



The Abdus Salam
International Centre for Theoretical Physics



SMR.1663- 1

SUMMER SCHOOL ON PARTICLE PHYSICS

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CP Violation and Rare Decays in the Standard Model and Beyond

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CP Violation and Rare Decays in the Standard Model and Beyond

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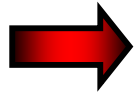
Trieste, June 13th-18th, 2005

Overture 1

$K^0 - \bar{K}^0$ Mixing (Oscillations)

$$K^0 = d\bar{s}$$

$$\bar{K}^0 = \bar{d}s$$



K^0 and \bar{K}^0 are not Mass Eigenstates

Mass Eigenstates :

$$K_L = \frac{K^0 + \bar{K}^0}{\sqrt{2}} \quad K_S = \frac{K^0 - \bar{K}^0}{\sqrt{2}}$$

$$M(K_L) \cong M(K_S) = 0.5 \text{ GeV}$$

(L = Long)

(S = Short)



$$M(K_L) - M(K_S) = 3.5 \cdot 10^{-15} \text{ GeV}$$

$$\frac{\tau(K_L)}{\tau(K_S)} \cong 600$$

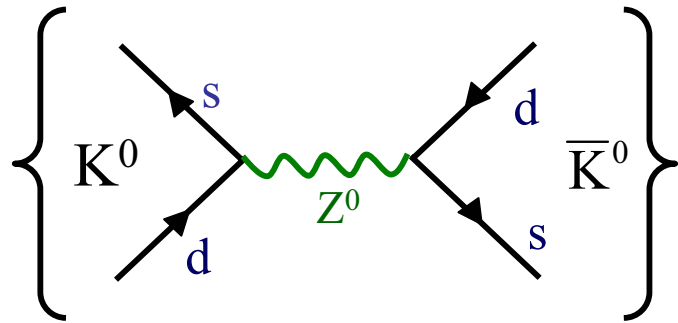


$M \equiv \text{mass}$

III
 ΔM_K

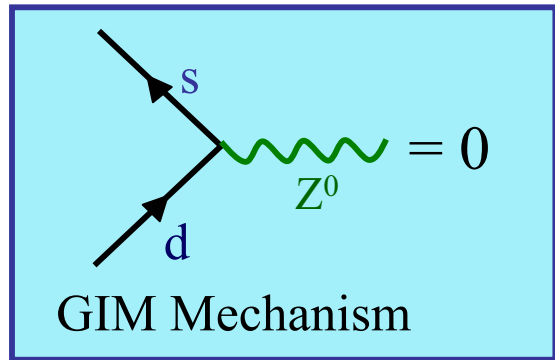
$\tau \equiv \text{Life Time}$

Could ordinary Weak Interactions explain ΔM_K ?



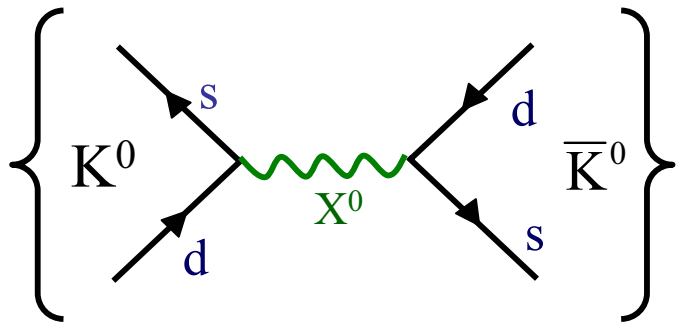
$$\left\{ \begin{aligned} \Delta M_K &= [2 \cdot 10^{-2} \text{ GeV}^3] G_F \left[\frac{M_W^2}{M_Z^2} \right] \\ G_F &\cong 1.1 \cdot 10^{-5} \text{ GeV}^{-2} \end{aligned} \right\}$$

$$M_Z \cong 90 \text{ GeV}$$



$$\Delta M_K \cong 2 \cdot 10^{-7} \text{ GeV}$$

Disaster !!!
Missed by
8 orders of
magnitude !!!



$$\left\{ \Delta M_K \cong 2 \cdot 10^{-7} \left[\frac{M_Z^2}{M_X^2} \right] \text{ GeV} = 3.5 \cdot 10^{-15} \text{ GeV} \right\}$$

New very heavy neutral boson !

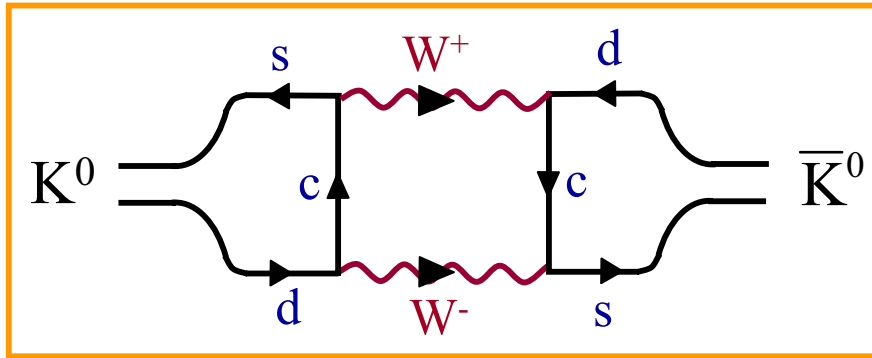
$$\{ M_X \cong 10^6 \text{ GeV} \}$$

Testing
 $10^{-22} \text{ m} !$

ΔM_K in the Standard Model

Gaillard-Lee (March 1974)

$$\lambda \cong 0.22$$



$$\left\{ \begin{array}{l} \Delta M_K = [1.4 \text{ GeV}^5] G_F^2 \lambda^2 \left[\frac{m_c^2}{M_W^2} \right] \\ G_F \cong 1.2 \cdot 10^{-5} \text{ GeV}^{-2} \end{array} \right\}$$



$$m_c = \sqrt{3.5 \cdot 10^{-2}} M_W = 1.5 \text{ GeV}$$

(Prediction !!)

$$\Delta M_K = 10^{-11} \text{ GeV} \left[\frac{m_c^2}{M_W^2} \right] = 3.5 \cdot 10^{-15} \text{ GeV}$$

{
 November Revolution
 1974
 Discovery of $\bar{c}c$ State
 (SLAC, Brookhaven)
 }

$$M_{\bar{c}c} \cong 3.1 \text{ GeV}$$

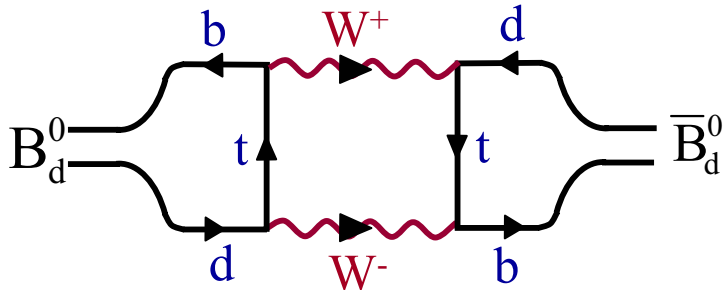


$$m_c \cong 1.5 \text{ GeV} \quad !!$$

Prediction confirmed !

Similar Studies: 1974-1994

$B_d^0 - \bar{B}_d^0$ Oscillations



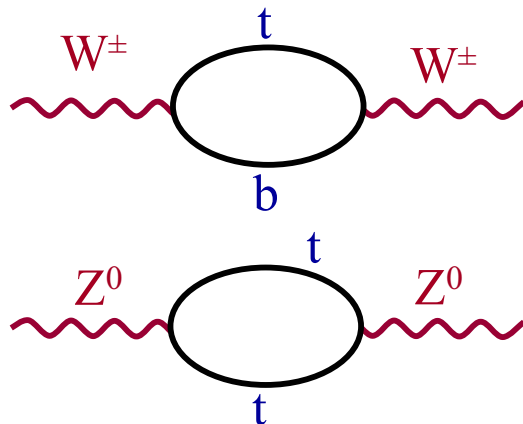
DESY 87

$$\Delta M_{B_d} \cong 4 \cdot 10^{-13} \text{ GeV}$$

(Prediction)

$$m_t \approx 150 \text{ GeV} \pm 30$$

Electroweak Precision Studies



CERN, SLAC
(1989-1994)

$$m_t \approx 150 \text{ GeV} \pm 20$$

(Prediction)

1994
Discovery of
the Top Quark
(Fermilab)

$$m_t = 178 \pm 4 \text{ GeV}$$

First Lessons

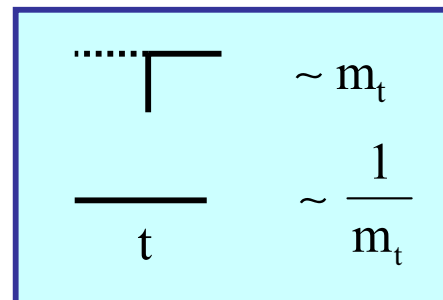
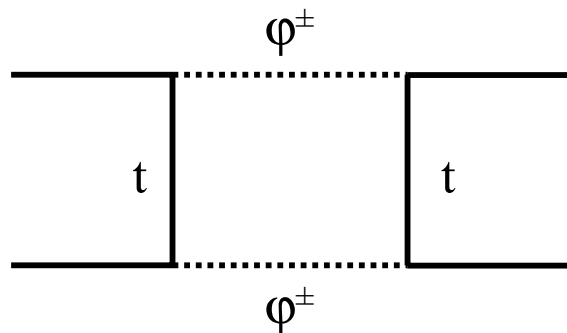
- 1.** Very rare processes allow to probe very short distance scales.
- 2.** Before claiming New Physics it is essential to make precise calculations (higher order corrections).
- 3.** Low Energy Processes can give information about heavy particles prior to their discovery.

Non-Decoupling of the Top Quark from Low Energy Processes

In QCD and QED very heavy particles ($m_H \rightarrow \infty$) do not influence low energy processes: Appelquist-Carazzone Decoupling Theorem

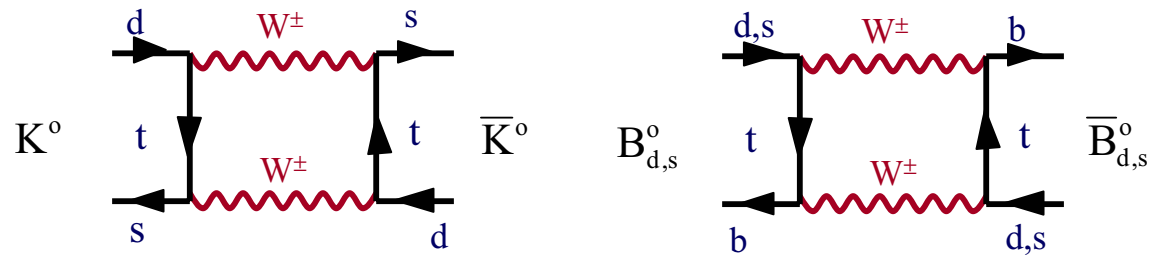
In the $SU(2)_L \otimes U(1)_Y$ the decoupling can be violated by couplings of heavy particles that increase with the heavy particle mass.

Goldstone-Boson



$$m_t^4 \cdot \frac{1}{m_t^2} = m_t^2$$

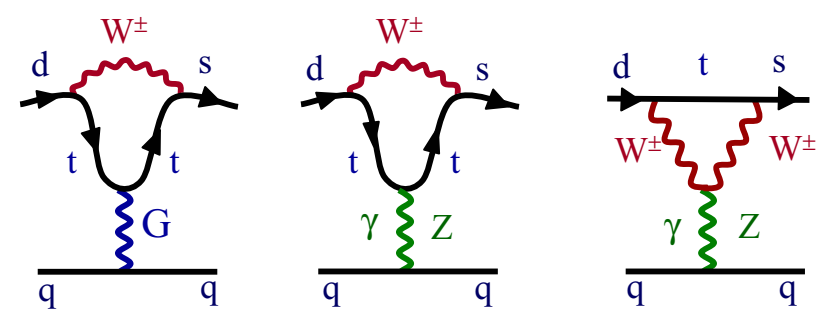
View at Short Distance Scales



★ \cancel{CP} ϵ_K -Parameter
 $\Delta M (K_L - K_S)$

$B_d^0 - \bar{B}_d^0$ Mixing ★

★ ϵ'



View at Short Distance Scales



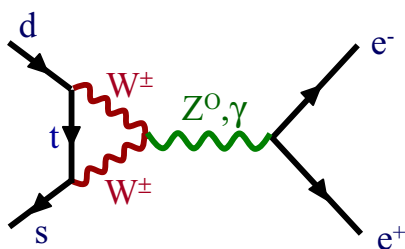
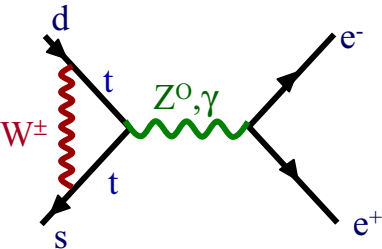
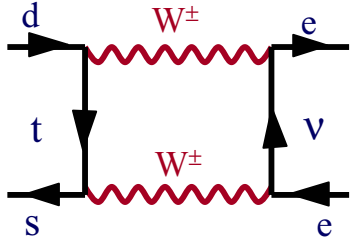
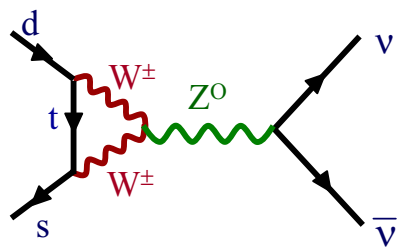
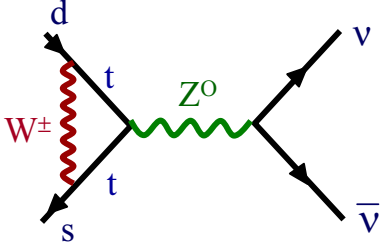
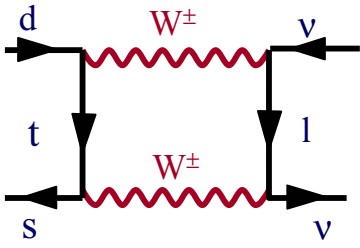
$$K^+ \rightarrow \pi^+ \nu \bar{\nu}$$

$$K_L \rightarrow \pi^0 \nu \bar{\nu}$$



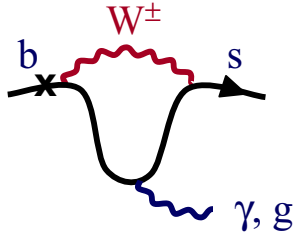
$$K_L \rightarrow \mu \bar{\mu}$$

$$B \rightarrow \mu \bar{\mu}, \quad B \rightarrow X_S \nu \bar{\nu}$$



$$K_L \rightarrow \pi^0 e^+ e^-$$

$$B \rightarrow X_S e^+ e^-, X_S \mu \bar{\mu}$$



$$B \rightarrow X_S \gamma \quad B \rightarrow K^* \gamma$$



$$B \rightarrow X_d \gamma \quad b \rightarrow s \text{ gluon}$$

Goals for these Lectures

- 1.** Develop Formalism for Rare Processes:
CP-Violating Transitions, CP-Asymmetries and
Rare Decays within Gauge Theories
- 2.** Apply this Formalism to the Standard Model and its
simplest Extensions
- 3.** Develop a systematic Procedure for Probing New Physics
with these Processes
- 4.** Identify most interesting Problems and Questions

Overture 2

Four Basic Properties in the SM

1. Charged Current Interactions only between left-handed Quarks

$$\frac{g_2}{2\sqrt{2}} \gamma_\mu (1 - \gamma_5) \cdot V_{td}$$

2. Quark Mixing { Weak Eigenstates } \neq { Mass Eigenstates }

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

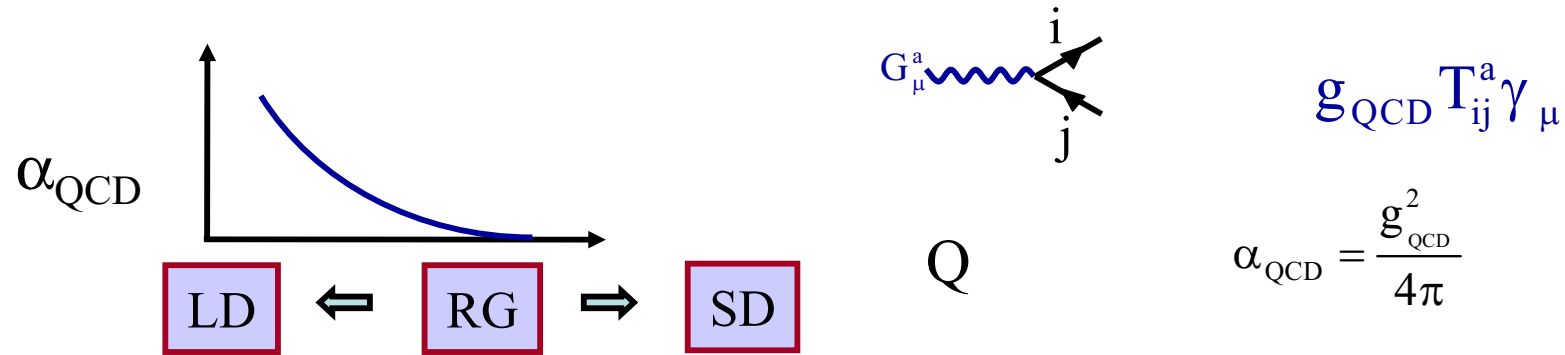
$$\left(\begin{array}{c} \text{Weak} \\ \text{Eigenstates} \end{array} \right) \left(\begin{array}{c} \text{Unitarity} \\ \text{CKM-Matrix} \end{array} \right) \left(\begin{array}{c} \text{Mass} \\ \text{Eigenstates} \end{array} \right)$$

3. GIM Mechanism

Natural suppression of FCNC

$$\left\{ \gamma, G, Z^0, H^0 \right\} \left\{ \begin{array}{c} i \\ \\ j \end{array} \right\} = 0 \quad \rightarrow \quad \left\{ \text{Loop Induced Decays, sensitive to} \right. \\ \left. \text{short distance flavour dynamics} \right\}$$

4. Asymptotic Freedom



$$\alpha_{\text{QCD}}(Q) = \frac{4\pi}{\beta_0 \ln(Q^2 / \Lambda_{\text{MS}}^2)} \left[1 - \frac{\beta_1}{\beta_0^2} \frac{\ln \ln(Q^2 / \Lambda_{\text{MS}}^2)}{\ln(Q^2 / \Lambda_{\text{MS}}^2)} + \dots \right]$$

$\Lambda_{\text{MS}}^{(5)} = 225 \pm 40 \text{ MeV} \quad \alpha_{\text{MS}}^{(5)}(M_Z) = 0.118 \pm 0.003$

SD = Short Distances (Perturbation Theory)



RG = Renormalization Group Effects



LD = Long Distances (Non-Perturbative Physics)

Kobayashi-Maskawa Picture of CP Violation

CP Violation arises from **a single phase δ**
in W^\pm interactions of Quarks

ud	$c_{12}c_{13}$	us	$s_{12}c_{13}$	ub	$s_{13}e^{-i\delta}$
cd	$-s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta}$	cs	$c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta}$	cb	$s_{23}c_{13}$
td	$s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta}$	ts	$-s_{23}c_{12}-s_{12}s_{23}s_{13}e^{i\delta}$	tb	$c_{23}c_{13}$

Four Parameters: $(\theta_{12} \approx \theta_{\text{cabibbo}})$

$s_{12} = |V_{us}|, \quad s_{13} = |V_{ub}|, \quad s_{23} = |V_{cb}|, \quad \delta$

$$c_{ij} \equiv \cos \theta_{ij} ; \quad s_{ij} \equiv \sin \theta_{ij} ; \quad c_{13} \cong c_{23} \cong 1$$

Wolfenstein Parametrization

Parameters:

$$\lambda, A, \rho, \eta$$

	d	s	b
u	$1 - \frac{\lambda^2}{2}$	λ	V_{ub}
c	$-\lambda$	$1 - \frac{\lambda^2}{2}$	V_{cb}
t	V_{td}	V_{ts}	1

$$\lambda = 0.22$$

$$V_{us} = \lambda + O(\lambda^7)$$

$$V_{cb} = A\lambda^2 + O(\lambda^8)$$

$$V_{ts} = -A\lambda^2 + O(\lambda^4)$$

$$(A = 0.83 \pm 0.02)$$

$$V_{ub} \equiv A\lambda^3(\rho - i\eta)$$

$$V_{td} = A\lambda^3(1 - \bar{\rho} - i\bar{\eta})$$

$$\bar{\rho} = \rho \left(1 - \frac{\lambda^2}{2} \right)$$

$$\bar{\eta} = \eta \left(1 - \frac{\lambda^2}{2} \right)$$

(AJB, Lautenbacher, Ostermaier, 94)

$$R_b \equiv \sqrt{\bar{\rho}^2 + \bar{\eta}^2} = \left(1 - \frac{\lambda^2}{2} \right) \frac{1}{\lambda} \left| \frac{V_{ub}}{V_{cb}} \right|$$

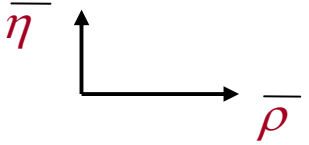
Circle around
 $(\bar{\rho}, \bar{\eta}) = (0, 0)$

$$R_t \equiv \sqrt{(1 - \bar{\rho})^2 + \bar{\eta}^2} = \frac{1}{\lambda} \left| \frac{V_{td}}{V_{cb}} \right|$$

Circle around
 $(\bar{\rho}, \bar{\eta}) = (1, 0)$

Unitarity Triangle

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$



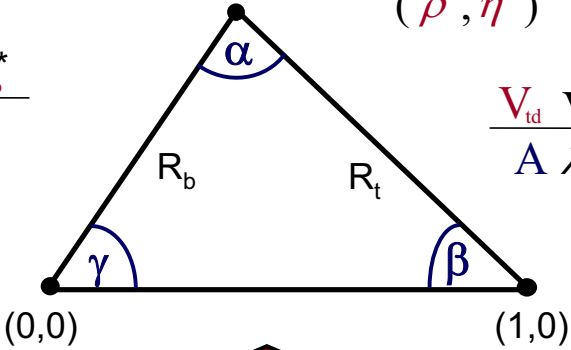
$\bar{\eta} \neq 0$ Signals CP Violation

$$\frac{V_{ud} V_{ub}^*}{A \lambda^3}$$

$$\frac{V_{td} V_{tb}^*}{A \lambda^3}$$

$$V_{ub} = |V_{ub}| e^{-i\gamma}$$

$$V_{td} = |V_{td}| e^{-i\beta}$$



An Important Target of Particle Physics

$$J_{CP} = \lambda^2 |V_{cb}|^2 \bar{\eta} = 2 \cdot \text{Area of unrescaled UT}$$

Area of unrescaled
UT

Particular Definition of λ , A , ρ , η

$$S_{12} \equiv \lambda$$

$$S_{23} \equiv A \lambda^2$$

$$S_{13} e^{i\delta} \equiv A \lambda^3 (\rho - i\eta)$$

BLO: Phys.Rev. (94); (Schmidtler, Schubert)

At $O(\lambda^5)$ equivalent to (Branco, Lavoura, 88)

Basic Virtues of this Definition:

$$V_{us} = \lambda + O(\lambda^7)$$

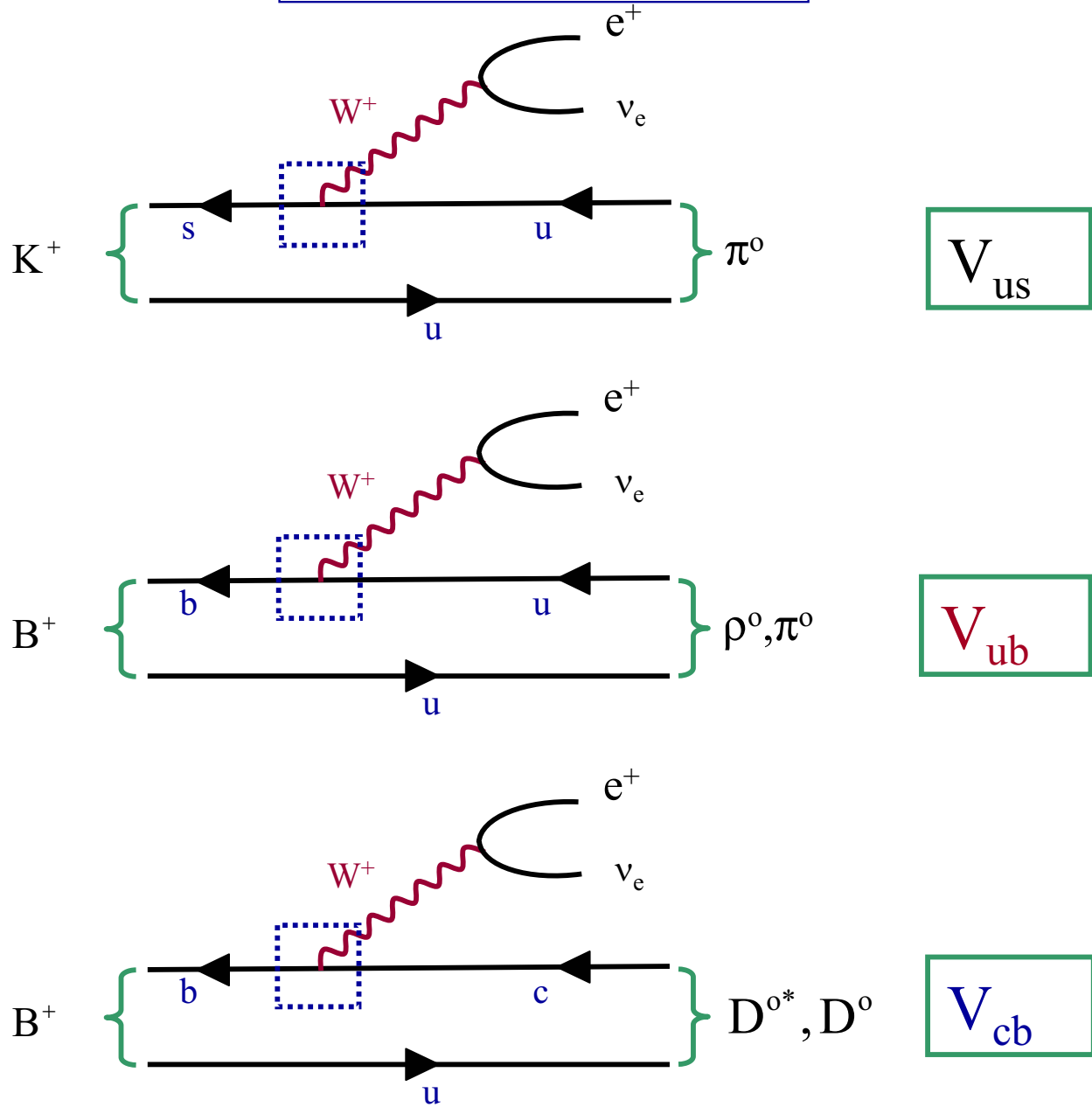
$$V_{ub} = A \lambda^3 (\rho - i\eta)$$

$$V_{cb} = A \lambda^2 + O(\lambda^8)$$

$$V_{td} = A \lambda^3 (1 - \bar{\rho} - i\bar{\eta})$$

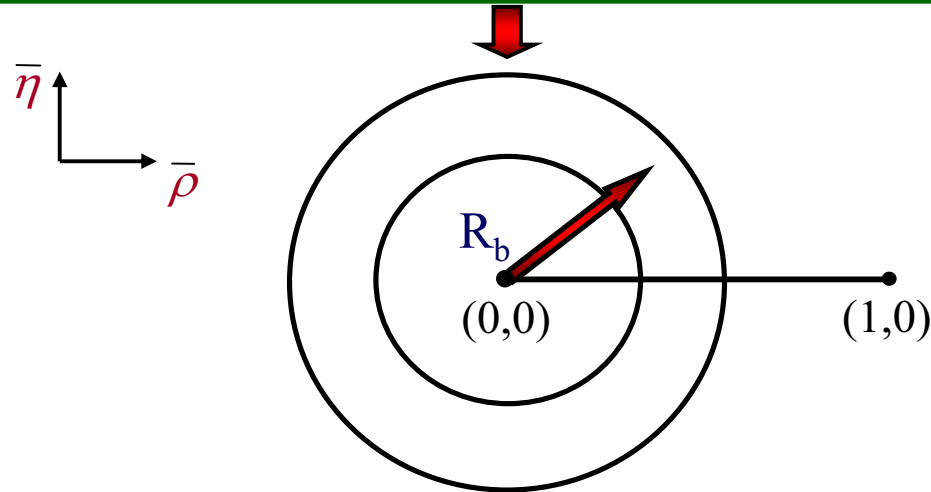
The apex of UT given by $(\bar{\rho}, \bar{\eta})$ (BLO)

Tree Level Decays



Information from Tree Level Decays

$$\begin{aligned}
 |V_{us}| &= 0.225 \pm 0.002 & = \lambda \\
 |V_{cb}| &= (41.5 \pm 0.8) \cdot 10^{-3} & (A = 0.83 \pm 0.02) \\
 \left| \frac{V_{ub}}{V_{cb}} \right| &= 0.092 \pm 0.012 & (R_b = 0.40 \pm 0.06)
 \end{aligned}$$



Apex of Unitarity Triangle somewhere on this Band

To find it **GO TO**

Loop Induced Decays

CP-Violation in K-Decays

CP-Violation in B-Decays

$$F_1^{\text{th}}(\lambda, A, \bar{\eta}, \bar{\rho}) = F_1^{\text{exp}}$$

$$F_2^{\text{th}}(\lambda, A, \bar{\eta}, \bar{\rho}) = F_2^{\text{exp}}$$

$$F_3^{\text{th}}(\lambda, A, \bar{\eta}, \bar{\rho}) = F_3^{\text{exp}}$$

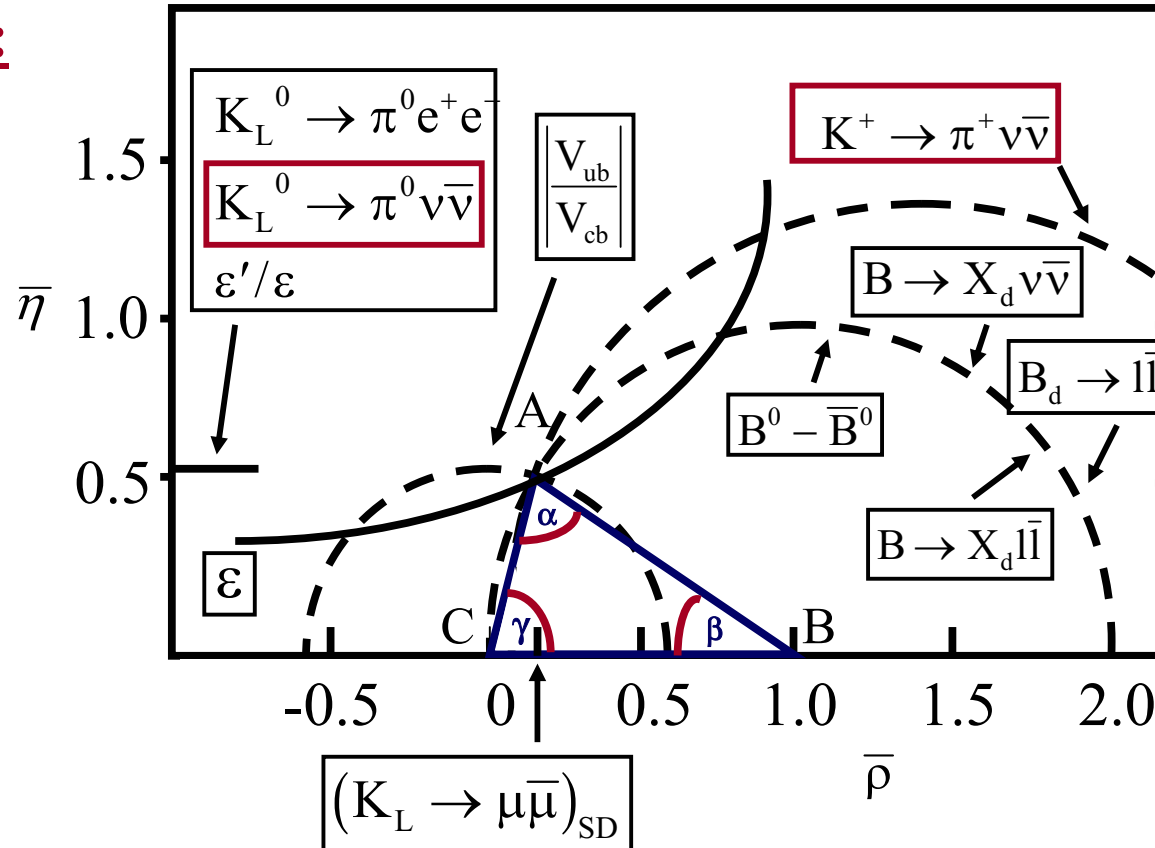
etc.



Determination of
the Unitarity Triangle

Hunting Δ with Rare and ~~CP~~ Decays

2012:



★ **Quark Mixing and CP Violation
closely related in the St. Model**

★ $\left\{ \begin{array}{l} \text{CP Asymmetries} \\ \text{in} \\ \text{B-Decays} \end{array} \right\} \rightarrow \left\{ \begin{array}{l} \alpha \\ \beta \\ \gamma \end{array} \right\}$

Lecture I

1. TH Framework
2. Various Types of ~~CP~~

Lecture II

3. Standard Analysis of Δ
4. α, β, γ from B's
5. $K^+ \rightarrow \pi^+ \nu \bar{\nu}, K_L \rightarrow \pi^0 \nu \bar{\nu}$

Lecture III

6. Rare B- and K-Decays
7. Models with MFV

Lecture IV

8. Going Beyond MFV
9. Probing New Physics in
10 Steps
10. Outlook

Literature

Buchalla, AJB, Lautenbacher

Rev. Mod. Phys. 68 (1996) 1125

AJB, Fleischer

in Heavy Flavours II (World Scientific) (1998) (hep-ph / 9704376)

AJB

★ Les Houches Lectures (1997) (hep-ph / 9806471)

Ericse Lectures (2000) (hep-ph / 0101336)

★ Spain Lectures (2004) (hep-ph / 0505175)

Y. Nir

Scottish Universities Summer School (hep-ph / 0109090)

The BABAR Physics Book

B-Physics at the LHC (hep-ph / 0003238)

Books: Branco, Lavoura, Silva;
Bigi, Sanda

B Physics at the Tevatron (Run II and Beyond) (hep-ph/0201071)

Fleischer:

Physics Reports (hep-ph/0207108)

1.

Theoretical Framework

Starting Point

:

$$\mathcal{L} = \mathcal{L}_{\text{SM}}(g_i, m_i, V_{\text{CKM}}^i) + \mathcal{L}_{\text{NP}}(g_i^{\text{NP}}, m_i^{\text{NP}}, V_{\text{NP}}^i)$$

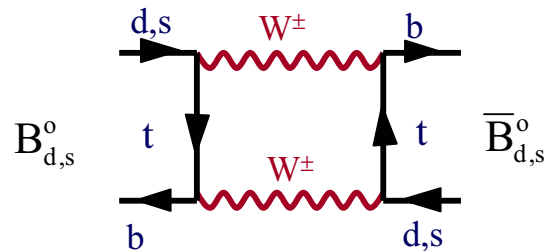
Goal

:

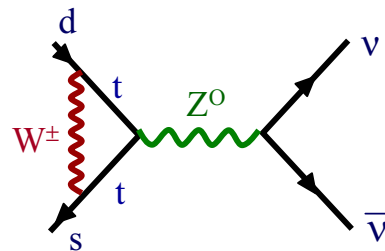
Identify the effects of \mathcal{L}_{NP} in weak decays in the presence of the background from \mathcal{L}_{SM}

First Implication from \mathcal{L}

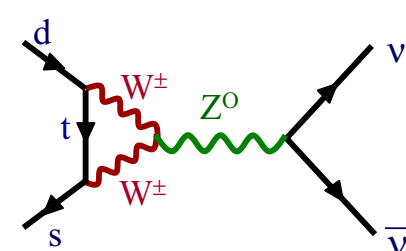
: Feynman Diagrams



$B_d^0 - \bar{B}_d^0$ Mixing



$K^+ \rightarrow \pi^+ \nu \bar{\nu}, K_L \rightarrow \pi^0 \nu \bar{\nu}$



+ NP

Two challenges :

1. Theory formulated in terms of quarks but experiments involve their bound states (K, B, D)
2. NP takes place at very short distance scales (10^{-19} - 10^{-18} m), while K, B, D live at 10^{-16} - 10^{-15} m.

Solution : Effective Theories, OPE, Renormalization Group



Separation of SD from LD
+ Summation of large $\log(\mu_{SD} / \mu_{LD})$

The Problem of Strong Interactions

$B_d^0 - \bar{B}_d^0$ Mixing

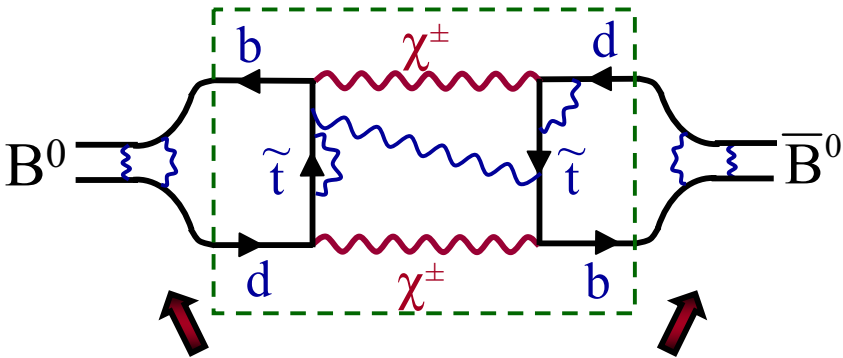
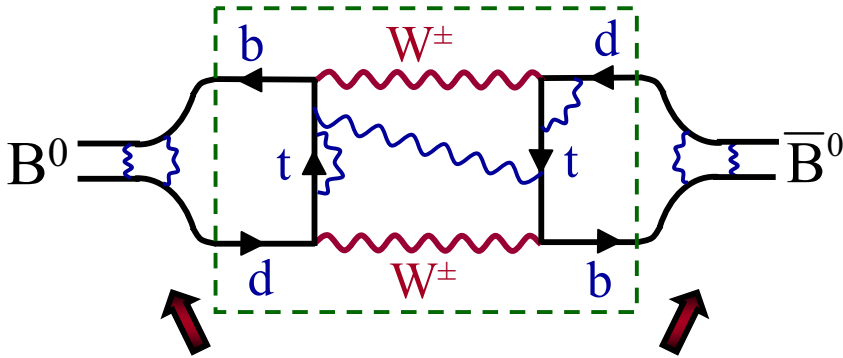
(SM)

$B_d^0 - \bar{B}_d^0$ Mixing

(MSSM)

Short Distance

Short Distance



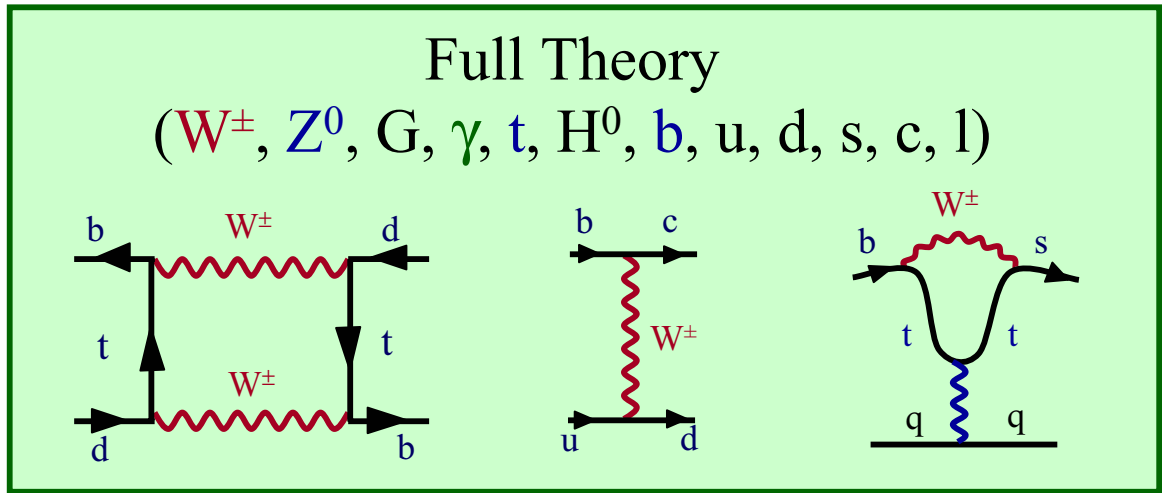
Long Distance

Long Distance

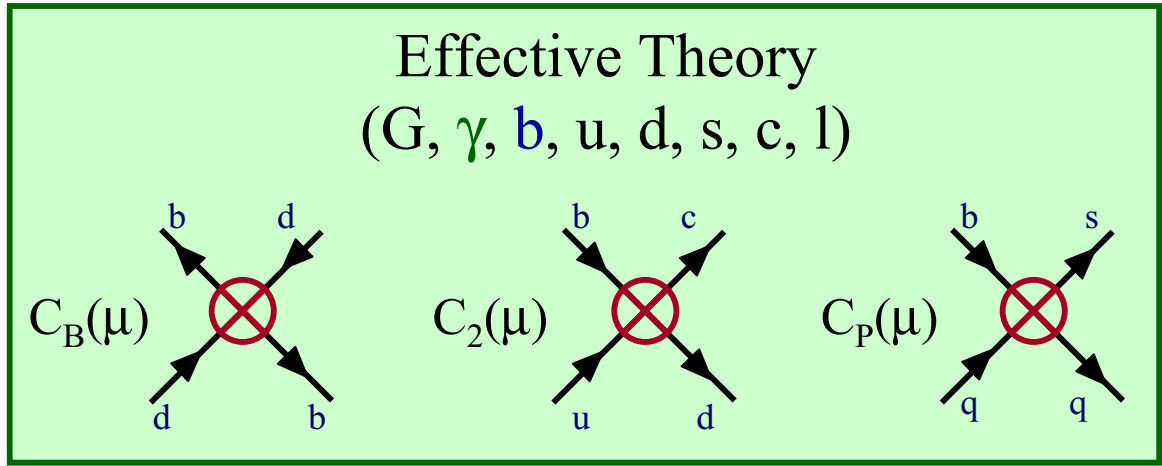
SD : Perturbative
(Asymptotic Freedom)

LD : Non-Perturbative
(Confinement)

Effective Field Theory



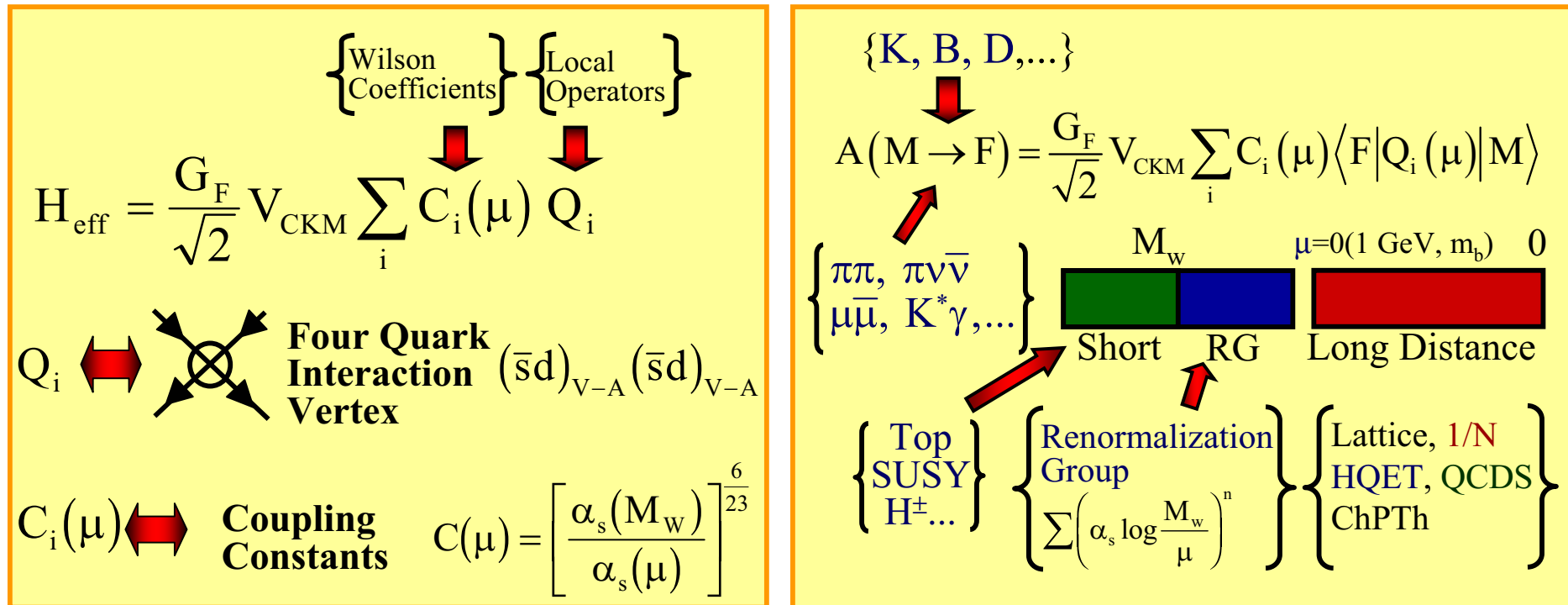
$\mu \geq M_W$



$\mu \cong 0(m_b)$

"Generalized Fermi Theory" with calculable "couplings" $C_B(\mu), C_2(\mu), \dots$

Operator Product Expansion



$$\langle \bar{K}^0 | (\bar{s}d)_{V-A} (\bar{s}d)_{V-A} | K^0 \rangle = \frac{8}{3} \hat{B}_K F_K^2 m_K^2 [\alpha_s(\mu)]^{2/9}$$

Operators

Current-Current

$$Q_1 = (\bar{s}_\alpha u_\beta)_{V-A} (\bar{u}_\beta d_\alpha)_{V-A} \quad Q_2 = (\bar{s}u)_{V-A} (\bar{u}d)_{V-A}$$

QCD-Penguins

$$Q_3 = (\bar{s}d)_{V-A} \sum_{q=u,d,s} (\bar{q}q)_{V-A} \quad Q_4 = (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V-A}$$
$$Q_5 = (\bar{s}d)_{V-A} \sum_{q=u,d,s} (\bar{q}q)_{V+A} \quad Q_6 = (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} (\bar{q}_\beta q_\alpha)_{V+A}$$

Electroweak-Penguins

$$Q_7 = \frac{3}{2} (\bar{s}d)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}q)_{V+A} \quad Q_8 = \frac{3}{2} (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\alpha)_{V+A}$$
$$Q_9 = \frac{3}{2} (\bar{s}d)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}q)_{V-A} \quad Q_{10} = \frac{3}{2} (\bar{s}_\alpha d_\beta)_{V-A} \sum_{q=u,d,s} e_q (\bar{q}_\beta q_\alpha)_{V-A}$$

View at Long Distance Scales



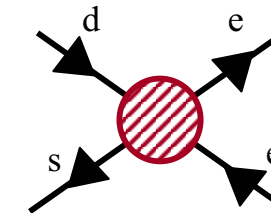
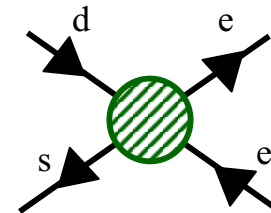
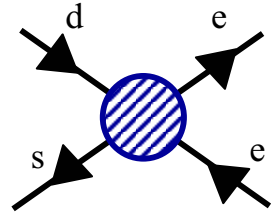
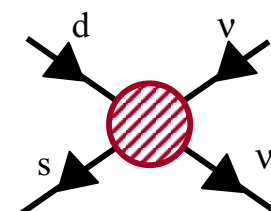
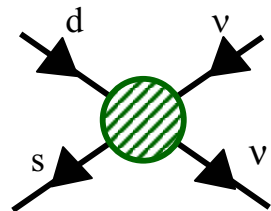
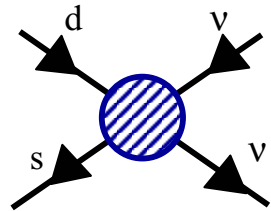
$$\boxed{K^+ \rightarrow \pi^+ \nu \bar{\nu}}$$

$$K_L \rightarrow \pi^0 \nu \bar{\nu}$$



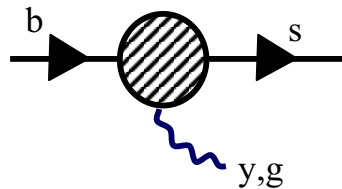
$$\boxed{K_L \rightarrow \mu \bar{\mu}}$$

$$B \rightarrow \mu \bar{\mu}, \quad B \rightarrow X_S \nu \bar{\nu}$$



$$K_L \rightarrow \pi^0 e^+ e^-$$

$$B \rightarrow X_S e^+ e^-, \quad X_S \mu \bar{\mu}$$

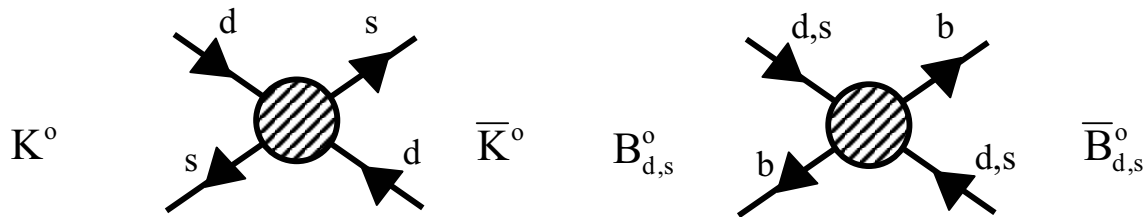


$$\boxed{B \rightarrow X_S \gamma} \quad \boxed{B \rightarrow K^* \gamma}$$

$$B \rightarrow X_d \gamma \quad b \rightarrow s \text{ gluon}$$

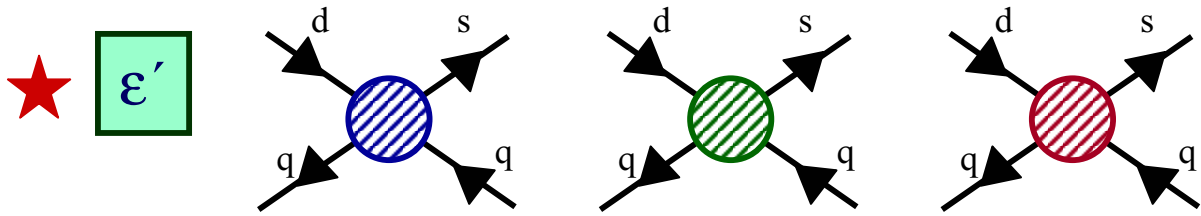


View at Long Distance Scales



★ \cancel{CP} ϵ_K -Parameter
 $\Delta M (K_L - K_S)$

$B_d^0 - \bar{B}_d^0$ Mixing ★



★

★

$K^+ \rightarrow \pi^+ \nu \bar{\nu}$, $K_L \rightarrow \pi^0 \nu \bar{\nu}$

$K_L \rightarrow \mu \bar{\mu}$, $B \rightarrow \mu \bar{\mu}$, $B \rightarrow X_S \nu \bar{\nu}$

Deriving $H_{\text{eff}}(B_d^0 - \bar{B}_d^0)$ and $\Delta M_d(B_d^0 - \bar{B}_d^0)$ in 7 Steps

Step 1 : Calculate Box Diagrams

Φ^\pm - Goldstone Bosons
Must be taken into account except in a Unitarity Gauge

$$\sum_{i,j=u,c,t} \sum_{\alpha,\beta=W^\pm,\Phi^\pm} \text{Diagram} \equiv \sum_{i,j=u,c,t} F(x_i, x_j) (V_{ib}^* V_{id}) (V_{jb}^* V_{jd}) Q$$

