The Abdus Salam
International Centre for Theoretical Physics

United Nations
Educational, Scientific
and Cultural Organization

International Atomic
Energy Agency

SMR.1663- 12

SUMMER SCHOOL ON PARTICLE PHYSICS

13 - 24 June 2005

Neutrino Physics - Part 2

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Lecture 2: Neutrino Mixing

John Beacom, The Ohio State University

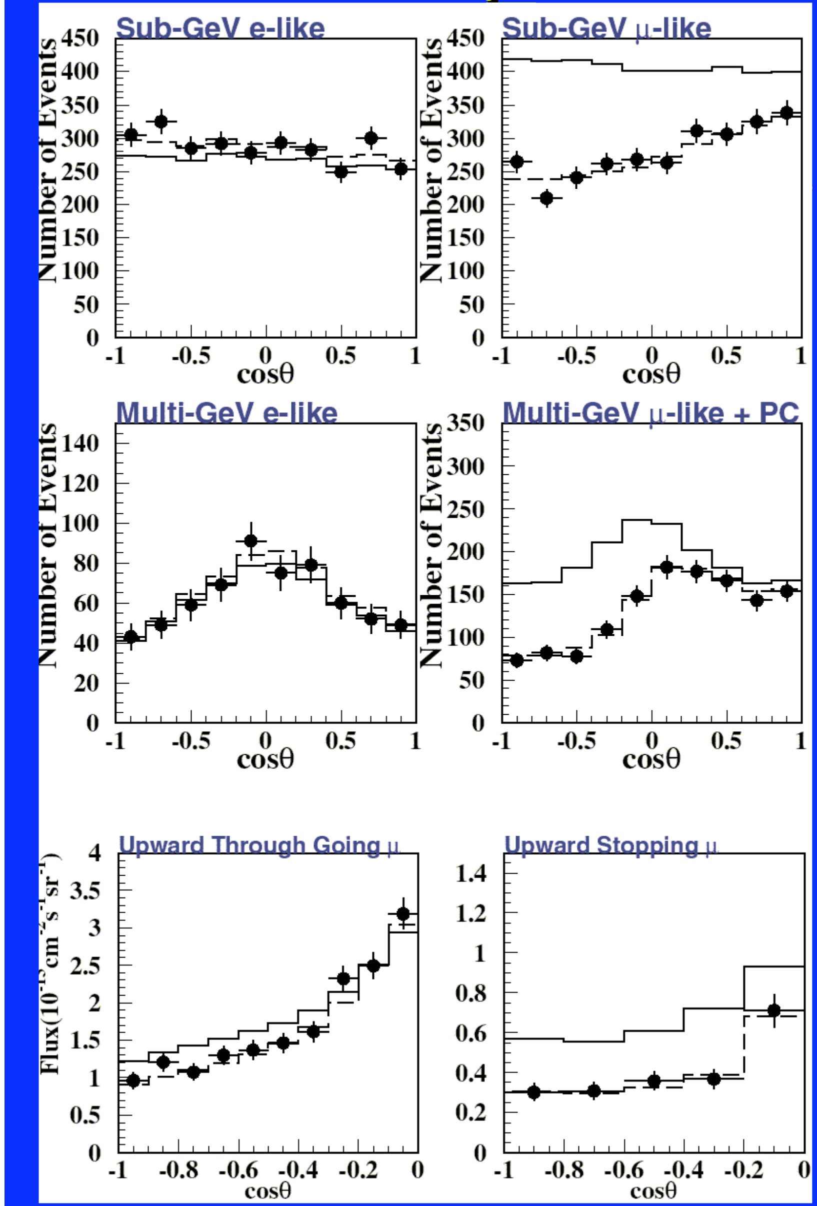
Elevator Pitch

- Neutrino mixing in vacuum is almost certain
- Neutrino mixing in matter is somewhat certain
- Smoking gun signatures are still needed
- As are precision mixing parameter measurements
- These are keys to understanding neutrino mass

Neutrino Vacuum Mixing

(On the blackboard)

Atmospheric Neutrino Results



Super – Kamiokande :

$\sin^2 2\theta \simeq 1$

$\delta m^2 \simeq 3 \times 10^{-3}\ \mathrm{eV}^2$

$\nu_\mu \to \nu_\tau$ preferred

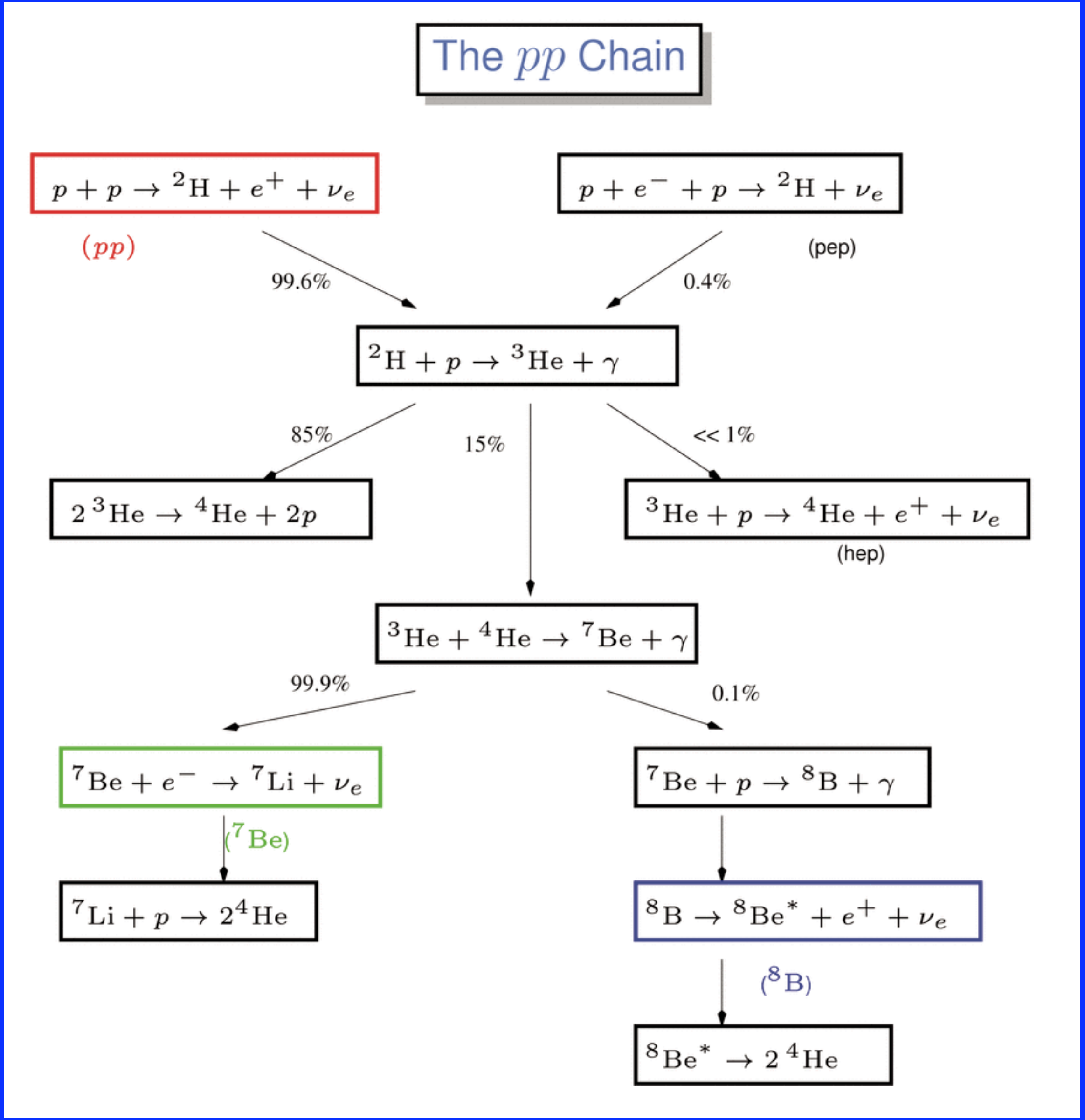
Soudan-2, MACRO, …

K2K, MINOS, off-axis, JHF, …

HK, UNO, …

Solar Neutrinos

Solar Nuclear Reactions



The pp Chain

$p + p \rightarrow {}^2\mathrm{H} + e^+ + \nu_e$

(pp)

$p + e^- + p \rightarrow {}^2\mathrm{H} + \nu_e$

(pep)

99.6%

0.4%

${}^2\mathrm{H} + p \rightarrow {}^3\mathrm{He} + \gamma$

85%

15%

<< 1%

$2\,{}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + 2p$

${}^3\mathrm{He} + p \rightarrow {}^4\mathrm{He} + e^+ + \nu_e$

(hep)

${}^3\mathrm{He} + {}^4\mathrm{He} \rightarrow {}^7\mathrm{Be} + \gamma$

99.9%

0.1%

${}^7\mathrm{Be} + e^- \rightarrow {}^7\mathrm{Li} + \nu_e$

$({}^7\mathrm{Be})$

${}^7\mathrm{Li} + p \rightarrow 2\,{}^4\mathrm{He}$

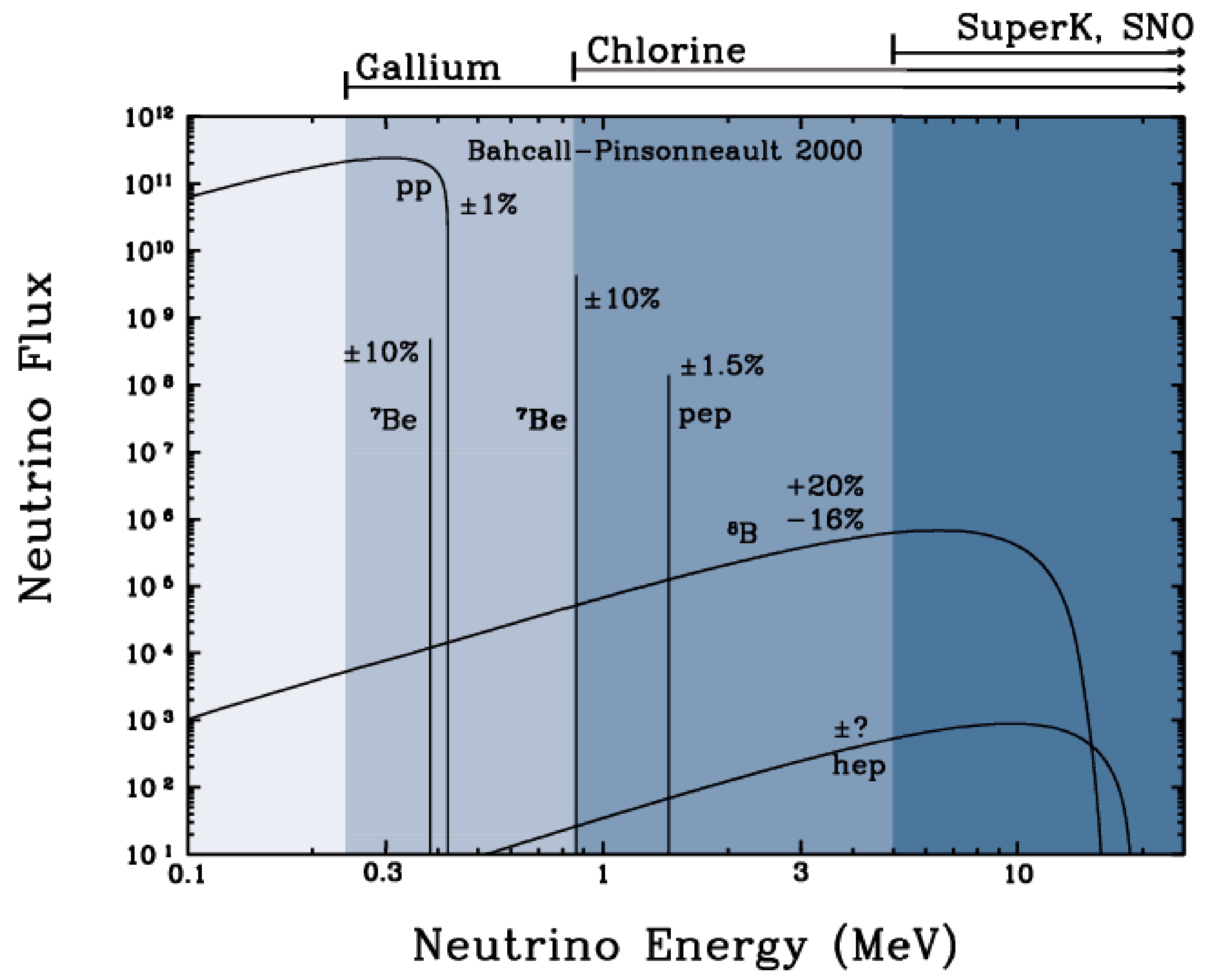
${}^7\mathrm{Be} + p \rightarrow {}^8\mathrm{B} + \gamma$

