\Downarrow$

INFLATION ENDS IN DIFFERENT PLACES AT DIFFERENT TIMES, WITH TIME DISPERSION

$$\delta t = \frac{\delta\varphi}{\dot{\varphi}}$$

$\Downarrow$

matter density different after inflation (if inflation ends earlier, energy density gets diluted more than on average, due to expansion)

$$\delta\rho \sim \dot{\rho}\cdot\delta t \simeq -H\rho\cdot\delta t$$

$\Downarrow$

$$\frac{\delta\rho}{\rho} \sim H\delta t \sim \frac{H\delta\varphi}{\dot{\varphi}}$$

$$\frac{\delta\rho}{\rho} \sim \frac{H^2}{\dot{\varphi}}$$

V: φ and ρ are Brane fields

$$H\dot{\varphi} = -V' \; ; \; H = \frac{V}{M^3}$$

$$\Downarrow$$

$$\left(\frac{\delta\rho}{\rho}\right)^2 \sim \frac{V^6}{M^{18} V'^2}$$

$$\frac{V'^2}{V^3} = \epsilon^2 \cdot \frac{1}{M^6}$$

$$\Downarrow$$

$$\left(\frac{\delta\rho}{\rho}\right)^2 = \frac{V^3}{\epsilon^2 M^{12}}$$

GRAVITON IS BULK FIELD. DOES NOT CARE ABOUT M_{PL}.

$$h^2 = \frac{H^3}{M^3} \sim \frac{V^3}{M^{12}}$$

$$\swarrow$$

$$\frac{\delta\rho}{\rho} \sim \frac{H^2}{\dot{\varphi}}$$ ← THESE SHOULD BE TAKEN TOWARS END OF INFLATION, some 60 e-folds before end

$$H\dot{\varphi} = -V'$$

$$H^2 = \frac{V}{M_{Pl}^2}$$

$$\Downarrow$$

$$\left(\frac{\delta\rho}{\rho}\right)^2 \sim \frac{V^3}{M_{Pl}^6 V'^2}$$

$$\frac{V'}{V} = \epsilon \frac{1}{M_{Pl}}$$

ϵ SMALL FROM SLOW ROLL CONDITION

$$\Downarrow$$

$$\boxed{\left(\frac{\delta\rho}{\rho}\right)^2 \sim \frac{V}{\epsilon^2 M_{Pl}^4}}$$ TOWARDS END OF INFLATION

Likewise, GRAVITON FIELD DEVELOPS QUANTUM FLUCTUATIONS

$$\hat{h} \sim H \quad \text{WHERE} \quad \hat{h} = M_{Pl} \cdot h$$

$$\Downarrow$$

GRAVITY WAVES

$$h^2 = \frac{H^2}{M_{Pl}^2} \qquad H^2 = \frac{V}{M_{Pl}^2}$$

$$h^2 \sim \frac{V}{M_{Pl}^4}$$ SOMEWHAT SMALLER THAN $\left(\frac{\delta\rho}{\rho}\right)^2$.

INFLATON LIVING ON THE BRANE. HOW DOES ALL THIS GETS MODIFIED FOR $H > k$ AT INFLATION?

Q1: DOES INFLATION OCCUR AT LOW ENERGY DENSITY,

$$V(\varphi) \ll M^4$$

(NB: M may be much smaller than M_{pl}).

$$H^2 \sim \rho / M_{pl}^2 \qquad 4 \text{ dim.}$$

$$H = \frac{\rho}{M_5^3} \qquad RS\text{-}2$$

$$ds^2 = e^{-2k|y|} \left[dt^2 - a^2(t)\, d\vec{x}^2 \right] - dy^2$$

$$H = \frac{\dot{a}(t)}{a(t)}$$

A1: YES, FOR SMALL $\lambda^{1/3}$

Q2: WHAT IS THE RELATION BETWEEN AMPLITUDES OF DENSITY PERTURBATIONS AND GRAVITY WAVES?

A2: CONSPIRACY!

GROSS FEATURES SIMILAR.

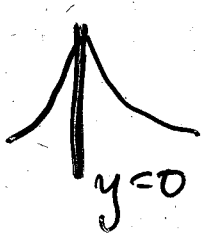
Details (tilt - tensor RELATION, ETC) MAY BE SLIGHTLY DIFFERENT

BUT NO SMOKING GUN.

FOR NO OBVIOUS GOOD REASON...

NATURE (THEORY, RATHER) IS NOT ALWAYS KIND.

QUASI-LOCALIZATION IN RS-2.



PROTOTYPE EXAMPLE:
SCALAR FIELD BOUND TO BRANE DUE TO A POTENTIAL

$$S_\phi = \int d^4x\, dy \left[\frac{1}{2} g^{AB} \partial_A \phi \partial_B \phi - \frac{1}{2} V(y) \phi^2 \right]$$

$V(y)$: BINDING POTENTIAL DUE TO DOMAIN WALL

- FLAT EXTRA DIMENSION, $g_{AB} = \eta_{AB}$

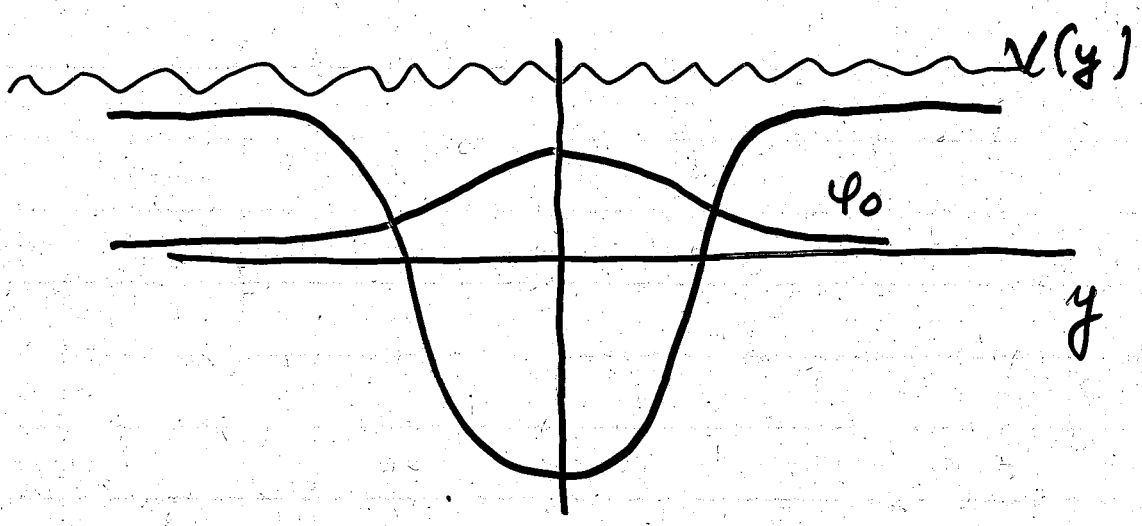
$$\partial_A \partial^A \phi + V(y)\phi = 0$$

$$\Downarrow$$

$$m^2 \phi = \partial_\mu \partial^\mu \phi = \left[-\partial_y^2 + V(y) \right] \phi$$

$\phi = e^{ip_\mu x^\mu}$; $p_\mu p^\mu = m^2$

BRANE-WORLD SCENARIO WORKS, provided THERE is one small EIGENVALUE $m^2 \equiv m_0^2$ WHILE OTHERS ARE LARGE ($\gtrsim (TeV)^2$).
TAKE $m_0^2 \neq 0$: AFTER ALL, MOST SM PARTICLES ARE MASSIVE



- Continuum STARTS AT $m^2 = V(\infty)$. ESCAPE INTO EXTRA DIMENSIONS IN HIGH ENERGY COLLISIONS.