Step 2 : Use

UG : see AJB, Poschenrieder, Uhlig; hep-ph/0410309

$$V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0$$

Multiply by i and keep only $V_{td} V_{tb}^*$ part

$$x_i \equiv \frac{m_i^2}{M_w^2}$$

$$H_{\text{eff}}^{(\Delta B=2)} = \frac{G_F^2}{16\pi^2} M_w^2 (V_{tb}^* V_{td})^2 S_0(x_t) Q(\Delta B=2)$$

$$S_0(x_t) = \tilde{F}(x_t, x_t) + \tilde{F}(x_u, x_u) - 2\tilde{F}(x_t, x_u)$$

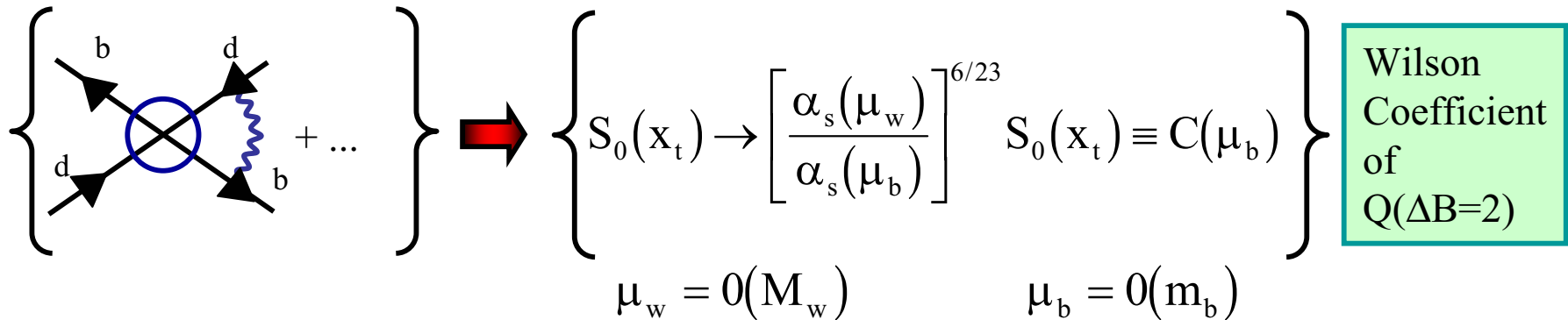
$$Q(\Delta B=2) = (\bar{b}d)_{V-A} (\bar{b}d)_{V-A}$$

$$x_u = 0$$

Step 3

: Include QCD Corrections in the Leading Logarithmic Approximation

 = Gluon



Problems with LO \equiv LLA

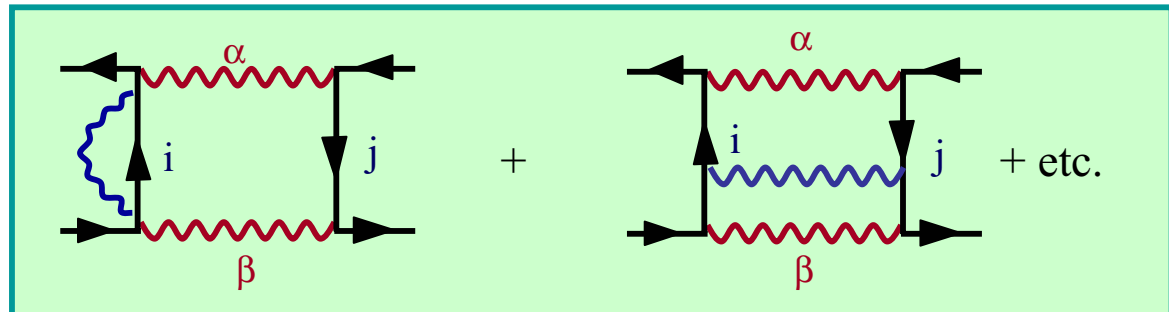
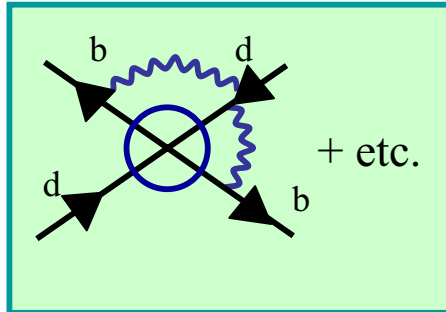
: Sensitivity to the choices of

- i) μ_w $80 \text{ GeV} < \mu_w < 300 \text{ GeV}$
- ii) μ_b $2.5 \text{ GeV} < \mu_b < 5 \text{ GeV}$
- iii) μ_t $x_t = \frac{m_t^2(\mu_t)}{M_w^2}$
 $80 \text{ GeV} < \mu_t < 300 \text{ GeV}$

Step 4 : Include Next to Leading QCD Corrections

(AJB, Jamin, Weisz 1990)

Requires:



~~~~~ = Gluon

Pages 101-103 Les Houches Lectures

$$H_{\text{eff}}^{(\Delta B=2)} = \frac{G_F^2}{16\pi^2} M_w^2 (V_{tb}^* V_{td})^2 S_0(x_t) \underbrace{\tilde{\eta}_B^{\text{QCD}} \left[ \frac{\alpha_s(\mu_w)}{\alpha_s(\mu_b)} \right]^{6/23} \left( 1 + J_5 \frac{\alpha_s(\mu_b) - \alpha_s(\mu_w)}{4\pi} \right)}_{\text{Independent of } \mu_w \text{ and } \mu_t \text{ but still dependent on } \mu_b} Q(\Delta B = 2)$$

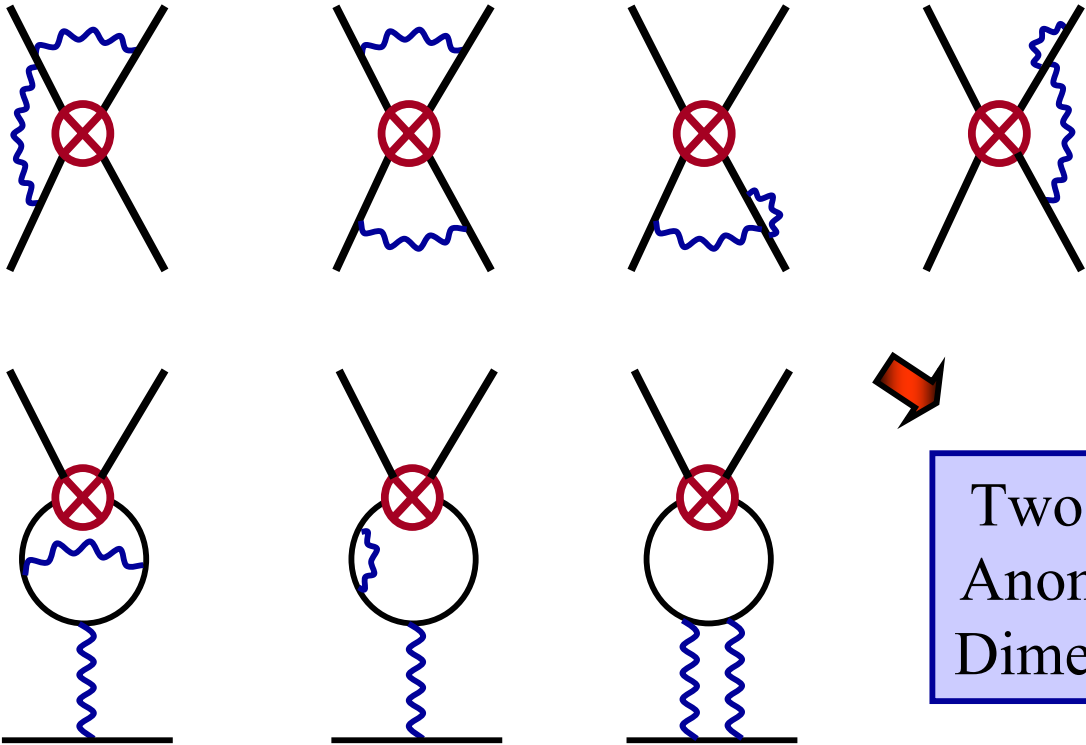
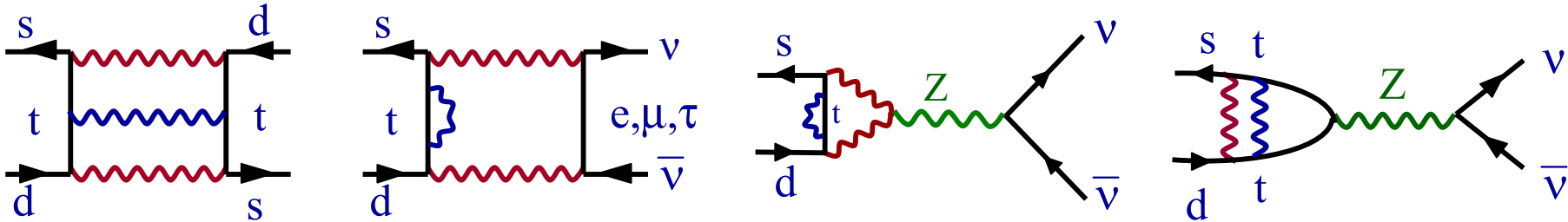
$$\tilde{\eta}_B^{\text{QCD}} = 1 + \frac{\alpha_s(\mu_w)}{4\pi} G(\mu_w, \mu_t)$$

Independent of  $\mu_w$  and  $\mu_t$  but still dependent on  $\mu_b$

$$J_5 = 1.627$$

# Typical Two-Loop Diagrams