${}^8\mathrm{B} \rightarrow {}^8\mathrm{Be}^* + e^+ + \nu_e$

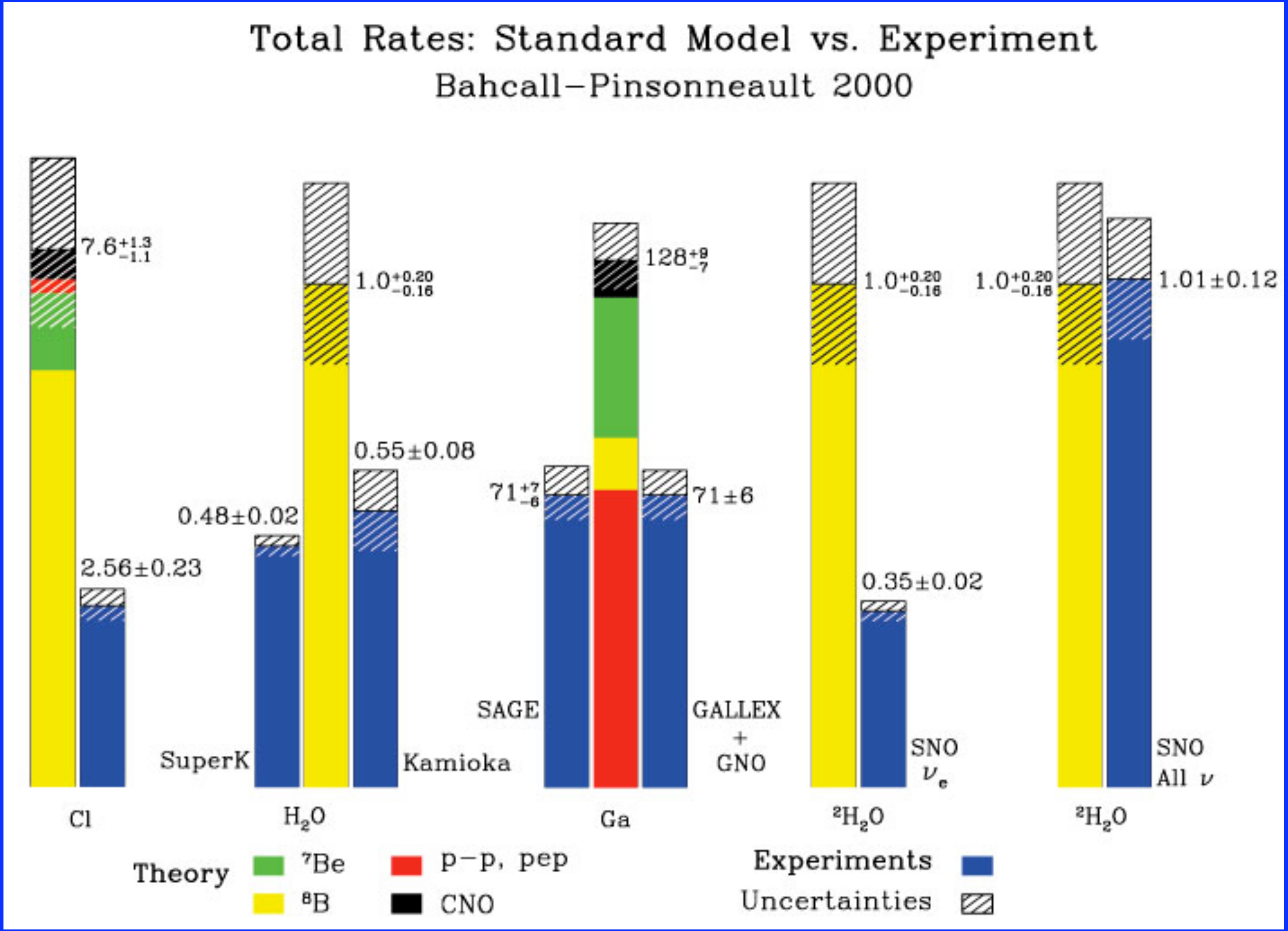
$({}^8\mathrm{B})$

${}^8\mathrm{Be}^* \rightarrow 2\,{}^4\mathrm{He}$

Solar Neutrino Fluxes



The Solar Neutrino Problem



Sudbury Neutrino Observatory

Inside SNO

SNO Detection Reactions

$$\mathrm{CC:}\quad \nu_e + d \rightarrow \boxed{e^-} + p + p$$

measures $\boxed{\phi_{\nu_e}}$

where $E_\nu \simeq E_e$ + few MeV

$$\mathrm{NC:}\quad \nu + d \rightarrow \nu + p + \boxed{n}$$

measures $\boxed{\phi_{\nu_e} + \phi_{\nu_\mu}}$

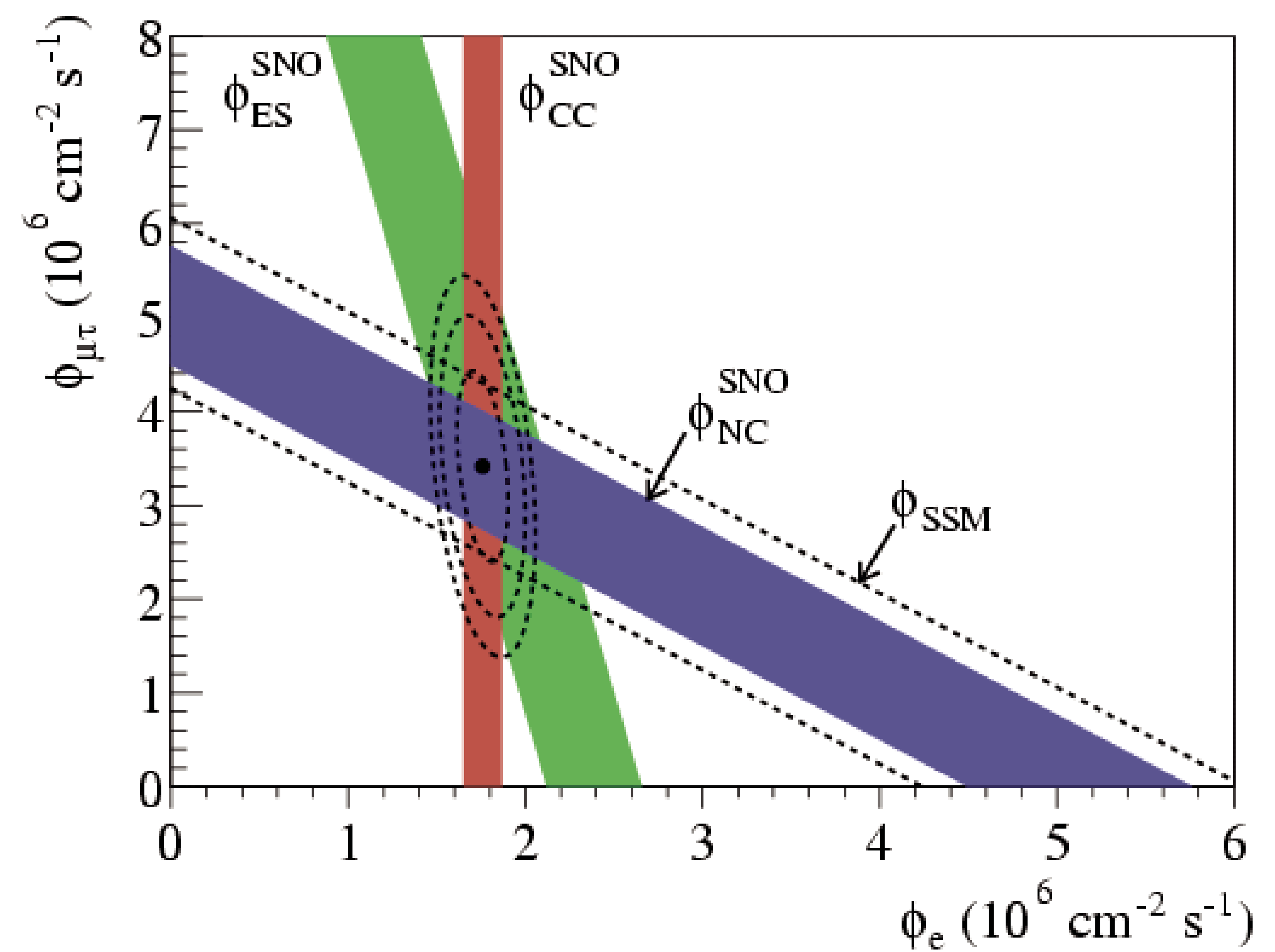
where E_ν unknown

$$\mathrm{ES:}\quad \nu + e^- \rightarrow \nu + \boxed{e^-}$$

