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SUMMER SCHOOL ON PARTICLE PHYSICS

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QCD Phase Transitions and Heavy ion Collisions - Part 2

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xQCD: QCD at high energies

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Outline of 4 lectures

- **Lecture I** : **General Introduction, QCD at high energies.**

- **Lecture II**: **QCD at high energies-continued.**

Outline of lecture I I

- **Quick review of key points in lecture I**

- **Light Cone preliminaries**

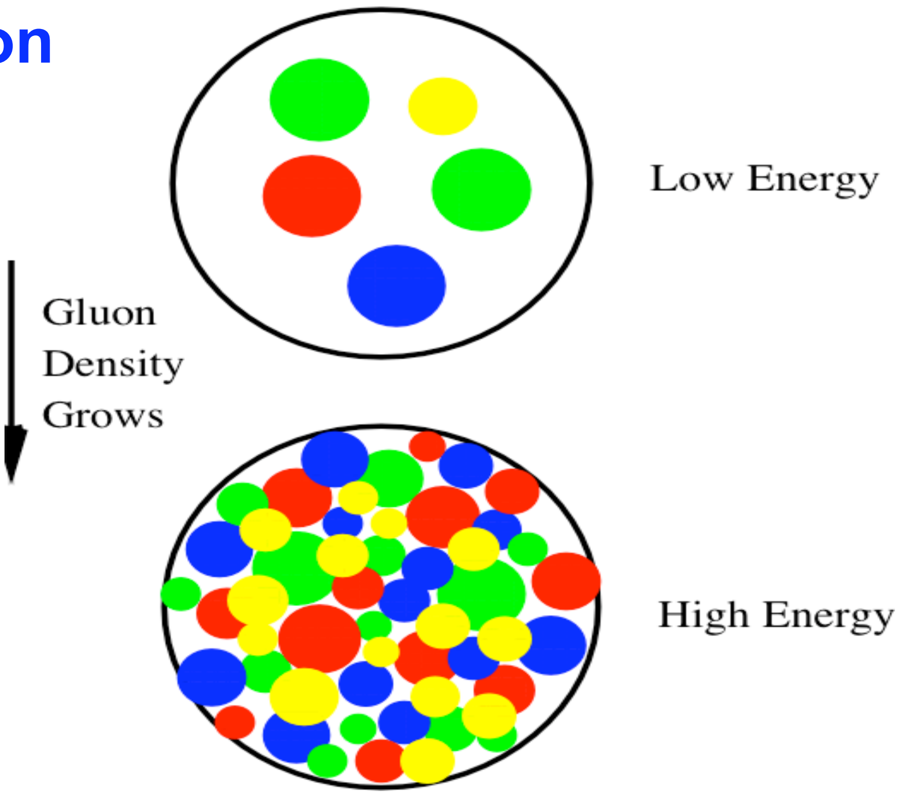
- **The Color Glass Condensate - an effective theory of wee parton dynamics at high energies**

Hadron structure in the Regge limit of QCD:

$$x_{\mathrm{Bj}} \to 0\,;\; s \to \infty\,;\; Q^2 (>> \Lambda^2_{\mathrm{QCD}}) = \mathrm{fixed}$$

Physics of strong fields in QCD,
multi-particle production-
novel universal properties of theory in this limit ?

Resolving the hadron
-BFKL evolution



– Large x

– Small x

Gluon density saturates at f= $$\frac{1}{\alpha_S}$$

Mechanism for parton saturation:

Competition between "attractive" bremsstrahlung and "repulsive" recombination effects.

Maximal phase space density =>

$$\frac{1}{2(N_c^2 - 1)} \frac{x\,G(x, Q^2)}{\pi R^2\, Q^2} = \frac{1}{\alpha_S(Q^2)}$$

Saturated for

$$Q = Q_s(x) >> \Lambda_{\rm QCD} \approx 0.2\ {\rm GeV}$$

- Higher twists (power suppressed-in Q^2) contribute equally when:

$$Q^2 \approx Q_s^2(x) >> \Lambda^2_{\mathrm{QCD}}$$

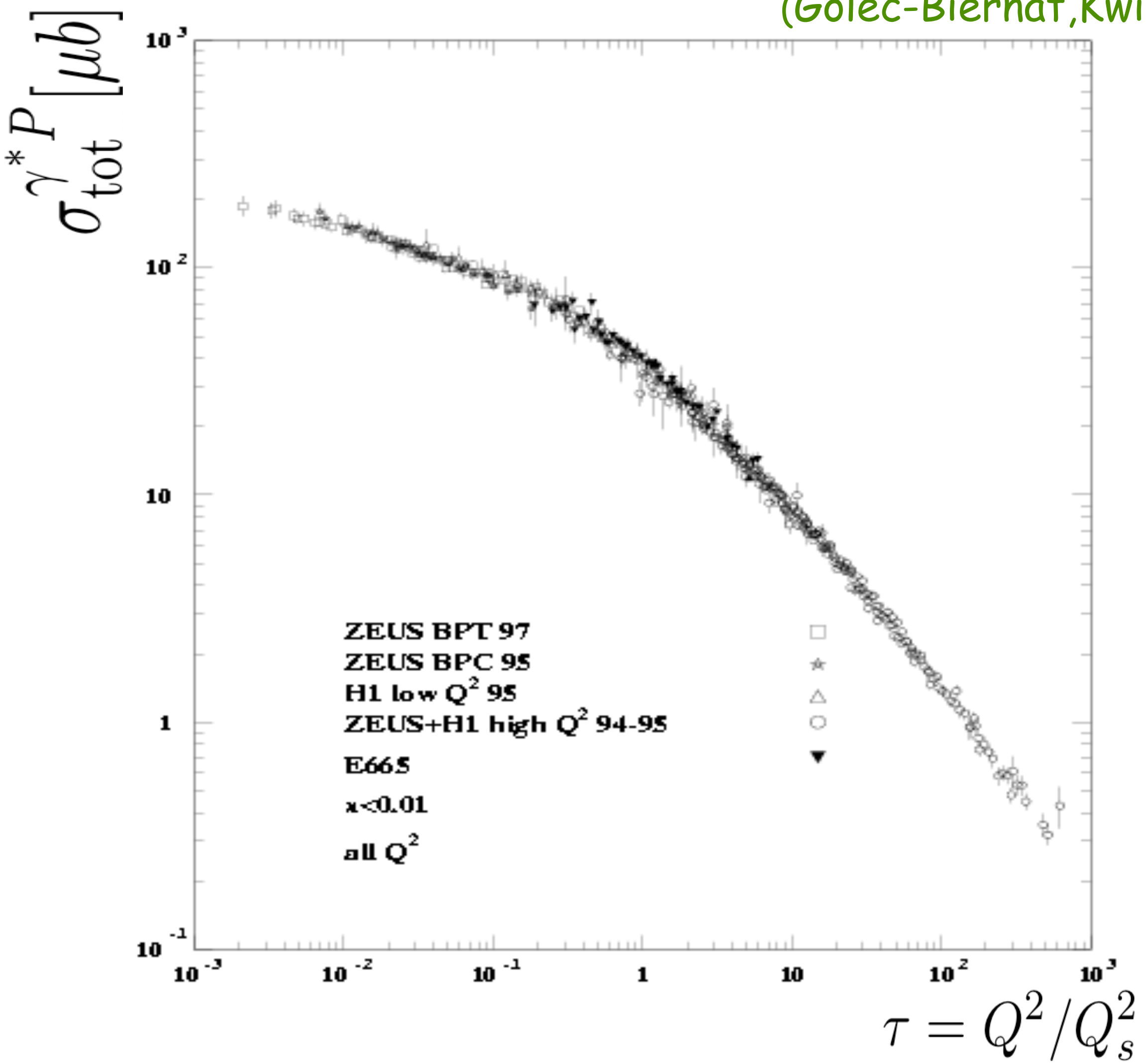
- Leading twist "shadowing" of these contributions can extend up to at small x.

$$Q^2 >> Q_s^2(x)$$

Need a new organizing principle- beyond the OPE- at small x.