- WARPED EXTRA DIMENSION:

$$g_{\mu\nu} = a^2(y)\,\eta_{\mu\nu} \qquad g_{yy} = -1$$

$$g^{\mu\nu} = \frac{1}{a^2(y)}\,\eta^{\mu\nu} = e^{2k|y|}\,\eta^{\mu\nu}$$

$$\Downarrow$$

$$S_\phi = \int d^4x\,dy \left[\frac{1}{2} g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi + \frac{1}{2} g^{yy}(\partial_y\phi)^2 - \frac{1}{2} V(y)\phi^2 \right] \cdot \sqrt{g^{(5)}}$$

$$= \int d^4x\,dy \left[\frac{1}{2} e^{2k|y|}\,\partial_\mu\phi\partial_\nu\phi \cdot \eta^{\mu\nu} - \frac{1}{2} V(y)\phi^2_{+\ldots} \right] \sqrt{g}$$

SMALL COMPARED TO KINETIC TERM.

BINDING POTENTIAL NEGLIGIBLE AT LARGE $|y|$.
EXPECT CONTINUUM STARTING FROM $m^2 = 0$

INDEED, EIGENVALUE EQUATION IS NOW

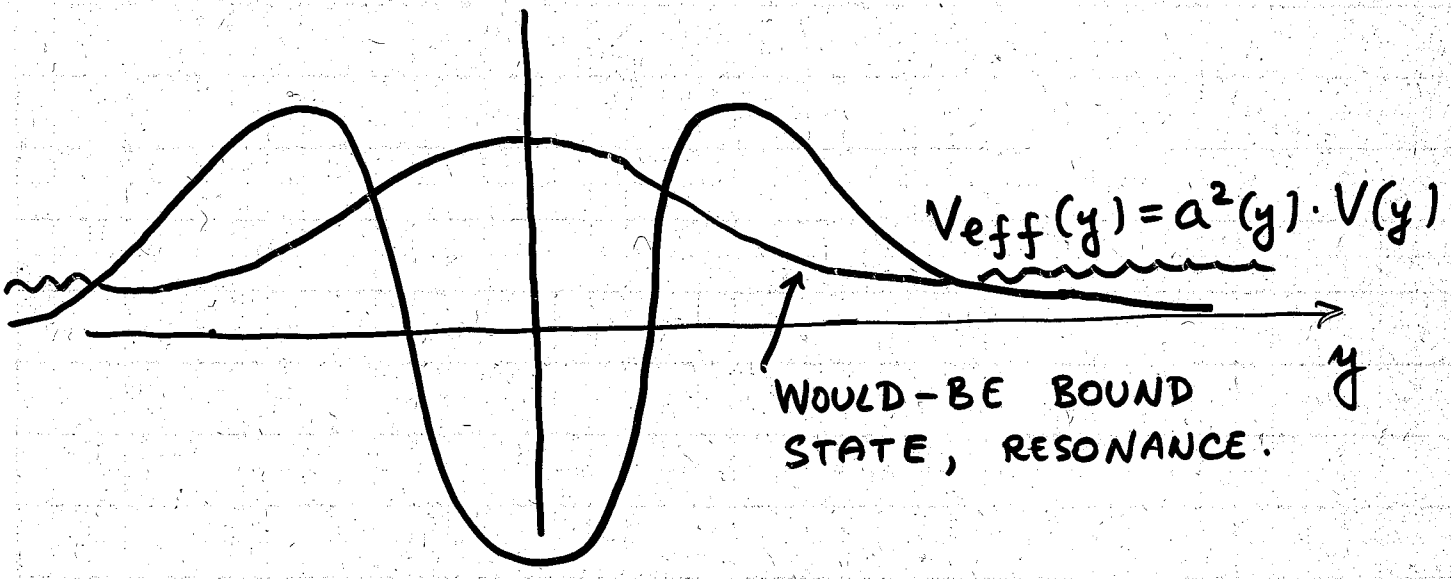
$$m^2\phi = -\frac{1}{a^2}\partial_y\left(a^4\partial_y\phi\right) + \underbrace{a^2(y)V(y)}_{\uparrow\ e^{-2k|y|}}\phi$$

NEGLIGIBLE AT LARGE y

Continuum indeed starts from $m^2=0$.
WOULD-BE BOUND STATE WITH $m^2=m_0^2$ EMBEDDED IN CONTINUUM

⇓

IT BECOMES QUASI-LOCALIZED STATE, RESONANCE.



FINITE WIDTH OF A MASSIVE BRANE PARTICLE AGAINST ESCAPE INTO BULK.

MAY BE LONG: TUNNELING.

EXAMPLE:

SCALAR FIELD BOUND TO BRANE BY GEOMETRY ITSELF

$$V(y) = 0$$

EIGENVALUE EQN:

$$m^2 \phi = -\frac{1}{a^2} \partial_y \left(a^4 \partial_y \phi \right)$$

NB: Normalization $\int a^2 dy \, |\phi|^2$

CONSTANT MODE, $\phi_0 = \text{const}$, NORMALIZABLE. MASSLESS BOUND STATE.

- ADD MASS IN BULK.

$$V(y) = \mu^2$$

$$\Downarrow$$

ϕ_0 BECOMES MASSIVE, BRANE MODE (in 4d sense)

$$m_0^2 = \frac{1}{2} \mu^2$$

WITH FINITE WIDTH AGAINST ESCAPE FROM BRANE

$$\Gamma = \frac{\pi}{16} m_0 \left(\frac{m_0}{k} \right)^2$$

NB: FINITE AT <u>SMALL</u> m_0. NOT A THRESHOLD EFFECT.

LESSON:

ESCAPE INTO EXTRA DIMENSION(S)
POSSIBLE EVEN AT LOW ENERGIES,

AT SMALL PROBABILITY. THE HEAVIER THE PARTICLE, THE LARGER THE WIDTH.

(positronium e^+e^-) $\longrightarrow$ NOTHING
nucleon $\longrightarrow$ NOTHING
Higgs, Z_0 $\longrightarrow$ NOTHING
....