measures $\boxed{\phi_{\nu_e} + \frac{1}{6}\phi_{\nu_\mu}}$

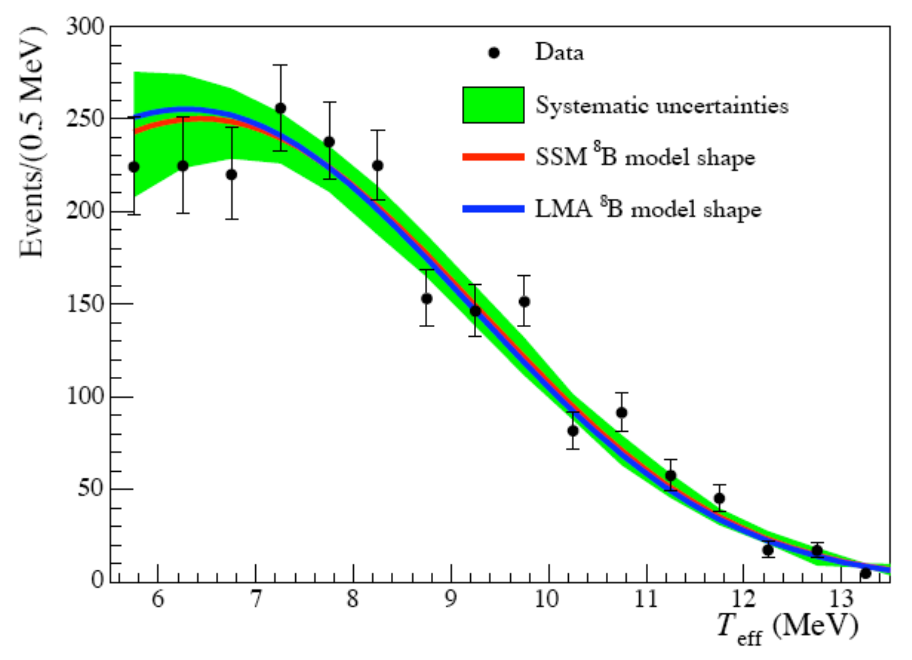
where $E_\nu \sim 2E_e$

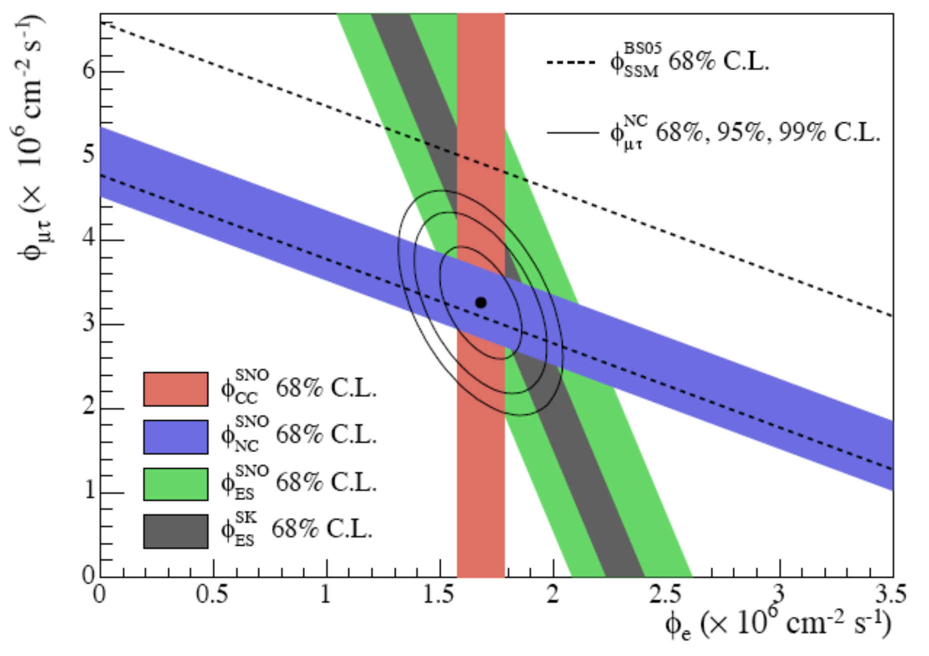
SNO Flux Measurements



$$\varphi_{\nu_e} = \frac{1}{3} \qquad \varphi_{\nu_e} + \varphi_{\nu_\mu} = 1$$

Latest SNO Results



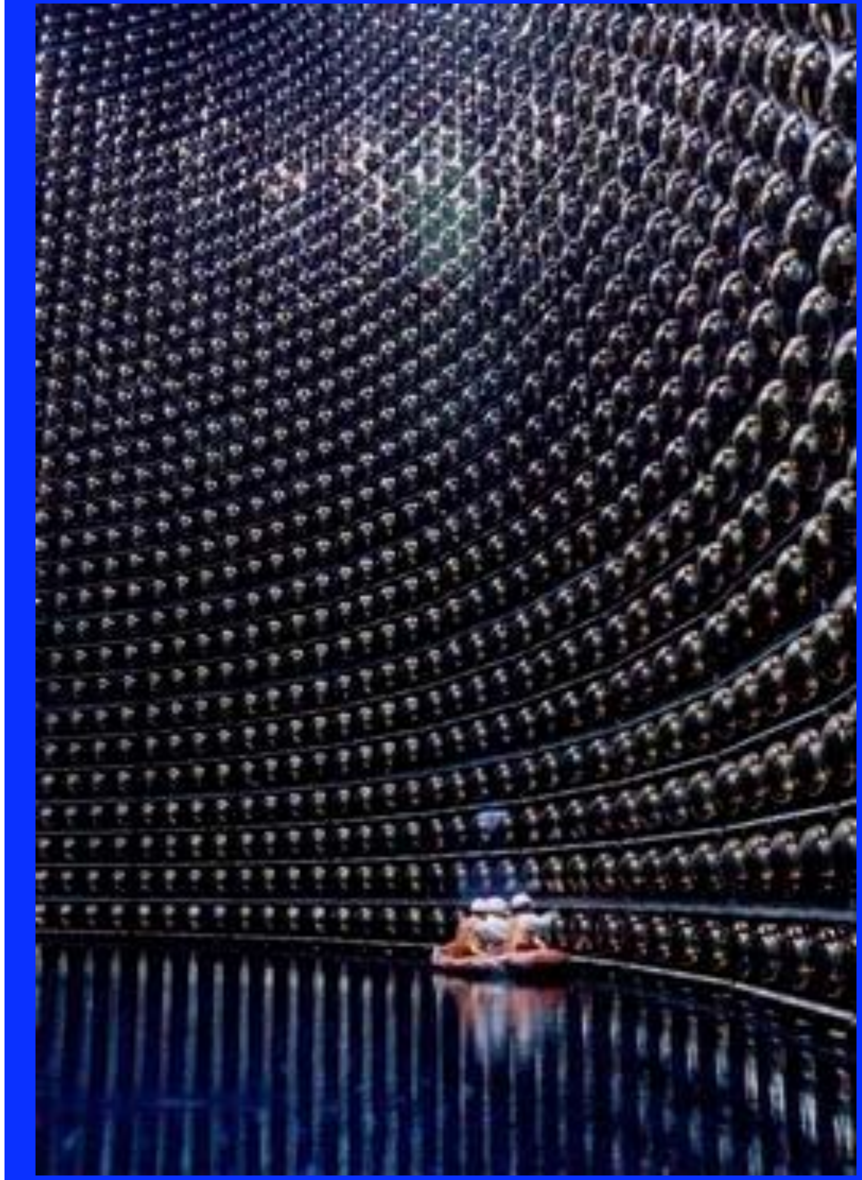


Aharmim et al, nucl-ex/0502021

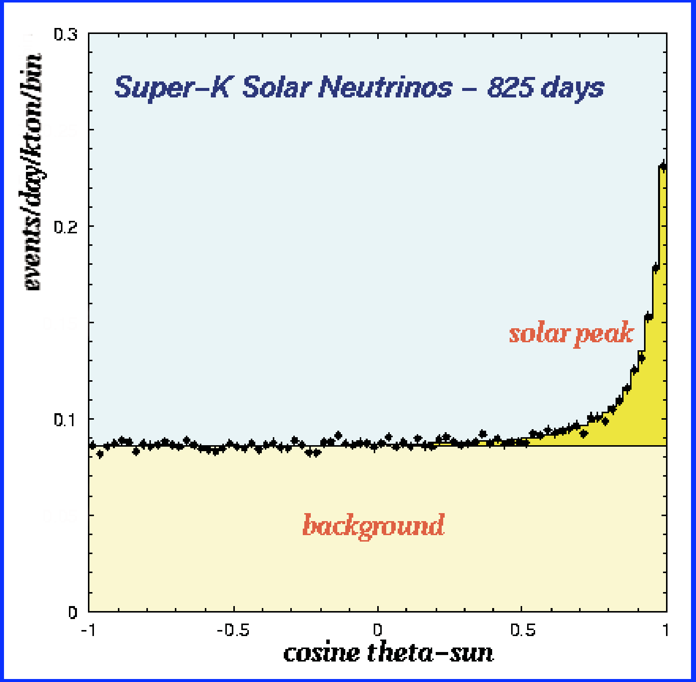
Still NO evidence of:

spectral distortion or day-night effect!

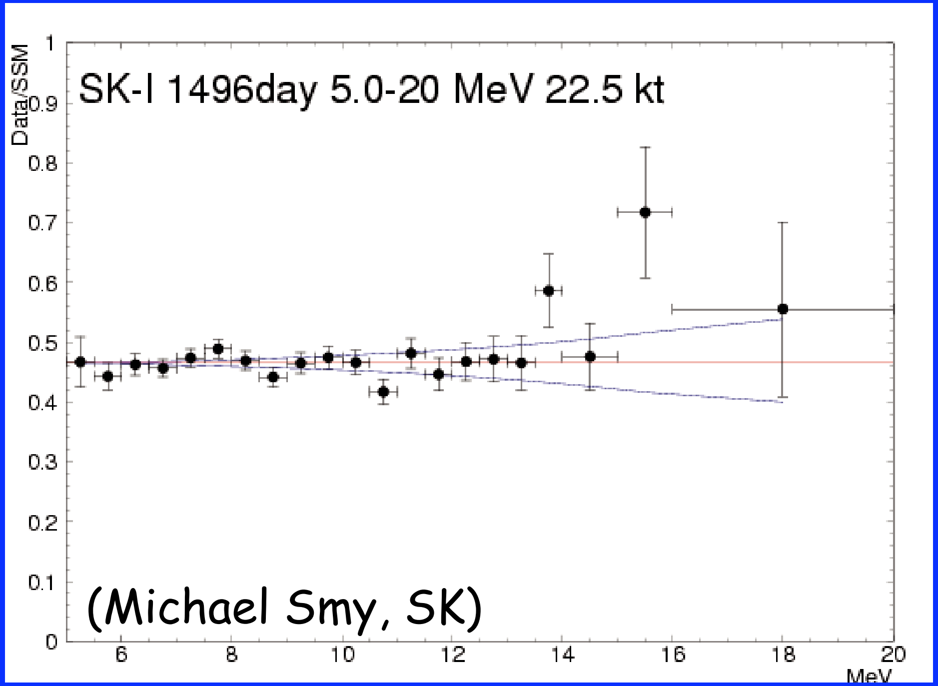
Super-Kamiokande



$$\nu + e^- \rightarrow \nu + e^-$$



Super-Kamiokande Results



(Michael Smy, SK)