~~~~~  $W^\pm$   
~~~~~  $G$



Two-Loop  
Anomalous  
Dimensions

**Step 5** : Calculate the Matrix Element  $\langle Q(\Delta B = 2) \rangle$

$$\langle \bar{B}_d^0 | Q(\Delta B = 2) | B_d^0 \rangle = \frac{8}{3} B_B(\mu_b) F_B^2 m_B^2 \quad F_B = \text{B-Meson Decay Constant}$$

This  $\mu_b$  – dependence cancels the one in  $H_{\text{eff}}^{\Delta B=2}$

**Step 6** : Put  $\langle H_{\text{eff}}^{\Delta B=2} \rangle$  in a manifestly  $\mu_w, \mu_t, \mu_b$  Form

$$\eta_B^{\text{QCD}} \equiv \tilde{\eta}_B^{\text{QCD}} [\alpha_s(\mu_w)]^{6/23} \left( 1 - J_5 \frac{\alpha_s(\mu_w)}{4\pi} \right) \quad \begin{array}{l} \text{with } \mu_t = m_t \\ \mu_w \text{ - independent} \end{array}$$

$$\hat{B}_B \equiv B_B(\mu_b) [\alpha_s(\mu_b)]^{-6/23} \left( 1 + J_5 \frac{\alpha_s(\mu_b)}{4\pi} \right) \quad \mu_b \text{ - independent}$$

$S_0(x_t)$  evaluated at  $\mu_t = m_t$

**Step 7**

: Calculation of  $\Delta M_d(B_d^0 - \bar{B}_d^0)$

Use

$$\Delta M_d = \frac{1}{m_b} \left| \langle \bar{B}_d^0 | H_{\text{eff}}^{(\Delta B=2)} | B_d^0 \rangle \right|$$



$$\Delta M_d = \frac{G_F^2}{6\pi^2} m_b M_w^2 \underbrace{(\hat{B}_d F_{B_d}^2)}_{\text{independent of } \mu_b} \underbrace{\eta_B^{\text{QCD}} S_0(x_t)}_{\text{independent of } \mu_w, \mu_t} |V_{td}|^2$$

$$\sqrt{\hat{B}_d} F_{B_d} = \left( 235^{+33}_{-41} \right) \text{MeV}$$
$$\eta_B^{\text{QCD}} = 0.551 \pm 0.006$$

$$S_0(x_t) = \frac{4x_t - 11x_t^2 + x_t^3}{4(1-x_t)^2} - \frac{3x_t \log x_t}{2(1-x_t)^3}$$
$$\approx 2.46 \left( \frac{m_t}{170 \text{GeV}} \right)^{1.52}$$



$$(\Delta M)_{d,s}, \quad |V_{td}|/|V_{ts}| \quad \text{and} \quad R_t$$



$$(\Delta M)_d = \frac{0.50}{\text{ps}} \left[ \frac{\sqrt{\hat{B}_d} F_{Bd}}{230 \text{MeV}} \right]^2 \left[ \frac{|V_{td}|}{7.8 \cdot 10^{-3}} \right]^2 \left[ \frac{\eta_B}{0.55} \right] \left[ \frac{S(x_t)}{2.34} \right]$$

$$(\Delta M)_s = \frac{18.4}{\text{ps}} \left[ \frac{\sqrt{\hat{B}_s} F_{Bs}}{270 \text{MeV}} \right]^2 \left[ \frac{|V_{ts}|}{0.040} \right]^2 \left[ \frac{\eta_B}{0.55} \right] \left[ \frac{S(x_t)}{2.34} \right]$$

$$S(x_t) = 2.42 \pm 0.12$$

$$\eta_B = 0.55 \pm 0.01$$

AJB, Jamin, Weisz

$$|V_{td}| = \lambda |V_{cb}| R_t$$

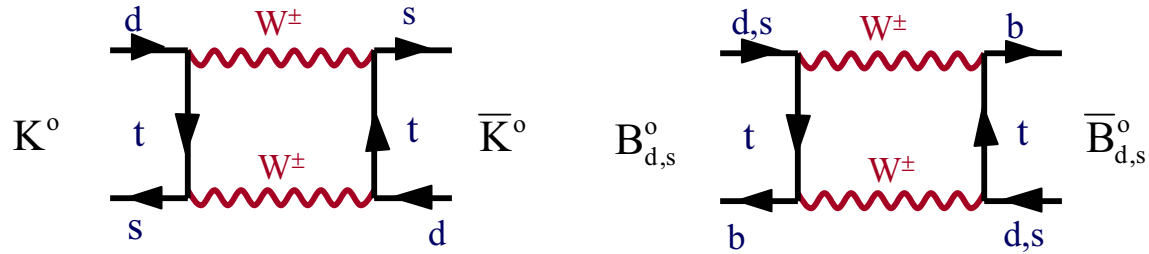
$$|V_{ts}| = |V_{cb}| \left( 1 - \frac{\lambda^2}{2} + \bar{\rho} \lambda^2 \right)$$

$$\xi = \frac{\sqrt{\hat{B}_s} F_{Bs}}{\sqrt{\hat{B}_d} F_{Bd}} = 1.22 \pm 0.07$$

$$\frac{|V_{td}|}{|V_{ts}|} = 1.01 \xi \sqrt{\frac{\Delta M_d}{\Delta M_s}}$$

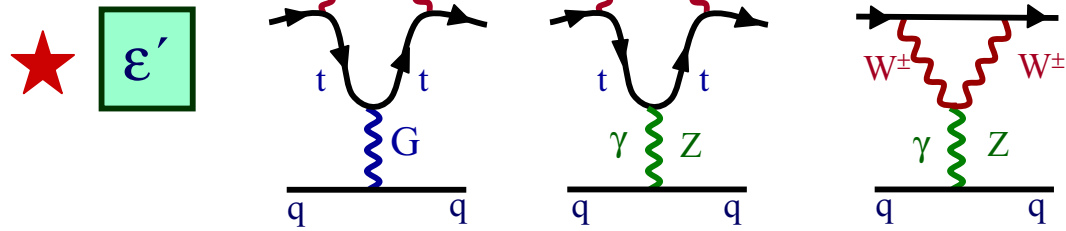
$$R_t = 0.90 \sqrt{\frac{\Delta M_d}{0.50 / \text{ps}}} \sqrt{\frac{18.4 / \text{ps}}{\Delta M_s}} \left[ \frac{\xi}{1.22} \right]$$

# View at Short Distance Scales



★  $\cancel{CP}$   $\epsilon_K$ -Parameter  
 $\Delta M (K_L - K_S)$

$B_d^0 - \bar{B}_d^0$  Mixing ★



# View at Short Distance Scales



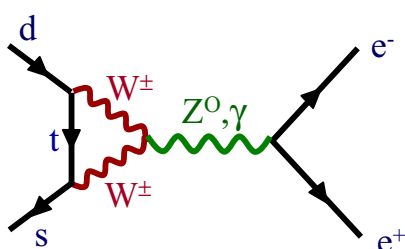
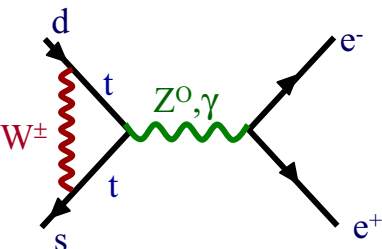
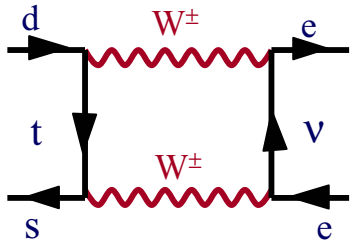
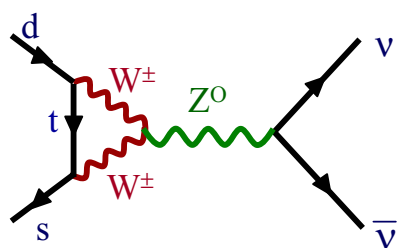
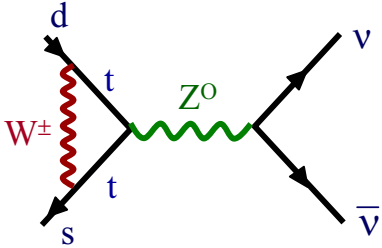
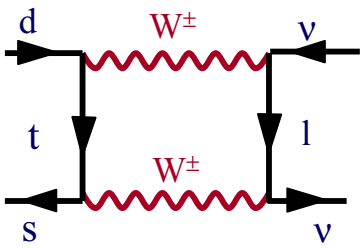
$$\boxed{K^+ \rightarrow \pi^+ \nu \bar{\nu}}$$

$$K_L \rightarrow \pi^0 \nu \bar{\nu}$$



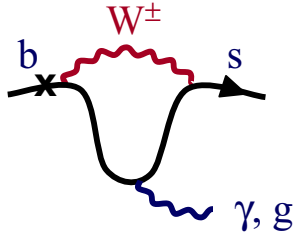
$$\boxed{K_L \rightarrow \mu \bar{\mu}}$$

$$B \rightarrow \mu \bar{\mu}, \quad B \rightarrow X_S \nu \bar{\nu}$$



$$K_L \rightarrow \pi^0 e^+ e^-$$

$$\boxed{B \rightarrow X_S e^+ e^-, X_S \mu \bar{\mu}}$$



$$\boxed{B \rightarrow X_S \gamma} \quad \boxed{B \rightarrow K^* \gamma}$$



$$B \rightarrow X_d \gamma \quad b \rightarrow s \text{ gluon}$$

# Penguin-Box Expansion (SM)

Buchalla, AJB, Harlander (90)

The  $m_t$  dependence of all K and B Decays resides in 7 Basic Universal Functions  $F_i(x_t)$

$$x_t = \frac{m_t^2}{M_W^2}$$

$$A(\text{Decay}) = \sum_i B_i \eta_{\text{QCD}}^i V_{\text{CKM}}^i F_i(x_t)$$

$F_i$  : S, X, Y, Z, E, E', D'

(Gauge Invariant set of functions)

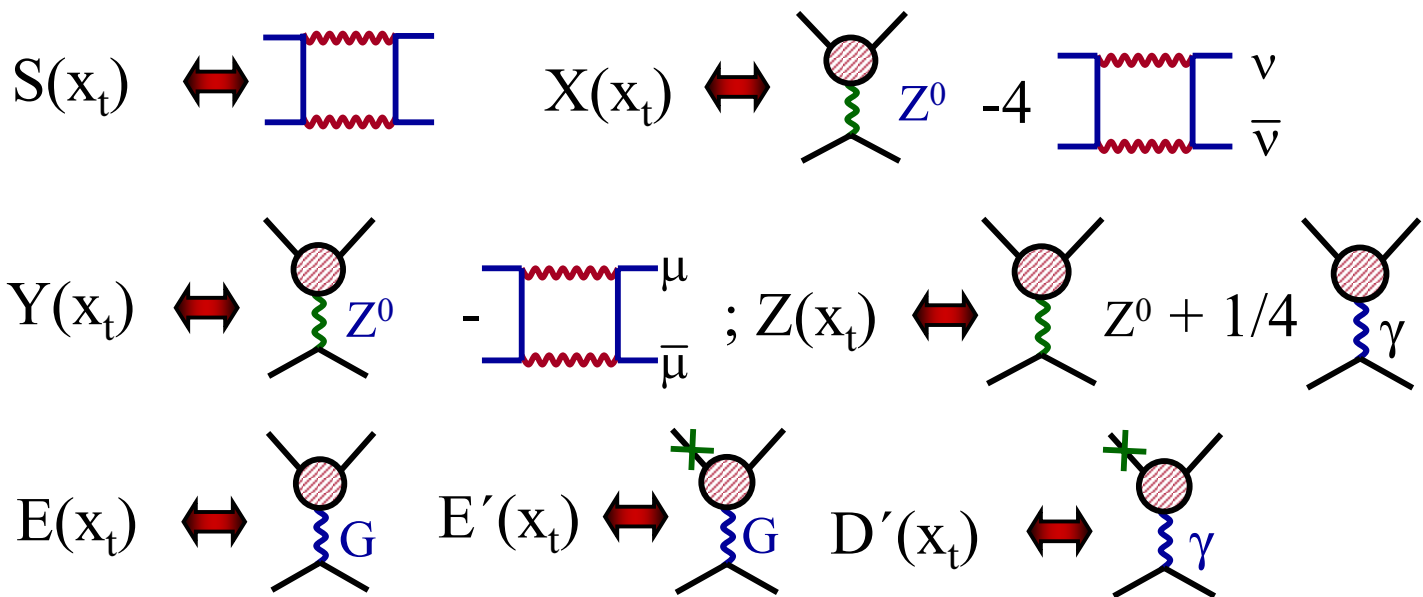
Relation to OPE:

RG

$$C_i(\mu) = \sum_j U_{ij}(\mu, M_W) \left[ \sum_r H_{jr} F_r(x_t) \right] = \sum_r \eta_{\text{ir}}^{\text{QCD}} F_r(x_t)$$

$\underbrace{\hspace{10em}}_{C_j(M_W)}$

| Decay                                                              | Contributing Functions                     |
|--------------------------------------------------------------------|--------------------------------------------|
| $B_d^0 - \bar{B}_d^0, B_s^0 - \bar{B}_s^0, \varepsilon$            | $S(x_t)$                                   |
| $K \rightarrow \pi \nu \bar{\nu}, B \rightarrow X_s \nu \bar{\nu}$ | $X(x_t)$                                   |
| $K_L \rightarrow \mu \bar{\mu}, B \rightarrow 1\bar{1}$            | $Y(x_t)$                                   |
| $\varepsilon'$                                                     | $Z(x_t), X(x_t), Y(x_t), E(x_t)$           |
| $K_L \rightarrow \pi^0 e^+ e^-$                                    | $Y(x_t), Z(x_t), E(x_t)$                   |
| $B \rightarrow X_s e^+ e^-$                                        | $Y(x_t), Z(x_t), D'(x_t), E'(x_t), E(x_t)$ |
| $B \rightarrow X_s \gamma$                                         | $D'(x_t), E'(x_t)$                         |



# $m_t$ Dependence of Basic Universal Functions

$$S(x_t) \equiv S_0(x_t) = 2.46 \left[ \frac{m_t}{170\text{GeV}} \right]^{1.52}$$

$$X(x_t) = 1.57 \left[ \frac{m_t}{170\text{GeV}} \right]^{1.15}$$

$$Y(x_t) = 1.02 \left[ \frac{m_t}{170\text{GeV}} \right]^{1.56}$$

$$Z(x_t) = 0.71 \left[ \frac{m_t}{170\text{GeV}} \right]^{1.86}$$

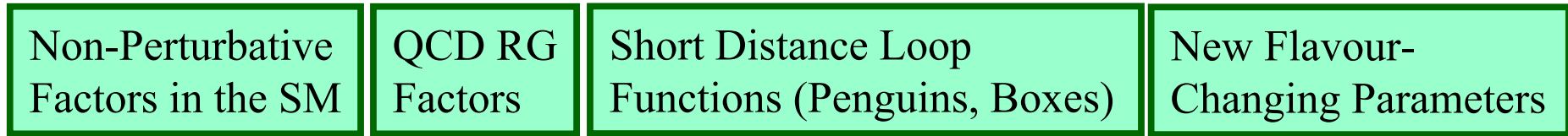
$$E(x_t) = 0.26 \left[ \frac{m_t}{170\text{GeV}} \right]^{-1.02}$$

$$D'(x_t) = 0.38 \left[ \frac{m_t}{170\text{GeV}} \right]^{0.60}$$

$$E'(X_t) = 0.19 \left[ \frac{m_t}{170\text{GeV}} \right]^{0.38}$$

# Master Formula for Weak Decays

AJB (2001)  
 hep-ph/0101336  
 hep-ph/0109197



Represent different  
 Dirac and Colour  
 Structures

$$A(\text{Decay}) = \sum_i B_i \eta_{\text{QCD}}^i V_{\text{CKM}}^i \left[ F_{\text{SM}}^i + F_{\text{New}}^i \right] + B_i^{\text{New}} \left[ \eta_{\text{QCD}}^i \right]^{\text{New}} V_{\text{New}}^i \left[ G_{\text{New}}^i \right]$$

(Summation over i)

New  $\equiv$  NP

Non-Perturbative  
 Factors beyond SM

Short Distance Loop  
 Functions Penguins, Boxes

|                                                                        |                                                                               |                                    |
|------------------------------------------------------------------------|-------------------------------------------------------------------------------|------------------------------------|
| $F_{\text{SM}}^i, F_{\text{New}}^i, G_{\text{New}}^i$                  | : Fully calculable in<br>Perturbation Theory                                  | } Fully<br>calculable<br>in the SM |
| $\eta_{\text{QCD}}^i, \left[ \eta_{\text{QCD}}^i \right]^{\text{New}}$ | : Fully calculable in RG<br>improved Perturbation Theory                      |                                    |
| $B_i, B_i^{\text{New}}$                                                | : Require Non-Perturbative Methods or<br>can be extracted from leading decays |                                    |

(represent  $\langle Q_i \rangle$ )

## Possible Dirac Structures in

$$K^0 - \bar{K}^0 \text{ and } B_{d,s}^0 - \bar{B}_{d,s}^0$$

**SM:**

$$\gamma_\mu (1 - \gamma_5) \otimes \gamma^\mu (1 - \gamma_5)$$

**Beyond SM:**

$$\begin{aligned} & \gamma_\mu (1 - \gamma_5) \otimes \gamma^\mu (1 + \gamma_5) \\ & (1 - \gamma_5) \otimes (1 + \gamma_5) \\ & (1 - \gamma_5) \otimes (1 - \gamma_5) \\ & \sigma_{\mu\nu} (1 - \gamma_5) \otimes \sigma^{\mu\nu} (1 - \gamma_5) \end{aligned}$$

**MSSM with large  $\tan\beta$**

**General Supersymmetric Models**

**Models with complicated Higgs System**

NLO  $[\eta_{\text{QCD}}^i]^{\text{New}}$  : Ciuchini, Franco, Lubicz,  
Martinelli, Scimemi, Silvestrini

AJB, Misiak, Urban, Jäger



## Three Simple Scenarios

Inami  
Lim Functions

**SM** : 
$$A(\text{Decay}) = \sum_i B_i \eta_{\text{QCD}}^i V_{\text{CKM}}^i \underbrace{F_{\text{SM}}^i(m_t)}_{\text{real}}$$

**MFV** : 
$$A(\text{Decay}) = \sum_i B_i \eta_{\text{QCD}}^i V_{\text{CKM}}^i \underbrace{\left[ F_{\text{SM}}^i + F_{\text{New}}^i \right]}_{\text{real}}$$

(Minimal  
Flavour  
Violation)

AJB, Gambino, Gorbahn, Jäger, Silvestrini  
D'Ambrosio, Giudice, Isidori, Strumia

**Enhanced  
Z<sup>0</sup>-Penguins** : 
$$A(\text{Decay}) = \sum_i B_i \eta_{\text{QCD}}^i V_{\text{CKM}}^i \left[ \underbrace{F_{\text{SM}}^i}_{\text{real}} + \underbrace{\Delta_{\text{New}}^i} \right]$$

AJB, Colangelo, Isidori, Romanino, Silvestrini  
Buchalla, Hiller, Isidori; Atwood, Hiller  
AJB, Fleischer, Recksiegel, Schwab

**Dominated by  
Z<sup>0</sup>-Penguins  
with a New  
Complex Phase**

## Two more complicated Scenarios

**MSSM (MFV)  
(large  $\tan\beta$ )**

(Higgs penguin)

$$\begin{aligned}
 A(\text{Decay}) = & \sum_i B_i \eta_{\text{QCD}}^i V_{\text{CKM}}^i \left[ \overbrace{F_{\text{SM}}^i + F_{\text{New}}^i}^{\text{real}} \right] \\
 & + \sum_i B_i^{\text{New}} \left[ \eta_{\text{QCD}}^i \right]^{\text{New}} V_{\text{CKM}}^i \underbrace{\left[ G_{\text{New}}^i \right]}_{\text{real}}
 \end{aligned}$$

**General  
MSSM**

$$\begin{aligned}
 A(\text{Decay}) = & \sum_i B_i \eta_{\text{QCD}}^i V_{\text{CKM}}^i \left[ \overbrace{F_{\text{SM}}^i + F_{\text{New}}^i}^{\text{complex}} \right] \\
 & + \sum_i B_i^{\text{New}} \left[ \eta_{\text{QCD}}^i \right]^{\text{New}} V_{\text{New}}^i \underbrace{\left[ G_{\text{New}}^i \right]}_{\text{complex}}
 \end{aligned}$$

**Z'-Models  
L-R Models  
Multi-Higgs  
Models**

# Inclusive Decays

(Generally TH  
cleaner than  
Exclusive Decays)

**Examples:**  $B \rightarrow X_s \gamma$ ,  $B \rightarrow X_s \mu^+ \mu^-$ ,  $B \rightarrow X_s \nu \bar{\nu}$

$X_s \equiv$  all final states with  $\Delta S = 1$  quantum number

1. Construction of  $H_{\text{eff}}$  as in the case of **Exclusive Decays**
2. The branching ratios calculated perturbatively from b-quark decay diagrams
3. Non-Perturbative Effects  $\sim \Lambda_{\text{QCD}}^2 / m_b^2$  ( $\sim 5\%$ )

## Heavy Quark Expansions:

Chay, Georgi, Grinstein (1990)  
Bigi, Shifman, Uraltsev, Vainshtein (1992)  
Manohar, Wise (1993); Mannel (1993)

# 2.

## Particle Mixing and Various Types of CP Violation

# Express Review of $B^0 - \bar{B}^0$ Mixing

## ◆ Flavour Eigenstates

$$B^0 = (\bar{b}d)$$

$$\bar{B}^0 = (b\bar{d})$$

$$CP|B^0\rangle = -|\bar{B}^0\rangle$$

$$CP|\bar{B}^0\rangle = -|B^0\rangle$$

In the absence of  $B^0 - \bar{B}^0$  Mixing:

$$\begin{array}{l}
 |B^0(t)\rangle = |B^0(0)\rangle \exp[-i H t] \\
 |\bar{B}^0(t)\rangle = |\bar{B}^0(0)\rangle \exp[-i H t]
 \end{array}
 \quad
 \begin{array}{l}
 H = M - i \frac{\Gamma}{2} \\
 \nearrow \quad \nearrow \\
 \text{Mass} \quad \text{Width}
 \end{array}$$

## ◆ Time Evolution in the Presence of Mixing

$$i \frac{d\psi(t)}{dt} = \hat{H} \psi(t) \quad \psi(t) = \begin{pmatrix} |B^0(t)\rangle \\ |\bar{B}^0(t)\rangle \end{pmatrix}$$

Hermitian Matrices  
with positive (real)  
eigenvalues

$$\hat{H} = \hat{M} - i \frac{\hat{\Gamma}}{2} = \begin{pmatrix} M_{11} - i \frac{\Gamma_{11}}{2} & M_{12} - i \frac{\Gamma_{12}}{2} \\ M_{21} - i \frac{\Gamma_{21}}{2} & M_{22} - i \frac{\Gamma_{22}}{2} \end{pmatrix}$$

$M_{ij}$ -transition with virtual  
intermediate states  
 $\Gamma_{ij}$ - transition with physical  
intermediate states

$$M_{21} = M_{12}^* \quad \Gamma_{21} = \Gamma_{12}^*$$

# Express Review of $B^0$ - $\bar{B}^0$ Mixing

## ◆ Flavour Eigenstates

$$B_d^0 = (\bar{b}d)$$

$$\bar{B}_d^0 = (b\bar{d})$$

$$B_s^0 = (\bar{b}s)$$

$$\bar{B}_s^0 = (b\bar{s})$$

see:  
 Erice (2000)  
 Spain (2004)  
 ( $K^0 - \bar{K}^0$ )

## ◆ Mass Eigenstates

$$B_{H,L} = p B^0 \pm q \bar{B}^0$$

$$p = \frac{(1 + \bar{\epsilon}_B)}{\sqrt{2(1 + |\bar{\epsilon}_B|^2)}} \quad q = \frac{(1 - \bar{\epsilon}_B)}{\sqrt{2(1 + |\bar{\epsilon}_B|^2)}}$$

$$\frac{q}{p} = \frac{2M_{12}^* - i\Gamma_{12}^*}{\Delta M - i\frac{\Delta\Gamma}{2}}$$

$$\Delta M = M(B_H) - M(B_L)$$

$$\Delta\Gamma = \Gamma(B_H) - \Gamma(B_L)$$

All exact formulae from  $K^0 - \bar{K}^0$  system apply  
 but now:

$$|M_{12}| \gg |\Gamma_{12}|$$



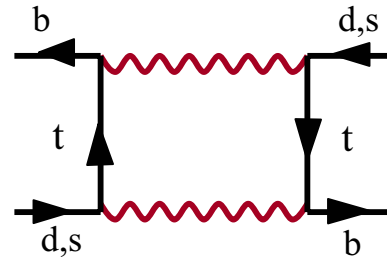
◆ Master Formulae ( $B^0$ - $\bar{B}^0$ )

$$\Delta M = 2|M_{12}|$$

$$\Delta\Gamma = 2\frac{\text{Re}(M_{12}\Gamma_{12}^*)}{|M_{12}|}$$

$$\frac{q}{p} \cong \frac{M_{12}^*}{|M_{12}|} \left[ 1 - \frac{1}{2} \text{Im} \left( \frac{\Gamma_{12}}{M_{12}} \right) \right]$$

$$M_{12}^* = \langle \bar{B}^0 | H_{\text{eff}} | B^0 \rangle \approx$$



$$\left( M_{12}^* \right)_d \sim \left( V_{td} V_{tb}^* \right)^2 \quad \left( M_{12}^* \right)_s \sim \left( V_{ts} V_{tb}^* \right)^2$$

$$V_{td} = |V_{td}| e^{-i\beta} \quad V_{ts} = |V_{ts}| e^{-i\beta_s} \quad (\beta_s \cong 0)$$

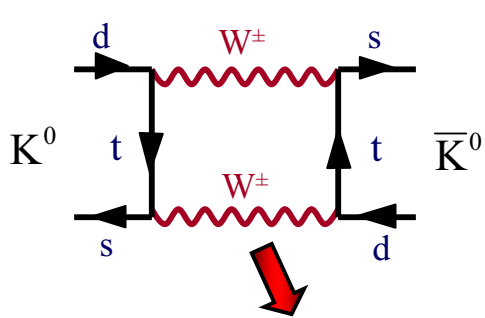
$$\frac{q}{p} \cong e^{i2\varphi_M} \quad \varphi_M = \begin{cases} -\beta & B_d^0 - \bar{B}_d^0 \\ -\beta_s & B_s^0 - \bar{B}_s^0 \end{cases}$$

(Pure Phase)

# Indirect and Direct $\mathcal{CP}$ in $K_L \rightarrow \pi\pi$

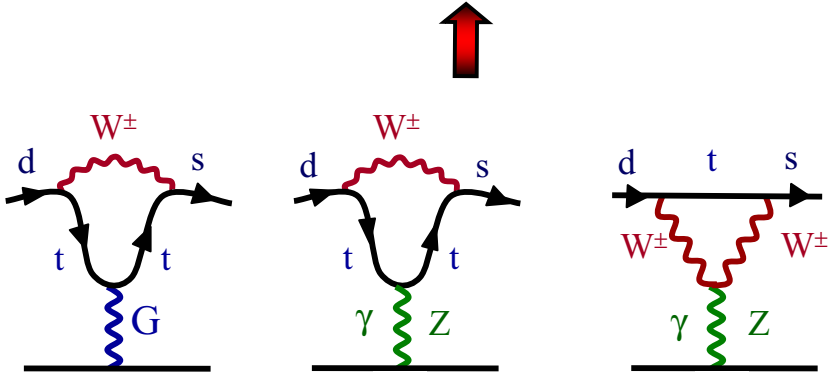
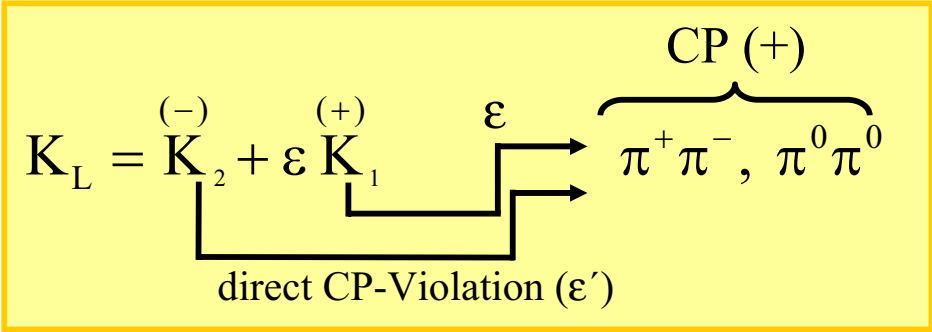
$$K_{1,2} = \frac{K^0 \mp \bar{K}^0}{\sqrt{2}}$$

$$CP |K^0\rangle = -|\bar{K}^0\rangle$$



Mass Eigenstates are not  
CP Eigenstates

indirect CP violation ( $\epsilon$ )



$$\eta_{+-} \equiv \frac{A(K_L \rightarrow \pi^+\pi^-)}{A(K_S \rightarrow \pi^+\pi^-)} = \epsilon + \epsilon'$$

$$\eta_{00} \equiv \frac{A(K_L \rightarrow \pi^0\pi^0)}{A(K_S \rightarrow \pi^0\pi^0)} = \epsilon - 2\epsilon'$$

$\epsilon' = 0$  in Superweak Models  
Wolfenstein (64)

$$\text{Re}\left(\frac{\epsilon'}{\epsilon}\right) = \frac{1}{6} \left( 1 - \left| \frac{\eta_{00}}{\eta_{+-}} \right|^2 \right)$$



**June 2005**

$$\Delta M_K = (0.5301 \pm 0.0016) \cdot 10^{-2} / \text{ps}$$

$$\Delta M_d = (0.503 \pm 0.006) / \text{ps}$$

$$\Delta M_s > 14.4 / \text{ps} \quad (95\% \text{ C.L.})$$

$$1 / \text{ps} = 6.582 \cdot 10^{-13} \text{ GeV}$$

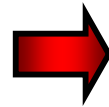
$$\varepsilon = (2.280 \pm 0.013) \cdot 10^{-3} e^{i\pi/4}$$

$$\text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right) = (16.6 \pm 1.6) \cdot 10^{-4}$$

# Modern Classification of CP Violation

We have:

Particle-Antiparticle  
Mixing



and

Decay

- 1.** CP Violation in Mixing
- 2.** CP Violation in Decay
- 3.** CP Violation in the Interference of Mixing and Decay

# Classification of $\mathcal{CP}$ in B- and K-Decays

(Nir 99),...

## 1. CP Violation in Mixing

$$B_{H,L} = p|B^0\rangle \pm q|\bar{B}^0\rangle \quad \left[ \begin{array}{c} \text{Mass} \\ \text{Eigenstates} \end{array} \right]$$

$$\mathcal{CP}: \quad |q/p| \neq 1 \quad \Rightarrow \quad (\text{Not CP Eigenstates})$$

$$a_{\text{SL}} = \frac{\Gamma(\bar{B}^0(t) \rightarrow l^+ \nu X) - \Gamma(B^0(t) \rightarrow l^- \nu X)}{\Gamma(\bar{B}^0(t) \rightarrow l^+ \nu X) + \Gamma(B^0(t) \rightarrow l^- \nu X)}$$

$$a_{\text{SL}} = \frac{1 - |q/p|^4}{1 + |q/p|^4} = \text{Im} \frac{\Gamma_{12}}{M_{12}}$$

$$\hat{H} = \hat{M} - i \frac{\hat{\Gamma}}{2}$$

Observed in K-system:  $\text{Re } \epsilon_K \neq 0$

$$\begin{array}{l} \bar{B}^0 \rightarrow B^0 \rightarrow l^+ \nu X \\ \updownarrow \text{ (Phase Difference) } \\ B^0 \rightarrow \bar{B}^0 \rightarrow l^- \nu X \end{array}$$

"wrong charge"  
leptons

$$a_{\text{SL}} \approx 0(10^{-3})$$

Hadronic Uncertainties in  $\Gamma_{12}, M_{12}$

## 2.

## CP Violation in Decay

$$A_f = \langle f | H^{\text{weak}} | B \rangle \quad \bar{A}_{\bar{f}} = \langle \bar{f} | H^{\text{weak}} | \bar{B} \rangle$$

$$\text{CP: } \boxed{|\bar{A}_{\bar{f}} / A_f| \neq 1} \quad f \xrightarrow{\text{CP}} \bar{f}$$

$$a_{f^\pm}^{\text{Decay}} = \frac{\Gamma(B^+ \rightarrow f^+) - \Gamma(B^- \rightarrow f^-)}{\Gamma(B^+ \rightarrow f^+) + \Gamma(B^- \rightarrow f^-)} = \frac{1 - |\bar{A}_{f^-} / A_{f^+}|^2}{1 + |\bar{A}_{f^-} / A_{f^+}|^2}$$

Requires at least two different contributions  
with different weak ( $\varphi_i$ ) and strong ( $\delta_i$ ) phases

$$A_f = \sum_i A_i e^{i(\delta_i + \varphi_i)} \quad \bar{A}_{\bar{f}} = \sum_i A_i e^{i(\delta_i - \varphi_i)} \quad (A_2 \ll A_1) \quad r \equiv \frac{A_2}{A_1} \ll 1$$

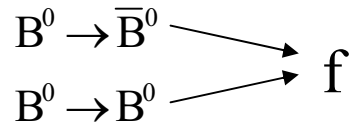
$i = 1, 2$

$$\boxed{a_{f^\pm}^{\text{Decay}} \approx -2r \sin(\delta_2 - \delta_1) \sin(\varphi_2 - \varphi_1)}$$

Observed in K-system:  $\boxed{\text{Re } \varepsilon'_K \neq 0}$

$\boxed{\text{Hadronic Uncertainties in } A_i, \delta_i}$

# B<sup>0</sup>-Decays into CP-Eigenstate



$\Delta M$  = Difference between Mass Eigenstates in (B<sup>0</sup>,  $\bar{B}^0$ ) System

$f \equiv f_{CP}$  = CP eigenstate  
 $\eta_f$  = CP-parity =  $\pm 1$

Time-dependent asymmetry:

$$a_{CP}(t, f) = \frac{\Gamma(B^0(t) \rightarrow f) - \Gamma(\bar{B}^0(t) \rightarrow f)}{\Gamma(B^0(t) \rightarrow f) + \Gamma(\bar{B}^0(t) \rightarrow f)}$$

$$a_{CP}(t, f) = a_{CP}^{Decay} \cos(\Delta Mt) + a_{CP}^{mix-ind} \sin(\Delta Mt)$$

$$a_{CP}^{Decay} = \frac{1 - |\xi_f|^2}{1 + |\xi_f|^2} \equiv C_f \quad a_{CP}^{mix-ind} = \frac{2 \operatorname{Im} \xi_f}{1 + |\xi_f|^2} \equiv -S_f$$

$$\xi_f = \underbrace{\exp[i2\phi_M]}_{\text{Mixing}} \frac{\bar{A}_f(\bar{B}^0 \rightarrow f)}{A_f(B^0 \rightarrow f)} \quad \left\{ \begin{array}{l} \text{Decay} \\ \text{Amplitudes} \end{array} \right.$$

For a **single** decay contribution or sum of contributions with **the same weak phase**

$$\begin{array}{l} \xi_f = -\eta_f \exp[i2\phi_M] \cdot \exp[-i2\phi_D] \\ |\xi_f|^2 = 1 \quad \text{weak phase } \phi_D: \text{ in the } B^0 \text{ decay} \end{array}$$



$\xi_f$  = given only in terms of CKM phase

$$a_{CP}^{decay} = 0$$

## Dominance of a single CKM Amplitude

|                                          |                                  |
|------------------------------------------|----------------------------------|
| $A_{\text{Tree}}, A_{\text{P}}$          | - hadronic matrix elements       |
| $\delta_{\text{T}}, \delta_{\text{P}}$   | - final state interaction phases |
| $\varphi_{\text{T}}, \varphi_{\text{P}}$ | - weak CKM phases                |

$$\frac{\overline{A}_f(\overline{B}^0 \rightarrow f)}{A_f(B^0 \rightarrow f)} = -\eta_f \left[ \frac{A_{\text{Tree}} e^{i(\delta_{\text{T}} - \varphi_{\text{T}})} + A_{\text{P}} e^{i(\delta_{\text{P}} - \varphi_{\text{P}})}}{A_{\text{Tree}} e^{i(\delta_{\text{T}} + \varphi_{\text{T}})} + A_{\text{P}} e^{i(\delta_{\text{P}} + \varphi_{\text{P}})}} \right]$$

### Tree Dominance

$$\frac{\overline{A}_f(\overline{B}^0 \rightarrow f)}{A_f(B^0 \rightarrow f)} = -\eta_f e^{-i2\varphi_{\text{T}}}$$

(Pure Phase)  
Very Clean !

### Penguin Dominance

$$\frac{\overline{A}_f(\overline{B}^0 \rightarrow f)}{A_f(B^0 \rightarrow f)} = -\eta_f e^{-i2\varphi_{\text{P}}}$$

(Pure Phase)  
Very Clean !

Also pure phase if  $\varphi_{\text{T}} = \varphi_{\text{P}}$  !! (Example:  $B_d^0 \rightarrow J/\psi K_S$ )

### 3. CP Violation in the Interference of Mixing and Decay

Misnomer: (“Mixing induced CP-Violation“)

$$a_{\text{CP}}(t, f) = \text{Im} \xi_f \sin(\Delta M t)$$

$$\text{Im} \xi_f = \eta_f \sin(2\varphi_D - 2\varphi_M) \equiv -S_f$$

Very clean  
TH

Measures the difference between the phases of  $B^0$ - $\bar{B}^0$  mixing ( $2\varphi_M$ ) and of decay amplitude ( $2\varphi_D$ )

Examples:

$$B_d^0 \rightarrow \psi K_S : \varphi_D = 0 \quad \varphi_M = -\beta \quad \eta_f = -1$$

$$\text{Im} \xi_{\psi K_S} = -\sin 2\beta$$

$$B_d^0 \rightarrow \pi^+ \pi^- : \varphi_D = \gamma \quad \varphi_M = -\beta \quad \eta_f = +1$$

$$\text{Im} \xi_{\pi\pi} = \sin(2(\gamma + \beta)) = -\sin 2\alpha$$

$$K_L \rightarrow \pi^0 \nu \bar{\nu} : \text{Measures the difference between the phases in } K^0\text{-}\bar{K}^0 \text{ mixing and } \bar{s} \rightarrow \bar{d} \nu \nu \text{ amplitude}$$

## $B^0$ -Decays into CP Eigenstates

(Two Contributions  $r = \frac{A_2}{A_1} \ll 1$ )

$$a_{\text{CP}}(t, f) = C_f \cos(\Delta Mt) - S_f \sin(\Delta Mt)$$

$$C_f = -2r \sin(\varphi_1 - \varphi_2) \sin(\delta_1 - \delta_2)$$

$$S_f = -\eta_f \left[ \sin 2(\varphi_1 - \varphi_M) + 2r \cos 2(\varphi_1 - \varphi_M) \sin(\varphi_1 - \varphi_2) \cos(\delta_1 - \delta_2) \right]$$

$\varphi_i =$  weak phases

$\delta_i =$  strong phases

$$\{r = 0\} \rightarrow C_f = 0 \quad S_f = -\eta_f \sin 2(\varphi_1 - \varphi_M)$$



## Comparison of Two-Languages

CP violation  
in mixing

≡

Manifestation of  
indirect  $\mathcal{CP}$

CP violation  
in decay

≡

Manifestation of  
direct  $\mathcal{CP}$

CP violation  