ESTIMATES VERY MODEL-DEPENDENT

NEW FEATURE:

LOW ENERGY / LARGE TIME INTERVAL,
LARGE DISTANCE
EFFECT OF EXTRA DIMENSIONS.

MORE ABOUT APPARENT ENERGY / ELECTRIC CHARGE NON-CONSERVATION.

HOW IS IT RECONCILED WITH 4-dim. GRAVITY / ELECTROMAGNETISM AT LARGE DISTANCES?

4-dim. ELECTROMAGNETISM:

GAUSS' LAW: $div\ \vec{E} = 4\pi \rho_{e.m.}$

"INSTANTANEOUS" EQUATION, NO TIME DERIVATIVES

[UNLIKE "CAUSAL" EQUATIONS DESCRIBING PROPAGATION OF FIELDS]

GIVEN $\rho_{e.m.}$, ELECTRIC FIELD DETERMINED EVERYWHERE IN SPACE AT SAME TIME.

$$\vec{E} = \frac{\vec{r}}{r^3} Q$$

IF CHARGE IS NOT CONSERVED, ITS CHANGE SHOULD GIVE RISE TO CHANGE OF $\vec{E}$ EVERYWHERE IN SPACE INSTANTANEOUSLY.

CONTRADICTION WITH CAUSALITY.

SAME WITH GRAVITY / ENERGY

$R^0_0 - \frac{1}{2} g^0_0 R = 8\pi G T^0_0 \Leftrightarrow$ GAUSS' LAW OF GR $\cong$ NEWTON'S LAW

YET RS2 COMBINES

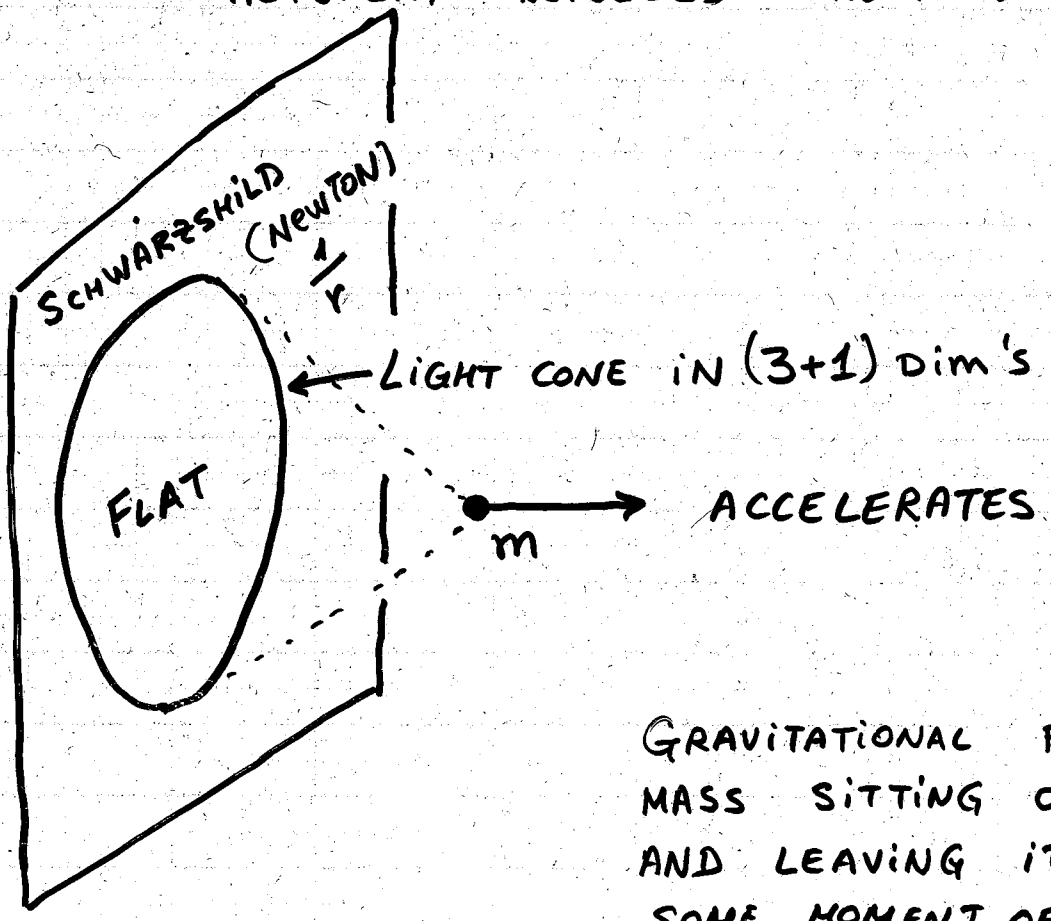
- 4-dim. GRAVITY ON BRANE AT LARGE DISTANCES

- POSSIBILITY OF ENERGY FLOWING AWAY from BRANE TO $y \to \infty$, i.e., APPARENT ENERGY NON-CONSERVATION FOR BRANE-BASED OBSERVERS, US.

IS THIS TRUE? CAN MATTER ESCAPE FROM BRANE?

YES. QUANTUM MECH.: "K-K" MODES FROM CONTINUUM $\Leftrightarrow$ PARTICLES MOVING TO $y \to \infty$.

CLASSICALLY: TEST MASS is ACTUALLY REPELLED FROM BRANE.



GRAVITATIONAL FIELD OF MASS SITTING ON BRANE AND LEAVING IT AT SOME MOMENT OF TIME.

NO PROBLEM WITH CAUSALITY

- VIEWPOINT #1; MULTIDIMENSIONAL:

  GRAVITY NO LONGER 4-DIMENSIONAL FOR SOURCES OUTSIDE BRANE

  KEY: CONTINUUM OF KK gravitons STARTING FROM $m^2 = 0$.

  $\Updownarrow$ adS/CFT

- VIEWPOINT #2, BRANE-BASED

  MASS DECAYED INTO CONFORMAL MATTER THAT DISSIPATES ALONG LIGHT CONE.

THIS CONFORMAL MATTER INTERACTS WITH BRANE PARTICLES GRAVITATIONALLY ONLY

EFFECTIVE 4D DESCRIPTION OF RS2 IN TERMS OF CONFORMAL MATTER.

ELECTRIC CHARGE NON-CONSERVATION:
SIMILAR, BUT INTERPRETATION IN TERMS OF CHARGED CONFORMAL MATTER QUESTIONABLE.

$e^- \rightarrow$ NOTHING

CONTRADICTS NOTHING.

WOULD BE STRONGEST INDICATION OF EXTRA DIMENSIONS

$\tau_{e \rightarrow \text{NOTHING}} > 6 \cdot 10^{24}$ yrs.