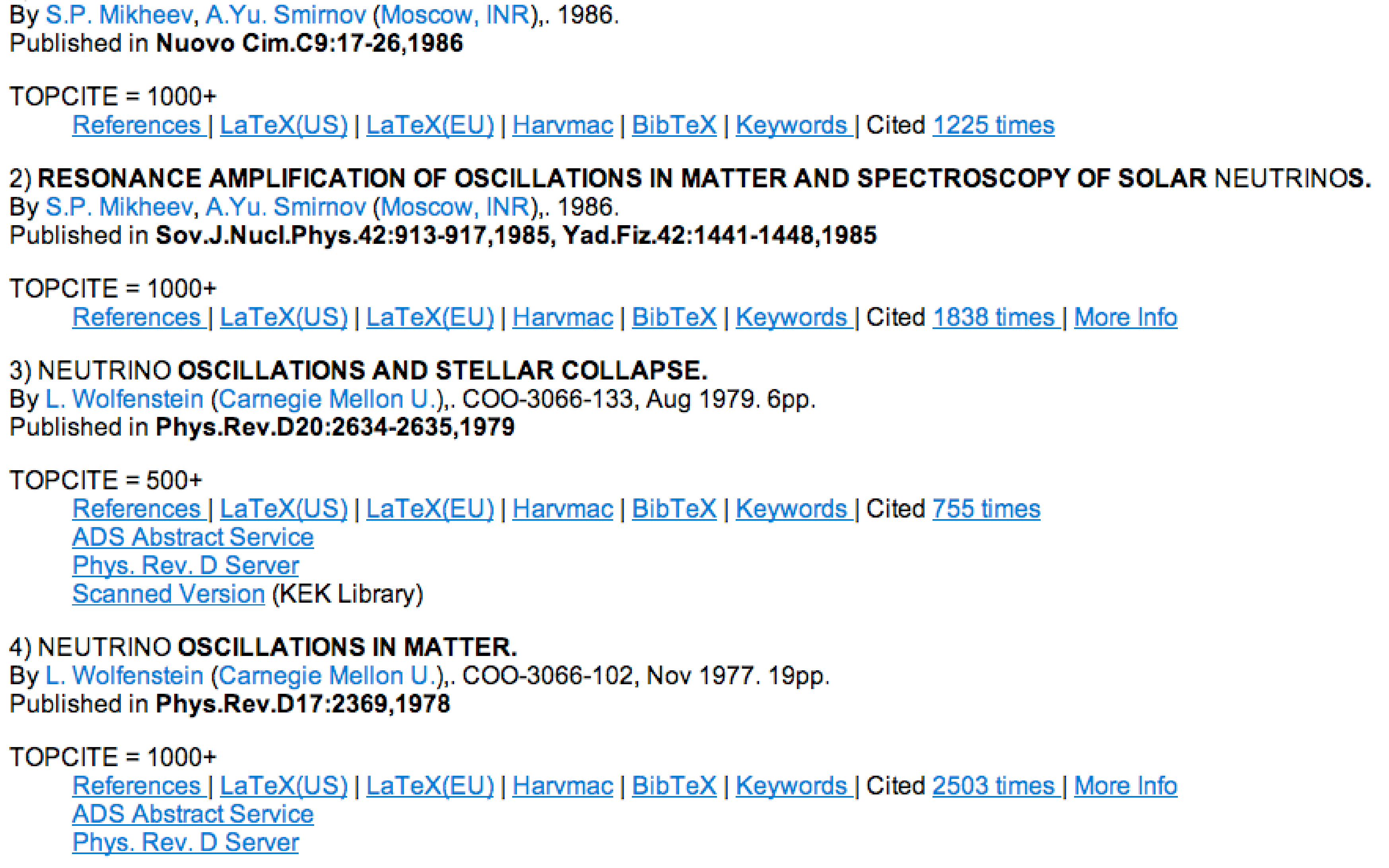
$$\nu + e^- \rightarrow \nu + e^-$$

$$\text{Ratio} = \frac{\text{measured } e^- \text{ spectrum}}{\text{expected } e^- \text{ spectrum}}$$

$$\boxed{\varphi_{\nu_e} + \tfrac{1}{6}\varphi_{\nu_\mu} = 0.34 + \tfrac{1}{6}0.66 = 0.45}$$

Perfect Agreement!

MS & W Papers



1) **RESONANT AMPLIFICATION OF** NEUTRINO **OSCILLATIONS IN MATTER AND SOLAR** NEUTRINO **SPECTROSCOPY.**
By S.P. Mikheev, A.Yu. Smirnov (Moscow, INR),. 1986.
Published in **Nuovo Cim.C9:17-26,1986**

TOPCITE = 1000+
References | LaTeX(US) | LaTeX(EU) | Harvmac | BibTeX | Keywords | Cited 1225 times

2) **RESONANCE AMPLIFICATION OF OSCILLATIONS IN MATTER AND SPECTROSCOPY OF SOLAR** NEUTRINOS.
By S.P. Mikheev, A.Yu. Smirnov (Moscow, INR),. 1986.
Published in **Sov.J.Nucl.Phys.42:913-917,1985, Yad.Fiz.42:1441-1448,1985**

TOPCITE = 1000+
References | LaTeX(US) | LaTeX(EU) | Harvmac | BibTeX | Keywords | Cited 1838 times | More Info

3) NEUTRINO **OSCILLATIONS AND STELLAR COLLAPSE.**
By L. Wolfenstein (Carnegie Mellon U.),. COO-3066-133, Aug 1979. 6pp.
Published in **Phys.Rev.D20:2634-2635,1979**

TOPCITE = 500+
References | LaTeX(US) | LaTeX(EU) | Harvmac | BibTeX | Keywords | Cited 755 times
ADS Abstract Service
Phys. Rev. D Server
Scanned Version (KEK Library)

4) NEUTRINO **OSCILLATIONS IN MATTER.**
By L. Wolfenstein (Carnegie Mellon U.),. COO-3066-102, Nov 1977. 19pp.
Published in **Phys.Rev.D17:2369,1978**

TOPCITE = 1000+
References | LaTeX(US) | LaTeX(EU) | Harvmac | BibTeX | Keywords | Cited 2503 times | More Info
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Phys. Rev. D Server

MSW Effect

$$H_{\text{flavor}} = \frac{\delta m^2}{4E} \begin{bmatrix} \zeta(t) - \cos 2\theta_v & \sin 2\theta_v \\ \sin 2\theta_v & -\zeta(t) + \cos 2\theta_v \end{bmatrix}$$

$$\zeta(t) = \frac{2\sqrt{2} G_F N_e(t)}{\delta m^2 / E}$$

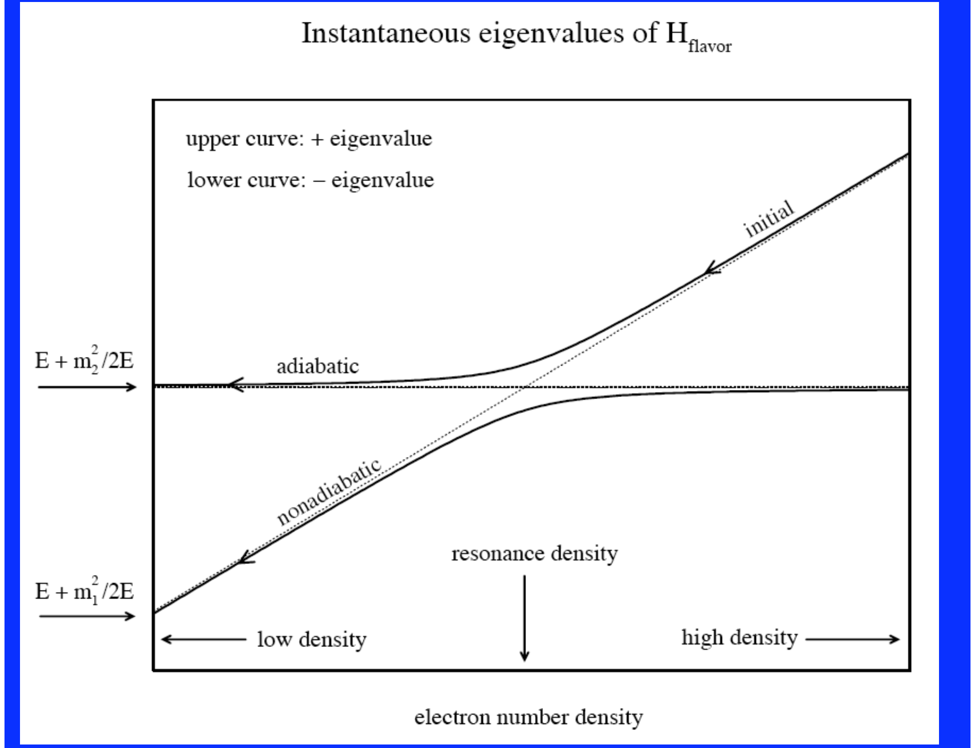
$$\begin{bmatrix} \Psi_1(t) \\ \Psi_2(t) \end{bmatrix} = \begin{bmatrix} \cos\theta(t) & -\sin\theta(t) \\ \sin\theta(t) & \cos\theta(t) \end{bmatrix} \begin{bmatrix} \Psi_e(t) \\ \Psi_\mu(t) \end{bmatrix}$$

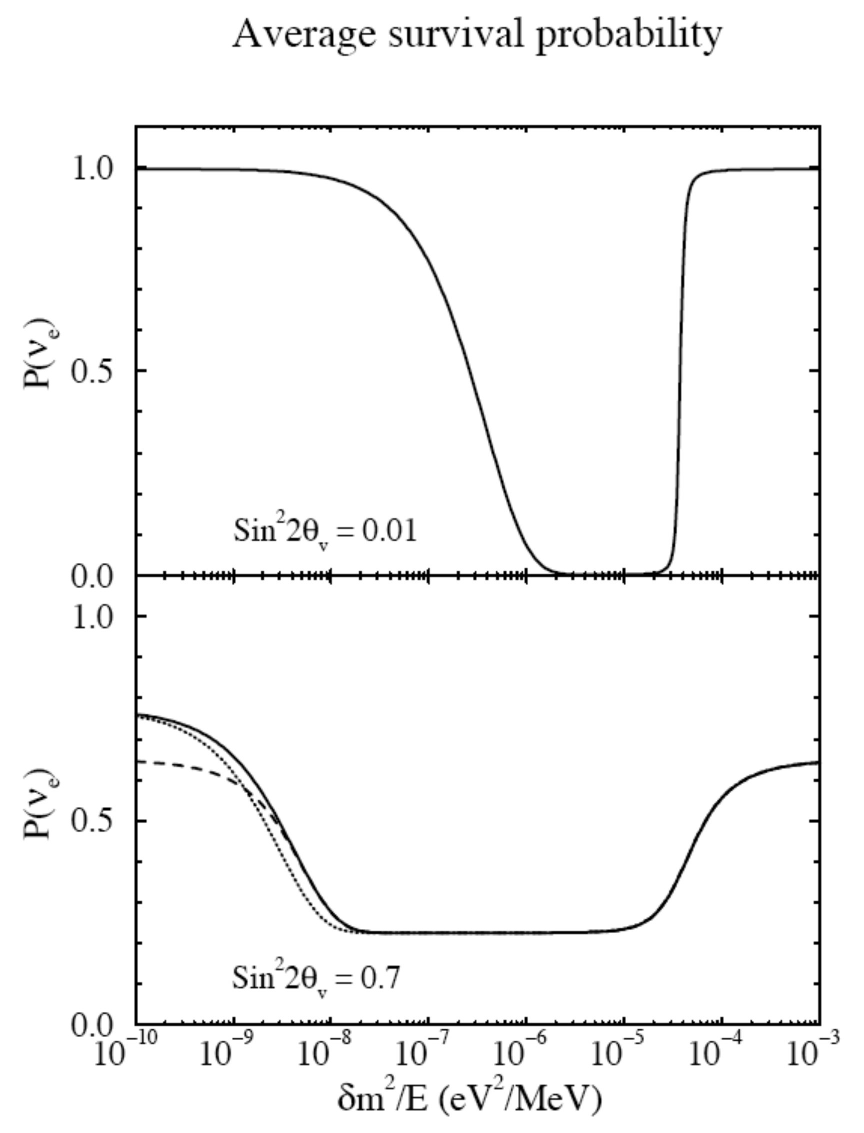
$$E + \frac{m_1^2 + m_2^2 + \delta m^2 \zeta(t)}{4E} \mp \frac{\delta m^2}{4E} \sqrt{\sin^2 2\theta_v + (\zeta(t) - \cos 2\theta_v)^2}$$