in interference  
of mixing and  
decay

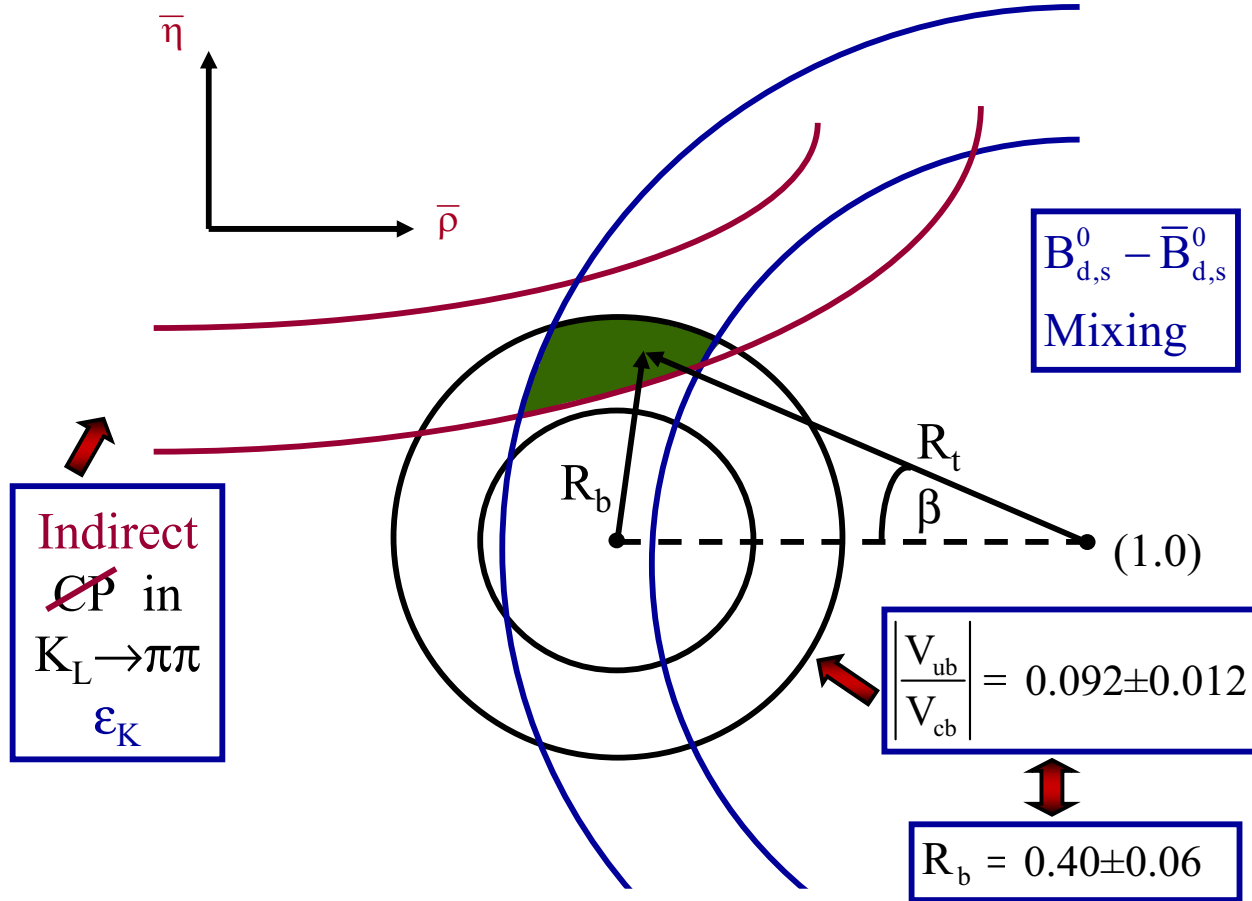
≡

With a single  
decay it is impossible  
to state whether  $\mathcal{CP}$   
in mixing or decay.  
But  $\text{Im } \xi_{f_1} \neq \text{Im } \xi_{f_2}$   
signals CP violation  
in decay (Direct  $\mathcal{CP}$ )

**3.**

**Standard Analysis  
of  
Unitarity Triangle**

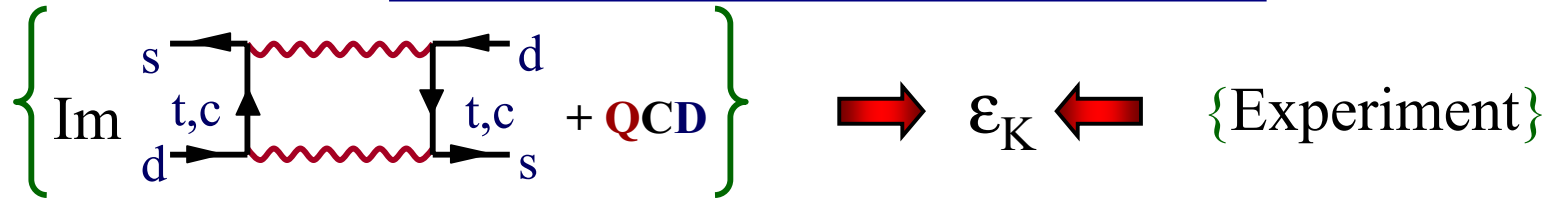
# Standard Analysis of UT



## Relevant Parameters

$$\hat{B}_K, F_{B_d} \sqrt{\hat{B}_{B_d}}, \xi = F_{B_s} \sqrt{\hat{B}_{B_s}} / F_{B_d} \sqrt{\hat{B}_{B_d}} \longleftrightarrow \epsilon_K, \Delta M_d, \Delta M_s / \Delta M_d$$

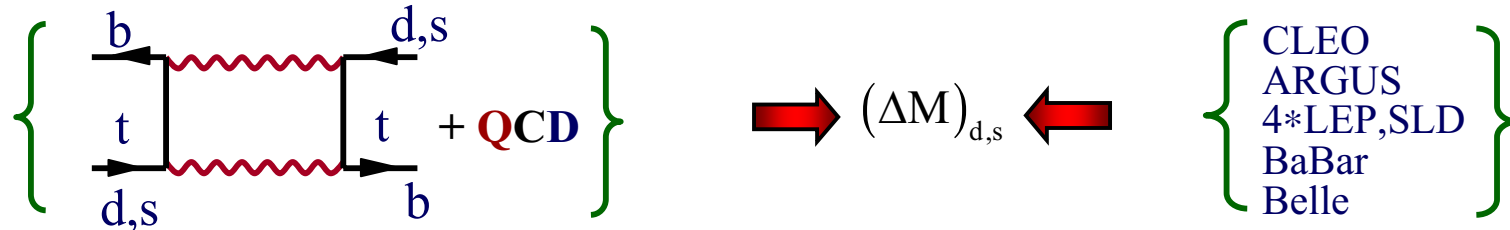
# Indirect CP in $K_L \rightarrow \pi\pi$



exp:

$$\epsilon_K = (2.280 \pm 0.013) \cdot 10^{-3} e^{i\frac{\pi}{4}}$$

# $B_{d,s}^0 - \bar{B}_{d,s}^0$ Mixing



$$(\Delta M)_{d,s} \equiv M(B_H^0)_{d,s} - M(B_L^0)_{d,s}$$

↙ ↘  
Mass Eigenstates

exp:



$$\begin{aligned}
 (\Delta M)_d &= (0.503 \pm 0.006) / \text{ps} \\
 (\Delta M)_s &> 14.4 / \text{ps} \quad (95\% \text{ C.L.}) \quad (\text{LEP/SLD})
 \end{aligned}$$

# Basic Formulae

1.

$\epsilon_K$  - Hyperbola

$$\bar{\eta} \left[ (1 - \bar{\rho}) A^2 F_{tt} \eta_{\text{QCD}}^{tt} + P_c(\epsilon) \right] A^2 \hat{B}_K = 0.213$$

$$\eta_{\text{QCD}}^{tt} = 0.57 \pm 0.01; \quad P_c(\epsilon) = 0.28 \pm 0.05; \quad F_{tt} = 2.42 \pm 0.12$$

$(F_{tt} \equiv S(x_t))$

2.

$B_d^0 - \bar{B}_d^0$  Mixing Constraint

$$R_t = 0.86 \left[ \frac{0.041}{|V_{cb}|} \right] \sqrt{\frac{2.34}{F_{tt}}} \sqrt{\frac{\Delta M_d}{0.50/\text{ps}}} \left[ \frac{230\text{MeV}}{\sqrt{\hat{B}_d} F_{B_d}} \right] \sqrt{\frac{0.55}{\eta_B^{\text{QCD}}}}$$

$$|V_{cb}| = 0.041 \pm 0.001; \quad \Delta M_d = (0.503 \pm 0.006)/\text{ps}; \quad \eta_B^{\text{QCD}} = 0.55 \pm 0.01$$

3.

$B_s^0 - \bar{B}_s^0$  Mixing Constraint ( $\Delta M_d/\Delta M_s$ )

$$R_t = 0.90 \sqrt{\frac{\Delta M_d}{0.50/\text{ps}}} \sqrt{\frac{18.4/\text{ps}}{\Delta M_s}} \left[ \frac{\xi}{1.22} \right]$$

$$\xi = \frac{\sqrt{\hat{B}_s} F_{B_s}}{\sqrt{\hat{B}_d} F_{B_d}}$$

$$\Delta M_s > 14.4 / \text{ps} \quad (95\% \text{ C.L.}) \quad \text{LEP (SLD)}$$

# 4.

## $\sin 2\beta$ from $A_{CP}(\psi K_S)$

$$A_{CP}(\psi K_S) \equiv -a_{\psi K_S} \sin(\Delta M_d t)$$

$$a_{\psi K_S} = \sin 2\beta \quad (\text{SM})$$

$$\sin 2\beta_{\psi K_S} = \begin{cases} 0.79 \pm \begin{matrix} 0.41 \\ 0.44 \end{matrix} & (\text{CDF}) \\ 0.741 \pm 0.067 \pm 0.033 & (\text{BaBar}) \\ \quad \quad \quad (\text{stat}) \quad (\text{syst}) \\ 0.719 \pm 0.074 \pm 0.035 & (\text{Belle}) \\ (\text{ALEPH} : 0.84 \begin{matrix} +0.82 \\ -1.04 \end{matrix} \pm 0.16) \end{cases}$$



$$\sin 2\beta = 0.726 \pm 0.037 \quad (a_{\psi K_S})$$



$$\beta = \begin{cases} (23.3 \pm 1.6)^\circ \\ (66.7 \pm 1.6)^\circ \quad (\text{excluded in the SM}) \end{cases} \quad (\sin \beta \cong 0.40 \pm 0.03)$$

## Crucial Parameters in SM and Beyond

|                                                                 |                                 |                                                |
|-----------------------------------------------------------------|---------------------------------|------------------------------------------------|
| $ V_{us}  = \lambda$                                            | $0.2240 \pm 0.0036$             |                                                |
| $ V_{ub} $                                                      | $(3.81 \pm 0.46) \cdot 10^{-3}$ |                                                |
| $ V_{cb} $                                                      | $(41.5 \pm 0.8) \cdot 10^{-3}$  | ★                                              |
| $\left  \frac{V_{ub}}{V_{cb}} \right $                          | $0.092 \pm 0.012$               | ★                                              |
|                                                                 |                                 |                                                |
| $m_t (m_t)$                                                     | $(168 \pm 4) \text{ GeV}$       |                                                |
| $\hat{B}_K$                                                     | $0.86 \pm 0.15$                 | $(\epsilon_K)$                                 |
| $\sqrt{\hat{B}_d} F_{Bd}$                                       | $(235^{+33}_{-41}) \text{ MeV}$ | $(\Delta M_d)$                                 |
| $\xi = \frac{\sqrt{\hat{B}_s} F_{Bs}}{\sqrt{\hat{B}_d} F_{Bd}}$ | $1.24 \pm 0.08$                 | $\left( \frac{\Delta M_s}{\Delta M_d} \right)$ |

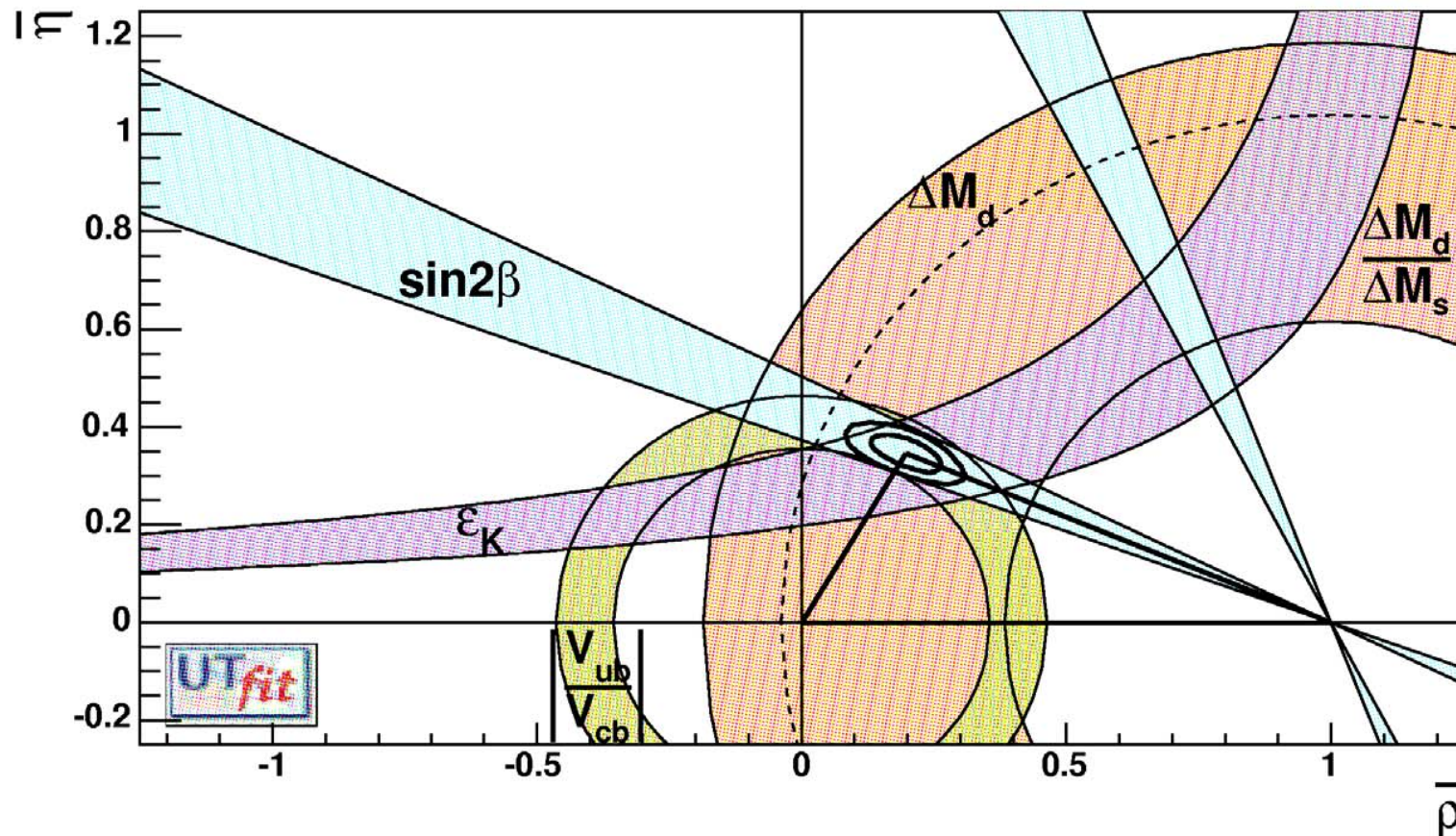
**Valid for all extensions of SM !!**





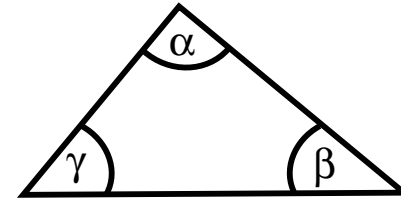
# Unitarity Triangle 2005

UTfit collaboration : Bona et al.



# 4.

$\alpha, \beta, \gamma$   
from  
B-Decays



$$V_{td} = |V_{td}| e^{-i\beta}$$

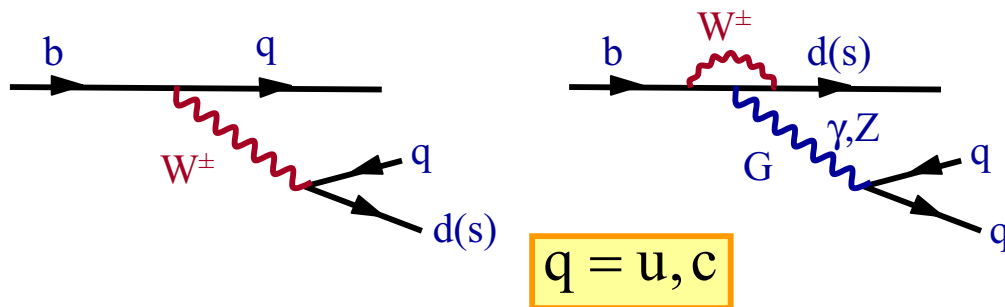
$$V_{ts} = |V_{ts}| e^{-i\beta_s}$$

$$V_{ub} = |V_{ub}| e^{-i\gamma}$$

# Basic Contributions

## Class I

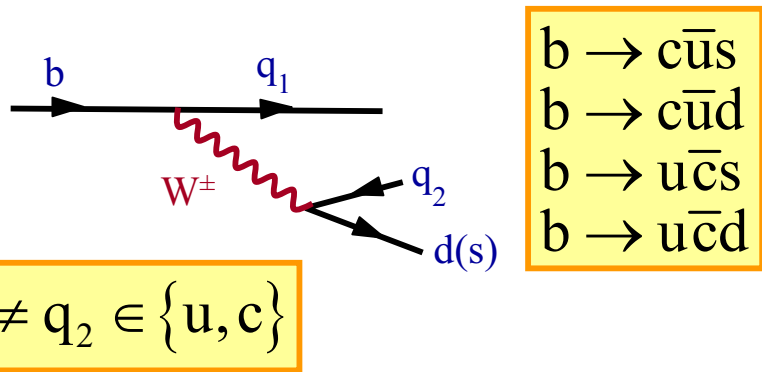
### Decays with Trees and Penguins



- $b \rightarrow c\bar{c}s$
- $b \rightarrow c\bar{c}d$
- $b \rightarrow u\bar{u}s$
- $b \rightarrow u\bar{u}d$

## Class II

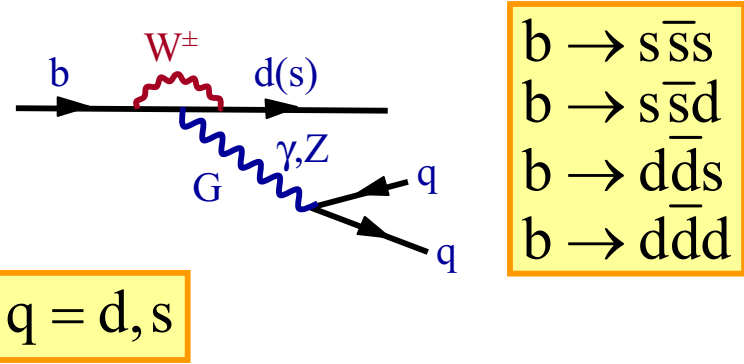
### Trees only



- $b \rightarrow c\bar{u}s$
- $b \rightarrow c\bar{u}d$
- $b \rightarrow u\bar{c}s$
- $b \rightarrow u\bar{c}d$

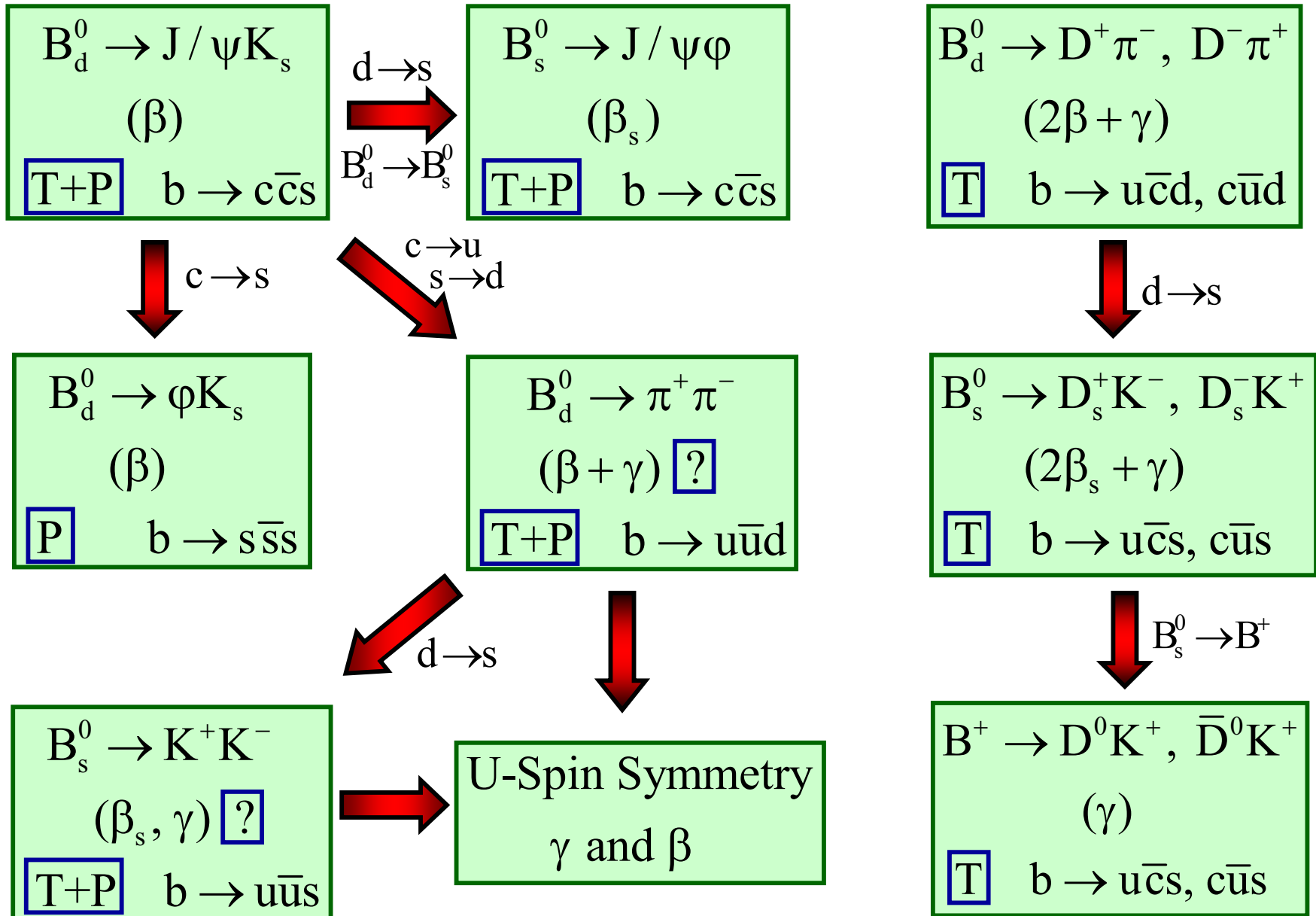
## Class III

### Penguins only

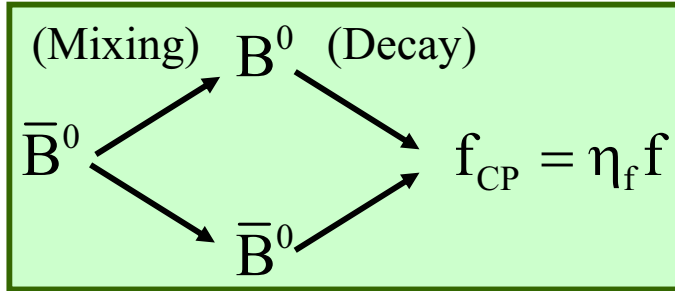
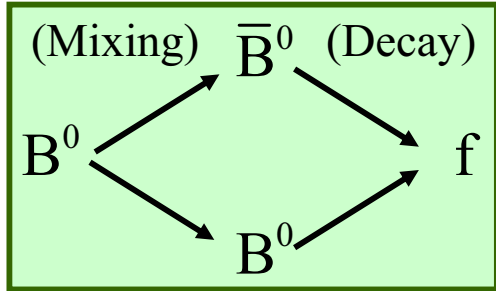


- $b \rightarrow s\bar{s}s$
- $b \rightarrow s\bar{s}d$
- $b \rightarrow d\bar{d}s$
- $b \rightarrow d\bar{d}d$

# α, β, γ from B-Decays



# B<sup>0</sup>(B̄<sup>0</sup>)-Decays into CP-Eigenstates



$$\eta_f = \pm 1 \quad \begin{matrix} \text{CP} \\ \text{Parity} \end{matrix}$$

Basic dynamical quantity:

$$\xi_f \equiv \underbrace{\exp(i2\varphi_M)}_{\text{Mixing}} \frac{\bar{A}_f(\bar{B}^0 \rightarrow f)}{\underbrace{A_f(B^0 \rightarrow f)}_{\text{Decay Amplitudes}}}$$

Weak phase in B<sup>0</sup> Decay

$$\varphi_D = 0, \beta, \gamma$$

$$= -\eta_f \exp(i2\varphi_M) \cdot \exp(-i2\varphi_D)$$

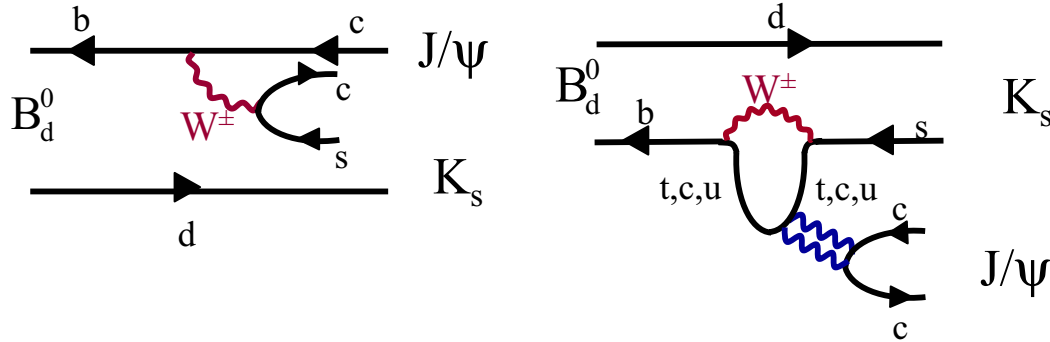
In the case of a single decay contribution or contributions with the same weak phase

$$\varphi_M = \begin{cases} -\beta & (B_d^0 - \bar{B}_d^0 \text{ mixing}) \\ -\beta_s & (B_s^0 - \bar{B}_s^0 \text{ mixing}) \end{cases} \approx 1^0$$

EXP

$$\text{Im } \xi_f = \eta_f \sin 2(\varphi_D - \varphi_M) \quad \star$$

# B<sub>d</sub><sup>0</sup> → J/ψ K<sub>S</sub> and β



$$V_{td} = |V_{td}| e^{-i\beta}$$

$$\begin{aligned} V_{cs} V_{cb}^* &\cong A\lambda^2 \\ V_{us} V_{ub}^* &\cong A\lambda^4 R_b e^{i\gamma} \\ V_{ts} V_{tb}^* &= -V_{cs} V_{cb}^* - V_{us} V_{ub}^* \end{aligned}$$

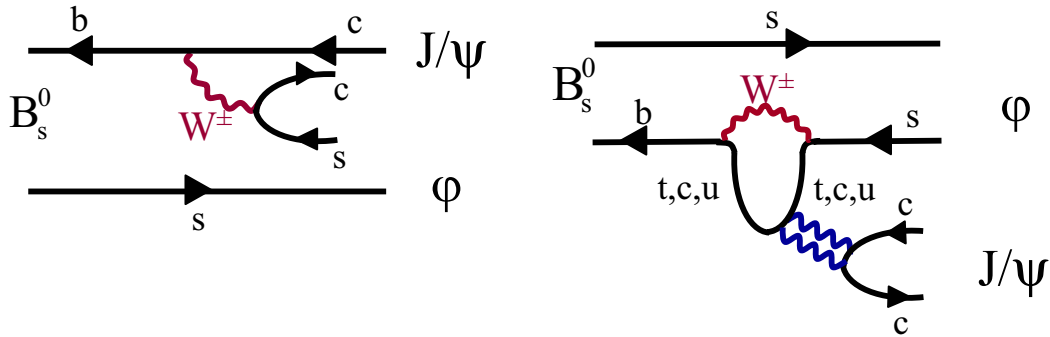
$$\begin{aligned} A(B_d^0 \rightarrow J/\psi K_S) &= V_{cs} V_{cb}^* (A_T + P_c) + V_{us} V_{ub}^* P_u + V_{ts} V_{tb}^* P_t \\ &= V_{cs} V_{cb}^* (A_T + P_c - P_t) + V_{us} V_{ub}^* (P_u - P_t) \end{aligned}$$

**(Dominance of a single phase)**

$$\left\{ \begin{array}{l} \left| \frac{V_{us} V_{ub}^*}{V_{cs} V_{cb}^*} \right| \leq 0.02 \\ \frac{P_u - P_t}{A_t + P_c - P_t} \ll 1 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} \varphi_D = 0 \\ \varphi_M = -\beta \\ |\xi_{\psi K_S}| = 1 \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} a_{CP}^{\text{mix}}(\psi K_S) = \eta_{\psi K_S} \sin 2(\varphi_D - \varphi_M) = -\sin 2\beta \\ a_{CP}^{\text{dir}}(\psi K_S) = 0 \quad a_{CP}(\psi K^+) \cong 0 \\ \boxed{C_{\psi K_S} = 0} \quad \boxed{S_{\psi K_S} = \sin 2\beta} \end{array} \right\}$$

$B_s^0 \rightarrow J/\psi \phi$  and  $\beta_s$

$V_{ts} = |V_{ts}| e^{-i\beta_s}$



Differs from  $B_d^0 \rightarrow J/\psi K_S$  only by "spectator" quark  $d \rightarrow s$

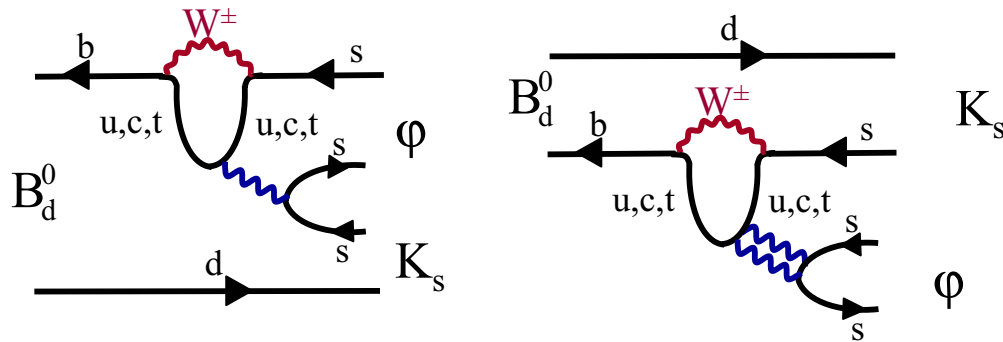
$(\varphi_D = 0)$

Complication:  $(J/\psi\phi)$  admixture of  $CP = +$  and  $CP = -$

(Can be resolved: see Page 40: "B-Decays at the LHC ")

$$\left\{ \begin{array}{l} \varphi_D = 0 \\ \varphi_M = -\beta_s \cong -\lambda^2 \eta \\ |\xi_{\psi\phi}| = 1 \end{array} \right\} \rightarrow \left\{ \begin{array}{l} a_{CP}^{mix} = \sin 2(\varphi_D - \varphi_M) \cong \underbrace{2\lambda^2 \eta}_{2\beta_s} \cong 0.03 \\ a_{CP}^{dir} \cong 0 \\ \text{A lot of room for New Physics!} \end{array} \right\}$$

# $B_d^0 \rightarrow \phi K_S$ and $\beta$ (Pure Penguin Decay)



$$\begin{aligned}
 V_{cs} V_{cb}^* &\cong A\lambda^2 \\
 V_{us} V_{ub}^* &\cong A\lambda^4 R_b e^{i\gamma} \\
 V_{ts} V_{tb}^* &= -V_{cs} V_{cb}^* - V_{us} V_{ub}^*
 \end{aligned}$$

$$\begin{aligned}
 A(B_d^0 \rightarrow \phi K_S) &= V_{cs} V_{cb}^* P_c + V_{us} V_{ub}^* P_u + V_{ts} V_{tb}^* P_t \\
 &= V_{cs} V_{cb}^* (P_c - P_t) + V_{us} V_{ub}^* (P_u - P_t)
 \end{aligned}$$

**(Dominance of a single phase)**

$$\left\{ \begin{array}{l} \left| \frac{V_{us} V_{ub}^*}{V_{cs} V_{cb}^*} \right| \leq 0.02 \\ \frac{P_u - P_t}{P_c - P_t} \approx 0(1) \end{array} \right\} \xrightarrow{\text{(neglecting)}} \left\{ \begin{array}{l} a_{CP}^{\text{mix}}(\phi K_S) = -\sin 2\beta = a_{CP}^{\text{mix}}(\psi K_S) \\ C_{\phi K_S} \approx 0 \\ S_{\psi K_S} = S_{\phi K_S} = \sin 2\beta \\ |S_{\psi K_S} - S_{\phi K_S}| \leq 0.04 \text{ (SM)} \end{array} \right\}$$

**Grossman, Isidori, Worah, London, Soni**



# First Results for $B_d^0 \rightarrow \phi K_S$

$$(\sin 2\beta)_{\phi K_S} = \begin{cases} +0.45 \pm 0.43 \pm 0.07 & \text{(BaBar)} \\ -0.96 \pm 0.50^{+0.11}_{-0.09} & \text{(Belle)} \end{cases}$$



World  
Averages

$$\begin{aligned} S_{\phi K_S} &= -0.05 \pm 0.24 \\ C_{\psi K_S} &= -0.15 \pm 0.33 \end{aligned}$$

(Belle)

$$\begin{aligned} S_{\eta' K_S} &= 0.76 \pm 0.36 \\ C_{\eta' K_S} &= -0.26 \pm 0.22 \end{aligned}$$

(BaBar)

$$S_{\eta' K_S} = 0.02 \pm 0.35$$

(fully consistent with SM)

$$\begin{aligned} |S_{\phi K_S} - S_{\psi K_S}| &\cong 0.88 \pm 0.34 \\ \text{(Violation of SM by } 2.6\sigma) \end{aligned}$$

but  $S_{\phi K_S} \neq S_{\eta' K_S}$  possible  
as non-leading terms  
could be different

Grossman,  
Isidori  
Worah  
Ciuchini  
Silvestrini

New Physics:

Enhanced QCD Penguins  
 $Z^0$  Penguins, ..

Hiller, Raidal, Ciuchini + Silvestrini  
Fleischer, Mannel

## Present Results for $B_d^0 \rightarrow \phi K_s$

$$(\sin 2\beta)_{\phi K_s} = \begin{cases} 0.50 \pm 0.25 \pm 0.06 & \text{(BaBar)} \\ 0.06 \pm 0.33 \pm 0.09 & \text{(Belle)} \end{cases}$$

World Averages:  $S_{\phi K_s} = 0.34 \pm 0.20$   $C_{\phi K_s} = -0.04 \pm 0.17$

To be compared with  $S_{\psi K_s} \approx 0.73$  New Physics?

# Decays to CP non-eigenstates and $\gamma$

$$\bar{B}_d^0 \rightarrow D^\pm \pi^\mp$$

(Dunietz+Sachs)

$d \rightarrow s$

$$\bar{B}_s^0 \rightarrow D_s^\pm K^\mp$$

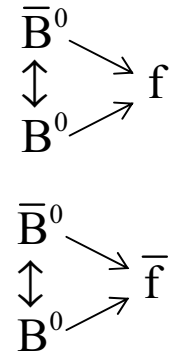
Aleksan, Dunietz, Kayser

- $B_d^0 (B_s^0)$  and  $\bar{B}_d^0 (\bar{B}_s^0)$  can decay to the same final state
- Requires full time-dependent analysis:  
4 time dependent rates
 
$$B_{d,s}^0(t) \rightarrow f, \quad \bar{B}_{d,s}^0(t) \rightarrow f,$$

$$B_{d,s}^0(t) \rightarrow \bar{f}, \quad \bar{B}_{d,s}^0(t) \rightarrow \bar{f},$$
- Tree diagrams only

$$\xi_f = e^{i2\varphi_M} \frac{A(\bar{B}^0 \rightarrow f)}{A(B^0 \rightarrow f)}$$

$$\xi_{\bar{f}} = e^{i2\varphi_M} \frac{A(\bar{B}^0 \rightarrow \bar{f})}{A(B^0 \rightarrow \bar{f})}$$

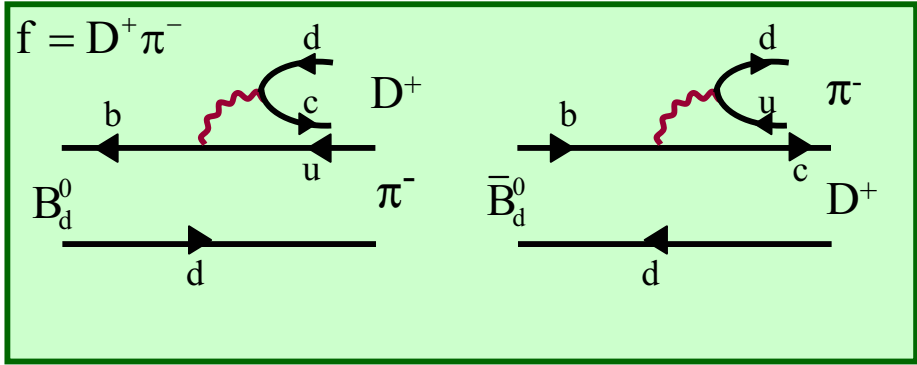


$$\varphi_M = \begin{cases} -\beta & B_d^0 \\ -\beta_s & B_s^0 \end{cases}$$

$$\xi_f \cdot \xi_{\bar{f}} = F(\gamma, \beta_{(s)})$$

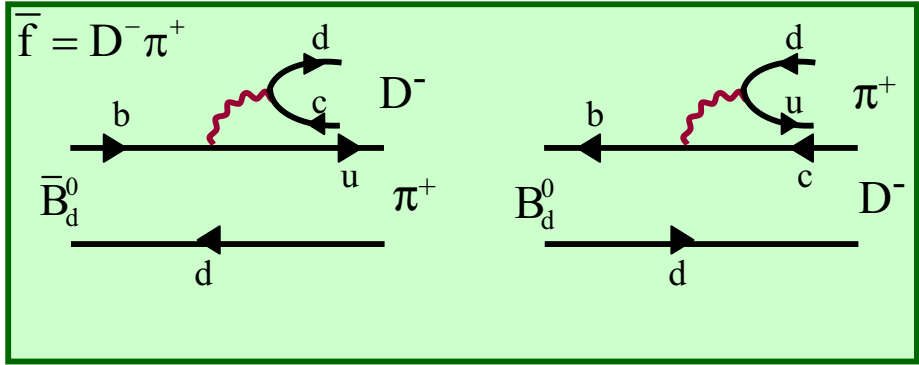
(Dunietz, Sachs)

$$\mathbf{B}_d^0 \rightarrow D^\pm \pi^\mp, \bar{\mathbf{B}}_d^0 \rightarrow D^\pm \pi^\mp \text{ and } \gamma$$



$$(M_f A \lambda^4 R_b e^{i\gamma})$$

$$(\bar{M}_f A \lambda^2)$$



$$(\bar{M}_{\bar{f}} A \lambda^4 R_b e^{-i\gamma})$$

$$(M_{\bar{f}} A \lambda^2)$$

$$\xi_{\mathbf{f}}^{(d)} = e^{-i2\beta} \frac{A(\bar{\mathbf{B}}_d^0 \rightarrow \mathbf{f})}{A(\mathbf{B}_d^0 \rightarrow \mathbf{f})} = e^{-i(2\beta+\gamma)} \frac{1}{\lambda^2 R_b} \frac{\bar{M}_f}{M_f}$$

$$\xi_{\bar{\mathbf{f}}}^{(d)} = e^{-i2\beta} \frac{A(\bar{\mathbf{B}}_d^0 \rightarrow \bar{\mathbf{f}})}{A(\mathbf{B}_d^0 \rightarrow \bar{\mathbf{f}})} = e^{-i(2\beta+\gamma)} \lambda^2 R_b \frac{\bar{M}_{\bar{f}}}{M_{\bar{f}}}$$



$$\xi_{\mathbf{f}}^{(d)} \cdot \xi_{\bar{\mathbf{f}}}^{(d)} = e^{-i2(2\beta+\gamma)}$$



$2\beta + \gamma$  without hadronic uncertainties

( $\beta$  known)



$$\gamma$$

$$\bar{M}_f = M_{\bar{f}} \quad M_f = \bar{M}_{\bar{f}}$$

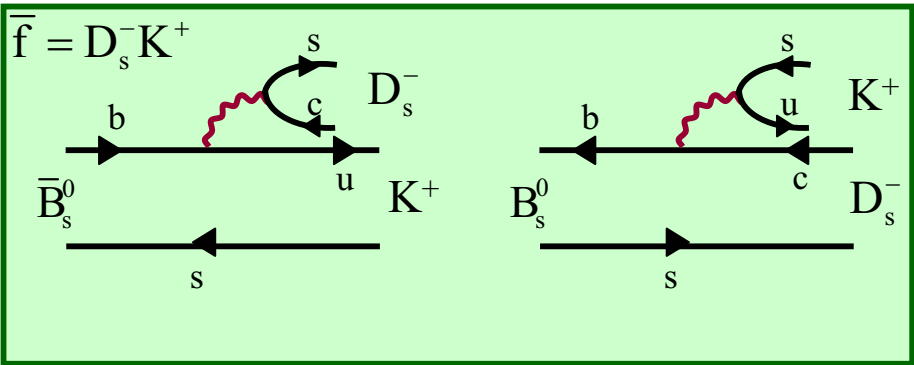
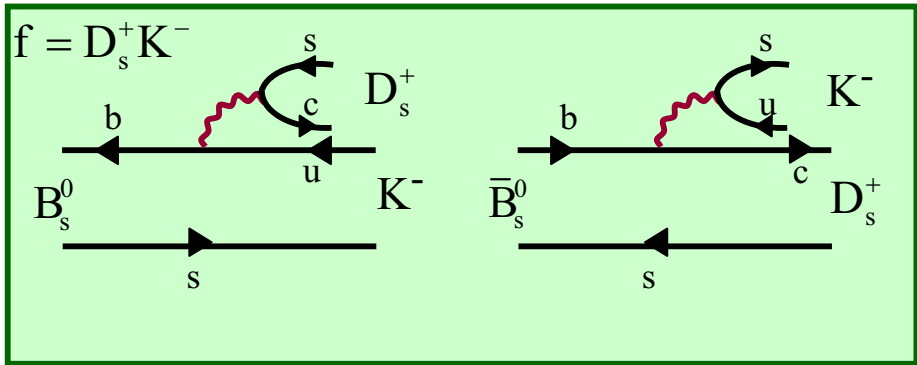
Hadronic Matrix Elements

Small Interference: difficult exp. task

Aleksan  
Dunietz  
Kayser

$$B_s^0 \rightarrow D_s^\pm K^\mp, \bar{B}_s^0 \rightarrow D_s^\pm K^\mp \text{ and } \gamma$$

Directly obtained from  $B_d^0, \bar{B}_d^0 \rightarrow D^\pm \pi^\mp$  through  $d \rightarrow s$



$$(M_f A \lambda^3 R_b e^{i\gamma})$$

$$(\bar{M}_f A \lambda^3)$$

$$(\bar{M}_{\bar{f}} A \lambda^3 R_b e^{-i\gamma})$$

$$(M_{\bar{f}} A \lambda^3)$$

In analogy to  $B_d^0, \bar{B}_d^0 \rightarrow D^\pm \pi^\mp$

$$\xi_f^{(s)} \cdot \xi_{\bar{f}}^{(s)} = e^{-i2(2\beta_s + \gamma)}$$



$2\beta_s + \gamma$  without hadronic uncertainties



$$\gamma$$

$\beta_s$  – phase in  $B_s^0 - \bar{B}_s^0$

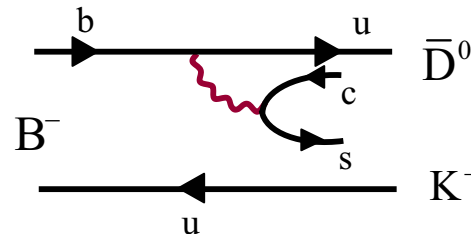
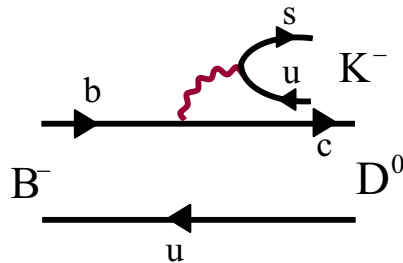
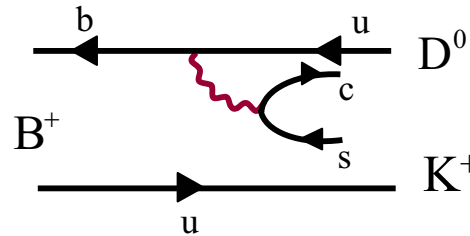
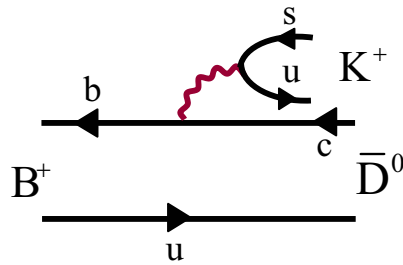
$\beta_s$  from  $B_s^0 \rightarrow \phi \psi$

Much bigger interference than in  $B_d^0, \bar{B}_d^0 \rightarrow D^\pm \pi^\mp$

$$B^\pm \rightarrow D^0 K^\pm, \bar{D}^0 K^\pm \text{ and } \gamma$$

(Gronau + Wyler)

Directly obtained from  $B_s^0, \bar{B}_s^0 \rightarrow D_s^\pm K^\pm$  through  $B_s \rightarrow B^\pm$



$$K^+ \bar{D}^0 \neq K^+ D^0$$



Need

$$B^+ \rightarrow D_+^0 K^+$$

$$D_+^0 = \frac{1}{\sqrt{2}} (D^0 + \bar{D}^0)$$

To each process only single diagram contributes

$$A(B^+ \rightarrow \bar{D}^0 K^+) = A(B^- \rightarrow D^0 K^-)$$

$$A(B^+ \rightarrow D^0 K^+) = A(B^- \rightarrow \bar{D}^0 K^-) e^{2i\gamma}$$

$$O(A\lambda^3)$$

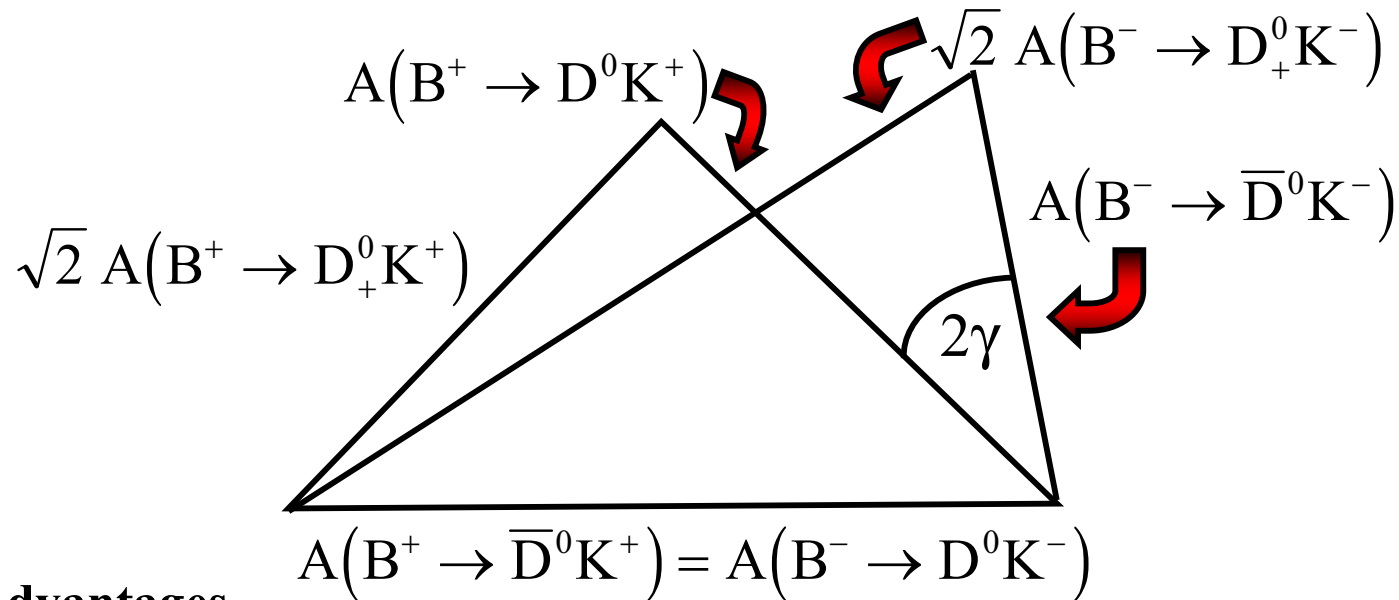
$$O(A\lambda^3 R_b) \text{ Colour suppressed}$$

## Gronau-Wyler Method for $\gamma$

$$\sqrt{2} A(B^+ \rightarrow D_+^0 K^+) = A(B^+ \rightarrow D^0 K^+) + A(B^+ \rightarrow \bar{D}^0 K^+)$$

$$\sqrt{2} A(B^- \rightarrow D_+^0 K^-) = A(B^- \rightarrow \bar{D}^0 K^-) + A(B^- \rightarrow D^0 K^-)$$

$$D_+^0 = \frac{1}{2} (|D^0\rangle + |\bar{D}^0\rangle) \quad \text{CP} = +$$



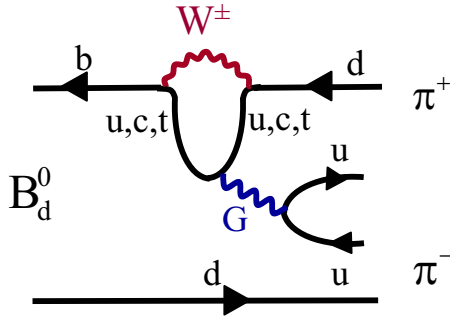
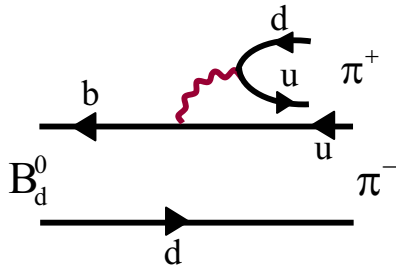
### Advantages

- ◆ Pure Trees
- ◆ No tagging
- ◆ No time dependent measurements
- ◆ Only rates

### Disadvantages

- ◆  $\text{Br}(B^+ \rightarrow D^0 K^+) \sim 0(10^{-6})$
- ◆  $\text{Br}(B^+ \rightarrow \bar{D}^0 K^+) \sim 0(10^{-4})$
- ◆ Detection of  $D_+^0$

# B<sub>d</sub><sup>0</sup> → π<sup>+</sup>π<sup>-</sup> and α



$$V_{ub}^* V_{ud} = A\lambda^3 R_b e^{i\gamma}$$

$$V_{cb}^* V_{cd} = A\lambda^3$$

$$V_{tb}^* V_{td} = -V_{ub}^* V_{ud} - V_{cb}^* V_{cd}$$

$$A(B_d^0 \rightarrow \pi^+ \pi^-) = V_{ub}^* V_{ud} (A_T + P_u) + V_{cb}^* V_{cd} P_c + V_{tb}^* V_{td} P_t$$

$$= V_{ub}^* V_{ud} (A_T + P_u - P_t) + V_{cb}^* V_{cd} (P_c - P_t)$$

$$\left| \frac{V_{cb}^* V_{cd}}{V_{ub}^* V_{ud}} \right| = \frac{1}{R_b} \approx 0(2)$$

$$\frac{P_c - P_t}{A_T + P_u - P_t} = \frac{P_{\pi\pi}}{T_{\pi\pi}} \quad ?$$

Assuming

$$\frac{P_{\pi\pi}}{T_{\pi\pi}} \ll 1$$



$$\varphi_D = \gamma$$

$$\varphi_M = -\beta$$

$$|\xi_{\pi\pi}| = 1$$

$$a_{CP}^{\text{mix}} = \eta_{\pi\pi} \sin 2(\varphi_D - \varphi_M) = \sin 2(\gamma + \beta) = -\sin 2\alpha$$

$$a_{CP}^{\text{dir}} = 0$$

$$C_{\pi\pi} = 0$$

$$S_{\pi\pi} = \sin 2\alpha$$

Dominance of a single amplitude uncertain



# Results for $B_d^0 \rightarrow \pi^+ \pi^-$

$$C_{\pi\pi} = \begin{cases} -0.09 \pm 0.15 \pm 0.04 & \text{(BaBar)} \\ -0.58 \pm 0.15 \pm 0.07 & \text{(Belle)} \end{cases}$$

Consistent with 0

~~CP~~

$$S_{\pi\pi} = \begin{cases} -0.30 \pm 0.17 \pm 0.03 & \text{(BaBar)} \\ -1.00 \pm 0.21 \pm 0.07 & \text{(Belle)} \end{cases}$$

Consistent with 0

~~CP~~

World Average:

$$C_{\pi\pi} = -0.37 \pm 0.11$$

$$S_{\pi\pi} = -0.61 \pm 0.16$$

Isospin analysis (Gronau + London)  
Model independent determination of  $\alpha$

Model independent upper bound  
(Grossman, Quinn; Charles)

$$\sin^2(\alpha_{\text{eff}} - \alpha) \leq \frac{\text{Br}(B^0 \rightarrow \pi^0 \pi^0)}{\text{Br}(B^+ \rightarrow \pi^+ \pi^0)}$$

$$\sin 2\alpha_{\text{eff}} \equiv \frac{\text{Im} \xi_{\pi\pi}}{|\xi_{\pi\pi}|}$$

Model dependent determination

of  $\alpha$  using  $(P_{\pi\pi} / T_{\pi\pi})_{\text{TH}}$

Beneke, Buchalla, Neubert, Sachrajda: small  $C_{\pi\pi}$

Keum, Li, Sanda: large  $C_{\pi\pi}$

Most recent:

Buchalla, Safir;

AJB, Fleischer, Recksiegel, Schwab

Gronau, Rosner et al.

Ali, Lunghi, Parkhomenko

# U-Spin Strategies (d↔s)

## Fleischer:

$$\left\{ \begin{array}{l} B_d^0 \rightarrow \pi^+ \pi^- \\ B_s^0 \rightarrow K^+ K^- \end{array} \right\} \Rightarrow \beta, \gamma$$

$$\left\{ \begin{array}{l} B_d^0 \rightarrow J / \psi K_s \\ B_s^0 \rightarrow J / \psi K_s \end{array} \right\} \Rightarrow \gamma$$

$$\left\{ \begin{array}{l} B_d^0 \rightarrow D^+ D^- \\ B_s^0 \rightarrow D_s^+ D_s^- \end{array} \right\} \Rightarrow \gamma$$

Uncertainty from  
U-Spin breaking

## Gronau + Rosner; Chiang Wolfenstein:

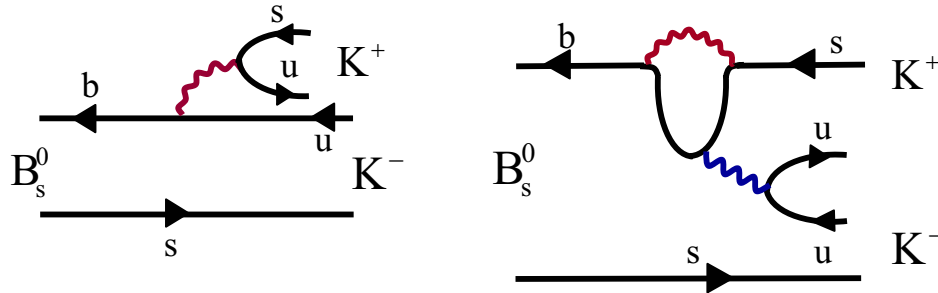
$$\left\{ \begin{array}{l} B_d^0 \rightarrow \pi^- K^+ \\ B_s^0 \rightarrow \pi^+ K^- \end{array} \right\} \Rightarrow \gamma$$

Uncertainty from U-Spin breaking,  
rescattering, colour suppressed  
EW-Penguins

$$B_d^0 \rightarrow \pi^+ \pi^- \text{ and } B_s^0 \rightarrow K^+ K^- \quad (\beta \text{ and } \gamma)$$

(Fleischer)

{ Replace in  $B_d^0 \rightarrow \pi^+ \pi^-$  :  $d \rightarrow s$  }



$$V_{ub}^* V_{us} = A\lambda^4 e^{i\gamma} R_b$$

$$V_{cb}^* V_{cs} = A\lambda^2$$

$$V_{tb}^* V_{ts} = -V_{ub}^* V_{us} - V_{cb}^* V_{cs}$$

$$A(B_s^0 \rightarrow K^+ K^-) = V_{ub}^* V_{us} (A'_T + P'_u - P'_t) + V_{cb}^* V_{cs} (P'_c - P'_t)$$

U-Spin Symmetry:

$$\frac{P_{\pi\pi}}{T_{\pi\pi}} = \frac{P_c - P_t}{A_T + P_u - P_t} = \frac{P'_c - P'_t}{A'_T + P'_u - P'_t} = \frac{P_{KK}}{T_{KK}} \equiv de^{i\delta}$$

strong phase

$$\begin{matrix} a_{CP}^{\text{mix}}(B_d^0 \rightarrow \pi^+ \pi^-) & a_{CP}^{\text{mix}}(B_s^0 \rightarrow K^+ K^-) \\ a_{CP}^{\text{dir}}(B_d^0 \rightarrow \pi^+ \pi^-) & a_{CP}^{\text{dir}}(B_s^0 \rightarrow K^+ K^-) \end{matrix}$$

( $\beta_s$  from  $B_s \rightarrow J/\psi\phi$ )



$d, \delta, \beta, \gamma$   
subject to U-Spin  
breaking corrections

$\beta$  present in  $B_d^0 - \bar{B}_d^0$  mixing

$$V_{td} = |V_{td}| e^{-i\beta}$$

$$V_{ub} = |V_{ub}| e^{-i\gamma}$$

$$V_{ts} = |V_{ts}| e^{-i\beta_s}$$

## Golden Measurements

(Essentially no TH uncertainties)

**1.**

Decays into CP-Eigenstates (time evolution)

$$B_d^0 (\bar{B}_d^0) \rightarrow \psi K_s \Rightarrow \beta \quad B_d^0 (\bar{B}_d^0) \rightarrow \psi \phi \Rightarrow \beta_s$$

**2.**

Decays into CP non-Eigenstates (time evolution)

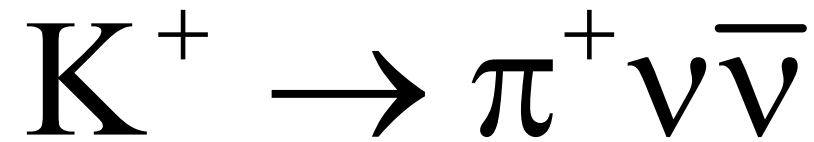
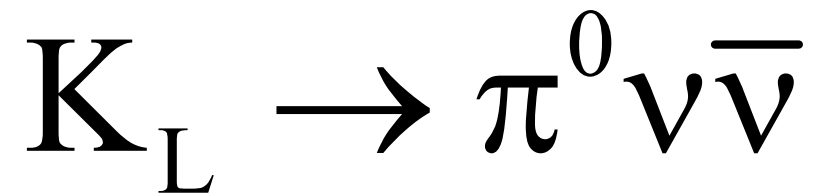
$$B_d^0 (\bar{B}_d^0) \rightarrow D^\pm \pi^\mp \Rightarrow 2\beta + \gamma \quad B_d^0 (\bar{B}_d^0) \rightarrow D_s^\pm K^\mp \Rightarrow 2\beta_s + \gamma$$

**3.**

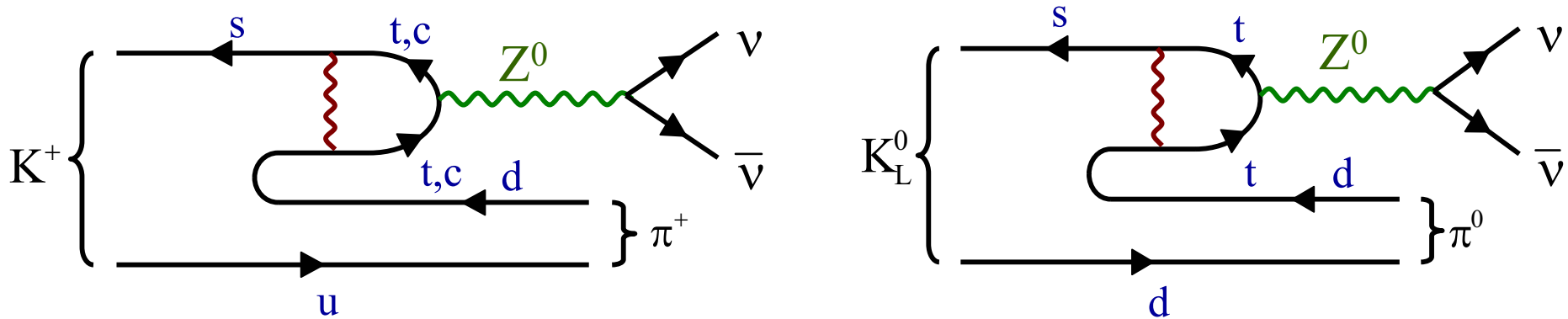
$K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  (branching ratios)

$$\begin{array}{l} \text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \\ \text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu}) \end{array} \Rightarrow \beta \text{ and } \gamma$$

5.



# Decays $K \rightarrow \pi \nu \bar{\nu}$



**Isospin Symmetry**

$$\langle \pi^+ | (\bar{s}d)_{V-A} | K^+ \rangle = \sqrt{2} \langle \pi^0 | (\bar{s}u)_{V-A} | K^+ \rangle$$

$$\langle \pi^0 | (\bar{s}d)_{V-A} | K^0 \rangle = \langle \pi^0 | (\bar{s}u)_{V-A} | K^+ \rangle$$

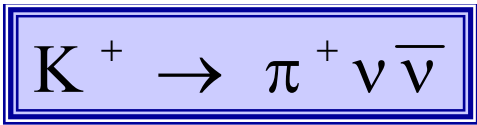
Leading Decay:  $K^+ \rightarrow \pi^0 e^+ \nu$

Isospin Breaking:

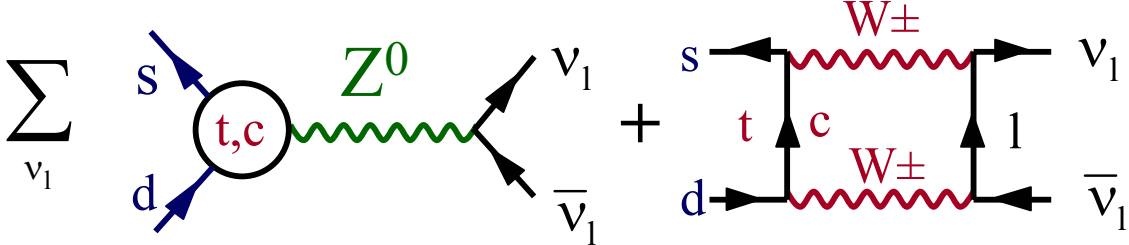
Marciano, Parsa  
 $K^+$ (10%)  $K_L$ (5%)

Long Distance:

$K^+$ :  $+(6 \pm 2)\%$  Isidori, Mescia, Smith (2005)  
 $K_L$ :  $\leq 1\%$  Buchalla, Isidori



$$V_{td}$$



+ QCD Corrections:  
 Dib, Dunietz, Gilman **LO (91)**  
 Buchalla + AJB **NLO (94)**

$$x_t = \bar{m}_t^2 / M_W^2$$

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2 \theta_W} \sum_{\nu_1} (\lambda_c X_{\text{NL}}^1 + \lambda_t X(x_t)) Q$$