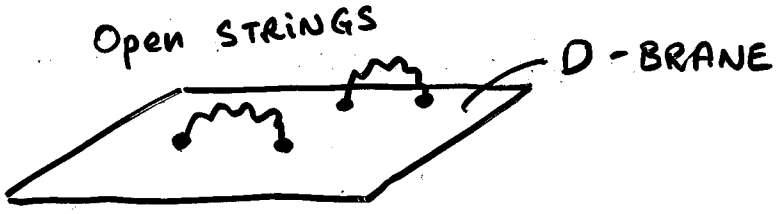
STRING/D-BRANE PICTURE OF QUASI-LOCALIZATION

FEW NOTIONS OF STRING THEORY

- IN QUANTUM THEORY, STRING HAS QUANTIZED SPECTRUM OF STATES $\Leftrightarrow$ PARTICLES. IN RELATIVISTIC THEORY, SOME OF THESE STATES (PARTICLES) MAY BE MASSLESS.

- GAUGE BOSONS, CHARGED PARTICLES ARE LOWEST STATES OF OPEN STRINGS.

- OPEN STRINGS HAVE ENDS ON D-BRANES.

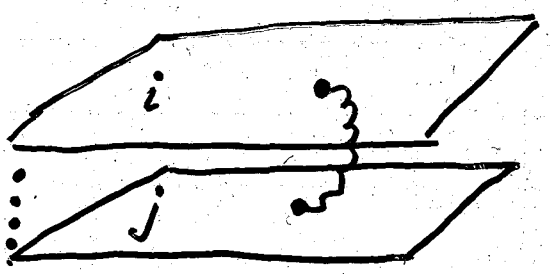


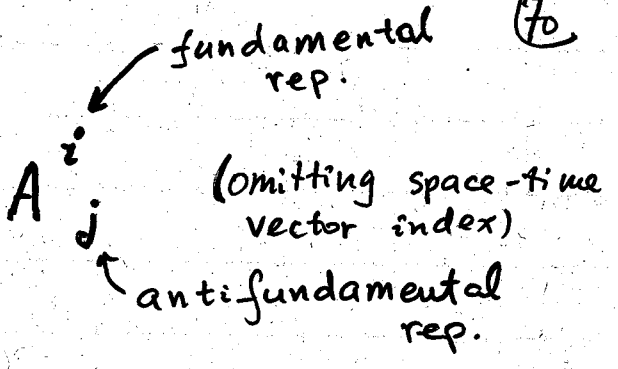
NB: D-BRANES MAY HAVE DIVERSE DIMENSIONALITIES. IN PARTICULAR, DIMENSIONALITY MAY BE THE SAME AS DIM'S OF ENTIRE SPACE $\Rightarrow$ D-BRANE FILLS WHOLE SPACE.

GAUGE BOSONS PROPAGATE ALONG D-BRANES.

- STACK OF K COINCIDENT D-BRANES HAS $U(K)$ GAUGE THEORY ON ITS HYPER SURFACE

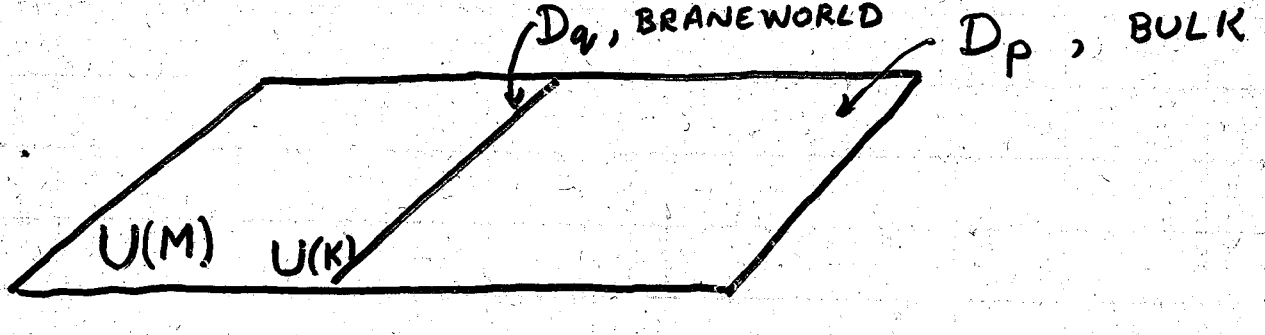
K^2 GAUGE BOSONS, ACCORDING TO # OF WAYS STRING ENDS CAN BE ATTACHED TO D-BRANES.





(fundamental) × (antifundamental) = adjoint

- D-BRANES OF SMALLER DIMENSIONALITY q MAY SIT WITHIN D-BRANES OF LARGER DIMENSIONALITY p.
  CALL D_q BRANEWORLD, D_p BULK
  $U(K)$, $U(M)$ - CORRESPONDING GAUGE GROUPS



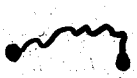
THEN THERE ARE

$U(K) \qquad U(M)$

p-p - STRINGS $\Leftrightarrow$ BULK GAUGE FIELDS

(1, ADJOINT)

q-q - STRINGS $\Leftrightarrow$ BRANEWORLD GAUGE FIELDS
(ADJOINT, 1)



p-q - STRINGS $\Leftrightarrow$ BRANEWORLD FIELDS
(FUNDAMENTAL, ANTIFUNDAM.)

THESE CONTAIN SCALARS WRT
BRANEWORLD LORENTZ GROUP.

IF GAUGE SYMMETRIES ARE UNBROKEN, CHARGED BRANEWORLD FIELDS, BRANEWORLD $U(K)$ GAUGE BOSONS CANNOT ESCAPE INTO BULK.

EFFECTIVE FIELD THEORY DESCRIPTION

$$S_{EFF} = \int_{D_p} d^p X \; F_{AB} F^{AB} \qquad U(M) \text{ IN BULK}$$

$$+ \int_{D_q} d^q x \; F_{\mu\nu} F^{\mu\nu} \qquad U(K) \text{ IN BRANEWORLD}$$

$$+ \int_{D_q} d^q x \, \mathrm{tr}\left[D_\mu \Phi\right]^+ \cdot D_\mu \Phi + \dots$$

$$D_\mu \phi = \partial_\mu \phi - i g_K A_\mu \phi - i g_M \phi A_\mu$$

$\Phi^{\alpha}{}_{m}$ — fundamental under $U(K)$, antifundam. under $U(M)$

Symmetry breaking in braneworld:
Φ gets v.e.v. (HIGGS PHASE ON D_q)

$$D_\mu \phi \rightarrow -i\left(g_K \mathcal{A}_\mu \langle\phi\rangle + g_M \langle\phi\rangle A_\mu\right)$$

$$\Downarrow$$

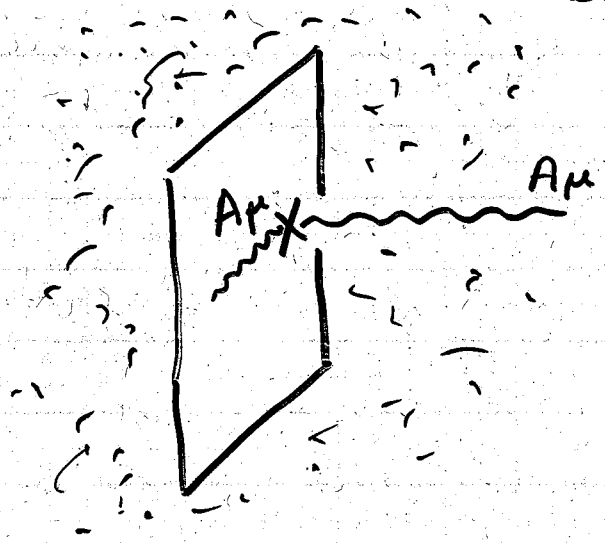
$$|D_\mu \Phi|^2 \rightarrow g_K^2 \langle\phi\rangle^2 \mathcal{A}_\mu^2 + g_M^2 \langle\phi\rangle^2 A_\mu^2 + g_K g_M \langle\phi\rangle^2 \mathcal{A}_\mu A_\mu$$