Parke,
PRL 1986:

$$P(\nu_e \to \nu_e) = \frac{1}{2}\left[1 + (1 - 2P_{\text{hop}}) \cos 2\theta_i \cos 2\theta_v\right]$$

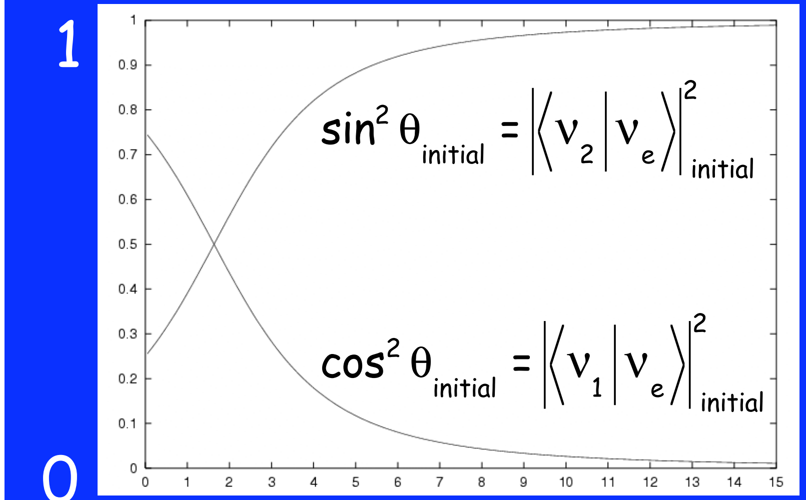
Neutrino Matter Mixing

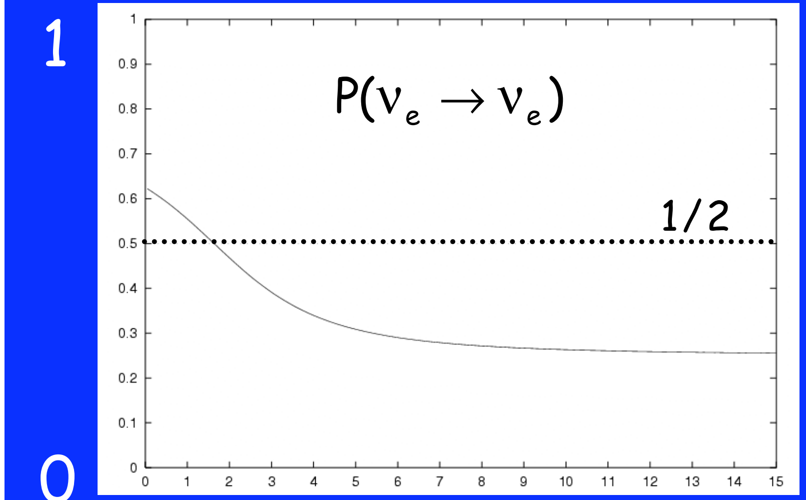




(graphics from Beacom and Balantekin, hep-th/9709117)

Solar Neutrinos Don't Oscillate





E in [0,15 MeV]

Matter effects $\sim \dfrac{G_F N_e}{\delta m^2 / E}$

Propagation is adiabatic

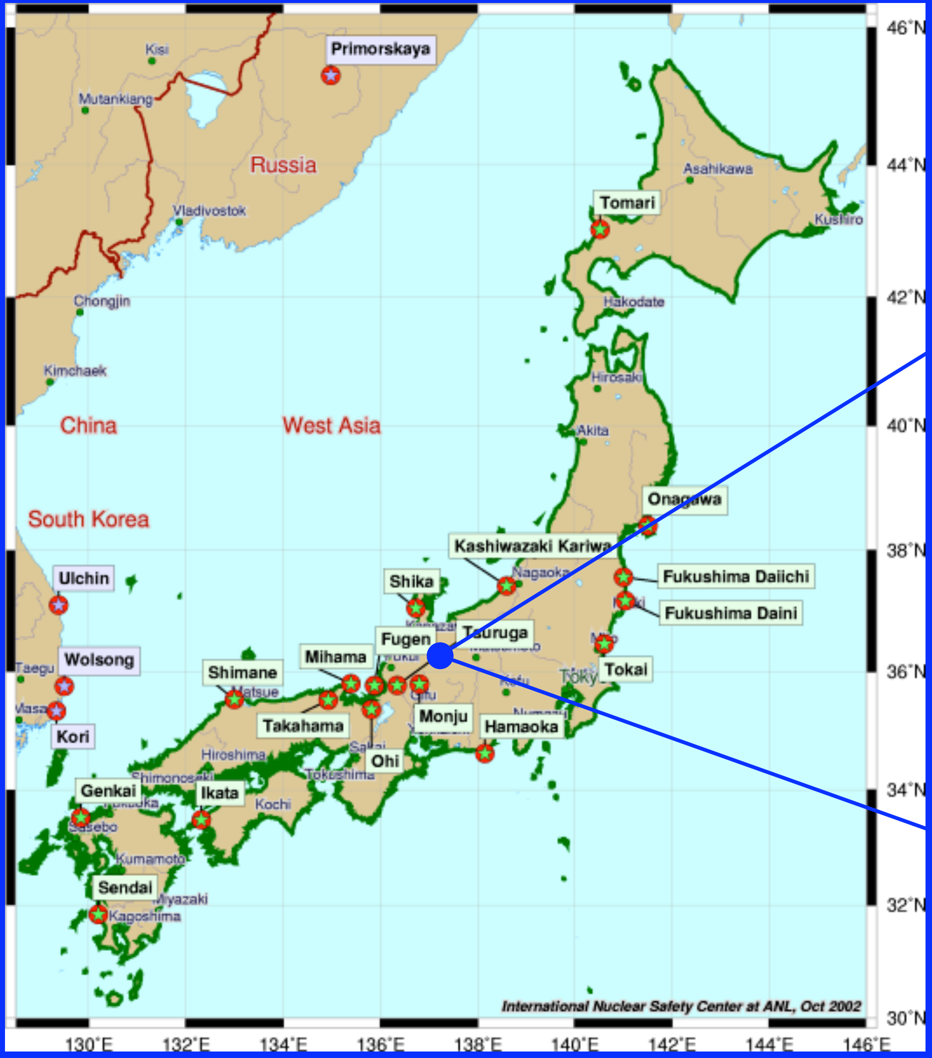
$$\varphi_{\nu_e} \sim \left|\langle \nu_e | \nu_2 \rangle\right|^2_{\text{vacuum}} = \sin^2\theta_{\text{vacuum}}$$

$$\varphi_{\nu_\mu} \sim \left|\langle \nu_\mu | \nu_2 \rangle\right|^2_{\text{vacuum}} = \cos^2\theta_{\text{vacuum}}$$

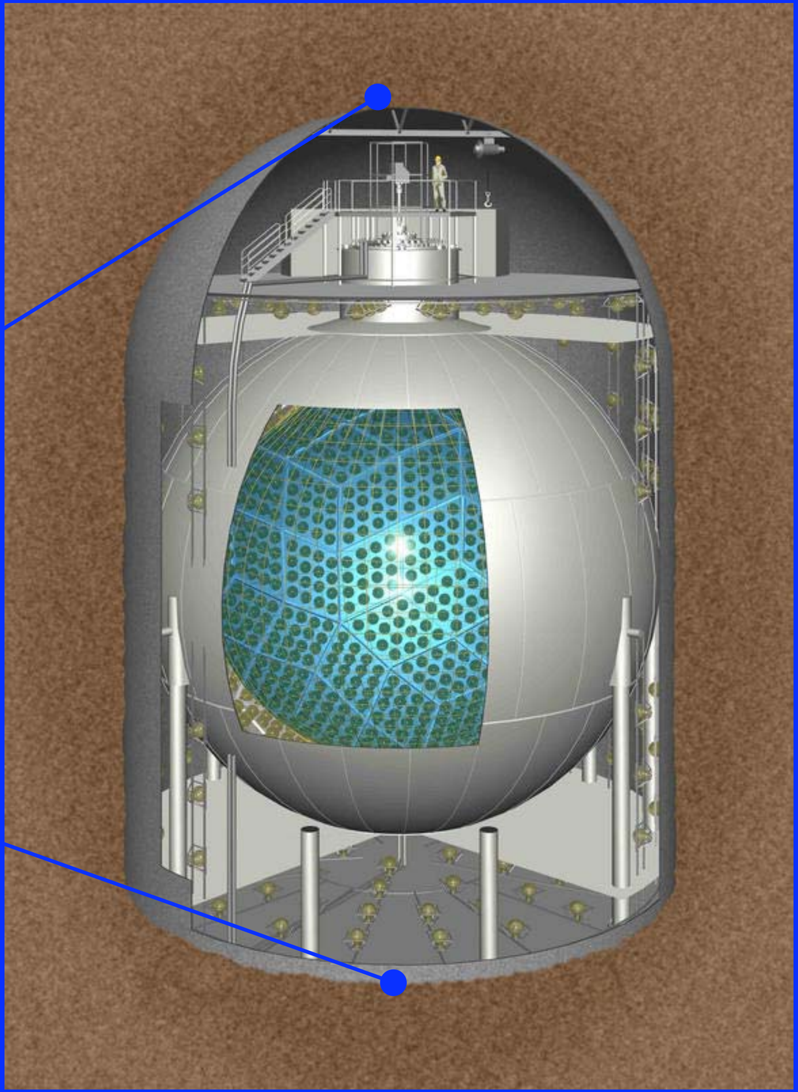
(above about 5 MeV)