$$\lambda_c = V_{cs}^* V_{cd} \quad \lambda_t = V_{ts}^* V_{td} \quad Q = (\bar{s}d)_{V-A} (\bar{\nu}\nu)_{V-A}$$

$$X(x_t) = X_0(x_t) + \frac{\alpha_s}{4\pi} X_1(x_t) \equiv \eta_x X_0(x_t)$$

$$\bar{m}_t(\mu_t)$$

$$X_1(x_t) = \tilde{X}_1(x_t) + 8x_t \underbrace{\frac{\partial X_0(t)}{\partial x_t} \ln \frac{\mu_t^2}{M_W^2}}_{\text{Cancels } \mu_t\text{-dependence in } X_0(x_t(\mu_t))}$$

$$X_{\text{NL}} \approx 10^{-3}$$

For  $\mu_t \cong m_t$   
 $\eta_x = 0.995$

$$X(x_t) = 0.65 \cdot x_t^{0.575}$$

$$\text{Br}(\text{K}^+ \rightarrow \pi^+ \nu \bar{\nu})$$

◆ **Take:** 
$$H_{\text{eff}}(\text{K}^+ \rightarrow \pi^0 e^+ \nu) = \frac{G_F}{\sqrt{2}} V_{us}^* (\bar{s}u)_{V-A} (\bar{\nu}e)_{V-A}$$

◆ **Use: Isospin Symmetry**

$$\langle \pi^+ | (\bar{s}d)_{V-A} | \text{K}^+ \rangle = \sqrt{2} \langle \pi^0 | (\bar{s}u)_{V-A} | \text{K}^+ \rangle$$

◆ **For single  $\nu$  with  $m_{\pi^+} = m_{\pi^0}$**

$$\frac{\text{Br}(\text{K}^+ \rightarrow \pi^+ \nu \bar{\nu})}{\text{Br}(\text{K}^+ \rightarrow \pi^0 e^+ \nu)} = \frac{\alpha^2}{V_{us}^2 2\pi^2 \sin^4 \theta_W} |\lambda_C X_{\text{NL}} + \lambda_t X(x_t)|^2$$

◆ **Include Isospin breaking corrections**

Marciano+ Parsa (95)

$m_{\pi^+} \neq m_{\pi^0}$   
 Isospin violation in  $\text{K} \rightarrow \pi$  formfactors  
 Electromagnetic corrections affecting  
 $\bar{s} \rightarrow \bar{u} e^+ \nu$  but not  $\bar{s} \rightarrow \bar{d} \nu \bar{\nu}$



Additional  
Factor

$$r_{\text{K}^+} = 0.901$$



## ◆ Summing over 3 $\nu$ 's

$$\text{Br}(K^+) = \kappa_+ \left[ \left( \frac{\text{Im} \lambda_t}{\lambda^5} X(x_t) \right)^2 + \left( \frac{\text{Re} \lambda_c}{\lambda} P_c + \frac{\text{Re} \lambda_t}{\lambda^5} X(x_t) \right)^2 \right]$$

$$\kappa_+ = r_{K^+} \frac{3\alpha^2 \text{Br}(K^+ \rightarrow \pi^0 e^+ \nu)}{2\pi^2 \sin^4 \theta_W} \lambda^8 = 4.84 \cdot 10^{-11} \left[ \frac{\lambda}{0.224} \right]^8$$

$$\alpha = 1/128; \quad \sin^2 \theta_W = 0.23; \quad \text{Br}(K^+ \rightarrow \pi^0 e^+ \nu) = 4.87 \cdot 10^{-2}$$

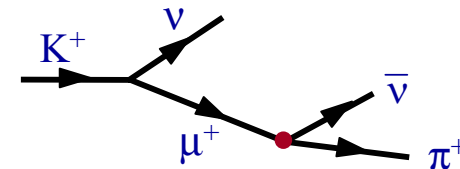
$$P_c = \frac{1}{\lambda^4} \left[ \frac{2}{3} X_{\text{NL}}^e + \frac{1}{3} X_{\text{NL}}^\tau \right] = 0.39 \pm \overbrace{0.07}^{(m_c, \mu_c)}$$

|   |                |   |   |                 |    |   |   |       |      |   |
|---|----------------|---|---|-----------------|----|---|---|-------|------|---|
| { | $\mu_c, \mu_t$ | } | : | Br              | LO | } | → | NLO   | }    |   |
|   | Uncertainty    |   |   |                 | {  |   |   | ± 22% |      | } |
|   | Br, $ V_{td} $ |   |   | V <sub>td</sub> |    |   |   | {     | ± 4% | } |

{  $\bar{m}_t(\mu_t), \bar{m}_c(\mu_c)$ : 100 GeV ≤  $\mu_t$  ≤ 300 GeV; 1 GeV ≤  $\mu_c$  ≤ 3 GeV }

LD Effects < 5%  
 Rein, Sehgal  
 Hagelin, Littenberg  
 Lu, Wise; Fajfer

Smallness of LD related to absence of internal  $\gamma$  contributions (present in  $K_L \rightarrow \pi^0 e^+ e^-, K_L \rightarrow \mu \bar{\mu}$ )



$$\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})$$

◆ Consider one  $\nu$ -flavour and denote:

$$F \equiv \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2 \theta_W} (\lambda_c X_{NL} + \lambda_t X(x_t))$$

$$H_{\text{eff}} = F(\bar{s}d)_{V-A} (\bar{\nu}\nu)_{V-A} + F^*(\bar{d}s)_{V-A} (\bar{\nu}\nu)_{V-A}$$

◆ Now:

$$K_L = \frac{1}{\sqrt{2}} \left( (1 + \bar{\varepsilon}) |K^0\rangle + (1 - \bar{\varepsilon}) |\bar{K}^0\rangle \right)$$

$$CP|K^0\rangle = -|\bar{K}^0\rangle; \quad C|K^0\rangle = |\bar{K}^0\rangle$$

$$A(K_L \rightarrow \pi^0 \nu \bar{\nu}) = \frac{1}{\sqrt{2}} \left( F(1 + \bar{\varepsilon}) \langle \pi^0 | (\bar{s}d)_{V-A} | K^0 \rangle \right. \\ \left. + F^*(1 - \bar{\varepsilon}) \langle \pi^0 | (\bar{d}s)_{V-A} | \bar{K}^0 \rangle \right) (\bar{\nu}\nu)_{V-A}$$

$$\langle \pi^0 | (\bar{d}s)_{V-A} | \bar{K}^0 \rangle = -\langle \pi^0 | (\bar{s}d)_{V-A} | K^0 \rangle$$

$$A(K_L \rightarrow \pi^0 \nu \bar{\nu}) = \frac{1}{\sqrt{2}} \underbrace{[F(1 + \bar{\varepsilon}) - F^*(1 - \bar{\varepsilon})]}_{\approx 2 \operatorname{Im} \lambda_t X(x_t)} \langle \pi^0 | (\bar{s}d)_{V-A} | K^0 \rangle \cdot (\bar{\nu}\nu)_{V-A} \quad (\operatorname{Im} \lambda_c = -\operatorname{Im} \lambda_t)$$

( $X_{NL} \ll X(x_t)$ )

$$\langle \pi^0 | (\bar{s}d)_{V-A} | K^0 \rangle = \langle \pi^0 | (\bar{s}u)_{V-A} | K^+ \rangle$$

$$\left\{ \begin{array}{l} \text{Isospin Breaking} \\ \text{(Marciano, Parsa)} \end{array} \right\} \rightarrow \left\{ r_{K_L} = 0.944 \right\}$$

### ◆ Summing over $\nu$

$$\operatorname{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = \kappa_{K_L} \left[ \frac{\operatorname{Im} \lambda_t}{\lambda^5} X(x_t) \right]^2 \quad \star$$

$$\kappa_{K_L} = \frac{\tau_{K_L}}{\tau_{K^+}} \frac{r_{K_L}}{r_{K^+}} \kappa_{K^+} = 2.12 \cdot 10^{-10} \left[ \frac{\lambda}{0.224} \right]^8$$

**Waiting for Precise Measurements  
of  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$**

AJB, Schwab, Uhlig (04)

1.

Present Status within SM (TH and Parametric Uncertainties)

2.

Impact of Present and Future Measurements of  $K \rightarrow \pi \nu \bar{\nu}$   
on CKM

3.

$K \rightarrow \pi \nu \bar{\nu}$  in Scenarios with New Complex Phases in EWP  
and  $B_d^0 - \bar{B}_d^0$  Mixing

4.

Interplay of  $K \rightarrow \pi \nu \bar{\nu}$  with

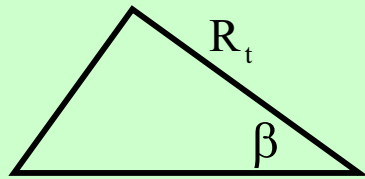
~~CP~~ in B Decays  
 $\Delta M_d / \Delta M_s$ , Rare Decays

# Basic Formulae for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

(SM)

$$X \equiv X(m_t)$$

$$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 4.8 \cdot 10^{-11} \left[ A^4 R_t^2 X^2 + 2P_c A^2 R_t X \cos \beta + P_c^2 \right]$$



$$= 10^{-11} \left[ \begin{array}{ccc} 4.1 & + & 3.0 & + & 0.7 \\ \text{(top)} & & \text{(top-charm)} & & \text{(charm)} \end{array} \right]$$

$$A = \frac{|V_{cb}|}{\lambda^2} \cong 0.83$$

Buchalla  
AJB (94)  
NLO\*)

$$P_c = 0.389 \pm \underbrace{0.033}_{\Delta m_c = 50 \text{ MeV}} \pm \underbrace{0.045}_{\text{scale } \mu_c} \pm \underbrace{0.010}_{\alpha_s} \cong 0.39 \pm 0.07$$

AJB  
Schwab  
Uhlig  
(04)

$$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{SM}} = \left[ 7.8 \pm \underbrace{0.8}_{P_c} \pm \underbrace{0.9}_{\text{CKM}} \right] 10^{-11} \cong (7.8 \pm 1.2) 10^{-11}$$

(Isidori)

$$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = \left[ 14.7 \begin{array}{c} +13.0 \\ -8.9 \end{array} \right] 10^{-11}$$

E787 (2)

E949 (1)

3 Events

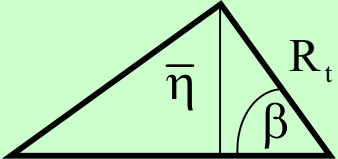
\*) NNLO : AJB, Gorbahn, Haisch, Nierste (05 Summer ?)

(Direct  $\mathcal{CP}$ )

**Basic Formulae for  $K_L \rightarrow \pi^0 \nu \bar{\nu}$**

(SM)

Buchalla  
AJB (NLO)

$$\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = 3.0 \cdot 10^{-11} \left[ \frac{\bar{\eta}}{0.35} \right]^2 \left[ \frac{|V_{cb}|}{41.5 \cdot 10^{-3}} \right]^4 \left[ \frac{X}{1.53} \right]^2$$


$$= (3.0 \pm 0.6) \cdot 10^{-11}$$

CKM

(AJB  
Schwab  
Uhlig)

KTeV :  $\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu}) < 5.9 \cdot 10^{-7}$

Future: E391a, KOPIO, JHF

Model independent bound (Grossman, Nir)

$$\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu}) \leq 4.4 \text{ Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \leq 1.4 \cdot 10^{-9} \text{ (90\% C.L.)}$$

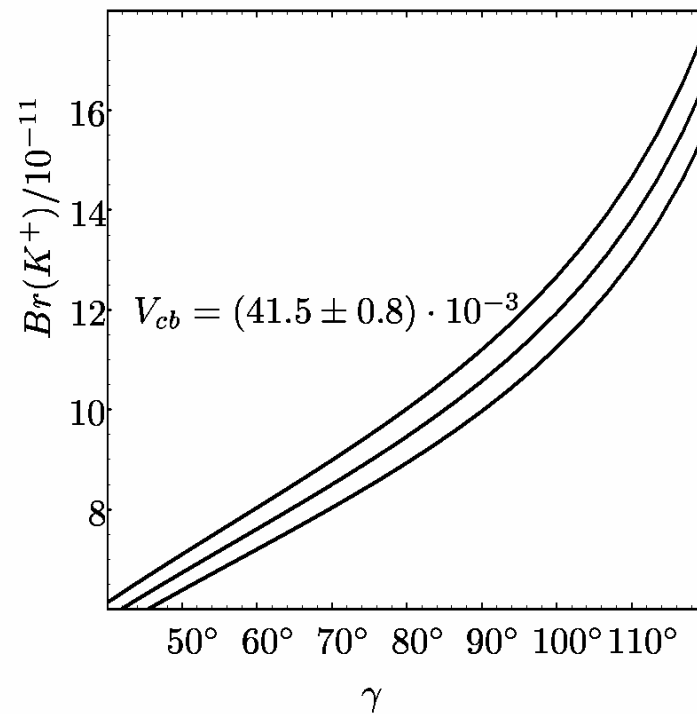
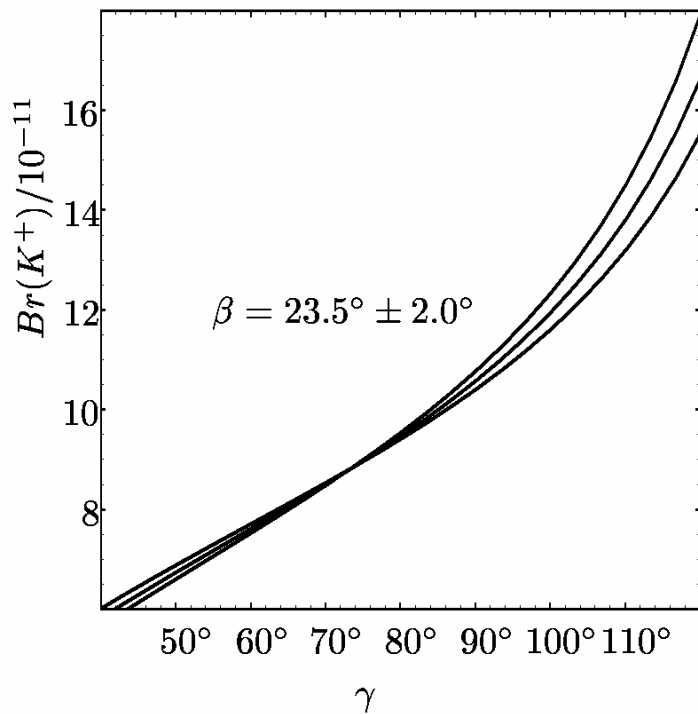
E391a could get the first non-trivial upper bound.

$$\bar{\eta} = R_t \sin \beta$$

KOPIO: ~ 50 Events  
E391 (JHF): ~ 1000 Events

$\beta$  and  $|V_{cb}|$  Dependence of  $\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$

BSU (04)



# Anatomy of $|V_{td}|$ from $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

AJB  
Schwab  
Uhlig

$$\frac{\sigma(|V_{td}|)}{|V_{td}|} = 0.39 \frac{\sigma(P_c)}{P_c} + 0.70 \frac{\sigma(\text{Br}(K^+))}{\text{Br}(K^+)} + \frac{\sigma(|V_{cb}|)}{|V_{cb}|}$$

Present:  $\pm 7\%$   $\pm$  (Very Large)  $\pm 2\%$

$\left. \begin{array}{l} \sigma(\text{Br}(K^+)) = 10\% \\ \sigma(P_c) = 0.03 \end{array} \right\} \pm 3\%$   $\pm 7\%$   $\pm 1.4\%$  (Scenario I)

$\left. \begin{array}{l} \sigma(\text{Br}(K^+)) = 5\% \\ \sigma(P_c) = 0.02 \end{array} \right\} \pm 2\%$   $\pm 3.5\%$   $\pm 1\%$  (Scenario II)

Determination  
at 4-5% possible

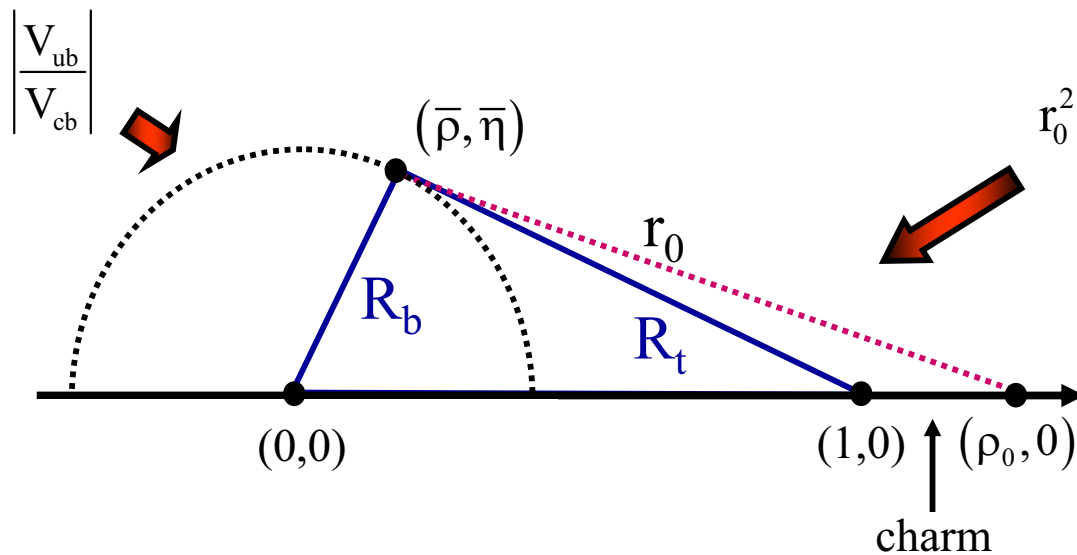
$|V_{td}|$  from UT fit:  
6-12% dependently  
on the error analysis



# K<sup>+</sup> → π<sup>+</sup>νν̄ in the (ρ̄, η̄) Plane

$$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 4.31 \cdot 10^{-11} A^4 X^2 (m_t) \frac{1}{\sigma} \left[ (\sigma \bar{\eta})^2 + (\rho_0 - \bar{\rho})^2 \right]$$

$$\sigma = \frac{1}{(1 - \lambda^2 / 2)^2} \quad \rho_0 = 1 + \frac{P_c}{A^2 X(m_t)} \approx 1.4$$



$$r_0^2 = \frac{1}{A^4 X^2(m_t)} \left[ \frac{\sigma \text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})}{4.31 \cdot 10^{-11}} \right]$$

$$R_t = 1 + R_b^2 - 2\bar{\rho}$$

$$V_{td} = A\lambda^3 (1 - \bar{\rho} - i\bar{\eta})$$

$$|V_{td}| = \lambda |V_{cb}| R_t$$

## Theoretically clean Relations

**D'Ambrosio + Isidori (02)**

$$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = \bar{\kappa}_+ |V_{cb}|^4 X^2 \left[ R_t^2 \sin^2 \beta + \left( R_t \cos^2 \beta + \frac{\lambda^4 P_c}{|V_{cb}|^2 X} \right)^2 \right]$$

$$R_t \sim \xi \frac{\sqrt{\Delta M_d}}{\sqrt{\Delta M_s}}$$

$$\bar{\kappa}_+ = 7.64 \cdot 10^{-6}$$

$$P_c = 0.39 \pm 0.07$$

**AJB, Schwab, Uhlig (04)**

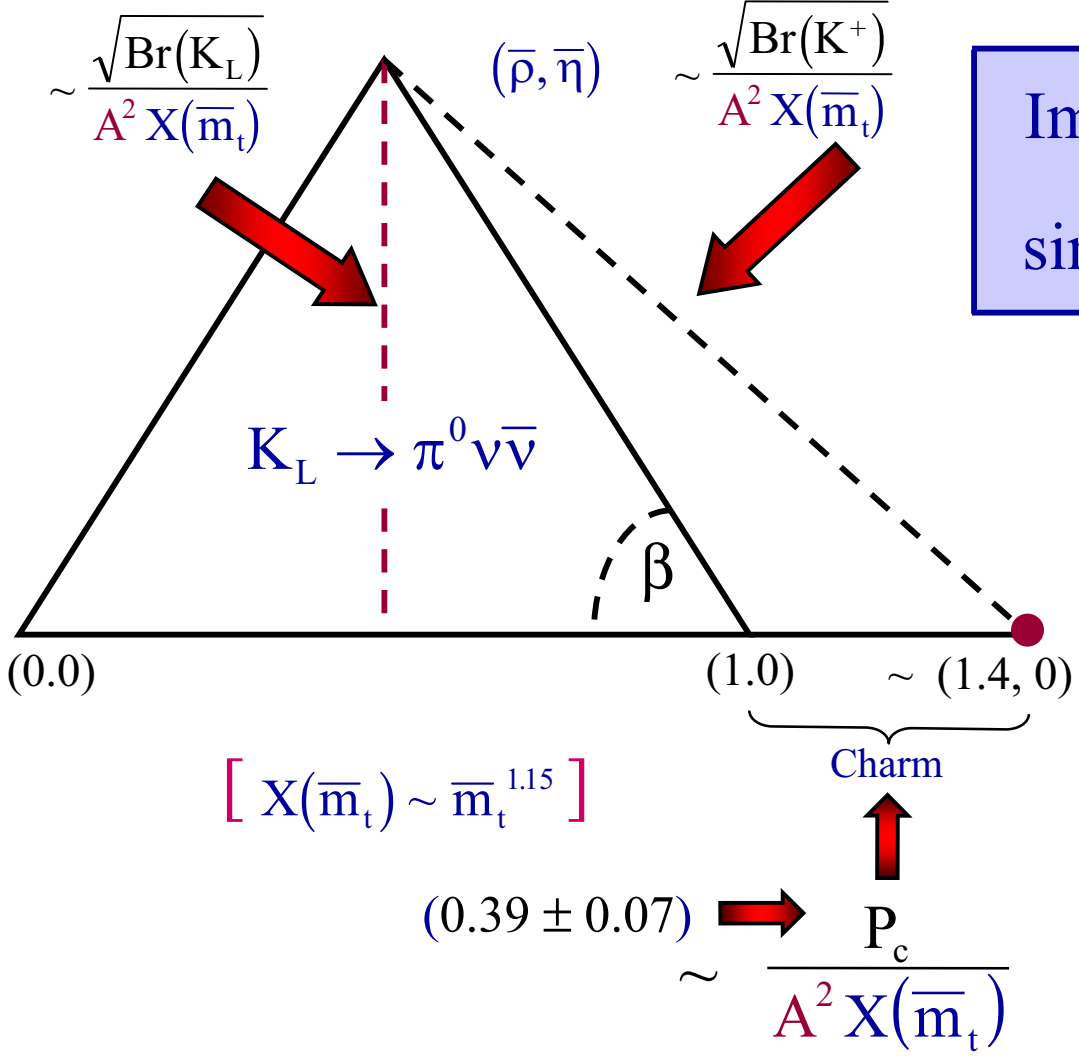
$$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = \bar{\kappa}_+ |V_{cb}|^4 X^2 \left[ T_1^2 + \left( T_2 + \frac{\lambda^4 P_c}{|V_{cb}|^2 X} \right)^2 \right]$$

$$T_1 = \frac{\sin \beta \sin \gamma}{\sin(\beta + \gamma)}$$

$$T_2 = \frac{\cos \beta \sin \gamma}{\sin(\beta + \gamma)}$$

# UT from $K \rightarrow \pi \nu \bar{\nu}$

Buchalla  
AJB



$$\text{Im } \lambda_t = F_1(\bar{m}_t, \text{Br}(K_L))$$

$$\sin 2\beta = F_2(\mathbf{P}_c, \text{Br}(K_L), \text{Br}(K^+))$$

$$\lambda_t = V_{ts}^* V_{td}$$

$$\sin 2\beta \quad \longleftrightarrow \quad \sin 2\beta$$

( $K \rightarrow \pi \nu \bar{\nu}$ )                      ( $B \rightarrow J / \psi, K_s \rightarrow \phi K_s$ )

K-Physics  $\longleftrightarrow$  B - Physics

Test of SM

and Beyond

## Golden Relations

(All involving  $K_L \rightarrow \pi^0 \nu \bar{\nu}$ )

**Buchalla, AJP (94)**

$$\cot \beta = \frac{\sqrt{B_1 - B_2} - P_c}{\sqrt{B_2}}$$



$$(\sin 2\beta)_{\pi\nu\bar{\nu}} = (\sin 2\beta)_{\psi K_S}$$

$$B_1 \sim \text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$$

$$B_2 \sim \text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})$$

$$J_{CP} \sim \text{triangle} \sim \frac{\sqrt{\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})}}{X(m_t)}$$

**AJB, Schwab, Uhlig (04)**

$$\frac{\sin \beta \sin \gamma}{\sin(\beta + \gamma)} = 0.35 \left[ \frac{1.53}{X(m_t)} \right] \left[ \frac{0.0415}{|V_{cb}|} \right]^2 \sqrt{\frac{\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})}{3 \cdot 10^{-11}}}$$

$\beta$  from  $a_{\psi K_S}$

$\gamma$  from  $B_s^0 \rightarrow D_s^\pm K^\mp$   
 $B_d^0 \rightarrow D^\pm \pi^\mp$

# The Angle $\beta$ from $K \rightarrow \pi\nu\bar{\nu}$

Buchalla, AJB (94)  
AJB, Schwab, Uhlig (04)

BSU: 
$$\frac{\sigma(\sin 2\beta)}{\sin 2\beta} = 0.31 \frac{\sigma(P_c)}{P_c} + 0.55 \frac{\sigma(\text{Br}(K^+))}{\text{Br}(K^+)} \pm 0.39 \frac{\sigma(\text{Br}(K_L))}{\text{Br}(K_L)}$$

$$\sigma(\sin 2\beta) = \pm 0.041 \quad \pm ? \quad \pm ? \quad (\text{Present})$$

$$\sigma(\sin 2\beta) = 0.017 \quad \pm 0.039 \quad \pm 0.028 \quad (\text{Scenario I})$$

Br's at 10%

$$\sigma(\sin 2\beta) = 0.011 \quad \pm 0.020 \quad \pm 0.014 \quad (\text{Scenario II})$$

Br's at 5%

TH  
very  
clean

$$\sigma(\sin 2\beta) \approx 0.02 - 0.03 \quad \text{requires } \sigma(\text{Br's}) \leq 5\%$$

# The Angle $\gamma$ from $K \rightarrow \pi\nu\bar{\nu}$

AJB, Schwab, Uhlig (04)

$$\frac{\sigma(\gamma)}{\gamma} = 0.75 \frac{\sigma(P_c)}{P_c} + 1.32 \frac{\sigma(\text{Br}(K^+))}{\text{Br}(K^+)} + 0.07 \frac{\sigma(\text{Br}(K_L))}{\text{Br}(K_L)} + 4.1 \frac{\sigma(|V_{cb}|)}{|V_{cb}|}$$

$$\sigma(\gamma) = \quad \pm 8.3^\circ \quad \pm ? \quad \pm ? \quad \pm 4.9^\circ \quad (\text{Present})$$

$$\sigma(\gamma) = \quad \pm 3.7^\circ \quad \pm 8.5^\circ \quad \pm 0.4^\circ \quad \pm 3.8^\circ \quad (\text{Scenario I})$$

Br's at 10%

$$\sigma(\gamma) = \quad \pm 2.5^\circ \quad \pm 4.2^\circ \quad \pm 0.2^\circ \quad \pm 2.5^\circ \quad (\text{Scenario II})$$

Br's at 5%

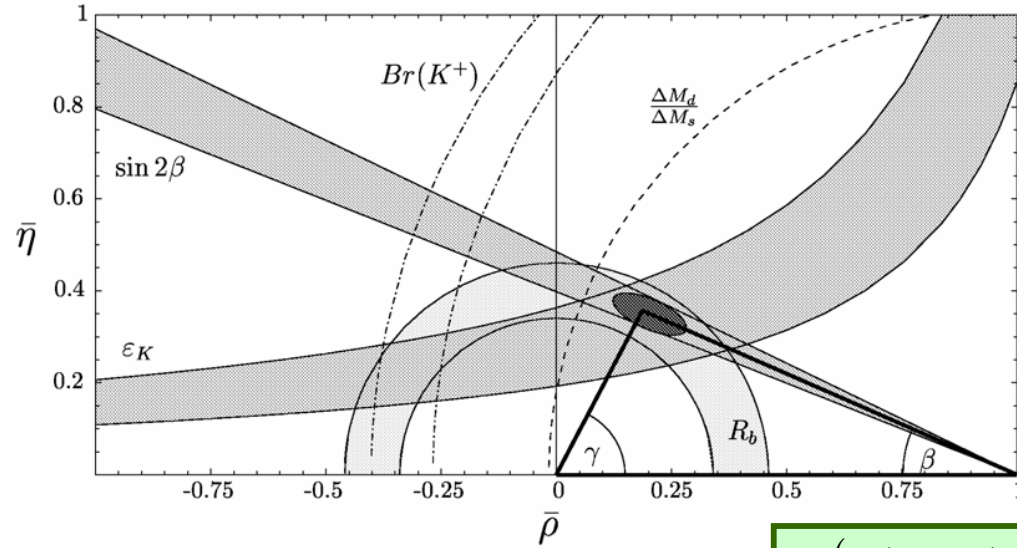
TH  
very  
clean

$$\sigma(\gamma) \approx \pm 5^\circ \quad \text{requires} \quad \sigma(\text{Br}(K^+)) \leq 5\%$$

# Unitarity Triangle 2004

(AJB, Schwab, Uhlig)

$$\text{Br}(K^+) \equiv \text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 14.7 \cdot 10^{-11}$$



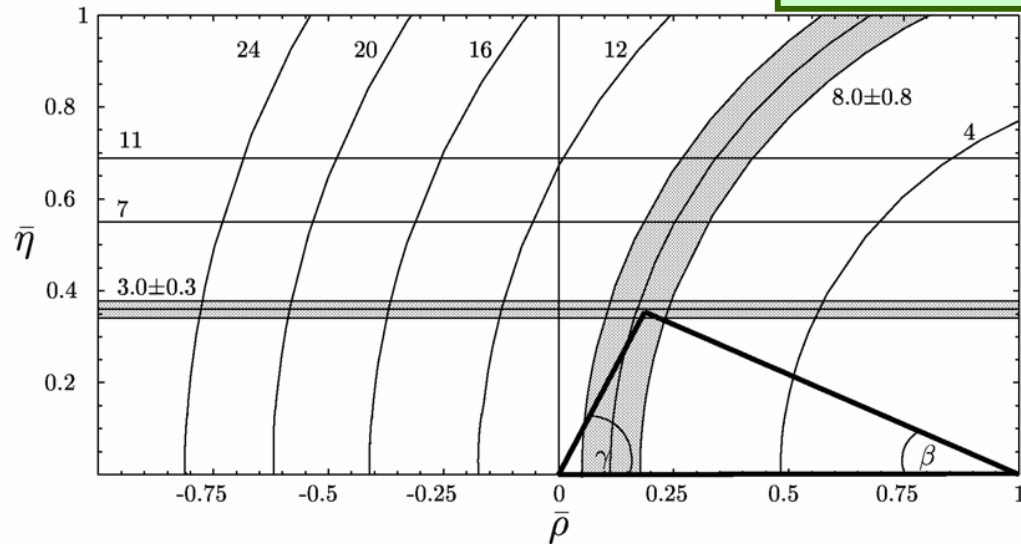
$$P_c = \underline{0.39 \pm 0.07}$$

$m_c, V_{cb}, \mu_c$

$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$

Unitarity Triangle from  $K \rightarrow \pi \nu \bar{\nu}$

(2012)



$\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})$

## Recent News

1.

Improved Calculation of LD  
Contributions to the Charm  
Component in  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ :

$$P_C = 0.39 \pm 0.07 \rightarrow 0.43 \pm 0.07$$

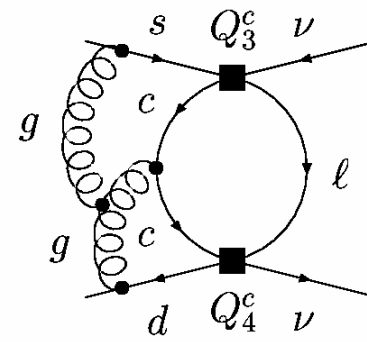
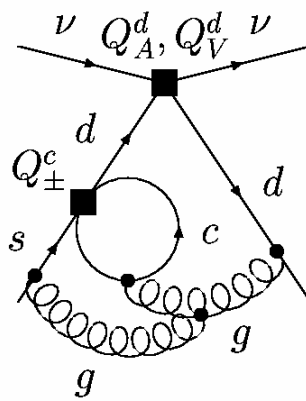
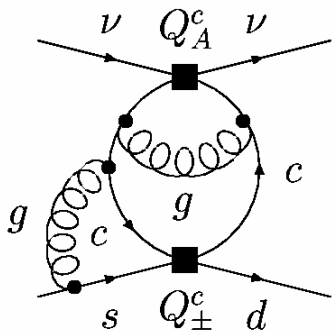
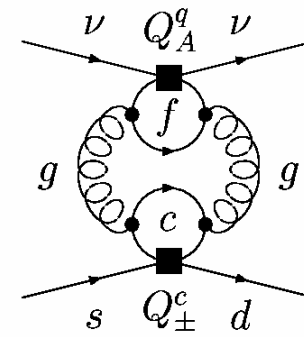
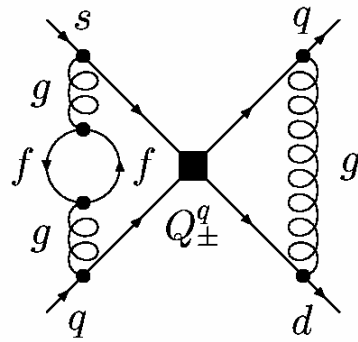
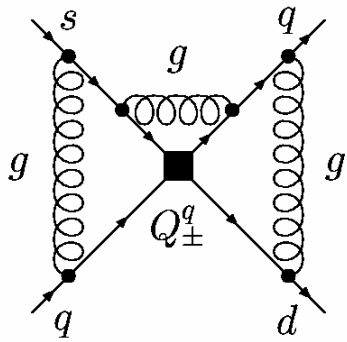
$$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (7.8 \pm 1.2) \cdot 10^{-11} \rightarrow (8.3 \pm 1.2) \cdot 10^{-11}$$

Isidori  
Mescia  
Smith

2.

$P_C$  at NNLO soon available !  
(AJB, Gorbahn, Haisch, Nierste)





# 6.

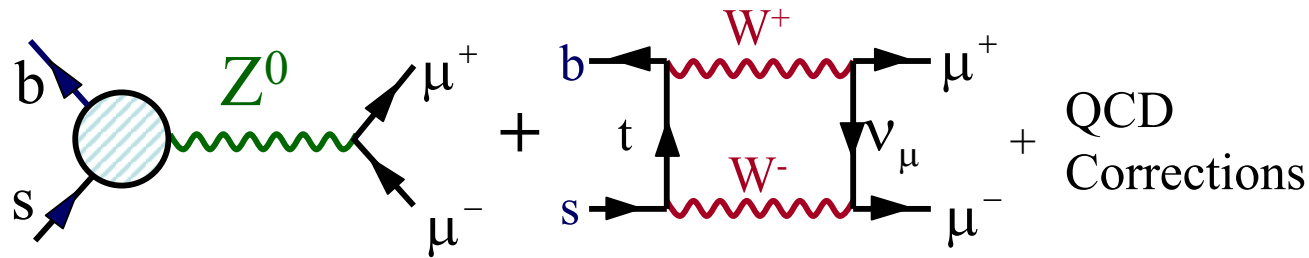
## Rare B and K Decays

$$B_{s,d} \rightarrow \mu^+ \mu^- \quad B \rightarrow X_{s,d} \nu \bar{\nu}$$

$$B \rightarrow X_s \gamma \quad B \rightarrow X_s l^+ l^-$$

$$K_L \rightarrow \pi^0 l^+ l^-$$

$$B_s \rightarrow \mu^+ \mu^-$$



QCD Corrections  
 Buchalla, AJB (93)  
 Misiak, Urban (98)

$$\text{Br}(B_s \rightarrow \mu^+ \mu^-) = 3.8 \cdot 10^{-9} \left[ \frac{\tau(B_s)}{1.46 \text{ps}} \right] \left[ \frac{F_{B_s}}{230 \text{MeV}} \right]^2 \left[ \frac{|V_{ts}|}{0.040} \right]^2 [Y(x_t)]^2$$

$$Y(x_t) = 1.02 \left[ \frac{m_t(m_t)}{170 \text{GeV}} \right]^{1.56} \approx 1$$

$$F_{B_s} = (230 \pm 30) \text{MeV}$$

$$\tau(B_s) = (1.46 \pm 0.05) \text{ps}$$

(Dominant uncertainty)

SM:  $\text{Br}(B_s \rightarrow \mu^+ \mu^-) = (3.7 \pm 1.0) \cdot 10^{-9}$

$$\text{Br}(B_s \rightarrow \mu^+ \mu^-) < 5 \cdot 10^{-7}$$

DØ, CDF

95% C.L.

$$B_d \rightarrow \mu^+ \mu^-$$

(just replace s→d)

$$\text{Br}(B_d \rightarrow \mu^+ \mu^-) = 1.0 \cdot 10^{-10} \left[ \frac{\tau(B_d)}{1.54 \text{ps}} \right] \left[ \frac{F_{B_d}}{190 \text{MeV}} \right]^2 \left[ \frac{|V_{td}|}{0.008} \right]^2 [Y(x_t)]^2$$

$$\tau(B_d) = (1.540 \pm 0.014) \text{ps}$$

$$F_{B_d} = (189 \pm 27) \text{MeV}$$

$$|V_{td}| = (8.24 \pm 0.54) \cdot 10^{-3}$$

**SM:**

$$\text{Br}(B_d \rightarrow \mu^+ \mu^-) = (1.04 \pm 0.34) \cdot 10^{-10}$$

**Belle:**

$$\text{Br}(B_d \rightarrow \mu^+ \mu^-) < 1.6 \cdot 10^{-7} \quad (95\% \text{ C.L.})$$

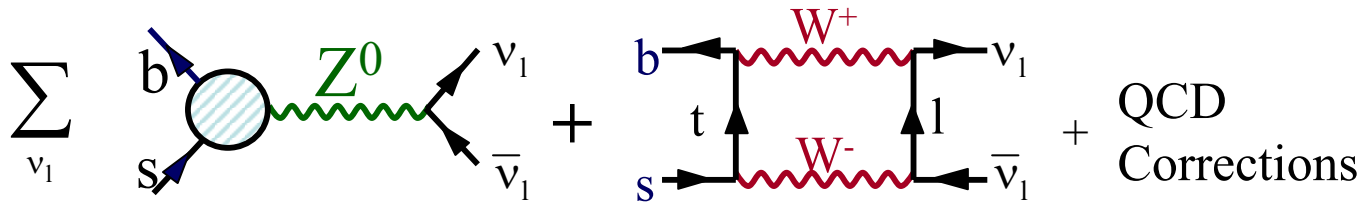
$$\frac{F_{B_s}}{F_{B_d}} = 1.22 \pm 0.06$$

$$\frac{\text{Br}(B_s \rightarrow \mu^+ \mu^-)}{\text{Br}(B_d \rightarrow \mu^+ \mu^-)} = \frac{\tau(B_s) m_{B_s}}{\tau(B_d) m_{B_d}} \left[ \frac{F_{B_s}}{F_{B_d}} \right]^2 \left[ \frac{|V_{ts}|}{|V_{td}|} \right]^2$$



Useful measurement  
of  $|V_{td}|$

$$B \rightarrow X_s \nu \bar{\nu}$$



QCD  
Corrections

Buchalla, AJB (93)  
Misiak, Urban (98)

$$\text{Br}(B \rightarrow X_s \nu \bar{\nu}) = 1.58 \cdot 10^{-5} \left[ \frac{|V_{ts}|}{0.040} \right]^2 [X(x_t)]^2 \quad [X(x_t)] = 1.57 \left( \frac{m_t(m_t)}{170 \text{ GeV}} \right)^{1.15}$$

SM:  $\text{Br}(B \rightarrow X_s \nu \bar{\nu}) = (3.66 \pm 0.21) \cdot 10^{-5}$

ALEPH:  $\text{Br}(B \rightarrow X_s \nu \bar{\nu}) < 6.4 \cdot 10^{-4}$

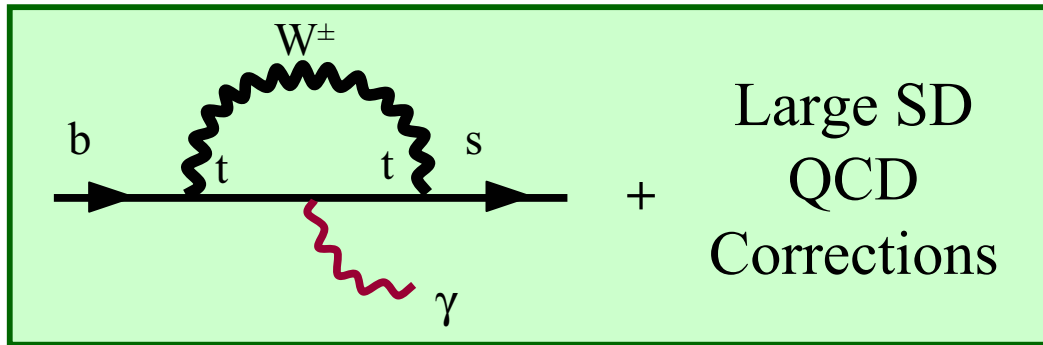
$$\frac{\text{Br}(B \rightarrow X_d \nu \bar{\nu})}{\text{Br}(B \rightarrow X_s \nu \bar{\nu})} = \frac{|V_{td}|^2}{|V_{ts}|^2}$$

Theoretically  
cleanest measurement  
of  $|V_{td}|/|V_{ts}|$



Long Distance Effects negligible: Buchalla, Isidori, Rey

$$B \rightarrow X_s \gamma$$



$\gamma = \text{on shell}$

Dominant Operator :

$$Q_7 = m_b \bar{s}_L \sigma_{\mu\nu} b_R F^{\mu\nu}$$

(Magnetic Penguins)

Bertolini, Borzumati, Masiero (1987)  
Deshpande, Lo, Trampetic, Eilam, Singer (1987)

QCD Enhancement ( $\sim 3$ )  
governed by the Mixing  
of  $Q_7$  with

$$Q_2 = (\bar{b}c)_{V-A} (\bar{c}s)_{V-A}$$

$$\text{Br}(B \rightarrow X_s \gamma) = \begin{cases} (3.52 \pm 0.30) \cdot 10^{-4} & \text{CLEO, BaBar, Belle} \\ (3.70 \pm 0.30) \cdot 10^{-4} & \text{SM} \end{cases}$$

Sensitive to New Physics! Important for constraining  
Supersymmetry !!

# NLO-QCD Corrections Saga 1994-2002

1993

: Identification of strong: Ali, Greub; AJB, Misiak, Münz, Pokorski  
 $\mu_b$  dependence ( $\sim 60\%$ )

Initial  
Conditions

: Adel, Yao (93); Greub, Hurth (97); AJB, Kwiatkowski, Pott (97)  
Ciuchini, Degrassi, Gambino, Giudice (97)

Two and  
Three-Loop  
Anomalous  
Dimensions

: AJB, Jamin, Lautenbacher, Weisz (92); Ciuchini, Franco,  
Martinelli, Reina (93)  
Misiak, Münz (95) (Two-Loop Mixing of Magnetic Operators)  
Chetyrkin, Misiak, Münz (97) (Three-Loop Mixing between  
 $Q_7$  and  $Q_2$ )

Operator  
Matrix  
Elements

: Greub, Hurth, Wyler (1996)  
AJB, Czarnecki, Misiak, Urban (2001)

Review:  
AJB, Misiak (2003)

Gluon  
Bremsstrahlung

: Ali, Greub (91)  
Pott (95)

# $B \rightarrow X_s \gamma$ beyond NLO $\rightarrow$ NNLO

2001: Gambino, Misiak (significant uncertainty due to  $m_c$ )

 Go beyond NLO

Considerable  
Progress  
made

Misiak, Steinhauser (2004)  
Bieri, Greub, Steinhauser (2003)

Gorbahn, Haisch (2005)  
Gorbahn, Haisch, Misiak (2005)

Initial Conditions

Matrix Elements  
(first steps)

Three-Loop Mixing  
of Magnetic Penguins

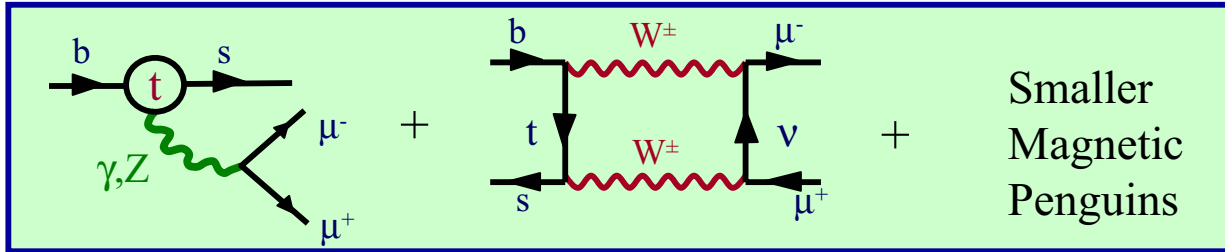
4-Loop  
Mixing  $Q_2 \leftrightarrow Q_7$

: Czakon, ...



$$\mathbf{B} \rightarrow X_s \mu^+ \mu^-$$

Hou, Willey, Soni (87)



QCD (LO): Grinstein, Savage, Wise (89)

QCD (NLO): Misiak (94)  
AJB + Münz (94)

Operators:

$$Q_{9V} = (\bar{s}b)_{V-A} (\bar{\mu}\mu)_V$$

$$Q_{10V} = (\bar{s}b)_{V-A} (\bar{\mu}\mu)_A$$

$$+ Q_7 = m_b \bar{s}_L \sigma_{\mu\nu} b_R F^{\mu\nu}$$

Known from  $B \rightarrow X_s \gamma$

$$\frac{d\sigma}{ds} \approx \frac{\alpha^2}{4\pi} (1-s)^2 |V_{ts}|^2 \left[ (1+2s)(|C_9(s)|^2 + |C_{10}|^2) + 4\left(1 + \frac{2}{s}\right) |C_7|^2 + 12C_7 C_9 \right]$$

$$s = \frac{(P_{\mu^+} + P_{\mu^-})^2}{m_b^2}$$

$$C_9(s) = P_0^{\text{NLO}}(s) + \frac{Y(x_t)}{\sin^2 \theta_w} - 4Z(x_t)$$

$$C_{10} = -\frac{Y(x_t)}{\sin^2 \theta_w}$$

[ ] = U(s)

# Recent Developments on $B \rightarrow X_s \mu^+ \mu^-$

NNLO  
QCD

Bobeth, Misiak, Urban (2000)  
 Ghinculov, Hurth, Isidori, Yao (2002-2004)  
 Asatryan, Asatrian, Greub, Walker (2002-2004)  
 Asatrian, Asatryan, Hovhannisyan, Poghosyan (2004)  
 Bobeth, Gambino, Gorbahn, Haisch (2004)

$$\text{Br}(B \rightarrow X_s l^+ l^-) = \begin{cases} (4.5 \pm 1.0) \cdot 10^{-6} & \text{Exp} \\ (4.4 \pm 0.7) \cdot 10^{-6} & \text{SM} \end{cases}$$

(low  $s$  region)

(below  $c\bar{c}$  resonance)

In order to test better look at

$A_{\text{FB}}(s) \equiv$  Forward – Backward  
Asymmetry

( see  
Section 6 )

$$A_{\text{FB}}(s) = -3C_{10} \frac{[sC_9(s) + 2C_7]}{U(s)}$$

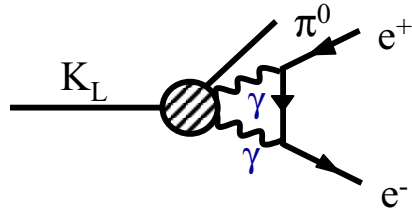
Vanishes at  $s_0$ :

$$s_0 C_9(s_0) + 2C_7 = 0$$

Theoretically clean; sensitive to New Physics.

# $K_L \rightarrow \pi^0 e^+ e^-$ (3 contributions)

①  $K_L \rightarrow \pi^0 \gamma \gamma \rightarrow \pi^0 e^+ e^-$

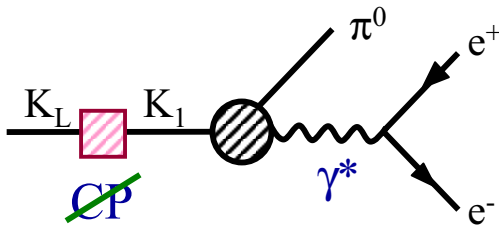


← CP conserving

Donoghue, Holstein, Valencia, Ecker,  
Pich, de Rafael, Flynn, Randall,  
Seghal, Heiliger, Fajfer (95)  
Cohen, Ecker, Pich (93)  
Donoghue, Gabbiani (95)  
Ambrosio, Portoles (97)

Using  
KTeV (99)  
 $K_L \rightarrow \pi \gamma \gamma$

②  $K_L \xrightarrow{K_1} \pi^0 \gamma^* \rightarrow \pi^0 e^+ e^-$



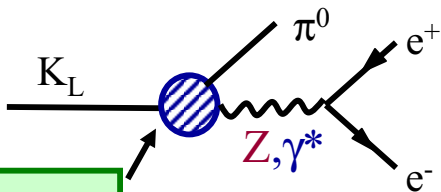
← indirect ~~CP~~

★ ( $K_S \rightarrow \pi^0 e^+ e^-$  helps here!)

Ecker, Pich, de Rafael (91)  
Heiliger, Seghal (93)  
Donoghue, Gabbiani (95)  
Fajfer (95)

$K_L \cong K_2 + \epsilon K_1$

③  $K_L \xrightarrow{K_2} \pi^0 \gamma^* \rightarrow \pi^0 e^+ e^-$



← direct ~~CP~~

★ (TH very clean!)

LO: { Dib, Dunietz, Gilman  
Flynn, Randall  
Buchalla, AJB, Harlander

NLO: AJB, Lautenbacher, Misiak,  
Münz (94)