MASS FOR brane ~~BULK~~ GAUGE FIELD

Potential for bulk gauge field on brane: $\langle\phi\rangle^2 \neq 0$ on brane.

AWAY FROM BRANE, A_μ still massless

↑ MIXING BETWEEN BRANE MASSIVE BRANEWORLD GAUGE FIELD AND MASSLESS BULK GAUGE FIELD.



"DECAY" WIDTH SMALL AT SMALL $\langle\phi\rangle$

$$\Gamma \sim g^4 \frac{\langle\phi\rangle^4}{M^3} \sim \frac{m_{\mathcal{A}}^4}{M^3}$$

↑ characteristic scale of UNDERLYING THEORY

$$\Downarrow$$

INVISIBLE DECAYS ARE CHARACTERISTIC TO D-BRANE CONSTRUCTIONS.

Fields carrying unbroken charges CANNOT ESCAPE. $e^- \not\rightarrow$ NOTHING

SUBSTANTIAL IF FUNDAMENTAL SCALE IN TeV RANGE.

SOME LESS UNDERSTOOD ISSUES:

1. VIOLATION OF LORENTZ INVARIANCE.

BACK TO WARPED GEOMETRY.

Imagine time and space HAVE DIFFERENT WARP FACTORS

HOW CAN THIS EMERGE IN NATURAL WAY?

$$ds^2 = a^2(y)\,dt^2 - b^2(y)\,d\vec{x}^2 - dy^2$$

- By rescaling t and $\vec{x}$:

$$a(0) = b(0) = 1 \quad , \text{ on the brane}$$

NO VIOLATION OF LORENTZ-INVARIANCE STRICTLY ON THE BRANE. 4-DIM. LORENTZ-INV. VIOLATED IN BULK.

BRANE PARTICLES HAVE WAVE FUNCTIONS EXTENDING SLIGHTLY INTO BULK ⇒ LORENTZ-INV. SLIGHTLY VIOLATED.

SAY, SCALAR FIELD IN BINDING POTENTIAL $V(y)$:

$$\Phi = e^{i\omega t - i\vec{k}\vec{x}} \cdot \phi_{m_0}(y)$$

Eigenvalue eqn. for frequency:

$$\omega^2 \phi = \frac{a^2(y)}{b^2(y)} \cdot \vec{k}^2 \phi + \mathcal{H}\phi$$

$\mathcal{H}$: TRANSVERSE OPERATOR

Our particle has WAVE FUNCTION ϕ_0 AT $\vec{k}=0$;

$\mathcal{H}\phi_0 = m_0^2 \phi_0$; $\omega(\vec{k}=0) = m_0$, rest mass.

- AT SMALL $\vec{k}$, TREAT

$$\frac{a^2(y)}{b^2(y)}\vec{k}^2 \quad \text{as perturbation}$$

$$\Downarrow$$

$$\omega^2(\vec{k}) = m_0^2 + c^2\vec{k}^2$$

$$c^2 = \frac{\int d\mu(y)\,\frac{a^2(y)}{b^2(y)}\,|\phi_0|^2}{\int d\mu(y)\,|\phi_0|^2}$$

NOT UNIVERSAL; depends on ϕ_0. DIFFERENT PARTICLES HAVE DIFFERENT $\omega(\vec{k})$.

c close to 1 FOR NARROW $|\phi_0|^2$

(since $\frac{a(0)}{b(0)} = 1$) $\Longrightarrow$ weak violation of 4-dim. Lorentz-invariance.

- SURPRIZING POSSIBILITY

$$\omega^2\phi = \frac{a^2(y)}{b^2(y)}\vec{k}^2\phi + \mathcal{H}\phi$$

SUPPOZE LOWEST EIGENVALUE IS ZERO, $\mathcal{H}\phi = 0 \Rightarrow$ MASSLESS PARTICLE ON BRANE. ASSUME

$$\frac{a^2}{b^2} \longrightarrow 0 \quad \text{AS} \quad |y| \to \infty$$

$$\Downarrow$$

CONTINUUM STARTS AT ZERO ENERGY ω EVEN FOR FINITE MOMENTUM $\vec{k}$. $\Rightarrow$ MOVING particles (even massless) get QUASI-LOCALIZED The larger spatial momentum/energy, the larger the width against escape into bulk.

DIFFICULTY : HOW TO GET

$$a(y) \neq b(y) \quad ?$$

* NOT adS in BULK

$\Downarrow$

$p \neq -\rho$ in BULK

WHAT KIND OF matter in BULK?

* Same for brane:

jumps of $a'(y)$ and $b'(y)$ correspond to different components of $T_{\mu\nu}$ of the brane (through 5D EINSTEIN EQS.)

$\Downarrow$

$p \neq -\rho$ on BRANE.

WHAT KIND OF matter on BRANE ?

KNOWN EXAMPLES : WEIRD...

↑ matter in our Universe!

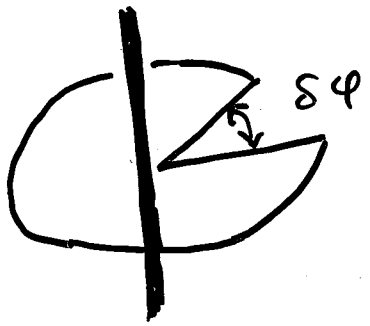
2. Cosmological constant problem.

- Vacuum energy density in SM = tension of our brane.

Let it curve <u>bulk</u>, not brane.

Version: 6D (two extra dim's):

codimension 2 object of finite tension ⇒ deficit angle in <u>flat</u> plane



$\delta\varphi \propto$ brane (string) tension

Does not work

E.g., 6D: to make gravity effectively 4-dimensional, one makes extra dim's compact ⇒ fine-tuning is back.

Appears hopeless...

LONG-DISTANCE MODIFICATION OF GRAVITY.

MOTIVATED BY ACCELERATION OF THE UNIVERSE TODAY:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \varkappa T_{\mu\nu}$$

SOMETHING NEW HERE?