Reactor Neutrinos

KamLAND

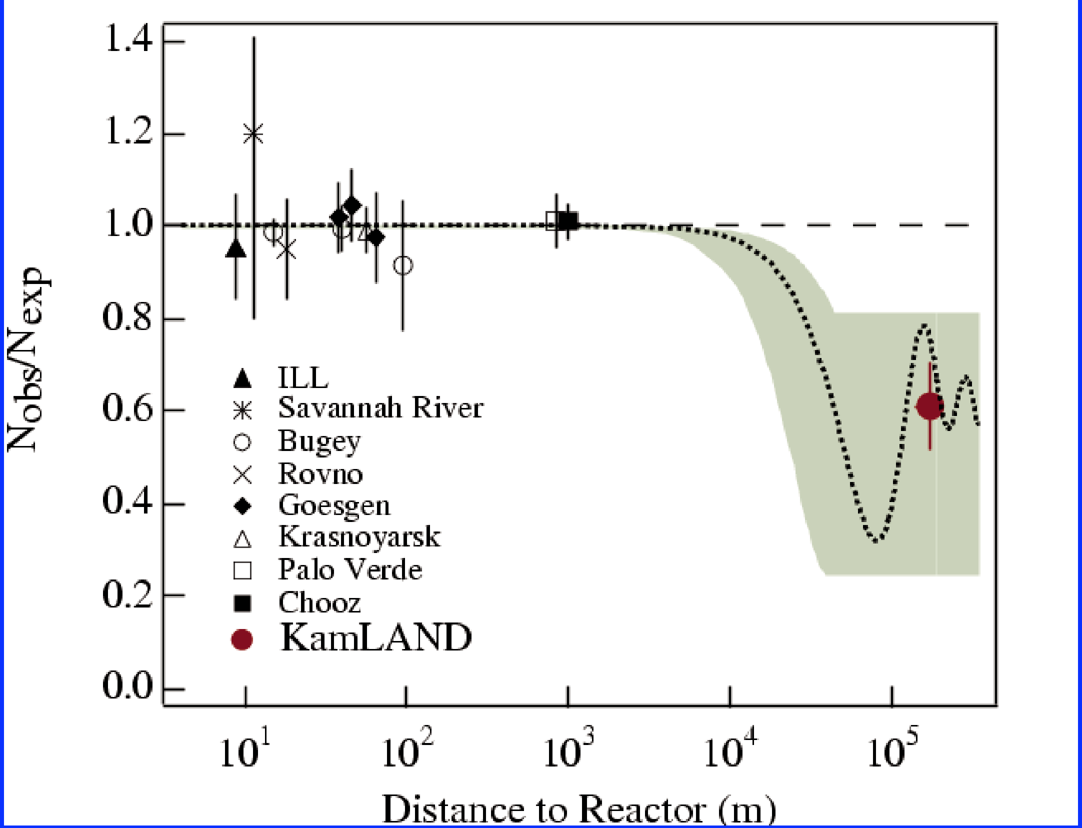


International Nuclear Safety Center at ANL, Oct 2002



Vacuum Oscillation Probability

$$P\left(\bar{\nu}_e \to \bar{\nu}_\mu\right) = \sin^2(2\theta)\ \sin^2\left(1.27\frac{\delta m^2 L}{E}\right)$$



$\sin^2\theta_{solar}$ is *large*

$\sin^2\theta_{13}$ is *small*

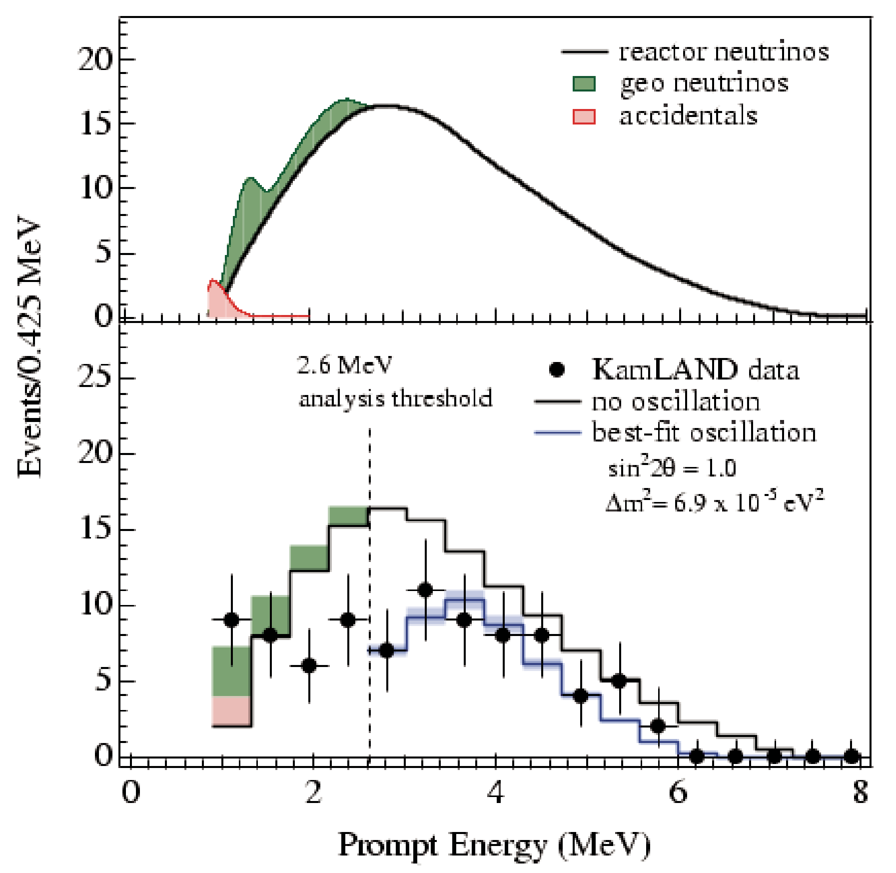
$\delta m^2 \simeq 5\times10^{-5}\ \mathrm{eV}^2$

$E \simeq 4$ MeV

$\Rightarrow$ Detector at $\simeq 10^5$ m

Eguchi et al., PRL 90, 021802 (2003)

KamLAND Neutrino Detection



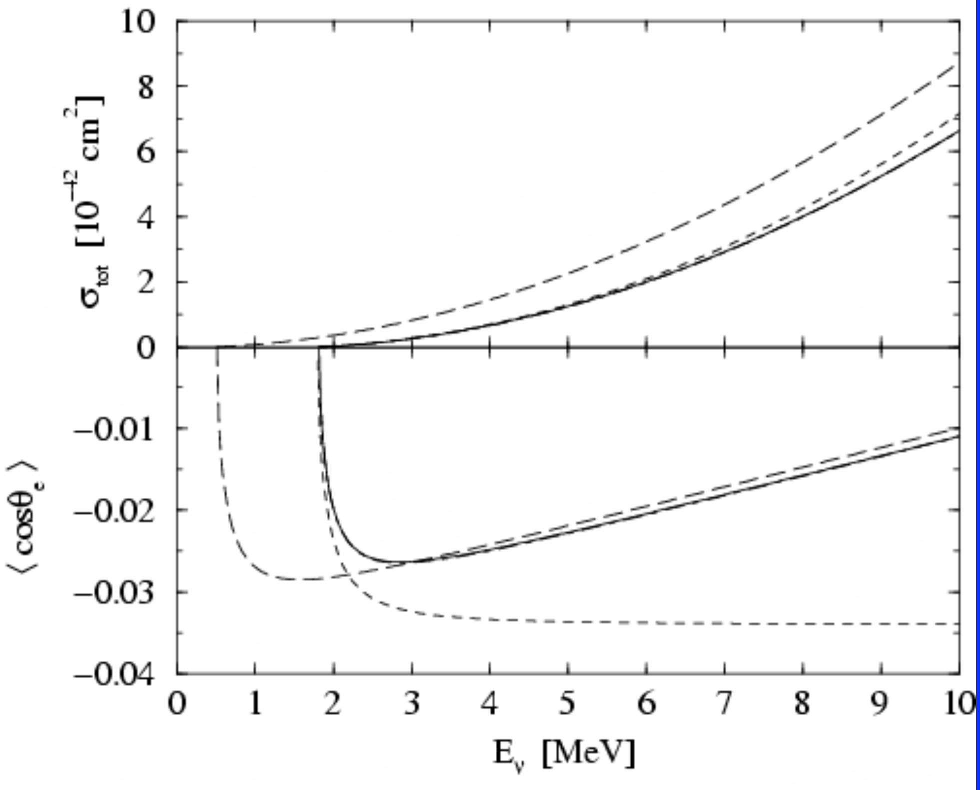
$$\bar{\nu}_e + p \rightarrow \boxed{e^+} + n$$

$$n + p \rightarrow d + \boxed{\gamma}$$

Detection rate
~ 1 event per day!

Eguchi et al., PRL 90, 021802 (2003)

Inverse Beta Cross Section



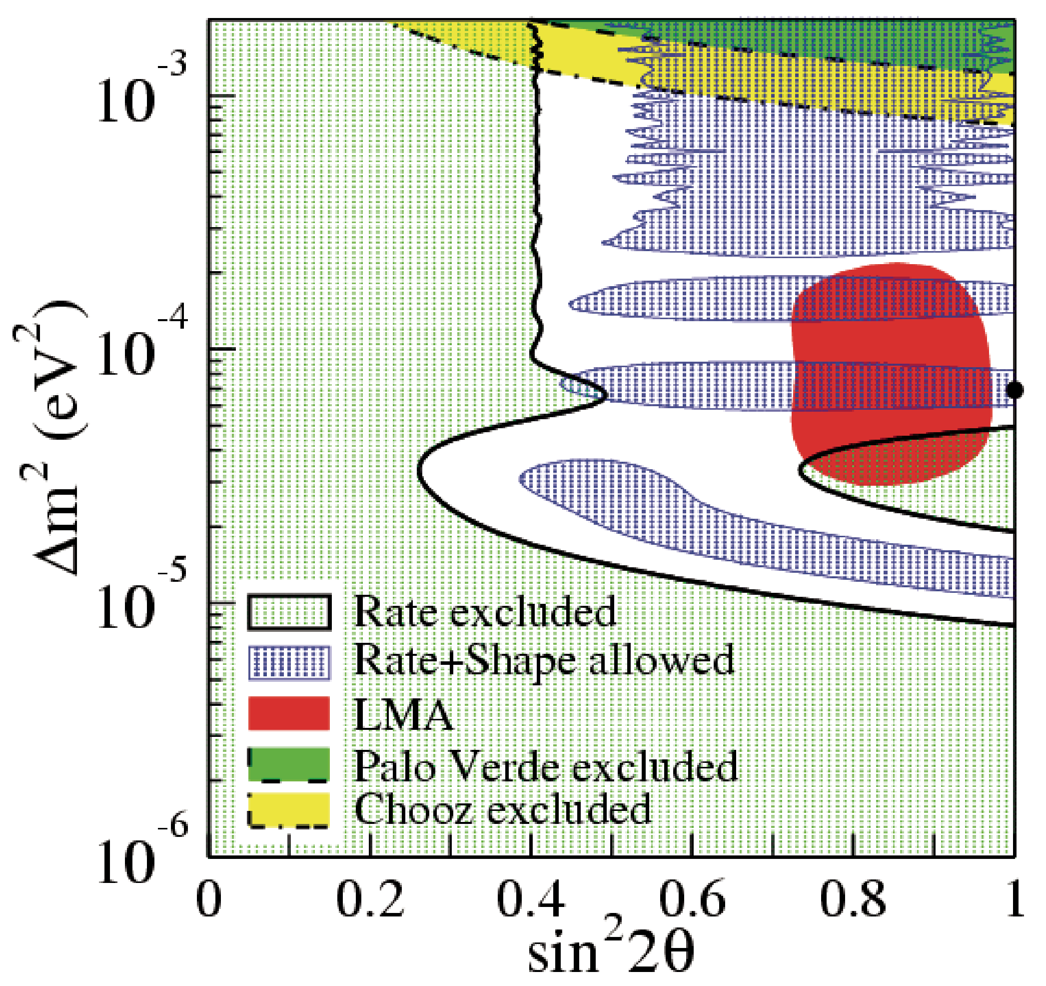
$$\sigma^{(0)} = \frac{2\pi^2 / m_e^5}{f^R \tau_n} E_e^{(0)} p_e^{(0)}$$

$$E_e^{(0)} = E_\nu - (M_n - M_p)$$

Corrections of order $1/M_p$ are very important

Vogel and Beacom, PRD 60, 053003 (1999)