New Physics  
can enter here

The action  
of  $Z^0, \gamma$   
Penguins

# Present Status on $K_L \rightarrow \pi^0 e^+ e^-$ , $K_L \rightarrow \pi^0 \mu^+ \mu^-$

Buchalla, D'Ambrosio, Isidori; Isidori, Smith, Unterdorfer

$l = e, \mu$

$$\text{Br}(K_L \rightarrow \pi^0 l^+ l^-) = \left[ \underbrace{C_{\text{mix}}^l}_{\substack{\text{indirect} \\ \mathcal{CP}}} + \underbrace{C_{\text{int}}^l \left( \frac{\text{Im} \lambda_t}{10^{-4}} \right)}_{\text{Interference of direct and indirect}} + \underbrace{C_{\text{dir}}^l \left( \frac{\text{Im} \lambda_t}{10^{-4}} \right)^2}_{\text{direct}} + \underbrace{C_{\text{CPC}}^l}_{\text{CP conserving}} \right] \cdot 10^{-12}$$

$$C_{\text{mix}}^e \cong 22.6 \pm 7.0$$

$$C_{\text{int}}^e \cong 7.4 \pm 1.5$$

$$C_{\text{dir}}^e \cong 2.4 \pm 0.2$$

$$C_{\text{CPC}}^e \cong 0$$

$$C_{\text{mix}}^\mu \cong 5.3 \pm 1.6$$

$$C_{\text{int}}^\mu \cong 1.9 \pm 0.4$$

$$C_{\text{dir}}^\mu \cong 1.0 \pm 0.1$$

$$C_{\text{CPC}}^\mu \cong 5.2 \pm 1.6$$

$$\text{Br}(K_L \rightarrow \pi^0 e^+ e^-) = \left( 3.7^{+1.1}_{-0.9} \right) \cdot 10^{-11}$$

$$\text{Br}(K_L \rightarrow \pi^0 \mu^+ \mu^-) = (1.5 \pm 0.3) \cdot 10^{-11}$$

# 7.

## Minimal Flavour Violation (MFV)

- a) Generalities
- b) Model with one Universal Extra Dimension
- c) Littlest Higgs Model
- d) MSSM at low  $\tan\beta$

Review: AJB    hep-ph/0310208

# Generalities

$$A(\text{Decay}) = \sum_i B_i \eta_{\text{QCD}}^i V_{\text{CKM}}^i \overbrace{\left[ F_{\text{SM}}^i + F_{\text{New}}^i \right]}^{F_i(\mathbf{v})}$$

real

AJB, Gambino, Gorbahn, Jäger, Silvestrini  
 D'Ambrosio, Giudice, Isidori, Strumia

K and B  
 Physics  
 related  
 to each  
 other

**1.**

All flavour changing processes governed by  $V_{\text{CKM}}^i$  .

**2.**

Only SM Operators are relevant.

**3.**

New Physics enters only through 7 Master Functions

$$F_i(\mathbf{v}) = S(\mathbf{v}), X(\mathbf{v}), Y(\mathbf{v}), Z(\mathbf{v}), D'(\mathbf{v}), E'(\mathbf{v}), E(\mathbf{v})$$

$\mathbf{v}$  = collects parameters specific to a given MFV model.

SM:

$$\mathbf{v} = \mathbf{x}_t$$

# Universal Unitarity Triangle

AJB, Gambino, Gorbahn, Jäger, Silvestrini (00)

In the full class of MFV-models it is possible to construct quantities that depend on CKM parameters but in which the dependence on new physics parameters cancels out



CKM Matrix determined without  
"New Physics Pollution"



Universal Unitarity Triangle

## Examples

$$R_t = 0.90 \sqrt{\frac{\Delta M_d}{0.50 / \text{ps}}} \sqrt{\frac{18.4 / \text{ps}}{\Delta M_s}} \left[ \frac{\xi}{1.22} \right]$$

$$a_{\psi K_s} = \sin 2\beta$$

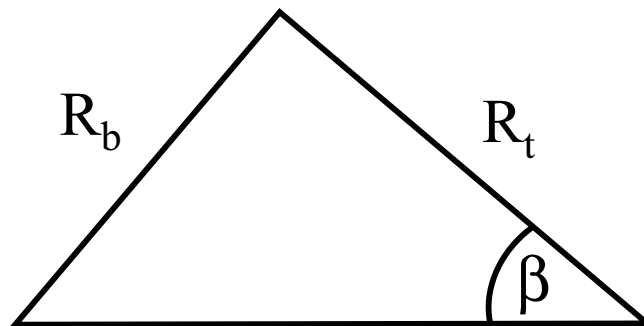
# Universal Unitarity Triangle 2004

AJB, Schwab, Uhlig

Use only quantities that are independent of parameters  
specific to a given Minimal Flavour Violation model

$$\left| \frac{V_{ub}}{V_{cb}} \right| \rightarrow R_b = \frac{(1 - \lambda^2 / 2)}{\lambda} \left| \frac{V_{ub}}{V_{cb}} \right|$$

$$\frac{\Delta M_d}{\Delta M_s} \rightarrow R_t = \frac{\xi_{\text{th}}}{\lambda} \sqrt{\frac{\Delta M_d}{\Delta M_s}} \quad a_{\psi K_s} \rightarrow \sin 2\beta$$



$$\xi_{\text{th}} = \frac{\sqrt{\hat{B}_s F_{B_s}}}{\sqrt{\hat{B}_d F_{B_d}}}$$



# SM UT versus UUT of MFV

BSU (04)

SM

$$\bar{\eta} = 0.354 \pm 0.027$$

$$\bar{\rho} = 0.187 \pm 0.059$$

$$\gamma = (62.2 \pm 8.2)$$

$$R_t = 0.887 \pm 0.059$$

$$R_b = 0.400 \pm 0.039$$

$$|V_{td}| = (8.24 \pm 0.54) \cdot 10^{-3}$$

$$\text{Im}\lambda_t = (1.40 \pm 0.12) \cdot 10^{-4}$$

$$\lambda_t = V_{ts}^* V_{td}$$

MFV

$$\bar{\eta} = 0.360 \pm 0.031$$

$$\bar{\rho} = 0.174 \pm 0.068$$

$$\gamma = (64.2 \pm 9.6)$$

$$R_t = 0.901 \pm 0.064$$

$$R_b = 0.400 \pm 0.044$$

$$|V_{td}| = (8.38 \pm 0.62) \cdot 10^{-3}$$

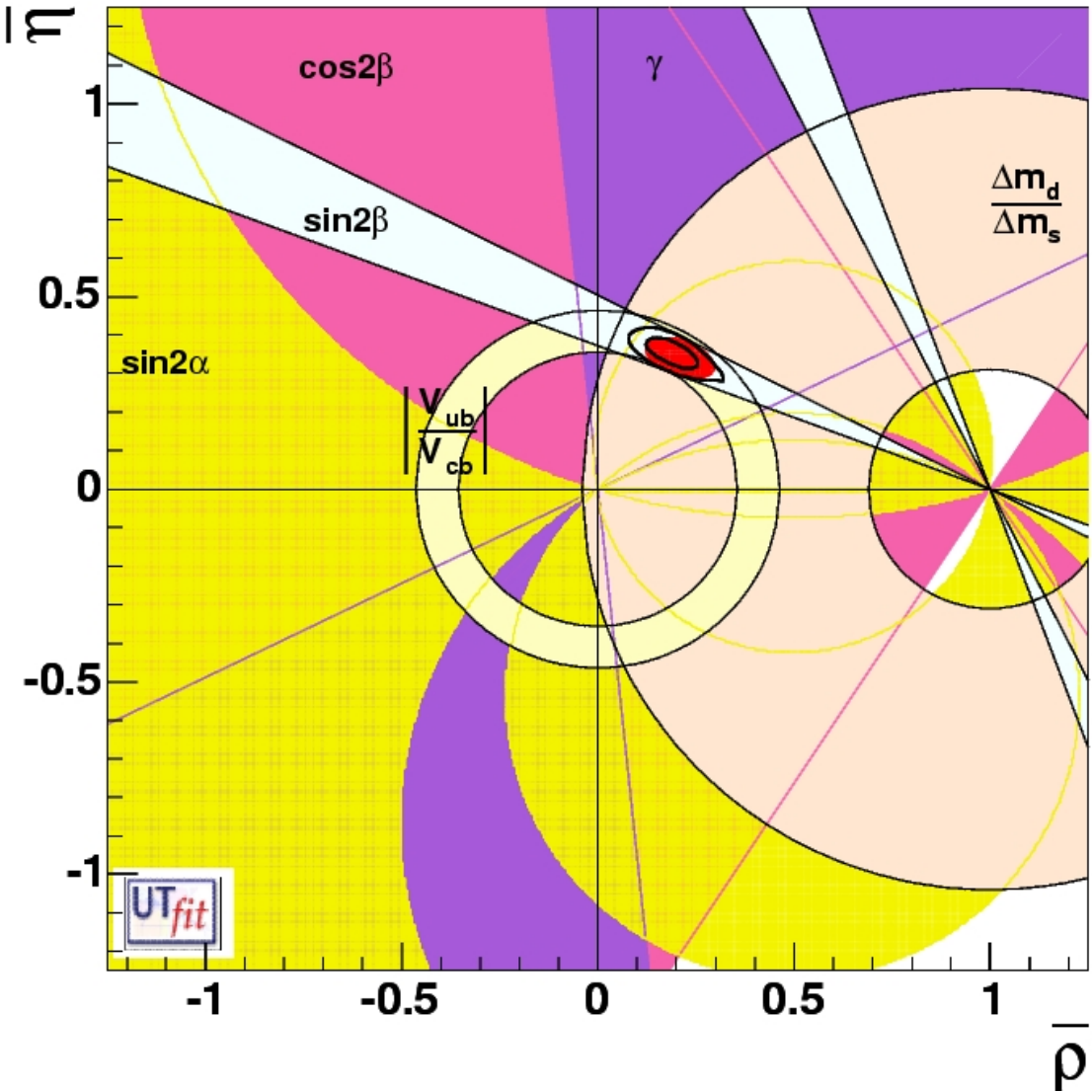
$$\text{Im}\lambda_t = (1.43 \pm 0.14) \cdot 10^{-4}$$

**UUT of MFV rather close to SM UT**



# Universal Unitarity Triangle (MFV)

UTfit Collaboration : Bona et al.



# MFV "Sum Rules"

Relations that do not involve the Master Functions  $X, Y, Z, S$ , etc.

Violation of these relations signals new flavour (CP) violating interactions beyond CKM or new operators that are strongly suppressed in SM

Examples

$$(\sin 2\beta)_{\pi\nu\bar{\nu}} = (\sin 2\beta)_{\psi K_s}$$

$$\frac{\text{Br}(B_s \rightarrow \mu^+ \mu^-)}{\text{Br}(B_d \rightarrow \mu^+ \mu^-)} = \frac{\tau(B_s) m_{B_s}}{\tau(B_d) m_{B_d}} \left[ \frac{F_{B_s}}{F_{B_d}} \right]^2 \left[ \frac{|V_{ts}|}{|V_{td}|} \right]^2$$

$$\frac{\Delta M_d}{\Delta M_s} = \frac{m_{B_d}}{m_{B_s}} \frac{\hat{B}_d}{\hat{B}_s} \frac{F_{B_d}^2}{F_{B_s}^2} \frac{|V_{td}|^2}{|V_{ts}|^2}$$

$$\frac{\text{Br}(B \rightarrow X_d \nu\bar{\nu})}{\text{Br}(B \rightarrow X_s \nu\bar{\nu})} = \frac{|V_{td}|^2}{|V_{ts}|^2}$$

## Impact of a Modified $|V_{td}|$

$$\left\{ \Delta M_d \approx |V_{td}|^2 \cdot [S_{SM}(m_t) + \Delta s] \right\} \Rightarrow |V_{td}|^2 \approx \frac{\Delta M_d}{[S(m_t) + \Delta s]}$$

$$\text{Br}(B_d \rightarrow \mu^+ \mu^-) \approx |V_{td}|^2 \cdot [Y_{SM}(m_t) + \Delta Y]^2 \approx \Delta M_d \frac{[Y_{SM}(m_t) + \Delta Y]^2}{[S(m_t) + \Delta s]}$$

But :

$$\text{Br}(B_s \rightarrow \mu^+ \mu^-) \approx |V_{ts}|^2 \cdot [Y_{SM}(m_t) + \Delta Y]^2$$

$$\uparrow$$

$$|V_{ts}| \approx |V_{cb}| \quad (\text{CKM Unitarity})$$

$$\left\{ \begin{array}{l} \Delta S > 0 \\ \text{in most} \\ \text{extensions} \end{array} \right\} \Rightarrow \left( \frac{\text{Br}(B_s \rightarrow \mu^+ \mu^-)}{\text{Br}(B_d \rightarrow \mu^+ \mu^-)} \right)_{NP} > \left( \frac{\text{Br}(B_s \rightarrow \mu^+ \mu^-)}{\text{Br}(B_d \rightarrow \mu^+ \mu^-)} \right)_{SM}$$

# Intriguing Property of Models with Minimal Flavour Violation

AJB, Fleischer (01)

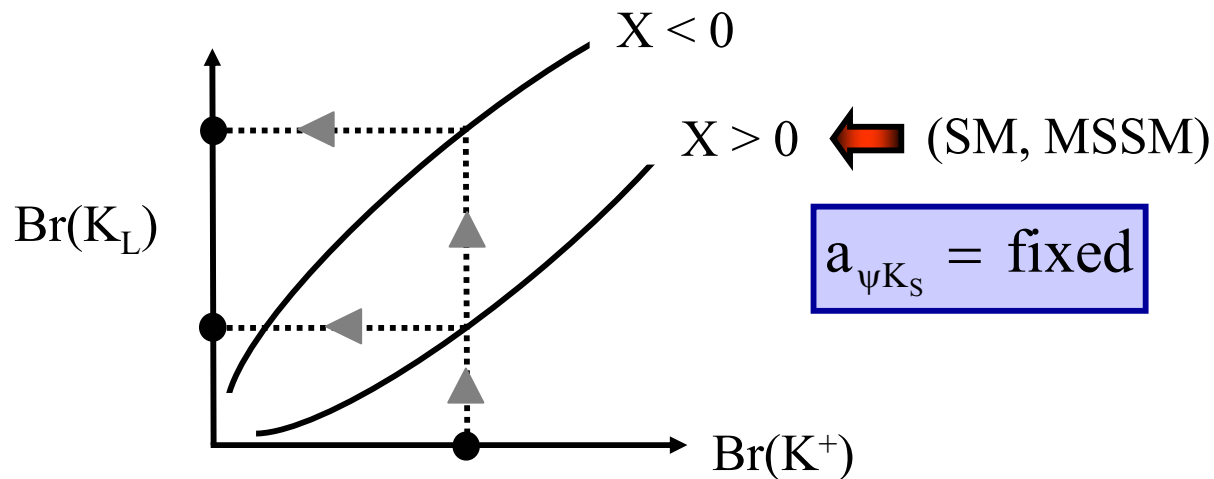
$$\text{Br}(K_L) = F\left(\text{Br}(K^+), a_{\psi K_S}, \text{sgn}(X)\right)$$

TH very clean

Independently of any parameters, for given  $\text{Br}(K^+)$  and  $a_{\psi K_S}$  only two values of  $\text{Br}(K_L)$  possible.



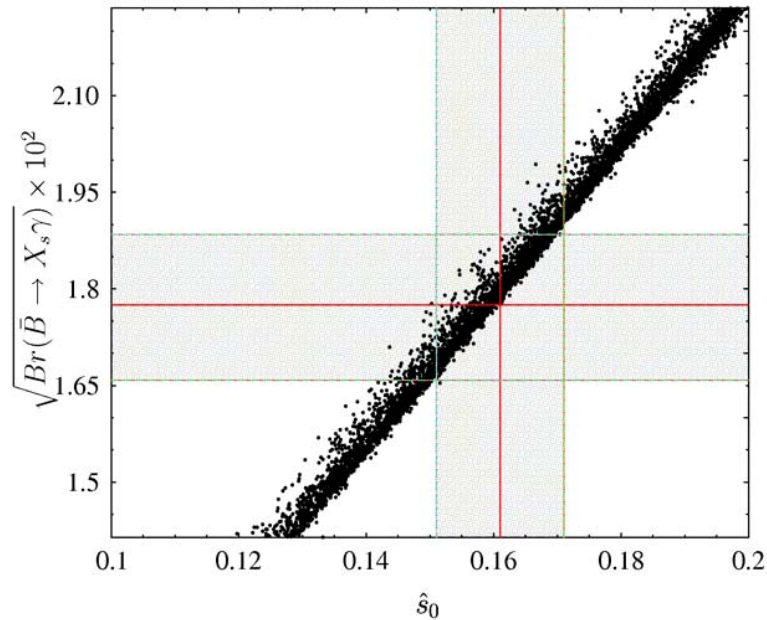
$X < 0$   
very unlikely



Correlation:  $\text{Br}(B \rightarrow X_s \gamma) \leftrightarrow \hat{s}_0$  in  $A_{\text{FB}}(B \rightarrow X_s l^+ l^-)$

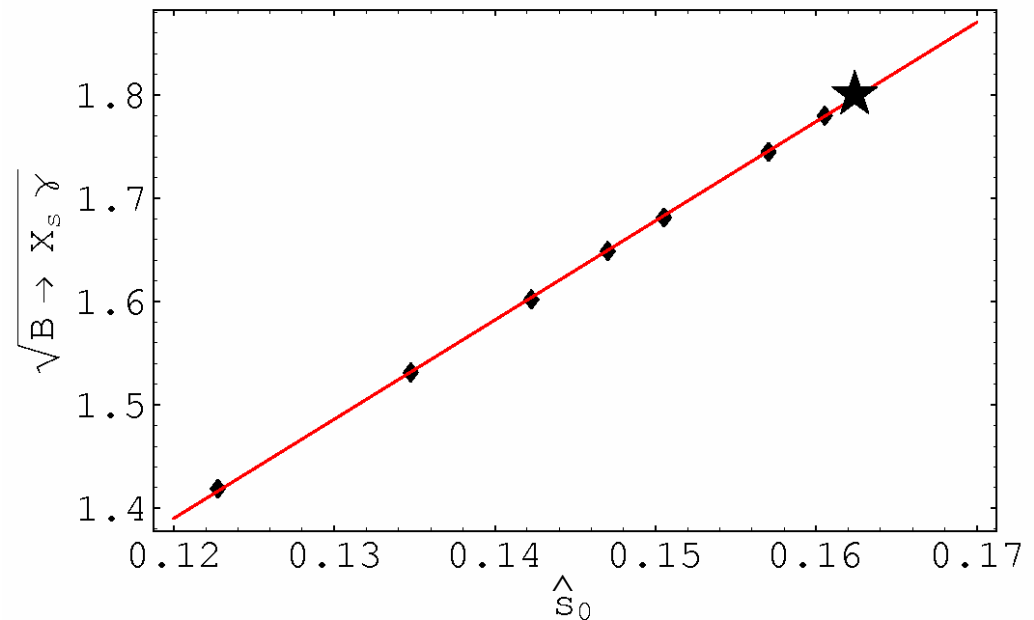
## MSSM (MFV)

(Bobeth, Ajb, Ewerth)



## Universal Extra Dimensions

(Ajb, Poschenrieder, Spranger, Weiler)



# Relations between $\Delta M_{s,d}$ and $B_{s,d} \rightarrow \mu\bar{\mu}$ in Models with Minimal Flavour Violation

(AJB, hep-ph/0303060)

$$\Delta M_q \sim \hat{B}_q F_{B_q}^2 |V_{tq}|^2 S(x_t, x_{\text{new}})$$

$$\text{Br}(B_q \rightarrow \mu\bar{\mu}) \sim F_{B_q}^2 |V_{tq}|^2 Y^2(x_t, \bar{x}_{\text{new}})$$

Large hadronic  
uncertainties  
due to  $F_{B_q}^2$

$$F_{B_d} \sqrt{\hat{B}_d} = \begin{pmatrix} 235 \pm 33 & +0 \\ & -24 \end{pmatrix} \text{MeV} \quad F_{B_d} = (189 \pm 27) \text{MeV}$$

$$F_{B_s} \sqrt{\hat{B}_d} = (276 \pm 38) \text{MeV} \quad F_{B_s} = (230 \pm 30) \text{MeV}$$

$$\hat{B}_d = 1.34 \pm 0.12$$

$$\hat{B}_s = 1.34 \pm 0.12$$

$$\frac{\hat{B}_s}{\hat{B}_d} = 1.00 \pm 0.03$$

(No problems with  
chiral logs and  
quenching)

$$\text{Br}(B_{s,d} \rightarrow \mu\bar{\mu}) \text{ from } \Delta M_{s,d}$$

$$\text{Br}(B_q \rightarrow \mu\bar{\mu}) = 4.36 \cdot 10^{-10} \frac{\tau(B_q)}{\hat{B}_q} \frac{Y^2(x_t, \bar{x}_{\text{new}})}{S(x_t, x_{\text{new}})} \Delta M_q$$

No dependence  
on  $F_{B_q}^2$

SM:

$$\text{Br}(B_s \rightarrow \mu\bar{\mu}) = 3.42 \cdot 10^{-9} \left[ \frac{\tau(B_s)}{1.46 \text{ps}} \right] \left[ \frac{1.34}{\hat{B}_s} \right] \left[ \frac{\bar{m}_t(m_t)}{167 \text{ GeV}} \right]^{1.6} \left[ \frac{\Delta M_s}{18.0 / \text{ps}} \right]$$

$$\text{Br}(B_d \rightarrow \mu\bar{\mu}) = 1.00 \cdot 10^{-10} \left[ \frac{\tau(B_d)}{1.54 \text{ps}} \right] \left[ \frac{1.34}{\hat{B}_d} \right] \left[ \frac{\bar{m}_t(m_t)}{167 \text{ GeV}} \right]^{1.6} \left[ \frac{\Delta M_d}{0.50 / \text{ps}} \right]$$

(Example)

$$\Delta M_s = (18.0 \pm 0.5 / \text{ps})$$



$$\text{Br}(B_s \rightarrow \mu\bar{\mu}) = (3.42 \pm 0.54) \cdot 10^{-9}$$

$$\Delta M_d = (0.503 \pm 0.006 / \text{ps})$$



$$\text{Br}(B_d \rightarrow \mu\bar{\mu}) = (1.00 \pm 0.14) \cdot 10^{-10}$$

Moreover new Physics Effects can be easier seen





## Testing MFV through $B_{s,d} \rightarrow \mu\bar{\mu}$ and $\Delta M_{s,d}$

$$\frac{\text{Br}(B_s \rightarrow \mu\bar{\mu})}{\text{Br}(B_d \rightarrow \mu\bar{\mu})} = \frac{\hat{B}_d}{\hat{B}_s} \frac{\tau(B_s)}{\tau(B_d)} \frac{\Delta M_s}{\Delta M_d}$$

$(1.00 \pm 0.03)$  Experiment

Valid in MFV models in which only SM operators relevant.

Violation of this relation would indicate the presence of new operators and generally of non-minimal flavour violation.

# The Impact of Universal Extra Dimensions on FCNC Processes

*Based on:*

*AJB, M. Spranger, A. Weiler  $\equiv$  (BSW) (hep-ph/0212143)*

*AJB, A. Poschenrieder, M. Spranger, A. Weiler (hep-ph/0306158)*

# The Next Steps

1. ACD Model in  $D = 5$
2. Impact of KK on Inami-Lim Functions
3. Impact on:  
 $\Delta, \varepsilon_K, \Delta M_{d,s}, K \rightarrow \pi \nu \bar{\nu}$   
 $K_L \rightarrow \mu \bar{\mu}, B \rightarrow X_{d,s} \nu \bar{\nu}, B_{s,d} \rightarrow \mu \bar{\mu}$   
 $B \rightarrow X_s \gamma, B \rightarrow X_s \text{gluon}, \varepsilon'/\varepsilon$   
 $B \rightarrow X_s \mu \bar{\mu}, K_L \rightarrow \pi^0 e^+ e^-,$
4. Conclusions

# Introduction to the Model

Kaluza (1921) and Klein (1926)  
Unification of gravity and electrodynamics  
in  $D = 5$  compactified on  $S^1$ .

Some extra dimensional Models:

- brane world: SM on brane, gravity in the bulk, localization mechanism
- gravity and gauge bosons in bulk, fermions on brane  $R^{-1} > \text{few TeV}$ , localization mechanism
- Universal extra dimensions (UED): **everything** in the bulk, no localization mechanism required, gravity not considered



# ACD Model

Appelquist, Cheng, and Dobrescu (ACD)

hep-ph/0012100

- All SM fields live in the bulk  $D = 4 + 1$ , Gravity not considered.
- Orbifold: Replace  $S^1$  by  $S^1/Z_2$
- Simple extension of SM, 1 extra parameter ( $R$ , radius of ED), boundary terms set to zero
- provides excellent dark-matter candidate  
Servant, Tait '02; Cheng, Feng, Matchev '02
- bounds on  $1/R$  are rather weak  
 $1/R \gtrsim 250 \text{ GeV}, M_H > 250 \text{ GeV},$   
 $1/R \gtrsim 300 \text{ GeV}, M_H < 250 \text{ GeV}.$  Appelquist, Yee '02



# Appelquist, Cheng, Dobrescu Model (ACD)

(D = 5)

Universal Extra Dimensions:

All SM fields live in extra dimensions

Particle Content

in an Effective D = 4 Theory

SM Fields (n = 0)  
(Zero Modes)

+

Corresponding KK Models  
(n = 1, 2, ...)  $W_{(n)}^{\pm}$ ,  $Z_{(n)}^0$ , etc.

+

Additional Physical Scalar  
Modes  $a_{(n)}^0$ ,  $a_{(n)}^{\pm}$ ; n = 1, 2, ...

Single New Parameter:  
Compactification Scale  
1/R

$1/R \geq$  250 GeV ( $M_H > 250$  GeV)  
300 GeV ( $M_H < 250$  GeV)

(ACD, AY: Electroweak Precision Observables)

## Mass Spectrum

$$M_{\gamma^{(n)}}^2 = \frac{n^2}{R^2}$$

$$M_{Z^{(n)}}^2 = \frac{n^2}{R^2} + M_Z^2$$

$$M_{W^{(n)}}^2 = \frac{n^2}{R^2} + M_W^2$$

$$m_{q^{(n)}}^2 = \frac{n^2}{R^2} + m_q^2$$

$$m_{l^{(n)}}^2 = \frac{n^2}{R^2} + m_l^2$$

$$m_{a^\pm}^2 = \frac{n^2}{R^2} + M_W^2 \quad n \geq 1$$

$$m_{a^0}^2 = \frac{n^2}{R^2} + M_Z^2 \quad n \geq 1$$

( $n = 0, 1, 2 \dots$ )

## Interactions

1. Full Set of Feynman Rules in BSW
2. Vertices depend on  $n/R$
3. Conservation of KK Parity  $\Rightarrow$  Absence of tree level KK contributions
4. Minimal Flavour Violation (CKM Matrix; no new operators)

# Properties Relevant for FCNC Processes

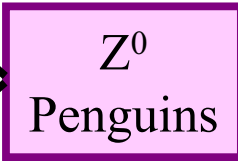
$$\boxed{\varepsilon_K, \Delta M_{d,s}} : S(x_t, 1/R) = S_0(x_t) + \sum_{n=1}^{\infty} S_n\left(x_t, \frac{n}{R}\right) \begin{pmatrix} \Delta F=2 \\ \text{Boxes} \end{pmatrix}$$

$$\left(x_t = \frac{m_t^2}{M_W^2}\right)$$

$$\begin{array}{l}
 \boxed{K^+ \rightarrow \pi^+ \nu \bar{\nu}} \\
 \boxed{K_L \rightarrow \pi^0 \nu \bar{\nu}} \\
 \boxed{B \rightarrow X_{s,d} \nu \bar{\nu}}
 \end{array}
 : X(x_t, 1/R) = X_0(x_t) + \sum_{n=1}^{\infty} C_n\left(x_t, \frac{n}{R}\right)$$

$$\underbrace{(C_0 - 4B_0)}_{\text{SM}}$$

$$\underbrace{(C_0 - B_0)}_{\text{SM}}$$

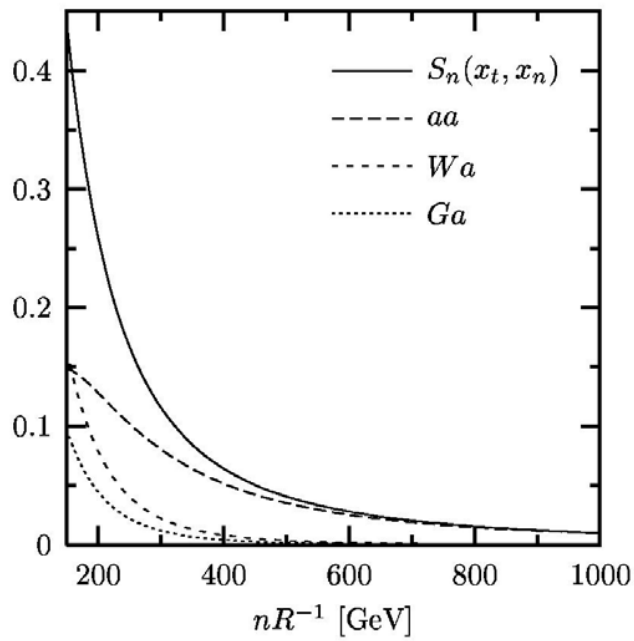
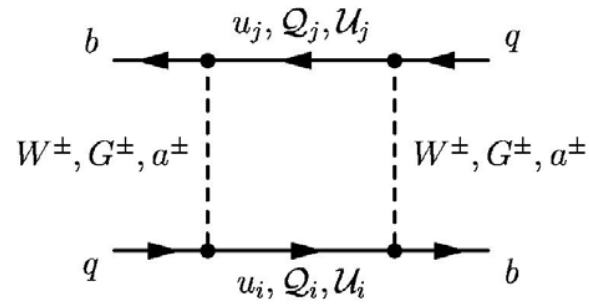
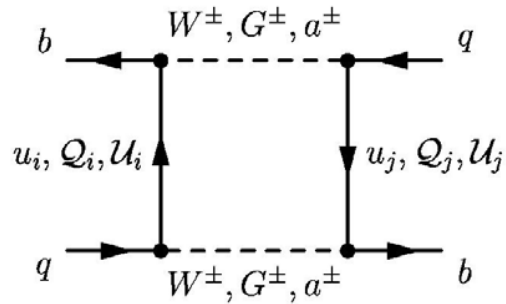


$$\begin{array}{l}
 \boxed{B_{s,d} \rightarrow \mu^+ \mu^-} \\
 \boxed{K_L \rightarrow \mu^+ \mu^-}
 \end{array}
 : Y(x_t, 1/R) = Y_0(x_t) + \sum_{n=1}^{\infty} C_n\left(x_t, \frac{n}{R}\right)$$

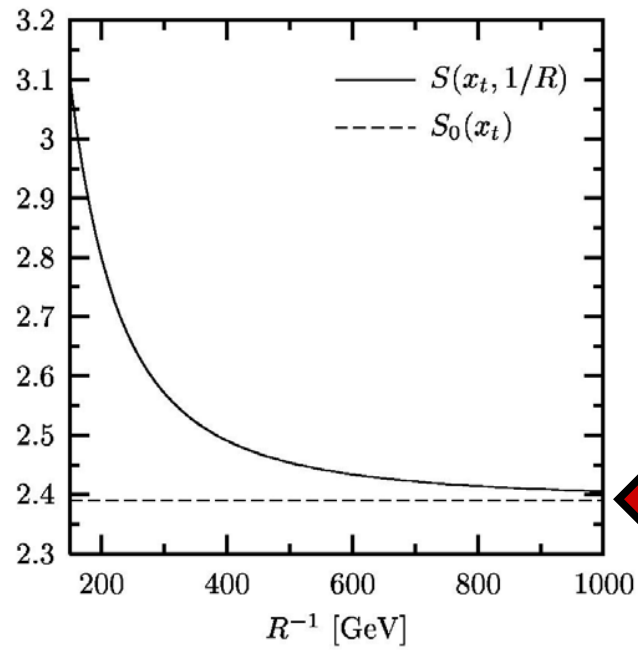
GIM mechanism improves significantly the convergence of the sum over the  $(KK)_t$  Modes and essentially removes the contributions of  $(KK)_{u,c}$  in the first two generations.



# Results for the Function $S(x_t, 1/R)$



(a)



**← SM**

(b)

# Basic Formulae for UT Analysis

1.

$\varepsilon_K$  - Hyperbola

$$\bar{\eta} \left[ (1 - \bar{\rho}) A^2 F_{tt} \eta_{\text{QCD}}^{tt} + P_c(\varepsilon) \right] A^2 \hat{B}_K = 0.213$$

$$\eta_{\text{QCD}}^{tt} = 0.57 \pm 0.01; \quad P_c(\varepsilon) = 0.28 \pm 0.05;$$

$$F_{tt}^{\text{SM}} = S_0(x_t)$$

$$F_{tt}^{\text{ACD}} = S(x_t, 1/R)$$

2.

$B_d^0 - \bar{B}_d^0$  Mixing Constraint

$$R_t = 0.86 \left[ \frac{0.041}{|V_{cb}|} \right] \sqrt{\frac{2.34}{F_{tt}}} \sqrt{\frac{\Delta M_d}{0.50/\text{ps}}} \left[ \frac{230\text{MeV}}{\sqrt{\hat{B}_d} F_{B_d}} \right] \sqrt{\frac{0.55}{\eta_B^{\text{QCD}}}}$$

$$\{ F_{tt}^{\text{ACD}} > F_{tt}^{\text{SM}} \}$$

$$\{ R_t^{\text{ACD}} < R_t^{\text{SM}} \}$$

$$|V_{cb}| = 0.041 \pm 0.001; \quad \Delta M_d = (0.503 \pm 0.006)/\text{ps}; \quad \eta_B^{\text{QCD}} = 0.55 \pm 0.01$$

3.

$B_s^0 - \bar{B}_s^0$  Mixing Constraint ( $\Delta M_d/\Delta M_s$ )

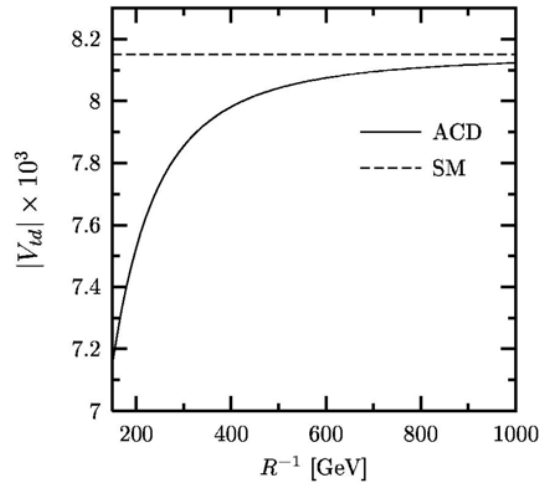
$$R_t = 0.90 \sqrt{\frac{\Delta M_d}{0.50/\text{ps}}} \sqrt{\frac{18.4/\text{ps}}{\Delta M_s}} \left[ \frac{\xi}{1.22} \right]$$

$$\xi = \frac{\sqrt{\hat{B}_s} F_{B_s}}{\sqrt{\hat{B}_d} F_{B_d}}$$

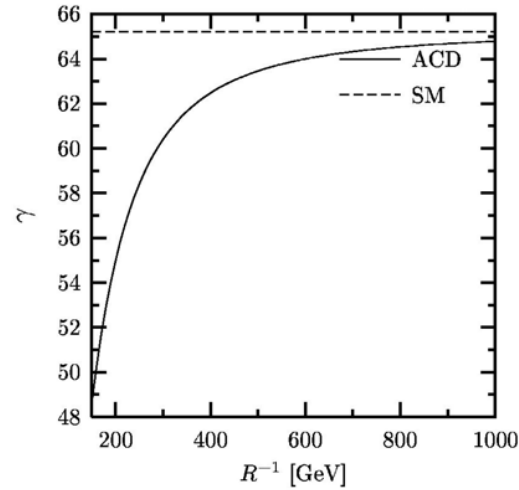
(No dependence on 1/R)

$$\Delta M_s > 14.4/\text{ps} \quad (95\% \text{ C.L.}) \quad \text{LEP (SLD)}$$

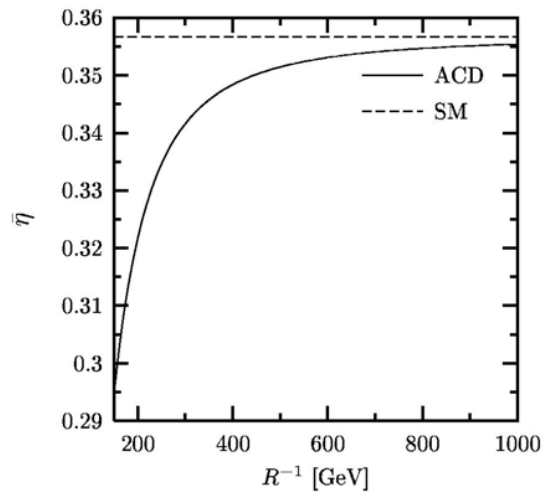
# Implications for Unitarity Triangle



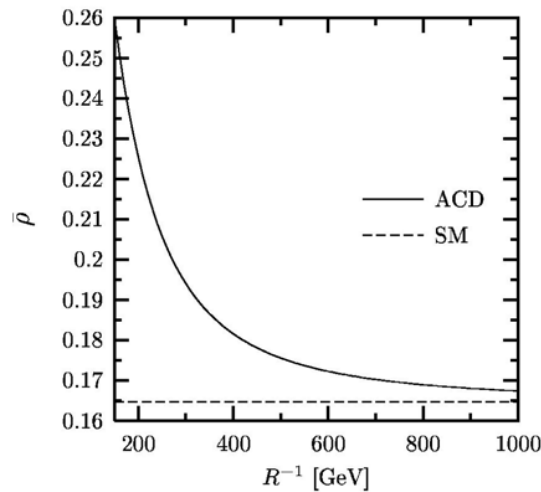
(a)



(b)



(c)



(d)

$1/R = 200$  (300) GeV

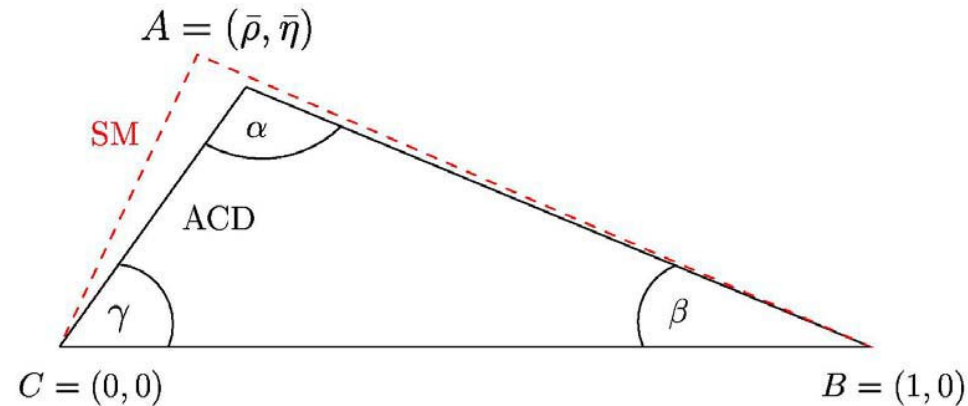


**Suppressions :**

|              |              |               |
|--------------|--------------|---------------|
| $ V_{td} $   | : 8%         | (4%)          |
| $\bar{\eta}$ | : 11%        | (4.5%)        |
| $\gamma$     | : $10^\circ$ | ( $5^\circ$ ) |

# Unitarity Triangle in the ACD Model

$$1/R = 200 \text{ GeV}$$

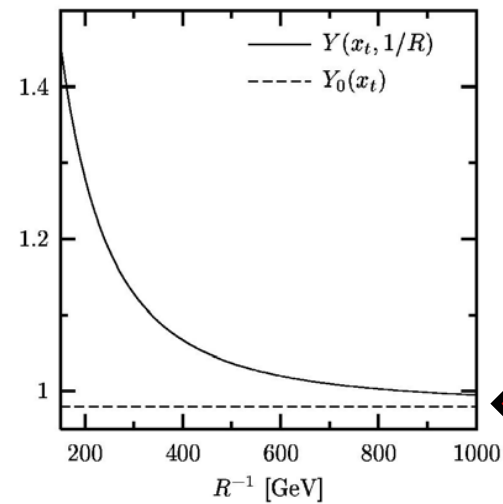
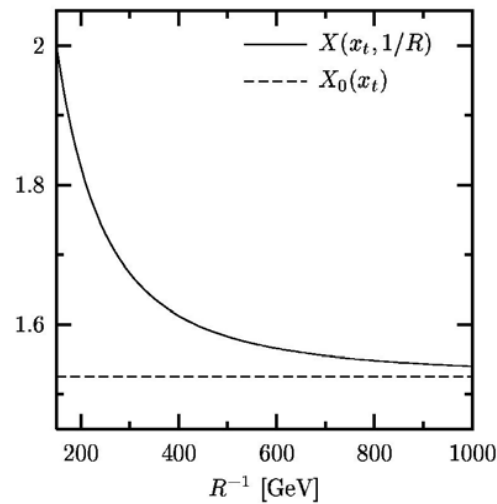
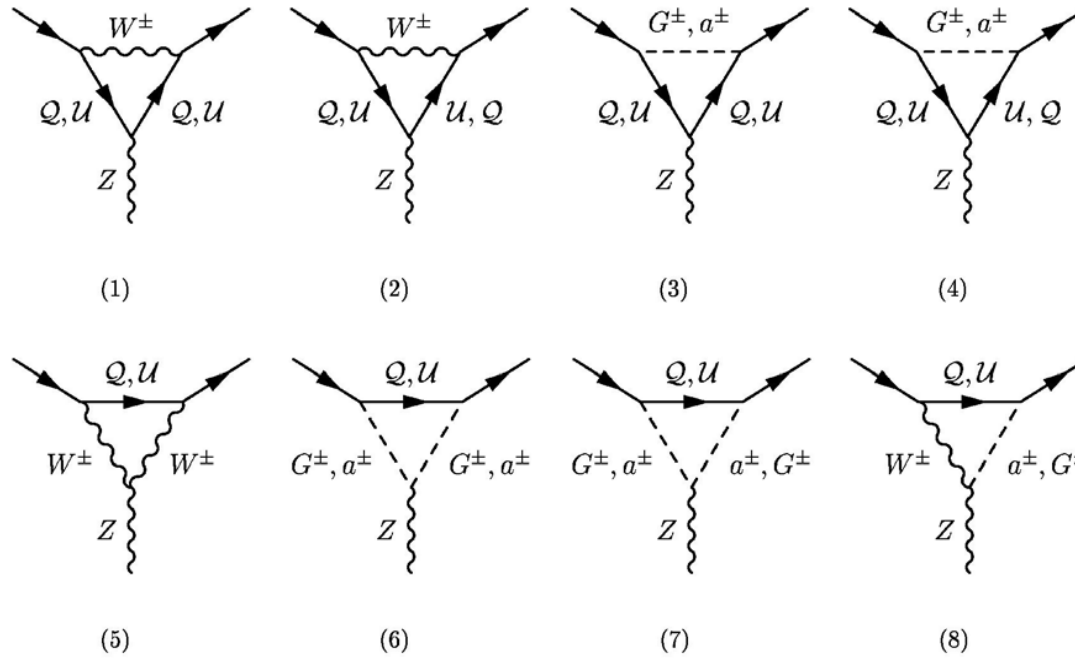


At  $1/R = 200 \text{ GeV}$   $\gamma_{\text{SM}} = 65^\circ \rightarrow \gamma_{\text{ACD}} = 49^\circ$

but at  $1/R = 300 (400) \text{ GeV}$   $\gamma_{\text{ACD}} = 60^\circ (63^\circ)$

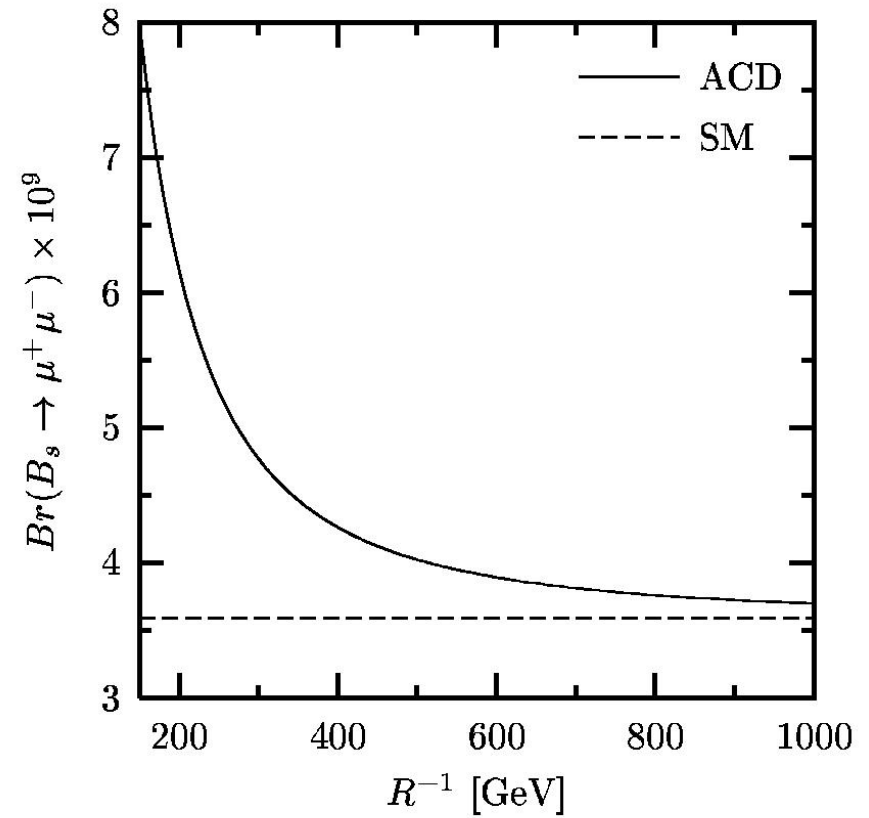
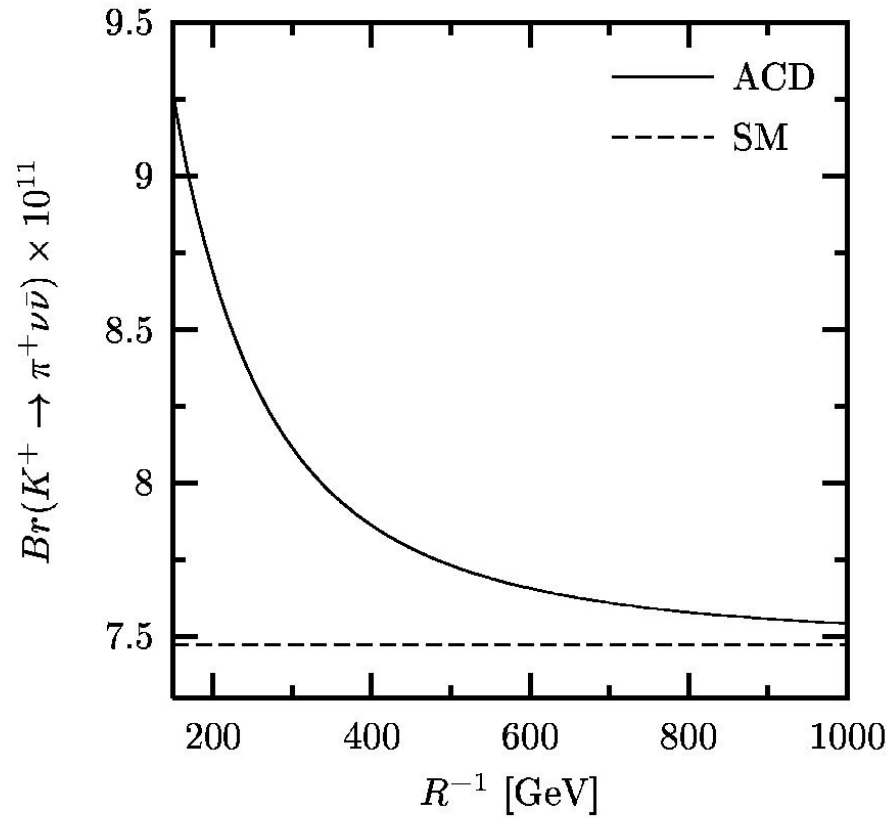
Very difficult to see the difference in view of hadronic uncertainties.

# Results for the Functions $X(x_t, 1/R)$ , $Y(x_t, 1/R)$



← SM

# Implications for Rare K and B Decays

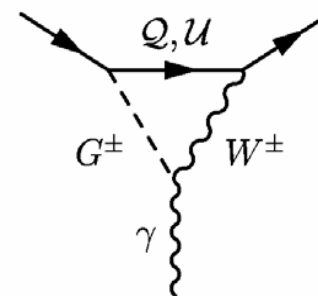
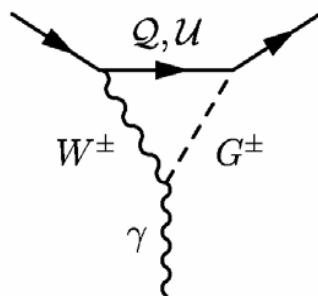
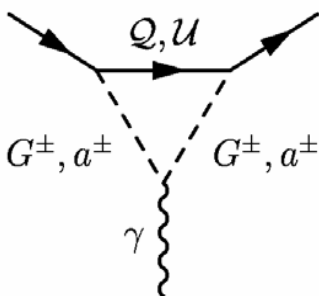
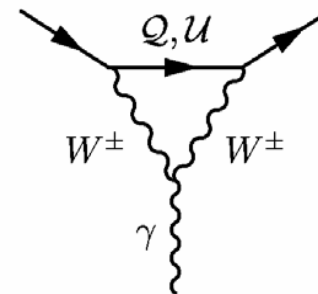
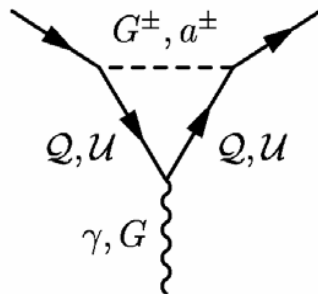
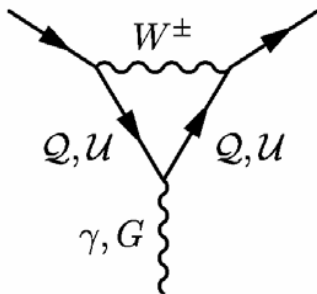


# The Impact of Universal Extra Dimensions on

$$B \rightarrow X_s \gamma, B \rightarrow X_s \mu^+ \mu^-, K_L \rightarrow \pi^0 e^+ e^-$$

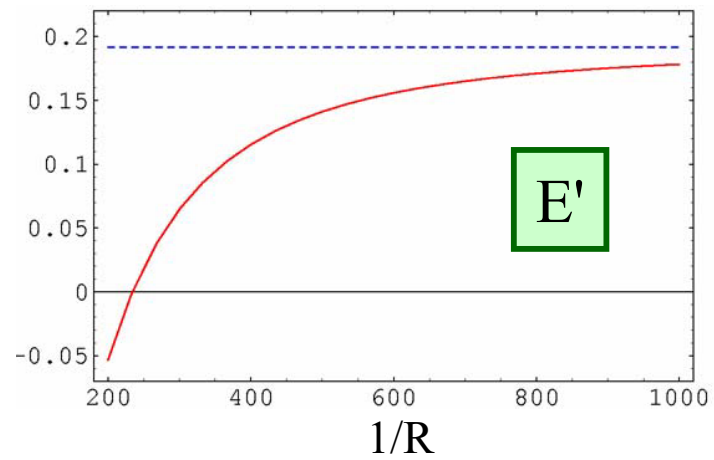
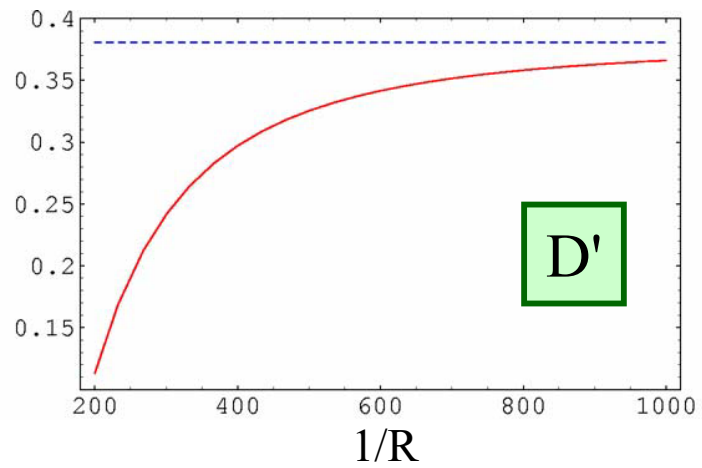
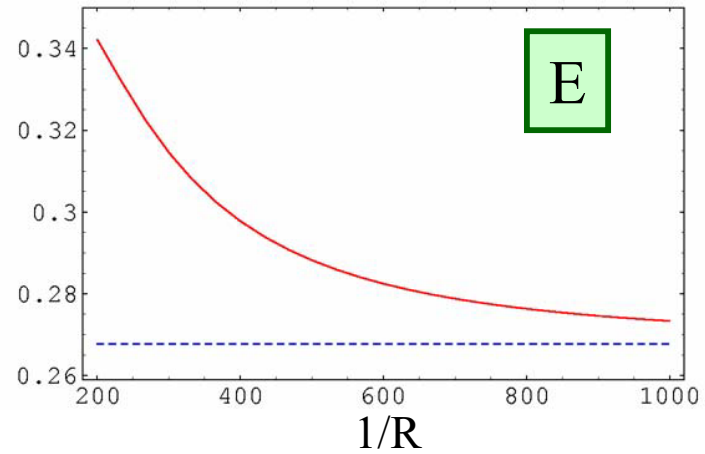
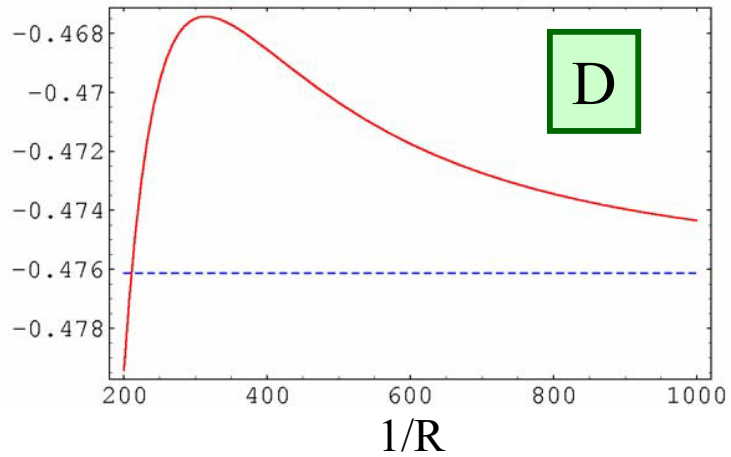
*Andrzej J. Buras, Anton Poschenrieder,  
Michael Spranger, Andreas Weiler*

# Diagrams Contributing to D, E, D', E'





# Results for D, E, D', E'



# Impact of KK on $B \rightarrow X_s \gamma$

CLEO  
ALEPH  
BaBar  
Belle

$$\text{Br}(B \rightarrow X_s \gamma) = \left( 3.28^{+0.41}_{-0.36} \right) \cdot 10^{-4}$$

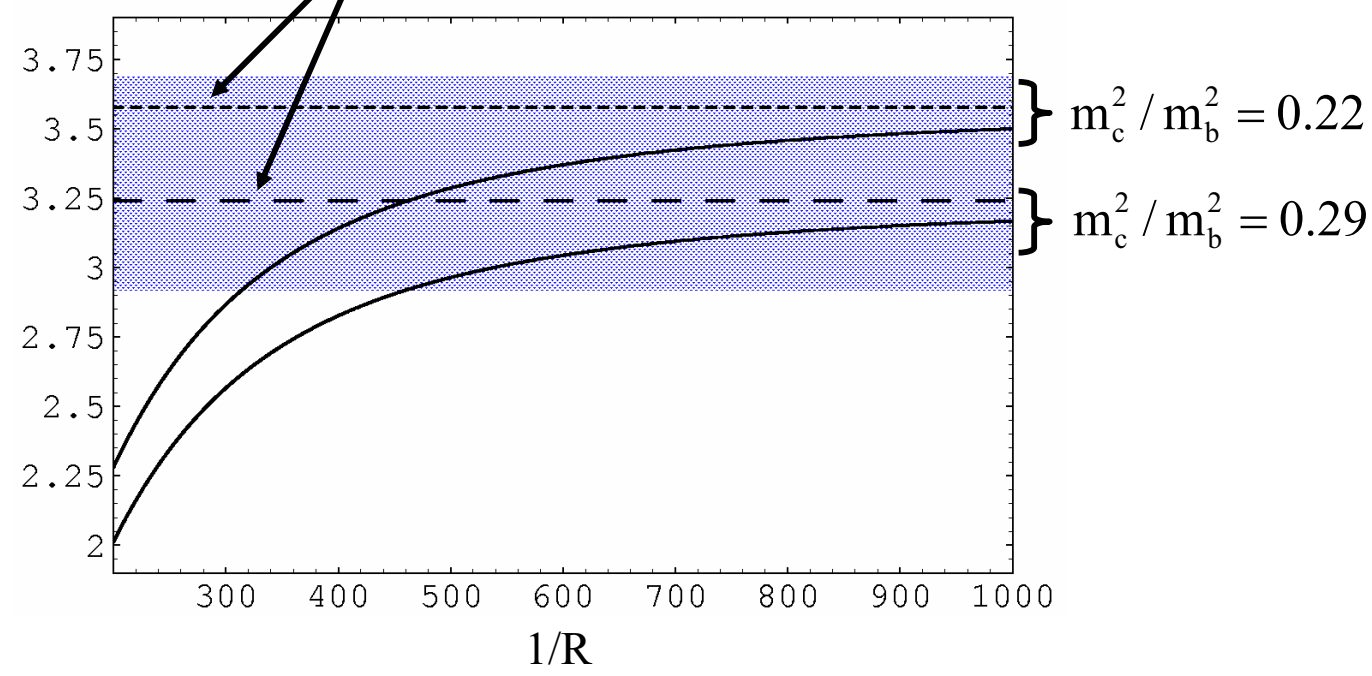
SM

$$(3.57 \pm 0.30) \cdot 10^{-4}$$

Gambino + Misiak  $\left( \frac{m_c^2}{m_b^2} = 0.22 \right)$

SM

$\text{Br}(B \rightarrow X_s \gamma)$   
[ $10^{-4}$ ]



# Impact of KK on $B \rightarrow X_s \mu \bar{\mu}$

{

Integration  
over  
full dilepton  
mass spectrum

}

$$\text{Br}(B \rightarrow X_s \mu \bar{\mu}) = \left( 7.9 \pm 2.1 \begin{matrix} +2.0 \\ -1.5 \end{matrix} \right) \cdot 10^{-6} \quad (\text{Belle})$$

SM:  $(4.1 \pm 0.7) \cdot 10^{-6}$  (Ali, Lunghi, Greub, Hiller)

ACD:  $(4.8 \pm 0.8) \cdot 10^{-6}$  (BPSW;  $1/R=300\text{GeV}$ )

$$\hat{s} = \frac{(P_{\mu^+} + P_{\mu^-})^2}{m_b^2}$$

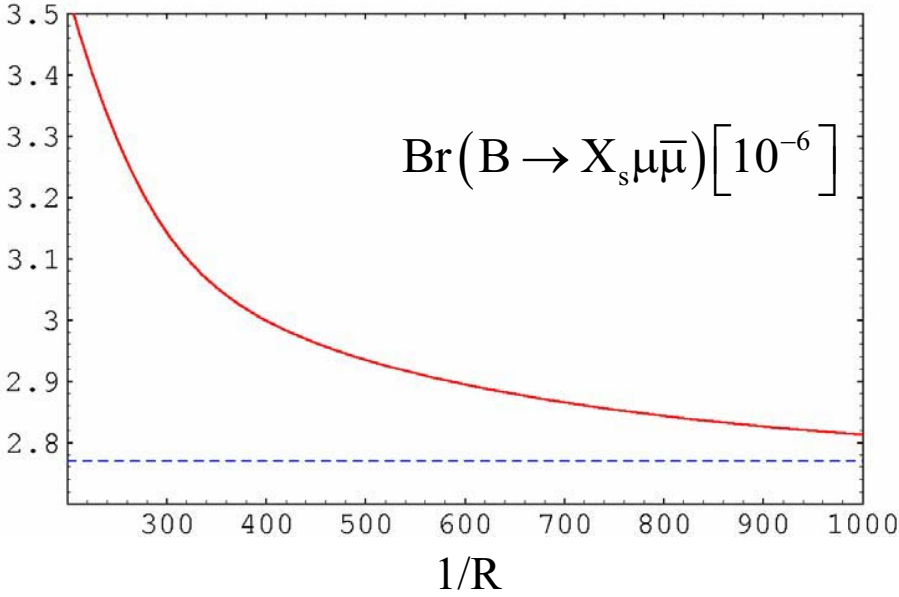
{

Integration  
over  
low dilepton  
mass spectrum

}

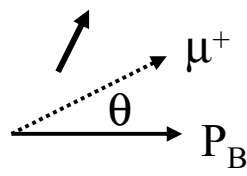
$$0.05 \lesssim \hat{s} \lesssim 0.25$$

SM:  $(2.75 \pm 0.45) \cdot 10^{-6}$



# Forward-Backward Asymmetry in $B \rightarrow X_s \mu^+ \mu^-$ (SM)

$$A_{\text{FB}}(\hat{s}) = \frac{1}{\Gamma(b \rightarrow c e \bar{\nu})} \int_{-1}^{+1} d \cos \theta_L \frac{d^2 \Gamma(b \rightarrow s \mu^+ \mu^-)}{d \hat{s} d \cos \theta_L} \text{sgn}(\cos \theta_L)$$

$$A_{\text{FB}}(\hat{s}) = -3 \tilde{C}_{10} \frac{\left[ \hat{s} \text{Re} \tilde{C}_9^{\text{eff}}(\hat{s}) + 2 C_{7\gamma}^{(0)\text{eff}} \right]}{U(\hat{s})}$$


$$\hat{s}_0 \cong - \frac{2 C_{7\gamma}^{(0)\text{eff}}}{\text{Re} \tilde{C}_9^{\text{eff}}(\hat{s}_0)}$$

$$C_9 \leftrightarrow (\bar{s}b)_{V-A} (\bar{\mu}\mu)_V$$

$$C_{10} \leftrightarrow (\bar{s}b)_{V-A} (\bar{\mu}\mu)_A$$

$$C_{7\gamma} \leftrightarrow B \rightarrow X_s \gamma$$

**SM** :

NLO:  $\hat{s}_0 \cong 0.14 \pm 0.02$  **Ali Mannel Morozumi**

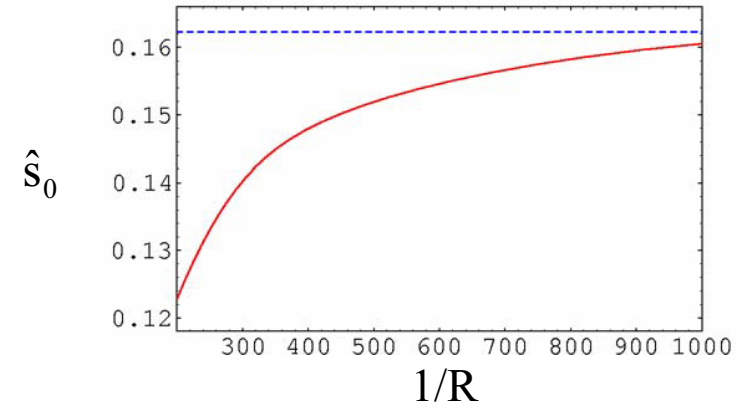
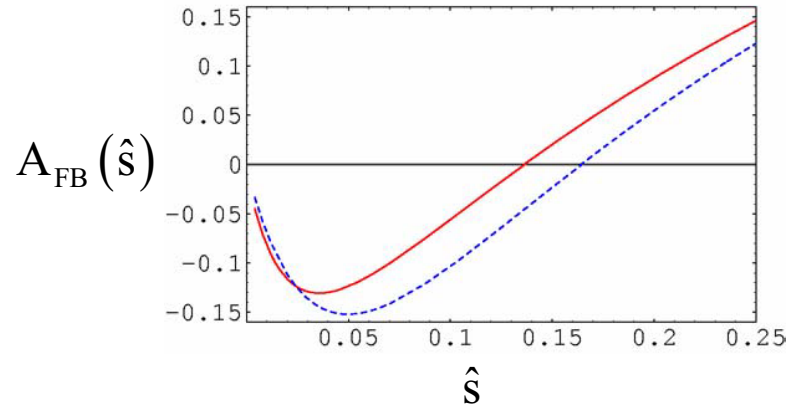
NNLO:  $\hat{s}_0 \cong 0.162 \pm 0.008$


**Asatrian, Asatrian, Greub, Walker, Bieri Hovhannisyan**

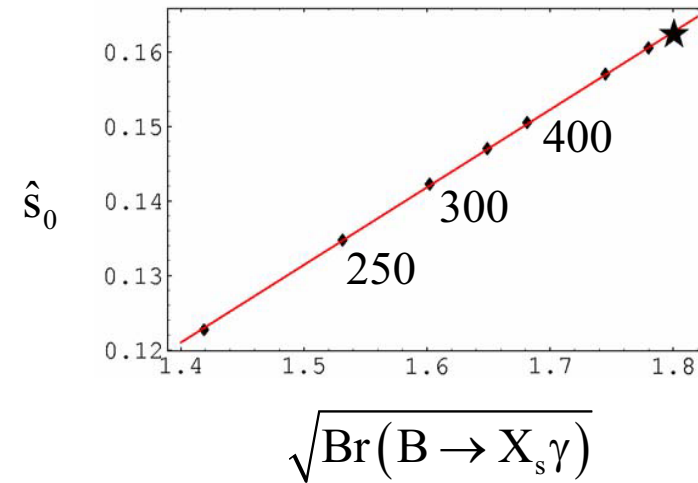
**Ghinculov, Hurth, Isidori, Yao**

# Impact of KK on $A_{\text{FB}}(\hat{s})$

--- SM  
— ACD



$\left\{ \begin{array}{l} \tilde{C}_9^{\text{eff}} \text{ very} \\ \text{weakly affected} \end{array} \right\}$   
  
 $\hat{s}_0 \sim \sqrt{\text{Br}(B \rightarrow X_s \gamma)}$



# Summary

**1.** ACD Model consistent with the data on FCNC Processes with  $1/R \cong 300 \text{ GeV}$

**2.** Only small impact on UT relative to the SM

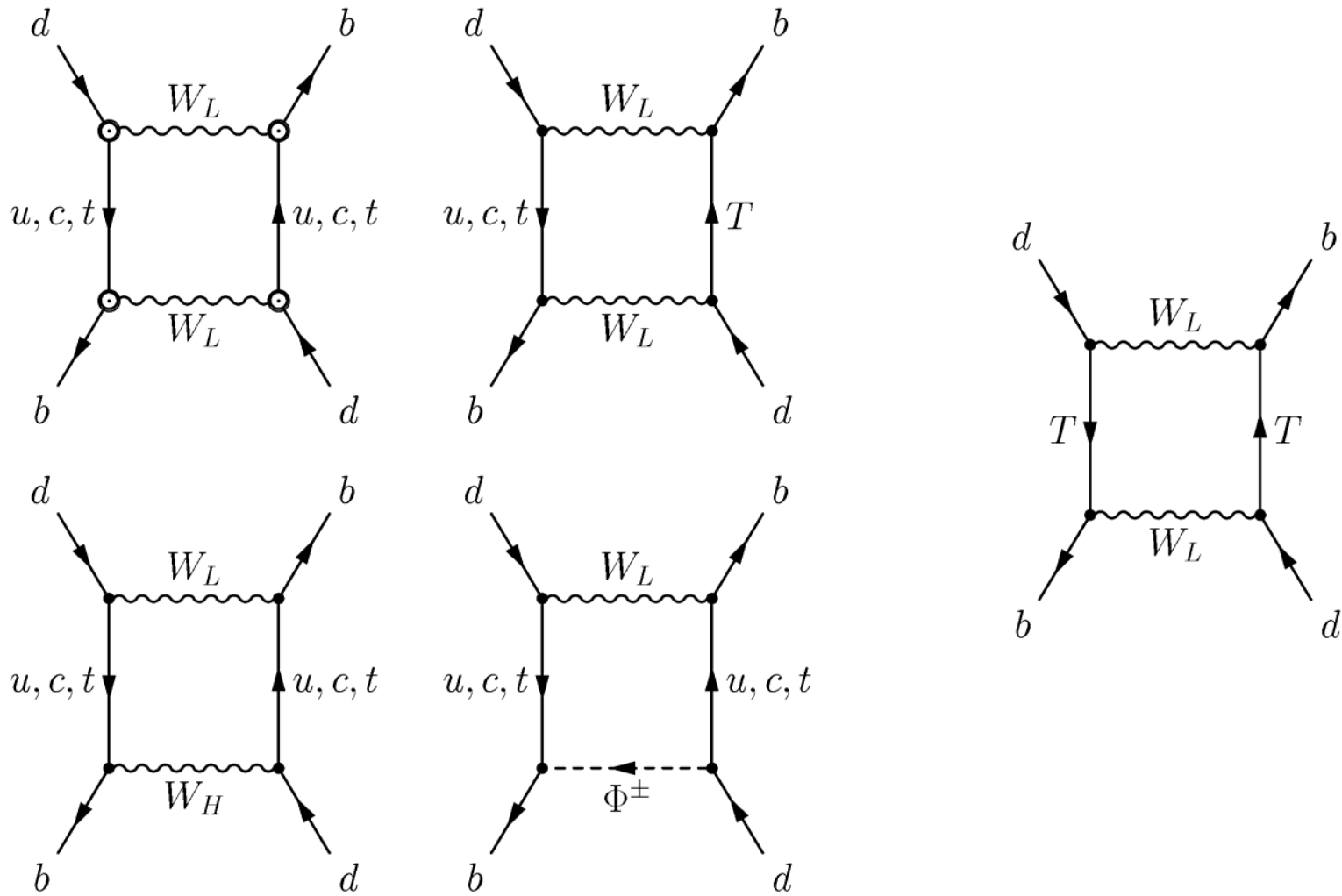
**3.** Enhancements :  $K^+ \rightarrow \pi^+ \nu \bar{\nu} (9\%)$ ,  $K_L \rightarrow \pi^0 \nu \bar{\nu} (10\%)$   
 $1/R = 300 \text{ GeV}$   $B \rightarrow X_s \mu \bar{\mu} (12\%)$ ,  $B \rightarrow X_d \nu \bar{\nu} (12\%)$   
 $B \rightarrow X_s \nu \bar{\nu} (21\%)$ ,  $K_L \rightarrow \mu^+ \mu^- (20\%)$   
 $B_d \rightarrow \mu^+ \mu^- (23\%)$ ,  $B_s \rightarrow \mu^+ \mu^- (33\%)$

**4.**Suppressions :  $B \rightarrow X_s \gamma (20\%)$ ,  $B \rightarrow X_s \text{gluon} (40\%)$   
 $\hat{S}_0 : 0.162 \rightarrow 0.142$  ;  $\epsilon'/\epsilon$

**5.** With improved Exp+Th for  $B \rightarrow X_s \gamma$  and  $\hat{S}_0$  strong lower bound on  $1/R$  could be obtained.

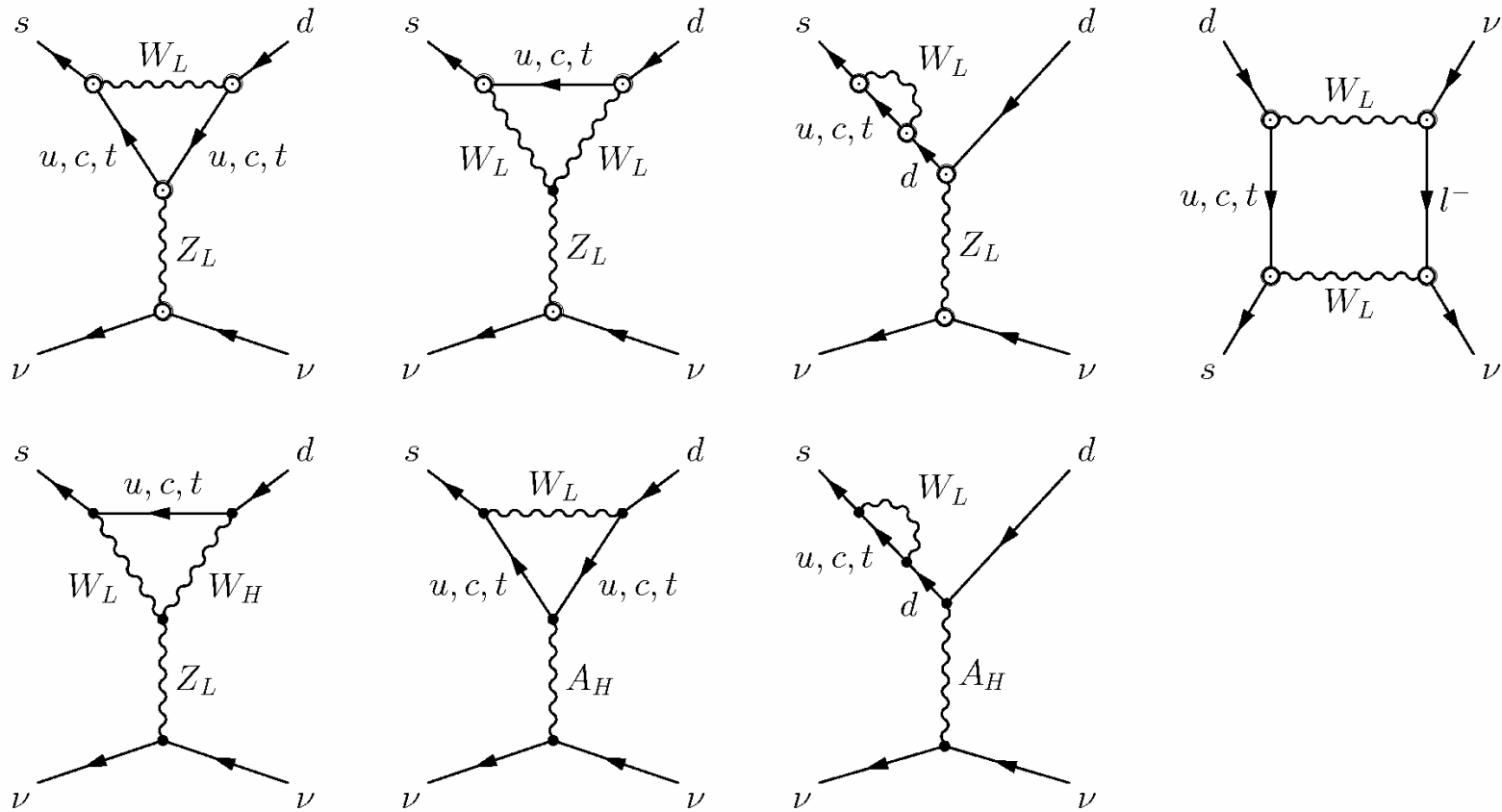


# Movement 1

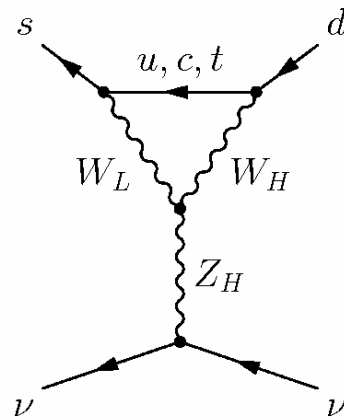
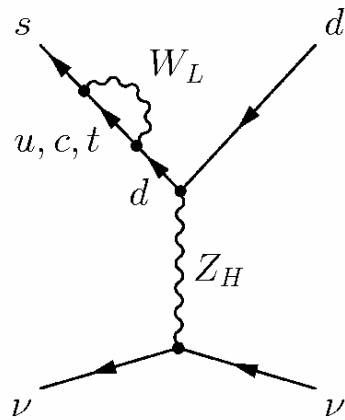
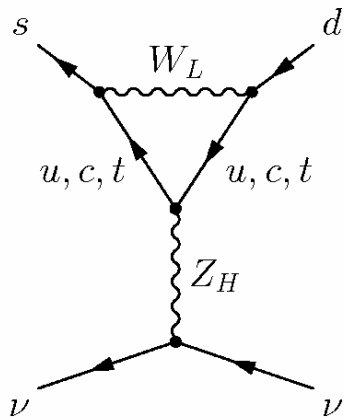
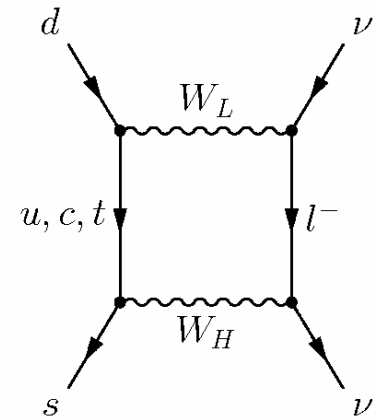
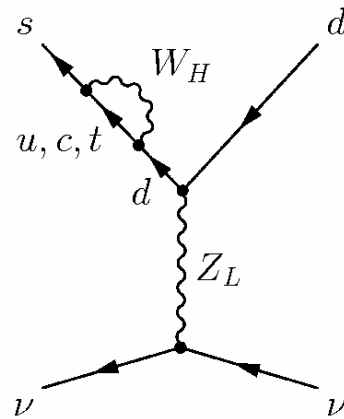
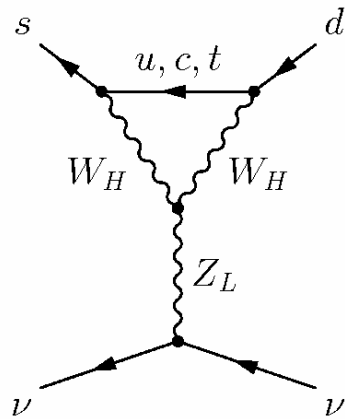
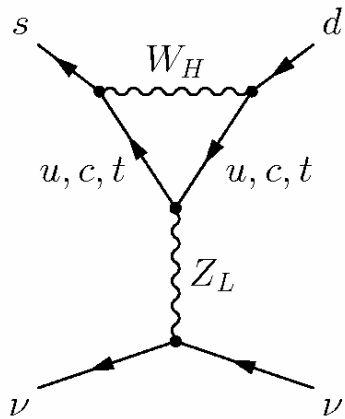




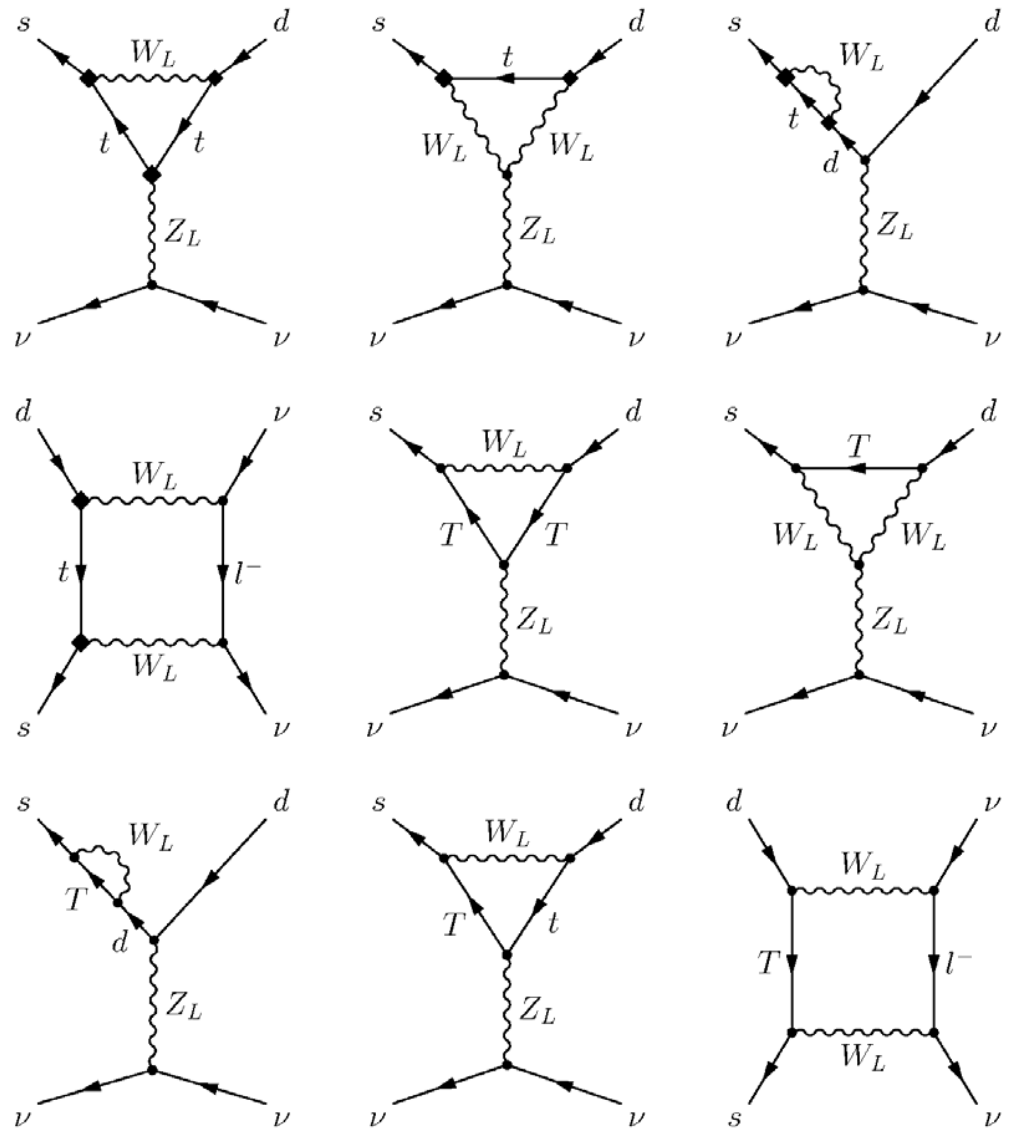
# Movement 2



# Movement 3



# Movement 4



# FCNC Processes in MSSM (MFV)

**Classic Paper** : Bertolini, Borzumati, Masiero, Ridolfi (1991)

**Last Analysis** : AJB, Gambino, Gorbahn, Jäger, Silvestrini (2000)

$$T(Q) \equiv \frac{Q_{\text{MSSM}}}{Q_{\text{SM}}}$$

$$0.65 \leq T(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \leq 1$$

$$0.41 \leq T(K_L \rightarrow \pi^0 \nu \bar{\nu}) \leq 1$$

Governed by the modification  
of  $X(\nu)$  and  $V_{td} \downarrow$   
enhanced or  
suppressed

$$0.73 \leq T(B \rightarrow X_s \nu \bar{\nu}) \leq 1.34$$

$$0.68 \leq T(B_s \rightarrow \mu \bar{\mu}) \leq 1.53$$

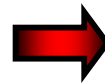
Governed by the modification  
of the functions  $X(\nu), Y(\nu)$   
 $V_{ts}$  not  
modified  
enhanced or  
suppressed

# MFVfit Collaboration

(BBBEPSW)  
hep-ph/0505110

Use the existing results for

1. UUTfit
2.  $B \rightarrow X_s \gamma$
3.  $B \rightarrow X_s l^+ l^-$
4.  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$



$$X_{\max}(\nu) = 1.95 \quad Y_{\max}(\nu) = 1.43$$