SOMETHING NEW HERE? (VACUUM ENERGY? QUINTESSENSE?)

GRAVITY GETS MODIFIED AT COSMOLOGICAL TIME/LENGTH SCALE?

AT FIRST SIGHT, EXTRA DIMENSIONS MAY PROVIDE A MECHANISM. BUT ONLY AT FIRST SIGHT (until now).

A PROSPECTIVE MECHANISM:
QUASI-LOCALISATION OF GRAVITON ON OUR BRANE.

* TWO MODELS AND TWO KINDS OF PROBLEMS.

• MODEL #1: GRS

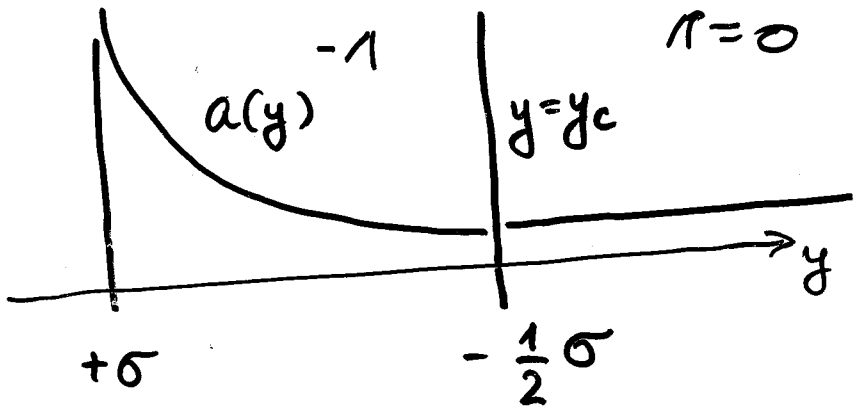
TWO BRANES WITH TENSIONS

$+\sigma$ and $-\frac{1}{2}\sigma$

AND FINE-TUNED COSMOLOGICAL CONSTANTS IN TWO PARTS OF BULK.

⇓

WARPED GEOMETRY (RS adS) BETWEEN BRANES; FLAT OUTSIDE.



ZERO MODE

$$h_{\mu\nu}(x,y) = a^2(y)\,\tilde{h}_{\mu\nu}(x)$$

NOT NORMALIZABLE any longer.

"ALMOST NORMALIZABLE" IF y_c IS LARGE.

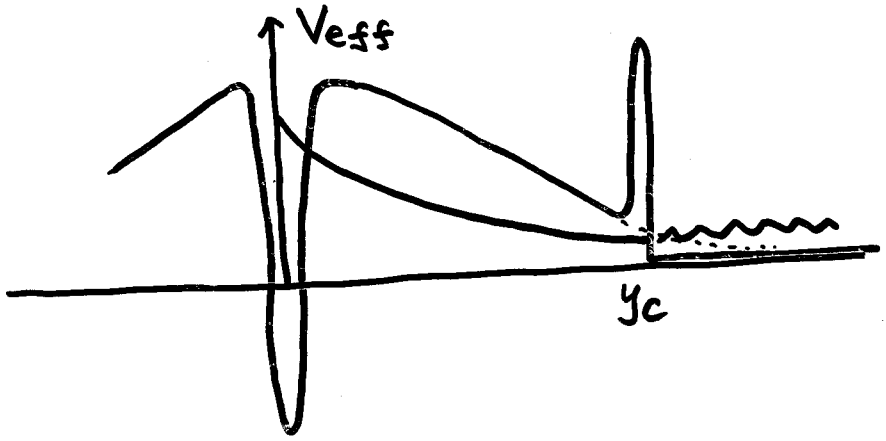
- FOR MATTER ON + BRANE:

  4D - NEWTON'S LAW at $k^{-1} \ll r \ll r_c$

  $r_c \sim \frac{1}{k} e^{+ky_c}$ ⇐ ONLY AT LARGE DISTANCES TINY EFFECTS DUE TO OUTER REGION BECOME RELEVANT.

  FOR $r \gtrsim r_c$ GRAVITY LAW FOR MATTER ON + BRANE GETS MODIFIED.

- IN TERMS OF EFFECTIVE POTENTIAL FOR GRAVITON



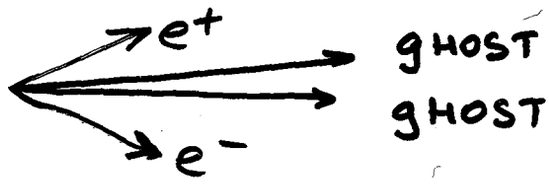
RS ZERO MODE GETS LIFTED, BECOMES RESONANCE IN CONTINUOUS SPECTRUM.

WHAT'S WRONG?

- DYNAMICAL NEGATIVE TENSION BRANE VIBRATIONS ⇒ NEGATIVE KINETIC ENERGY GHOST (SCALAR) IN SPECTRUM OF PERTURBATIONS:

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial_\mu\phi \Rightarrow E = -\left[\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\vec{\nabla}\phi)^2\right]$$

VERY BAD (ESPECIALLY IN LORENTZ-INVARIANT THEORY). PAIR CREATION FROM VACUUM



- CONSISTENT WITH ENERGY-momentum CONSERVATION
- PROPORTIONAL TO VOLUME OF LORENTZ GROUP (BOOST ALL PARTICLES) = ∞.

VACUUM ABSOLUTELY UNSTABLE.

[ANOTHER OPTION: NEGATIVE PROBABILITY. NO BETTER.]

MODEL #2: DGP

FLAT 5D BACKGROUND. NO COSMO. CONSTANTS.

BRANE TERM FOR GRAVITY.

$$S = M^3 \int d^5x \sqrt{g^{(5)}}\, R^{(5)} + M_{Pl}^2 \int_{y=0} d^4x \sqrt{g^{(4)}}\, R^{(4)}$$

↑ induced metric on BRANE.

* GRAVITATIONAL FIELD PRODUCED BY SOURCE ON BRANE, LINEARIZED THEORY.

OMITTING INDICES (BUT LATER ON...)

$$M^3 \Box^{(5)} h + M_{Pl}^2 \Box^{(4)} h \cdot \delta(y) = T(x)\delta(y)$$

In 4-dim. momentum rep.