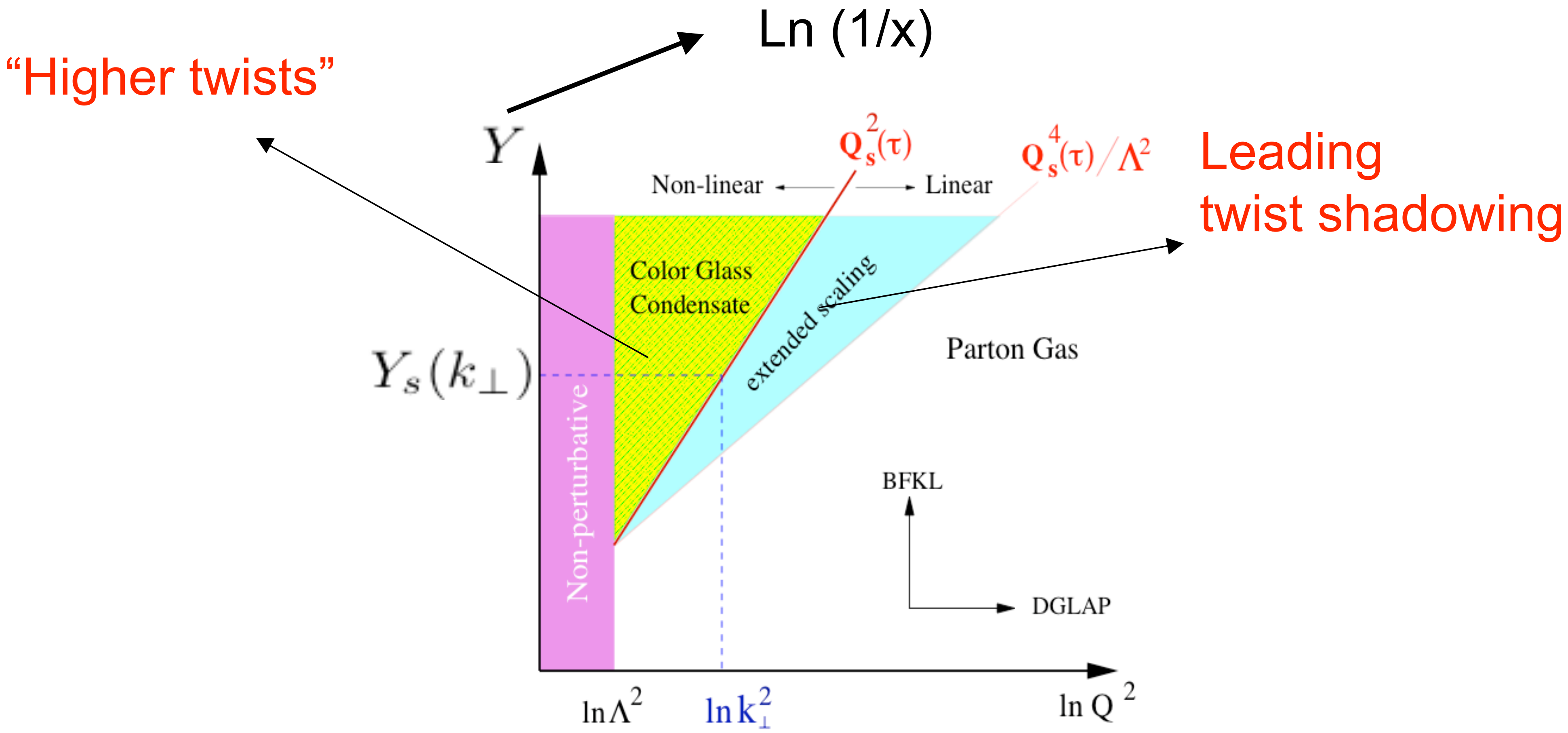
Geometrical scaling at HERA

(Golec-Biernat,Kwiecinski,Stasto)



Scaling seen for all x < 0.01 and $0.045 < Q^2 < 450\,\mathrm{GeV}^2$

Novel regime of QCD evolution at high energies



Light cone preliminaries

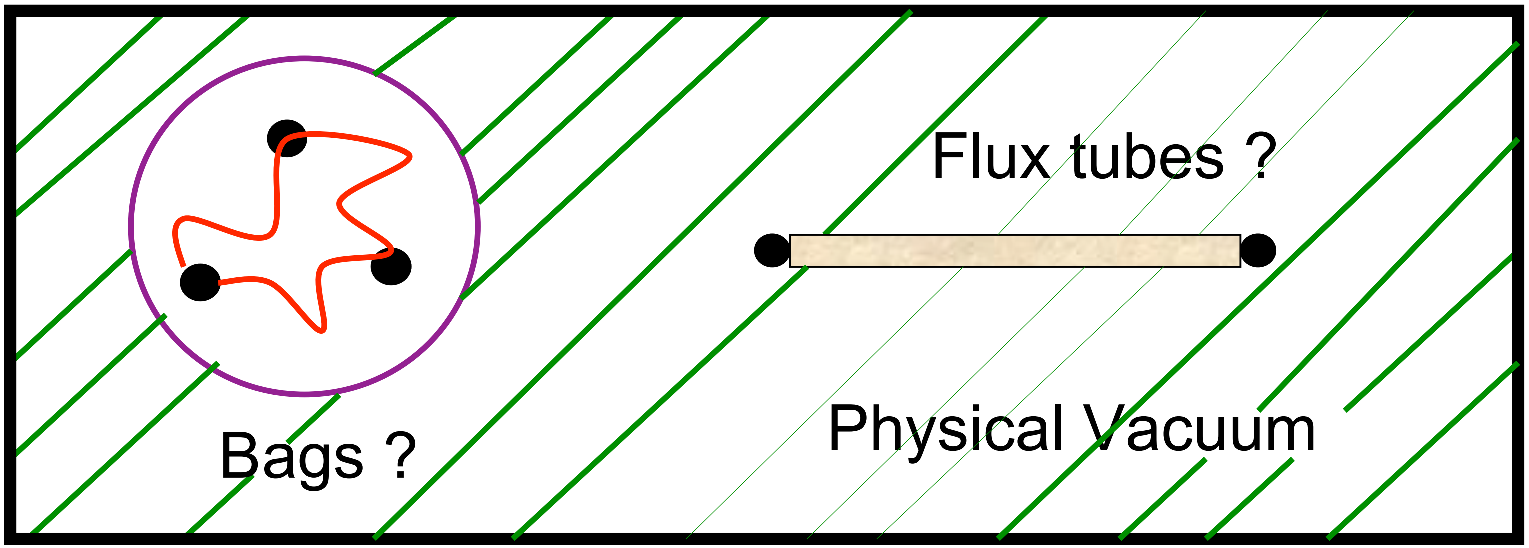
The QCD vacuum is very complicated:

Instantons, Monopoles, Skyrmions, …

Hadrons - bags or flux tubes or solitons:

Complex phenomena - Chiral symmetry breaking,

Confinement,…



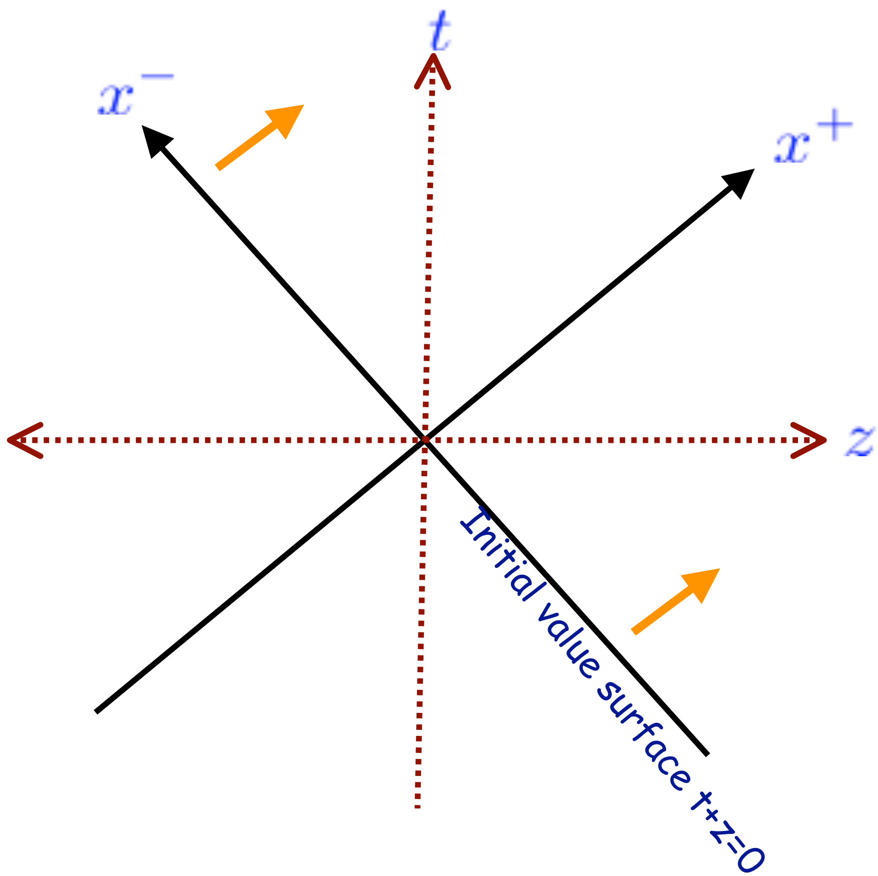
- **Given this, how does one describe the structure of hadrons in high energy scattering?**