$$(X_{\text{SM}} \cong 1.53) \quad (Y_{\text{SM}} \cong 0.99)$$



Model Independent  
Upper Bounds  
within MFV Scenario

Conclusion :

Large Departures from  
SM within MFV not  
possible

## Upper Bounds on Rare K and B Decays from MFV

Bobeth, Bona, AJB, Ewerth, Pierini, Silvestrini, Weiler hep-ph/0505110

| Branching Ratios                                               | MFV<br>(95%) | SM<br>(95%) | SM<br>(68%)   | Exp                   |
|----------------------------------------------------------------|--------------|-------------|---------------|-----------------------|
| $\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \cdot 10^{11}$ | $<11.9$      | $<10.9$     | $8.3 \pm 1.2$ | $14.7^{+13.0}_{-8.9}$ |
| $\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu}) \cdot 10^{11}$ | $<4.6$       | $<4.2$      | $3.1 \pm 0.6$ | $<5.9 \cdot 10^4$     |
| $\text{Br}(B \rightarrow X_S \nu \bar{\nu}) \cdot 10^5$        | $<5.2$       | $<4.1$      | $3.7 \pm 0.2$ | $<64$                 |
| $\text{Br}(B_s \rightarrow \mu^+ \mu^-) \cdot 10^9$            | $<7.4$       | $<5.9$      | $3.7 \pm 1.0$ | $<5.0 \cdot 10^2$     |
| $\text{Br}(B_d \rightarrow \mu^+ \mu^-) \cdot 10^{10}$         | $<2.2$       | $<1.8$      | $1.1 \pm 0.4$ | $<1.6 \cdot 10^3$     |

# 8.

## Going beyond MFV

1. MSSM at large  $\tan\beta$
2. New Complex Phase in  $Z^0$  Penguins

## Three Simple Scenarios

Inami  
Lim Functions

**SM** : 
$$A(\text{Decay}) = \sum_i B_i \eta_{\text{QCD}}^i V_{\text{CKM}}^i \underbrace{F_{\text{SM}}^i(m_t)}_{\text{real}}$$

**MFV** : 
$$A(\text{Decay}) = \sum_i B_i \eta_{\text{QCD}}^i V_{\text{CKM}}^i \underbrace{\left[ F_{\text{SM}}^i + F_{\text{New}}^i \right]}_{\text{real}}$$

(Minimal Flavour Violation)

AJB, Gambino, Gorbahn, Jäger, Silvestrini  
D'Ambrosio, Giudice, Isidori, Strumia

**Enhanced Z<sup>0</sup>-Penguins** : 
$$A(\text{Decay}) = \sum_i B_i \eta_{\text{QCD}}^i V_{\text{CKM}}^i \left[ \underbrace{F_{\text{SM}}^i}_{\text{real}} + \underbrace{\Delta_{\text{New}}^i}_{\text{real}} \right]$$

AJB, Colangelo, Isidori, Romanino, Silvestrini  
Buchalla, Hiller, Isidori; Atwood, Hiller  
AJB, Fleischer, Recksiegel, Schwab

**Dominated by  
Z<sup>0</sup>-Penguins  
with a New  
Complex Phase**



## Two more complicated Scenarios

**MSSM (MFV)  
(large  $\tan\beta$ )**

(Higgs penguin)

$$\begin{aligned}
 A(\text{Decay}) = & \sum_i B_i \eta_{\text{QCD}}^i V_{\text{CKM}}^i \left[ \overbrace{F_{\text{SM}}^i + F_{\text{New}}^i}^{\text{real}} \right] \\
 & + \sum_i B_i^{\text{New}} \left[ \eta_{\text{QCD}}^i \right]^{\text{New}} V_{\text{CKM}}^i \underbrace{\left[ G_{\text{New}}^i \right]}_{\text{real}}
 \end{aligned}$$

**General  
MSSM**

$$\begin{aligned}
 A(\text{Decay}) = & \sum_i B_i \eta_{\text{QCD}}^i V_{\text{CKM}}^i \left[ \overbrace{F_{\text{SM}}^i + F_{\text{New}}^i}^{\text{complex}} \right] \\
 & + \sum_i B_i^{\text{New}} \left[ \eta_{\text{QCD}}^i \right]^{\text{New}} V_{\text{New}}^i \underbrace{\left[ G_{\text{New}}^i \right]}_{\text{complex}}
 \end{aligned}$$

**Z'-Models  
L-R Models  
Multi-Higgs  
Models**

# The Case of large $\tan\beta$ in MSSM

$h_i =$  Yukawa couplings

**SM** :  $\left\{ \begin{matrix} m_b = h_b v \\ m_t = h_t v \end{matrix} \right\} \Rightarrow \{h_b \ll h_t\} \Rightarrow$  Couplings of b, s, d,  $\mu$ ,  $\tau$ , e to Higgs particles can be neglected

**MSSM** :  $\left\{ \begin{matrix} m_b = h_b v_D \\ m_t = h_t v_U \end{matrix} \right\} \Rightarrow \left\{ \begin{matrix} h_b \approx h_t \\ \text{if} \\ \tan\beta = \frac{v_U}{v_D} \gg 1 \end{matrix} \right\} \Rightarrow$  Couplings of b, s,  $\tau$ ,  $\mu$  to Higgs particles **cannot** be neglected

$v = \sqrt{v_U^2 + v_D^2}$

- Implications** :
- 1.** Enhancement Factors in Higgs-Fermion Vertices  $\sim \tan\beta \cdot m_b$
  - 2.** Higgs Penguins cannot be neglected
  - 3.** New relevant Vertices imply new Operators

# MSSM with MFV but large $\tan\beta$

$$\tan\beta = \frac{v_2}{v_1}$$

$$\begin{aligned}
 A(\text{Decay}) = & \sum_i B_i \eta_{\text{QCD}}^i V_{\text{CKM}}^i \left[ \overbrace{F_{\text{SM}}^i}^{\text{real}} + \overbrace{F_{\text{New}}^i}^{\text{real}} \right] \\
 & + \sum_i B_i^{\text{New}} \left[ \eta_{\text{QCD}}^i \right]^{\text{New}} V_{\text{CKM}}^i \underbrace{\left[ G_{\text{New}}^i \right]}_{\text{real}}
 \end{aligned}$$

$B_s \rightarrow \mu^+ \mu^-$

$\Delta M_s$

$$O_A = [\bar{b}\gamma_\mu(1-\gamma_5)s][\bar{\mu}\gamma^\mu\gamma_5\mu] \quad \text{SM Operators}$$

$$\begin{aligned}
 O_S &= m_b [\bar{b}(1-\gamma_5)s][\bar{\mu}\mu] \\
 O_P &= m_b [\bar{b}(1-\gamma_5)s][\bar{\mu}\gamma_5\mu]
 \end{aligned}$$

New Operators

$$\begin{aligned}
 Q^{\text{VLL}} &= [\bar{b}\gamma_\mu(1-\gamma_5)s][\bar{\mu}\gamma^\mu(1-\gamma_5)s] \\
 Q_1^{\text{LR}} &= [\bar{b}\gamma_\mu(1-\gamma_5)s][\bar{\mu}\gamma^\mu(1+\gamma_5)s] \\
 Q_2^{\text{LR}} &= [\bar{b}(1-\gamma_5)s][\bar{\mu}(1+\gamma_5)s] \\
 Q_1^{\text{SLL}} &= [\bar{b}(1-\gamma_5)s][\bar{\mu}(1-\gamma_5)s] \\
 Q_2^{\text{SLL}} &= [\bar{b}\sigma_{\mu\nu}(1-\gamma_5)s][\bar{\mu}\sigma_{\mu\nu}(1-\gamma_5)s]
 \end{aligned}$$

*Correlation between  $\Delta M_s$  and  $B_{s,d}^0 \rightarrow \mu^+ \mu^-$   
in Supersymmetry at Large  $\tan\beta$*

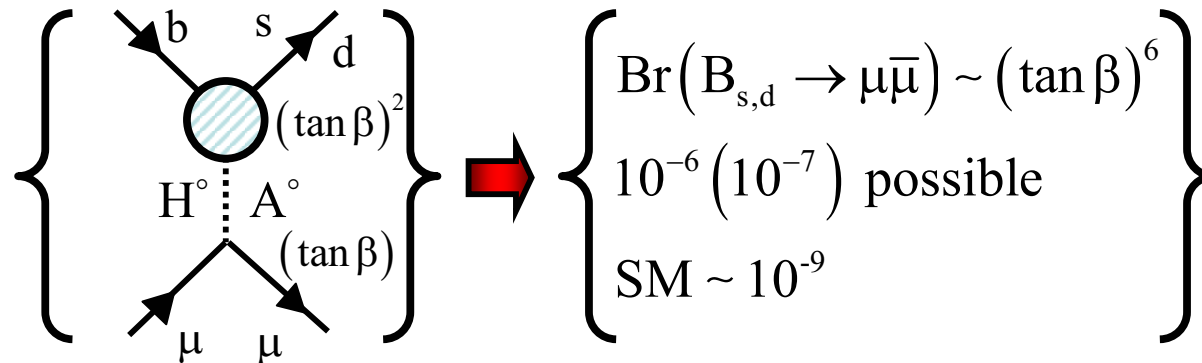
Based on: *AJB, Chankowski, Rosiek, Slawianowska*  
*(hep-ph/0207241)*  
*(hep-ph/0210121)*

# MSSM at large $\tan\beta$

$$(B_{s,d} \rightarrow \mu^+ \mu^-)$$

In MSSM at large  $\tan\beta$   
 (CKM still the only source of Flavour and CP Violation)

Strong Enhancement



Babu, Kolda  
 Chankowski, Slawianowska  
 Bobeth, Ewerth, Krüger, Urban  
 Huang, Liao, Yan, Zhu  
 Isidori, Retico  
 Dedes, Dreiner, Nierste  
 Dedes, Pilaftis  
 Chankowski, Rosiek  
 Foster, Okumura, Roszkowski

$$\text{Br}(B_s \rightarrow \mu\bar{\mu}) < \begin{matrix} 5.0 \cdot 10^{-7} (D\Phi) \\ 7.5 \cdot 10^{-7} (CDF) \end{matrix} \quad 95\% \text{ C.L.}$$

$$\text{Br}(B_d \rightarrow \mu\bar{\mu}) < \begin{matrix} 1.9 \cdot 10^{-7} (CDF) \\ 8.3 \cdot 10^{-8} (\text{BaBar}) \end{matrix} \quad \begin{matrix} 95\% \text{ C.L.} \\ 90\% \text{ C.L.} \end{matrix}$$

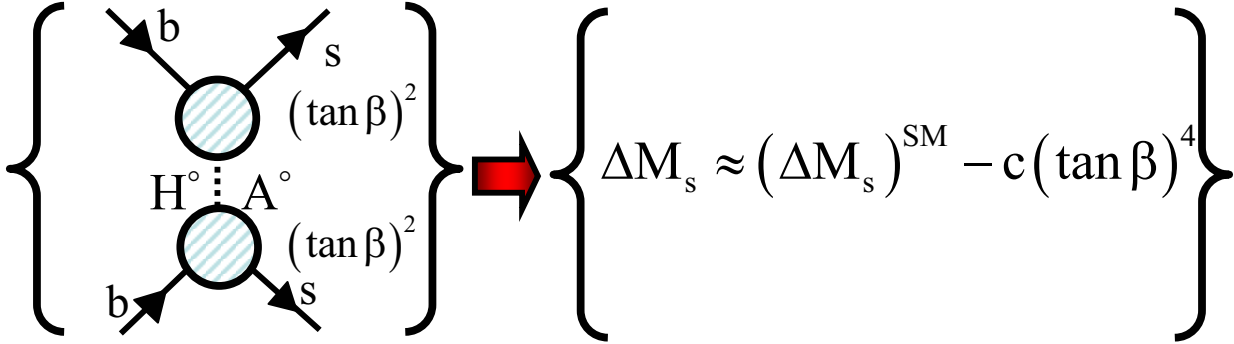
# MSSM at large $\tan\beta$ (cont.)

(Double-Higgs Penguin)

$(\Delta M_s)$

$B_s^0 - \bar{B}_s^0$  Mixing

Suppression



AJB, Chankowski, Rosiek  
Slawianowska (2001, 2002)

Correlation between  
SUSY effects in  
 $Br(B_{s,d} \rightarrow \mu\bar{\mu})$  and  $\Delta M_s$

Negligible contributions to  $\Delta M_d, \epsilon_K$

Clear violation  
of  $K \leftrightarrow B$  MFV  
relations

Subsequent analyses: D'Ambrosio, Giudice, Isidori, Stumia  
Dedes, Pilaftis

$$\left\{ \begin{array}{l} \text{Double} \\ \text{Higgs - Penguin} \end{array} \right\} \Rightarrow \left\{ \Delta M_S^{\text{DP}} \sim -(\tan \beta)^4 \frac{F_{B_s}^2}{M_A^2} \frac{\varepsilon_Y^2}{(1 + \tilde{\varepsilon}_3 \tan \beta)^2 (1 + \varepsilon_0 \tan \beta)^2} \right\}$$

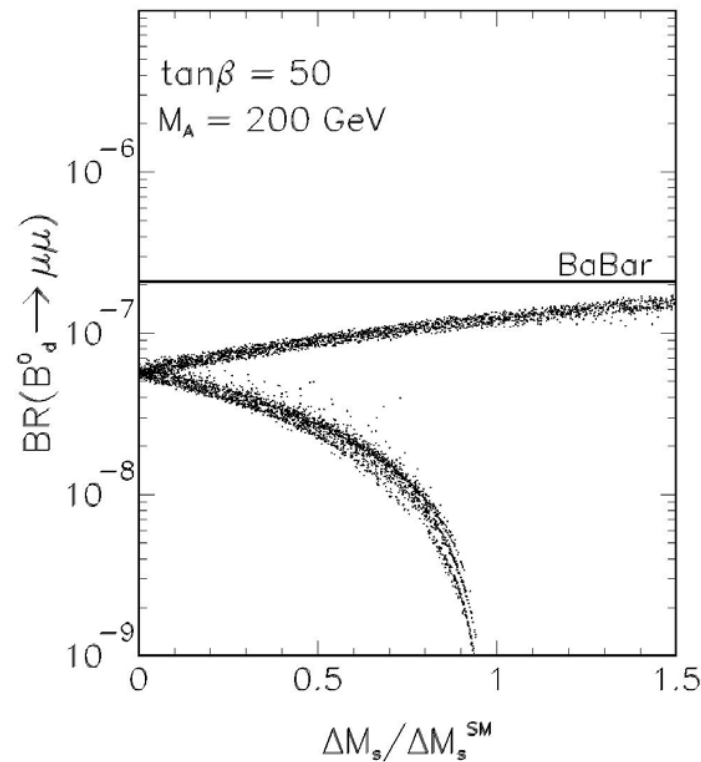
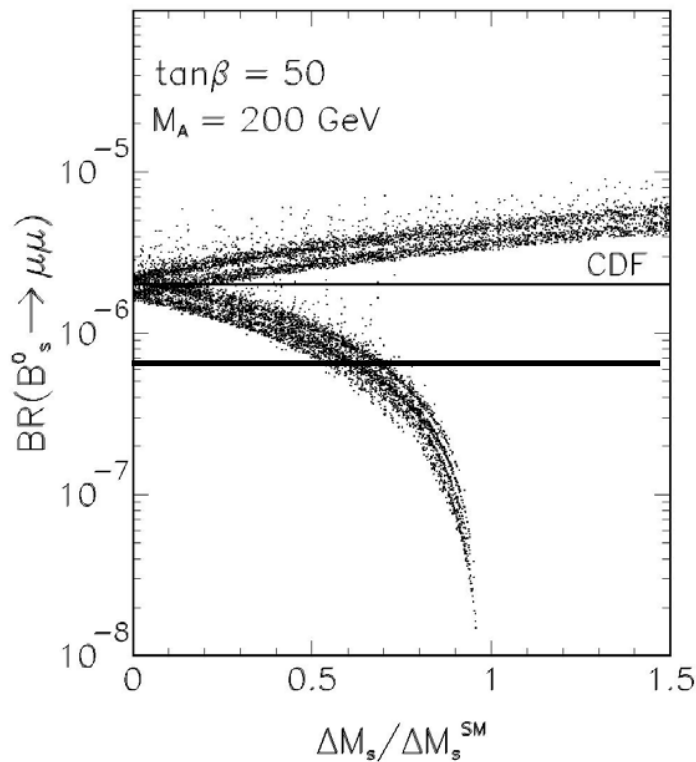
$$\left\{ [\text{Higgs - Penguin}]^2 \right\} \Rightarrow \left\{ \text{Br}(B_s \rightarrow \mu \bar{\mu}) \sim (\tan \beta)^6 \frac{F_{B_s}^2}{M_A^4} \frac{\varepsilon_Y^2}{(1 + \tilde{\varepsilon}_3 \tan \beta)^2 (1 + \varepsilon_0 \tan \beta)^2} \right\}$$

$\varepsilon_Y, \varepsilon_0, \bar{\varepsilon}_3$  - Functions of SUSY parameters

$$\frac{\text{Br}(B_d^0 \rightarrow \mu^+ \mu^-)}{\text{Br}(B_s^0 \rightarrow \mu^+ \mu^-)} \cong \left[ \frac{F_{B_d}}{F_{B_s}} \right]^2 \left| \frac{V_{td}}{V_{ts}} \right|^2 \left| \frac{M_{B_d}}{M_{B_s}} \right|^5$$

# $\text{Br}(B_{s,d} \rightarrow \mu^+ \mu^-)$ vs $(\Delta M_s)^{\text{exp}} / (\Delta M_s)^{\text{SM}}$ in SUSY at Large $\tan \beta$

AJB, Chankowski, Rosiek, Slawianowska, hep-ph/0207241



2002

2002  
2004



# Numerical Results

$$0.8 \leq \frac{(\Delta M_s)^{\text{exp}}}{(\Delta M_s)^{\text{SM}}} \leq 0.95$$



$$6 \cdot 10^{-7} \geq \text{Br}^{\text{max}}(B_s \rightarrow \mu^+ \mu^-) \geq 3 \cdot 10^{-8}$$
$$1.8 \cdot 10^{-8} \geq \text{Br}^{\text{max}}(B_d \rightarrow \mu^+ \mu^-) \geq 1 \cdot 10^{-9}$$

$$(\Delta M_s)^{\text{exp}} > 15 / \text{ps}$$



$$\text{Br}(B_s \rightarrow \mu^+ \mu^-) < 8 \cdot 10^{-7}$$
$$\text{Br}(B_d \rightarrow \mu^+ \mu^-) < 2 \cdot 10^{-8}$$

$$\text{Br}(B_s \rightarrow \mu^+ \mu^-)_{\text{exp}} < 5 \cdot 10^{-7} \quad (\text{CDF})$$

$$\text{Br}(B_d \rightarrow \mu^+ \mu^-)_{\text{exp}} < 1.6 \cdot 10^{-7} \quad (\text{BaBar})$$

# Conclusions

1.

For  $(\Delta M_s)^{\text{exp}} \geq (\Delta M_s)^{\text{SM}}$  substantial enhancements of  $\text{Br}(B_{s,d} \rightarrow \mu^+ \mu^-)$  not possible without new sources of flavour violation (beyond CKM)

2.

Observation of  $\text{Br}(B_s \rightarrow \mu^+ \mu^-) \geq 10^{-8}$   
 $\text{Br}(B_d \rightarrow \mu^+ \mu^-) \geq 10^{-9}$   
will imply either  $(\Delta M_s)^{\text{exp}} < (\Delta M_s)^{\text{SM}}$   
and/or new sources of flavour violation (beyond CKM)

3.

In order to find out  $\frac{(\Delta M_s)^{\text{exp}}}{(\Delta M_s)^{\text{SM}}}$   $\leftarrow$  (CDF, DØ)  
 $\leftarrow$   $F_{B_s}, B_i$

# News on $B \rightarrow \pi\pi$ , $B \rightarrow \pi K$ and Rare $K$ and $B$ Decays

hep-ph/0312259    hep-ph/0410407  
hep-ph/0402112

# New Complex Phase in $Z^0$ Penguins

## Enhanced $Z^0$ -Penguins

$$: A(\text{Decay}) = \sum_i B_i \eta_{\text{QCD}}^i V_{\text{CKM}}^i \left[ \underbrace{F_{\text{SM}}^i}_{\text{real}} + \underbrace{\Delta_{\text{New}}^i}_{\text{complex}} \right]$$

AJB, Colangelo, Isidori, Romanino, Silvestrini  
 Buchalla, Hiller, Isidori; Atwood, Hiller  
 AJB, Fleischer, Recksiegel, Schwab

**Dominated by  
 $Z^0$ -Penguins  
 with a New  
 Complex Phase**

Just replace:

$$C(x_t) \rightarrow |C(v)| e^{i\varphi_C}$$

$\varphi_C = \text{new complex  
 Phase}$

$$X(x_t) \rightarrow |X(v)| e^{i\theta_X}$$

$$Y(x_t) \rightarrow |Y(v)| e^{i\theta_Y}$$



# Basic Structure

**BFRS**

**Basic Scenario**


New Physics enters dominantly through EW Penguins including new complex phases

**AJB, Colangelo, Isidori, Romanino, Silvestrini (00)  
Buchalla, Hiller, Isidori (01)**

**Simple, Predictive**

**Step 1.**

$B \rightarrow \pi\pi$  Decays described within SM  
(EW -Penguins small)

Isospin Symmetry  
  
 $B \rightarrow \pi\pi$  Data

Hadronic Parameters in  $B \rightarrow \pi\pi$   
( $d, \theta, x, \Delta$ )

Sizable Departures from QCDF, PQCD

**Step 2.**

$d, \theta, x, \Delta$   
+  $SU(3)_F$



Hadronic Parameters in  $B \rightarrow \pi K$

$B \rightarrow \pi K$  Data

Enhanced EWP with large New Complex Phase

+

$\gamma$   
( $65 \pm 7$ )°



**Step 3.**

Correlations between  $B \rightarrow \pi K$ , Rare K and B Decays and other Processes



Implications for Rare K and B Decays sensitive to EWP

## The $B \rightarrow \pi K$ Puzzle

$$R \equiv \left[ \frac{\text{Br}(B_d^0 \rightarrow \pi^- K^+) + \text{Br}(\bar{B}_d^0 \rightarrow \pi^+ K^-)}{\text{Br}(B^+ \rightarrow \pi^+ K^0) + \text{Br}(\bar{B}^- \rightarrow \pi^- \bar{K}^0)} \right] \frac{\tau(B^+)}{\tau(B_d^0)} = 0.91 \pm 0.07$$

$$R_c \equiv 2 \left[ \frac{\text{Br}(B^+ \rightarrow \pi^0 K^+) + \text{Br}(B^- \rightarrow \pi^0 K^-)}{\text{Br}(B^+ \rightarrow \pi^+ K^0) + \text{Br}(B^- \rightarrow \pi^- \bar{K}^0)} \right] = 1.17 \pm 0.12$$

$$R_n \equiv \frac{1}{2} \left[ \frac{\text{Br}(B_d^0 \rightarrow \pi^- K^+) + \text{Br}(\bar{B}_d^0 \rightarrow \pi^+ K^-)}{\text{Br}(B_d^0 \rightarrow \pi^0 K^0) + \text{Br}(\bar{B}_d^0 \rightarrow \pi^0 \bar{K}^0)} \right] = 0.76 \pm 0.10$$

CLEO  
Belle  
BaBar

Status  
before  
ICHEP 04



Pattern  $R_c > 1$  and  $R_n < 1$  very puzzling!

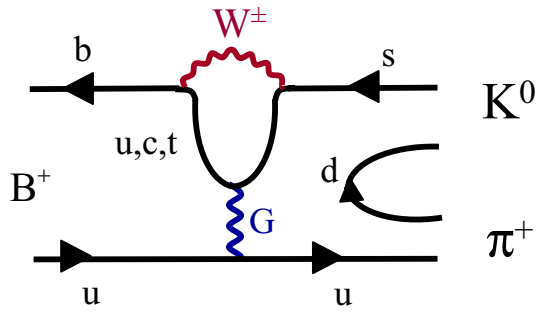
seen by all  
experiments

Could it come from enhanced EWP ?

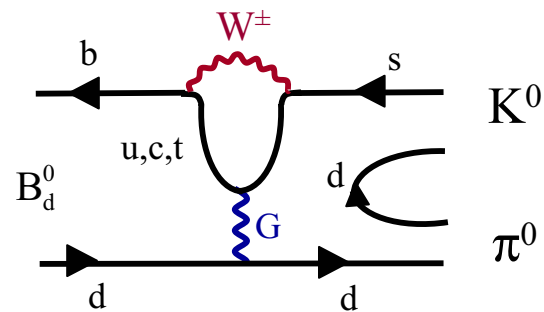
AJB + Fleischer (2000)

Recently also: Yoshikawa, Beneke + Neubert, Gronau + Rosner

$$B^+ \rightarrow \pi^+ K^0$$

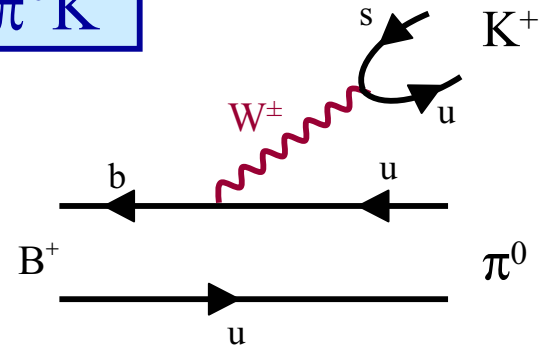
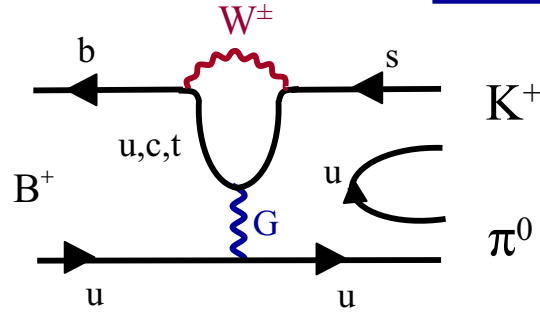


$$B_d^0 \rightarrow \pi^0 K^0$$



QCD Penguins  
at work

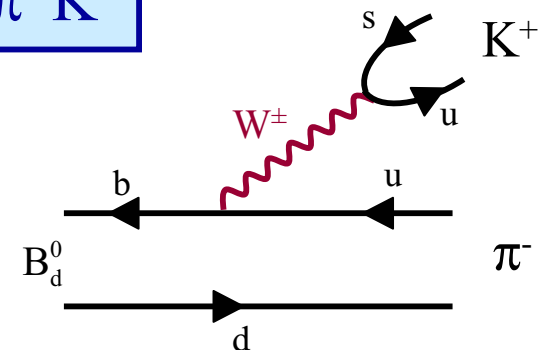
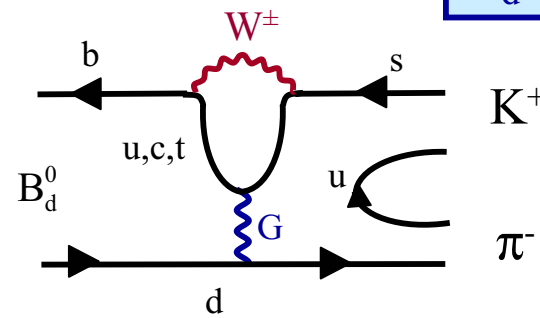
$$B^+ \rightarrow \pi^0 K^+$$



Penguins:  $\lambda^2$  (c,t)  
(P)  $\lambda^4 e^{i\gamma}$  (u)

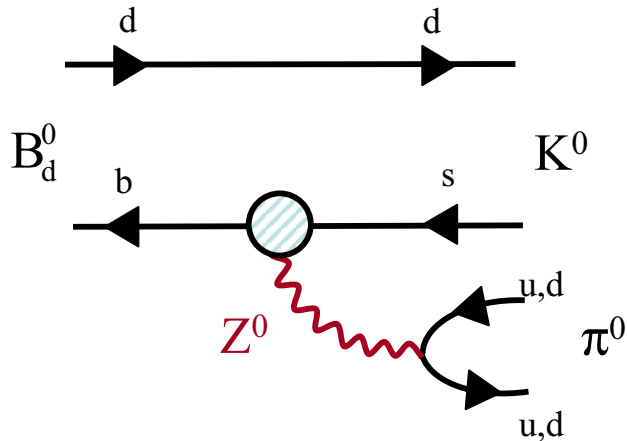
Trees:  $\lambda^4 e^{i\gamma}$   
(T)

$$B_d^0 \rightarrow \pi^- K^+$$

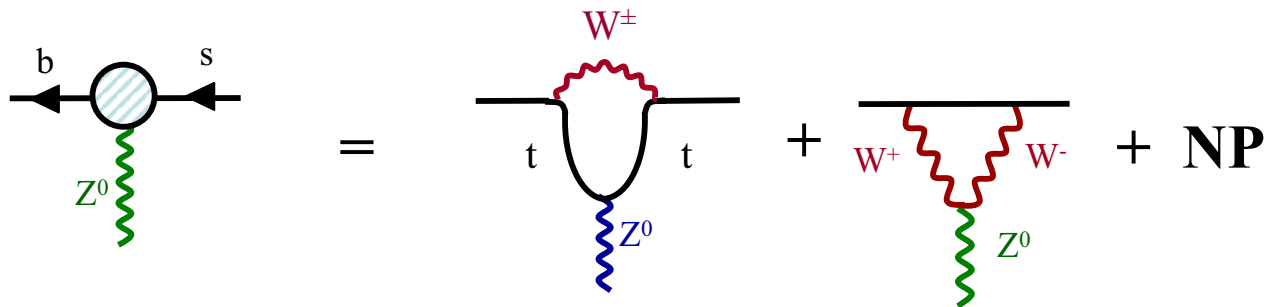
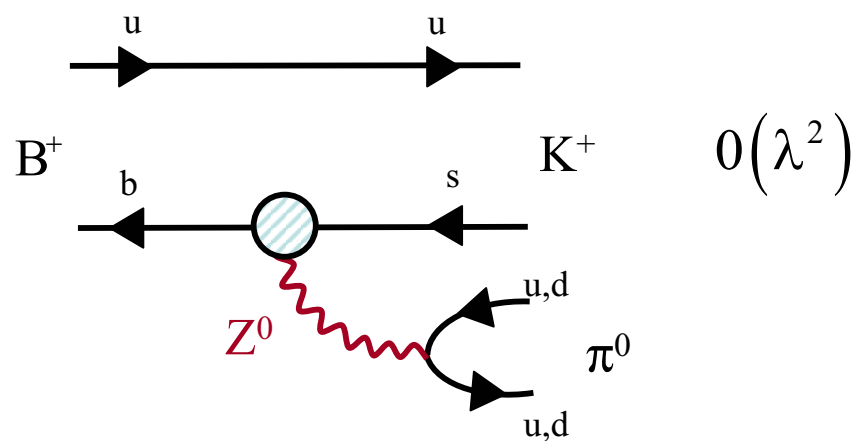


# Electroweak Penguins in $B \rightarrow \pi K$

$$B_d^0 \rightarrow \pi^0 K^0$$



$$B^+ \rightarrow \pi^0 K^+$$



EWP's in  $B^+ \rightarrow \pi^+ K^0$  and  $B_d^0 \rightarrow \pi^- K^+$  colour suppressed.



# B → πK Amplitudes

(BFRS 04)

**Basic Assumptions**

:

**Neglect colour-suppressed EW penguins  
SU(3) Flavour Symmetry**

$$A(B_d^0 \rightarrow \pi^- K^+) = P' [1 - r e^{i\delta} e^{i\gamma}]$$

$$A(B^+ \rightarrow \pi^+ K^0) = -P' = \text{QCP Penguin}$$

colour suppressed EW penguins tiny

$$\sqrt{2} A(B_d^0 \rightarrow \pi^0 K^0) = -P' [1 + \rho_n e^{i\theta_n} e^{i\gamma} - r_c e^{i\delta_c} q e^{i\varphi}]$$

$$\sqrt{2} A(B^+ \rightarrow \pi^0 K^+) = P' [1 - (e^{i\gamma} - q e^{i\varphi}) r_c e^{i\delta_c}]$$

EW penguins significant

$$\begin{matrix} r, \delta, \rho_n, \theta_n \\ r_c, \delta_c \end{matrix}$$

Non-Perturbative Parameters

SU(3)

$$B \rightarrow \pi\pi$$

$$q e^{i\varphi}$$

EW-Penguin Parameter

$$R_n, R_c$$

# B → πK in the SM

SM EWP

$$q = 0.69 \quad \varphi = 0$$

Neubert  
Rosner (98)

|          | Exp                         |                                  |
|----------|-----------------------------|----------------------------------|
| (BFRS) { | $R _{SM} = 0.94 \pm 0.03$   | $(0.91 \pm 0.07)$ ok             |
|          | $R_c _{SM} = 1.14 \pm 0.08$ | $(1.17 \pm 0.12)$ ok             |
|          | $R_n _{SM} = 1.11 \pm 0.07$ | $(0.76 \pm 0.10)$ Disagreement!! |



Modify EWP:  $qe^{i\varphi}$        $\varphi = \text{new } \cancel{CP} \text{ phase}$   
 to fit  $R_n$  and  $R_c$

(BFRS)

# The $B \rightarrow \pi K$ Puzzle

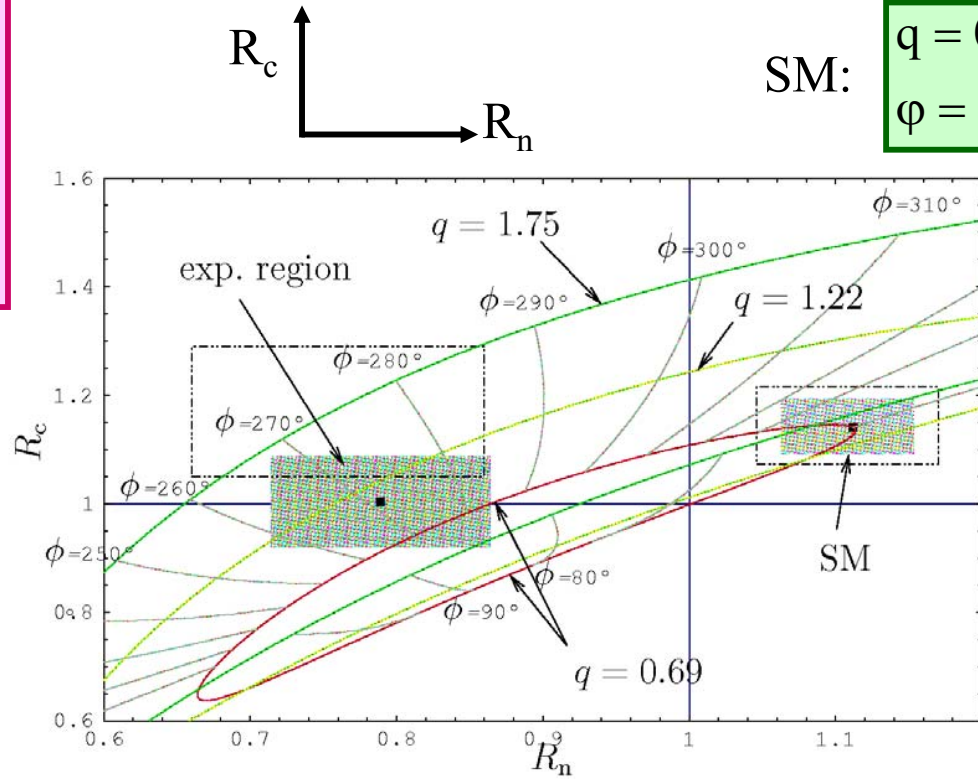
$EWP = qe^{i\phi}$

$$R_c = 2 \left[ \frac{\text{Br}(B^\pm \rightarrow \pi^0 K^\pm)}{\text{Br}(B^\pm \rightarrow \pi^\pm K^0)} \right]$$

$$R_n = \frac{1}{2} \left[ \frac{\text{Br}(B^0 \rightarrow \pi^- K^+)}{\text{Br}(B^0 \rightarrow \pi^0 K^0)} \right]$$

SM:  $q = 0.69$  Neubert  
 $\phi = 0$  Rosner

$(R_c)_{SM} = 1.14 \pm 0.05$   
 $(R_n)_{SM} = 1.11 \pm 0.05$



Best Values  
(including rare  
decay constraint)

$q \approx 0.92$   
 $\phi \approx -85^\circ$

BFRS

$R_c = 1.17 \pm 0.12$   
 $R_n = 0.76 \pm 0.10$

Rare Decays



$R_c = 1.00^{+0.12}_{-0.08}$   
 $R_n = 0.82^{+0.12}_{-0.11}$

BFRS Expectation  
of June 04

$R_c = 1.00 \pm 0.08$   
 $R_n = 0.79 \pm 0.08$

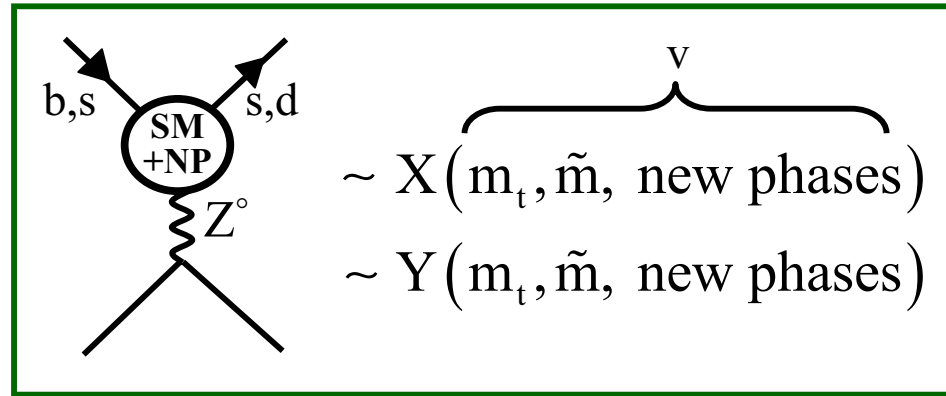
Exp: after ICHEP 04

CLEO  
BaBar  
Belle

# Relation: $B \rightarrow \pi K \leftrightarrow$ Rare K and B Decays

BFRS

Fundamental  
Short Distance  
Physics of EWP  
 $\mu = 0(m_t)$



$\nu\bar{\nu}$   
 $\mu\bar{\mu}$

RG

$B \rightarrow \pi K$   
 $(q, \varphi)$   
 $\mu = 0(m_b)$

$K \rightarrow \pi\nu\bar{\nu}, B \rightarrow X_S\mu^+\mu^-$   
 $K_L \rightarrow \pi^0\mu^+\mu^-, B_{d,s} \rightarrow \mu^+\mu^-$   
 $B \rightarrow X_{d,s}\nu\bar{\nu}$

Renormalization  
group :

$|X(\nu)|e^{i\theta_X} = 2.35 qe^{i\varphi} - 0.09 \approx 2.2 \cdot e^{-i86^\circ}$   
 $|Y(\nu)|e^{i\theta_Y} = 2.35 qe^{i\varphi} - 0.64 \approx 2.2 \cdot e^{-i100^\circ}$

$X_{SM} = 1.53$

$Y_{SM} = 0.98$

# Implications for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$

$$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{SM} = (7.8 \pm 1.2) \cdot 10^{-11} \Rightarrow (7.5 \pm 2.1) \cdot 10^{-11}$$

Enhancement of  $|X|$  compensated by destructive "top-charm" interference

★  $\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})_{SM} = (3.0 \pm 0.6) \cdot 10^{-11} \Rightarrow (3.1 \pm 1.0) \cdot 10^{-10}$

★ Strong Violation of the "Golden" Relation

Buchalla, AJB (94)

SM:  $(\sin 2\beta)_{\pi\nu\bar{\nu}} = (\sin 2\beta)_{\psi K_S}$   
 Here:  $-\left(0.69^{+0.23}_{-0.41}\right) \neq 0.74 \pm 0.05$   
 $\sin 2\beta_X$

$\beta_X = \beta - \theta_X$   
 $X = |X| e^{i\theta_X}$   
 $\beta_X \cong 110^\circ$

★ Saturation of the model-independent Grossman-Nir bound

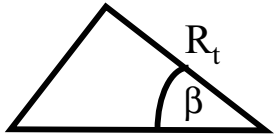
FIG

$\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu}) \leq 4.4 \quad \text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$

Here:

$\frac{\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})}{\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})} \approx 4.4 \quad \sin^2(\beta_X) \approx 4.2 \pm 0.2$

**Impact of  $X = |X|e^{i\theta_X}$  on  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$**



(A ≈ 0.83)

(P<sub>c</sub> ≈ 0.39)

$$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 4.8 \cdot 10^{-11} \left[ A^4 R_t^2 |X|^2 + 2P_c A^2 R_t |X| \overbrace{\cos(\beta - \theta_X)}^{\beta_X} + P_c^2 \right]$$

SM: = 10<sup>-11</sup> [ 4.1 + 3.0 + 0.7 ] X = 1.53

Here: = 10<sup>-11</sup> [ 8.5 - 1.7 + 0.7 ] X = 2.2e<sup>-i86°</sup>

(top) (charm-top) (charm)

β - θ<sub>X</sub> ≈ 110°

**Br(K<sup>+</sup> → π<sup>+</sup> ν ν̄) = (7.8 ± 1.2) · 10<sup>-11</sup> → (7.5 ± 2.1) · 10<sup>-11</sup>**

(SM)

**Impact of  $X = |X|e^{i\theta_X}$  on  $K_L \rightarrow \pi^0 \nu \bar{\nu}$**

$$\frac{\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})}{\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})_{\text{SM}}} = \underbrace{\left| \frac{X}{X_{\text{SM}}} \right|^2}_{\approx 2} \underbrace{\left[ \frac{\sin(\beta - \theta_X)}{\sin \beta} \right]^2}_{\approx 5}$$

$\beta - \theta_X \approx 110^\circ$   
 $\beta \approx 24^\circ$

$$\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = (3.0 \pm 0.6) \cdot 10^{-11} \quad \Rightarrow \quad (3.1 \pm 1.0) \cdot 10^{-10}$$

(SM)

(Here)

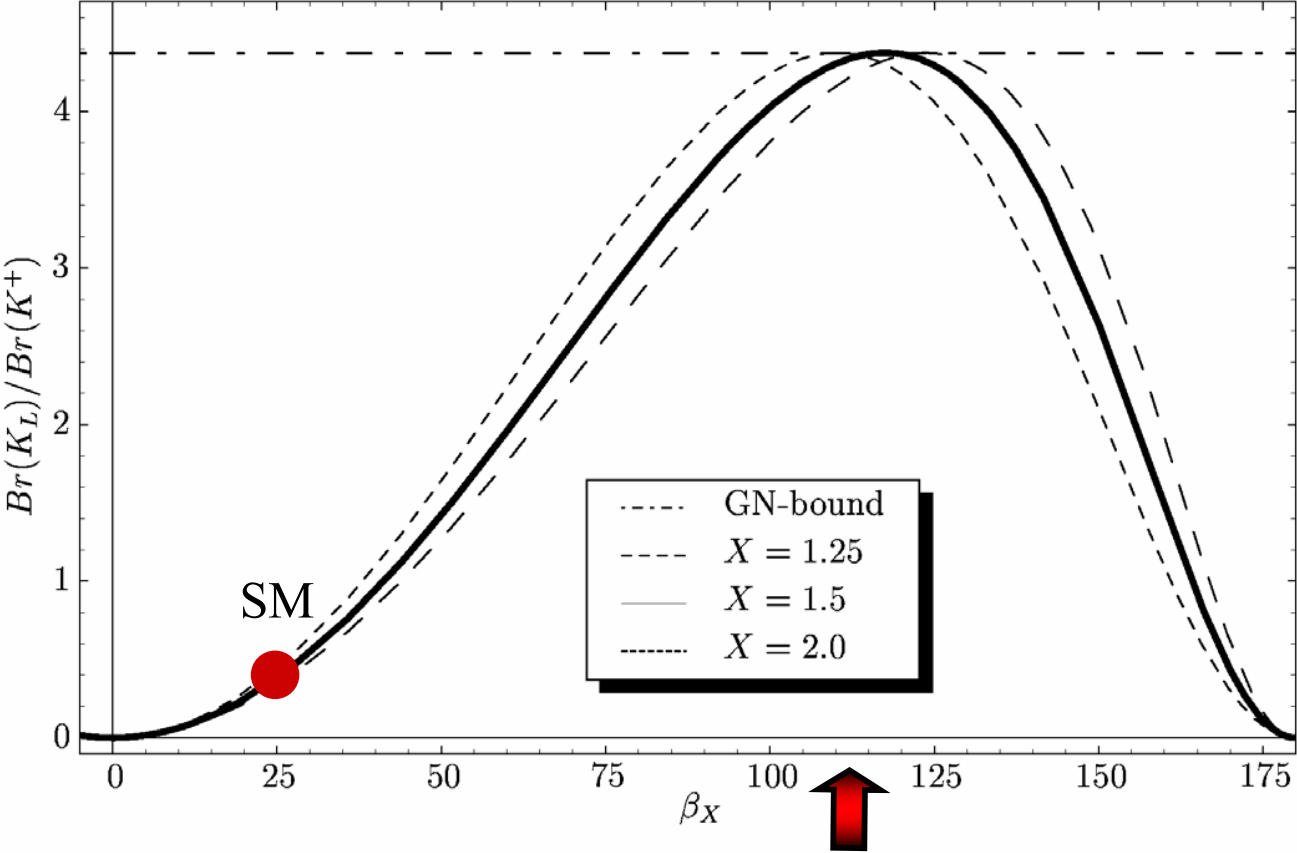
$$\frac{\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})}{\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{SM}}} = 0.4 \quad \Rightarrow \quad 4.2$$

< 4.4  
Grossman-Nir bound

**Order of magnitude enhancement !!**

$$\frac{\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})}{\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})} \text{ versus } \beta_X$$

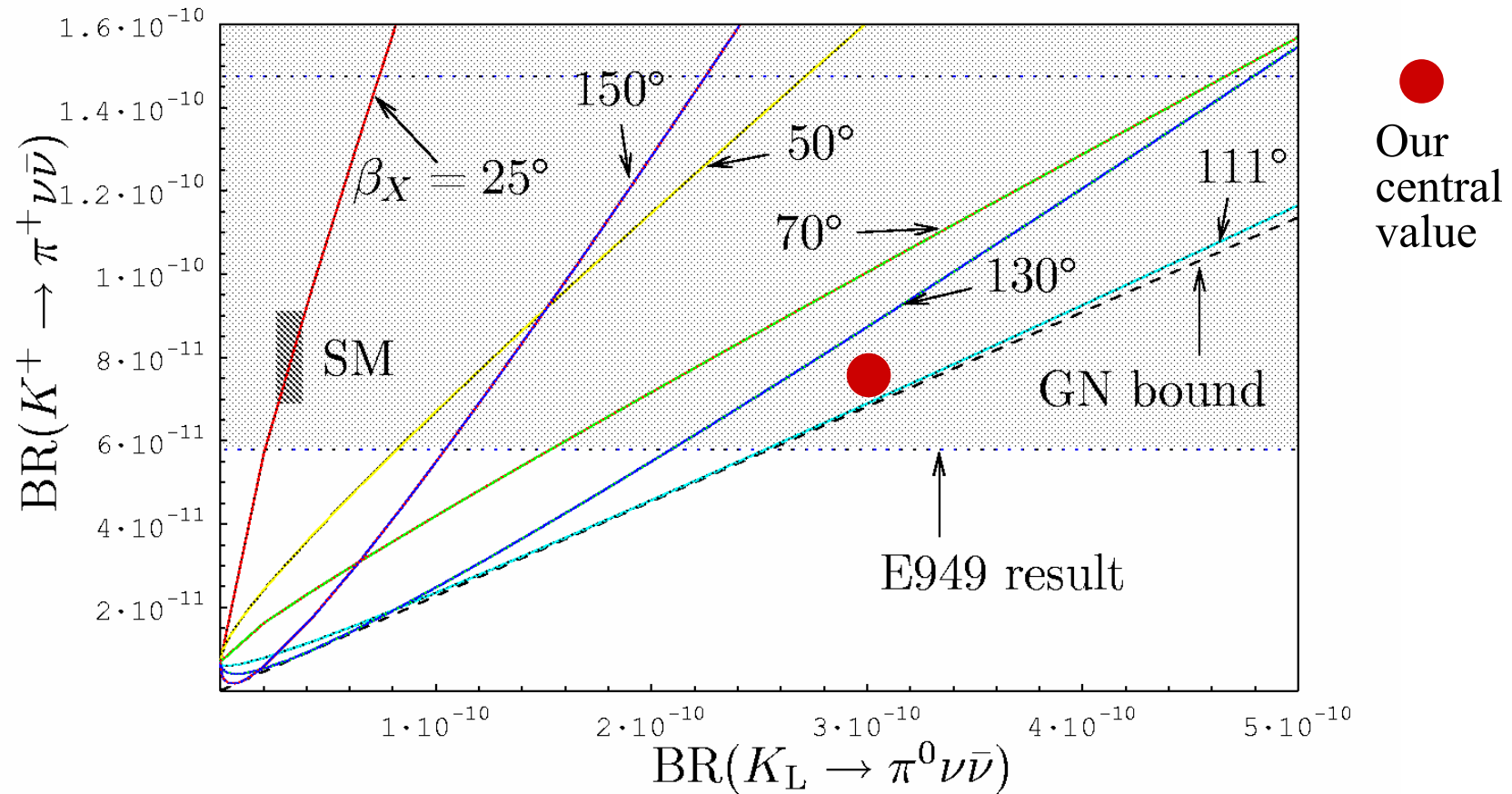
BSU (04)





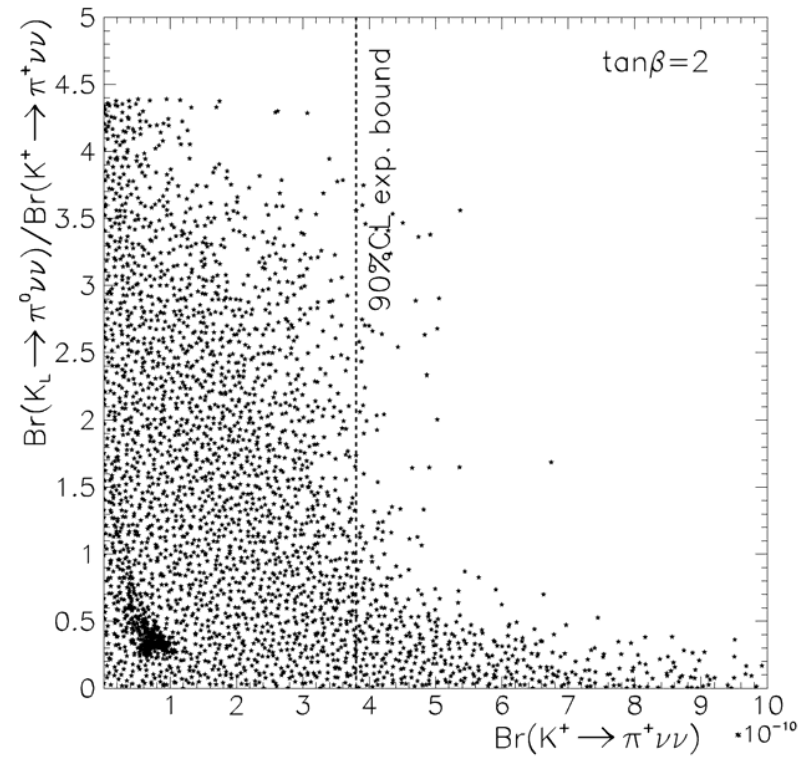
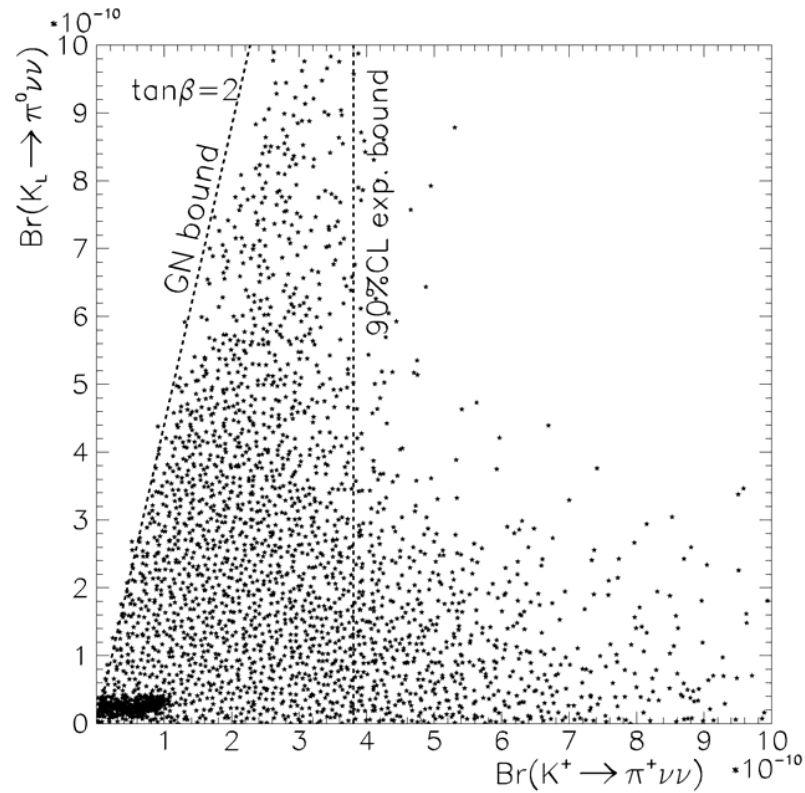
$K^+ \rightarrow \pi^+ \nu \bar{\nu}$  and  $K_L \rightarrow \pi^0 \nu \bar{\nu}$  with  $X = |X| e^{i\theta_x}$  (BFRS)

$$\beta_x = \beta - \theta_x$$



# $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^0 \nu \bar{\nu}$ from a general MSSM

AJB, Ewerth, Jäger, Rosiek (04)



## Other Impacts on Rare K and B Decays

Could it be seen  
by Belle, BaBar ?

Could it be seen  
at Tevatron ?  
Will be seen at LHC!

$$\frac{\text{Br}(B \rightarrow X_{s,d} \nu \bar{\nu})}{\text{Br}(B \rightarrow X_{s,d} \nu \bar{\nu})_{\text{SM}}} \approx 2.0$$

$$\text{Br}(B \rightarrow X_s \nu \bar{\nu}) \approx 7 \cdot 10^{-5}$$

$$\frac{\text{Br}(B_{s,d} \rightarrow \mu^+ \mu^-)}{\text{Br}(B_{s,d} \rightarrow \mu^+ \mu^-)_{\text{SM}}} \approx 2.5$$

$$\text{Br}(B_s \rightarrow \mu^+ \mu^-) \approx 1 \cdot 10^{-8}$$

$$\text{Br}(B_d \rightarrow \mu^+ \mu^-) \approx 3 \cdot 10^{-10}$$

Spectacular Effects in  
FB CP Asymmetry in  
 $B_d \rightarrow K^* \mu^+ \mu^-$

Lepton polarization  
asymmetries in  $b \rightarrow s l^+ l^-$   
(Choudhury, Gaur, Cornell (04))

BUT:  $(\sin 2\beta)_{\phi_{K_S}} > (\sin 2\beta)_{\psi_{K_S}} \approx 0.73$

Consistent with BaBar  $(0.50 \pm 0.25 \pm 0.06)$

but

$0.06 \pm 0.33 \pm 0.09$  (Belle)

Z'-models can explain both  $\pi K$  and Belle (Barger et al, 0406126)  
but no relation to  $K \rightarrow \pi \nu \bar{\nu}$ ,  $B \rightarrow \mu \mu$

# Impact on $K_L \rightarrow \pi^0 e^+ e^-$ and $K_L \rightarrow \pi^0 \mu^+ \mu^-$

$$\text{Br}(K_L \rightarrow \pi^0 e^+ e^-)_{\text{SM}} = \left( 3.7^{+1.1}_{-0.9} \right) \cdot 10^{-11}$$

Dominated by indirect  $\mathcal{CP}$   
 (Buchalla, D'Ambrosio, Isidori)  
 (Friot, Greynat, de Rafael)



$$(9.0 \pm 1.6) \cdot 10^{-11}$$

Dominated by direct  $\mathcal{CP}$   
 BFRS

Isidori  
 Smith  
 Underdorfer

$$\text{Br}(K_L \rightarrow \pi^0 \mu^+ \mu^-) = (1.5 \pm 0.3) \cdot 10^{-11}$$

Dominated by CP-conserving  
 + indirect  $\mathcal{CP}$



$$(4.3 \pm 0.7) \cdot 10^{-11}$$

Dominated by direct  $\mathcal{CP}$   
 ISU

KTeV:

$$\text{Br}(K_L \rightarrow \pi^0 e^+ e^-) < 2.8 \cdot 10^{-10}$$

$$\text{Br}(K_L \rightarrow \pi^0 \mu^+ \mu^-) < 3.8 \cdot 10^{-10}$$