NB: $p^2 = \vec{p}^2 - \omega^2 > 0$, e.g. STATIC SOURCE

$$M^3 (\partial_y^2 - p^2) h + M_{Pl}^2 \delta(y)(-p^2) h = T(p)\cdot\delta(y)$$

$$\Downarrow$$

$$h = e^{-p|y|} h^{(4)}(x).$$

$$-M^3 p \cdot \delta(y) h^{(4)} \Downarrow - p^2 M_{Pl}^2 \delta(y) h^{(4)} = T(p)\delta(y)$$

$$\Downarrow$$

$$h^{(4)} = -\frac{T(p)}{p^2 M_{Pl}^2 + p M^3}$$

$$h^{(4)} = - \frac{T(p)}{p^2 M_{Pl}^2 + p M^3}$$

- GRAVITY ON BRANE 4-Dim. DOWN TO ENERGY SCALE

$$\boxed{\mu_g = \frac{M^3}{M_{Pl}^2}}$$

LENGTH SCALE

$$\ell_g = \frac{1}{\mu_g} = \frac{M_{Pl}^2}{M^3}$$

ABOVE THIS SCALE: 5 Dim. NEWTON'S LAW.

$$h^{(4)}(p) = \frac{T(p)}{p M^3} \Rightarrow h^{(4)}(r) = \frac{T}{r^2 M^3}$$

NB: WANT $\ell_g \sim 10^{28}$ cm $(\equiv H_0^{-1}) \Rightarrow M \simeq 10$ MeV
NOT BAD.

WHAT'S WRONG?

STRONG COUPLING IN UV.
BUT "UV" ACTUALLY MEANS VERY LOW ENERGY HERE!

CANNOT TRUST THEORY AT DISTANCES EXPERIMENTALLY ACCESSED. Modulo Nicolis, RATTAZZI

Too early quantum gravity.

4D SCALAR MODES (and vector)

$$h_{\mu\nu} = \partial_\mu \pi_\nu + \partial_\nu \pi_\mu + \eta_{\mu\nu}\phi$$

BECOME DYNAMICAL IN 5-DIM'S.

E.g., π_μ IS NOT PURE GAUGE.

$\mu\nu$-component of "Einstein" eqs. for arbitrary SOURCE (i.e., graviton propagator eqn.)

$$M^3\left(R^{(5)}_{\mu\nu} - \tfrac{1}{2} g^{(5)}_{\mu\nu} R^{(5)}\right)$$

$$+ M_{pl}^2 \left(R^{(4)}_{\mu\nu} - \tfrac{1}{2} g^{(4)}_{\mu\nu} R^{(4)}\right)\delta(y) = T_{\mu\nu}(x,y)$$

↗ No longer 4D conserved

Consider linearised.

∂_μ ( ) $\Rightarrow$

$$M^3\left(\partial_\mu R^{(5)}_{\mu\nu} - \tfrac{1}{2}\partial_\mu g^{(5)}_{\mu\nu} R^{(5)}\right) = \partial_\mu T_{\mu\nu}$$

$\Downarrow$

Some fields ARE $\sim \dfrac{T_{\mu\nu}}{M^3}$

INCLUDING ϕ. COUNT M and M_{pl} only.

$$\phi \sim \frac{1}{M^3}$$

BACK INTO ORIGINAL EQN.

$$R^{(4)}_{\mu\nu} - \frac{1}{2} g^{(4)}_{\mu\nu} R^{(4)} = \eta_{\mu\nu} \Box^{(4)} \phi + \ldots$$

$\phi \leftarrow \frac{1}{M^3}$

$$\Downarrow$$

$$M^3 \left(R^{(5)}_{\mu\nu} - \frac{1}{2} g^{(5)}_{\mu\nu} \right) + M_{Pl}^2 \, \eta_{\mu\nu} \Box^{(4)} \phi \, \delta(y) = T_{\mu\nu}$$

$$\Downarrow$$

SOME FIELDS ARE $\dfrac{M_{Pl}^2}{M^6}$!

THESE ARE LONGITUDINAL π_μ, $\pi_\mu^L = \partial_\mu \chi$

$$\chi \leftarrow \partial_\mu \pi_\mu \sim \frac{M_{Pl}^2}{M^6} \cdot T \quad \leftarrow \text{SOURCE}$$

$$\Downarrow$$

GRAVITON PROPAGATOR HAS PIECE

$$G_{\mu\nu\rho\sigma} \sim \frac{M_{Pl}^2}{M^6} \frac{p_\mu p_\nu \, p_\rho p_\sigma}{p^4} \propto \langle \chi \chi \rangle$$

instead of $\frac{1}{M_{Pl}^2}$ in GR

CANONICALLY NORMALISED

$$\chi^{can} = \frac{M^3}{M_{Pl}} \chi$$

i.e. $\chi = \dfrac{M_{Pl}}{M^3} \chi^{can}$; $\langle \chi^{can} \chi^{can} \rangle \sim 1$

$$\Downarrow$$

χ large.

Interactions:

again power counting

$\pi_\mu = \partial_\mu \chi$ does not enter $R^{(4)}_{\mu\nu}$ $\Rightarrow$

SELF-INTERACTIONS OF χ ONLY FROM BULK TERM $M^3 \int d^5x \, R^{(5)} \sqrt{g^{(5)}}$

CUBIC INTERACTION

$$M^3 \chi^3 = M^3 \left(\frac{M_{pl}}{M^3}\chi^{can}\right)^3 = \frac{M_{pl}^3}{M^6}\left(\chi^{can}\right)^3$$

(χ: derivatives...)

Recall $\mu_g = \dfrac{M^3}{M_{pl}^2}$

$\Downarrow$

Interaction $\dfrac{1}{\mu_g^2 M_{pl}}\left(\chi^{can}\right)^3$

$\Downarrow$

STRONG COUPLING AT UV SCALE

$$E_{uv} = \left(\mu_g^2 M_{pl}\right)^{1/3}$$

$$\ell_{uv} = \left(\ell_g^2 \cdot \ell_{pl}\right)^{1/3} = 1000 \text{ km}$$

($\ell_g \sim 10^{28}$ cm, $\ell_{pl} \sim 10^{-33}$ cm)

CANNOT TRUST THEORY AT $\ell \lesssim 1000$ km.

UNACCEPTABLE.

STORY REPEATS ITSELF IN OTHER MODELS:

- EITHER GHOST
- OR STRONG COUPLING AT UNACCEPTABLY LOW ENERGIES.

TO UNDERSTAND BETTER:
CONSIDER MASSIVE GRAVITY IN 4-DIM'S ABOUT FLAT BACKGROUND

$$\mathcal{L} = M_{Pl}^2 \left[\sqrt{-g}\, R + \mu_1^2\, h_{\mu\nu} h^{\mu\nu} + \mu_2^2\, h_\mu^\mu h_\nu^\nu \right]$$

NEW DYNAMICAL FIELDS: WOULD-BE PURE GAUGE (a lá Stückelberg)

$$h_{\mu\nu} = \partial_\mu \pi_\nu + \partial_\nu \pi_\mu + \partial_\mu \pi_\lambda \cdot \partial_\nu \pi^\lambda + \dots$$

DO NOT ENTER EINSTEIN-HILBERT TERM.