- **How does one construct a Lorentz invariant wave fn for a hadron?**

Partial answer: formulate the theory on the light cone

Life on the light cone

RV, nucl-th/9808023



Quantize theory on light like surface: $x^+ = 0$

Quantum field theories quantized on light like surfaces have remarkable properties **Dirac**

Light cone algebra

Co-ordinates: $$x^\mu \equiv (x^0, x^1, x^2, x^3) = (t, \vec{x})$$

$$x^\pm = \frac{(t \pm z)}{\sqrt{2}} \; ; \; \partial_\pm = \frac{(\partial_t \pm \partial_z)}{\sqrt{2}} \; ; \; A^\pm = \frac{(A^0 \pm A^z)}{\sqrt{2}}$$

$$g^{++} = g^{--} = 0 \; ; \; g^{-+} = g^{+-} = 1 \; ; \; g^{xx} = g^{yy} = -1$$

$$=> A_\pm = A^\mp \; ; \; A_{x,y} = -A^{x,y}$$

For spinors, define projection operators: $$\alpha^{\pm} = \frac{\gamma^{\mp}\gamma^{\pm}}{2}$$

Project out two component spinors: $$\psi_{\pm} = \alpha^{\pm}\psi$$

$$\begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} \longrightarrow \psi_+ = \begin{pmatrix} \psi_1 \\ 0 \\ 0 \\ \psi_4 \end{pmatrix} \; ; \; \psi_- = \begin{pmatrix} 0 \\ \psi_2 \\ \psi_3 \\ 0 \end{pmatrix}$$

ψ_+ and A_{x,y} : dynamical "good" fields

- express physical content of the theory

Light cone quantization:

$$\psi_+ = \int \frac{d^3k}{(2\pi)^3} \sum_{s=\pm 1/2} \left[e^{ik\cdot x} b_s(k; x^+) + e^{-ik\cdot x} d_s^\dagger(k; x^+) \right]$$

$$\left\{ b_s(k; x^+), b_{s'}^\dagger(k'; x^+) \right\} = \left\{ d_s(k; x^+), d_{s'}^\dagger(k'; x^+) \right\} = (2\pi)^3 \delta^{(3)}(k - k') \delta_{ss'}$$

$$A_i^a(x) = \int \frac{d^3k}{\sqrt{2k^+}(2\pi)^3} \sum_{\lambda=1,2} \delta_{\lambda i} \left[e^{ik\cdot x} a_\lambda^a(k;\, x^+) + c.c \right]$$

$$\left[a_\lambda^a(k; x^+), a_{\lambda'}^{a\,\dagger} \right] = (2\pi)^3 \delta^{(3)}(k - k') \delta_{\lambda\lambda'}$$

Light cone QCD Hamiltonian in light cone gauge: $A^+ = 0$

$$P^-_{\rm QCD} = P^-_0 + V_{\rm QCD}$$

$$P^-_{0,\rm fermi} = \int \frac{d^3k}{(2\pi)^3} \sum_{s=\pm 1/2} \frac{(k_\perp^2 + M^2)}{2k^+} \left(b_s^\dagger(k) b_s(k) + d_s^\dagger(k) d_s(k)\right)$$

$$P^-_{0,\rm bose} = \int \frac{d^3k}{(2\pi)^3} \sum_{\lambda=1,2} \frac{(k_\perp^2 + M^2)}{2k^+} a_\lambda^{a\,\dagger} a_\lambda^a$$

- **The QCD vacuum is ``trivial” in light cone quantization. It is an eigenstate of both**

$$P^-_{\mathrm{QCD}} \;\&\; P^-_0$$

Physical states therefore expressed in terms of Fock states of bare quanta => PARTON MODEL

Weinberg, 1966
Susskind, 1968

Q.F.T on the light cone

↓ isomorphism

Two dimensional quantum mechanics

Light cone dispersion relation:

$$P^{-} = \frac{(P_t^2 + M^2)}{2P^{+}}$$

Energy

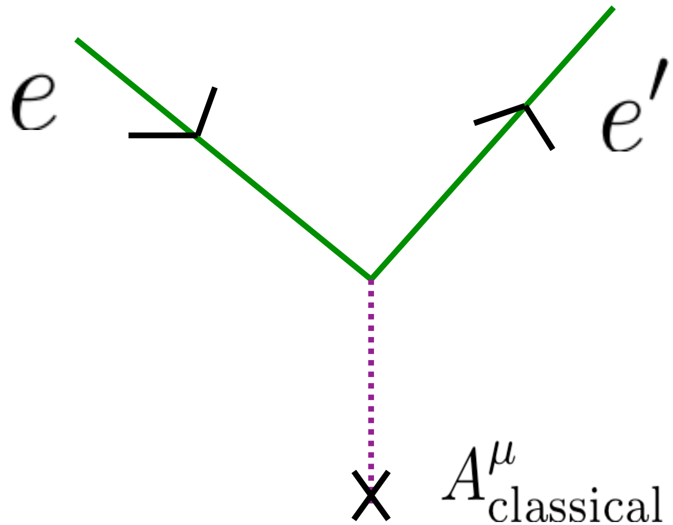
Momenta

Mass

Light cone pert. theory = Rayleigh-Schrodinger pert. theory

Example: electron scattering off an external potential

Bjorken,Kogut,Soper, 1971



$$|e^-_{\text{phys.}}\rangle = a_1\,|e^-_{\text{bare}}\rangle + a_2\,|e^-\gamma^*\rangle + a_3\,|e^-\gamma^* e^+ e^-\rangle + \cdots$$

Scattering of physical state is complex at high energies - many interacting quanta.

Mutual interactions of the quanta ("partons") is simple - slowed by time dilation.

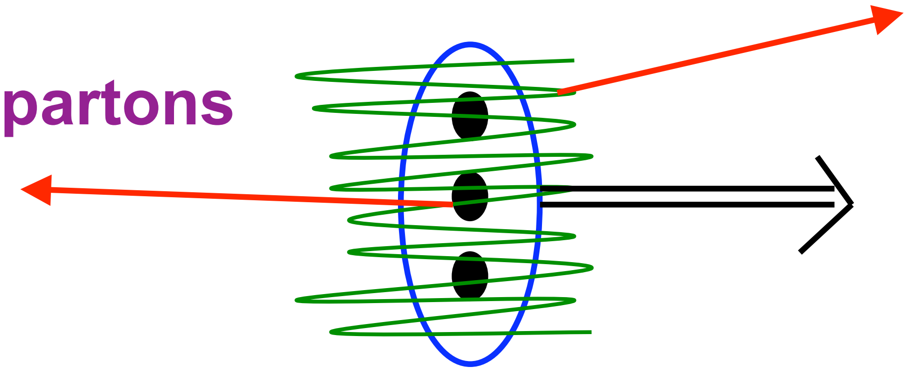
Scattering of the partons off the potential is simple- they acquire a phase - **eikonal** scattering