KamLAND Oscillation Results



Very different technique

No solar physics

No MSW effects

$\bar{\nu}_e$, not ν_e

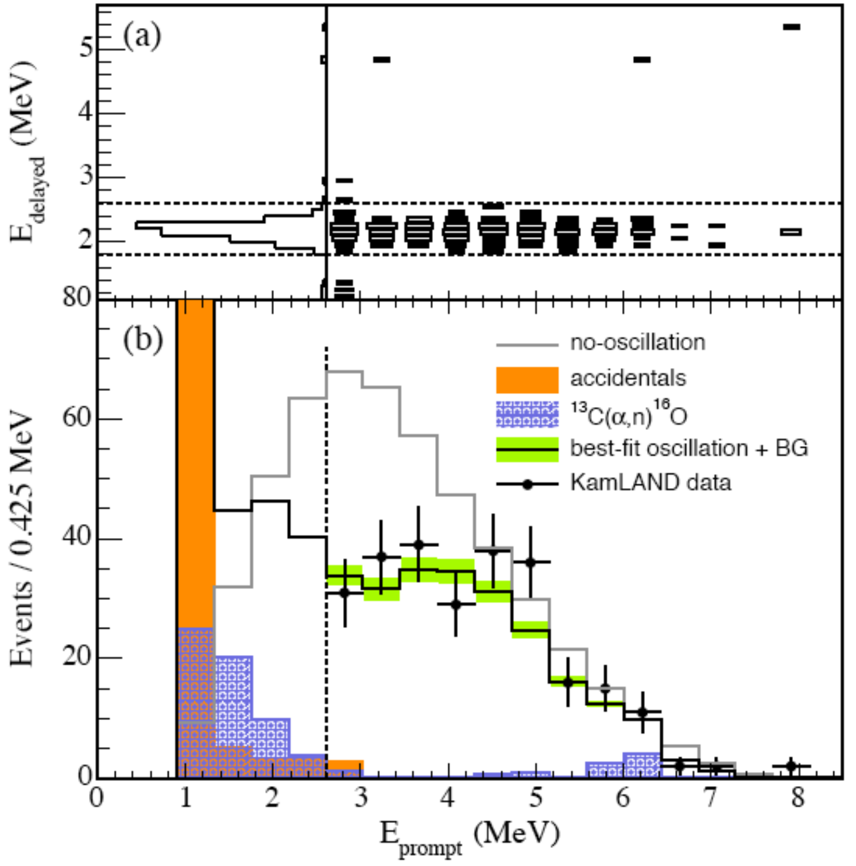
No neutrino decay

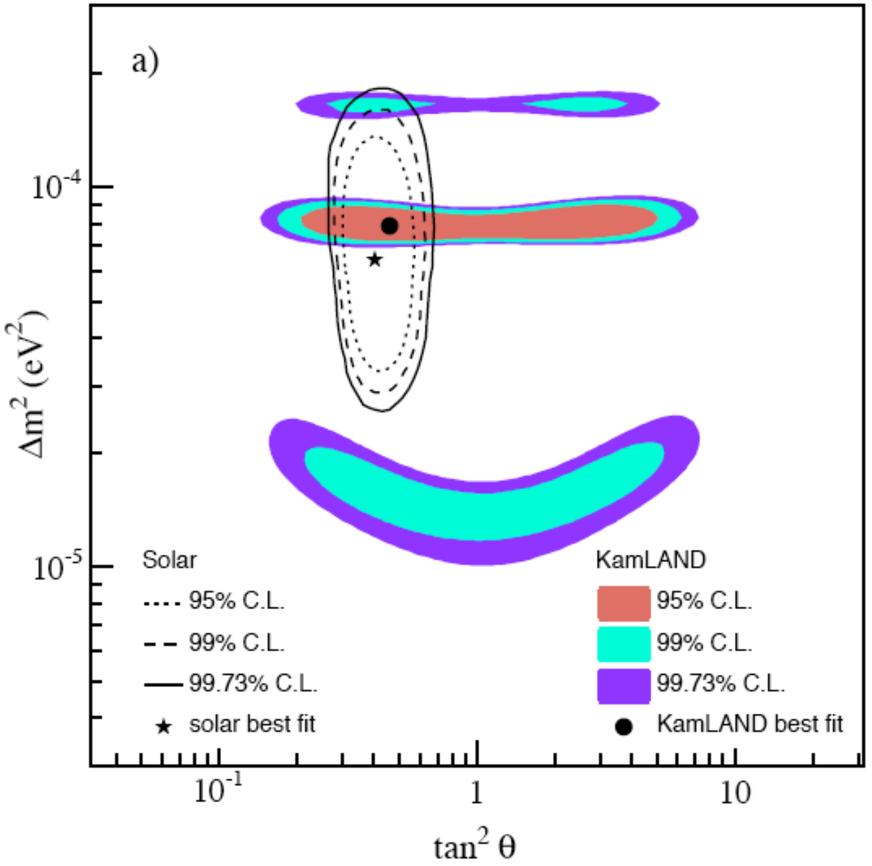
No magnetic fields

No matter FCNC

Eguchi et al., PRL 90, 021802 (2003)

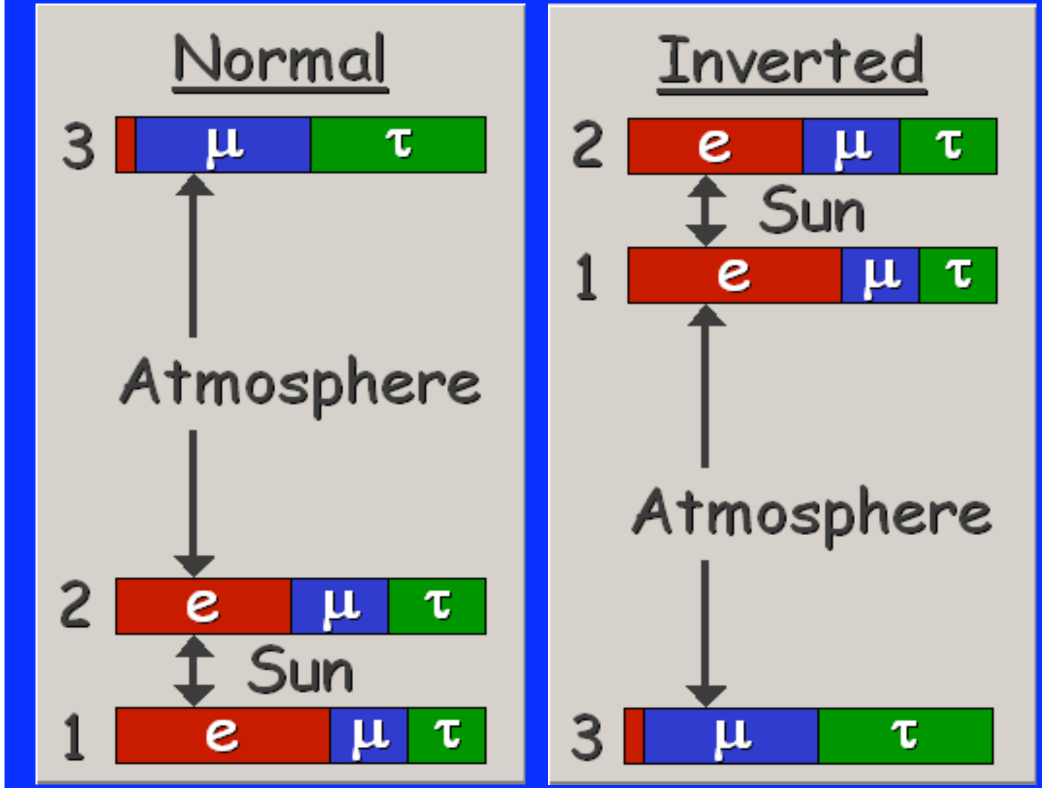
New KamLAND Data





Araki et al., PRL 94, 081801 (2005)

Neutrino Mixing



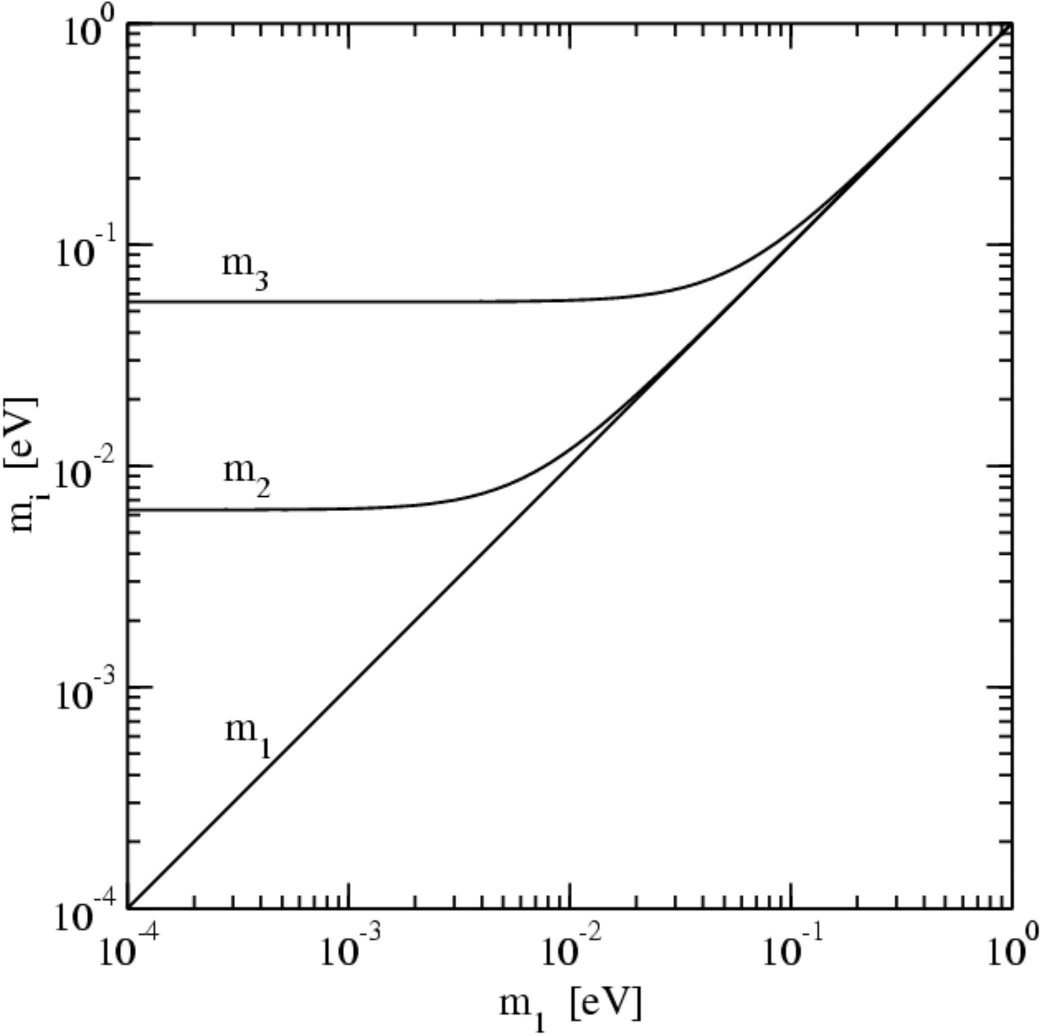
(graphic from Georg Raffelt)

$$\begin{bmatrix}\nu_e\\ \nu_\mu\\ \nu_\tau\end{bmatrix} = \begin{bmatrix}U_{\alpha j}\end{bmatrix}\begin{bmatrix}\nu_1\\ \nu_2\\ \nu_3\end{bmatrix}$$

$$U \simeq \begin{bmatrix} c_\odot & s_\odot & s_{13}e^{-i\delta} \\ -s_\odot/\sqrt{2} & c_\odot/\sqrt{2} & 1/\sqrt{2} \\ s_\odot/\sqrt{2} & -c_\odot/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$$\theta_{atm} \simeq 45^\circ, \quad \theta_{solar} \simeq 35^\circ, \quad \theta_{13} \leq 10^\circ$$