## Six messages

**1.**  $B \rightarrow \pi\pi$  data described within SM with hadronic parameters differing significantly from QCDF and PQCD

**2.**  $B \rightarrow \pi K$  data give a hint for enhanced EWP with new Large negative CP phase.

**3.** The angle  $\gamma$  found in agreement with UT fits.

**4.** 
$$\frac{\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})}{\text{Br}(K^+ \rightarrow \pi \nu \bar{\nu})_{\text{SM}}} \approx 1$$

$$\frac{\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})}{\text{Br}(K^+ \rightarrow \pi^0 \nu \bar{\nu})_{\text{SM}}} \approx 10$$

$$\frac{\text{Br}(B_{s,d} \rightarrow \mu^+ \mu^-)}{\text{Br}(B_{s,d} \rightarrow \mu^+ \mu^-)_{\text{SM}}} \approx 2.5$$

**5.**  $(\sin 2\beta)_{\varphi_{K_S}} \geq (\sin 2\beta)_{\psi_{K_S}}$  in disagreement with Belle but consistent with BaBar

**6.** General MSSM can be made consistent with this pattern (AJB, Ewerth, Jäger, Rosiek)

**9.**

# **Probing New Physics in 10 Steps**

**Starting Point**

:

$$\mathcal{L} = \mathcal{L}_{\text{SM}}(g_i, m_i, V_{\text{CKM}}^i) + \mathcal{L}_{\text{NP}}(g_i^{\text{NP}}, m_i^{\text{NP}}, V_{\text{NP}}^i)$$

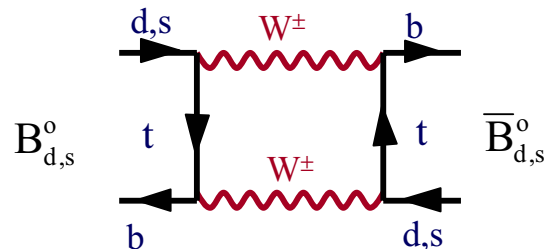
**Goal**

:

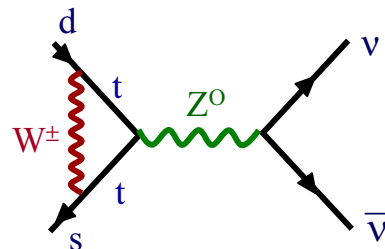
Identify the effects of  $\mathcal{L}_{\text{NP}}$  in weak decays in the presence of the background from  $\mathcal{L}_{\text{SM}}$

**First Implication from  $\mathcal{L}$**

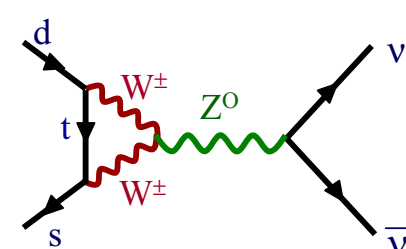
: Feynman Diagrams



$B_d^0 - \bar{B}_d^0$  Mixing



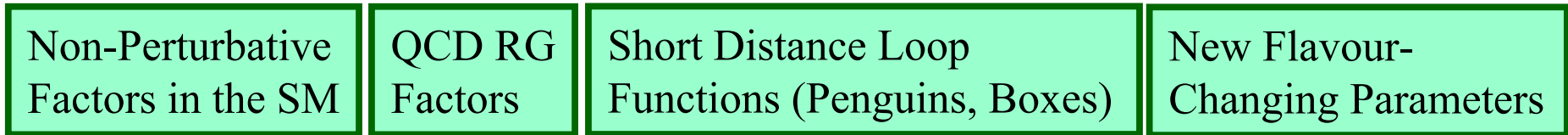
$K^+ \rightarrow \pi^+ \nu \bar{\nu}, K_L \rightarrow \pi^0 \nu \bar{\nu}$



+ NP

# Master Formula for Weak Decays

AJB (2001)  
 hep-ph/0101336  
 hep-ph/0109197



Represent different Dirac and Colour Structures

$$A(\text{Decay}) = \sum_i B_i \eta_{\text{QCD}}^i V_{\text{CKM}}^i [F_{\text{SM}}^i + F_{\text{New}}^i] + B_i^{\text{New}} [\eta_{\text{QCD}}^i]^{\text{New}} V_{\text{New}}^i [G_{\text{New}}^i]$$

(Summation over i)

New  $\equiv$  NP

Non-Perturbative Factors beyond SM

Short Distance Loop Functions Penguins, Boxes

|                                                           |                                                                            |
|-----------------------------------------------------------|----------------------------------------------------------------------------|
| $F_{\text{SM}}^i, F_{\text{New}}^i, G_{\text{New}}^i$     | : Fully calculable in Perturbation Theory                                  |
| $\eta_{\text{QCD}}^i, [\eta_{\text{QCD}}^i]^{\text{New}}$ | : Fully calculable in RG improved Perturbation Theory                      |
| $B_i, B_i^{\text{New}}$                                   | : Require Non-Perturbative Methods or can be extracted from leading decays |
| (represent $\langle Q_i \rangle$ )                        |                                                                            |

Fully calculable in the SM



## Three Simple Scenarios

Inami  
Lim Functions

**SM** :

$$A(\text{Decay}) = \sum_i B_i \eta_{\text{QCD}}^i V_{\text{CKM}}^i \underbrace{F_{\text{SM}}^i(m_t)}_{\text{real}}$$

**MFV** :

$$A(\text{Decay}) = \sum_i B_i \eta_{\text{QCD}}^i V_{\text{CKM}}^i \underbrace{\left[ F_{\text{SM}}^i + F_{\text{New}}^i \right]}_{\text{real}}$$

(Minimal Flavour Violation)

AJB, Gambino, Gorbahn, Jäger, Silvestrini  
D'Ambrosio, Giudice, Isidori, Strumia

**Enhanced Z<sup>0</sup>-Penguins** :

$$A(\text{Decay}) = \sum_i B_i \eta_{\text{QCD}}^i V_{\text{CKM}}^i \left[ \underbrace{F_{\text{SM}}^i}_{\text{real}} + \underbrace{\Delta_{\text{New}}^i}_{\text{real}} \right]$$

AJB, Colangelo, Isidori, Romanino, Silvestrini  
Buchalla, Hiller, Isidori; Atwood, Hiller  
AJB, Fleischer, Recksiegel, Schwab

**Dominated by  
Z<sup>0</sup>-Penguins  
with a New  
Complex Phase**

## Two more complicated Scenarios

**MSSM (MFV)  
(large  $\tan\beta$ )**

(Higgs penguin)

$$\begin{aligned}
 A(\text{Decay}) = & \sum_i B_i \eta_{\text{QCD}}^i V_{\text{CKM}}^i \left[ \overbrace{F_{\text{SM}}^i + F_{\text{New}}^i}^{\text{real}} \right] \\
 & + \sum_i B_i^{\text{New}} \left[ \eta_{\text{QCD}}^i \right]^{\text{New}} V_{\text{CKM}}^i \underbrace{\left[ G_{\text{New}}^i \right]}_{\text{real}}
 \end{aligned}$$

**General  
MSSM**

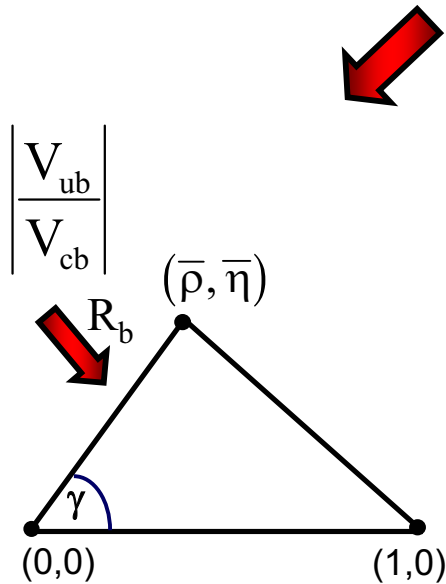
$$\begin{aligned}
 A(\text{Decay}) = & \sum_i B_i \eta_{\text{QCD}}^i V_{\text{CKM}}^i \left[ \overbrace{F_{\text{SM}}^i + F_{\text{New}}^i}^{\text{complex}} \right] \\
 & + \sum_i B_i^{\text{New}} \left[ \eta_{\text{QCD}}^i \right]^{\text{New}} V_{\text{New}}^i \underbrace{\left[ G_{\text{New}}^i \right]}_{\text{complex}}
 \end{aligned}$$

**Z'-Models  
L-R Models  
Multi-Higgs  
Models**

# Step 1

:

Determine  $|V_{us}|, |V_{cb}|, |V_{ub}|, \gamma$   
 by means of tree level decays insensitive to NP  
 (requires TH at a certain level)



Reference  
 Unitarity  
 Triangle

Goto, Kitazawa, Okada, Tanaka  
 Cohen et al.  
 Grossman et al.  
 Ciuchini et al.

{ Semi - leptonic  
 { K, B decays }



{  $|V_{us}|, |V_{cb}|, |V_{ub}|$  }

{  $B^\pm \rightarrow D^0 K^\pm$  }



{  $\gamma$  } (Gronau-London-Wyler)

{  $B_d^0 \rightarrow D^+ \pi^- (2\beta + \gamma)$   
 $B_d^0 \rightarrow \psi K_s (2\beta)$  }



{  $\gamma$  } (Dunietz, Sachs)

{  $B_s^0 \rightarrow D^+ K^- (2\beta_s + \gamma)$   
 $B_s^0 \rightarrow \psi \phi (2\beta_s)$  }



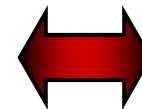
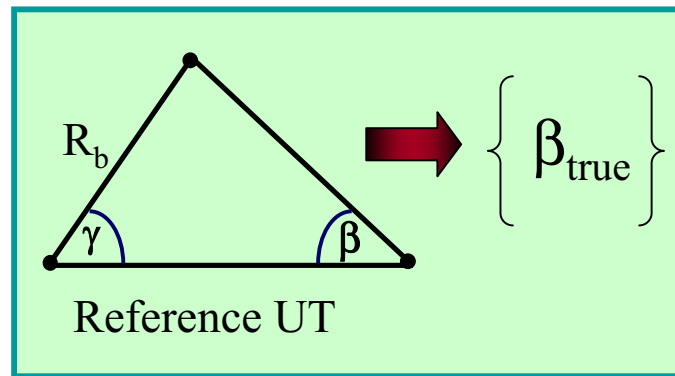
{  $\gamma$  } (Aleksan, Dunietz, Sachs)

Generalizations: Fleischer, Wyler, Gronau, Atwood, Soni, ...

# Comment to Step 1

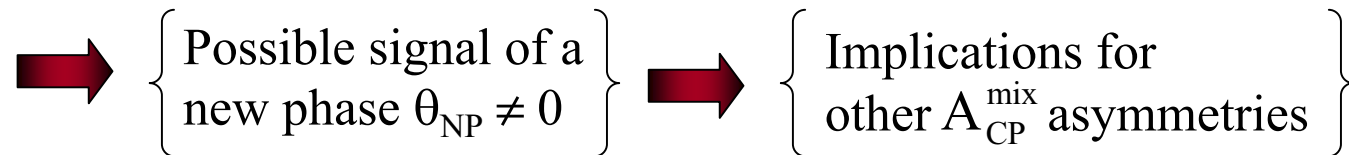
Having  $\gamma$  from tree-level decays allows the first test of SM and NP

Up to discrete ambiguities

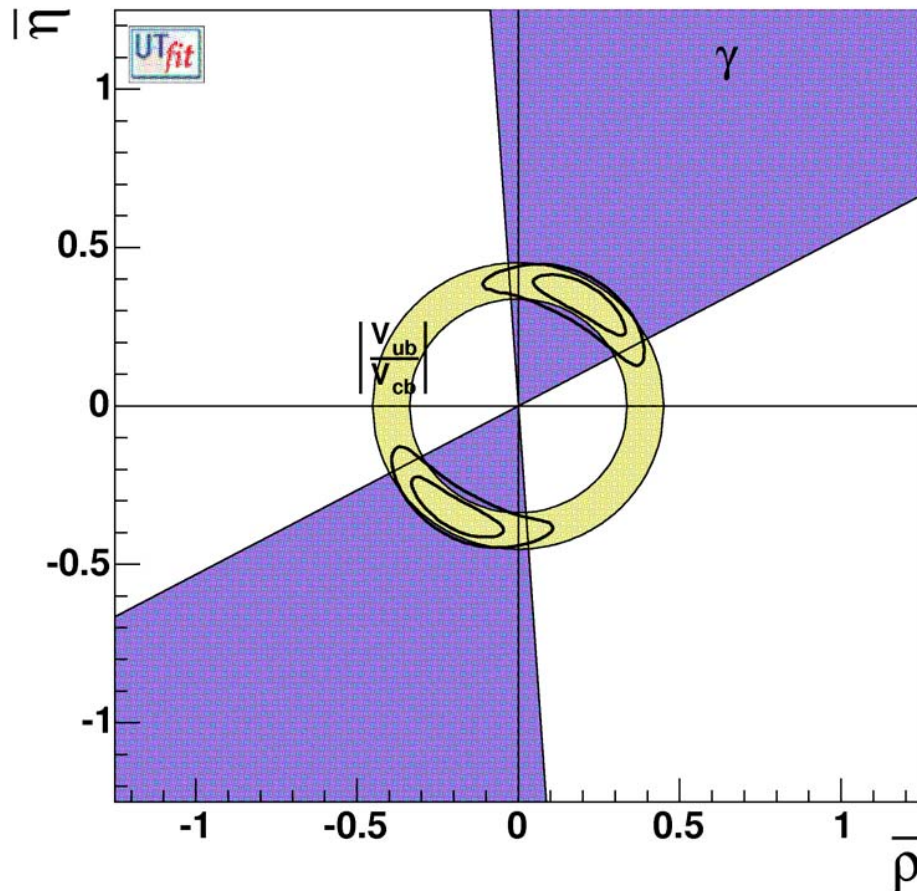


$\beta_{\psi K_s}$  from  
 $A_{CP}^{\text{mix}}(B_d^0 \rightarrow \psi K_s)$   
 $\beta_{\psi K_s} = \beta_{\text{true}} + \theta_{NP}$

New Phase in  $B_d^0 - \bar{B}_d^0$  mixing



# First Steps towards RUT



Obtained using only tree-level processes:

$$\left| \frac{V_{ub}}{V_{cb}} \right| \text{ and } \gamma \text{ from } B \rightarrow D^{(*)} K^{(*)}$$

Ufit collaboration: M. Bona et al.  
hep-ph/0501199.

## Step 2

Determine experimentally matrix elements of weak currents in tree-level decays

marginal impact of NP

Ideally:

Calculate these matrix elements by Non-Perturbative Methods

No impact of NP



$$F_{\pi}, F_K, F_{B_d}, F_{B_s}, \left\langle \pi^+ \left| (\bar{s}d)_{V-A} \right| K^+ \right\rangle, \left\langle \pi^0 \left| (\bar{s}d)_{V-A} \right| K_L^0 \right\rangle$$

known very well

$\pm 15\%$   
Lattice, QCDS

$K^+ \rightarrow \pi^0 e^+ \nu_e +$  Isospin Breaking Corrections

(Marciano  
Parsa)

$\pm 2-3\%$

### Step 3

use input from  
Steps 1 and 2

Calculate  $\text{Br}(\text{K}^+ \rightarrow \pi^+ \nu \bar{\nu})$ ,  $\text{Br}(\text{K}_L \rightarrow \pi^0 \nu \bar{\nu})$   
 $\text{Br}(\text{B}_s \rightarrow \mu^+ \mu^-)$ ,  $\text{Br}(\text{B}_d \rightarrow \mu^+ \mu^-)$   
 $\text{Br}(\text{B} \rightarrow X_s \nu \bar{\nu})$ ,  $\text{Br}(\text{B} \rightarrow X_d \nu \bar{\nu})$   
in the Standard Model

Comparison with experiment will hopefully give  
hints for NP in a clean TH environment.



or

use input from  
Step 2 to  
determine

$$|V_{td}|, |V_{ts}|$$
$$\beta, \gamma, \triangle$$

$$\left\{ \text{Br}(\text{B}_{s,d} \rightarrow \mu^+ \mu^-) \propto F_{\text{B}_{d,s}}^2 \right\} \Rightarrow \left\{ \text{Need precise values of } F_{\text{B}_{d,s}}^2 \right\}$$

# Express Review of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$

AJB  
Schwab  
Uhlig

hep-ph/0405132

NLO: Buchalla + AJB (94); NNLO: AJB, Gorbahn, Haisch, Nierste (05)

SM:  $\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = (7.8 \pm 1.2) \cdot 10^{-11}$   $\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu}) = (3.0 \pm 0.6) \cdot 10^{-11}$

Exp:  $\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = \left(14.7^{+13.0}_{-8.9}\right) \cdot 10^{-11}$   $\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu}) < 5.9 \cdot 10^{-7} \text{ (KTeV)}$

Brookhaven: E787, E949  
(CKM, NA48, JPARC, ..)

Soon improved by E391a !!!  
(KOPIO, J-PARC, ...)

TH very clean

:  $\left( \begin{array}{l} \text{With improved} \\ \text{CKM parameters} \\ \sim 2008 \end{array} \right) \rightarrow$

$\sigma(\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})) < 5\%$   
 $\sigma(\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})) < 5\%$

Very clean  
determination  
of Unitarity  
Triangle

$\sigma(\text{Br}) \cong 10\%$   
 $\sigma(\text{Br}) \cong 5\%$

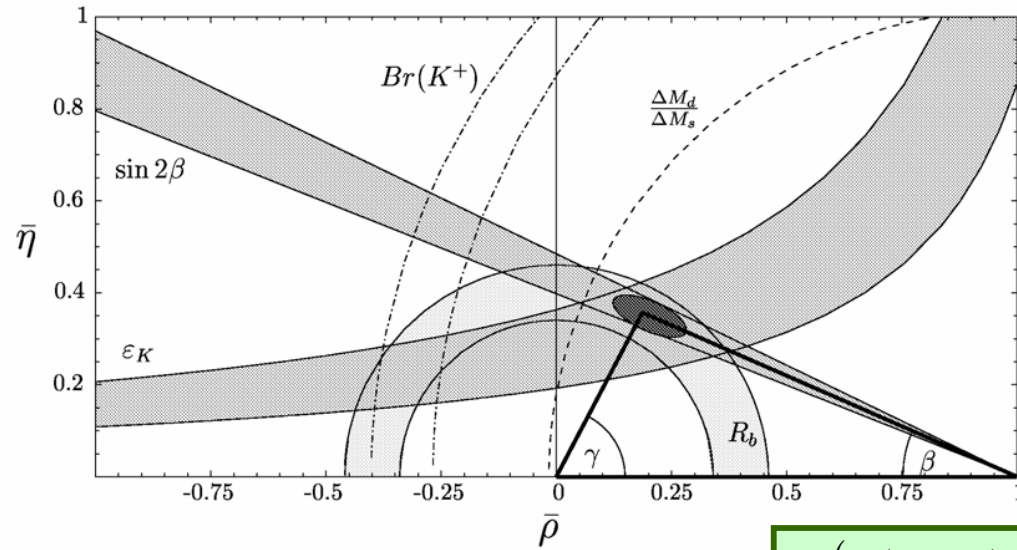
$\sigma(\sin 2\beta \cong 0.04) \mid \sigma(\gamma) = 9^\circ \mid \sigma(|V_{td}|) = 7\%$   
 $\sigma(\sin 2\beta \cong 0.025) \mid \sigma(\gamma) = 5^\circ \mid \sigma(|V_{td}|) = 4\%$



# Unitarity Triangle 2004

(AJB, Schwab, Uhlig)

$$\text{Br}(K^+) \equiv \text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 14.7 \cdot 10^{-11}$$



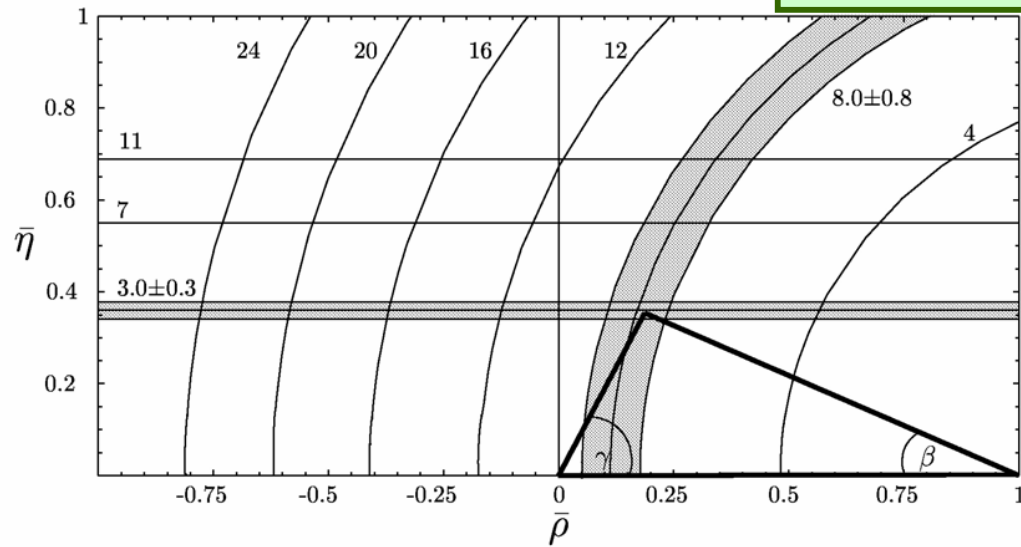
$$P_c = \underline{0.39 \pm 0.07}$$

↑  
 $m_c, V_{cb}, \mu_c$

$\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$

Unitarity Triangle from  $K \rightarrow \pi \nu \bar{\nu}$

(2012)



$\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})$

# Possible Enhancements

**1.** MFV :  $\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu}), \text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu}), \text{Br}(B \rightarrow X_s \nu \bar{\nu}), \text{Br}(B \rightarrow X_d \nu \bar{\nu})$

by at most a factor of 1.5 relative to SM (MFV fit)

$\text{Br}(B_s \rightarrow \mu^+ \mu^-), \text{Br}(B_d \rightarrow \mu^+ \mu^-)$  up to a factor of 2

**2.**  $\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})$  can be enhanced by 10 in models with enhanced EW penguins with a large complex phase.

(Hints from  $B \rightarrow \pi K$ ) (AJB, Fleischer, Recksiegel, Schwab)

(AJB, Ewerth, Jäger, Rosiek)

general MSSM

$3 \cdot 10^{-10}$

**3.**  $\text{Br}(B_{s,d} \rightarrow \mu^+ \mu^-)$  can be enhanced by 100 in MSSM with large  $\tan\beta \approx 40$  (Higgs penguins)

As high as present experimental bounds

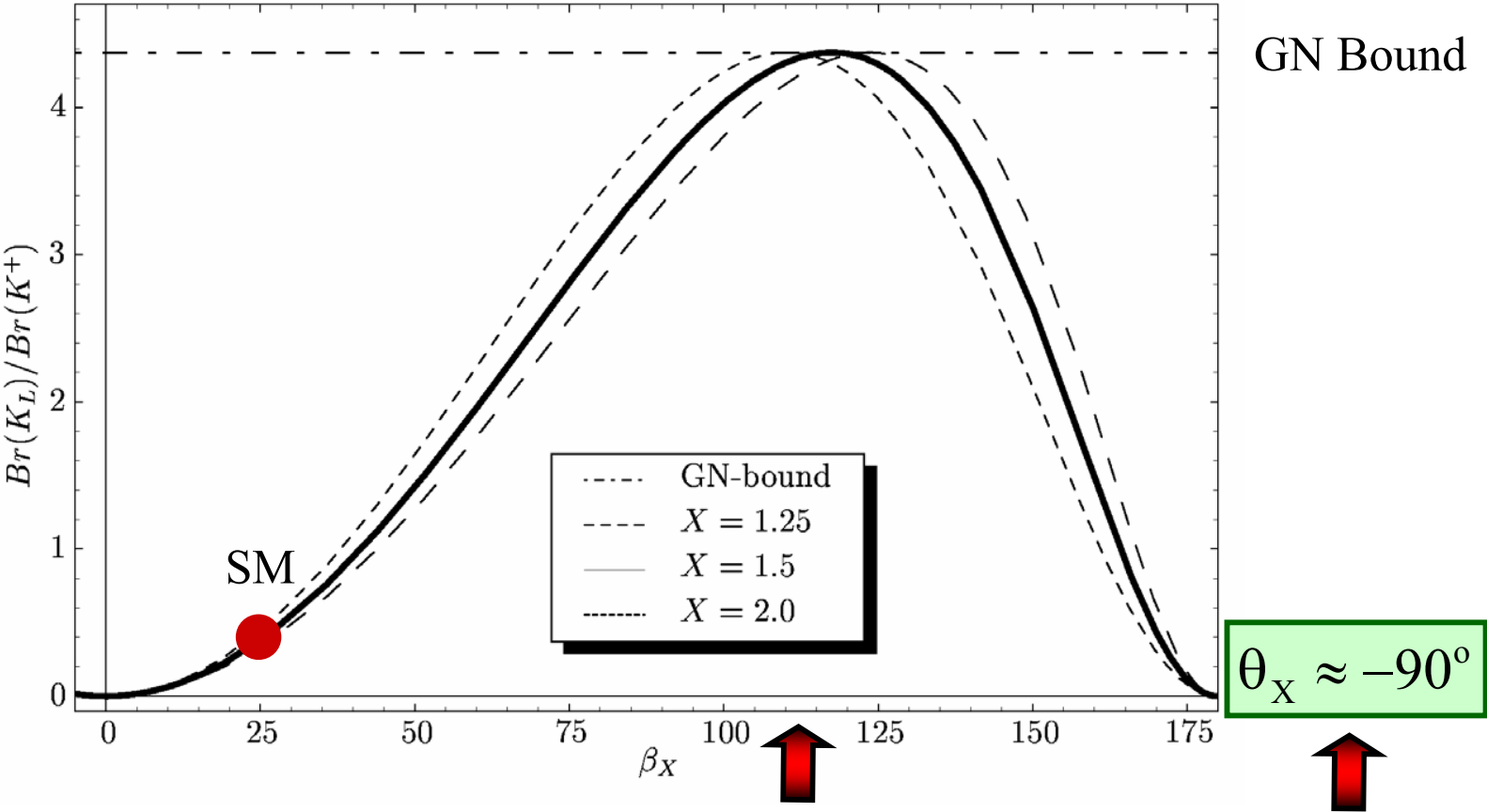
$\sim 10^{-7} (10^{-8})$

Babu, Kolda  
 Chankowski, Slawianowska  
 Bobeth, Ewerth, Krüger, Urban  
 Huang, Liao, Yan, Zhu  
 Isidori, Retico  
 Dedes, Dreiner, Nierste  
 Dedes, Pilaftis  
 Chankowski, Rosiek

$$\frac{\text{Br}(K_L \rightarrow \pi^0 \nu \bar{\nu})}{\text{Br}(K^+ \rightarrow \pi^+ \nu \bar{\nu})} \text{ versus } \beta_X$$

BSU (04)

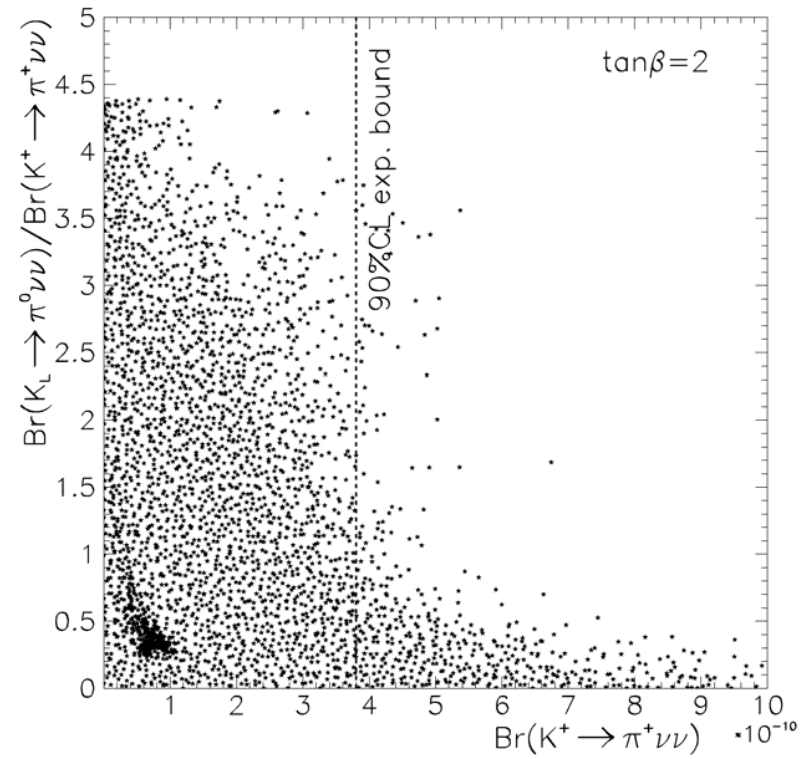
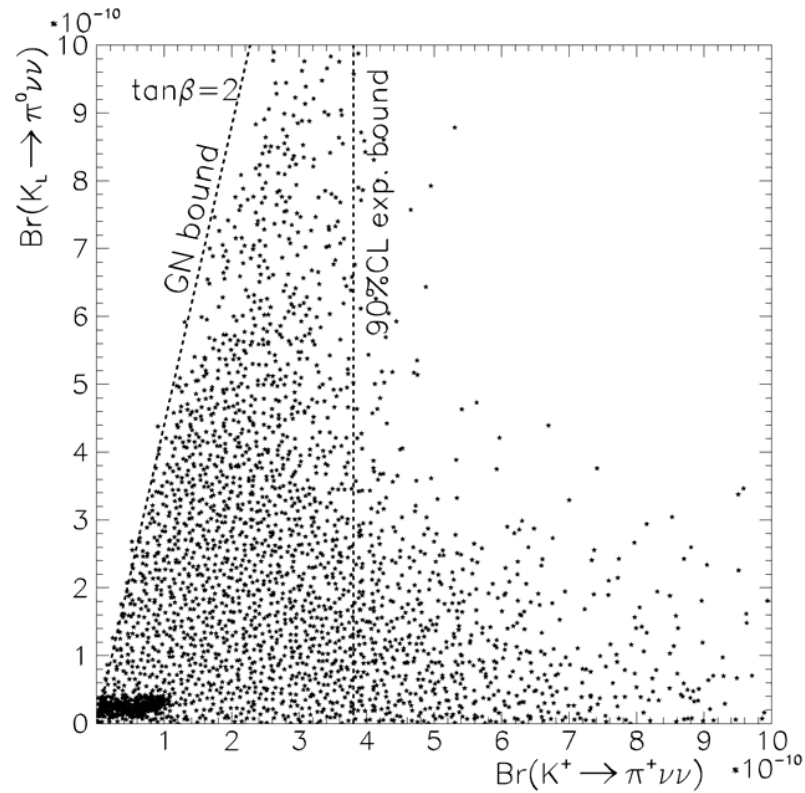
$$\beta_X = \beta - \theta_X \leftarrow \text{New Phase in EWP}$$



Necessary to fit  $B \rightarrow \pi K$  data (BFRS)

# $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^0 \nu \bar{\nu}$ from a general MSSM

AJB, Ewerth, Jäger, Rosiek (04)

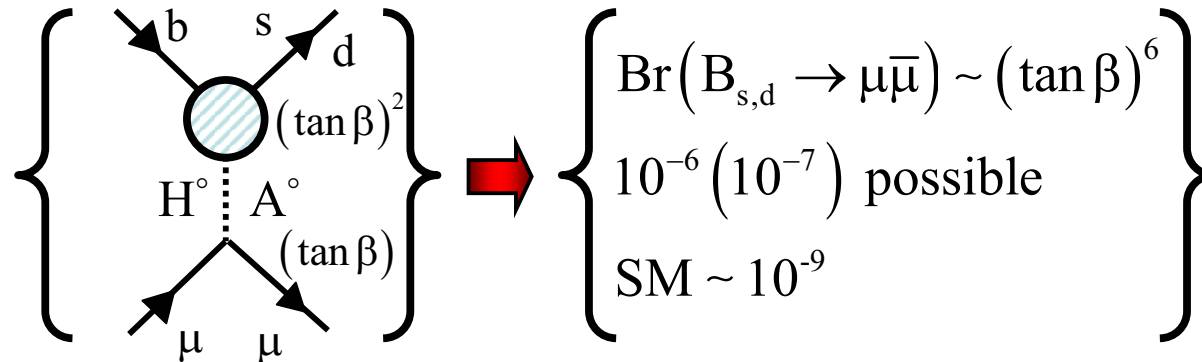


# MSSM at large $\tan\beta$

$$(B_{s,d} \rightarrow \mu^+ \mu^-)$$

In MSSM at large  $\tan\beta$   
 (CKM still the only source of Flavour and CP Violation)

Strong Enhancement



$$\left. \begin{array}{l} \text{Br}(B_{s,d} \rightarrow \mu\bar{\mu}) \sim (\tan\beta)^6 \\ 10^{-6} (10^{-7}) \text{ possible} \\ \text{SM} \sim 10^{-9} \end{array} \right\}$$

Babu, Kolda  
 Chankowski, Slawianowska  
 Bobeth, Ewerth, Krüger, Urban  
 Huang, Liao, Yan, Zhu  
 Isidori, Retico  
 Dedes, Dreiner, Nierste  
 Dedes, Pilaftis  
 Chankowski, Rosiek  
 Foster, Okumura, Roszkowski

$$\text{Br}(B_s \rightarrow \mu\bar{\mu}) < \begin{array}{l} 5.0 \cdot 10^{-7} (D\Phi) \\ 7.5 \cdot 10^{-7} (CDF) \end{array} \quad 95\% \text{ C.L.}$$

$$\text{Br}(B_d \rightarrow \mu\bar{\mu}) < \begin{array}{l} 1.9 \cdot 10^{-7} (CDF) \\ 8.3 \cdot 10^{-8} (\text{BaBar}) \end{array} \quad \begin{array}{l} 95\% \text{ C.L.} \\ 90\% \text{ C.L.} \end{array}$$

## Step 4

Calculate  $\hat{B}_K, \hat{B}_{B_d}, \hat{B}_{B_s}$   
using Lattice or other methods

(no chiral logs present)

## Steps 2+ 4

$$\hat{B}_K F_K$$

$$\hat{B}_{B_d} F_{B_d}$$

$$\hat{B}_{B_s} F_{B_d}$$



$$\varepsilon_K$$

$$\Delta M_d$$

$$\Delta M_s$$

## SM Box Diagrams

UT to be compared with RUT

+

Possible signals of NP in  
 $\Delta S = 2, \Delta B = 2$  Transitions

## Possible Effects in $\Delta M_s$

1. MFV : Enhancement by at most 1.4
2. General MSSM and Models with new FCNC sources 2-3
3. MSSM (CKM) with large  $\tan\beta$  0.8  
Suppression of  $\Delta M_s$  correlated with enhancement of  $\text{Br}(B_{s,d} \rightarrow \mu^+ \mu^-)$   
(AJB, Chankowski, Rosiek, Slawianowska)
4. In a General MSSM this correlation can be avoided.  
(Dedes, Pilaftis; Chankowski, Rosiek; Isidori, Reticco)

## Step 5

CP Asymmetries  
in  $B \rightarrow X_{s,d} \gamma$

Calculate  $\text{Br}(B \rightarrow X_{s,d} \gamma)$ ,  $\text{Br}(B \rightarrow X_{s,d} l^+ l^-)$   
 $\text{Br}(K_L \rightarrow \pi^0 \mu^+ \mu^-)$ ,  $\text{Br}(K_L \rightarrow \pi^0 e^+ e^-)$   
 and related CP Asymmetries,  
 FB asymmetries, etc.  $K_L \rightarrow \mu^+ \mu^-$

$\text{Br}(B \rightarrow (K^*, \rho) \gamma)$ ,  $\text{Br}(B \rightarrow (K^*, \rho) l^+ l^-)$  (Larger TH uncertainties)

$$\text{Br}(B \rightarrow X_s \gamma) = \begin{cases} (3.52 \pm 0.30) \cdot 10^{-4} & \text{EXP} \\ (3.70 \pm 0.30) \cdot 10^{-4} & \text{SM} \end{cases}$$

CLEO, BaBar, Belle

GMH, BCMU, GH

$$\text{Br}(B \rightarrow X_s l^+ l^-) = \begin{cases} (4.5 \pm 1.0) \cdot 10^{-6} & \text{EXP} \\ (4.4 \pm 0.7) \cdot 10^{-6} & \text{SM} \end{cases}$$

Belle, BaBar

BGGH, GHIY, AAGW (NNLO)

 (low  $\hat{s}$  region)

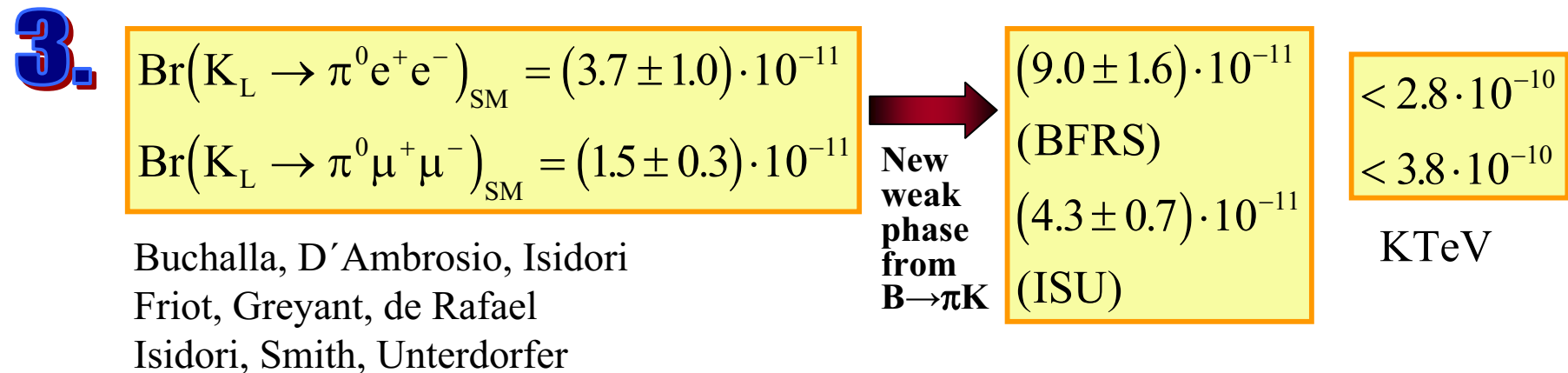
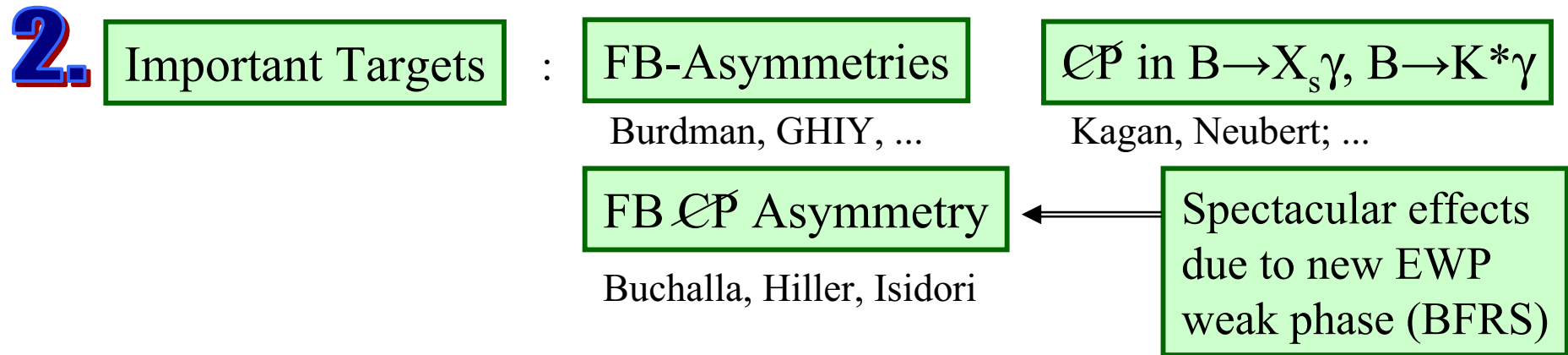
ALGH

SUSY (NNLO) : BBE



# Comments on Step 5

**1.** Very valuable Constraints on SUSY and other models  
 (Ali, Hiller, Greub, Lunghi, Handoko, Morozumi; Krüger)  
 {Gambino, Haisch, Misiak (2004)} →  $C_7^{NP} = -C_7^{SM}$  essentially excluded!



## Step 6

Calculate Hadronic Matrix Elements relevant for

$$B \rightarrow \pi\pi, \boxed{B \rightarrow \pi K}, B \rightarrow K\bar{K}, \boxed{B \rightarrow \phi K_s}, \dots$$

QCDF  
PQCD  
SCET  
LCSR

Many, many  
papers ...

Study simultaneously many channels using  
Flavour Symmetries

GHLR  
Fleischer  
(U-Spin)  
Rome  
BFRS  
⋮

CKM input

Input from  
RUT

Predictions for Branching Ratios  
and CP asymmetries possible

WICK CONTRACTIONS  
Rome; AJB, Silvestrini

New Complex  
Phase from  
EWP in  $B \rightarrow \pi K$ ?

AJB, Fleischer (2000)  
AJB, Fleischer, Recksiegel, Schwab (2003)  
Yoshikawa (2003), Gronau + Rosner (2003), Beneke, Neubert (2003)  
Barger, Chiang, Langacker, Lee (2004)  
He, McKellar (2004) Wu, Zhou  
London, Matias et al. (2004)  
Baek, Hamel, London, Datta, Suprun (2004)  
Ciuchini et al.  
Grossman, Neubert  
Kagan (1999)

(BFRS)

# The B → πK Puzzle

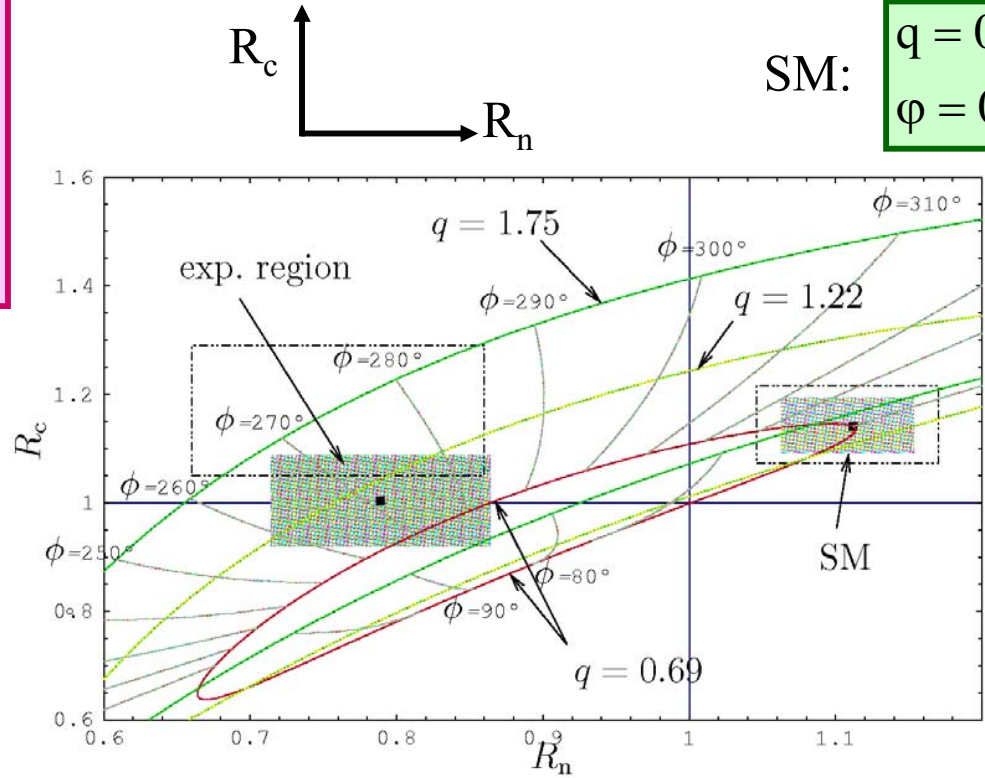
$EWP = qe^{i\phi}$

$$R_c = 2 \left[ \frac{\text{Br}(B^\pm \rightarrow \pi^0 K^\pm)}{\text{Br}(B^\pm \rightarrow \pi^\pm K^0)} \right]$$

$$R_n = \frac{1}{2} \left[ \frac{\text{Br}(B^0 \rightarrow \pi^- K^+)}{\text{Br}(B^0 \rightarrow \pi^0 K^0)} \right]$$

SM:  $q = 0.69$  Neubert  
 $\phi = 0$  Rosner

$(R_c)_{SM} = 1.14 \pm 0.05$   
 $(R_n)_{SM} = 1.11 \pm 0.05$



Best Values  
(including rare  
decay constraint)

$q \approx 0.92$   
 $\phi \approx -85^\circ$

BFRS

$R_c = 1.17 \pm 0.12$   
 $R_n = 0.76 \pm 0.10$

Rare Decays



$R_c = 1.00^{+0.12}_{-0.08}$   
 $R_n = 0.82^{+0.12}_{-0.11}$

BFRS Expectation  
of June 04

$R_c = 1.00 \pm 0.08$   
 $R_n = 0.79 \pm 0.08$

Exp: after ICHEP 04

CLEO  
BaBar  
Belle

# Confront the Avalanche of magic Numbers from BaBar and Belle

$$\text{Br}(B^\pm \rightarrow \pi^\pm \pi^0)$$

$$\text{Br}(B_d \rightarrow \pi^+ \pi^-)$$

$$\text{Br}(B_d \rightarrow \pi^0 \pi^0)$$

$$A_{\text{CP}}^{\text{dir}}(B_d \rightarrow \pi^+ \pi^-)$$

$$A_{\text{CP}}^{\text{mix}}(B_d \rightarrow \pi^+ \pi^-)$$

$$A_{\text{CP}}^{\text{dir}}(B_d \rightarrow \pi^0 \pi^0)$$

$$A_{\text{CP}}^{\text{mix}}(B_d \rightarrow \pi^0 \pi^0)$$

$$\text{Br}(B^\pm \rightarrow \pi^\pm K)$$

$$\text{Br}(B_d \rightarrow \pi^\mp K^\pm)$$

$$\text{Br}(B^\pm \rightarrow \pi^0 K^\pm)$$

$$\text{Br}(B_d \rightarrow \pi^0 K^0)$$

$$A_{\text{CP}}^{\text{dir}}(B^\pm \rightarrow \pi^\pm K)$$

$$A_{\text{CP}}^{\text{dir}}(B_d \rightarrow \pi^\mp K^\pm)$$

$$A_{\text{CP}}^{\text{dir}}(B^\pm \rightarrow \pi^0 K^\pm)$$

$$A_{\text{CP}}^{\text{dir}}(B^0 \rightarrow \pi^0 K_s)$$

$$A_{\text{CP}}^{\text{mix}}(B^0 \rightarrow \pi^0 K_s)$$

Simultaneous study of all these channels  
and also of  $B_s$  decays (DØ, CDF, LHC)  
should offer valuable insight into

QCD

and Flavour Dynamics

$\mathcal{CP}$

## Step 7

Calculate Hadronic Matrix Elements relevant for

$$K_L \rightarrow \pi\pi$$

$$\rightarrow \epsilon'/\epsilon$$

NA48  
KTeV

$$(\epsilon'/\epsilon)_{\text{exp}} = (1.6 \pm 0.16) \cdot 10^{-3} \quad \text{and}$$

Large sensitivity  
to NP  
in the Electroweak  
Penguin Sector

Estimates from:

Munich, Rome, Trieste, Dortmund  
Lund, Valencia, ...

within the SM, consistent with EXP.

**But:**

Large hadronic uncertainties in  
 $\langle Q_6 \rangle$ ,  $\langle Q_8 \rangle$  preclude reasonable  
tests of New Physics at present.

**Step 8**

Look at Charm Decays

## Final Messages

It is essential to study simultaneously as many processes as possible in a given NP scenario.

- ★ Identify correlations between various quantities that generally depend on fewer parameters. (MFV "Sum Rules")
- ★ In the presence of many quantities already the pattern of enhancements and suppressions relative to the SM can rule out a given NP or give hints for it. (Example)
- ★ In particular identify quantities that vanish or are tiny in the SM but have a definite sign in a given NP.

# Comparison of different Models

|                                                 | universal<br>extra<br>dimensions | MSSM (CKM)<br>large $\tan\beta$ | MSSM (CKM)<br>low $\tan\beta$ | Littlest Higgs |
|-------------------------------------------------|----------------------------------|---------------------------------|-------------------------------|----------------|
| $\text{Br}(K^+ \rightarrow \pi^+ \nu\bar{\nu})$ | ↑                                | No effect                       | ↓                             | ↓              |
| $\text{Br}(K_L \rightarrow \pi^0 \nu\bar{\nu})$ | ↑                                | No effect                       | ↓                             | ↓              |
| $\text{Br}(B_s \rightarrow \mu\bar{\mu})$       | ↑                                | ↑↑↑                             | ↑↓                            | ↑              |
| $\Delta M_s$                                    | ↑                                | ↓                               | ↑                             | ↑              |

Collaboration with:

Spranger  
Weiler (02)

see previous  
refs.

Gambino  
Gorbahn  
Jäger  
Silvestrini (01)

Poschenrieder  
Uhlig (04)



**10.**

# Short Outlook

# Targets for 2005-2012

$|V_{us}|, |V_{cb}|, |V_{ub}|, \gamma$   
from tree level decays

Improved  $F_{B_d}, F_{B_s}$   
 $\hat{B}_d, \hat{B}_s, B_i^{NP}, \xi, \dots$

$K^+ \rightarrow \pi^+ \nu \bar{\nu}$   
 $K_L \rightarrow \pi^0 \nu \bar{\nu}$   
 $B \rightarrow X_{d,s} \nu \bar{\nu}$

$B_{d,s} \rightarrow l^+ l^-$   
 $\Delta M_s$

Improved  
 $\text{Br}(B \rightarrow X_s \gamma)$   
 $\text{Br}(B \rightarrow X_s l^+ l^-)$   
+ Exclusive Modes

FB - Asymmetries ( $B \rightarrow X_s l^+ l^-, K^* l^+ l^-$ )  
 $\mathcal{CP}$  in  $B \rightarrow X_s \gamma, K^* \gamma$   
 $\mathcal{CP}$  - FB Asymmetries ( $B \rightarrow X_s l^+ l^-, K^* l^+ l^-$ )

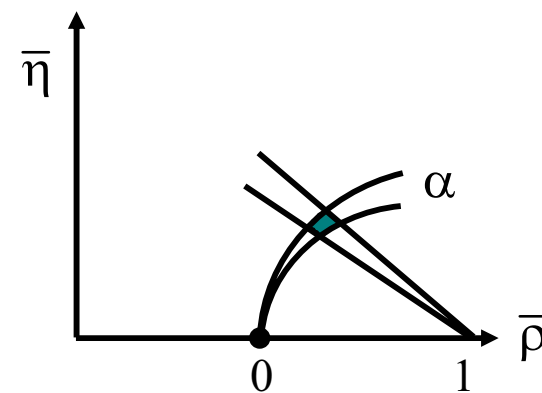
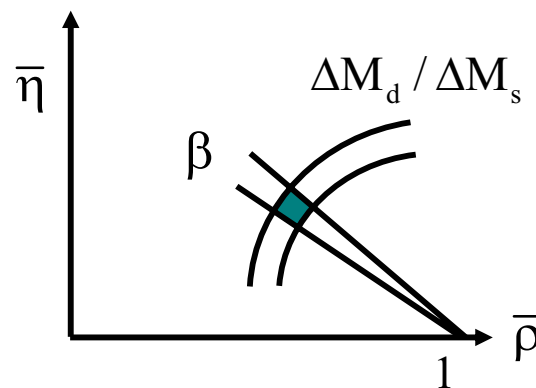
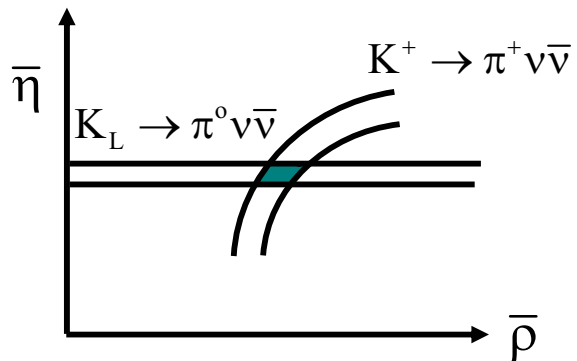
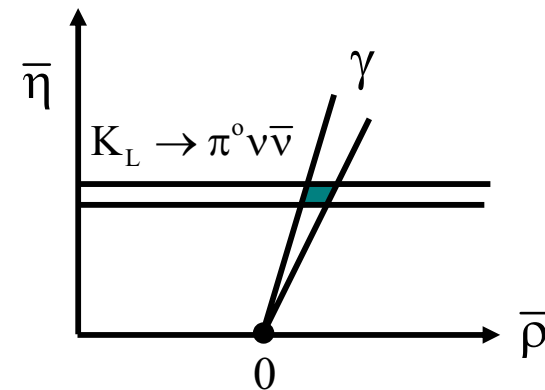
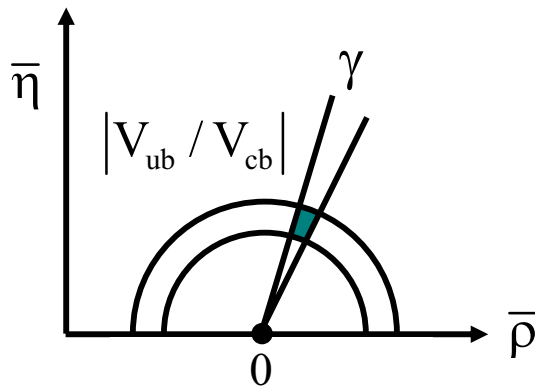
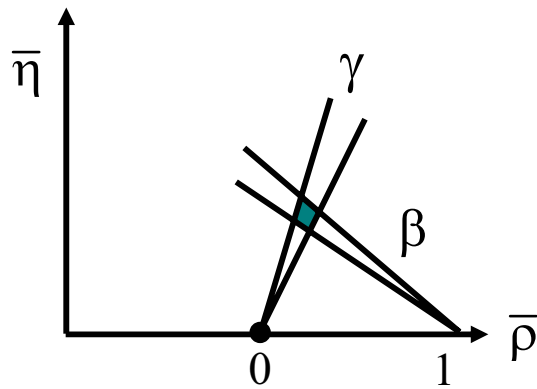
$K_L \rightarrow \pi^0 e^+ e^-$   
 $K_L \rightarrow \pi^0 \mu^+ \mu^-$   
 $K_L \rightarrow \mu^+ \mu^-$  (TH)  
 $\varepsilon' / \varepsilon$  (TH)

Resolution of  
 $B \rightarrow \pi K$  Puzzle  
 $B \rightarrow \pi \pi$  Puzzle  
 $B \rightarrow \phi K_s$  Puzzle  
 $B \rightarrow \eta' K$  Puzzle

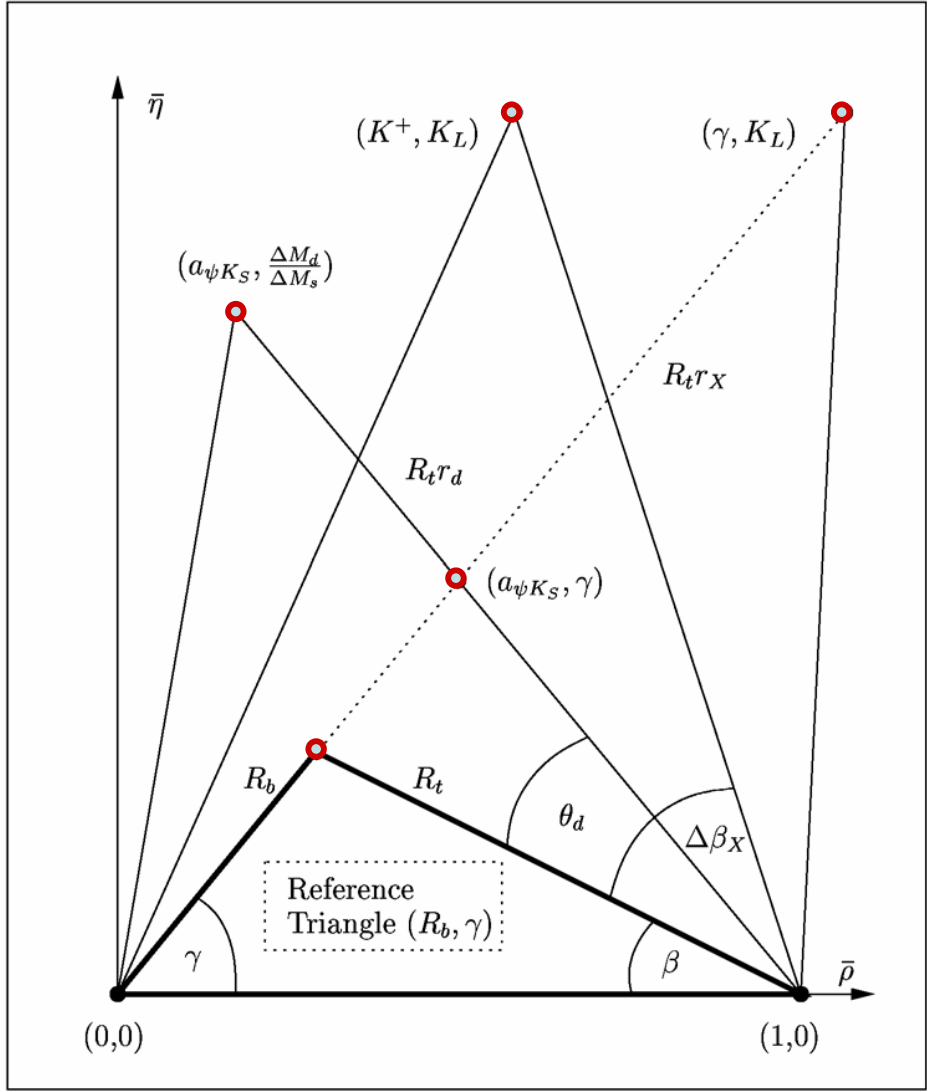
$A_{CP}^{\text{dir}}, A_{CP}^{\text{mix}}$   
in  
2 - Body  
 $B_{d,s}, B^\pm$   
decays

Correlations with  
Electric Dipole Moments  
 $\mu \rightarrow e \gamma$   
 $(g-2)_\mu$   
Lepton Flavour Violation

# Searching for New Physics



# The 2012 Vision of the Unitarity Triangle



AJB  
Schwab  
Uhlig

**The Future  
until 2012  
should be  
very exciting**