QFT basis of Bj scaling

Now armed with light cone tools, return to

- **hadron structure at high energies**

The Hadron at high energies: II



$$P^+ \rightarrow \infty$$

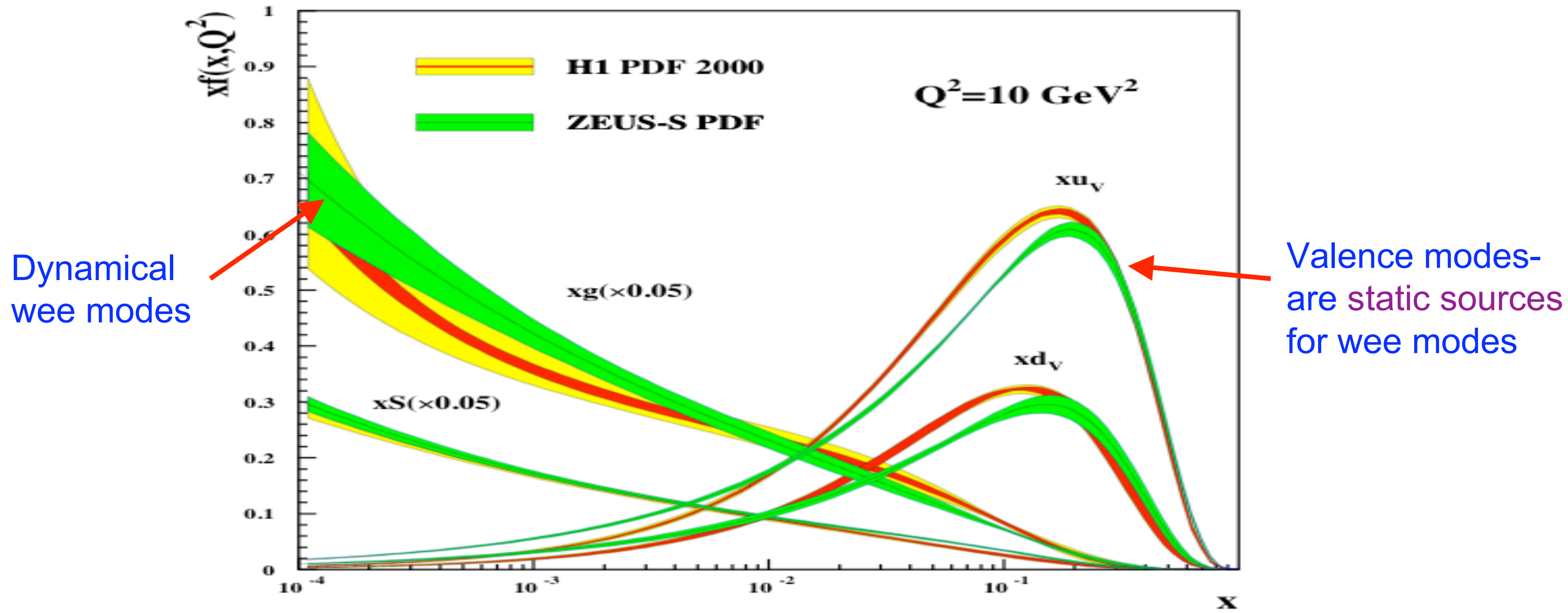
In infinite momentum frame (IMF) ,

$$|h>=|qqq>+|qqqg>+\cdots|qqqgg\cdots q\bar{q}g>$$

Construct "effective" theory of wee parton modes

$$x = k^+/P^+$$

Born-Oppenheimer: separation of large x and small x modes



$$\tau_{\rm wee} \sim \frac{1}{k^-} = \frac{2\,k^+}{k_\perp^2} \equiv \frac{2\,xP^+}{k_\perp^2}$$

$$\tau_{\rm valence} = \frac{2P^+}{k_\perp^2} >> \tau_{\rm wee} \text{ for } x << 1$$

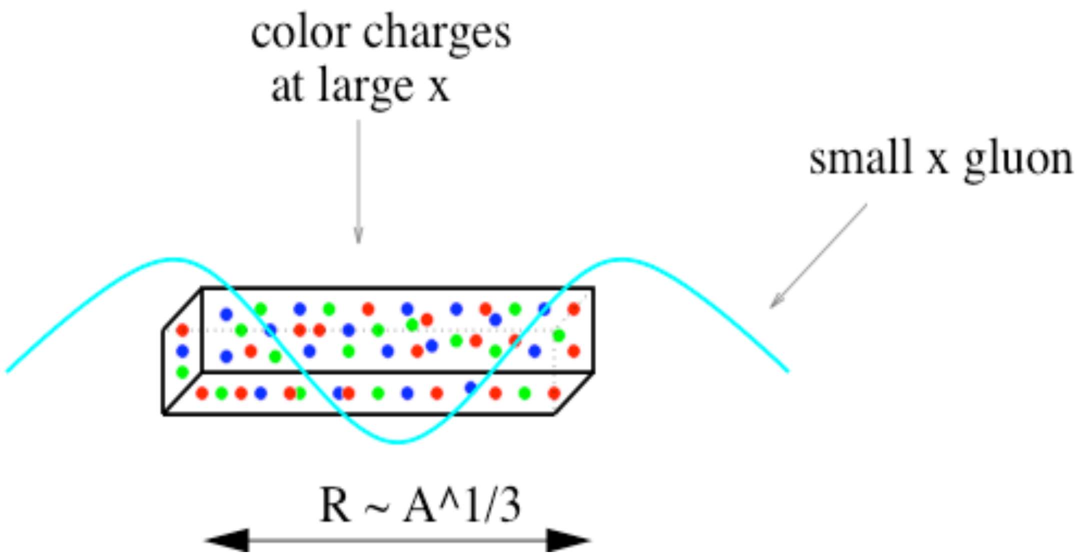
Valence partons are static over wee parton life times

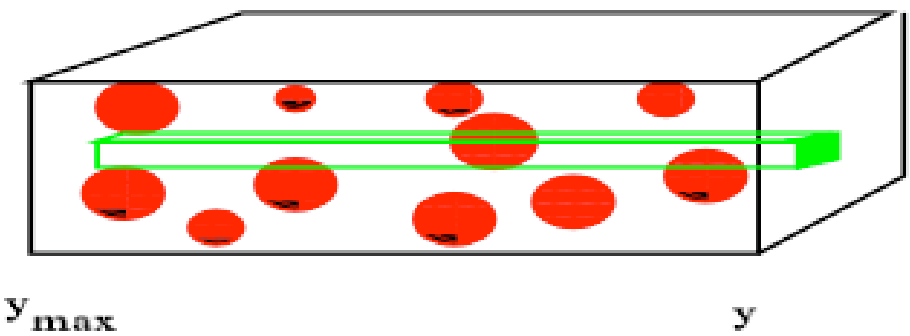
Random sources

$$\lambda_{\rm wee} \approx \frac{1}{k^+} \equiv \frac{1}{xP^+}$$

$$>> \lambda_{\rm valence} \equiv \frac{R\, m_p}{P^+}$$

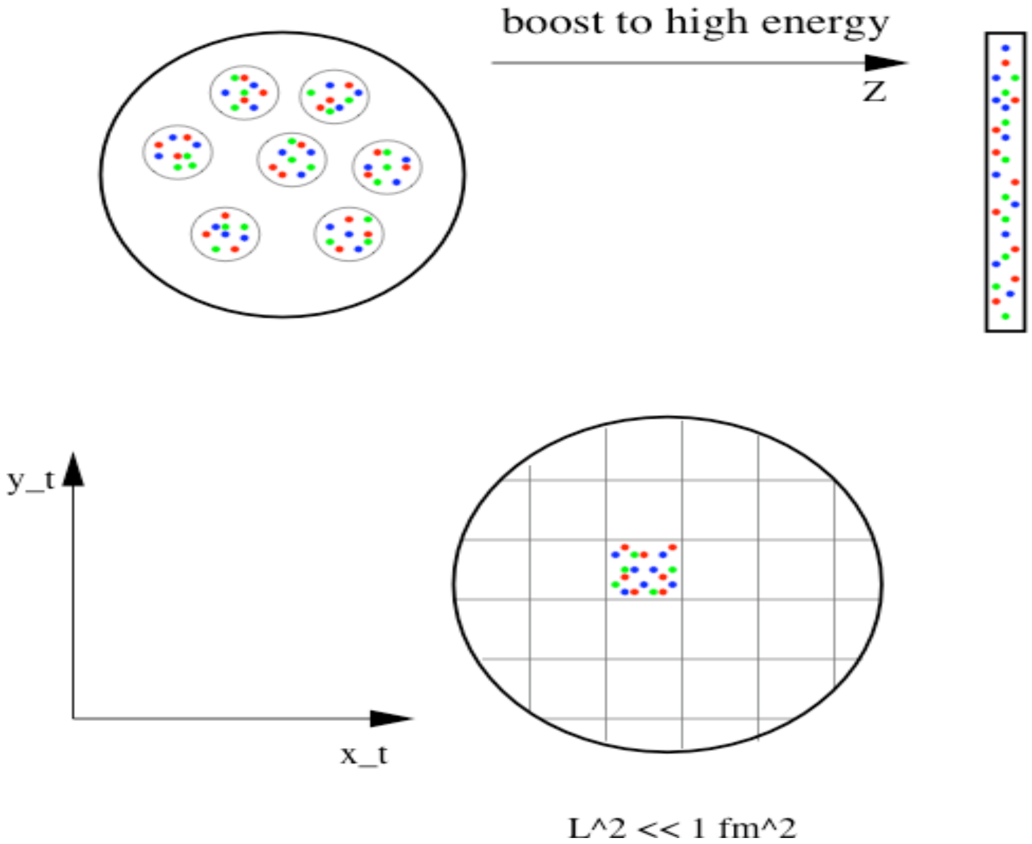
$$=> x << A^{-1/3}$$





Gaussian random sources

Consider large nucleus in the IMF frame: $P^+ \to \infty$



One large component of the current-others suppressed by $\frac{1}{P^+}$

Wee partons "see" a large density of valence color charges at small transverse resolutions.