Neutrino Mass Splittings



Beacom and Bell, PRD 65, 113009 (2002)

Normal Hierarchy

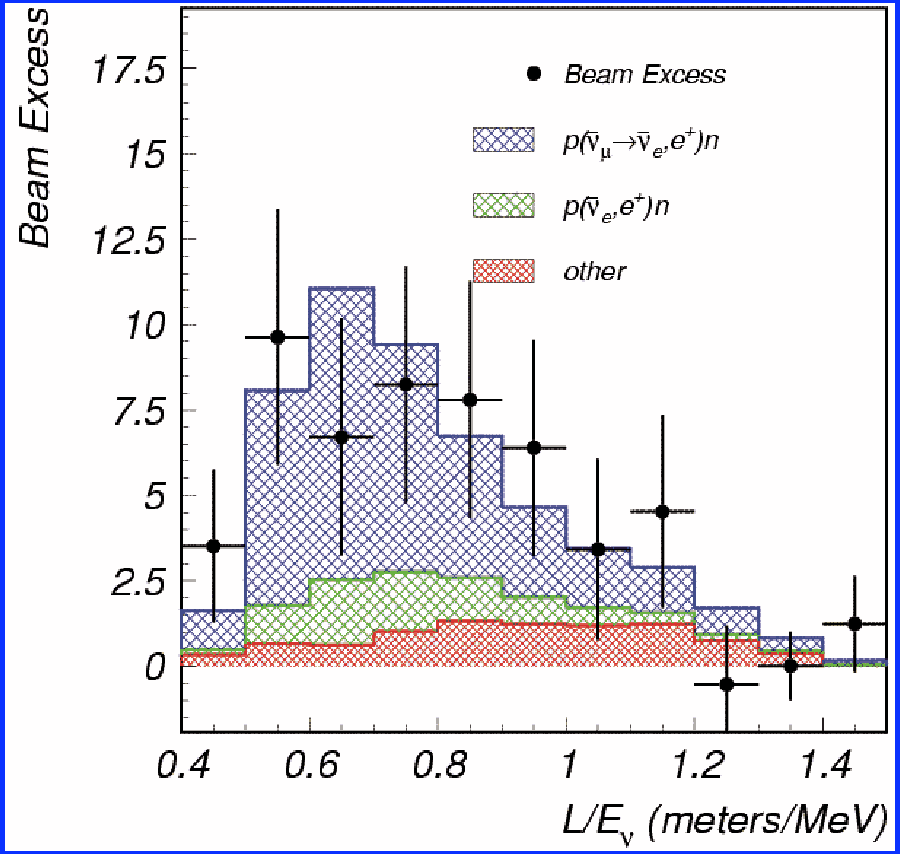
$$m_1 = m_1$$

$$m_2 = \sqrt{m_1^2 + \delta m^2_{solar}}$$

$$m_3 = \sqrt{m_1^2 + \delta m^2_{solar} + \delta m^2_{atm}}$$

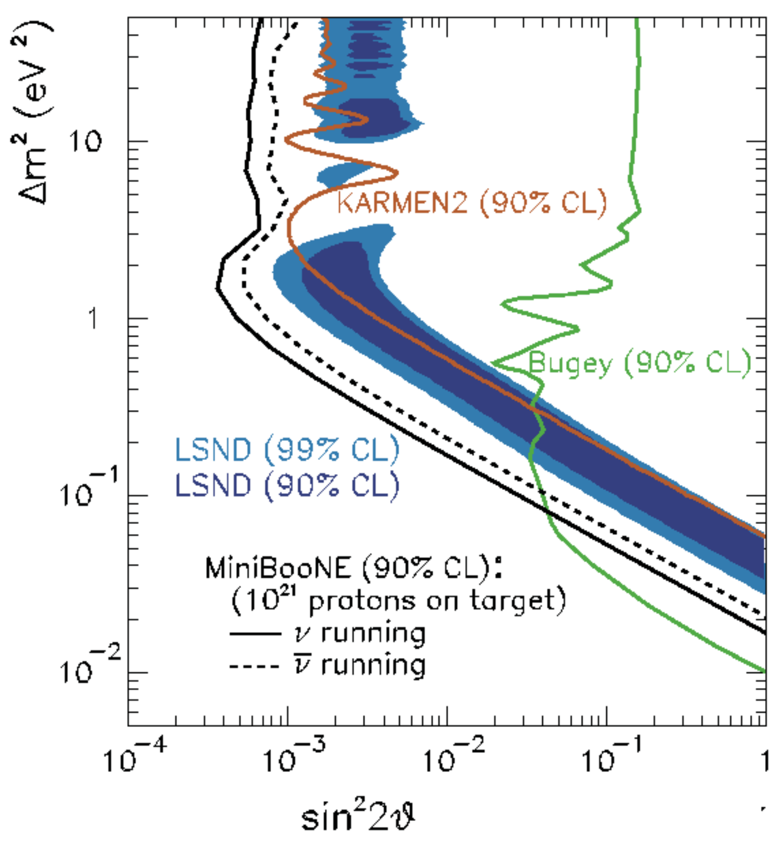
$$\frac{m_3}{m_2} \leq \frac{\sqrt{\delta m^2_{atm}}}{\sqrt{\delta m^2_{solar}}} \leq 10$$

What About LSND?



LSND :

$\bar{\nu}_\mu \to \bar{\nu}_e$ appearance signal



MiniBooNE :

$\nu_\mu \to \nu_e$ appearance search

Conclusions

- The atmospheric neutrino problem was solved by neutrino oscillations

- And the solar neutrino problem was solved by neutrino oscillations

- Ok, well, just almost sure...

- Essential to complete the mixing parameter measurements and to find the final answers on the above claims (and LSND)!

Further Reading

- See yesterday's list
- Smirnov, *all of his papers*

- Mass eigenstates:

$$i\frac{\partial}{\partial t}\begin{bmatrix}\psi_1(t)\\ \psi_2(t)\end{bmatrix} = H_{mass}\begin{bmatrix}\psi_1(t)\\ \psi_2(t)\end{bmatrix}$$

$$H_{mass} = \begin{bmatrix}E_1 & 0\\ 0 & E_2\end{bmatrix}$$

~~[illegible]~~

$$|\nu(t)\rangle = \psi_1(t)\,|\nu_1\rangle + \psi_2(t)\,|\nu_2\rangle$$

fixed, orthonormal kets

$$\begin{aligned}\text{Prob}(m_1) &= \left|\langle\nu_1|\nu(t)\rangle\right|^2\\ &= |\psi_1(t)|^2 = \left|e^{-iE_1t}\right|^2|\psi_1(0)|^2\\ &= |\psi_1(0)|^2\end{aligned}$$

Born in M.E., stay in M.E.

No off-diagonal couplings.

$$E_1 \simeq E + \frac{M_1^2 + M_2^2}{4E} - \frac{\delta m^2}{4E}$$

$$E_2 \simeq E + \frac{M_1^2 + M_2^2}{4E} + \frac{\delta m^2}{4E}$$

$$\delta m^2 = m_2^2 - M_1^2$$

$E \simeq p$, expand at lowest order around common momentum

$$H_{mass} \simeq \left(E + \frac{M_1^2 + M_2^2}{4E} \right) \mathbb{1} + \frac{\delta m^2}{4E} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\psi_1(t) \sim e^{+i\delta m^2 t/4E} \psi_1(0)$$

$$\psi_2(t) \sim e^{-i\delta m^2 t/4E} \psi_2(0)$$

$$t \sim L/c \sim L$$

Easy, dull.

Neutrino masses basically irrelevant.

- Flavor basis:

Other bases related by unitary transformation, which is just a rotation in 2d.

Rotate amplitudes, with fixed kets.

Useful if states are <u>born</u> and <u>detected</u> in another basis.

~~$\begin{bmatrix} \psi_e \\ \psi_\mu \end{bmatrix} = R$~~

$$\begin{bmatrix} \psi_1(t) \\ \psi_2(t) \end{bmatrix} = R(\theta) \begin{bmatrix} \psi_e(t) \\ \psi_\mu(t) \end{bmatrix} = \begin{bmatrix} +\cos\theta & -\sin\theta \\ +\sin\theta & -\cos\theta \end{bmatrix} \begin{bmatrix} \psi_e(t) \\ \psi_\mu(t) \end{bmatrix}$$

Physical definition of this basis.

Steps to

$$i \frac{\partial}{\partial t} \begin{bmatrix} \psi_e(t) \\ \psi_\mu(t) \end{bmatrix} = \underbrace{A^\dagger H_{mass} A}_{H_{e\mu}} \begin{bmatrix} \psi_e(t) \\ \psi_\mu(t) \end{bmatrix}$$

Specifically get

$$H_{e\mu} = \frac{\delta m^2}{4E}\begin{bmatrix} -\cos 2\theta & +\sin 2\theta \\ +\sin 2\theta & +\cos 2\theta \end{bmatrix} + (\quad)\,\mathbb{1}$$

$$\psi_e(t) = \cos\theta\,\underset{\uparrow}{\psi_1(t)} + \sin\theta\,\underset{\uparrow}{\psi_2(t)}$$

put in solutions

$\psi_e(0) = 1$ means $\psi_1(0) = \cos\theta$, $\psi_2(0) = \sin\theta$

projection, propagation, deprojection

Grind out $|\psi_e(t)|^2$ or $|\psi_\mu(t)|^2$

$$\boxed{|\psi_e(t)|^2 = 1 - \sin^2 2\theta \, \sin^2\left(\frac{\delta m^2 L}{4E}\right)}$$

put in units

- Need both mass and mixing
- Explain limits, allowed/disallowed regions
- $\boxed{\text{Importance of } \tfrac{1}{2} \text{ for } P_{1/2}}$