Linearised theory: INTEGRATE BY PARTS

$$\mathcal{L}_\pi = \left[2\mu_1^2 \left[(\partial_\mu \pi_\nu)^2 + (\partial_\mu \pi_\mu)^2 \right] + 4\mu_2^2 (\partial_\mu \pi_\mu)^2 \right] \cdot M_{Pl}^2$$

- TRANSVERSE SECTOR

$$\partial_\mu \pi_\mu^T = 0 \Downarrow$$

$$\mathcal{L}_{\pi T} = 2 M_{Pl}^2 \mu_1^2 (\partial_\mu \pi_\nu^T)^2$$

HEALTHY KINETIC TERM. $\pi_\mu^{can,T} = M_{Pl} \mu_1 \cdot \pi_\mu^T$

$$\pi_\mu^T = \frac{1}{M_{Pl} \cdot \mu_1} \pi_\mu^{can,T}$$

Interactions:

AGAIN COUNT POWERS OF MASSES

$$M_{Pl}^2 \mu^2 (\pi^T)^3 = \frac{1}{M_{Pl} \cdot \mu} (\pi_\mu^{can,\,T})^3$$

STRONG COUPLING SCALE

$$E_{UV} = (M_{Pl} \cdot \mu)^{1/2}$$

$$\Downarrow$$

$$\ell_{UV} = (\ell_{Pl} \cdot \ell_g)^{1/2} \sim 0.1 \text{ mm}$$

$\ell_{Pl} \sim 10^{-33}$ cm, $\ell_g \sim 10^{28}$ cm

NOT THAT DANGEROUS!

EVEN INTERESTING!

- LONGITUDINAL SECTOR

$$\pi_\mu = \partial_\mu \chi$$

$$\Downarrow$$

$$\mathcal{L}_\chi = 4 M_{Pl}^2 (\mu_1^2 + \mu_2^2)(\Box \chi)^2$$

THEORY WITH HIGH DERIVATIVES $\Rightarrow$ GHOST

PAULI-FIERZ: $\mu_2^2 = -\mu_1^2 \equiv \mu^2$

No genuine kinetic term for χ.

RECALL ANOTHER SCALAR:

$$h_{\mu\nu} = 2\,\partial_\mu\partial_\nu\chi + \eta_{\mu\nu}\phi$$

does enter Einstein-Hilbert action.

$$\Downarrow$$

$$\mathcal{L} = M_{Pl}^2\left[-(\partial_\mu\phi)^2 + \mu^2\,\phi\cdot\Box\chi\right]$$

$$\Downarrow$$

diagonalise (shift ϕ) $\Rightarrow$ $\phi \to \phi + \mu^2\chi$

$$\mathcal{L}_\chi = M_{Pl}^2\,\mu^4\,(\partial_\mu\chi)^2$$

$$\Downarrow$$

$$\chi = \frac{1}{M_{Pl}\cdot\mu^2}\,\chi^{can}$$

NB: $h_{\mu\nu}T^{\mu\nu} = \partial_\mu\partial_\nu\chi\cdot T^{\mu\nu} + \eta_{\mu\nu}T^{\mu\nu}\phi$

$\partial_\mu\partial_\nu\chi\cdot T^{\mu\nu} = 0$

$$\Downarrow$$

$\eta_{\mu\nu}T^{\mu\nu}\mu^2\chi$

$$\Downarrow$$

$$T^\mu_\mu\cdot\frac{1}{M_{Pl}}\,\chi^{can}$$

χ interacts with matter at gravitational strength; does not decouple in limit $\mu \to 0$. vDVZ.

STRONG COUPLING SCALE

$$\mathcal{L}_{int} = M_{Pl}^2 \cdot \mu^2 \chi^3$$

$$= M_{Pl}^2 \mu^2 \left(\frac{1}{M_{Pl} \mu^2} \chi^{can} \right)^3$$

$$= \frac{1}{M_{Pl} \mu^4} (\chi^{can})^3$$

$$\Downarrow$$

$$E_{UV} = (M_{Pl} \cdot \mu^4)^{1/5}$$ WAY TOO LOW.

AT BEST (cancelling dangerous terms by extra operators)

$$E_{UV} = (M_{Pl} \cdot \mu^2)^{1/3} \Rightarrow \ell_{UV} = 1000 \text{ km}.$$

IS THERE A WAY OUT?

BREAKING OF LORENTZ INVARIANCE: MAY HELP.

NB: GRAVITY $\approx$ GAUGE THEORY OF LORENTZ GROUP.

FURTHER HINT:

4-DIM. GRAVITY WITH LORENTZ VIOLATING GRAVITON MASSES.

$$\mathcal{L} = M_{Pl}^2 \left[\sqrt{-g}\, R + m_0^2\, h_{00} h_{00} + m_1^2 h_{00} h_{ii} + m_2^2 h_{ij} h_{ij} + m_3^2\, h_{ii} h_{jj} + m_4^2\, h_{0i} h_{0i} \right]$$

CASES WITH NO GHOSTS

- $m_0 \neq 0$

  $m_1 = m_2 = m_3 = m_4 \neq 0$

  "GHOST CONDENSATE"

  DOES NOT GIVE RISE TO ACCELERATION OF UNIVERSE TODAY

- $m_0 = 0$, OTHERS NON-ZERO, MILD INEQUALITIES TO ENSURE NO GHOSTS OR TACHYONS.

NEW FIELDS

$$h_{\mu\nu} = \partial_\mu \pi_\nu + \partial_\nu \pi_\mu + \dots$$

- π_0 NOT DYNAMICAL (LAGRANGE MULTIPLIER)
- π_i GET HEALTHY KINETIC TERMS FROM GRAVITON MASS TERMS

$$\sim M_{Pl}^2\, m^2 \left[\alpha \cdot \dot{\pi}_i^2 + \beta \cdot (\vec{\nabla} \pi_i)^2 \right]$$

DIFFERENT FOR DIFFERENT POLARISATIONS

$$E_{UV} = \sqrt{M_{Pl} \cdot m} \Rightarrow \ell_{UV} = 0.1 \text{ mm}.$$

- No consistent model producing this mass pattern known yet

⇓

cosmology not studied.

But speculation:

Accelerated expansion of Universe may tell us that Lorentz invariance is violated

and

Something happens to gravity at $\ell_{UV} \simeq 0.1$ mm.

Extra Dimensions may help!

TO SUMMARIZE:

EXTRA DIMENSIONS + BRANE WORLD

- OFFER FRAMEWORK TO ASK EXOTIC QUESTIONS (ENERGY/CHARGE NON-CONSERVATION, PROPERTIES OF CONFORMAL MATTER, VIOLATION OF NEWTON'S LAW...)
- SUGGEST A VARIETY OF EXOTIC PHENOMENA BOTH AT HIGH AND LOW ENERGIES

  WHETHER ANY OF THOSE WILL BE FOUND IS A MATTER OF EFFORT AND LUCK.

Reviews on various aspects of extra dimensions/brane worlds

Incomplete list; please tell if you know others

Inverse temporal order; many published in journals/Proceedings

- R. Maartens, Brane world gravity gr-qc/0312059
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