The effective action

McLerran, RV

Generating functional:

Scale separating sources and fields

$$\mathcal{Z}[j] = \int [d\rho]\, W_{\Lambda^+}[\rho] \left\{ \frac{\int^{\Lambda^+} [dA]\, \delta(A^+)\, e^{iS[A,\rho] - \int j\cdot A}}{\int^{\Lambda^+} [dA]\, \delta(A^+)\, e^{iS[A,\rho]}} \right\}$$

Gauge invariant weight functional for distribution of sources

$$S[A,\rho] = \frac{-1}{4} \int d^4x\, F_{\mu\nu}^2 + \frac{i}{N_c} \int d^2x_\perp dx^- \delta(x^-) \mathrm{Tr}\left(\rho(x_\perp)\, U_{-\infty,\infty}[A^-]\right)$$

Dynamical wee fields

Coupling of wee fields to classical sources

where $U_{-\infty,\infty}[A^-] = \mathcal{P} \exp\left(ig \int dx^+ A^{-,a} T^a \right)$

In large A limit of QCD:

McLerran, RV; Kovchegov;
Jeon, RV

$$W_{\Lambda^+} = \exp\left(-\int d^2x_\perp \left[\frac{\rho^a \rho^a}{2\,\mu_A^2} - \frac{d_{abc}\,\rho^a \rho^b \rho^c}{\kappa_A}\right]\right)$$

$$\mu_A^2 = \frac{g^2 A}{2\pi R^2} \propto A^{1/3}$$

$$\kappa_A = \frac{g^3 A^2 N_c}{\pi^2 R^4} \propto A^{2/3}$$

$$\mu_A^2 \approx Q_s^2 \; ; \; \alpha_S(Q_s^2) << 1$$

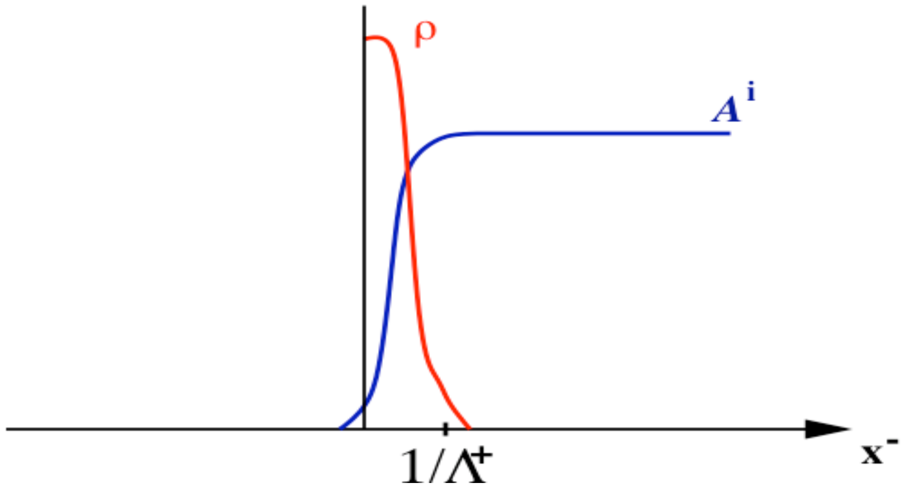
Effective action describes a weakly coupled albeit non-perturbative system

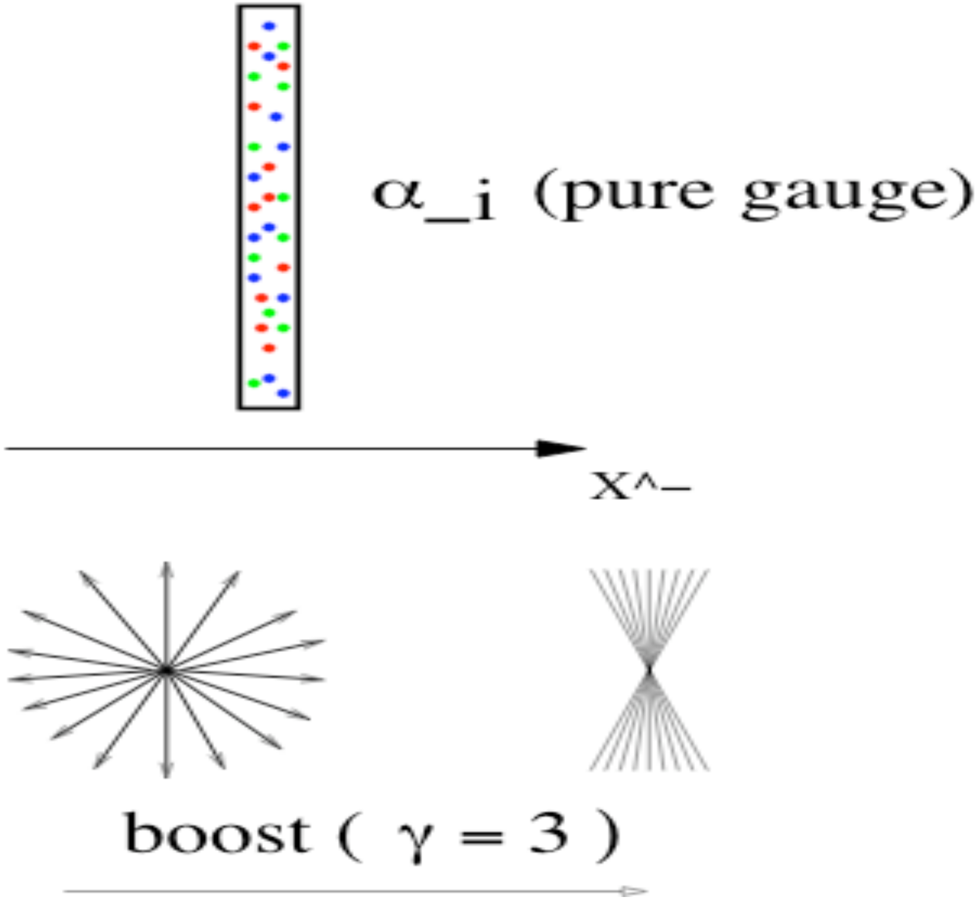
classical color field of a nucleus at high energies

Yang-Mills equations

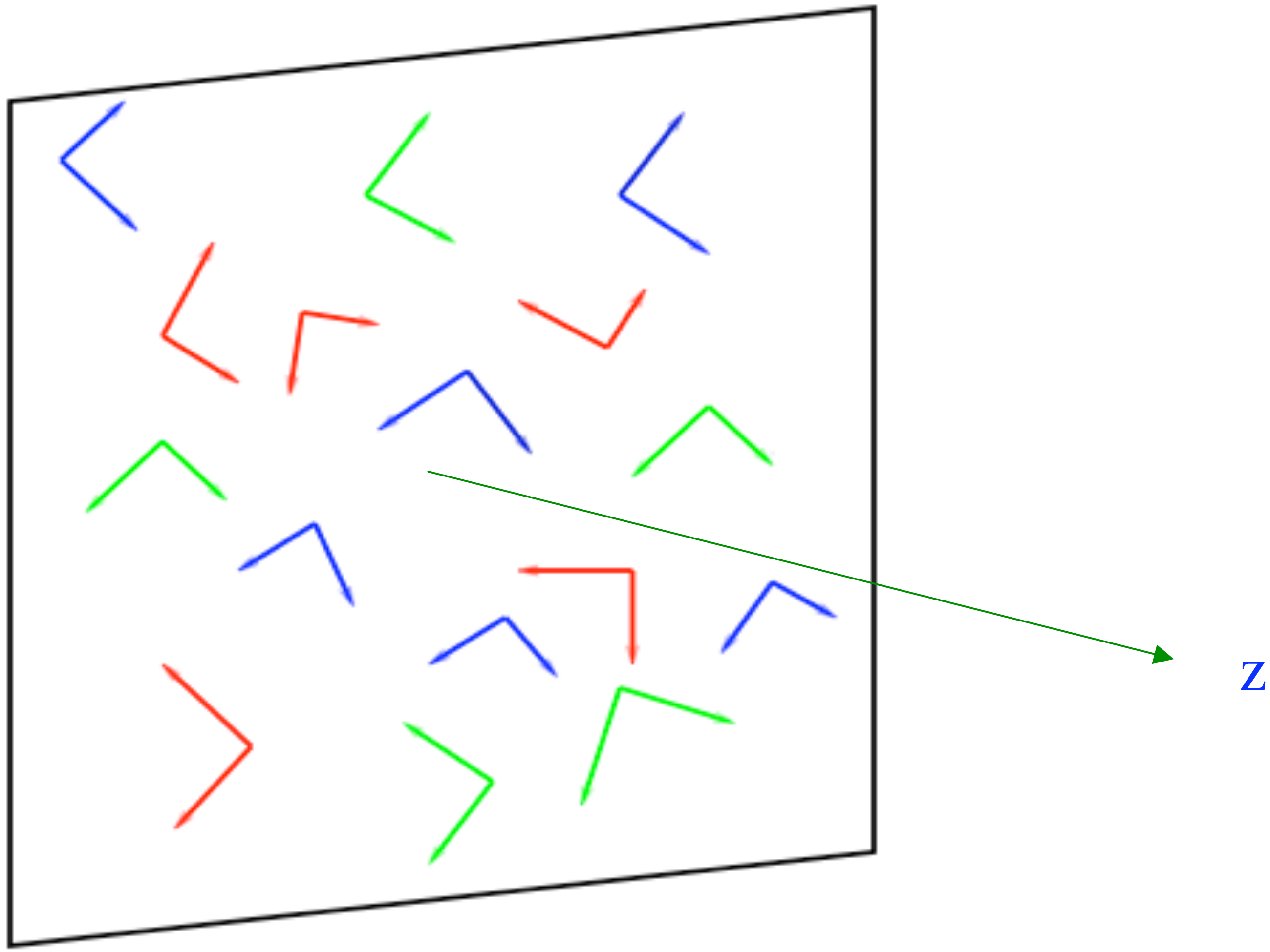
$$(D_\mu F^{\mu\nu})^a = J^{\nu,a} \equiv \delta^{\nu+}\,\delta(x^-)\,\rho^a(x_\perp)$$

Solutions are non-Abelian Weizsäcker-Williams fields



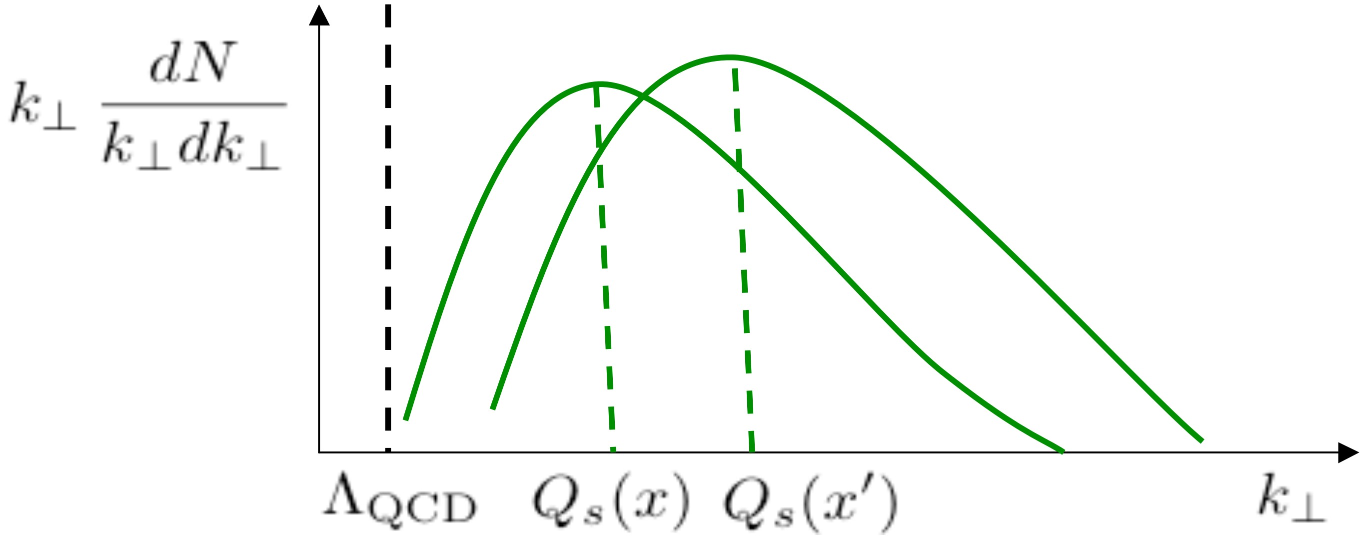


Random Electric & Magnetic fields
in the plane of the fast moving nucleus

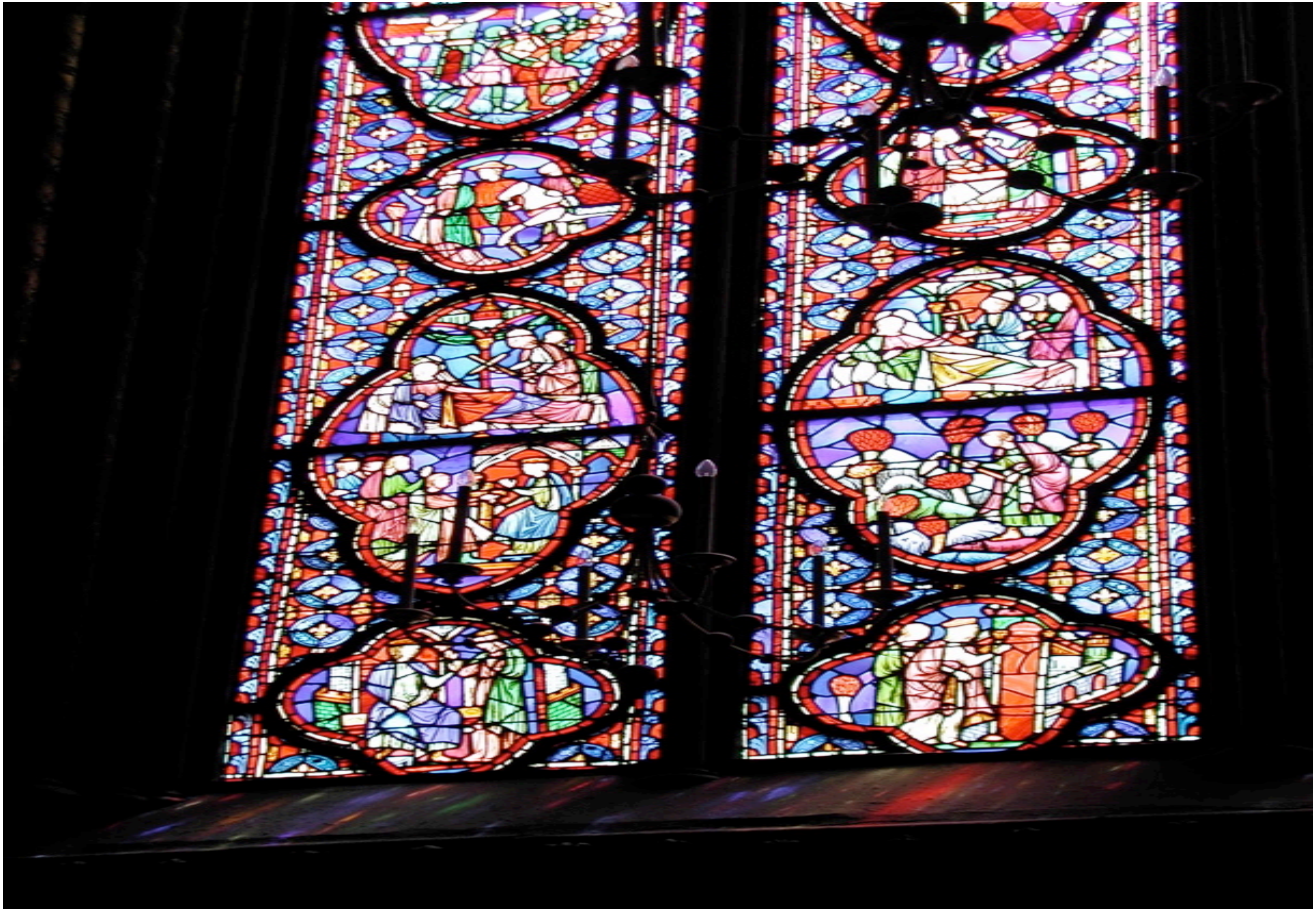


$$x^{-} = 0$$

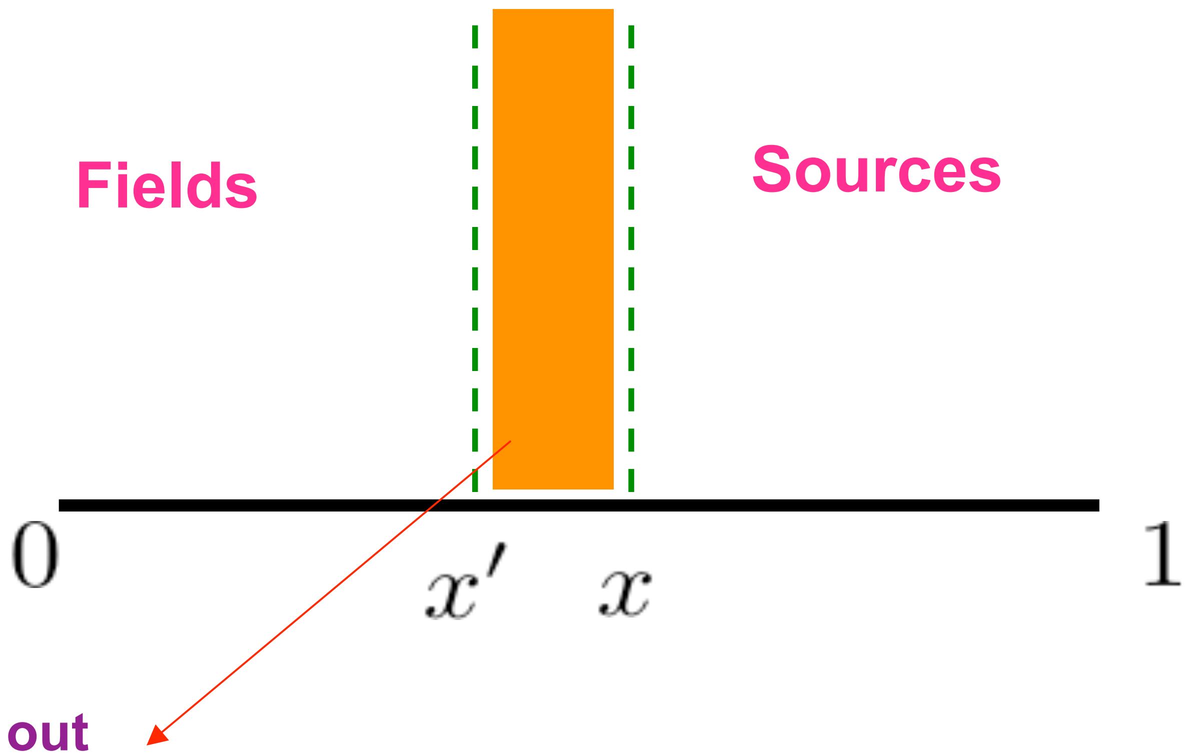
Hadron at high energies is a Color Glass Condensate



- ✓ **Gluons are colored**
- ✓ **Random sources evolving on time scales much larger than natural time scales-very similar to spin glasses**
- ✓ **Bosons with large occupation ~** $\frac{1}{\alpha_S}$ **- form a condensate**
- ✓ **Typical momentum of gluons is** Q_s



Quantum evolution of classical theory: Wilsonian RG



Integrate out
Small fluctuations => Increase color charge of sources

Wilsonian RG equations describe evolution of all N-point correlation functions with energy

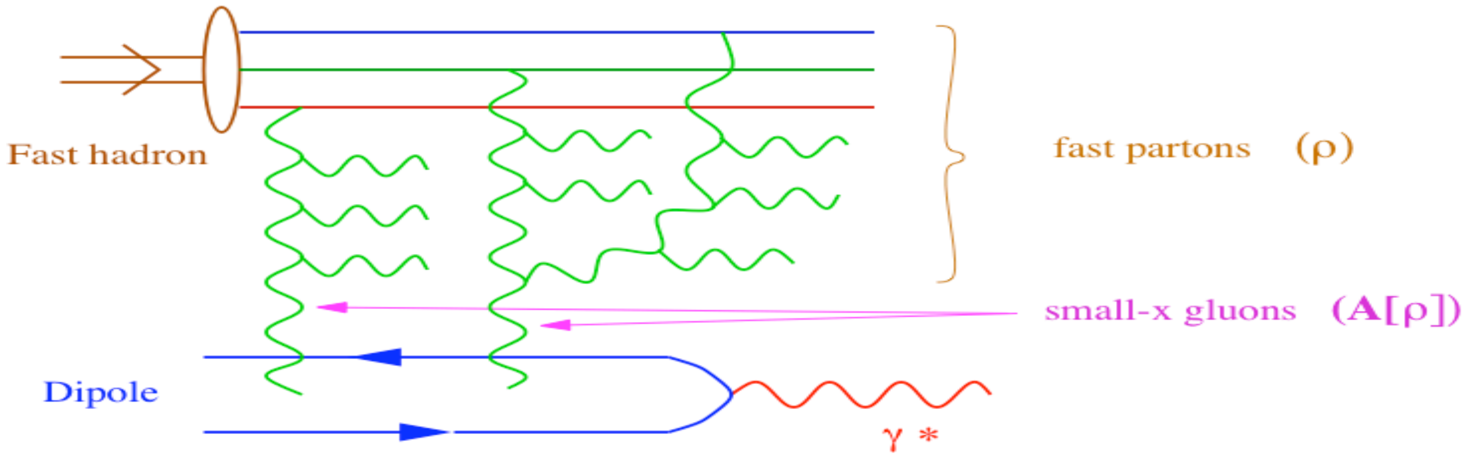
JIMWLK Jalilian-marian, Iancu, McLerran,Weigert, Leonidov,Kovner

The hadron at high energies - III

Mean field solution of JIMWLK = B-K equation

Balitsky-Kovchegov

DIS:



Dipole amplitude N satisfies

BFKL kernel

$$\frac{\partial \mathcal{N}}{\partial \ln(1/x)} = K * [\mathcal{N} - \mathcal{N}^2]$$

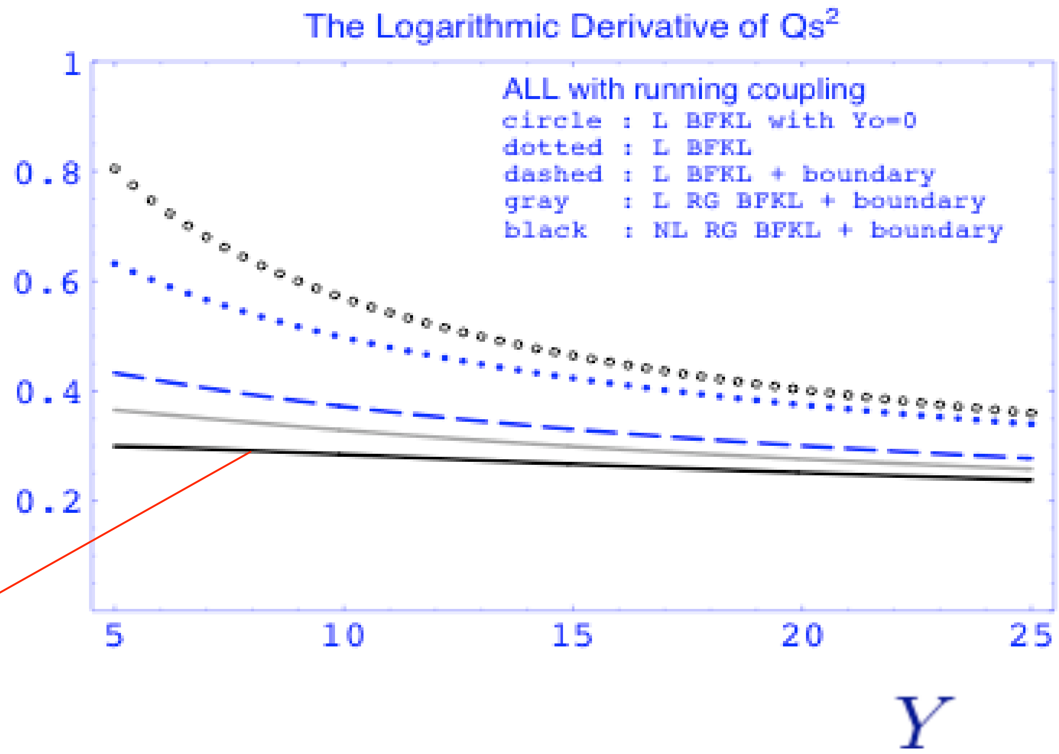
How does Q_s behave as function of Y?

Fixed coupling LO BFKL: $$Q_s^2 = Q_0^2 e^{c\,\bar{\alpha}_s Y}$$

LO BFKL+ running coupling: $$Q_s^2 = \Lambda_{\text{QCD}}^2 e^{\sqrt{2b_0 c(Y+Y_0)}}$$

Re-summed NLO BFKL + CGC:

$$\lambda \equiv \frac{d \ln Q_s^2}{dY}$$



Triantafyllopolous

Very close to HERA result!

Remarkable correspondence of high energy QCD With Stat. Mech. :

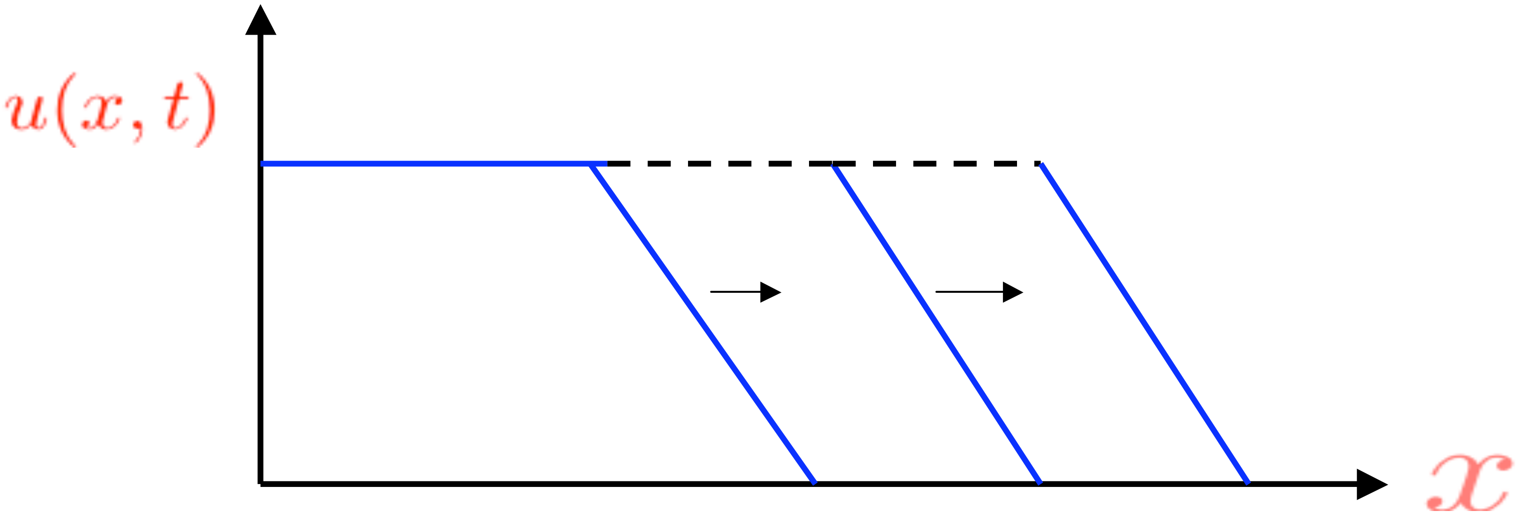
Munier-Peschanski

B-K same universality class as FKPP equation

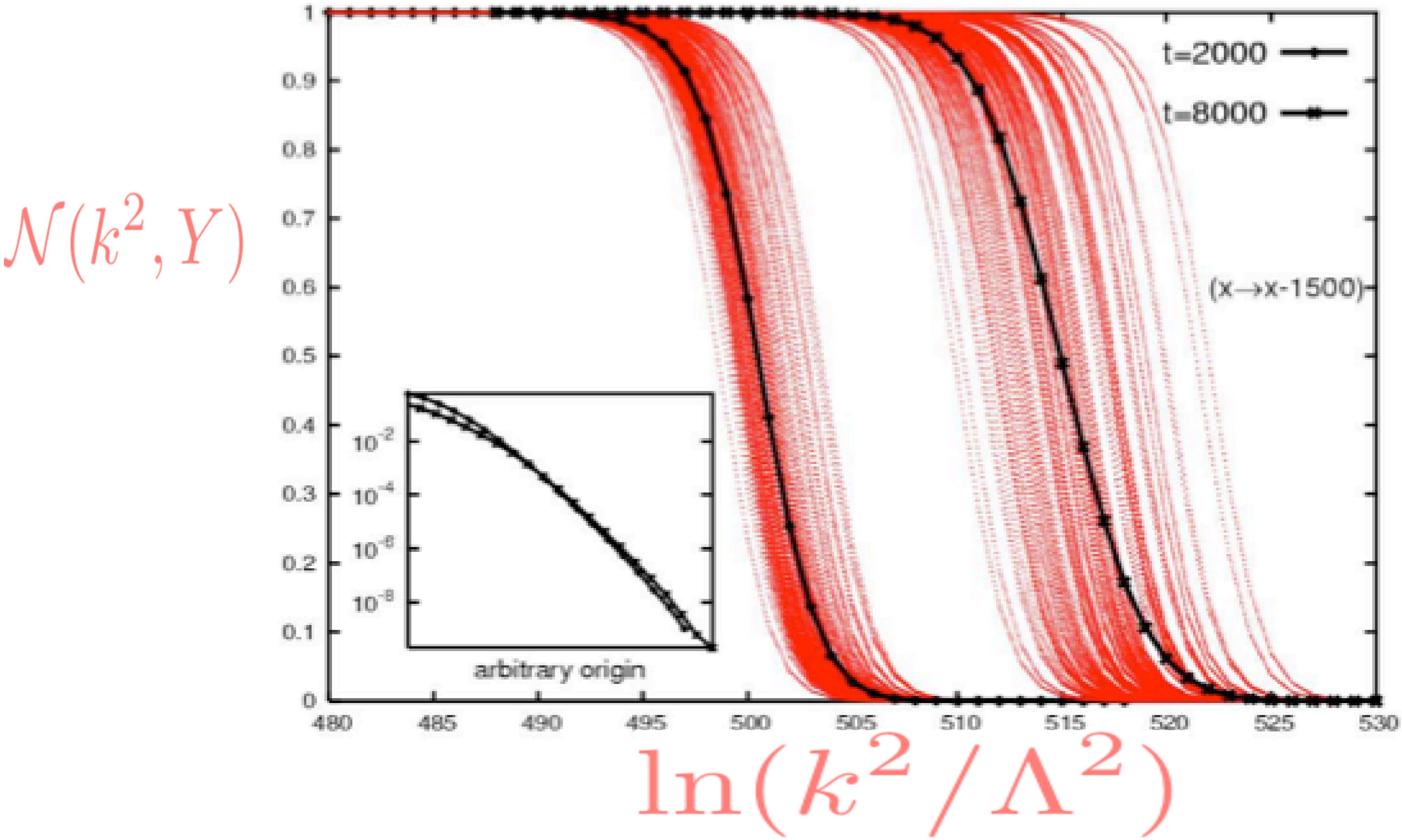
FKPP = Fisher-Kolmogorov-Petrovsky-Piscunov

FKPP-describes unstable travelling wave fronts -

B-K correspond to spin glass phase of FKPP



$$v(t) \to \ln(Q_s^2(Y))/dY$$



Fluctuations in high energy QCD described by stochastic FKPP Eq.

Iancu, Itakura, Munier

Melting Colored Glass in Heavy Ion Collisions